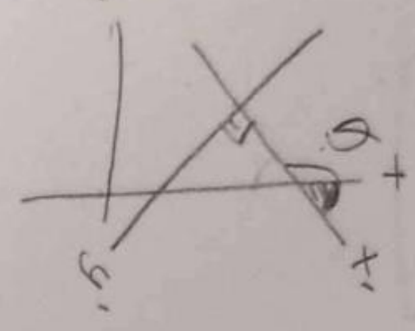
CS460
Homework - 1

1. 1. Translate origin to (x_0, y_0)

$$T(x_0, y_0) = \begin{bmatrix} 1 & 0 & x_0 \\ 0 & 1 & y_0 \\ 0 & 0 & 1 \end{bmatrix}$$

2. Rotate coordinate system

$$\frac{y_2 - y_0}{x_2 - x_0} = m \qquad \tan(\theta) = m \qquad \theta = \tan^{-1} m$$



turn clock-wise

$$\begin{bmatrix} \cos(180-\theta) & \sin(180-\theta) & 0 \\ -\sin(180-\theta) & \cos(180-\theta) & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

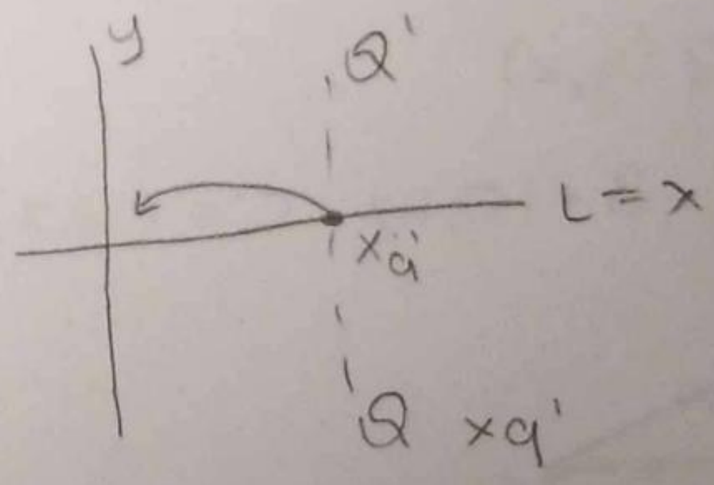
$$T = T_{rot} \cdot T_{tr}$$

$$\begin{bmatrix} \cos(180-\theta) & \sin(180-\theta) & 0 \\ -\sin(180-\theta) & \cos(180-\theta) & 0 \\ 0 & 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} 1 & 0 & x_0 \\ 0 & 1 & y_0 \\ 0 & 0 & 1 \end{bmatrix} =$$

$$= \begin{bmatrix} \cos(180-\theta) & \sin(180-\theta) & \cos(180-\theta)x_0 + \sin(180-\theta)y_0 \\ -\sin(180-\theta) & \cos(180-\theta) & -\sin(180-\theta)x_0 + \cos(180-\theta)y_0 \\ 0 & 0 & 1 \end{bmatrix}$$

2. 1. Rotate $-\theta$ (clockwise)

$$\begin{bmatrix} \cos\theta & \sin\theta & 0 \\ -\sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{bmatrix} = T_{rot}$$



$$\begin{bmatrix} \cos\theta & \sin\theta & 0 \\ -\sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} x_q \\ y_q \\ 1 \end{bmatrix} = \begin{bmatrix} x_{q'} \\ y_{q'} \\ 1 \end{bmatrix}$$

$$= \begin{bmatrix} \cos\theta x_q + \sin\theta y_q \\ -\sin\theta x_q + \cos\theta y_q \\ 1 \end{bmatrix} = \begin{bmatrix} x_{q'} \\ y_{q'} \\ 1 \end{bmatrix}$$

$$x_{q'} = \cos\theta x_q + \sin\theta y_q$$

(1)

2. Translate $x_q' \rightarrow (x_0, y_0)$

$$T_{tr} = \begin{bmatrix} 1 & 0 & -(\cos\theta x_q + \sin\theta y_q) \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

3. Reflection $P \rightarrow P'$ around y-axis

$$T_{ref} = \begin{bmatrix} -1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

4. Translate back

$$-T_r = \begin{bmatrix} 1 & 0 & \cos\theta x_q + \sin\theta y_q \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

