Approximating uncertainty around indices from stratified-random trawl surveys using the Gamma distribution

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2022-06-20

# Introduction

A primary objective of fisheries-independent trawl surveys is to obtain indices of stock size and quantify the uncertainty around these indices. Such information plays a critical role in the assessment and management of fish stocks around the world as they often serve as a leading indicator of change ([Kimura and Somerton, 2006](#ref-kimura2006); [Pennington and Strømme, 1998](#ref-pennington1998)). Surveys indices also influence risk-based decision making; however, such information is typically provided indirectly via estimates of uncertainty from stock assessment models that are calibrated using point estimates of trawlable abundance or biomass. Data limitations often preclude the use of complex assessment models and, as such, many stocks are assessed using survey indices. While model-based indices produced using geostatistical approaches (e.g., [Anderson et al., 2022](#ref-anderson2022); [Thorson et al., 2015](#ref-thorson2015)) are growing in popularity, design-based estimators continue to be widely used. In the Northwest Atlantic, surveys typically follow a stratified-random sampling design with proportional allocation (e.g., [González-Troncoso et al., 2022](#ref-gonzalez2022); [Rideout et al., 2022](#ref-rideout2022)) and indices are obtained using stratified analyses (e.g., [S. Smith and Somerton, 1981](#ref-smith1981)). Unfortunately, the quantification of uncertainty around these estimates remains a challenge. Quantiles from a Student’s t distribution are often used to approximate the uncertainty around stratified estimates; however, the lower limits of this approximation can result in negative values, which is unrealistic ([Cadigan, 2011](#ref-cadigan2011)). We propose an alternate approximation of uncertainty using the Gamma distribution which accounts for the positive and skewed nature of survey indices.

# Methods

Provided data from a stratified-random survey, average trawlable abundance or biomass () and sampling variance () can be estimated using standard design-based formula ([Cochran, 1977](#ref-cochran1977); [S. J. Smith, 1990](#ref-smith1990); [S. Smith and Somerton, 1981](#ref-smith1981)). Instead of using a Student’s t distribution to describe uncertainty and allow negative values, we apply the Gamma distribution by translating and to scale () and shape () parameters as follows:

Provided these values, density, quantile, and random functions for the Gamma distribution can be used to calculate probabilities. For instance, the probability that the index increased from one year to the next can be quantified. For some cases there might also be a need to calculate the probability that the current index is above or below an average level from a reference period, . If the reference period is based on the index, then the level cannot be perfectly known. To account for uncertainty around this reference, , it is necessary to combine the variances across the indices. This is accomplished by averaging the means and summing equally weighted variances across a reference set of years , where is the size of the set,

This assumes that the estimates being averaged are independent. As above, and can be converted to and parameters to approximate uncertainty.

## Simulation

We simulated a redfish-like population using the R package SimSurvey ([Regular et al., 2020](#ref-regular2020)). The simulated population was based on the exponential decay cohort model where parameter settings for mortality, recruitment, and growth were based on assessments of redfish on the Grand Bank. The simulated population were distributed through an area according to the age-year-space covariance with a parabolic relationship with depth. This survey area was 300 x 300 km with 10 km2 cell size and had 30 depth-based strata. We simulated stratified random sampling with a 2 m wide trawl hauled for a distance of 1.5 km. The population and survey were simulated over 20 years. The number of sets in a stratum was proportional to its area (approximately 1 set per 1000 km2) and the minimum set per stratum was 2. The survey simulation was replicated five times over the same population.

Average trawlable abundance () and sampling variance () was calculated by year and replicate (20 years across 5 surveys) using standard design-based estimators ([S. J. Smith, 1990](#ref-smith1990); [S. Smith and Somerton, 1981](#ref-smith1981)) and these estimates were translated to scale () and shape () parameters for the Gamma distribution as described above. To compare densities obtained from the Gamma distribution with densities based on an empirical approach, we applied a non-parametric bootstrap to resample the observations (sets) independently within each stratum with replacement. The resampling and calculation of the mean bootstrap estimator were repeated 1000 times with the R package boot ([Canty and Ripley, 2021](#ref-canty2021)). Densities from these boostrap samples were computed for each year and survey replicate for comparison to the Gamma approximation.

