Automated Variable Selection

Singh, Gurjeet

This report contains analysis done while building regression models for the for the home sale price on Ames housing data to understand and provide estimates of home values for typical homes in Ames, lowa.

Table of Contents

| Introduction: | 3 |
|---|------------------------------|
| Section 1: Sample Definition and Data Split | |
| | |
| Section 1.1: Sample Definition | |
| Section 1.2: The Train/Test Split | 5 |
| Section 2: Model Identification and In-Sample Model Fit | 6 |
| Section 2.1: Forward Variable Selection | |
| Section 2.2: Backward Variable Selection | <u>C</u> |
| Section 2.3: Stepwise Variable Selection | 12 |
| Section 2.4: Model Comparison | 15 |
| Section 3: Predictive Accuracy | 17 |
| Section 4: Operational Validation | 18 |
| Conclusion: | 19 |
| Annondiy 1 · P. Codo | Errorl Poolemark not defined |

Table of Figures

| Figure 1: Drop Conditions with count | |
|---|----------|
| Figure 2: Train/Test data Partition | |
| Figure 3: Response, Predictor and Indicator variables | |
| Figure 4: forward.lm model: Last Step | |
| Figure 5: forward.lm model: Output | |
| Figure 6: forward.lm model: VIF values | 8 |
| Figure 7: Backward.lm model: Last Step | <u>c</u> |
| Figure 8: backward.lm model: Output | 10 |
| Figure 9: backward.lm model: VIF values | |
| Figure 10: Stepwise.lm model: Last Step | 12 |
| Figure 11: stepwise.lm model: Output | 13 |
| Figure 12: stepwise.lm model: VIF values | 14 |
| Figure 13: Four Models: Forward, Backward, Stepwise, and Junk | 15 |
| Figure 14: VIF Values: Forward, Backward, Stepwise, and Junk | 16 |
| Figure 15: Metrics: Forward, Backward, Stepwise, and Junk | 17 |
| Figure 16: Out-of-Sample Metrics | 17 |
| Figure 17: Prediction Grades | 18 |

Introduction:

The purpose of this assignment is to build statistical models i.e. regression model to predict the value of a property or home. In this assignment, we will set up a predictive modeling framework, explore the use of automated variable selection techniques for model identification, assess the predictive accuracy of our model using cross-validation, and compare and contrast the difference between statistical model validation and an application (or business) model validation. There will be three different models namely forward variable selection (forward.lm), backward variable selection (backward.lm), stepwise variable selection (stepwise.lm), and one additional model i.e. junk model (junk.lm). Each of the first three models is discussed in their own section. Junk model is compared and discussed in section 2.4, Model Comparison.

For this assignment, we use the data set that contains information from the Ames Assessor's Office used in computing assessed values for individual residential properties sold in Ames, IA from 2016 to 2010. The data set contains 2930 observations and 82 explanatory variables which include 23 nominal, 23 ordinal, 14 discrete, and 20 continuous variables, and 2 additional observation identifiers.

Now that we have some context for our analysis and dataset, let's look at the results in the next section.

Section 1: Sample Definition and Data Split

Section 1.1: Sample Definition

For the purposes of the end goal, we have created a sample population. This eligible sample excludes the following kind of properties using the drop conditions. Figure 1 shows the drop conditions with the total count. Each of the drop conditions used to eliminate a property is explained below.

- Building type is not a single family.
- It not a normal sale
- Street are not paved
- Any house built prior to 1950
- There is no basement
- Living room square feet is below 800 and above 4000.
- There is no bedroom
- There is no kitchen
- There is no full bath
- There is no public utilities

Figure 1: Drop Conditions with count

| | | Total Count |
|-----|----------------------|-------------|
| 01: | Not SFR | 505 |
| 02: | Non-Normal Sale | 423 |
| 03: | Street Not Paved | 6 |
| 04: | Built Pre-1950 | 489 |
| 05: | No Basement | 28 |
| 06: | LT 800 SqFt | 9 |
| 07: | LT 4000 SqFt | 1 |
| 08: | No Bedroom | 4 |
| 09: | No Kitchen | 1 |
| 11: | Not Public Utilities | 1 |
| 99: | Eligible Sample | 1463 |

Next, I deleted any observations with any missing values after creating a list of predictor variables. Lastly, I created some discrete and indicator variables and saved the sample population as an .RData data object for the later use for the remainder of the assignment.

