

# 4.

# PROBABILITY [IT2110]

*By Department of Mathematics and Statistics  
Faculty of Humanities and Sciences, SLIIT*

# TERMINOLOGY

- ***Experiment:*** A process leading to a well-defined observations or outcomes that generates a set of data.
- ***Trial:*** Each repetition, if the experiment can be repeated any number of times under identical conditions.
- ***Sample Space ( $S$ ):*** The set containing all possible outcomes of an experiment
- ***Finite sample space:*** Sample space that contains a finite number of outcomes
- ***Continuous Sample space:*** Sample space that contains an interval of values

# EVENTS

- **Event:** A subset of the sample space. Usually denoted in upper case letters.
- **Simple Event:** An event that corresponds to a single possible outcome.
- The null subset ( $\emptyset$ ) of  $S$  is called an impossible event.
- The event  $A \cup B$  consists of all outcomes that are in  $A$  or in  $B$  or in both.
- The event  $A \cap B$  consists of outcomes that are both in  $A$  and  $B$ .
- The event  $A^c$  (the complement of  $A$  in  $S$ ) consists of all outcomes not in  $A$ , but in  $S$ .

- ***Mutually Exclusive Events:*** Two events A and B are said to be mutually exclusive or disjoint if  $A \cap B = \emptyset$ . They cannot happen together.
- ***Collectively exhaustive events:*** One of the events must occur. The set of events covers the entire sample space.
- ***Independent Events:*** If the occurrence of one event not affect on the occurrence of other event, then both events are said to be independent with each other.
- ***Joint Events (Compound Events):*** An event that corresponds to more than a single possible outcome is known as compound event.

***Eg:-*** Getting an odd number by rolling  
a die

# Example

1. A balanced/fair die (with all outcomes equally likely) is rolled. Let  $A$  be the event that an even number occurs.

*Experiment* : Rolling a balanced die.

*Sample Space* :  $S = \{1, 2, 3, 4, 5, 6\}$

*Event (A)* :  $A = \{2, 4, 6\}$

*Type of the event*: Compound event

2. Consider a deck of cards. Let  $A$  – Aces,  $B$  – Black cards,  $C$  – Diamonds and  $D$  – Hearts. Find collectively exhaustive events and mutually exclusive events.

# PROBABILITY

- **Probability:** Measure of the chance that an uncertain event will occur.
- The notation for the statement “Probability of the event  $A$ ” is denoted as  $Pr(A)$  or  $P(A)$ .
- The value for the probability is between 0 and 1.
- A probability of 1 means that we are 100% sure of the occurrence of an event.
- A probability of 0 means that we are 100% sure of the non-occurrence of an event.
- The probability of  $S$  is always 1 ( $Pr(S) = 1$ ).
- The probability of an impossible event is always 0 ( $Pr(\emptyset) = 0$ ).

## Classical Definition of Probability

If there are  $N$  equally likely outcomes, of which one must occur, and  $n$  of these are regarded as favourable to an event, then the probability of the event is given by  $\frac{n}{N}$ .

## Frequency (Empirical) Definition of Probability

The probability of an event is the proportion of times the event would occur in a long run of repeated experiments.

Probability of the Event =  $\frac{\text{Number of favourable outcomes observed}}{\text{Total number of outcomes observed}}$

## Subjective Probability

An individual judgement or opinion about the probability of occurrence.

# Examples

1. A balanced/fair die (with all outcomes equally likely) is rolled. Let  $A$  be the event that an even number occurs. What is the probability of  $A$ ?

$$\Pr(A) = \frac{\text{Number of outcomes in } A}{\text{Total number of outcomes in } S} = \frac{3}{6} = 0.5$$

# Examples

2. Suppose we toss two coins. Assume that all the outcomes are equally likely (fair coins). Let  $A$  be the event that at least one of the coins shows up heads. Find  $P(A)$ .

$$\Pr(A) = \frac{\text{Number of outcomes in } A}{\text{Total number of outcomes in } S} = \frac{3}{4} = 0.75$$

