PROJECT 1
Linear Programming

Submit your project through *WISEflow*. The submission deadline is Friday September 23rd, at 14:00 hrs. The project can be done individually or in a group of at most 2 students. No cooperation between people who are not submitting this project as a group is allowed. It is possible to change groups throughout the semester and it is also possible to do some project(s) alone and other project(s) in a group. Provide **all your AMPL files** (model code, data, running commands, solution file, etc.) compressed in a single file (.zip). Include all files needed to run all parts of the project, even if from one to another task the changes are just marginal (we need all files to be able to run without modifying what you submitted). In addition, provide a written report as a pdf file with your model formulations and the answers to the questions required in each part. The formulation of your models can be typed in a text editor (e.g. Word, LaTeX), written by hand and scanned, or copied directly as text or screenshot from the AMPL code files when it applies (please just be careful the presentation must be clear enough for a reader). In the written report, it is fine that when there is just a marginal change from one task to another, in the latter you include just the modified part of the formulation (e.g., in task 2 you just defined a new variable or modified one constraint of the model you formulated in task 1, then it is fine that you included the full model formulation in task 1 and only the new variable definition and new constraint that you modified in task 2). Recall using the solver *cplex* to solve the models. Provide a short description (no more than two sentences, e.g. "demand fulfillment") for every objective function and constraint in your formulations. Expected (not required) length of your report: Parts A and D between one and two pages each; Parts B and C between 2 and 3 pages each. All model formulations in this project must be linear.

Part A

A fishing company has three production facilities along the Norwegian coast (F1, F2, and F3). Each facility emits two types of pollutants (P1 and P2) into the sea. The government is promoting the use of a new technology for processing fish products, in order to reduce the pollution in the sea. ~~It costs $ 30 to process a ton of fish with this technology at Facility 1~~, and each ton processed reduces the amount of pollutant ~~1 by 0.10 ton and the amount of pollutant 2 by 0.45 ton. It costs $ 20 to process a ton of fish with the new technology at Facility 2, and each ton processed will reduce the amount of pollutant 1 by 0.20 ton and the amount of pollutant 2 by 0.25 ton. It costs $ 40 to process a ton of fish with the new technology at Facility 3, and each ton processed will reduce the amount of pollutant 1 by 0.40 ton and the amount of pollutant 2 by 0.30 ton.~~ The government sets as targets to reduce the amount of pollutant 1 in the sea by at least 25 tons and the amount of pollutant 2 in the sea by at least 35 tons, and the company needs to conform to these conditions by incorporating the new technology to process part of their fish products. Assume the amount of fish processed by the company is large enough to satisfy the requirement of the government (that is, there is no upper limit on the amount of fish to be processed using the new technology).

1. Formulate a linear programming model for deciding the amount of fish to be processed with the new technology, such that the optimal solution minimizes the cost of respecting the targets set by the government. Implement the model in AMPL and solve it. What is the optimal solution? What is the optimal objective value?

2. Briefly (no more than a couple of paragraphs) discuss how the constraints are satisfied in the optimal solution.

3. How sensitive is the optimal cost to the targets of the government? In particular, if the target of pollutant 2 decreases to 25 tons, can you conclude what is the effect in the optimal cost without running the model again? And if the target of pollutant 2 increases to 70 tons, can you conclude what is the effect in the optimal cost without running the model again?

4. Investigate the effect of changes of the cost coefficients on the optimal solution. How sensitive are the decisions to the accuracy in these coefficients? In particular, if the cost of processing at Facility 1 would

be reduced from $ 30 to $ 20: Would your optimal decisions remain the same? Would your optimal cost remain the same?

