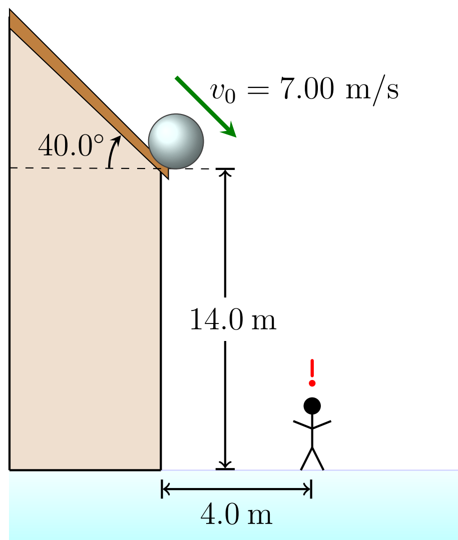
Compound Projectile Motion Problem #4

Motion in Two or Three Dimensions

A snowball rolls off a barn roof that slopes downward at an angle of 40.0°. The edge of the roof is 14.0 m above the ground, and the snowball has a speed of 7.00 m/s as it rolls off the roof. Ignore air resistance.



a.) How far from the edge of the barn does the snowball strike the ground if it doesn't strike anything else while falling?

b.) Draw x-t, y-t, v_x-t and v_y-t graphs for the motion in part (a).

c.) A man 1.9 m tall is standing 4.0 m from the edge of the barn. Will he be hit by the snowball?

Solution

a.) To determine the range of the snowball, the time of flight for the object is needed. This can be found by using the vertical version of the kinematic equation for displacement:

$$y_{\mathrm{f}} = y_{\mathrm{i}} + v_{\mathrm{i}_y} t + \frac{1}{2} a_y t^2 \tag{1}$$

Where $y_{\mathrm{f}} = 0$ and $a_y = -g$:

$$0 = y_{\mathrm{i}} + v_{\mathrm{i}} \sin(\theta) t - \frac{1}{2} g t^2$$

This is a quadratic expression in t. The quadratic coefficients are:

$$A = -\frac{1}{2}g$$

$$B = v_\mathrm{i}\sin(\theta)$$

$$C = y_\mathrm{i}$$

Plugging these into the quadratic equation:

$$t = \frac{-\left(v_\mathrm{i}\sin(\theta)\right) \pm \sqrt{\left(v_\mathrm{i}\sin(\theta)\right)^2 - 4\left(-\frac{1}{2}g\right)(y_\mathrm{i})}}{\cancel{2}\left(-\frac{1}{\cancel{2}}g\right)}$$

$$= \frac{-\left(v_\mathrm{i}\sin(\theta)\right) \pm \sqrt{\left(v_\mathrm{i}\sin(\theta)\right)^2 + 2gy_\mathrm{i}}}{-g}$$

$$= \frac{v_\mathrm{i}\sin(\theta) \mp \sqrt{\left(v_\mathrm{i}\sin(\theta)\right)^2 + 2gy_\mathrm{i}}}{g}$$

Using the minus sign results in a negative time which can be discarded.

The positive time is:

$$= \frac{\left(7.00\,\frac{\mathrm{m}}{\mathrm{s^2}}\right)\sin(-40.0^\circ) + \sqrt{\left(\left(7.00\,\frac{\mathrm{m}}{\mathrm{s^2}}\right)\sin(-40.0^\circ)\right)^2 + 2\left(9.80\,\frac{\mathrm{m}}{\mathrm{s^2}}\right)(14.0\,\mathrm{m})}}{9.80\,\frac{\mathrm{m}}{\mathrm{s^2}}}$$

$$\approx 1.29\,\mathrm{s}$$

** **Note:** The calculation above could also be done with $\theta = 40.0^\circ$ and $v_\mathrm{i} = -7.00\,\frac{\mathrm{m}}{\mathrm{s^2}}$. Both approaches are mathematically equivalent.

The time above will be used with the horizontal version of (1):

$$x_{\mathrm{f}} = x_{\mathrm{i}} + v_{\mathrm{i}_x} t + \frac{1}{2} a_x t^2$$

Where $x_{\mathrm{i}} = 0$ and $a_{\mathrm{x}} = 0$. Let the range of the snowball be R such that:

$$\mathrm{R} = v_{\mathrm{i}} \cos(\theta) t \tag{2}$$

$$= \left(7.00\,\frac{\mathrm{m}}{\mathrm{s}}\right) \cos(-40.0°)(1.29\,\mathrm{s})$$

$$\approx \boxed{6.93\,\mathrm{m}}$$

b.) Plots for the time-dependent behavior of the snowball:

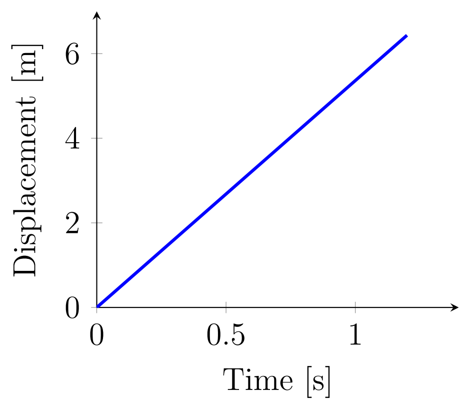


Figure 1: x vs. t

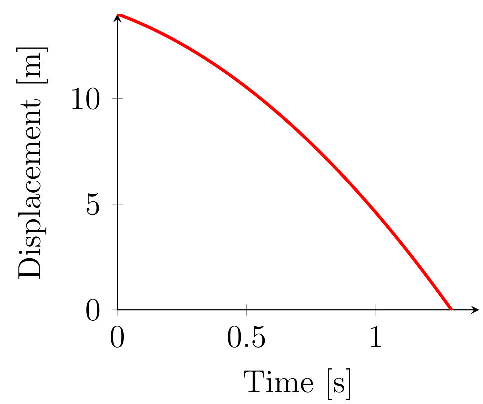


Figure 2: y vs. t

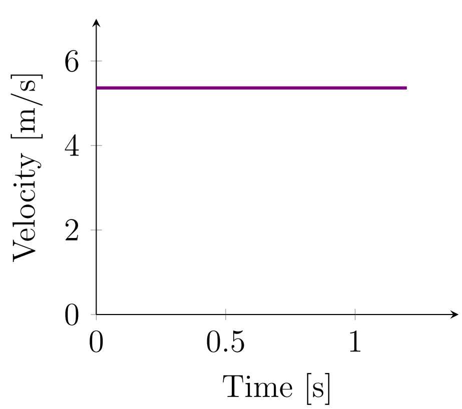


Figure 3: v_x vs. t

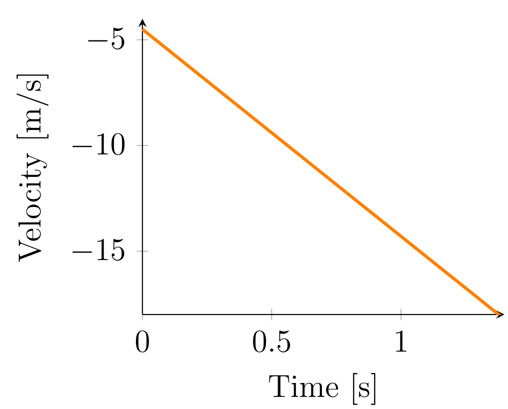


Figure 4: v_y vs. t

Functions used for each plot:

$$x(t) = \left(7.00\,\frac{\mathrm{m}}{\mathrm{s}}\right)\cos(-40.0^\circ)t$$

$$y(t) = (14.0\,\mathrm{m}) + \left(7.00\,\frac{\mathrm{m}}{\mathrm{s}}\right)\sin(-40.0^\circ)t - \frac{1}{2}\left(9.80\,\frac{\mathrm{m}}{\mathrm{s}^2}\right)t^2$$

$$v_x(t) = \left(7.00\,\frac{\mathrm{m}}{\mathrm{s}}\right)\cos(-40.0^\circ)$$

$$v_y(t) = \left(7.00\,\frac{\mathrm{m}}{\mathrm{s}}\right)\sin(-40.0^\circ) - \left(9.80\,\frac{\mathrm{m}}{\mathrm{s}^2}\right)t$$

c.) To determine if the man will be hit by the snowball, rearrange **(2)** for time:

$$t = \frac{\mathrm{R}}{v_\mathrm{i}\cos(\theta)}$$

Plugging this into **(1)**:

$$y_\mathrm{f} = y_\mathrm{i} + \cancel{v_\mathrm{i}}\sin(\theta)\left(\frac{R}{\cancel{v_\mathrm{i}}\cos(\theta)}\right) - \frac{1}{2}g\left(\frac{\mathrm{R}}{v_\mathrm{i}\cos(\theta)}\right)^2$$

$$= y_\mathrm{i} + \mathrm{R}\frac{\sin(\theta)}{\cos(\theta)} - \frac{g}{2}\left(\frac{R\sec(\theta)}{v_\mathrm{i}}\right)^2$$

$$= y_\mathrm{i} + \mathrm{R}\tan(\theta) - \frac{g}{2}\left(\frac{R\sec(\theta)}{v_\mathrm{i}}\right)^2$$

$$= (14.0\,\mathrm{m}) + (4.0\,\mathrm{m})\tan(-40.0^\circ) - \frac{9.80\,\frac{\mathrm{m}}{\mathrm{s}^2}}{2}\left(\frac{(4.0\,\mathrm{m})\sec(-40.0^\circ)}{7.00\,\frac{\mathrm{m}}{\mathrm{s}}}\right)^2$$

$$\approx 7.9\,\mathrm{m}$$

This distance is much higher than the vertical position of the man's head (1.9 m). Therefore, he will not be hit by the snowball.