In this sample population, we have <u>1132 observations</u> and <u>56 explanatory variables</u>.

Section 1.2: The Train/Test Split

The sample population that we created in section 1.1, we will use that to split the sample into a 70/30 train/test split using the uniform random number. The 70/30 training/test split that we are using is the most basic form of cross-validation. With the train/test split, we now have two data sets: one for in-sample model development and the other one for out-of-sample model assessment. We will use train data set for our in-sample model development and test data set for our out-of-sample model assessment. The train data set was used in section 2 to develop the three automated selection models i.e. forward, backward, and stepwise. The test data set was used in section 3 to assess the predictive accuracy of the same models.

Figure 2: Train/Test data Partition

| | Train_DF | Test_DF |
|-------------|----------|---------|
| Observation | 802 | 330 |

Section 2: Model Identification and In-Sample Model Fit

This section explains the model identification of the three models i.e. Forward Variable Selection, Backward Variable Selection, and Stepwise Variable Selection. At the end, we perform the model comparison. For these model identification, we created a new data frame that only contains our response variable and the predictor variables that we include as our pool of predictor variables. Figure 3 shows the list of all the variables (Response, Predictor, and Indicator variables) that we used in our model development process.

Figure 3: Response, Predictor and Indicator variables

| | Field_Names $^{\diamondsuit}$ |
|----|-------------------------------|
| 1 | LotFrontage |
| 2 | LotArea |
| 3 | BedroomAbvGr |
| 4 | TotRmsAbvGrd |
| 5 | Fireplaces |
| 6 | GarageCars |
| 7 | GarageArea |
| 8 | WoodDeckSF |
| 9 | OpenPorchSF |
| 10 | EnclosedPorch |
| 11 | ThreeSsnPorch |
| 12 | ScreenPorch |
| 13 | SalePrice |
| 14 | TotalSqftCalc |
| 15 | TotalBathCalc |
| 16 | CornerLotInd |
| 17 | CentralAirInd |
| 18 | BrickInd |
| 19 | VinylSidingInd |
| 20 | PoolInd |
| 21 | WoodDeckInd |
| 22 | Porchind |
| 23 | QualityIndex |
| | |

Section 2.1: Forward Variable Selection

In the forward variable selection approach, we start with no regressors and continue to add terms until adding another term makes the criterion of interest worse i.e. increase AIC. Figure 4 shows the last step at which the function stopped because adding any of the five remaining predictors (LotFrontage, TotalBathCalc, ScreenPorch, ThreeSsnPorch, and CentralAirInd) would increase the AIC.

Figure 4: forward.lm model: Last Step

```
Step: AIC=16511.91
SalePrice ~ TotalSqftCalc + GarageCars + QualityIndex + VinylSidingInd +
TotRmsAbvGrd + BedroomAbvGr + LotArea + GarageArea + PoolInd +
OpenPorchSF + CornerLotInd + WoodDeckInd + Fireplaces + BrickInd +
EnclosedPorch + WoodDeckSF

Of Sum of Sq RSS AIC
<none> 671739547728 16512
+ LotFrontage 1 1196772312 670542775416 16513
+ TotalBathCalc 1 493610235 671245937493 16513
+ ScreenPorch 1 490773684 671248774044 16513
+ ThreeSsnPorch 1 414397756 671325149973 16513
+ CentralAirInd 1 133167295 671606380433 16514
```

Figure 5 shows the summary output of the forward selection model i.e. forward.lm. We look at the R-Squared statistic to measure how well our model is fitting the actual data. The R-Squared measure the linear relationship between our predictor variables and our response variable (SalePrice). The R-Squared we got is **0.8437** which is roughly 84% (approx.) of the variance found in the response variable (SalePrice) can be explained by the predictor variables.

