COMP2005

Compression

Image Compression

- Types of redundancy:
  - Coding redundancy
  - Spatial redundancy
  - Psychovisual redundancy
- Structure of compression systems
- Components and complete schemes: Huffman coding, GIF, JPEG

?

Why?

- Compression affects image quality
- Need to understand/choose methods/parameters
- New methods are based on core concepts

Coding Redundancy

- The grey level histogram of an image gives the probability (frequency) of occurrence of grey level r_k

$$p(r_k) = \frac{n_k}{n} \qquad k = 0,2,\ldots,L-1$$

- If the number of bits used to represent each value of r_k is $l(r_k)$, the **average number of bits required to represent a pixel** is

$$L_{avg} = \sum_{k=0}^{L-1} l(r_k)p(r_k)$$

- To code an MxN image requires MNL_{avg} bits

Coding Redundancy

- If an m-bit natural binary code is used to represent grey level then
  - all pixels take the same amount of space,
  - $P(r_k)$ values sum to 1, so

$$L_{avg} = \sum_{k=0}^{L-1} l(r_k)p(r_k) = \sum_{k=0}^{L-1} mp(r_k) = m$$

  - and an image occupies MNm bits
- But some pixel values are more common than others…..

Coding Redundancy : Variable Length Encoding

- Assigning fewer bits to the more probable grey levels than to less probable ones can achieve data compression, e.g:

r_k	$p_r(r_k)$	Code 1	$l_1(r_k)$	Code 2	$l_2(r_k)$
$r_{87} = 87$	0.25	01010111	8	01	2
$r_{128} = 128$	0.47	10000000	8	1	1
$r_{186} = 186$	0.25	11000100	8	000	3
$r_{255} = 255$	0.03	11111111	8	001	3
r_k for $k \neq 87, 128, 186, 255$	0	—	8	—	0

- Build a codebook, replace ‘true’ pixel values with code
- Lossless: the process can be reversed by inverting the codebook

Huffman Coding : Entropy

- The idea: associate information with probability
- A random event *E* with probability *P*(*E*) contains:

$$I(E) = log(\frac{1}{P(E)}) = -log(P(E))$$

units of information

- **Suppose that grey level values are generated by a random variable, then r_k contains:**

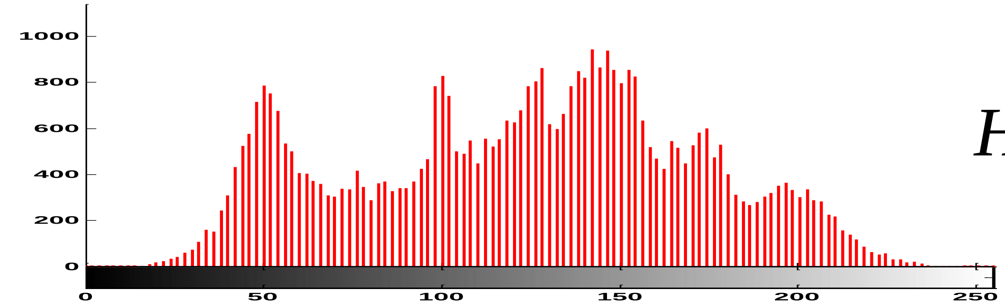
$$I(r_k) = -log(P(r_k))$$

Note: I(E)=0 when P(E)=1

units of information

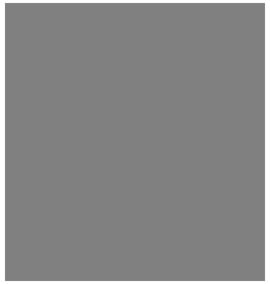
Entropy

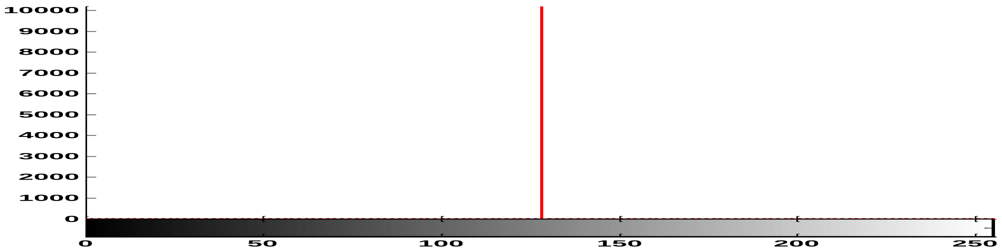
- Entropy is the average information content of an image, a measure of histogram dispersion



$$H = -\sum_{k=0}^{L-1} p(r_k)\log_2 p(r_k)$$

entropy=7.4635





entropy=0

- **Can’t compress to less than H bits/pixel without losing information**

Huffman Coding

- Compute probabilities of each symbol by histogramming the source
- Process probabilities to pre-compute codebook: code(i)
  - codebook is static (fixed)
- Encode source symbol-by-symbol:
  symbol(i) -> code(i)
- Transmit coded signal and codebook
- The need to pre-process (histogram) the source before encoding begins is a disadvantage

