Practice Problem #5

Newton's Laws of Motion

Question: *The position of a* $2.75 \cdot 10^5$ *N training helicopter under test is given by:*

$$\vec{r} = \left(0.020\,\frac{m}{s^3}\right)t^3\hat{\boldsymbol{i}} + \left(2.2\,\frac{m}{s}\right)t\hat{\boldsymbol{j}} - \left(0.060\,\frac{m}{s^2}\right)t^2\hat{\boldsymbol{k}}$$

Find the net force on the helicopter at $t = 5.0$ *s.*

Solution: Apply the vector form of Newton's 2nd Law:

$$\sum \vec{F} = m\,\vec{a} \tag{1}$$

Where $\vec{a}$ is the second-order derivative of $\vec{r}$ with respect to time:

$$\vec{a} = \frac{d^2\vec{r}}{dt^2} = \frac{d}{dt}\left(\frac{d}{dt}\left(\left(0.020\,\frac{m}{s^3}\right)t^3\hat{\boldsymbol{i}} + \left(2.2\,\frac{m}{s}\right)t\hat{\boldsymbol{j}} - \left(0.060\,\frac{m}{s^2}\right)t^2\hat{\boldsymbol{k}}\right)\right)$$

$$= \frac{d}{dt}\left(3\left(0.020\,\frac{m}{s^3}\right)t^2\hat{\boldsymbol{i}} + \left(2.2\,\frac{m}{s}\right)\hat{\boldsymbol{j}} - 2\left(0.060\,\frac{m}{s^2}\right)t\hat{\boldsymbol{k}}\right)$$

$$= 6\left(0.020\,\frac{m}{s^3}\right)t\hat{\boldsymbol{i}} - 2\left(0.060\,\frac{m}{s^2}\right)\hat{\boldsymbol{k}}$$

$$= \left(0.120\,\frac{m}{s^3}\right)t\hat{\boldsymbol{i}} - \left(0.120\,\frac{m}{s^2}\right)\hat{\boldsymbol{k}}$$

At $t = 5.0\ s$:

$$\vec{a} = \left(0.120\,\frac{m}{s^3}\right)(5.0\ s)\hat{\boldsymbol{i}} - \left(0.120\,\frac{m}{s^2}\right)\hat{\boldsymbol{k}}$$

$$= \left(0.60\,\frac{m}{s^2}\right)\hat{\boldsymbol{i}} - \left(0.12\,\frac{m}{s^2}\right)\hat{\boldsymbol{k}}$$

This acceleration can be plugged into (1) to obtain the net force:

$$\sum \vec{F} = \frac{W}{g}\,\vec{a}$$

$$= \frac{(2.75 \cdot 10^5\ N)}{(9.80\ \frac{m}{s^2})}\left((0.60\ \frac{m}{s^2})\hat{\boldsymbol{i}} - (0.12\ \frac{m}{s^2})\hat{\boldsymbol{k}}\right)$$

$$\approx \boxed{(1.7 \cdot 10^4\ N)\hat{\boldsymbol{i}} - (3.4 \cdot 10^3\ N)\hat{\boldsymbol{k}}}$$