Competition sign-ups

Difficulty: Moderate
Total Points:
Author: Jonas

Jon is planning to start an economics competition in his school, which has a total of 5000 students (excluding himself). To fund the competition, Jon must charge an entry fee. It is in his interest to earn as much as possible from the entry fees to maximise the quality of the competition. Jon offers a full refund to students who decide to withdraw later.

The students in the school can be categorised as follows:

1. **Economics Enthusiasts (1000 students):** These students have a direct interest in Economics and will decide independently whether to participate. Their demand is given by:
   $Q_I = 1000 - 10p$
   where Q_I is the number of interested students participating, and p is the fee.
2. **Uninterested Students (4000 students):** These students are not inherently interested in Economics but may experience a Fear of Missing Out (FOMO) due to the participation of their peers, causing them to be influenced by a network externality. Their demand is given by:
   $Q_{NI} = 4000 - 200p + 5\sqrt{N}$
   where Q_{NI} is the number of uninterested students participating, p is the fee, and N is the total number of participants in the competition, $N = Q_I + Q_{NI}$

Assumptions:

- The 1000 Economics Enthusiasts decide their participation first.
- The market always reaches an equilibrium.
- Supply is perfectly price elastic.

Jon's total earning is given by T, where $T = Np$

Sample Question 2

<u>Question</u> <u>Score</u>

i. Deduce $\lfloor p \rfloor$, where p is the price that Jon should charge to maximise his self-interest.
$\lfloor\,\rfloor$ denotes the floor function—$\lfloor p \rfloor$ is the greater integer number less than or equal to p.

Integer (4)

ii. Deduce $\lfloor T \rfloor$, where T is the total revenue made if Jon charges a price equal to $\lfloor p \rfloor$ as calculated in (i).
$\lfloor\,\rfloor$ denotes the floor function—$\lfloor T \rfloor$ is the greater integer number less than or equal to T.

Integer (2)

Solution

<u>Solution to (i)</u>
Answer: 12
Explanation:
Sub $Q_I = 1000 - 10p$ and $N = Q_I + Q_{NI}$ to $Q_{NI} = 4000 - 200p + 5\sqrt{N}$,
$N - (1000 - 10p) = 4000 - 200p + 5\sqrt{N}$

Manipulate the equation to obtain,
$N - 5\sqrt{N} = 5000 - 210p$
$(\sqrt{N} - 2.5)^2 - 2.5^2 = 5000 - 210p$
$\sqrt{N} = \sqrt{5006.25 - 210p} + 2.5$
$N = (\sqrt{5006.25 - 210p} + 2.5)^2$

Let total earnings = T
$T = Np$

Therefore, obtain an equation with T as the subject,
$T = (\sqrt{5006.25 - 210p} + 2.5)^2 p$
Differentiate T w.r.t. p, using a differentiation calculator,

$$\left(\sqrt{5006.25 - 210p} + 2.5\right)^2 - \frac{210\left(\sqrt{5006.25 - 210p} + 2.5\right)p}{\sqrt{5006.25 - 210p}}$$

Therefore, to maximise T, take the value of p where $\frac{dT}{dp} = 0$, which is when
$p = 12.21375086$
Rounding down,
$p = 12$

<u>Solution to (ii)</u>
Answer: 32902
Explanation:
To determine the T, sub $p = 12$ to $T = (\sqrt{5006.25 - 210p} + 2.5)^2 p$,

Sample Question 2

$T = (\sqrt{5006.25 - (210 * 12)} + 2.5)^2 * 12$
$T = 32901.7386$

Rounding down,
$T = 32902$