Huffman Coding

- Builds a binary tree in which symbols to be coded are nodes
- The algorithm:
  - Create a list of nodes, one per for symbol, sorted in order of symbol frequency (or probability)
  - REPEAT (until only one node left)
    - Pick the two nodes with the lowest frequencies/probabilities and create a parent of them
    - **Randomly** assign the codes 0,1 to the two new branches of the tree and delete the children from the list
    - Assign the sum of the children's probabilities to their parent and insert it in the list
- Path from root to node gives code for corresponding symbol

Huffman Coding

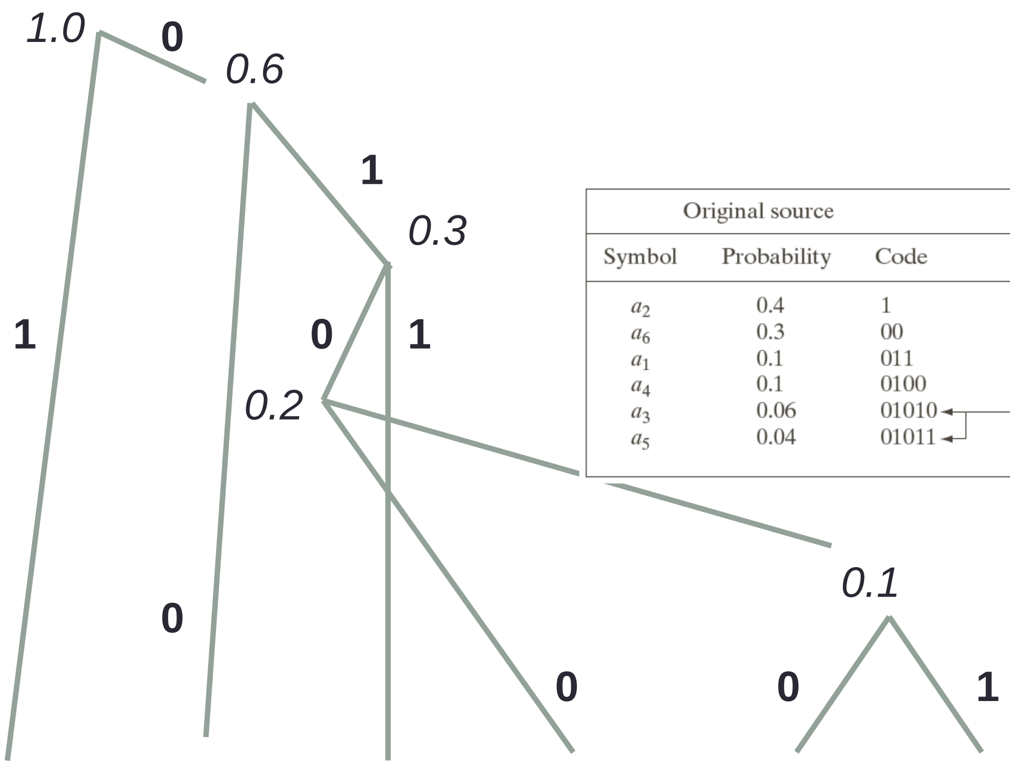
Original source		Source reduction			
Symbol	Probability	1	2	3	4
a_2	0.4	0.4	0.4	0.4	0.6
a_6	0.3	0.3	0.3	0.3	0.4
a_1	0.1	0.1	0.2	0.3	
a_4	0.1	0.1	0.1		
a_3	0.06	0.1			
a_5	0.04				

- Create parent node of a_3 and a_5

- Create parent of new node and a_4

- New parent is higher up list (0.2) than a_1 (0.1)

Huffman Coding



Original source			Source reduction							
Symbol	Probability	Code	1		2		3		4	
a_2	0.4	1	0.4	1	0.4	1	0.4	1	0.6	0
a_6	0.3	00	0.3	00	0.3	00	0.3	00	0.4	1
a_1	0.1	011	0.1	011	0.2	010	0.3	01		
a_4	0.1	0100	0.1	0100	0.1	011				
a_3	0.06	01010	0.1	0101						
a_5	0.04	01011								

$Prob_i$	0.4	0.3	0.1	0.1	0.06	0.04
$Symbol_i$	a_2	a_6	a_1	a_4	a_3	a_5
$Code_i$	**1**	**00**	**011**	**0100**	**01010**	**01011**

Huffman Coding

- The algorithm systematically places nodes representing high probability symbols further up the tree: their paths (and so codes) are shorter
- No code is a prefix to any other – don't need to mark boundaries between codes
  - e.g. 01101010 must be a_1a_3
- In this example
  - Average length of the code is 2.2.bits/symbol
  - The entropy of the source is 2.14 bits/symbol
- In general
  - Break image into small (e.g. 8 x 8) blocks
  - Each block is a symbol to be encoded