This simulation can be replicated using code in [Appendix A](#app:appendix-a).

## Application

During the 2022 assessment of Redfish in NAFO division 3O, candidate biomass reference points were examined using indices derived from the Canadian spring and fall surveys of 3O. Given relative stability in catches through the history of the fishery, and trends in survey indices, the survey time series is considered to represent normal conditions for this stock (i.e. no apparent prolonged period of collapse). The average of the survey time series was therefore considered a reasonable proxy for B\_MSY\_ [REF?] and, following the NAFO precautionary approach framework ([NAFO, 2004](#ref-nafo2004)), 30% of B\_MSY\_ would be considered the limit reference point (LRP).

To combine indices from the spring and fall surveys, and account for uncertainty associated with estimates from both surveys, annual stratified means and variances from each survey were integrated using the properties of the variance and translated to shape and scale parameters for use in the gamma distribution following the abovementioned equations. In years when a survey index is missing, the available survey is used in place of the mean and variance estimate. This same approach was applied to account for the uncertainty in the B\_MSY\_ proxy by applying the gamma distribution informed by averaged point estimates of mean and variance.

# Results and Discussion

The Gamma probability density distribution showed high variability among survey simulations, as did the bootstrap samples (Figure 1. Nevertheless, the shape of both the Gamma density and the bootstrap samples were similar across all years and survey replicates, indicating that the Gamma distribution provides a reasonable approximation of the uncertainty around the stratified estimates. The similarity holds when survey indices are aggregated (Figure 2). Though further quantitative analysis is required to assess the performance of these methods for calculating the confidence intervals, these results indicate that confidence intervals from the Gamma approach would be similar to those obtained using bootstrap samples. At the very least, confidence intervals from the Gamma distribution represents an improvement over the sometimes negative intervals derived from the t-distribution.

The B\_MSY\_ proxy and associated limit reference point (30% B\_MSY\_ proxy) proposed for redfish was accepted as an interim reference point as was the Gamma-based method for quantifying uncertainty. Since this proxy is based on survey indices, it is not considered perfectly known. Estimates were therefore aggregated to account for this uncertainty. Determining status relative to the LRP considering uncertainty in both the proxy-BMSY and the current Biomass level provides was considered the most fulsome formulation of uncertainty in stock status and the most precautionary approach to advice.

It is important to note, however, that survey indices can show unrealistic fluctuations year over year. This is apparent in both the simulations and across the refish indices. Large inter-annual fluctuations may be a consequence of sampling noise or distribution shifts. A single year may therefore be insufficient to indicate a true change in stock status.