Part B

Consider an oil refinery company producer of 5 types of gasoline (G1, G2, G3, G4 and G5). Each gasoline is produced by blending 4 types of crude oil (C1, C2, C3 and C4). T~~he maximum weekly purchases of crude oil are 25,000 barrels of C1, 15,000 barrels of C2, 1800 barrels of C3 and 3200 barrels of C4~~. ~~The purchase prices per barrel are $45 for C1, $40 for C2, $55 for C3 and $50 for C4. The production cost per barrel of gasolines are $6 for G1, $6 for G2, $5 for G3, $7 for G4 and $8 for G5.~~

The production processes require the supervision of specialised staff. This staff is composed by 12 full time and 2 part time employees. Each full time employee works 40 hours per week and each part time employee works 20 hours per week. T~~he required time of supervision is 0.01 hr per barrel of G1, 0.01 hr per barrel of G2, 0.01 hr per barrel of G3, 0.02 hr per barrel of G4 and 0.02 hr per barrel of G5~~.

~~The company sells the final products in three different markets (M1, M2, M3). There are minimum demand quantities that must be satisfied for each product in each market, as shown in Table 1. Any production above these minimum quantities can also be sold in any of the markets.~~

	~~G1~~	~~G2~~	~~G3~~	~~G4~~	~~G5~~
~~M1~~	~~3000~~	~~3000~~	~~1500~~	~~2000~~	~~1000~~
~~M2~~	~~2500~~	~~2000~~	~~1000~~	~~2000~~	~~1000~~
M3	5000	4250	3000	4000	2000

Table 1: Minimum demand quantities (in barrels) that must be satisfied for each gasoline in each market.

~~The sales prices per barrel are shown in Table 2.~~

	~~G1~~	~~G2~~	~~G3~~	~~G4~~	~~G5~~
~~M1~~	~~75~~	~~75~~	~~75~~	~~85~~	~~90~~
M2	75	80	85	90	92
M3	80	85	90	95	95

Table 2: Sale price (in $) per barrel of each gasoline in each market.

~~The crude oils differ in quality. The gasoline quality is dependent on the proportion of each crude oil used for blending (assume linearity of yield). The quality of crude oils and gasolines is measured by two characteristics: average octane rating, and sulfur content. Table 3 shows the quality of the crude oils and Table 4 shows the minimum average octane rating and maximum sulfur content required for each type of gasoline.~~

~~Crude~~	~~Octane rating~~	~~Sulfur content~~
~~C1~~	~~94~~	~~1.0%~~
~~C2~~	~~90~~	~~2.0%~~
~~C3~~	~~98~~	~~0.1%~~
~~C4~~	~~96~~	~~0.3%~~

Table 3: Quality of crude oils in octane rating and sulfur content.

1. Formulate a linear programming model to determine a weekly blending plan such that the refinery maximizes profit. Implement the model in AMPL and solve it. What is the optimal plan and how much profit it provides?

2. Modify your implementation for each of the following four scenarios (each scenario is independent from each other). In your report, explain which modifications you have introduced. How do your new results (decisions and objective value) compare to the results obtained in question 1? Briefly explain why any differences or similarities occur.
   Scenario 1: The price of gasoline G1 in market M1 increases to $78 and the minimum demand quantity of G1 in M1 decreases to 2700 barrels.
   Scenario 2: The minimum demand quantity for gasoline G3 in market M3 increases to 4000 barrels, for gasoline G1 in market M1 increases to 3100 barrels and for gasoline G1 in market M2 decreases to 2400 barrels.
   Scenario 3: The maximum number of barrels available for purchases decreases to 1350 for crude C3 and to 2050 for crude C4.

Gasoline	Octane rating	Sulfur content
G1	88	2.0%
G2	90	1.5%
G3	92	1.2%
G4	94	0.8%
G5	96	0.5%

Table 4: Minimum average octane rating and maximum sulfur content of the different types of gasoline.

Scenario 4: A new supplying source of crude C2 is available but at higher cost. This alternative source offers up to 1900 barrels of C2 at $41 per barrel. How many barrels would you purchase from this source?

Part C

The *FreshFruits Company* obtains apples from two regions. In Region R1 the company can obtain as many as 150,000 kg per week, and in Region R2 it can obtain as many as 200,000 kg per week. It is possible to ship the apples directly from these two regions to 10 markets denoted as K1, K2,..., K10. It is also possible for FreshFruits to transport apples from regions to ports P1 or P2, and then ship them by vessel from these ports to the 10 markets. The weekly demands of the markets are shown in Table 5 and the costs of shipping 1000 kg between two points are shown in Table 6.