Figure 5: forward.lm model: Output

```
call:
lm(formula = SalePrice ~ TotalSqftCalc + GarageCars + QualityIndex +
    VinylSidingInd + TotRmsAbvGrd + BedroomAbvGr + LotArea +
    GarageArea + Poolind + OpenPorchSF + CornerLotind + WoodDeckInd +
   Fireplaces + BrickInd + EnclosedPorch + WoodDeckSF, data = train.clean)
Residuals:
            1Q Median
                           3Q
                                  Max
-105858 -17726
                -1486
                       16045 187230
Coefficients:
                Estimate Std. Error t value
                                                      Pr(>|t|)
                         (Intercept)
              -86053.549
                                   40.791
TotalSqftCalc
                             2,206
               16278.894
                          3357.294
                                              0.00000149759766
                                    4.849
GarageCars
                                   11.954 < 0.00000000000000000
                2065.796
                           172.808
QualityIndex
VinylsidingInd 16865.987
                          2413.119
                                     6.989
                                              0.0000000000591
TotRmsAbvGrd
               11430.989
                                    8.729 < 0.000000000000000 ***
                          1309.495
              -13577.072
                          2211.228
                                              0.0000000130914 ***
BedroomAbvGr
                                    -6.140
                          0.292
LotArea
                  1.610
                                    5.514
                                              0.00000004762055 ***
                                              0.00001234415980 ***
GarageArea
                  52.410
                            11.912
                                     4,400
                                                      0.000109 ***
               45317.547
                         11651.206
PoolInd
                                     3.890
OpenPorchSF
                 59.662
                            18.913
                                                      0.001669
                                    3.155
               -6295.554
                          2829.401
CornerLotInd
                                    -2.225
                                                     0.026361
                8704.135
                          3425.116
                                    2.541
                                                     0.011236
WoodDeckInd
Fireplaces
                3238.958
                          1969.306
                                    1.645
                                                     0.100428
                9776.671
                          6455.831
BrickInd
                                    1.514
EnclosedPorch
                 -28.082
                            18.146
                                    -1.548
                                                      0.122129
WoodDeckSF
                -19.490
                            13.182
                                   -1.479
                                                      0.139663
Signif. codes: 0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' '1
Residual standard error: 29250 on 785 degrees of freedom
Multiple R-squared: 0.8437,
                             Adjusted R-squared:
F-statistic: 264.8 on 16 and 785 DF, p-value: < 0.0000000000000022
```

Figure 6 shows the Variance Inflation Factor (VIF) values of the forward.lm model for the predictor variables. The VIF values of the predictor variables indicate the strength of the linear relationship between the variable and remaining predictor variables. A good rule of thumb is that VIF values greater than 10 give some cause for concern. A low VIF value means that there are no high correlations among some or all predictor variables. In Figure 6, we see that all the VIF values are below 5. Hence, we can conclude that in this model that multicollinearity is not a problem. In section 2.4, we do the side-by-side comparison of these values.

Figure 6: forward.lm model: VIF values

| | VIF_Values |
|----------------|------------|
| GarageCars | 4.673355 |
| GarageArea | 4.214643 |
| TotRmsAbvGrd | 2.792751 |
| WoodDeckInd | 2.717133 |
| WoodDeckSF | 2.660281 |
| TotalSqftCalc | 2.157398 |
| BedroomAbvGr | 1.693272 |
| Fireplaces | 1.590113 |
| QualityIndex | 1.450879 |
| VinylSidingInd | 1.361003 |
| OpenPorchSF | 1.210311 |
| LotArea | 1.176445 |
| EnclosedPorch | 1.111708 |
| PoolInd | 1.100782 |
| BrickInd | 1.088088 |
| CornerLotInd | 1.025362 |
| | |

Section 2.2: Backward Variable Selection

In the backward variable selection approach, we start with all the regressors in the model and continue to remove terms until removing another term makes the criterion of interest worse i.e. increase AIC. Figure 7 shows the last step at which the function stopped because removing any of the sixteen remaining predictors would increase the AIC. Hence, selected this model as our backward selection.