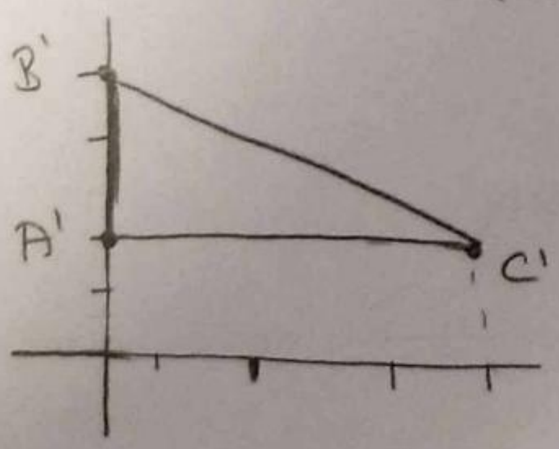
5. Rotate counter - clock wise $+\theta$

$$\begin{bmatrix} \cos\theta & -\sin\theta & 0 \\ \sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$M = T(\theta) \cdot -T_{tr} \cdot T_{ref} \cdot T_{tr} \cdot T(-\theta)$$

3. a) 1. $Rf(y,x)$

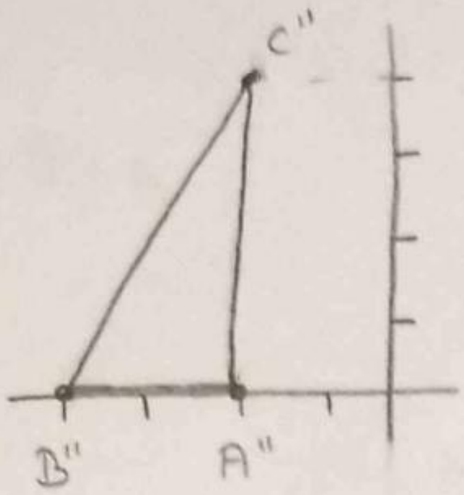
$A' = (0,2)$ $B' = (0,4)$ $C' = (4,2)$



cont.

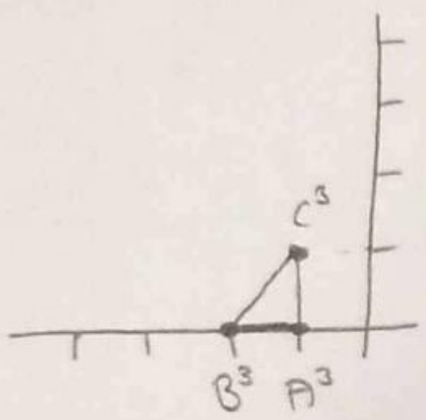
2. $R(90°) \rightarrow$ counter-clockwise rot $\Rightarrow (x,y) \rightarrow (-y,x)$