A Huffman Code Example

From a past exam paper:

An image has the following normalized histogram. Derive a Huffman code for each pixel value, showing how you obtained your code

10 mins

Pixel Value	Normalised Frequency
0	0.1
1	0.1
2	0.15
3	0.35
4	0.2
5	0
6	0.05
7	0.05

A Huffman Code Example

Pixel Value	Normalised Frequency
0	0.1
1	0.1
2	0.15
3	0.35
4	0.2
5	0
6	0.05
7	0.05

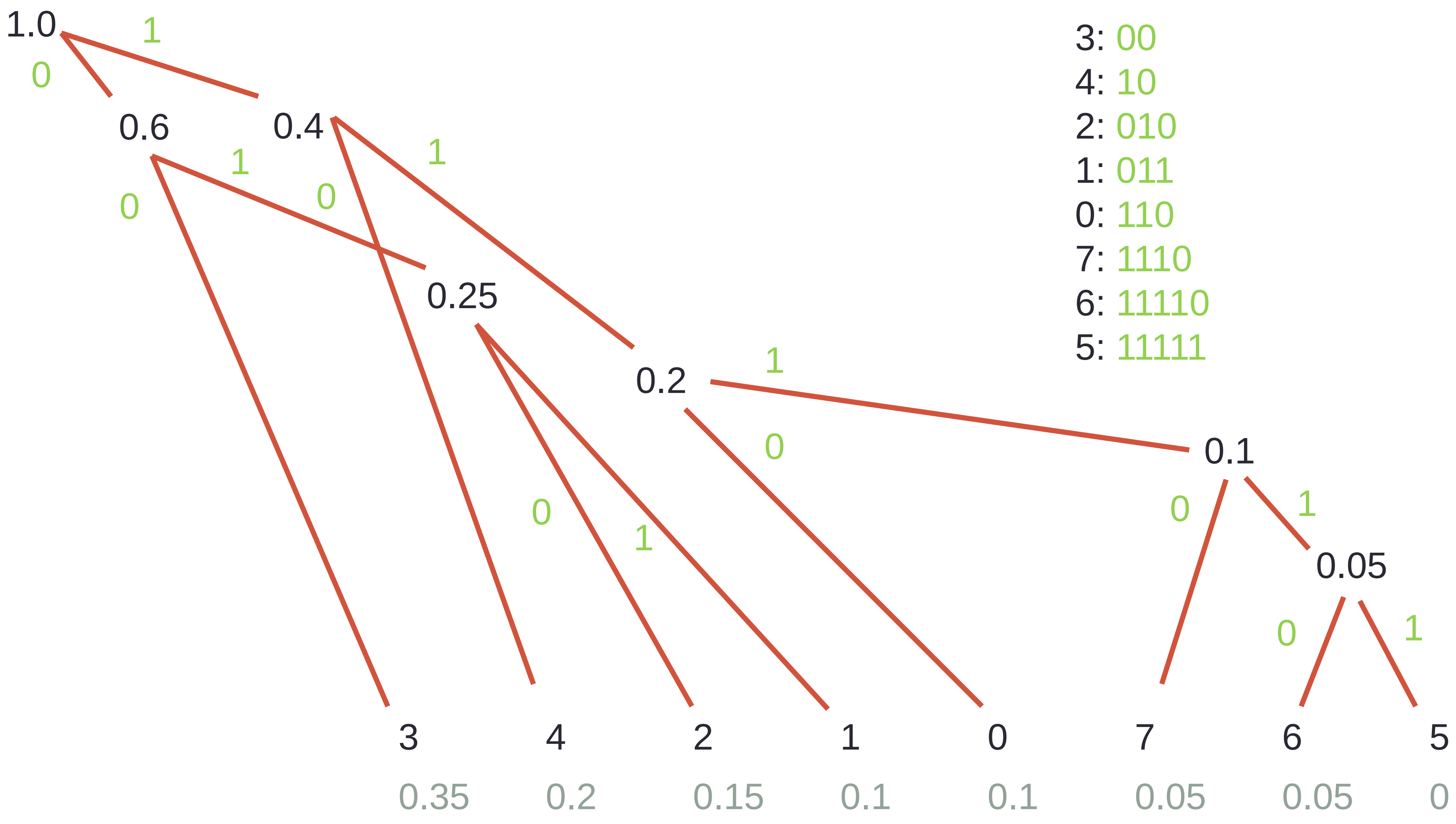
Sort →

Pixel Value	Normalised Frequency
3	0.35
4	0.2
2	0.15
1	0.1
0	0.1
7	0.05
6	0.05
5	0