Figure 7: Backward.lm model: Last Step

```
Step: AIC=16511.91
SalePrice ~ LotArea + BedroomAbvGr + TotRmsAbvGrd + Fireplaces +
     GarageCars + GarageArea + WoodDeckSF + OpenPorchSF + EnclosedPorch +
     TotalSqftCalc + CornerLotInd + BrickInd + VinylSidingInd +
     PoolInd + WoodDeckInd + QualityIndex
                    Df
                            Sum of Sq
                                                   RSS
<none>
                                        671739547728 16512

    WoodDeckSF

                          1870685634 673610233363 16512
- BrickInd
                          1962496999 673702044727 16512

    EnclosedPorch

                    1 2049425977 673788973705 16512
- Fireplaces
                      1 2314809472 674054357200 16513
                      1 4236528232 675976075960 16515

    CornerLotInd

- WoodDeckInd 1 5526268384 677265816112 16517

- OpenPorchSF 1 8515251286 680254799014 16520

- PoolInd 1 12945602893 684685150621 16525

- GarageArea 1 16563508471 688303056200 16529

- GarageCars 1 20118805075 691858352803 16534

- LotArea 1 26017518699 697757066427 16540
- BedroomAbvGr 1 32260885681 704000433410 16548
- vinylsidingInd 1 41802020309 713541568037 16558

    TotalSqftCalc

                      1 292569667386 964309215115 16800
```

Figure 8 shows the summary output of the backward selection model i.e. backward.lm. We look at the R-Squared statistic to measure how well our model is fitting the actual data. The R-Squared measure the linear relationship between our predictor variables and our response variable (SalePrice). The R-Squared we got is **0.8437** which is roughly 84% (approx.) of the variance found in the response variable (SalePrice) can be explained by the predictor variables.

The reason we have the same R-Squared as of forward.Im model is because of both of the models, forward.Im and backward.Im, have selected the same predictor variables. Hence, our models are the same.

Figure 8: backward.lm model: Output

```
lm(formula = SalePrice ~ LotArea + BedroomAbvGr + TotRmsAbvGrd +
   Fireplaces + GarageCars + GarageArea + WoodDeckSF + OpenPorchSF +
    EnclosedPorch + TotalSqftCalc + CornerLotInd + BrickInd +
    VinylSidingInd + PoolInd + WoodDeckInd + QualityIndex, data = train.clean)
Residuals:
   Min
            1Q Median
                            3Q
                                  Max
-105858 -17726 -1486
                         16045 187230
Coefficients:
               Estimate Std. Error t value
                                                       Pr(>|t|)
(Intercept)
              -86053.549 7612.039 -11.305 < 0.0000000000000000 ***
LotArea 1.610 0.292 5.514
BedroomAbvGr -13577.072 2211.228 -6.140
                                               0.00000004762055 ***
                                               0.0000000130914 ***
TotRmsAbvGrd 11430.989 1309.495 8.729 < 0.0000000000000000 ***
Fireplaces
              3238.958 1969.306 1.645
                                                      0.100428
               16278.894 3357.294 4.849
52.410 11.912 4.400
GarageCars
                                               0.00000149759766 ***
                                               0.00001234415980 ***
GarageArea
                 -19.490
                            13.182 -1.479
WoodDeckSF
                                                      0.139663
OpenPorchSF
                            18.913 3.155
                                                      0.001669 **
                 59.662
                 -28.082
EnclosedPorch
                            18.146
                                    -1.548
                                                      0.122129
TotalSqftCalc
                 40.791
                             2.206 18.491 < 0.0000000000000000 ***
                         2829.401
             -6295.554
CornerLotInd
                                    -2.225
                                                      0.026361 *
                                     1.514
BrickInd
                9776.671
                           6455.831
                                                      0.130328
                                    6.989
                          2413.119
                                               0.0000000000591 ***
VinylsidingInd 16865.987
                                                      0.000109 ***
               45317.547 11651.206 3.890
PoolInd
WoodDeckInd
                8704.135
                          3425.116
                                    2.541
                                                      0.011236 *
               2065.796
                           172.808 11.954 < 0.00000000000000000 ***
QualityIndex
Signif. codes: 0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
Residual standard error: 29250 on 785 degrees of freedom
Multiple R-squared: 0.8437,
                              Adjusted R-squared: 0.8405
F-statistic: 264.8 on 16 and 785 DF, p-value: < 0.00000000000000022
```

Figure 9 shows the Variance Inflation Factor (VIF) values of the backward.Im model for the predictor variables. The VIF values of the predictor variables indicate the strength of the linear relationship between the variable and remaining predictor variables. A good rule of thumb is that VIF values greater than 10 give some cause for concern. A low VIF value means that there are no high correlations among some or all predictor variables. In Figure 9, we see that all the VIF values are again below 5. Hence, we can conclude that in this model that multicollinearity is not a problem.