$A'' = (-2,0)$ $B'' = (-4,0)$ $C'' = (-2,4)$



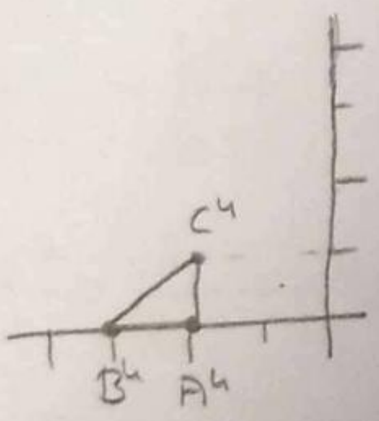
3. $S(1/2, 1/4) \rightarrow (x/h, y/k)$

$A^3 = (-1,0)$ $B^3 = (-2,0)$ $C^3 = (-1,1)$



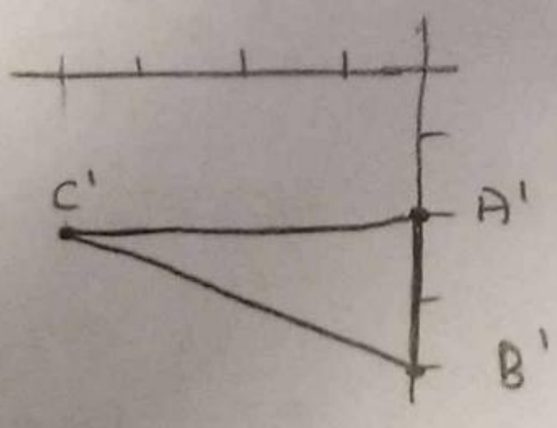
4. $T(-1,0)$

$A^4 = (-2,0)$ $B^4 = (-3,0)$ $C^4 = (-2,1)$



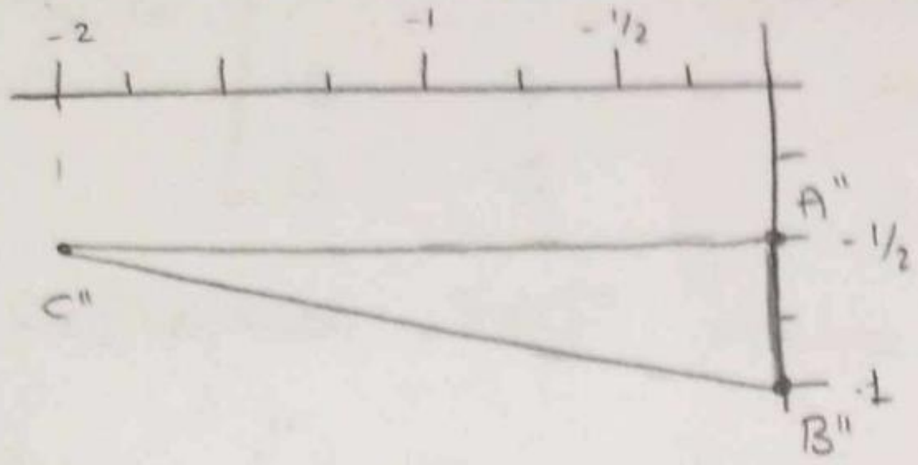
b.) 1. $Rf(y,-x) \rightarrow (-y,-x)$

$A' = (0,-2)$ $B' = (0,-4)$ $C' = (-4,-2)$



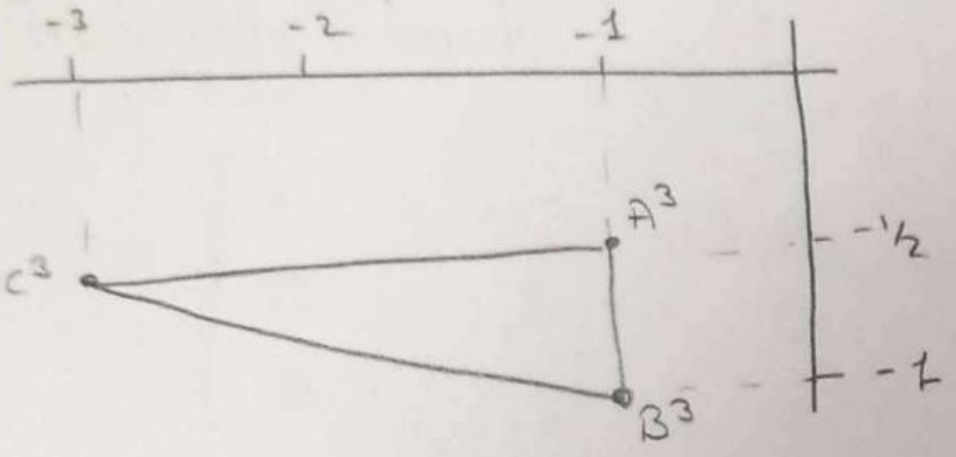
2. S (1/2 , 1/4)

A" = (0, -1/2) B" = (0, -1) C" = (-2, -1/2



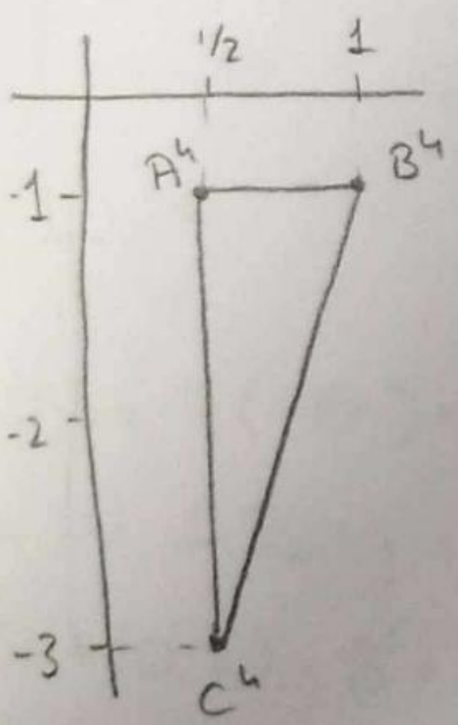
3. T (-1, 0)

