Practice Problem #4

Newton's Laws of Motion

Question: *A hot-air balloon consists of a basket, one passenger, and some cargo. Let the total mass be M. Even though there is an upward lift force on the balloon, the balloon is initially accelerating downward at a rate of g/3.*

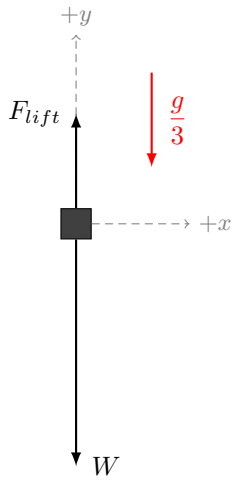
a.) Draw a free-body diagram for the descending balloon.

b.) Find the upward lift force in terms of the initial total weight Mg.

c.) The passenger notices that he is heading straight for a waterfall and decides he needs to go up. What fraction of the total weight must he drop overboard so that the balloon accelerates upward at a rate of g/2? Assume that the upward lift force remains the same.

Solution:

a.) Free-body diagram of the balloon:



b.) To find the lift force, apply Newton's 2nd Law in the vertical direction:

$$\sum F_y = Ma_{net}$$

$$(F_{lift} - W) = -M\left(\frac{g}{3}\right)$$

$$F_{lift} - Mg = -\frac{Mg}{3}$$

$$F_{lift} = Mg - \frac{Mg}{3}$$

$$= Mg\left(1 - \frac{1}{3}\right)$$

$$= \boxed{\frac{2}{3}Mg}$$

c.) Let m be the mass remaining after a portion is thrown overboard. Applying Newton's 2nd Law:

$$\sum F_y = ma_{net}$$

$$(F_{lift} - mg) = m\Big(\frac{g}{2}\Big)$$

$$\frac{2}{3}Mg - mg = \frac{mg}{2}$$

$$\frac{2}{3}Mg = \frac{mg}{2} + mg$$

$$\frac{2}{3}M\cancel{g} = m\cancel{g}\Big(\frac{1}{2} + 1\Big)$$

$$\frac{2}{3}M = \frac{3}{2}m$$

$$\frac{4}{9} = m$$

Thus, if m is the amount of mass that's *remaining*, then the amount that was thrown overboard was $M - m$:

$$M - m = M - \frac{4}{9}M$$

$$= \frac{9}{9}M - \frac{4}{9}M$$

$$= \frac{9-4}{9}M$$

$$= \boxed{\frac{5}{9}M}$$