Compound Projectile Motion Problem #3

Motion in Two or Three Dimensions

Question: *According to the Guinness Book of World Records, the longest home run ever measured was hit by Roy "Dizzy" Carlyle in a minor league game. The ball traveled 188 m (618 ft) before landing on the ground outside a ballpark.*

a.) Assuming the ball's initial velocity was in a direction 45.0° above the horizontal and ignoring air resistance, what did the initial speed of the ball need to be to produce such a home run if the ball was hit at a point 0.90 m (3.00 ft) above ground level? Assume the ground was perfectly flat.

b.) How far would the ball be above a fence 3.00 m (10.0 ft) high if the fence was 116 m (380 ft) from home plate?

Solution:

a.) Use the kinematic equation for vertical displacement:

$$y_f = y_i + v_{iy}t + \frac{1}{2}a_y t^2 \tag{1}$$

Where $y_f = 0$, $v_{iy} = v_i sin(\theta)$, and $a_y = -g$.

Let $y_i = H$, such that:

$$0 = H + v_i sin(\theta)t - \frac{1}{2}gt^2$$

This is a quadratic expression in t, but there is not enough information to use the quadratic equation. We can express t in terms of known quantities by using the horizontal version of equation (1):

$$x_f = x_i + v_{ix}t + \frac{1}{2}a_x t^2 \tag{2}$$

Where $x_i = 0$, $v_{ix} = v_i cos(\theta)$, and $a_x = 0$.

Let the range of the ball be R, such that:

$$R = v_i cos(\theta)t$$

$$\implies t = \frac{R}{v_i cos(\theta)}$$

Plugging this expression for time into the vertical displacement equation:

$$0 = H + \cancel{v_i} sin(\theta)\frac{R}{\cancel{v_i} cos(\theta)} - \frac{1}{2}g\Big(\frac{R}{v_i cos(\theta)}\Big)^2$$

$$= H + R\frac{sin(\theta)}{cos(\theta)} - \frac{1}{2}g\frac{R^2}{v_i^2 cos^2(\theta)}$$

$$= H + Rtan(\theta) - \frac{gR^2}{2v_i^2}sec^2(\theta)$$

For $\theta = 45.0°$, $\tan(\theta) = 1$. Therefore:

$$0 = (H + R) - \frac{gR^2}{2v_i^2}sec^2(\theta)$$

Solving for v_i:

$$\frac{gR^2}{2v_i^2}sec^2(\theta) = (H + R)$$

$$\frac{gR^2 sec^2(\theta)}{2} = (H + R)v_i^2$$

$$\frac{g[Rsec(\theta)]^2}{2(H + R)} = v_i^2$$

$$\sqrt{\frac{g[Rsec(\theta)]^2}{2(H + R)}} = v_i$$

$$\sqrt{\frac{(9.8\,\frac{m}{s^2})[(188\,m)sec(45.0°)]^2}{2(0.90\,m + 188\,m)}} \approx \boxed{42.8\,\frac{m}{s}}$$

b.) The vertical displacement of the ball at the range of 116 m can be expressed as:

$$y_{total} = y_{fence} + \Delta y \tag{3}$$

Where Δy is the vertical distance between the ball and the fence.

To obtain the value of y_{total}, we can use a similar process from part (a), starting from equation (1). Let $y_f = y_{total}$ such that:

$$y_{total} = H + v_i sin(\theta)t - \frac{1}{2}gt^2$$

Inserting the expression for time found at the bottom of page 1:

$$y_{total} = H + \cancel{v_i} sin(\theta)\frac{R}{\cancel{v_i} cos(\theta)} - \frac{1}{2}g\Big(\frac{R}{v_i cos(\theta)}\Big)^2$$

$$= H + Rtan(\theta) - \frac{g}{2}\Big(\frac{Rsec(\theta)}{v_i}\Big)^2$$

$$= (H + R) - \frac{g}{2}\Big(\frac{Rsec(\theta)}{v_i}\Big)^2$$

$$= (0.90\,m + 116\,m) - \frac{(9.80\,\frac{m}{s^2})}{2}\Big(\frac{(116\,m)sec(45.0°)}{42.8\,\frac{m}{s}}\Big)^2$$

$$\approx 44.9\,m$$

By equation (3):

$$\Delta y = y_{total} - y_{fence}$$

$$= (44.9\,m) - (3.00\,m)$$

$$= \boxed{41.9\,m}$$