A Huffman Code Example: Table View

Pixel Value	Normalised Frequency							
3	0.35	0.35	0.35	0.35	0.35	0.4	0.6	1.0
4	0.2	0.2	0.2	0.2	0.25	0.35	0.4	
2	0.15	0.15	0.15	0.2	0.2	0.25		
1	0.1	0.1	0.1	0.15	0.2			
0	0.1	0.1	0.1	0.1				
7	0.05	0.05	0.1					
6	0.05	0.05						
5	0							

A Huffman Code Example: Tree View



3: 00
4: 10
2: 010
1: 011
0: 110
7: 1110
6: 11110
5: 11111

A Huffman Code Example: The Code

		Code length	Normalised Freq,
3: 00	0: 110	3	0.1
4: 10	1: 011	3	0.1
2: 010	2: 010	3	0.15
1: 011 →	3: 00	2	0.35
0: 110	4: 10	2	0.2
7: 1110	5: 11111	5	0
6: 11110	6: 11110	5	0.05
5: 11111	7: 1110	4	0.05

Mean bits/pixel L_{avg} = 0.3 +0.3 +0.45 +0.70 + 0.4 + 0 + 0.25 + 0.2 = 2.6

$$L_{avg} = \sum_{k=0}^{L-1} l(r_k)p(r_k)$$

Without compression L_{avg} = 3

Compression ratio = 2.6/3 = 0.86

$l(r_k)$: Number of bits used to represent each value of r_k is

L_{avg} : **average number of bits required to represent a pixel** is

Spatial Redundancy

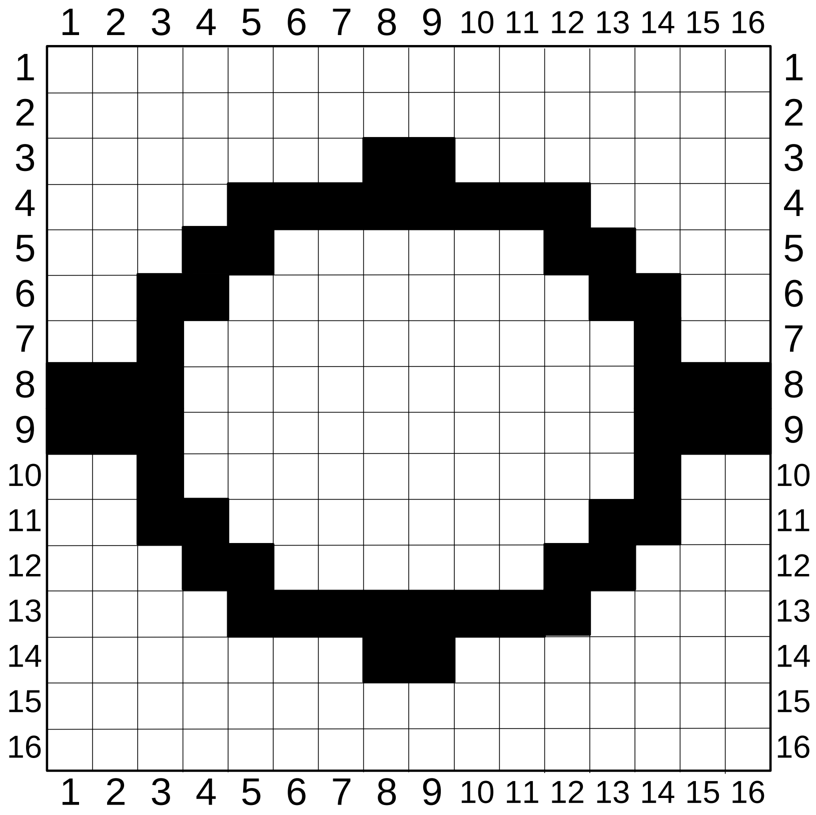
- Sometimes called *Interpixel Redundancy*
- Neighbouring pixels often have similar values
- Compression based on spatial redundancy involves some element of **pixel grouping, or transformation**
- Simplest is ***Run-Length Encoding***
  - maps the pixels along each scan line into a sequence of pairs $(g_1, r_1), (g_2, r_2), \ldots,$
  - where g_i is the ith grey level, r_i is the run length of ith run

A Binary Example

Row 1: (0, 16)
Row 2: (0, 16)
Row 3: (0, 7) (1, 2) (0, 7)
Row 4: (0, 4), (1, 8) (0, 4)
Row 5: (0, 3) (1, 2) (0, 6) (1, 3) (0, 2)
Row 6: (0,2) (1, 2) (0,8) (1, 2) (0, 2)
Row 7: (0, 2) (1,1) (0, 10) (1,1) (0, 2)
Row 8: (1, 3) (0, 10) (1,3)
Row 9: (1, 3) (0, 10) (1, 3)
Row 10: (0,2) (1, 1) (0,10) (1, 1) (0, 2)
Row 11: (0, 2) (1, 2) (0, 8) (1, 2) (0, 2)
Row 12: (0, 3) (1, 2) (0, 6) (1, 3) (0, 2)
Row 13: (0, 4) (1,8) (0, 4)
Row 14: (0, 7) (1, 2) (0, 7)
Row 15: (0, 16)
Row 16: (0, 16)

encode ←

decode →



Psychovisual Redundancy – e.g. GIF compression

- Some grey level and colour differences are imperceptible; goal is to compress without noticeable change to the image