If we compare the backward.Im VIF values (Figure 9) with forward.Im (Figure 6), we will notice the values are the same. This is because both of these models have selected the same predictor variables. Hence, they are the same models. Therefore, we got the same VIF values. In section 2.4, we do the side-by-side comparison of these values.

Figure 9: backward.lm model: VIF values

| | VIF_Valueŝ |
|----------------|------------|
| GarageCars | 4.673355 |
| GarageArea | 4.214643 |
| TotRmsAbvGrd | 2.792751 |
| WoodDeckInd | 2.717133 |
| WoodDeckSF | 2.660281 |
| TotalSqftCalc | 2.157398 |
| BedroomAbvGr | 1.693272 |
| Fireplaces | 1.590113 |
| QualityIndex | 1.450879 |
| VinylSidingInd | 1.361003 |
| OpenPorchSF | 1.210311 |
| LotArea | 1.176445 |
| EnclosedPorch | 1.111708 |
| PoolInd | 1.100782 |
| BrickInd | 1.088088 |
| CornerLotInd | 1.025362 |

Section 2.3: Stepwise Variable Selection

In the stepwise variable selection approach, we start with one regressor in the model and continue to remove or add terms until removing or adding another term makes the criterion of interest worse i.e. increase AIC. Figure 10 shows the last step at which the function stopped because removing or adding any of the twenty-one remaining predictors would increase the AIC. Hence, selected this model as our stepwise selection.

Figure 10: Stepwise.lm model: Last Step

```
Step: AIC=16511.91
SalePrice ~ TotalSqftCalc + GarageCars + QualityIndex + VinylSidingInd +
    TotRmsAbvGrd + BedroomAbvGr + LotArea + GarageArea + PoolInd +
    OpenPorchSF + CornerLotInd + WoodDeckInd + Fireplaces + BrickInd +
    EnclosedPorch + WoodDeckSF
                   Df
                          Sum of Sq
<none>
                                      671739547728 16512

    WoodDeckSF

                    1 1870685634 673610233363 16512

    BrickInd

                    1 1962496999 673702044727 16512

    EnclosedPorch 1 2049425977 673788973705 16512

+ LotFrontage
                    1 1196772312 670542775416 16513
- Fireplaces
                    1 2314809472 674054357200 16513
+ TotalBathCalc
                       493610235 671245937493 16513
                    1
+ ScreenPorch 1
+ ThreeSsnPorch 1
                         490773684 671248774044 16513
                    1
                         414397756 671325149973 16513
+ CentralAirInd 1
                         133167295 671606380433 16514

    CornerLotInd 1

                         4236528232 675976075960 16515

    WoodDeckInd

                   1 5526268384 677265816112 16517
- OpenPorchSF 1 8515251286 680254799014 16520
- PoolInd 1 12945602893 684685150621 16525
- GarageArea 1 16563508471 688303056200 16529

- GarageCars 1 20118805075 691858352803 16534

- LotArea 1 26017518699 697757066427 16540

- BedroomAbvGr 1 32260885681 704000433410 16548

    vinylsidingInd 1 41802020309 713541568037 16558

    TotRmsAbvGrd

                    1 65206572366 736946120094 16584
- QualityIndex
                    1 122286638244 794026185973 16644
- TotalSqftCalc
                    1 292569667386 964309215115 16800
```

Figure 11 shows the summary output of the stepwise selection model i.e. forward.lm. We look at the R-Squared statistic to measure how well our model is fitting the actual data. The R-Squared measure the linear relationship between our predictor variables and our response variable (SalePrice). The R-Squared we got is **0.8437** which is roughly 84% (approx.) of the variance found in the response variable (SalePrice) can be explained by the predictor variables.

