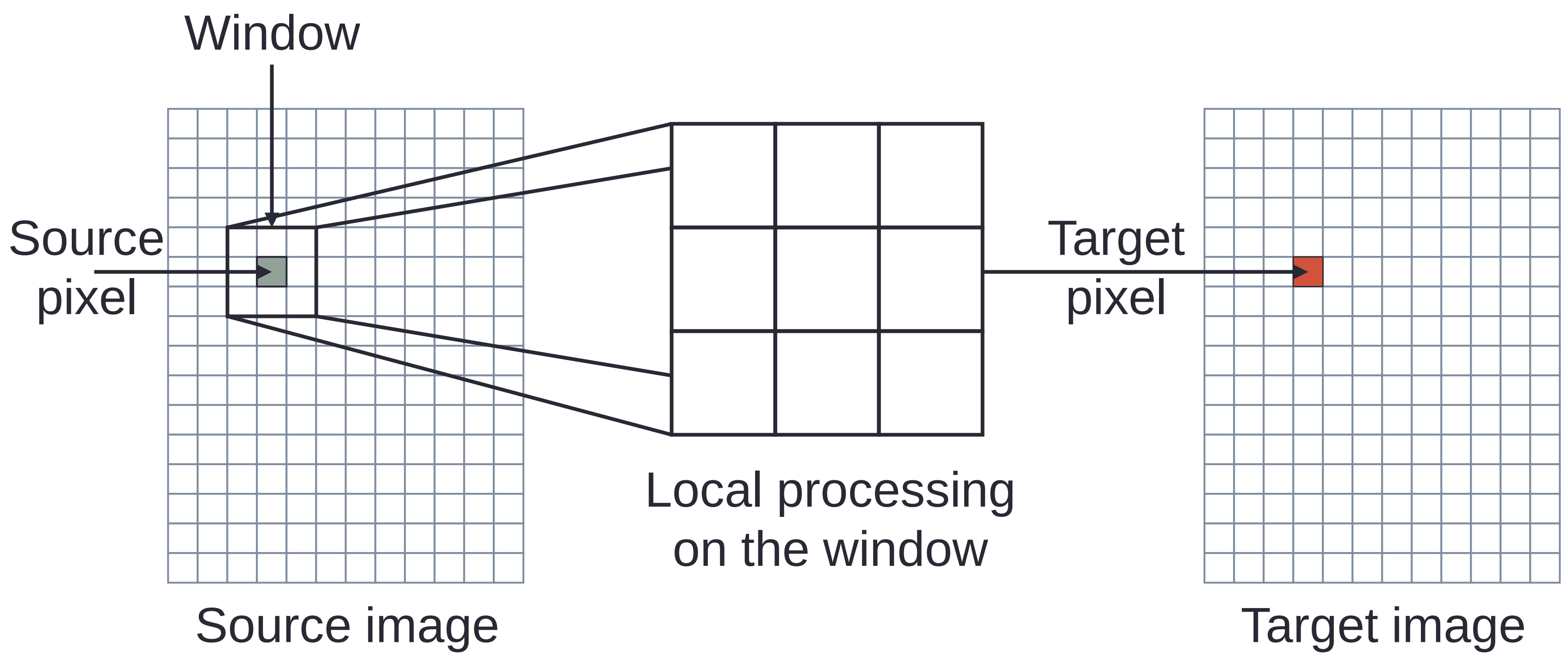
COMP2005

Linear Filters :

- Convolution, Mean Filtering and Noise
- Gaussian Filtering

Spatial Filtering



Why?

- Intensity Transforms read and affect only a single pixel, their power is limited
- Images are spatially organised data structures, many important attributes vary slowly across the image
  - Object identity
  - Viewed surface orientation, colour, etc
  - Illumination
- Processes restricted to a small, compact area have access to more information but are still likely to consider a single object, surface, illumination pattern, etc.