# References

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# Figures

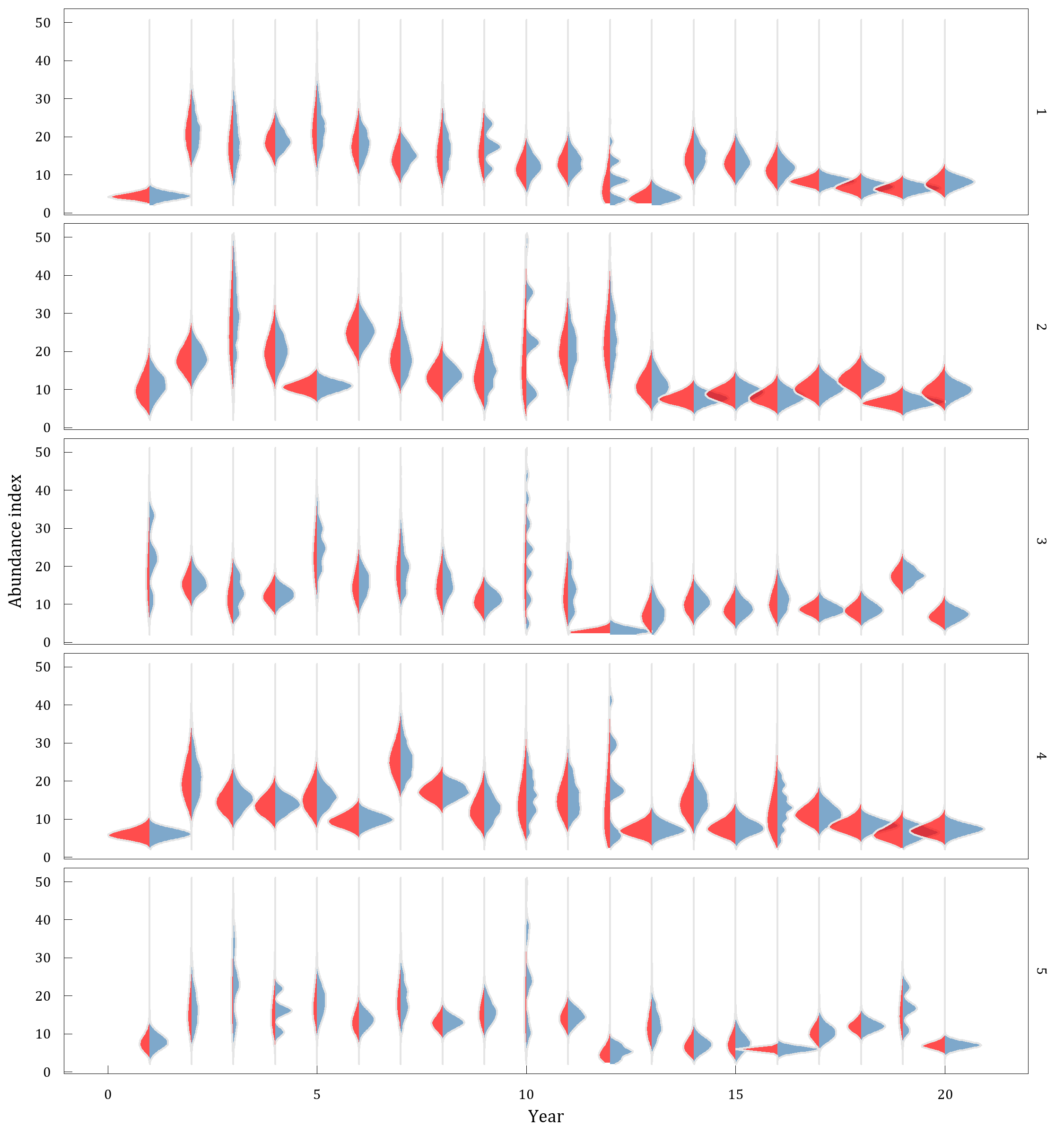


Fig 1: The bootstrap and gamma distributions estimated using simulated data from five independent surveys conducted over the same population across 20 years. The red area shows the density distribution from 1000 bootstrapped samples from each year and survey replicate. The blue area shows the gamma probability distribution from each year and survey replicate based on the mean and standard deviation of the design-based index.