The reason we have the same R-Squared as of forward.Im and backward models is because of all three models, forward.Im, backward.Im, and stepwise.Im, have selected the same predictor variables. Hence, our models are the same as well as our R-Squared values.

Figure 11: stepwise.lm model: Output

```
call:
lm(formula = SalePrice ~ TotalSqftCalc + GarageCars + QualityIndex +
    VinylSidingInd + TotRmsAbvGrd + BedroomAbvGr + LotArea +
    GarageArea + PoolInd + OpenPorchSF + CornerLotInd + WoodDeckInd +
    Fireplaces + BrickInd + EnclosedPorch + WoodDeckSF, data = train.clean)
Residuals:
   Min
            1Q Median
                            3Q
                                  Max
                         16045 187230
-105858 -17726
                 -1486
Coefficients:
                Estimate Std. Error t value
                                                      Pr(>|t|)
(Intercept)
                          7612.039 -11.305 < 0.0000000000000000 ***
              -86053.549
TotalSoftCalc
                             2.206 18.491 < 0.0000000000000000 ***
                  40.791
                                               0.00000149759766 ***
GarageCars
               16278.894
                           3357.294
                                   4.849
QualityIndex
               2065.796
                          172.808 11.954 < 0.00000000000000000 ***
VinylSidingInd 16865.987
                          2413.119 6.989
                                               0.0000000000591 ***
TotRmsAbvGrd
               11430.989
                          1309.495 8.729 < 0.0000000000000000 ***
              -13577.072
                                               0.0000000130914 ***
BedroomAbvGr
                           2211.228 -6.140
                             0.292 5.514
                                               0.00000004762055 ***
LotArea
                  1.610
                                    4.400
                  52.410
                            11.912
                                               0.00001234415980 ***
GarageArea
PoolInd
               45317.547 11651.206 3.890
                                                      0.000109 ***
OpenPorchSF
                  59.662
                            18.913
                                     3.155
                                                      0.001669 **
CornerLotInd
               -6295.554
                          2829.401 -2.225
                                                      0.026361 *
WoodDeckInd
                8704.135
                          3425.116
                                     2.541
                                                      0.011236 *
Fireplaces
                3238.958
                          1969.306
                                     1.645
                                                      0.100428
BrickInd
                9776.671
                           6455.831
                                     1.514
                                                      0.130328
EnclosedPorch
              -28.082
                            18.146 -1.548
                                                      0.122129
                 -19.490
WoodDeckSF
                            13.182 -1.479
                                                      0.139663
Signif. codes: 0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
Residual standard error: 29250 on 785 degrees of freedom
Multiple R-squared: 0.8437, Adjusted R-squared: 0.8405
F-statistic: 264.8 on 16 and 785 DF, p-value: < 0.00000000000000022
```

Figure 12 shows the Variance Inflation Factor (VIF) values of the stepwise.Im model for the predictor variables. The VIF values of the predictor variables indicate the strength of the linear relationship between the variable and remaining predictor variables. A good rule of thumb is that VIF values greater than 10 give some cause for concern. A low VIF value means that there are no high correlations among some or all predictor variables. In Figure 12, we see that all the VIF values are again below 5. Hence, we can conclude that in this model that multicollinearity is not a problem.

If we compare the stepwise.Im VIF values (Figure 12) with backward.Im (Figure 9) and forward.Im (Figure 6), we will notice the VIF values are the same for each of these models. This is because all three of these models have selected the same predictor variables. Hence, they are the same models. Therefore, we got the same VIF values. In section 2.4, we do the side-by-side comparison of these values.