Image Noise

- Noise = small errors in image values
- Imperfect sensors introduce noise
- Image compression methods are lossy: repeated coding & decoding adds noise

Original

JPEG

Difference (Enhanced)

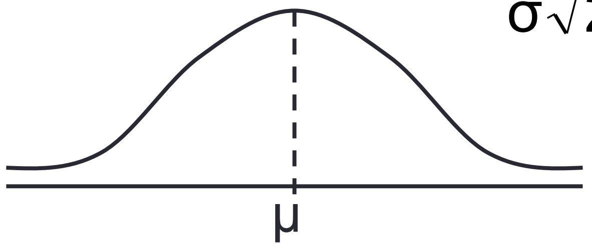
- Noise is often modelled as additive:

  Recorded value = true value + random noise value

Gaussian Noise

- Sensors often give a measurement a little off the true value
  - On average they give the right value
  - They tend to give values near the right value rather than far from it
- We model this with a Gaussian
  - Mean (μ) = 0
  - Variance (σ^2) indicates how much noise there is

$$P(x) = \frac{1}{\sigma\sqrt{2\pi}} e^{\frac{-(x-\mu)^2}{2\sigma^2}}$$



Gaussian Noise

The level of noise is related to the Gaussian parameter σ

σ = 1 σ = 10 σ = 20

Image with varying degrees of Gaussian noise added

Noise Reduction

- If you have multiple images, taking the mean value of each pixel will reduce noise
  - Noise is randomly added to each value
  - Mean value added is 0
  - If you average a large set of estimates of the same pixel the random noise values will cancel out

42 43 44 41 40 42 42 44 40

➔ 42

- Given only a single image, averaging over a local region has a similar effect

42 43 44

41 40 42 ➔ 42

42 44 40

- Ideally, we would choose the region to only include pixels that should have the same value
- **We need a spatial filter……**

Spatial Filtering: Convolution

- Many filters follow a similar pattern - multiplying each image value by a corresponding filter entry, and summing the results

$F_{(-1,-1)}$	$F_{(0,-1)}$	$F_{(+1,-1)}$
$F_{(-1,0)}$	$F_{(0,0)}$	$F_{(+1,0)}$
$F_{(-1,+1)}$	$F_{(0,+1)}$	$F_{(+1,+1)}$

Filter Window

$P_{(x-1,y-1)}$	$P_{(x,y-1)}$	$P_{(x+1,y-1)}$
$P_{(x-1,y)}$	$P_{(x,y)}$	$P_{(x+1,y)}$
$P_{(x-1,y+1)}$	$P_{(x,y+1)}$	$P_{(x+1,y+1)}$

Picture Window

$F_{(-1,-1)} \times P_{(x-1,y-1)}$

$+ F_{(0,-1)} \times P_{(x,y-1)}$

$+ F_{(+1,-1)} \times P_{(x+1,y-1)}$

$+ F_{(-1,0)} \times P_{(x-1,y)}$

$+ \ldots$

$+ F_{(+1,+1)} \times P_{(x+1,y+1)}$

Result

Filtering

- More generally, with a filter with radius *r*
  - $p_{x,y}$ is the original image value at (x,y)
  - $p'_{x,y}$ is the new image value at (x,y)

$$p'(x,y) = \sum_{dx=-r}^{+r} \sum_{dy=-r}^{+r} f_{dx,dy} \times p_{x+dx,y+dy}$$

- Many, though not all, filters work this way, e.g. the mean filter:

3 × 3 mean filter

1/9	1/9	1/9
1/9	1/9	1/9
1/9	1/9	1/9

1/25	1/25	1/25	1/25	1/25
1/25	1/25	1/25	1/25	1/25
1/25	1/25	1/25	1/25	1/25
1/25	1/25	1/25	1/25	1/25
1/25	1/25	1/25	1/25	1/25

5 × 5 mean filter

The Mean Filter

Original

Gaussian

Key Points

- Spatial filters operate on local image regions
- Many can be formulated as convolution with a suitable mask
- Noise reduction via mean filtering is a classic example