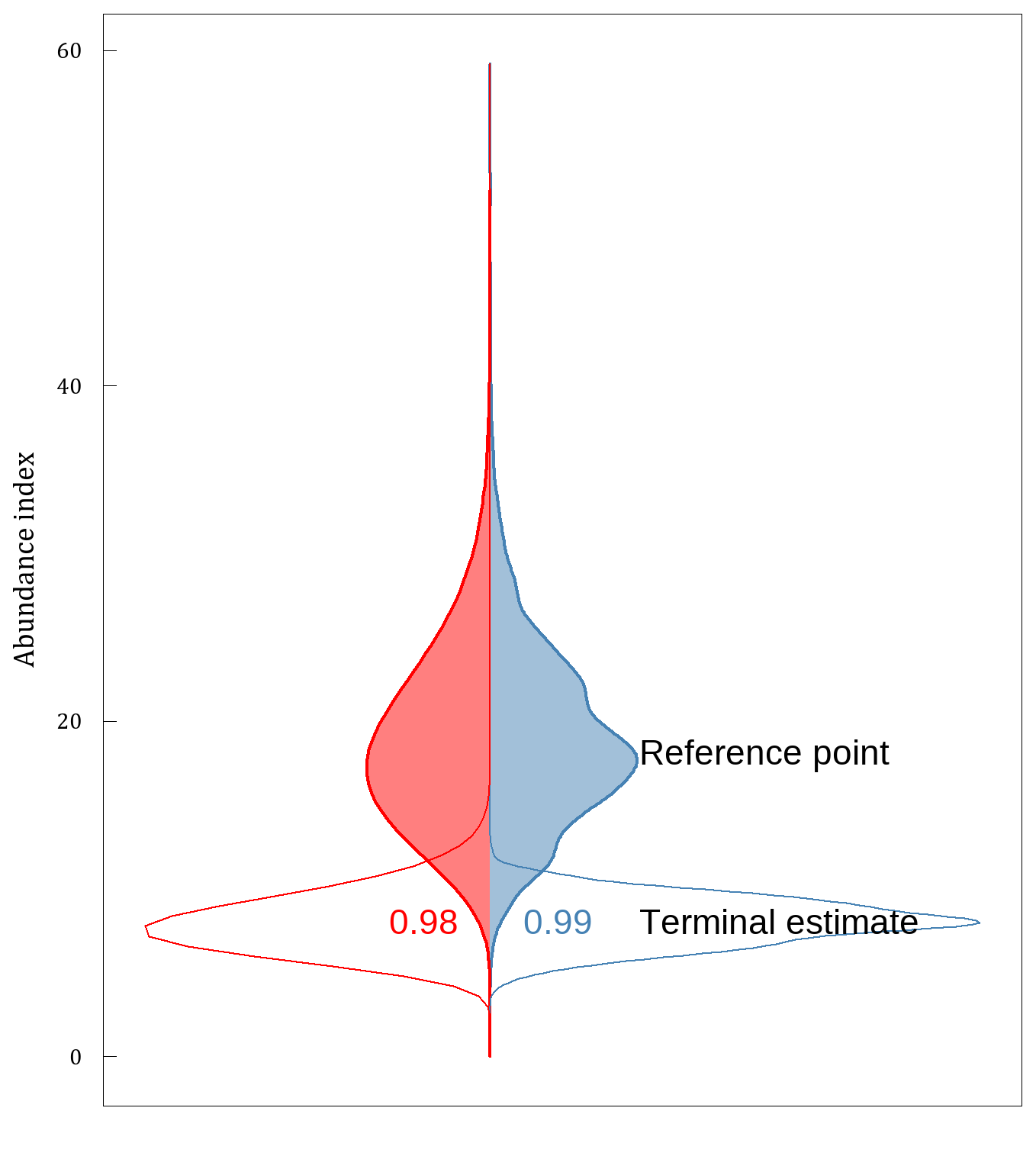


Fig 2: Bootstrap (blue) and gamma (red) distributions estimated from simulation 1 of a redfish-like population, where terminal estimates (year 20; open area) are compared to a reference period (aggregate estimates from years 2-9; shaded area). Densities for the reference period were obtained by combining the bootstrap samples and by aggregate parameters across the reference period (see Methods section). Probability that the terminal value is below the reference point is indicated.