Figure 12: stepwise.lm model: VIF values

| | VIF_Values |
|----------------|------------|
| GarageCars | 4.673355 |
| GarageArea | 4.214643 |
| TotRmsAbvGrd | 2.792751 |
| WoodDeckInd | 2.717133 |
| WoodDeckSF | 2.660281 |
| TotalSqftCalc | 2.157398 |
| BedroomAbvGr | 1.693272 |
| Fireplaces | 1.590113 |
| QualityIndex | 1.450879 |
| VinylSidingInd | 1.361003 |
| OpenPorchSF | 1.210311 |
| LotArea | 1.176445 |
| EnclosedPorch | 1.111708 |
| PoolInd | 1.100782 |
| BrickInd | 1.088088 |
| CornerLotInd | 1.025362 |

Section 2.4: Model Comparison

Figure 13 shows each of our four models i.e. forward selection, backward selection, stepwise selection, and junk model. We have created a fourth model i.e. junk mode for model comparison purposes. When comparing our models, the junk model here outperformed all the other three models by every measure. However, there's no guarantee that the junk model will be predictive. Hence, we check for the multicollinearity for each to the models to make sure the predictor variables used in these models have no high correlation among themselves.

In Figure 13, we see that our forward, backward, and stepwise selection have selected the same model. Hence, the metrics in these models are the same as seen in section 2.1, section 2.2, and section 2.3.

Figure 13: Four Models: Forward, Backward, Stepwise, and Junk

```
> forward_selection_model
lm(formula = SalePrice ~ TotalSqftCalc + GarageCars + QualityIndex +
    VinylSidingInd + TotRmsAbvGrd + BedroomAbvGr + LotArea +
    GarageArea + Poolind + OpenPorchSF + CornerLotind + WoodDeckInd +
    Fireplaces + BrickInd + EnclosedPorch + WoodDeckSF, data = train.clean)
> backward_selection_model
lm(formula = SalePrice ~ LotArea + BedroomAbvGr + TotRmsAbvGrd +
    Fireplaces + GarageCars + GarageArea + WoodDeckSF + OpenPorchSF +
    EnclosedPorch + TotalSqftCalc + CornerLotInd + BrickInd +
    VinylSidingInd + PoolInd + WoodDeckInd + QualityIndex, data = train.clean)
> stepwise_selection_model
lm(formula = SalePrice ~ TotalSqftCalc + GarageCars + QualityIndex +
    VinylSidingInd + TotRmsAbvGrd + BedroomAbvGr + LotArea +
    GarageArea + PoolInd + OpenPorchSF + CornerLotInd + WoodDeckInd +
    Fireplaces + BrickInd + EnclosedPorch + WoodDeckSF, data = train.clean)
> junk_selection_model
lm(formula = SalePrice ~ OverallQual + OverallCond + QualityIndex +
    GrLivArea + TotalSqftCalc, data = train.df)
```

Figure 14 shows the Variance Inflation Factor (VIF) values of all the models for the predictor variables. The VIF values of the predictor variables indicate the strength of the linear relationship between the variable and remaining predictor variables. As stated before a good rule of thumb is that VIF values greater than 10 give some cause for concern. A low VIF value means that there are no high correlations among some or all predictor variables. In Figure 14, we see that all the VIF values are below 5 for forward, backward, and stepwise selection. However, we see large VIF values i.e. greater than 10, for three of the five predictor variables of the junk mode. Therefore, multicollinearity is not a problem for forward, backward, and stepwise selection models but it is for junk model. Hence, we call the junk model junk because it leads to unreliable and unstable estimates of regression coefficients.

We should not be concerned with VIF values for indicator variables because if they are not considered to be important as compared to the other variables, they will be dropped automatically.

Figure 14: VIF Values: Forward, Backward, Stepwise, and Junk

| Forward, | Backward, a | nd Stepwise N | <u>1odels</u> | Junk I | <u>Model</u> |
|----------------|-------------|---------------|---------------|---------------|--------------|
| | forward.VIF | backward.VIF | stepwise.VIF | | junk.VIF |
| GarageCars | 4.673355 | 4.673355 | 4.673355 | QualityIndex | 67.082864 |
| GarageArea | 4.214643 | 4.214643 | 4.214643 | OverallQual | 60.469459 |
| TotRmsAbvGrd | 2.792751 | 2.792751 | 2.792751 | OverallCond | 33.396955 |
| WoodDeckInd | 2.717133 | 2.717133 | 2.717133 | GrLivArea | 3.109841 |
| WoodDeckSF | 2.660281 | 2.660281 | 2.660281 | TotalSqftCalc | 2.320355 |
| TotalSqftCalc | 2.157398 | 2.157398 | 2.157398 | | |
| BedroomAbvGr | 1.693272 | 1.693272 | 1.693272 | | |
| Fireplaces | 1.590113 | 1.590113 | 1.590113 | | |
| QualityIndex | 1.450879 | 1.450879 | 1.450879 | | |
| VinylSidingInd | 1.361003 | 1.361003 | 1.361003 | | |
| OpenPorchSF | 1.210311 | 1.210311 | 1.210311 | | |
| LotArea | 1.176445 | 1.176445 | 1.176445 | | |
| EnclosedPorch | 1.111708 | 1.111708 | 1.111708 | | |
| PoolInd | 1.100782 | 1.100782 | 1.100782 | | |
| BrickInd | 1.088088 | 1.088088 | 1.088088 | | |
| CornerLotInd | 1.025362 | 1.025362 | 1.025362 | | |

