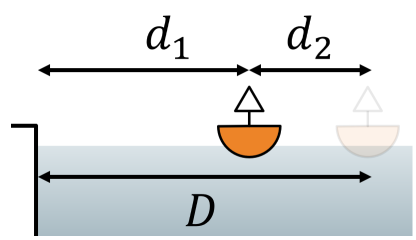
Compound Projectile Motion Problem #2

Motion in Two or Three Dimensions

Question: *As a ship is approaching a dock at 45.0 cm/s, a piece of landing equipment needs to be thrown to it before it can dock. This equipment is thrown at 15.0 m/s at 60.0°above the horizontal from the top of a tower at the edge of the water, 8.75 m above the ship's deck. For this equipment to land at the front of the ship, at what distance D from the dock should the ship be when the equipment is thrown? Air resistance can be neglected.*

Solution: The distance D will be the sum of two distances, d_1 and d_2, as seen below:



Where d_1 is the distance between the dock and the ship when the equipment is caught, and d_2 is the distance covered by the boat as moves towards the dock. From the picture, it is clear that:

$$D = d_1 + d_2 \tag{1}$$

To determine the value of d_1, we need to know the amount of time it takes for the equipment to travel from the top of the tower down to the ship. This will be the same amount of time that's used to determine the value of d_2.

The time can be found by using the kinematic equation for vertical displacement:

$$y_f = y_i + v_{iy}t + \frac{1}{2}a_y t^2 \tag{2}$$

Let H be the height of the tower, θ be the launch angle of the equipment, and $-g$ the acceleration due to gravity, such that:

$$0 = H + v_i \sin(\theta)t - \frac{1}{2}gt^2$$

This is a quadratic expression in t. The quadratic coefficients are:

$$A = -\frac{1}{2}g$$

$$B = v_i \sin(\theta)$$

$$C = H$$

Plugging these into the quadratic equation:

$$t = \frac{-(v_i \sin(\theta)) \pm \sqrt{(v_i \sin(\theta))^2 - 4(-\frac{1}{2}g)(H)}}{\cancel{2}(-\frac{\cancel{1}}{\cancel{2}}g)}$$

$$= \frac{-(v_i \sin(\theta)) \pm \sqrt{(v_i \sin(\theta))^2 + 2gH}}{-g}$$

$$= \frac{v_i \sin(\theta) \mp \sqrt{(v_i \sin(\theta))^2 + 2gH}}{g}$$

Using the minus sign results in a negative time, which can be discarded. The positive result is:

$$= \frac{15.0\,\frac{m}{s}\sin(60.0^\circ) + \sqrt{(15.0\,\frac{m}{s}\sin(60.0^\circ))^2 + 2(9.80\,\frac{m}{s^2})(8.75\,m)}}{9.80\,\frac{m}{s^2}}$$

$$\approx 3.21\,s$$

With the time known, distance d_1 is found by using the horizontal version of equation (2):

$$x_f = x_i + v_{ix}t + \frac{1}{2}a_x t^2$$

Where $x_f = d_1$, $x_i = 0$, and $a_x = 0$:

$$d_1 = v_{ix}t$$

$$= (15.0\frac{m}{s})\cos(60.0^\circ)(3.21\,s)$$

$$\approx 24.1\,m$$

Distance d_2 is found in a similar manner as d_1, but no angle is necessary and the ship's velocity is used for v_i instead. Converting the boat's velocity to meters per second:

$$45.0 \, \frac{\cancel{cm}}{s} \cdot \frac{1 \, m}{100 \, \cancel{cm}} = 0.45 \, \frac{m}{s}$$

Therefore:

$$d_2 = (0.45 \, \frac{m}{s})(3.21 \, s)$$

$$\approx 1.44 \, m$$

So then, by equation (1):

$$D = 24.1 \, m + 1.44 \, m$$

$$\boxed{\approx 25.5 \, m}$$