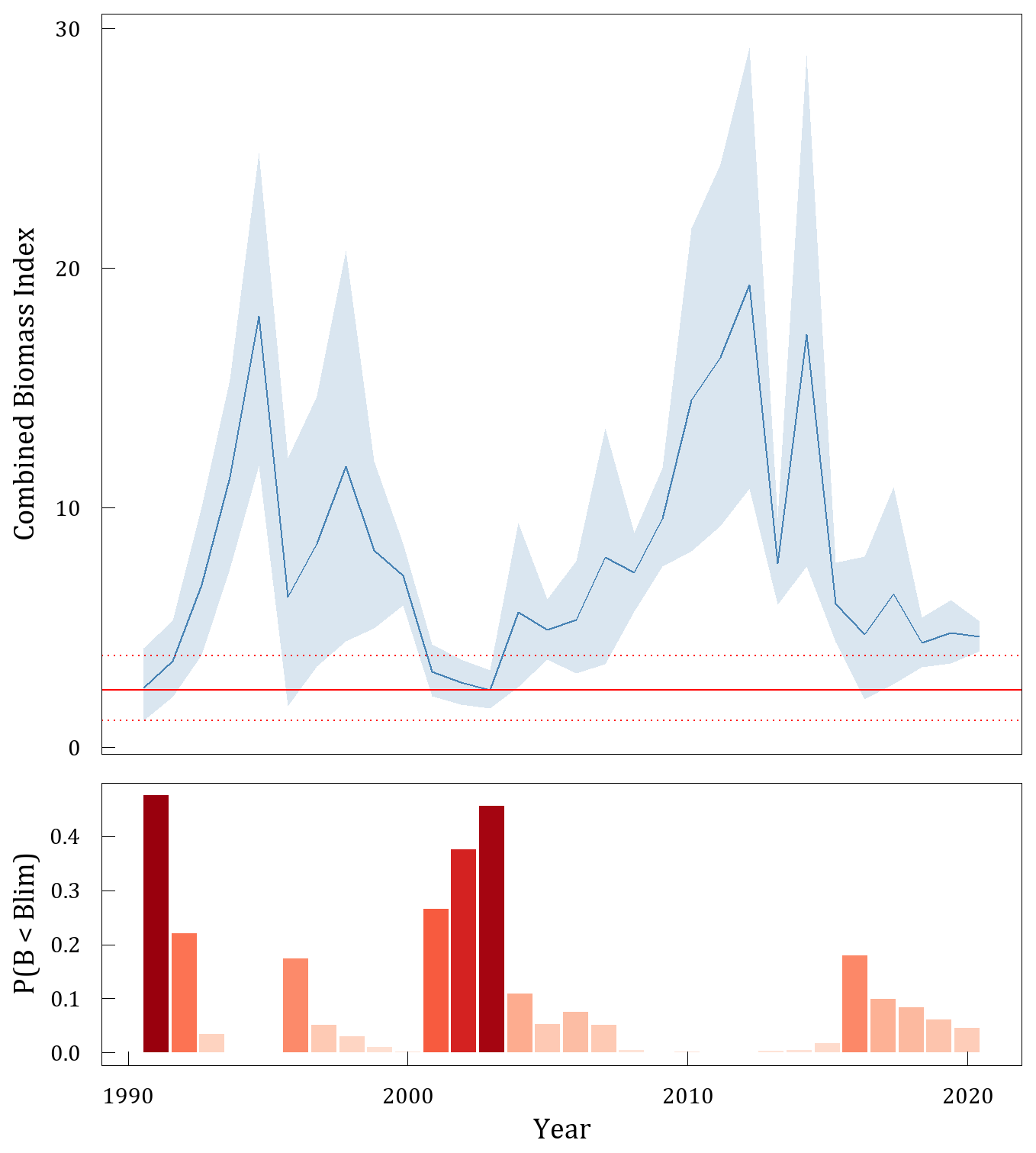


Fig 3: Combined CAN-Spring and CAN-Autumn biomass index (top; blue line) with 80% confidence intervals (blue shaded area) calculated using a Gamma distribution. Horizontal line (red) indicates interim Blim = 0.3 Bmsy-proxy with 80% confidence intervals (red dashed lines). Probability of By < Blim is presented below.

# Appendix A

Simulation results can be replicated using the below code.

library(SimSurvey)  
library(tidyr)  
library(future)  
library(tictoc)  
library(ggplot2)  
library(ggridges)  
library(dplyr)  
library(purrr)  
library(data.table)  
library(NAFOdown)  
  
plan(multisession, workers = floor(availableCores()/2))  
  