256 gray levels

16 gray levels

16 gray levels

A simple method: add a small random number to each pixel before quantization

GIF : Vector Quantisation

- Palettised images (gif)
  - Map vector values (R,G,B) onto scalar values
  - Multiple vectors map to each scalar

Vector quantisation

True colour R,G,B
8 bits each
1677216 possible colours

gif
8 bits per pixel
256 possible colours

Paletised Images

(r, g, b)

For each pixel in the original image

Find the closest colour in the Colour Table

r_0	g_0	b_0
r_1	g_1	b_1
$\vdots$		
r_{255}	g_{255}	b_{255}

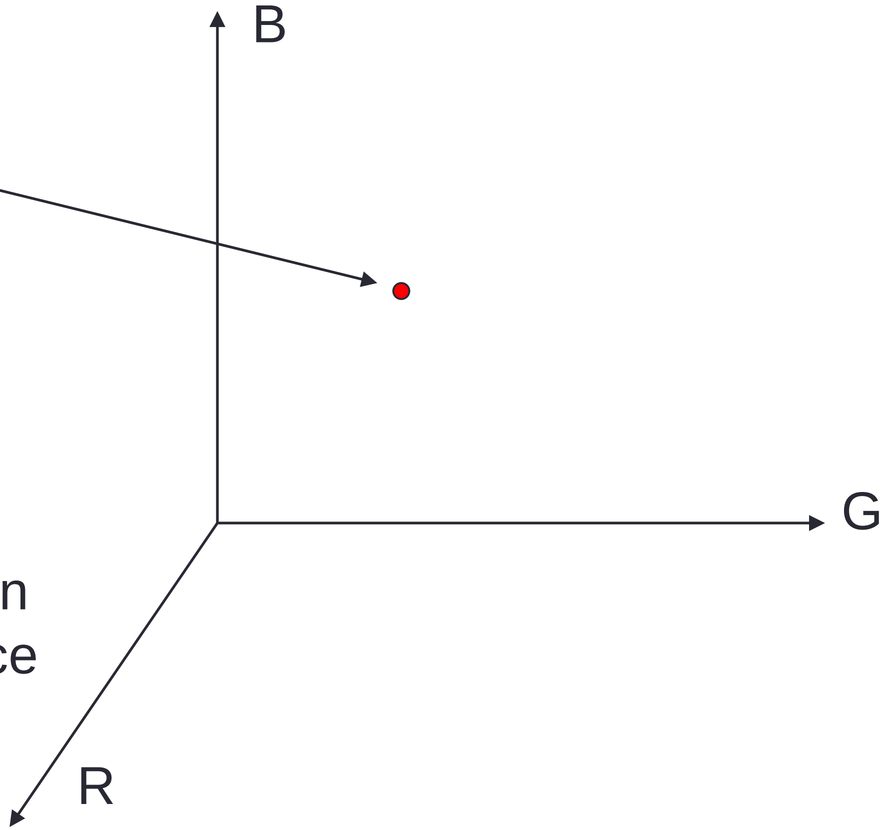
Colour Table

Record the index of that colour (for storage or transmission)

To reconstruct the image, place the indexed colour from the Colour Table at the corresponding spatial location

