Compound Projectile Motion Problem #3

Motion in Two or Three Dimensions

According to the Guinness Book of World Records, the longest home run ever measured was hit by Roy "Dizzy" Carlyle in a minor league game. The ball traveled 188 m (618 ft) before landing on the ground outside a ballpark.

a.) Assuming the ball's initial velocity was in a direction 45.0° above the horizontal and ignoring air resistance, what did the initial speed of the ball need to be to produce such a home run if the ball was hit at a point 0.90 m (3.00 ft) above ground level?
Assume the ground was perfectly flat.

b.) How far would the ball be above a fence 3.00 m (10.0 ft) high if the fence was 116 m (380 ft) from home plate?

Solution

a.) Use the kinematic equation for vertical displacement:

$$y_\mathrm{f} = y_\mathrm{i} + v_{\mathrm{i_y}} + \frac{1}{2}a_\mathrm{y}t^2 \tag{1}$$

Where $y_\mathrm{f} = 0$, $v_{\mathrm{i_y}} = v_\mathrm{i}\sin(\theta)$, and $a_\mathrm{y} = -g$.

Let $y_\mathrm{i} = \mathrm{H}$ to represent the initial height of the ball above ground level, such that:

$$0 = \mathrm{H} + v_\mathrm{i}\sin(\theta)t - \frac{1}{2}gt^2 \tag{2}$$

This is a quadratic expression in t, but there isn't enough information to apply the quadratic equation. The get around this issue, the goal is to express t in terms of known quantities using the horizontal version of (1):

$$x_\mathrm{f} = x_\mathrm{i} + v_{\mathrm{i_x}} + \frac{1}{2}a_\mathrm{x}t^2$$

Where $x_\mathrm{i} = 0$, $v_{\mathrm{i_x}} = v_\mathrm{i}\cos(\theta)$, and $a_\mathrm{x} = 0$.

Let $x_\mathrm{f} = \mathrm{R}$ such that:

$$\mathrm{R} = v_\mathrm{i}\cos(\theta)t$$

Solving for t:

$$t = \frac{\mathrm{R}}{v_\mathrm{i}\cos\theta} \tag{3}$$

Plugging **(3)** into **(2)**:

$$
\begin{aligned}
0 &= \mathrm{H} + \cancel{v_\mathrm{i}}\sin(\theta)\left(\frac{\mathrm{R}}{\cancel{v_\mathrm{i}}\cos(\theta)}\right) - \frac{1}{2}g\left(\frac{\mathrm{R}}{v_\mathrm{i}\cos(\theta)}\right)^2 \\
&= \mathrm{H} + \mathrm{R}\frac{\sin(\theta)}{\cos(\theta)} - \frac{g\mathrm{R}^2}{2v_\mathrm{i}^2\cos^2(\theta)} \\
&= \mathrm{H} + \mathrm{R}\tan(\theta) - \frac{g\mathrm{R}^2\sec^2(\theta)}{2v_\mathrm{i}^2}
\end{aligned}
$$

For $\theta = 45.0°$, $\tan(\theta) = 1$. Therefore:

$$0 = (\mathrm{H} + \mathrm{R}) - \frac{g\Big(\mathrm{R}\sec(\theta)\Big)^2}{2v_\mathrm{i}^2}$$

Solving for v_i:

$$\frac{g\Big(\mathrm{R}\sec(\theta)\Big)^2}{2v_\mathrm{i}^2} = (\mathrm{H} + \mathrm{R})$$

$$\frac{g\Big(\mathrm{R}\sec(\theta)\Big)^2}{2} = (\mathrm{H} + \mathrm{R})v_\mathrm{i}^2$$

$$\frac{g\Big(\mathrm{R}\sec(\theta)\Big)^2}{2(\mathrm{H} + \mathrm{R})} = v_\mathrm{i}^2$$

$$\sqrt{\frac{g\Big(\mathrm{R}\sec(\theta)\Big)^2}{2(\mathrm{H} + \mathrm{R})}} = v_\mathrm{i}$$

$$\sqrt{\frac{\left(9.80\,\frac{\mathrm{m}}{\mathrm{s}^2}\right)\Big((188\,\mathrm{m})\sec(45.0°)\Big)^2}{2(0.90\,\mathrm{m} + 188\,\mathrm{m})}} \approx \boxed{42.8\,\frac{\mathrm{m}}{\mathrm{s}}}$$

b.) The vertical displacement of the ball at the range of 116 m can be expressed as:

$$y_{\text{total}} = y_{\text{fence}} + \Delta y \tag{4}$$

Where Δy represents the vertical distance between the ball and the fence.

The value of y_{total} can be found using a process similar to part (a) starting from **(2)**. Let $y_{\text{f}} = y_{\text{total}}$ such that:

$$y_{\text{total}} = \mathrm{H} + v_{\text{i}} \sin(\theta) t - \frac{1}{2} g t^2$$

Substituting t with **(3)**:

$$\begin{aligned}
y_{\text{total}} &= \mathrm{H} + \cancel{v_{\text{i}}} \sin(\theta) \frac{\mathrm{R}}{\cancel{v_{\text{i}}} \cos(\theta)} - \frac{1}{2} g \left(\frac{\mathrm{R}}{v_{\text{i}} \cos(\theta)} \right)^2 \\
&= \mathrm{H} + \mathrm{R} \tan(\theta) - \frac{g}{2} \left(\frac{\mathrm{R} \sec(\theta)}{v_{\text{i}}} \right)^2 \\
&= (\mathrm{H} + \mathrm{R}) - \frac{g}{2} \left(\frac{\mathrm{R} \sec(\theta)}{v_{\text{i}}} \right)^2 \\
&= (0.90\,\mathrm{m} + 116\,\mathrm{m}) - \frac{\left(9.80\,\frac{\mathrm{m}}{\mathrm{s}^2}\right)}{2} \left(\frac{(116\,\mathrm{m}) \sec(45.0^\circ)}{42.8\,\frac{\mathrm{m}}{\mathrm{s}}} \right)^2 \\
&\approx 44.9\,\mathrm{m}
\end{aligned}$$

Then by **(4)**:

$$\begin{aligned}
\Delta y &= y_{\text{total}} - y_{\text{fence}} \\
&= (44.9\,\mathrm{m}) - (3.00\,\mathrm{m}) \\
&= \boxed{41.9\,\mathrm{m}}
\end{aligned}$$