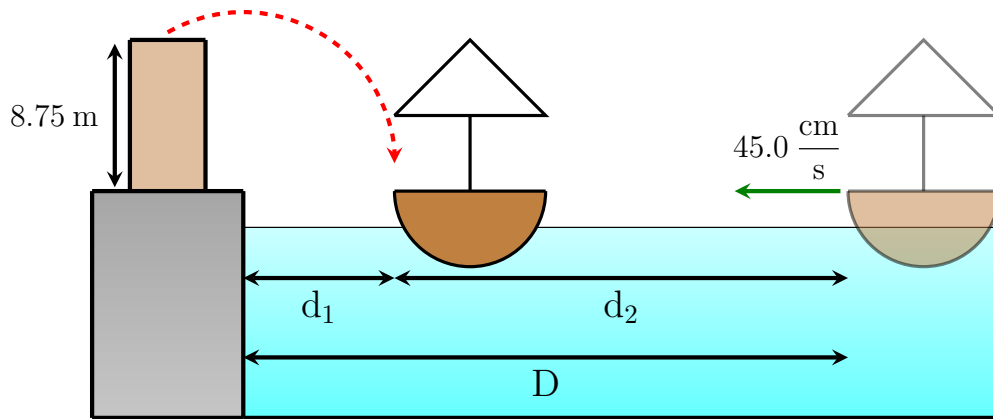


Compound Projectile Motion Problem #2

Motion in Two or Three Dimensions

As a ship is approaching a dock at 45.0 cm/s , a piece of landing equipment needs to be thrown to it before it can dock. This equipment is thrown at 15.0 m/s at 60.0° above the horizontal from the top of a tower near the edge of the water, 8.75 m above the ship's deck. For this equipment to land at the front of the ship, at what distance D from the dock should the ship be when the equipment is thrown? Air resistance can be neglected.



Solution

The distance D will be the sum of the two distances d_1 and d_2 , where d_1 is the distance between the dock and the ship when the equipment lands on the boat, and d_2 is the distance between the boat's initial and final positions.

To determine the value of d_1 and d_2 , the time of flight for the equipment must be known. This can be found by using the kinematic equation for vertical displacement:

$$y_f = y_i + v_{iy}t + \frac{1}{2}a_yt^2 \quad (1)$$

Let $y_i = H$, $a_y = -g$, and θ be the launch angle of the equipment such that:

$$0 = H + v_i \sin(\theta)t - \frac{1}{2}gt^2$$

This is a quadratic expression in t .

The quadratic coefficients are:

$$A = -\frac{1}{2}g$$

$$B = v_i \sin(\theta)$$

$$C = H$$

Plugging these into the quadratic equation:

$$\begin{aligned} t &= \frac{-\left(v_i \sin(\theta)\right) \pm \sqrt{\left(v_i \sin(\theta)\right)^2 - 4\left(-\frac{1}{2}g\right)(H)}}{2\left(-\frac{1}{2}g\right)} \\ &= \frac{-\left(v_i \sin(\theta)\right) \pm \sqrt{\left(v_i \sin(\theta)\right)^2 + 2gH}}{-g} \\ &= \frac{\left(v_i \sin(\theta)\right) \mp \sqrt{\left(v_i \sin(\theta)\right)^2 + 2gH}}{g} \end{aligned}$$

Using the minus sign results in a negative time which can be discarded.

The positive time is:

$$\begin{aligned} &= \frac{\left(15.0 \frac{\text{m}}{\text{s}}\right) \sin(60.0^\circ) + \sqrt{\left(\left(15.0 \frac{\text{m}}{\text{s}}\right) \sin(60.0^\circ)\right)^2 + 2\left(9.80 \frac{\text{m}}{\text{s}^2}\right)(8.75 \text{ m})}}{9.80 \frac{\text{m}}{\text{s}^2}} \\ &\approx 3.21 \text{ s} \end{aligned}$$

With the time known, d_1 is found by using the horizontal version of (1):

$$x_f = x_i + v_{i_x}t + \frac{1}{2}a_x t^2$$

Where $x_f = d_1$, $x_i = 0$, and $a_x = 0$:

$$\begin{aligned}d_1 &= v_{i_x}t \\&= v_i \cos \theta t \\&= \left(15.0 \frac{\text{m}}{\text{s}}\right) \cos(60.0^\circ)(3.21 \text{ s}) \\&\approx 24.1 \text{ m}\end{aligned}$$

To find d_2 , use the 1D version of (1) in the horizontal direction applied to the boat at its initial position, where $x_f = d_2$, $x_i = 0$, and $a_x = 0$:

$$\begin{aligned}d_2 &= \left(0.45 \frac{\text{m}}{\text{s}}\right)(3.21 \text{ s}) \\&\approx 1.44 \text{ m}\end{aligned}$$

Therefore, the final answer is:

$$\begin{aligned}D &= (24.1 \text{ m}) + (1.44 \text{ m}) \\&\approx \boxed{25.5 \text{ m}}\end{aligned}$$