K1	K2	K3	K4	K5	K6	K7	K8	K9	K10
24	30	40	35	15	52	42	12	20	40

Table 5: Demand of apples by each market expressed in 1000 kg.

$	P1	P2	K1	K2	K3	K4	K5	K6	K7	K8	K9	K10
R1	10	13	78	108	216	120	210	144	252	252	126	30
R2	15	12	108	96	108	36	108	180	144	150	162	72
P1	-	-	12	12	28	56	56	28	120	116	24	32
P2	-	-	112	68	20	32	28	80	36	28	128	112

Table 6: Costs of shipping 1000 kg of apples between regions, ports and markets.

1. Formulate a linear programming model to minimize the weekly transport costs in meeting demands of all markets. Implement the model in AMPL and solve it. What is the optimal shipping plan?

The following three sections introduce some modifications. Each section is independent from each other.

2. The port authorities are evaluating a renovation project in port P1. If the project is carried out, it will be impossible for FreshFruits to use this port. Determine what would be the optimal shipping plan of the company in this new scenario. What is the impact in costs for the company?

3. Due to adverse weather conditions, the weekly supply of apples has decreased to 125,000 kg in region R1 and to 175,000 kg in region R2. Run your model using these new data, what do you obtain as result? Because of the decrease of supply, a manager of the company has proposed to define a maximum level of unsatisfied demand for each market, taking into account the relevance that each market represents for FreshFruits. The levels are shown in Table 7 (for example, at most 10% of the demand from market K1 can remain unsatisfied). The manager asks you to formulate and solve a model for this new scenario, in order to minimize weekly cost of transport while satisfying these maximum levels of unsatisfied demand. Check your result and briefly discuss the proposal of the manager.

K1	K2	K3	K4	K5	K6	K7	K8	K9	K10
10 %	10 %	5 %	10 %	20 %	5 %	5 %	20 %	20 %	5 %

Table 7: Maximum percentage of unsatisfied demand for each market.

Part D

The article "Potential savings and cost allocations for forest fuel transportation in Sweden: A country-wide study" (available on a link in the *Complementary readings* folder in *Canvas*), utilizes several models for the assessment of the transportation of forest fuels in Sweden. In what follows, we focus on the linear programming model outlined in the mathematical expressions (1) – (5) on page 357 of the article. We will assume that the set of time periods relevant for the decisions involved in the problem is defined as $T = \{1, 2, \dots, 12\}$, where each element of the set represents a month. Note that in particular for y_{ikt} we need an initial value to represent the inventory available right before the first month in the time horizon. Assume that such starting inventory quantity is $y_{i,k,0} = 100$ units for all $i \in I, k \in K$. Also, assume that the cost coefficient c_{ijkt} is positive for all $i \in I, j \in J, k \in K, t \in T$. Address the following tasks, clearly justifying your answers.

1. a) Could this model be infeasible? If yes, what is (are) the possible reason(s) for that? If not, why?
   b) Could this model be unbounded? If yes, what is (are) the possible reason(s) for that? If not, why?

   In tasks 2 and 3 below, assume that the model has been solved and the optimal objective value is 234,560.

2. Suppose that $c_{ijkt} = 1$ for all $i \in I, j \in J, k \in K, t \in T$. Without having access to the detailed data, but only to such information and the optimal objective value given above, address the tasks a) to d) below.
   a) Can you conclude what is the total demand quantity (aggregating all demand points, assortment groups, and time periods)?
   b) Can you conclude what is the total demand quantity (aggregating all demand points and assortment groups) specifically for period 6?
   c) Can you conclude what is the total supply availability (aggregating all supply points, assortments, and time periods)?
   d) Can you conclude what is the total supply availability (aggregating all supply points and assortments) specifically for period 6?

3. Suppose that a new condition is required by the transport operators, for each of the time periods after the first month: for all supply point $i \in I$, the total flow exiting supply point i in a time period cannot be higher by 10% neither lower by 10% than the total flow exiting supply point i in the previous time period. How would you modify the original linear programming model to capture this new condition, while keeping its linearity? What are the possible consequences that this modification could imply on the results?