A^3 = (-1, -1/2) B^3 = (-1, -1) C^3 = (-3, -1/2)



4. R(90) (x, y) -> (-y, x)

A^4 = (1/2, -1) B^4 = (1, -1) C^4 = (1/2, -3)



4. ViewPort 1

$$\frac{x_v - x_{v_{min}}}{x_{v_{max}} - x_{v_{min}}} = \frac{x_w - x_{w_{min}}}{x_{w_{max}} - x_{w_{min}}}$$

$x_{v_{min}} = 1 \quad x_{w_{max}} = 3$
$y_{v_{min}} = 2 \quad y_{w_{max}} = 6$

A(3,5)

$$\frac{x_v - 1}{3-1} = \frac{3-2}{6-2}$$

$$\frac{x_v - 1}{2} = \frac{1}{4} \Rightarrow x_v - 1 = 1/2 \Rightarrow x_v = 3/2$$

$$\frac{y_v - 3}{6-3} = \frac{5-2}{6-2}$$

$$\frac{y_v - 3}{3} = \frac{3}{4} \Rightarrow y_v - 3 = \frac{9}{4} \Rightarrow y_v = \frac{21}{4}$$

A(3,5) → (3/2, 21/4)

B(4, 5.5)

$$\frac{x_v - 1}{3-1} = \frac{4-2}{6-2}$$

$$\frac{x_v - 1}{2} = \frac{2}{4} \Rightarrow x_v - 1 = 1 \Rightarrow x_v = 2$$

$$\frac{y_v - 3}{6-3} = \frac{5.5 - 2}{6-2}$$

$$\frac{y_v - 3}{3} = \frac{3.5}{4} \Rightarrow \frac{y_v - 3}{3} = \frac{7}{8} \Rightarrow y_v - 3 = \frac{21}{8} \Rightarrow y_v = \frac{45}{8}$$

B(4, 5.5) → (2, 45/8)

C(5, 4)

$$\frac{x_v - 1}{3-1} = \frac{5-2}{6-2}$$

$$\frac{x_v - 1}{2} = \frac{3}{4} \Rightarrow x_v - 1 = \frac{3}{2} \Rightarrow x_v = \frac{5}{2}$$

$$\frac{y_v-3}{6-3}=\frac{4-2}{6-2}$$

$$\frac{y_v-3}{3}=\frac{2}{4}\Rightarrow\frac{y_v-3}{3}=\frac{1}{2}\Rightarrow y-3=\frac{3}{2}\Rightarrow y=\frac{9}{2}$$

$C(5,4)\rightarrow(5/2,\ 9/2)$

viewport 2

$x_{vmin}=4 \quad x_{vmax}=7$

$y_{vmin}=5 \quad y_{vmax}=7$

$A(3,5)$

$$\frac{x_v-4}{7-4}=\frac{3-2}{6-2}$$

$$\frac{x-4}{3}=\frac{1}{4}\Rightarrow x-4=\frac{3}{4}\Rightarrow x=\frac{19}{4}$$

$$\frac{y_v-5}{7-5}=\frac{5-2}{6-2}$$

$$\frac{y_v-5}{2}=\frac{3}{4}\Rightarrow y_v-5=\frac{3}{2}\Rightarrow y_v=\frac{13}{2}$$

$A(3,5)\rightarrow(19/4,\ 13/2)$

$B(4,5.5)$

$$\frac{x_v-4}{7-4}=\frac{4-2}{6-2}$$

$$\frac{x-4}{3}=\frac{2}{4}\Rightarrow x-4=\frac{3}{2}\Rightarrow x=\frac{11}{2}$$

$$\frac{y_v-5}{7-5}=\frac{5.5-2}{6-2}$$

$$\frac{y_v-5}{2}=\frac{3.5}{4}\Rightarrow y_v-5=\frac{7}{4}\Rightarrow y_v=\frac{27}{4}$$

C(5,4)

$\frac{x_v - 4}{7-4} = \frac{5-2}{6-2}$

$\frac{x_v - 4}{3} = \frac{3}{4} \Rightarrow x_v - 4 = \frac{9}{4} \Rightarrow x_v = \frac{25}{4}$