Building a Palette

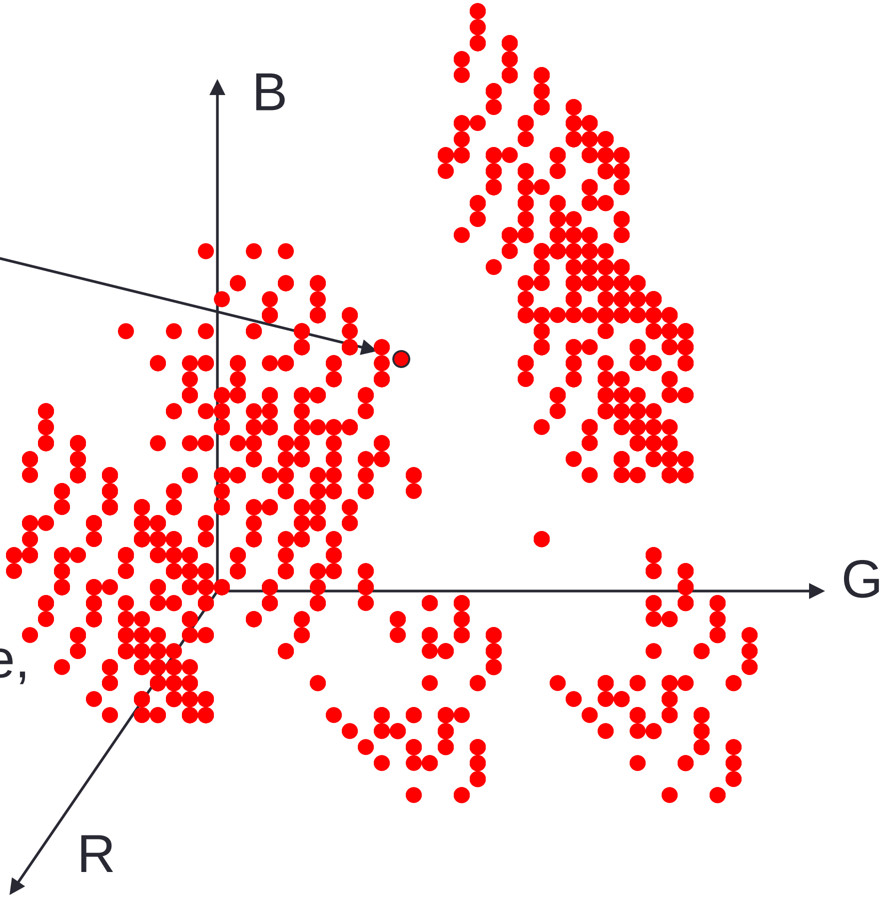
(r, g, b)



A pixel corresponds to a point in the 3 dimensional R, G, B space

Building a Palette

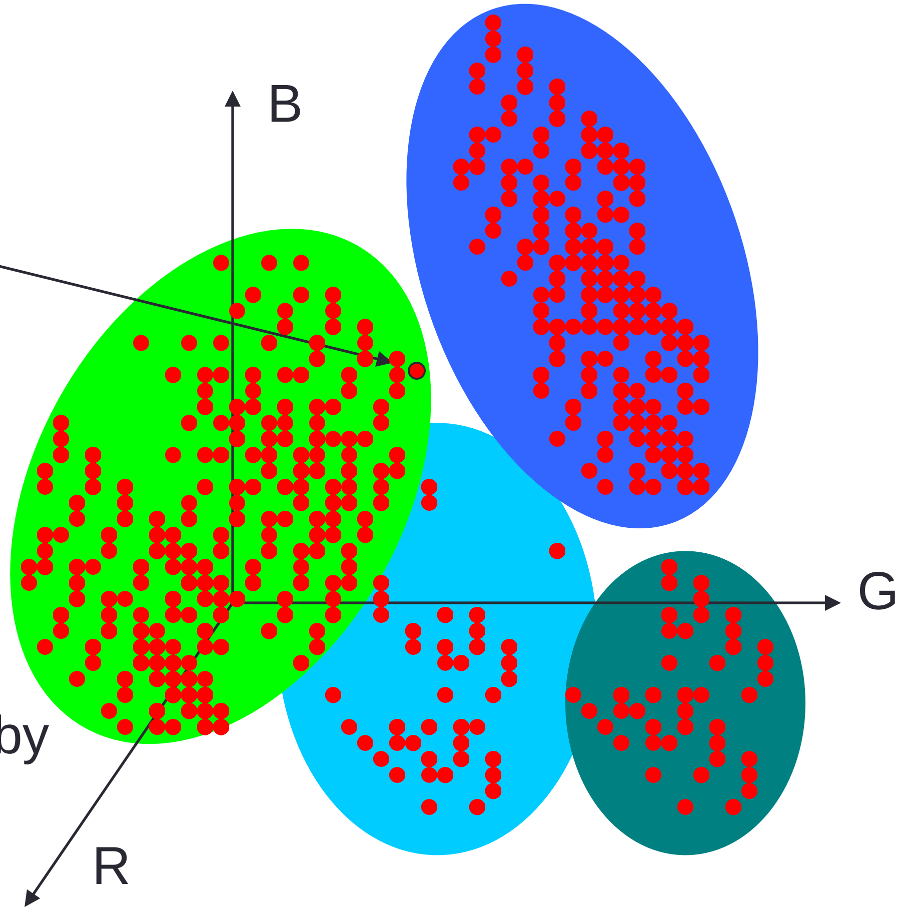
(r, g, b)



