Computer Vision: Representation and Recognition
Assignment 2

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1 Question 1

1.1 question 1.1

The whole proof and details are written below.

Proof.

1. In the first place, we can obtain the rotation equation by using a given rotation matrix.

$$x^{'} = x\cos\theta - y\sin\theta$$
$$y^{'} = x\sin\theta + y\cos\theta$$

2. Next, we can get the gradient amplitude after the rotation.

$$G(x^{'}, y^{'}) = \sqrt{G_{x^{'}}^2 + G_{y^{'}}^2} \tag{1.1}$$

We can simplify the equation above.

$$\begin{aligned} G(x^{'}, y^{'})^2 =& \frac{\partial^2 f}{\partial^2 x^{'2}} + \frac{\partial^2 f}{\partial^2 y^{'2}} \\ =& \frac{\partial}{\partial x^{'}}(\frac{\partial f}{\partial x}\frac{\partial x}{\partial x^{'}} + \frac{\partial f}{\partial y}\frac{\partial y}{\partial x^{'}}) + \frac{\partial}{\partial y^{'}}(\frac{\partial f}{\partial x}\frac{\partial x}{\partial y^{'}} + \frac{\partial f}{\partial y}\frac{\partial y}{\partial y^{'}}) \end{aligned}$$

3. Finally, we can use the rotation equation to simplify the equation above.

$$\begin{aligned} G(x^{'}, y^{'})^2 =& \frac{\partial}{\partial x^{'}}(\frac{\partial f}{\partial x}\cos\theta + \frac{\partial f}{\partial y}\sin\theta) + \frac{\partial}{\partial y^{'}}(\frac{\partial f}{\partial x}\sin\theta + \frac{\partial f}{\partial y}\cos\theta) \\ =& \frac{\partial}{\partial x}\frac{\partial f}{\partial x} + \frac{\partial}{\partial y}\frac{\partial f}{\partial y} \\ =& \frac{\partial^2 f}{\partial^2 x^2} + \frac{\partial^2 f}{\partial^2 y^2} \\ =& G_x^2 + G_y^2 \end{aligned}$$

4. In conclusion, we can know that:

$$G(x^{'}, y^{'}) = G(x, y) \tag{1.2}$$

So that we can prove that the gradient amplitude of (x, y) is the same as that of $(x^{'}, y^{'})$.

□

1.2 question 1.2

The solution to the problem is to make **low threshold** smaller(lower) and make **high threshold** bigger(higher).

1. The reason why long edges are broken into short segments is that the **low threshold** is so high that some weak edge points, which are often near some strong edge points, are abandoned and removed. Therefore, the algorithm fails to connect the segments together. On the other hand, if we lower the value of **low threshold**, those weak points will be adopted and saved in the result.
2. The reason why some spurious edges appear is that the **high threshold** is so low that some points, which are not on the edge, will be considered as edge points. T herefore, the algorithm will mistake some points as edge points and connect them together to form an edge. On the other hand, if we make the **high threshold** higher, those fake edge points will be removed and abandoned.

2 Question 2

2.1 question 2.1

The 2nd derivative of 1-D Gaussian is written below. Also, the image of it is shown below.

$$g_\sigma^{''}(x) = -\frac{1}{\sqrt{2\pi}\sigma^3}(1 - \frac{x^2}{\sigma^2})\exp(-\frac{x^2}{2\sigma^2}) \tag{2.1}$$

Image. 2nd Derivative of 1-D Gaussian.

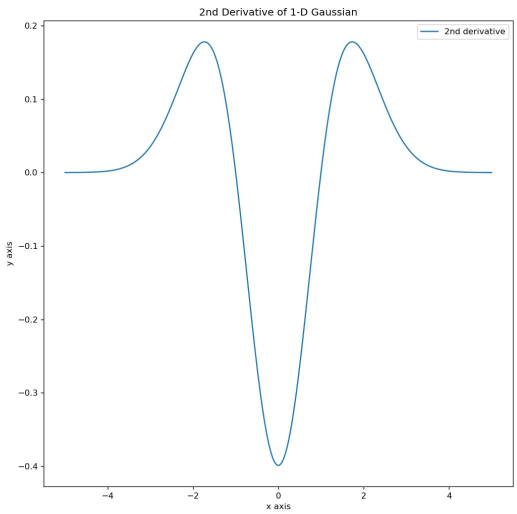

Figure 1: 2nd Derivative of 1-D Gaussian

2.2 question 2.2

The image of the difference of 1-D Gaussians (DoG) is shown below, including the comparison bewteen different value of k and also the 2nd derivative of 1-D Gaussian.

Image. DoG.

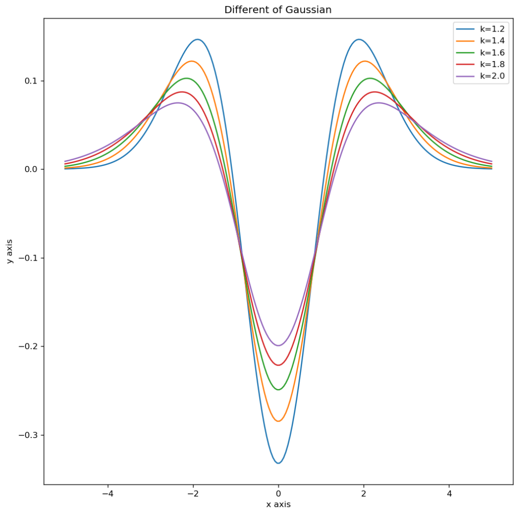

Figure 2: Difference of Gaussian

Image. Comparison between DoG and 2nd Derivative.