# Basic Properties

Consider two events  $A$  and  $B$  in  $S$ . Then,

- $Pr(A^c) = 1 - Pr(A)$
- $Pr(A \cup B) = Pr(A) + Pr(B) - Pr(A \cap B)$ .
- If  $A \cap B = \emptyset$  ( $A$  and  $B$  are mutually exclusive) then,  
 $Pr(A \cup B) = Pr(A) + Pr(B)$
- If  $A_1, A_2, \dots, A_k$  are mutually exclusive events then  
 $Pr(A_1 \cup A_2 \cup \dots \cup A_k) = Pr(A_1) + Pr(A_2) + \dots + Pr(A_k)$
- If  $A$  and  $B$  are independent then,  
 $Pr(A \cap B) = Pr(A) * Pr(B)$

# Example

1. In a large university, the freshman profile for one year's fall admission says that 40% of the students were in the top 10% of their high school class, and that 65% are white, 25% of the students were white as well as were in the top 10% of their high school class. What is the probability that a freshman student selected randomly from this class either was in the top 10% of his or her high school class or is white?

# Joint Probability

The probability of events  $A$  and  $B$  occurring together is defined as Joint probability of  $A$  and  $B$ .

The probability of a joint event,  $A$  and  $B$  [ $Pr(A \cap B)$ ]:

$$Pr(A \text{ and } B) = \frac{\text{Number of outcomes satisfying } A \text{ and } B}{\text{Total number of outcomes in } S}$$

# Examples

1. Find the probability that you will get a Black-Ace from a playing deck of cards, if a card is drawn at random.
2. Find the probability that you will get a Red-Jack from a playing deck of cards, if a card is drawn at random.

# Marginal Probability

The probability of a single event occurring ( $Pr(A)$ ), without the interference of another event (not conditioned on another event) is known as marginal probability.

This can be thought of as an unconditional probability

# Examples

1. Find the probability that you will get a King from a playing deck of cards, if a card is drawn at random.
2. Find the probability that you will get a Black card from a playing deck of cards, if a card is drawn at random.