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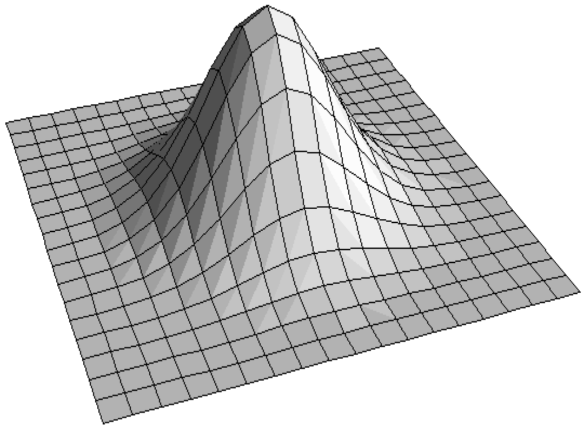
Gaussian Filtering

Gaussian Filters

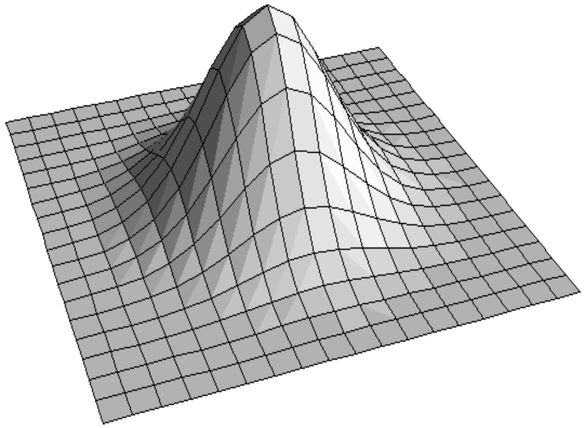
- Convolution with a mask whose weights are determined by a 2D Gaussian function
- Higher weight is given to pixels near the source pixel
- These are more likely to lie on the same object as the source pixel



$$P(x,y) = \frac{1}{\sigma^2 2\pi} e^{\frac{-(x^2+y^2)}{2\sigma^2}}$$

Discrete Gaussian Fitlers

- The Gaussian
  - Extends infinitely in all directions, but we want to process just a local window
  - Has a volume underneath it of 1, which we want to maintain



- We can approximate the Gaussian with a discrete filter
  - We restrict ourselves to a square window and sample the Gaussian function
  - We normalise the result so that the filter entries add to 1