Map all pixels into R,G,B space,
Clouds of pixels are formed

Building a Palette

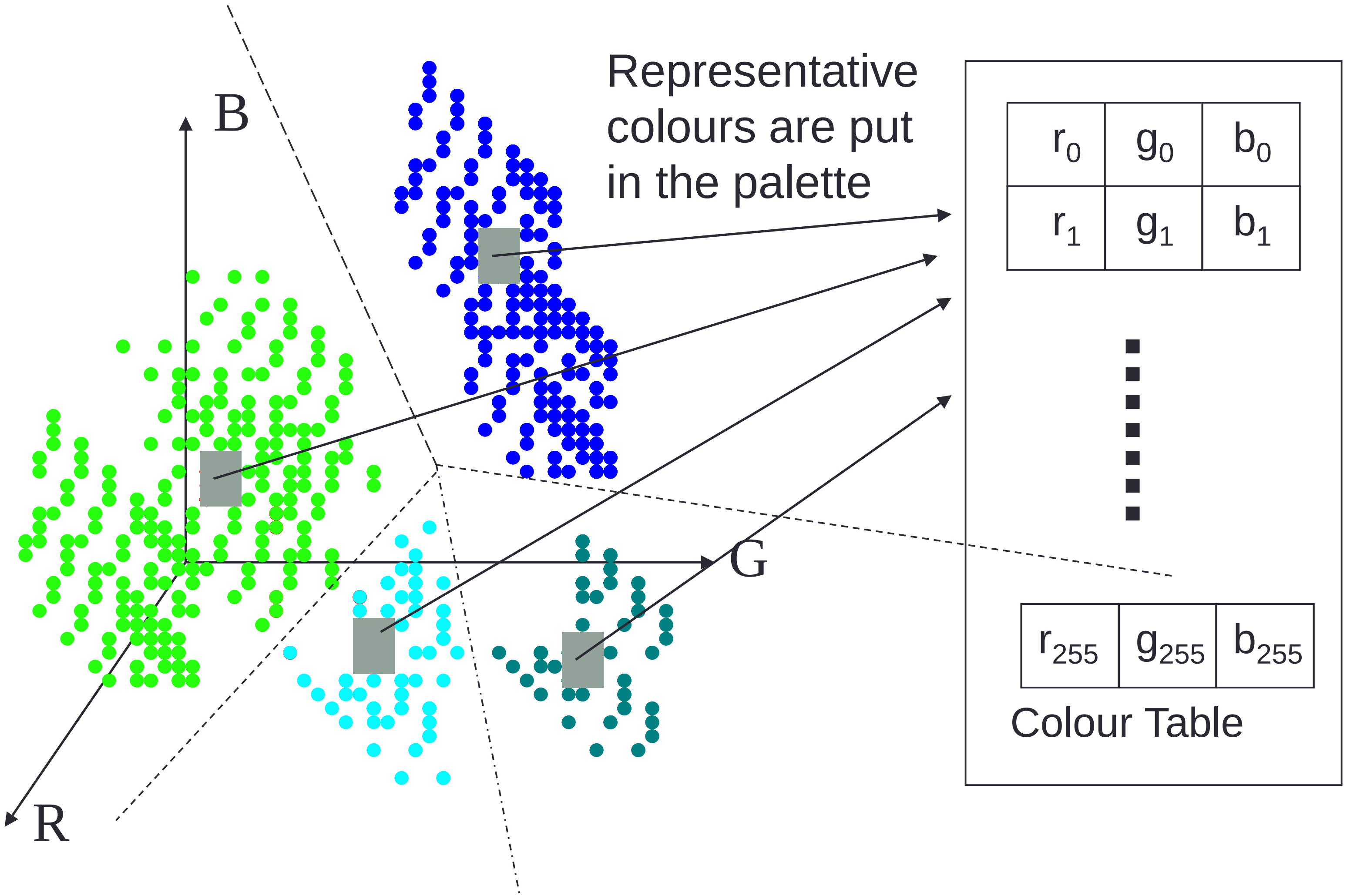
(r, g, b)



Group pixels that are close to each other, and replace them by one single colour

Close to each other means of a similar colour

Building a Palette



Building a Palette

- Many clustering algorithms exist
  - supervised
  - unsupervised
- We know how many clusters we need: one per palette entry
- We need clusters that are spread across the colour space
- A supervised method….

- K-Means Clustering
  - Start with estimates of the mean of each cluster

    $$\mu_1, \mu_2, \ldots, \mu_k$$

  - Assign each point, p, to the cluster where

    $$|p - \mu_i|$$

    is smallest
  - Recompute the means
  - Repeat until no changes are made to the clusters

Evaluating Compression

- **Fidelity Criteria**: success is judged by comparing original and compressed versions
- Some measures are objective, e.g. root **mean square error (e_{rms}) and signal to noise ratio (SNR)**
- Let f(x,y) be the input image, f'(x,y) be reconstructed input image from compressed bit stream, then

$$e_{rms} = \left(\frac{1}{MN} \sum_{x=0}^{M-1} \sum_{y=0}^{N-1} \left(f'(x,y) - f(x,y) \right)^2 \right)^{1/2} \qquad SNR = \frac{\sum_{x=0}^{M-1} \sum_{y=0}^{N-1} \left(f'(x,y) \right)^2}{\sum_{x=0}^{M-1} \sum_{y=0}^{N-1} \left(f'(x,y) - f(x,y) \right)^2}$$

