

Compound Projectile Motion Problem #3

Motion in Two or Three Dimensions

According to the Guinness Book of World Records, the longest home run ever measured was hit by Roy “Dizzy” Carlyle in a minor league game. The ball traveled 188 m (618 ft) before landing on the ground outside a ballpark.

a.) Assuming the ball’s initial velocity was in a direction 45.0° above the horizontal and ignoring air resistance, what did the initial speed of the ball need to be to produce such a home run if the ball was hit at a point 0.90 m (3.00 ft) above ground level?

Assume the ground was perfectly flat.

b.) How far would the ball be above a fence 3.00 m (10.0 ft) high if the fence was 116 m (380 ft) from home plate?

Solution

a.) Use the kinematic equation for vertical displacement:

$$y_f = y_i + v_{iy} + \frac{1}{2}a_y t^2 \quad (1)$$

Where $y_f = 0$, $v_{iy} = v_i \sin(\theta)$, and $a_y = -g$. Let $y_i = H$ such that:

$$0 = H + v_i \sin(\theta)t - \frac{1}{2}gt^2 \quad (2)$$

This is a quadratic expression in t , but there isn’t enough information to apply the quadratic equation. To get around this issue, the goal is to express t in terms of known quantities using the horizontal version of (1):

$$x_f = x_i + v_{ix} + \frac{1}{2}a_x t^2$$

Where $x_i = 0$, $v_{ix} = v_i \cos(\theta)$, and $a_x = 0$.

Let $x_f = R$ such that:

$$R = v_i \cos(\theta)t$$

Solving for t :

$$t = \frac{R}{v_i \cos \theta} \quad (3)$$

Plugging (3) into (2):

$$\begin{aligned}
 0 &= H + v_i \sin(\theta) \left(\frac{R}{v_i \cos(\theta)} \right) - \frac{1}{2} g \left(\frac{R}{v_i \cos(\theta)} \right)^2 \\
 &= H + R \frac{\sin(\theta)}{\cos(\theta)} - \frac{g R^2}{2 v_i^2 \cos^2(\theta)} \\
 &= H + R \tan(\theta) - \frac{g R^2 \sec^2(\theta)}{2 v_i^2}
 \end{aligned}$$

For $\theta = 45.0^\circ$, $\tan(\theta) = 1$. Therefore:

$$0 = (H + R) - \frac{g \left(R \sec(\theta) \right)^2}{2 v_i^2}$$

Solving for v_i :

$$\frac{g \left(R \sec(\theta) \right)^2}{2 v_i^2} = (H + R)$$

$$\frac{g \left(R \sec(\theta) \right)^2}{2} = (H + R) v_i^2$$

$$\frac{g \left(R \sec(\theta) \right)^2}{2(H + R)} = v_i^2$$

$$\sqrt{\frac{g \left(R \sec(\theta) \right)^2}{2(H + R)}} = v_i$$

$$\sqrt{\frac{\left(9.80 \frac{\text{m}}{\text{s}^2} \right) \left((188 \text{ m}) \sec(45.0^\circ) \right)^2}{2(0.90 \text{ m} + 188 \text{ m})}} \approx \boxed{42.8 \frac{\text{m}}{\text{s}}}$$

b.) The vertical displacement of the ball at the range of 116 m can be expressed as:

$$y_{\text{total}} = y_{\text{fence}} + \Delta y \quad (4)$$

Where Δy represents the vertical distance between the ball and the fence.

The value of y_{total} can be found using a process similar to part (a) starting from (2).
Let $y_{\text{f}} = y_{\text{total}}$ such that:

$$y_{\text{total}} = H + v_i \sin(\theta)t - \frac{1}{2}gt^2$$

Substituting t with (3):

$$\begin{aligned} y_{\text{total}} &= H + v_i \sin(\theta) \frac{R}{v_i \cos(\theta)} - \frac{1}{2}g \left(\frac{R}{v_i \cos(\theta)} \right)^2 \\ &= H + R \tan(\theta) - \frac{g}{2} \left(\frac{R \sec(\theta)}{v_i} \right)^2 \\ &= (H + R) - \frac{g}{2} \left(\frac{R \sec(\theta)}{v_i} \right)^2 \\ &= (0.90 \text{ m} + 116 \text{ m}) - \frac{\left(9.80 \frac{\text{m}}{\text{s}^2} \right)}{2} \left(\frac{(116 \text{ m}) \sec(45.0^\circ)}{42.8 \frac{\text{m}}{\text{s}}} \right)^2 \\ &\approx 44.9 \text{ m} \end{aligned}$$

Then by (4):

$$\begin{aligned} \Delta y &= y_{\text{total}} - y_{\text{fence}} \\ &= (44.9 \text{ m}) - (3.00 \text{ m}) \\ &= \boxed{41.9 \text{ m}} \end{aligned}$$