$\frac{y_v - 5}{7-5} = \frac{4-2}{6-2}$

$\frac{y_v - 5}{2} = \frac{2}{4} \Rightarrow y_v - 5 = 1 \Rightarrow y_v = 6$

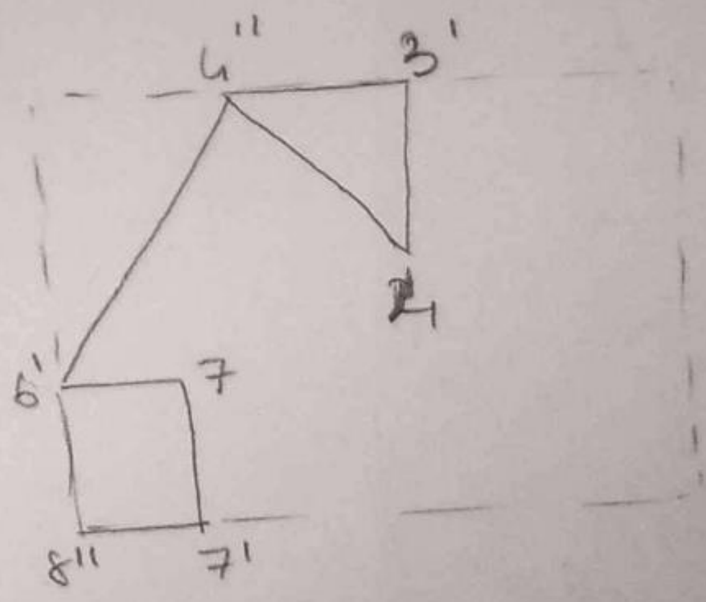
C(5,4) → (25/4, 6)

5. a.) Left Clipper → Right Clipper → Bottom Clipper

{1,2}	out-out	{ }	{2',3}	i-i	{3}	{2',3}	i-i	{3}
{2,3}	o-in	{2',3}	{3,4}	i-i	{4}	{3,4}	i-i	{4}
{3,4}	i-i	{4}	{4,4'}	i-i	{4}	{4,4'}	i-i	{4'}
{4,5}	i-o	{4'}	{4',6'}	i-i	{6'}	{4',6'}	i-i	{6''}
{5,6}	o-o	{ }	{6',7}	i-i	{7}	{6',7}	i-i	{7}
{6,7}	o-i	{6',7}	{7,8}	i-i	{8}	{7,8}	i-o	{7'}
{7,8}	i-i	{8}	{8,8'}	i-i	{8'}	{8,8'}	o-o	{ }
{8,1}	i-o	{8'}	{8',2}	i-i	{2'}	{8',2}	o-i	{8'',2'}

→ Top Clipper

{2',3}	o-o	{ }
{3,4}	o-i	{3'}
{4,4'}	i-o	{4''}
{4',6'}	o-i	{6'} → in these lines because
{6',7}	i-i	{7} algorithm only
{7,7'}	i-i	{7'} can put point
{7',8''}	i-i	{8''} only at intersection
{8'',2'}	i-o	{ } → these parts won't work ideally