Fidelity Criteria

$e_{rms} = 6.93$ $\quad$ $e_{rms} = 6.78$

$SNR_{rm} = 10.25$ $\quad$ $SNR_{rm} = 10.39$

Fidelity Criteria

- e_{rms} and SNR are convenient objective measures
- Most decompressed images are viewed by human beings
- Subjective evaluation of compressed image quality by human observers are often more appropriate

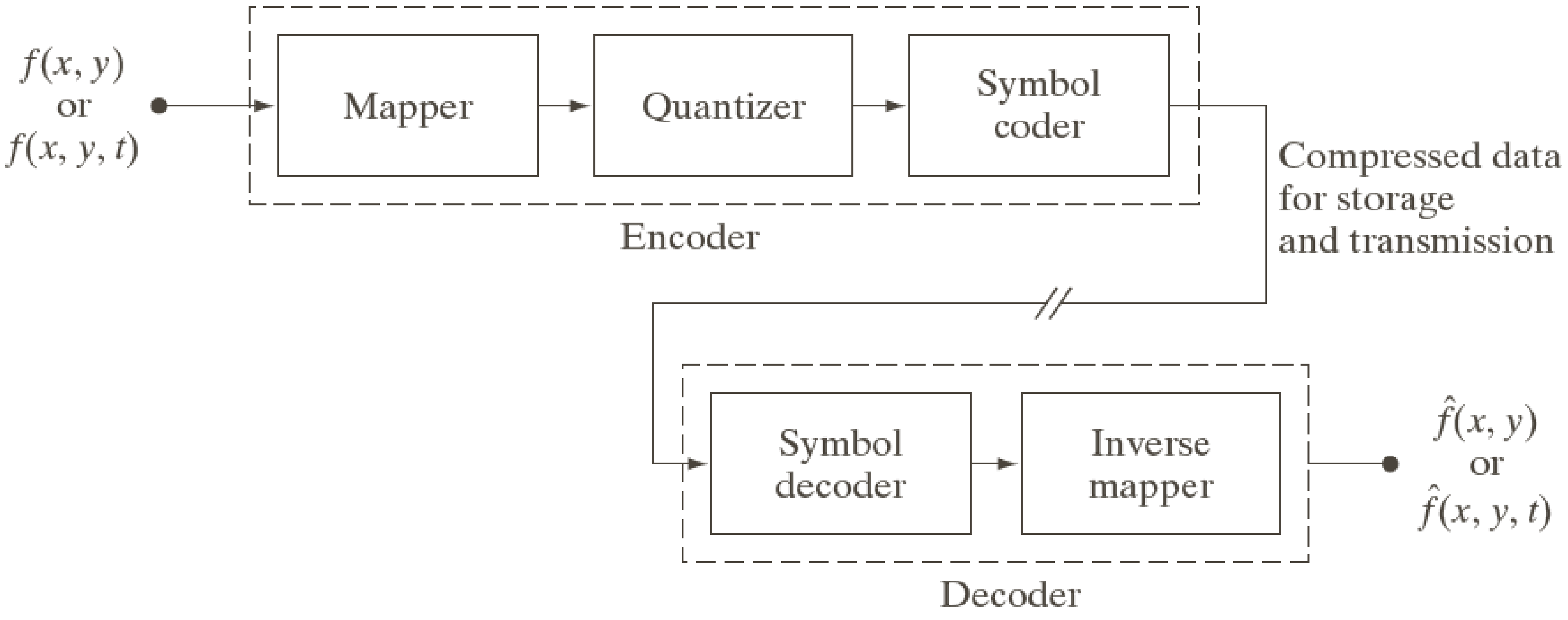
Value	Rating	Description
1	Excellent	An image of extremely high quality, as good as you could desire.
2	Fine	An image of high quality, providing enjoyable viewing. Interference is not objectionable.
3	Passable	An image of acceptable quality. Interference is not objectionable.
4	Marginal	An image of poor quality; you wish you could improve it. Interference is somewhat objectionable.
5	Inferior	A very poor image, but you could watch it. Objectionable interference is definitely present.
6	Unusable	An image so bad that you could not watch it.

Rating scale of the Television Allocations Study Organization. (Frendendall and Behrend.)

Image Compression Systems

- Mapper
  - transforms input data in a way that facilitates reduction of interpixel redundancies
  - reversible
- Quantiser
  - transforms input data in a way that facilitates reduction of psychovisual redundancies
  - not reversible
- Symbol coder
  - assigns the shortest code to the most frequently occurring output values
  - reversible

Image Compression Systems



Functional block diagram of a general image compression system

Image Compression in Matlab

- Imwrite provides control of compression methods and parameters
- [peaksnr, snr] = psnr(image, reference image) supports evaluation

JPEG quality 25/100
SNR = 30.0

JPEG quality 100/100
SNR = 53.8

Next Week

Conclusion and Exam Revision