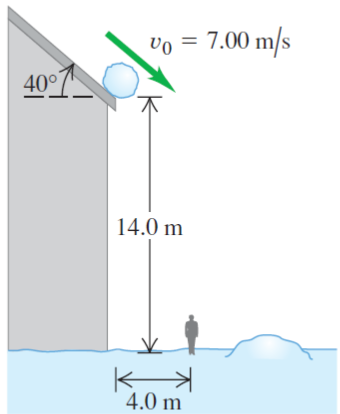
Compound Projectile Motion Problem #4

Motion in Two or Three Dimensions

Question: *A snowball rolls off a barn roof that slopes downward at an angle of 40.0°. The edge of the roof is 14.0 m above the ground, and the snowball has a speed of 7.00 m/s as it rolls off the roof. Ignore air resistance.*



a.) How far from the edge of the barn does the snowball strike the ground if it doesn't strike anything else while falling?

b.) Draw x-t, y-t, v_x-t and v_y-t graphs for the motion in part (a).

c.) A man 1.9 m tall is standing 4.0 m from the edge of the barn. Will he be hit by the snowball?

Solution:

a.) To determine the range of the snowball, the time it takes to hit the ground is needed. This can be found by using the vertical version of the kinematic equation for displacement:

$$y_f = y_i + v_{iy}t + \frac{1}{2}a_y t^2 \tag{1}$$

Where $y_f = 0$ and $a_y = -g$:

$$0 = y_i + v_i sin(\theta)t - \frac{1}{2}gt^2$$

This is a quadratic expression in t. The quadratic coefficients are:

$$A = -\frac{1}{2}g$$

$$B = v_i sin(\theta)$$

$$c = y_i$$

Plugging these into the quadratic equation:

$$t = \frac{-\left(v_i sin(\theta)\right) \pm \sqrt{\left(v_i sin(\theta)\right)^2 - 4(-\frac{1}{2}g)(y_i)}}{\cancel{2}(-\frac{\cancel{1}}{\cancel{2}}g)}$$

$$= \frac{-v_i sin(\theta) \pm \sqrt{\left(v_i sin(\theta)\right)^2 + 2gy_i}}{-g}$$

$$= \frac{v_i sin(\theta) \mp \sqrt{\left(v_i sin(\theta)\right)^2 + 2gy_i}}{g}$$

Using the minus sign results in a negative time, which can be discarded. The positive result is:

$$= \frac{(7.00\ \frac{m}{s})sin(-40.0°) + \sqrt{\left(7.00\ \frac{m}{s}sin(-40.0°\right)^2 + 2(9.80\ \frac{m}{s^2})(14.0\ m)}}{(9.80\ \frac{m}{s^2})}$$

$$\approx 1.29\ s$$

Notice that the opposite angle of -40.0° was used for θ to produce a negative y-component of the velocity. This could be done using negative v_i and positive θ as well. Both approaches are equivalent.

This time will be used with the horizontal version of equation (1):

$$x_f = x_i + v_{ix}t + \frac{1}{2}a_x t^2$$

Where $x_f = R$, $x_i = 0$, and $a_x = 0$:

$$\begin{aligned} R &= v_i cos(\theta)t \\ &= (7.00\,\frac{m}{s})cos(-40.0^\circ)(1.29\,s) \\ &\approx \boxed{6.93\,m} \end{aligned}$$

b.) Figures for the time-dependent behavior of the snowball are provided below:

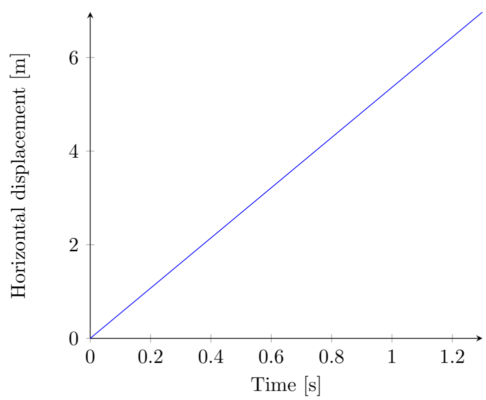


Figure 1: x vs. t

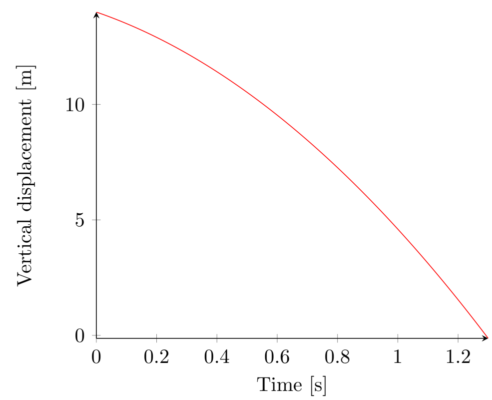


Figure 2: y vs. t

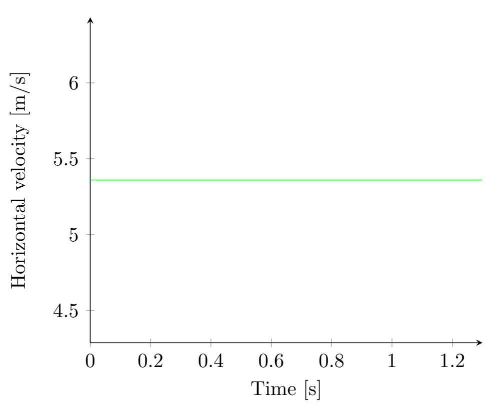


Figure 3: v_x vs. t

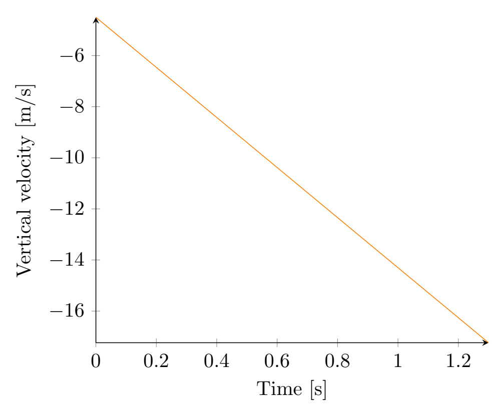


Figure 4: v_y vs. t

c.) First get an expression for the amount of time it would take for the snowball to fall and hit the top of the man's head using the range equation from the end of part (a):

$$t = \frac{R}{v_i cos(\theta)}$$

Plugging this expression into equation (1):

$$y_f = y_i + \cancel{v_i} sin(\theta)\left(\frac{R}{\cancel{v_i} cos(\theta)}\right) - \frac{1}{2}g\left(\frac{R}{v_i cos(\theta)}\right)^2$$

$$= y_i + R\frac{sin(\theta)}{cos(\theta)} - \frac{g}{2}\left(\frac{Rsec(\theta)}{v_i}\right)^2$$

$$= y_i + Rtan(\theta) - \frac{g}{2}\left(\frac{Rsec(\theta)}{v_i}\right)^2$$

$$= (14.0\, m) + (4.0\, m)tan(-40.0°) - \frac{9.80\,\frac{m}{s^2}}{2}\left(\frac{(4.0\, m)sec(-40.0°)}{7.00\,\frac{m}{s}}\right)^2$$

$$\approx 7.9\, m$$

This value is higher than the man's head. Therefore, he will not be hit by the snowball.