Practice Problem #3

Newton's Laws of Motion

Question: *A 4.9 N hammer head is stopped from an initial downward velocity of 3.2 m/s in a distance of 0.45 cm by a nail in a pine board. In addition to its weight, there is a 15 N downward force on the hammer head applied by the person using the hammer. Assume that the acceleration of the hammer head is constant while it is in contact with the nail and moving downward.*

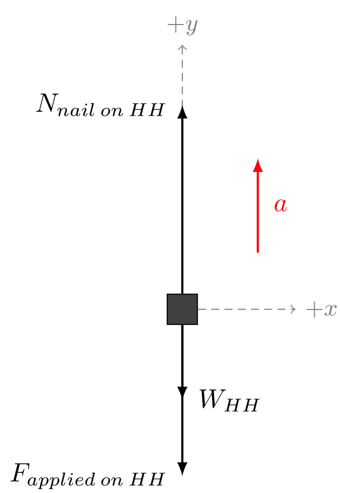
a.) Draw a free-body diagram for the hammer head. Identify the reaction force to each action force in the diagram.

b.) Calculate the downward force F exerted by the hammer head on the nail while the hammer head is in contact with the nail and moving downward.

c.) Suppose the nail is in hardwood and the distance the hammer head travels in coming to rest is only 0.12 cm. The downward forces on the hammer head are the same as in part (b). What then is the force F exerted by the hammer head on the nail while the hammer head is in contact with the nail and moving downward?

Solution:

a.) Free-body diagram of the hammer head:



The acceleration vector points in the $+y$ direction due to the hammer head initially moving downwards and then coming to a stop once it hits the nail.

The action forces are:

1. The downward gravitional force exerted by the Earth on the hammer head (W_{HH})

   Reaction force: The upwards gravitational force exerted by the hammer head on the Earth.

2. The upwards normal force of the nail acting on the hammer head ($N_{nail\ on\ HH}$)

   Reaction force: The downwards force of the hammer head acting on the nail.

3. The downward applied force of the person acting on the hammer head ($F_{applied\ on\ HH}$)

   Reaction force: The upwards force of the hammer head acting back onto the person.

b.) To calculate the force exerted by the hammer head on the nail, use Newton's 2nd Law:

$$\sum F = ma_{net}$$

$$(N_{nail} - W_{HH} - F_{applied}) = ma_{net}$$

The value of the normal force of the nail acting on the hammer head is unknown, but we have identified that the reaction force to $N_{nail\ on\ HH}$ is $F_{HH\ on\ nail}$. Thus, $N_{nail\ on\ HH} = -F_{HH\ on\ nail}$. Substituting that above:

$$(-F_{HH} - W_{HH} - F_{applied}) = ma_{net}$$

Solving for F_{HH}:

$$-ma_{net} - W_{HH} - F_{applied} = F_{HH}$$

The acceleration of the hammer head can be found by the following kinematic equation:

$$v_f^2 = v_i^2 + 2a\Delta y$$

$$0 = v_i^2 + 2a\Delta y$$

$$-2a\Delta y = v_i^2$$

$$a = -\frac{v_i^2}{2\Delta y}$$

Thus, the value of the force of the hammer head is:

$$\cancel{-}(\frac{W_{HH}}{g})(\cancel{-}\frac{v_i^2}{2\Delta y}) - W_{HH} - F_{applied} = F_{HH}$$

$$W_{HH}\Big(\frac{v_i^2}{2g\Delta y} - 1\Big) - F_{applied} = F_{HH}$$

$$(4.9\ N)\left(\frac{\Big(-3.2\ \dfrac{m}{s}\Big)^2}{2(9.8\ \dfrac{m}{s^2})\Big(0 - 0.45\ \cancel{cm} \cdot \dfrac{1\ m}{100\ \cancel{cm}}\Big)} - 1\right) - (15\ N) \approx \boxed{-5.9 \cdot 10^2\ N}$$

A negative answer is expected due to the hammer pushing downward on the nail upon contact.

c.) We can use the same expression from the end of part (b), but change $0.45\ cm$ to $0.12\ cm$:

$$(4.9\ N)\left(\frac{\Big(-3.2\ \dfrac{m}{s}\Big)^2}{2(9.8\ \dfrac{m}{s^2})\Big(0 - 0.12\ \cancel{cm} \cdot \dfrac{1\ m}{100\ \cancel{cm}}\Big)} - 1\right) - (15\ N) \approx \boxed{-2.2 \cdot 10^3\ N}$$

Due to the smaller distance, the acceleration of the hammer head is much larger, which in turn causes the force to be larger as well.