b.) Polygone

{1,2}	o-o	{ }
{2,3}	o-o	{ }
{3,4}	o-i	{3'}
{4,5}	i-o	{4'}

→ Window

{4',3}	i-i	{4'}
{3',4}	i-i	{3'}
{4,4'}	i-i	{4}

→ Polygone

{4',5}	i-o	{ }
{5,6}	o-o	{ }
{6,7}	o-i	{6'}
{7,8}	i-o	{7'}

Window

→

{7',7''}	i-i	{7''}
{7'',6'}	i-i	{6'}
{6',7}	i-i	{7}
{7,7'}	i-i	{7'}

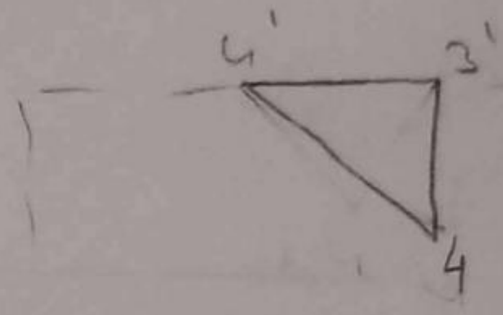
→ Polygone

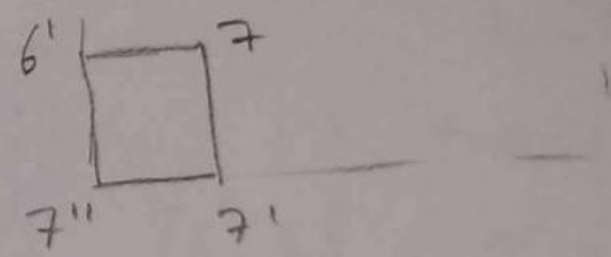
{7',8}	i-o	{ }
{8,1}	—	{ }

Main array

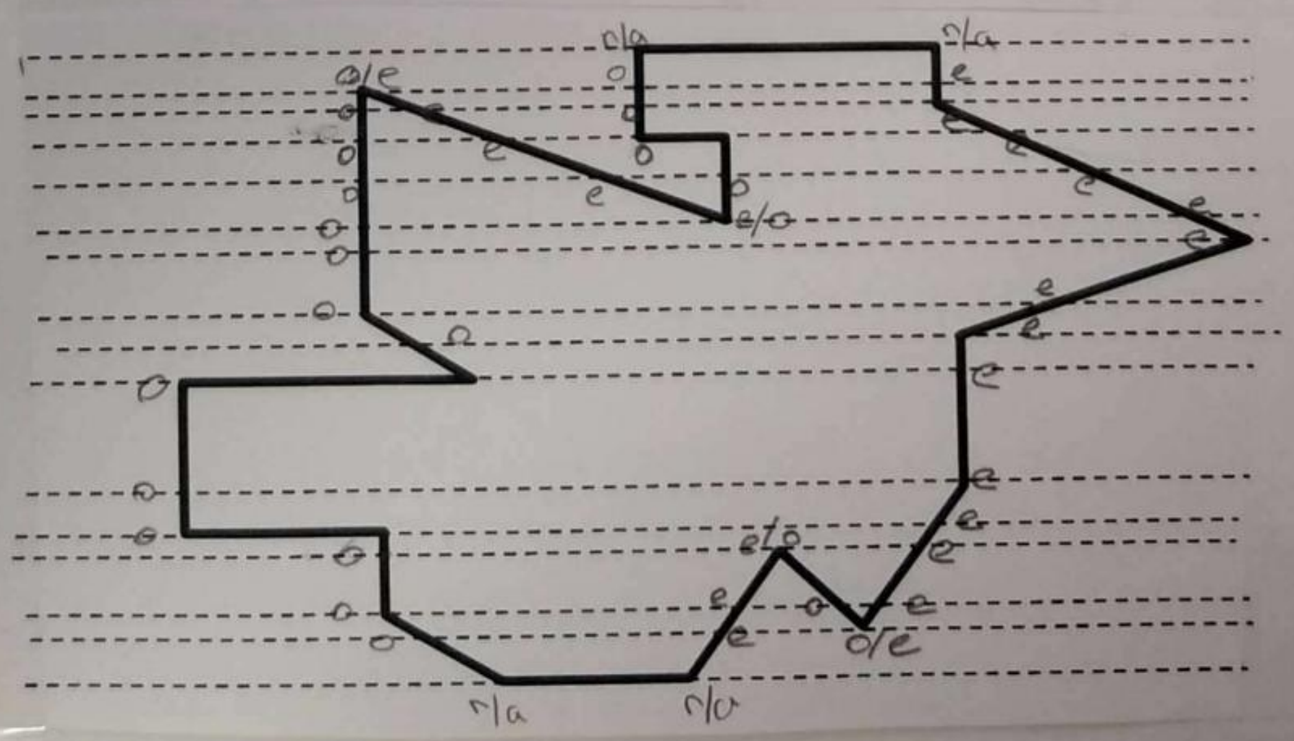
Main array

{4',3',4} {7'',6',7,7'}





6.



7.

Polygone 1

{1,2}	o-o {}
{2,3}	o-o {}
{3,4}	o-i {3',4}
{4,5}	i-o {4'}

P 2

{4',4"}	i-i {4"}
{4",6}	i-i {6}
{6,7}	i-o {6'}

P 1

P2

{6',6"}	i-i {6"}
{6",6³}	i-i {6³}
{6³,6⁴}	i-i {6⁴}
{6⁴,3'}	i-i {}

{3', 4, 4', 4", 6, 6', 6", 6³, 6⁴}

Turning vertexes = {4', 4", 6'}

↑ P1-P2 ↑ P2-P1 ↑ P1-P2