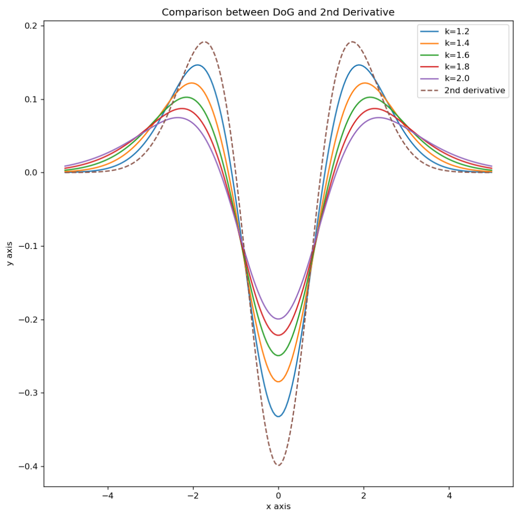

Figure 3: Comparison between DoG and 2nd Derivative

2.3 question 2.3

From the questions above we can know that a difference of Gaussian (DoG) has a pole at the edge point where the pixel changes obviously, leading to a DoG responding maximally.

3 Question 3

3.1 question 3.1

Image. Gaussian Pyramid.

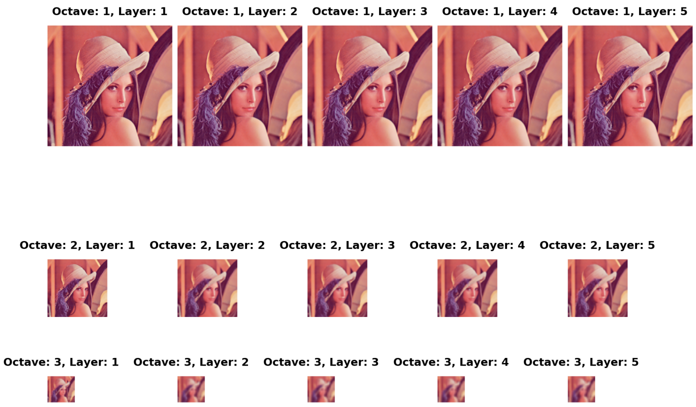

Figure 4: Gaussian Pyramid

Image. Downsampling.

Figure 5: Down Sampling Results

Image. Difference of Gaussian (DoG).

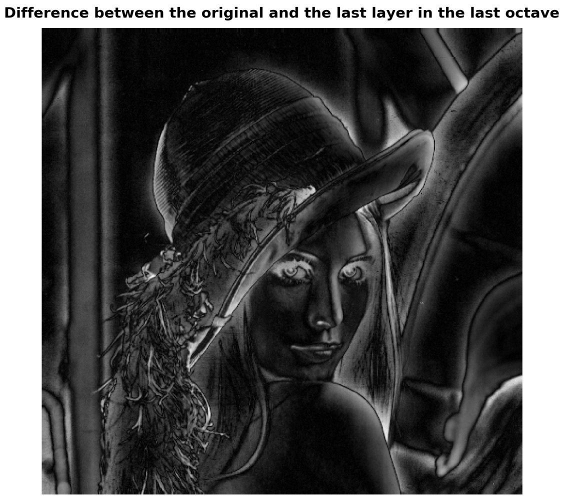

Figure 6: The Difference to the Original

3.2 question 3.2

Image. Edge Detection.

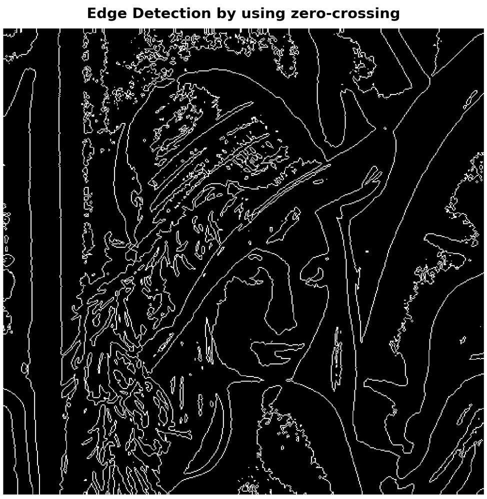

Figure 7: My Edge Detection

3.3 question 3.3

Image. Edge Detection by using Canny operator.

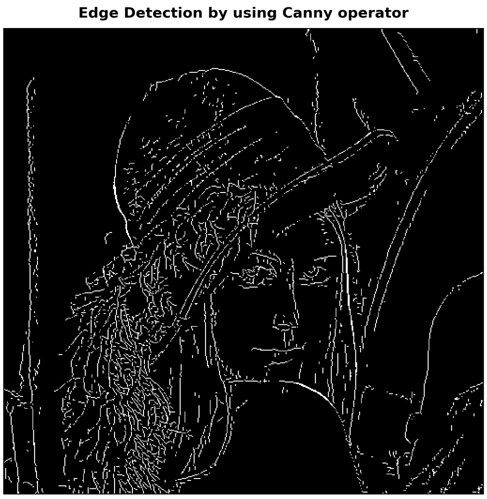

Figure 8: My Canny Edge Detection

Image. Comparison with other's results.

Figure 9: Other Results