n\_sims <- 5  
n\_boot <- 1000  
  
  
## Simulation ----------  
  
set.seed(794)  
population <- sim\_abundance(ages = 1:50,  
 years = 1:20,  
 R = sim\_R(log\_mean = log(600000000),  
 log\_sd = 0.6,  
 random\_walk = F),  
 Z = sim\_Z(log\_mean = log(0.2),  
 log\_sd = 0.2,  
 phi\_age = 0.4,  
 phi\_year = 0.4),  
 N0 = sim\_N0(N0 = "exp", plot = FALSE),  
 growth = sim\_vonB(Linf = 30, L0 = 0,   
 K = 0.1, log\_sd = 0.13,  
 length\_group = 1,   
 digits = 0)) |>  
 sim\_distribution(grid = make\_grid(x\_range = c(-150, 150),  
 y\_range = c(-150, 150),  
 res = c(10, 10),  
 shelf\_depth = 60,  
 shelf\_width = 170,  
 depth\_range = c(0, 1600),  
 n\_div = 2,  
 strat\_breaks = seq(0, 1600,   
 by = 65),  
 strat\_splits = 4,  
 method = "bezier"),  
 ays\_covar = sim\_ays\_covar(sd = 2,  
 range = 200,  
 phi\_age = 0.5,  
 phi\_year = 0.9),  
 depth\_par = sim\_parabola(mu = log(190),  
 sigma = 0.3,  
 log\_space = TRUE))  
  
  
survey <- sim\_survey(population,  
 n\_sims = n\_sims,  
 q = sim\_logistic(k = 1, x0 = 6.5),  
 trawl\_dim = c(1.5, 0.02),  
 resample\_cells = FALSE,  
 binom\_error = TRUE,  
 min\_sets = 2,  
 set\_den = 1/1000,  
 lengths\_cap = 250,  
 ages\_cap = 20,  
 age\_sampling = "stratified",  
 age\_length\_group = 1,  
 age\_space\_group = "division") |>  
 run\_strat()  
  
  
## Density from the Gamma distribution ----------  
  
total\_strat <- survey$total\_strat |>  
 mutate(sigma = sampling\_units \* sd,  
 scale = sigma ^ 2 / total,  
 shape = total / scale)  
  
## Use gamma to generate density by sim and year  
rng <- c(0.001, max(total\_strat$total) \* 2)  
x <- seq(rng[1], rng[2], length.out = 100)  
total\_strat\_den <- lapply(seq.int(nrow(total\_strat)),   
 function(i) {  
 data.frame(sim = total\_strat$sim[i],  
 year = total\_strat$year[i],  
 total = x,  
 den = dgamma(x, shape = total\_strat$shape[i],  
 scale = total\_strat$scale[i]))  
}) |> dplyr::bind\_rows()  
  
  
### Density from bootstrapping ----------  
  
setdet <- survey$setdet  
  
split\_setdet <- split(setdet, paste0(setdet$year, "-", setdet$sim))  
  
sumYst <- function(data, i = seq\_len(nrow(data))) {  
 data[i, ] |>  
 ### stratum level  
 group\_by(year, strat, strat\_area) |>  
 summarise(meanYh = mean(n), tow\_area = mean(tow\_area),   
 .groups = "drop\_last") |>  
 mutate(Nh = strat\_area/(tow\_area)) |>  
 group\_by(year) |>  
 mutate(N = sum(Nh), Wh = Nh/N, WhmeanYh = Wh \* meanYh)|>  
 ### year level  
 summarise(sumYst= mean(N) \* sum(WhmeanYh),   
 .groups = "drop\_last") |>  
 pull(sumYst)  
}  
  
boot\_one\_year <- function(data, reps) {  
 b <- boot::boot(data, statistic = sumYst,   
 strata = data$strat, R = reps)  
 boot <- data.table(b$t) |> dplyr::rename(total = V1) |>  
 mutate(sim = mean(data$sim), year = mean(data$year))  
 return(boot)  
}  
  
tic()  
boot\_index <- furrr::future\_map\_dfr(split\_setdet, boot\_one\_year,   
 reps = n\_boot,  
 .options = furrr::furrr\_options(seed = TRUE))  
toc()  
  
quantile(boot\_index$total, prob = c(0.001, 0.999))  
  
den\_plot <- ggplot() +  
 geom\_density\_ridges(aes(x = total, y = as.numeric(year),   
 group = factor(year)),  
 color = "grey90", fill = "steelblue",   
 alpha = 0.7,  
 data = boot\_index, scale = 1) +  
 geom\_density\_ridges(aes(x = total, y = year, height = den,   
 group = factor(year)),  
 stat = "identity", color = "grey90", fill = "red",   
 alpha = 0.7,  
 data = total\_strat\_den, scale = -1) +  
 coord\_flip() + guides(fill = "none") +  
 scale\_x\_continuous(labels = scales::label\_number(suffix = "",  
 scale = 1e-8),  
 limits = c(194587641, 5116017391)) +  
 ylab("Year") + xlab("Abundance index") +  
 facet\_grid(rows = "sim") +  
 theme\_nafo()  
  
