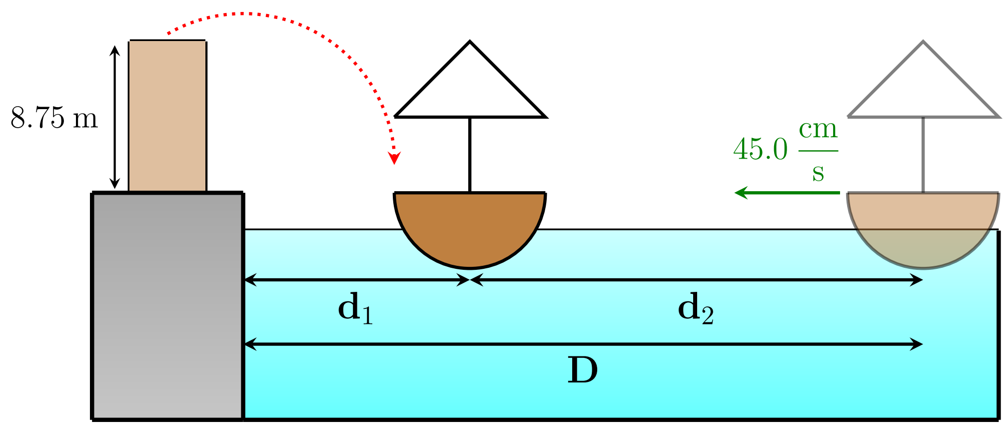
Compound Projectile Motion Problem #2

Motion in Two or Three Dimensions

As a ship is approaching a dock at 45.0 cm/s, a piece of landing equipment needs to be thrown to it before it can dock. This equipment is thrown at 15.0 m/s at 60.0° above the horizontal from the top of a tower near the edge of the water, 8.75 m above the ship's deck. For this equipment to land at the front of the ship, at what distance D from the dock should the ship be when the equipment is thrown? Air resistance can be neglected.



Solution

The distance D will be the sum of the two distances d_1 and d_2, where d_1 is the distance between the dock and the ship when the equipment lands on the boat, and d_2 is the distance between the boat's initial and final positions.

To determine the value of d_1 and d_2, the time of flight for the equipment must be known. This can be found by using the kinematic equation for vertical displacement:

$$y_{\mathrm{f}} = y_{\mathrm{i}} + v_{\mathrm{i_y}} t + \frac{1}{2} a_{\mathrm{y}} t^2 \tag{1}$$

Let H be the height of the tower, θ be the launch angle of the equipment, and $-g$ the acceleration due to gravity, such that:

$$0 = \mathrm{H} + v_{\mathrm{i}} \sin(\theta) t - \frac{1}{2} g t^2$$

This is a quadratic expression in t.

The quadratic coefficients are:

$$A = -\frac{1}{2}g$$

$$B = v_{\mathrm{i}} \sin(\theta)$$

$$C = \mathrm{H}$$

Plugging these into the quadratic equation:

$$t = \frac{-\Big(v_{\mathrm{i}} \sin(\theta)\Big) \pm \sqrt{\Big(v_{\mathrm{i}} \sin(\theta)\Big)^2 - 4(-\frac{1}{2}g)(\mathrm{H})}}{\cancel{2}\Big(-\frac{1}{\cancel{2}}g\Big)}$$

$$= \frac{-\Big(v_{\mathrm{i}} \sin(\theta)\Big) \pm \sqrt{\Big(v_{\mathrm{i}} \sin(\theta)\Big)^2 + 2g\mathrm{H}}}{-g}$$

$$= \frac{\Big(v_{\mathrm{i}} \sin(\theta)\Big) \mp \sqrt{\Big(v_{\mathrm{i}} \sin(\theta)\Big)^2 + 2g\mathrm{H}}}{g}$$

Using the minus sign results in a negative time which can be discarded.

The positive time is:

$$= \frac{\Big(15.0\ \frac{\mathrm{m}}{\mathrm{s}}\Big) \sin(60.0°) \mp \sqrt{\Bigg(\Big(15.0\ \frac{\mathrm{m}}{\mathrm{s}}\Big) \sin(60.0°)\Bigg)^2 + 2\Big(9.80\ \frac{\mathrm{m}}{\mathrm{s}^2}\Big)(8.75\ \mathrm{m})}}{\Big(9.80\ \frac{\mathrm{m}}{\mathrm{s}^2}\Big)}$$

$$\approx 3.21\ \mathrm{s}$$

With the time known, d_1 is found by using the horizontal version of **(1)**:

$$x_{\mathrm{f}} = x_{\mathrm{i}} + v_{\mathrm{i_x}} t + \frac{1}{2} a_{\mathrm{x}} t^2$$

Where $x_\mathrm{f} = \mathrm{d}_1$, $x_\mathrm{i} = 0$, and $a_\mathrm{x} = 0$:

$$\begin{aligned}
\mathrm{d}_1 &= v_{\mathrm{i_x}} t \\
&= \left(15.0\,\frac{\mathrm{m}}{\mathrm{s}}\right)\cos(60.0^\circ)(3.21\,\mathrm{s}) \\
&\approx 24.1\,\mathrm{m}
\end{aligned}$$

To find d_2, use the 1D version of **(1)** in the horizontal direction applied to the boat at its initial position. First convert the boat's initial velocity to meters per second:

$$45.0\,\frac{\cancel{\mathrm{cm}}}{\mathrm{s}} \cdot \frac{1\,\mathrm{m}}{100\,\cancel{\mathrm{cm}}} = 0.45\,\frac{\mathrm{m}}{\mathrm{s}}$$

Therefore:

$$\begin{aligned}
\mathrm{d}_2 &= \left(0.45\,\frac{\mathrm{m}}{\mathrm{s}}\right)(3.21\,\mathrm{s}) \\
&\approx 1.44\,\mathrm{m}
\end{aligned}$$

The final answer is:

$$\begin{aligned}
\mathrm{D} &= (24.1\,\mathrm{m}) + (1.44\,\mathrm{m}) \\
&\approx \boxed{25.5\,\mathrm{m}}
\end{aligned}$$