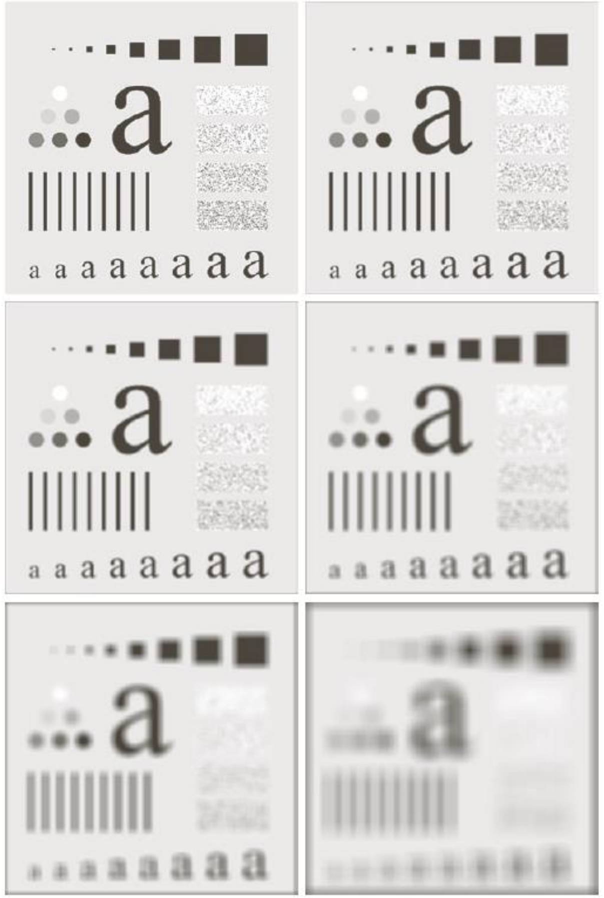
Example

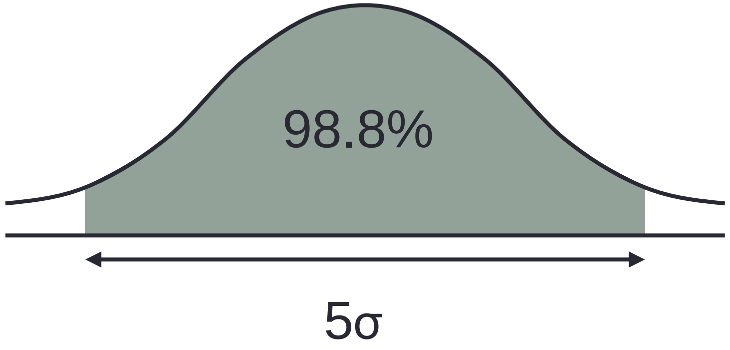
- Suppose we want to use a 5x5 window to apply a Gaussian filter with $\sigma^2 = 1$
  - The centre of the window has x = y = 0
  - We sample the Gaussian at each point
  - We then normalise it

$$
\begin{array}{c|c|c|c|c|c|}
\cline{2-6}
-2 & 0.00 & 0.01 & 0.02 & 0.01 & 0.00 \\
\cline{2-6}
-1 & 0.01 & 0.06 & 0.10 & 0.06 & 0.01 \\
\cline{2-6}
0 & 0.02 & 0.10 & 0.16 & 0.10 & 0.02 \\
\cline{2-6}
1 & 0.01 & 0.06 & 0.10 & 0.06 & 0.01 \\
\cline{2-6}
2 & 0.00 & 0.01 & 0.02 & 0.01 & 0.00 \\
\cline{2-6}
\multicolumn{1}{c}{} & \multicolumn{1}{c}{-2} & \multicolumn{1}{c}{-1} & \multicolumn{1}{c}{0} & \multicolumn{1}{c}{1} & \multicolumn{1}{c}{2}
\end{array}
\times \frac{1}{0.96}
$$

Gaussian Filters

- How big should the filter window be?
  - With Gaussian filters this depends on the variance (σ^2)
  - Under a Gaussian curve 98% of the area lies within 2σ of the mean
  - A filter width of 5σ gives more than 98% of the values we want





The Gaussian Filter

Original

Gaussian

Separable Filters

- The Gaussian filter is separable
  - A 2D Gaussian is equivalent to two 1D Gaussians
  - First you filter with a 'horizontal' Gaussian
  - Then with a 'vertical' Gaussian…..

0.06
0.24
0.40
0.24
0.06

0.06	0.24	0.40	0.24	0.06

$$P(x,y) = \frac{1}{\sigma^2 2\pi} e^{\frac{-(x^2+y^2)}{2\sigma^2}}$$

$$= \frac{1}{\sigma\sqrt{2\pi}} \frac{1}{\sigma\sqrt{2\pi}} e^{\frac{-x^2}{2\sigma^2}} e^{\frac{-y^2}{2\sigma^2}}$$

$$= \left(\frac{1}{\sigma\sqrt{2\pi}} e^{\frac{-x^2}{2\sigma^2}}\right)\left(\frac{1}{\sigma\sqrt{2\pi}} e^{\frac{-y^2}{2\sigma^2}}\right)$$

$$= P(x) \times P(y)$$

Separable Filters

- The separated filter is more efficient
  - Given an N×N image and a n×n filter we need to do $O(N^2n^2)$ operations
  - Applying two n×1 filters to a N×N image takes $O(2N^2n)$ operations
- Example
  - A 600×400 image and a 5×5 filter
  - Applying it directly takes around 6,000,000 operations
  - Using a separable filter takes around 2,400,000 - less than half as many

Key Points

- Like mean filtering, and Gaussian smoothing can be used to remove additive noise
- Gaussian smoothing emphasises pixels near the source pixel, where it is more likely image properties are fixed
- Gaussian smoothing is separable, and so efficient