**Note:** *Contingency Tables and Tree Diagrams* can be used to visualize events and make calculations easier.

# Example

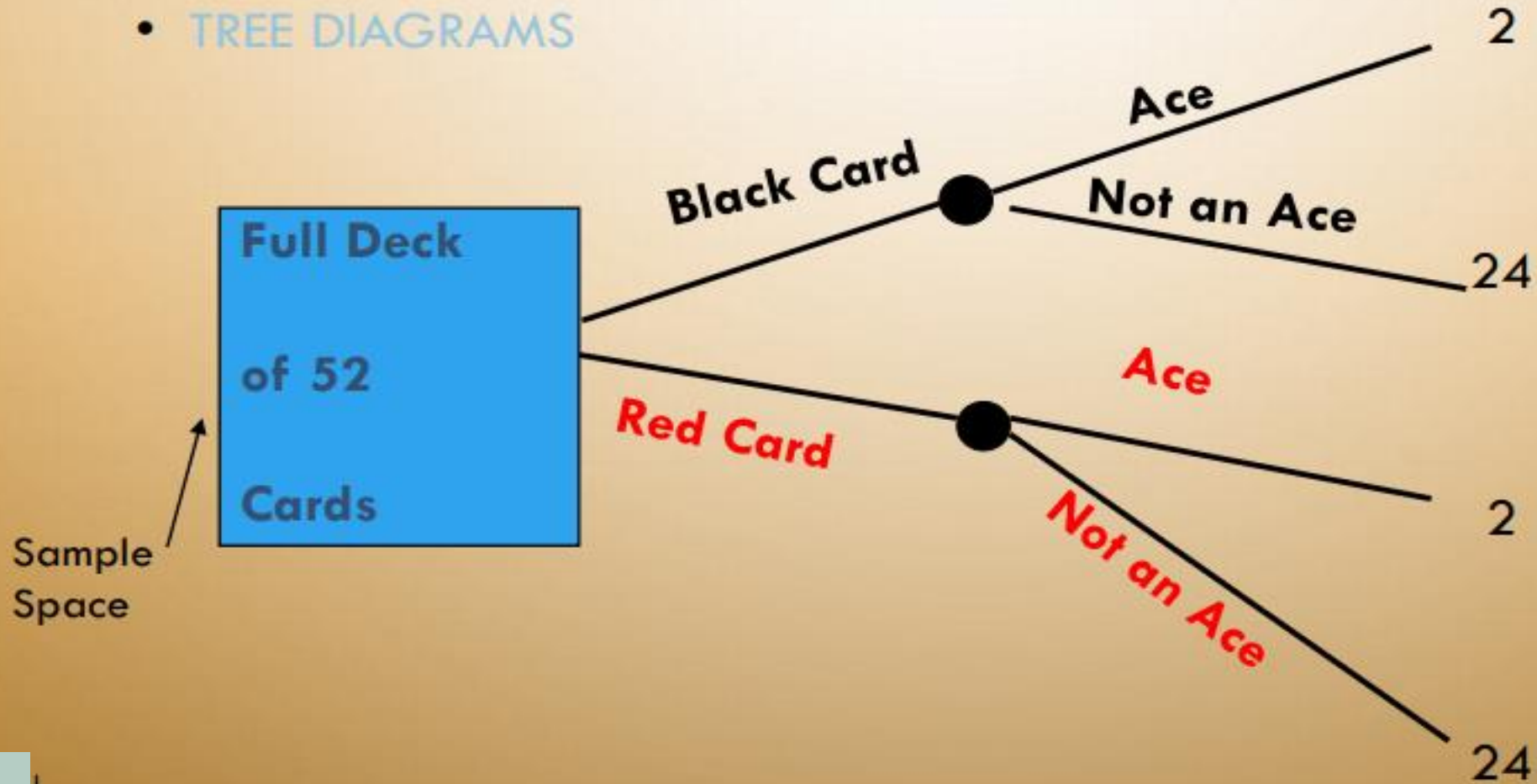
Type	Color		Total
	Red	Black	
Ace	2	2 [2/52]	4
Non-Ace	24 [24/52]	24	48 [48/52]
Total	26	26 [26/52]	52

*Joint Probabilities*

*Marginal Probabilities*

# Example

- TREE DIAGRAMS



# Conditional Probability

- This is the probability of one event, given that another event has already occurred.
- The conditional probability of an event  $A$ , given that an event  $B$  has already occurred is denoted by  $Pr(A | B)$ .

$$Pr(A | B) = \frac{Pr(A \cap B)}{Pr(B)} \quad ; Pr(B) > 0$$

$$Pr(B | A) = \frac{Pr(A \cap B)}{Pr(A)} \quad ; Pr(A) > 0$$

;Where  $P(A \cap B)$  = Joint probability of  $A$  and  $B$

$P(A)$  = Marginal probability of  $A$

$P(B)$  = Marginal probability of  $B$

# Example

1. Of the cars on a used car lot, 70% have air conditioning and 40% have a CD player. 20% of the cars have both. What is the probability that a car has a CD player, given that it has AC ?
2. If two balanced dice are tossed, find the probability that the sum of the face values is 8, if the face value of the first one is 3.

# Properties of Conditional Probability

➤  $Pr(A | B) = 1 - Pr(A^c | B)$

$$Pr(B \cup C | A) = Pr(B | A) + Pr(C | A) - Pr(B \cap C | A)$$

➤ Multiplication law:

$$Pr(A \cap B) = Pr(B) * Pr(A | B) = Pr(A) * Pr(B | A)$$

➤ If  $A$  and  $B$  are independent then,

$$Pr(A | B) = Pr(A) \quad \text{or} \quad Pr(B | A) = Pr(B)$$

$$Pr(A \cap B) = Pr(A) * Pr(B)$$

➤ For independent events  $A_1, A_2, \dots, A_k$ ,

$$Pr(A_1 \cap A_2 \cap \dots \cap A_k) = Pr(A_1) * Pr(A_2) * \dots * Pr(A_k)$$

# THANK YOU!

Any questions?