  
## Relative status ----------  
  
  
sub\_total\_strat <- total\_strat |>  
 filter(sim == 1)  
  
ref\_est <- total\_strat |>  
 filter(sim == 1, year %in% 2:9) |>  
 summarise(total = mean(total),  
 sigma = sqrt(mean(sigma ^ 2)),  
 scale = sigma ^ 2 / total,  
 shape = total / scale)  
  
ref\_boot <- boot\_index |>  
 filter(sim == 1, year %in% 2:9)  
  
x <- seq(min(ref\_boot), max(ref\_boot), length.out = 100)  
ref\_den <- data.frame(total = x, den = dgamma(x, shape = ref\_est$shape,   
 scale = ref\_est$scale))  
  
t\_est <- total\_strat |>  
 filter(sim == 1, year == 20)  
  
t\_den <- total\_strat\_den |>  
 filter(sim == 1, year == 20)  
  
t\_boot <- boot\_index |>  
 filter(sim == 1, year == 20)  
  
boot\_prob <- mean((t\_boot$total - ref\_boot$total) < 0)  
n\_samp <- 100000  
ref\_samp <- rgamma(n\_samp, shape = ref\_est$shape, scale = ref\_est$scale)  
t\_samp <- rgamma(n\_samp, shape = t\_est$shape, scale = t\_est$scale)  
gamma\_prob <- mean((t\_samp - ref\_samp) < 0)  
  
ggplot() +  
 geom\_density(aes(x = total), data = ref\_boot, fill = "steelblue",   
 color = "steelblue", alpha = 0.5) +  
 geom\_area(aes(x = total, y = -den), data = ref\_den, fill = "red",   
 color = "red", alpha = 0.5) +  
 geom\_density(aes(x = total), data = t\_boot, fill = NA,   
 color = "steelblue", size = .nafo\_lwd) +  
 geom\_area(aes(x = total, y = -den), data = t\_den, fill = NA,   
 color = "red", size = .nafo\_lwd) +  
 geom\_text(aes(x = t\_est$total, y = max(ref\_den$den) \* 1.2,   
 label = "Terminal estimate"), hjust = 0, vjust = 0.5) +  
 geom\_text(aes(x = ref\_est$total, y = max(ref\_den$den) \* 1.2,   
 label = "Reference point"), hjust = 0, vjust = 1) +  
 geom\_text(aes(x = t\_est$total, y = 0, label = round(boot\_prob, 2)),   
 hjust = -0.5, color = "steelblue") +  
 geom\_text(aes(x = t\_est$total, y = 0, label = round(gamma\_prob, 2)),   
 hjust = 1.5, color = "red") +  
 theme\_nafo() +  
 coord\_flip() +  
 scale\_x\_continuous(labels = scales::label\_number(suffix = "",   
 scale = 1e-8)) +  
 ylab("") + xlab("Abundance index") +  
 theme(axis.ticks.x = element\_blank(),  
 axis.text.x = element\_blank())

# Colophon

This version of the document was generated on 2022-06-20 14:35:12 using the R markdown template for SCR documents from [NAFOdown](https://github.com/nafc-assess/NAFOdown).

The computational environment that was used to generate this version is as follows:

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#> withr 2.4.3 2021-11-30 [1] CRAN (R 4.1.2)  
#> xfun 0.26 2021-09-14 [1] CRAN (R 4.1.0)  
#> xml2 1.3.2 2020-04-23 [1] CRAN (R 4.1.1)  
#> yaml 2.2.1 2020-02-01 [1] CRAN (R 4.1.0)  
#> zip 2.2.0 2021-05-31 [1] CRAN (R 4.1.1)  
#>   
#> [1] C:/Users/RegularP/Documents/R/win-library/4.1  
#> [2] C:/Program Files/R/R-4.1.2/library  
#>   
#> ------------------------------------------------------------------------------