Figure 15 shows the adjusted R-Squared, AIC, BIC, mean squared error (MSE), and the mean absolute error (MSE) for each of these models i.e. junk, forward, backward, and stepwise. In addition, Figure 15 provides the rank for each model based on the metric values. Since my forward, backward, and stepwise models selected the same predictor variables, all these models fell into rank 2 because each metric values were the same for those three models. Even though junk model is unreliable, for the purposes of ranking, it takes the rank one due to metric values. I expected each metric to give the same ranking of model fit.

Rank Adjusted_R_Squared AIC_Values BIC_Values MSE_Values MAE_Values 0.8538010 junk.lm 1 18709.21 18742.02 778488283 20795.79 Forward.Im 2 0.8404994 18789.89 18874.26 837580483 21221.65 backward.Im 2 0.8404994 18789.89 18874.26 837580483 21221.65 2 0.8404994 18789.89 18874.26 837580483 21221.65 stepwise.lm

Figure 15: Metrics: Forward, Backward, Stepwise, and Junk

Section 3: Predictive Accuracy

Next, we test the predictive accuracy of our model in the out-of-sample population. Figure 16 shows the Mean Squared Error (MSE) and the Mean Absolute Error (MAE) for each of the four models in the test sample. Based on the criteria in Figure 16, we see that the junk model fits the best. However, since we know that junk model is unstable, we consider other models to evaluate the fitness based on the criteria. All three of the models i.e. forward, backward, and stepwise, fit the best on these criteria. The reason these all fit because they have selected the same predictor variables.

Based on the output, I noticed that the junk model fit best in-sample predicted the best out-of-sample as well. If we compare the MAE values in Figure 15 (in-sample) and Figure 16 (out-of-sample), we notice that in Figure 16 (out-of-sample) MAE value is less than the Figure 15 (in-sample). However, this might be pure chance and luck due to the out-of-sample data. For the purposes of discussion, let's ignore junk model for now. We notice that remaining three models did not predict the best out-of-sample as compared to insample since the MAE and MSE values are higher in out-of-sample. However, the difference is not that high. Looking at the values in Figure 16 and Figure 15, we only have the difference of \$2,433 (approx.) which is not the best nor the worst, in my opinion. My own preference is always to use MAE for comparison and explanation because its unit is same as of response variable. Therefore, it helps to convey the message and make the point across easily. Since we know that we have a difference in MAE, our models have better predictive accuracy in-sample then out-of-sample. Hence, it means that our model is slightly overfitting. I say slightly here because the difference is not a lot.

MSE_Values MAE_Values junk.Im.test 877239140 19827.30
Forward.Im.test 1287709268 23665.08
backward.Im.test 1287709268 23665.08
stepwise.Im.test 1287709268 23665.08

Figure 16: Out-of-Sample Metrics

Section 4: Operational Validation

So far we have validated our models in the statistical sense. In this section, we validate these models in the business sense as well. For our business rule, we state the same policy that GSEs use to rate an AVM model as 'underwriting quality' i.e. we need our model to be accurate to within ten percent (10%) more than fifty percent (50%) of the time. In order to test our model against the business rule, we categorize the predicted value into different grades: 'Grade 1', if it is within ten percent of the actual value, 'Grade 2', if it is not Grade 1 but within fifteen percent of the actual value, 'Grade 3', if it is not Grade 2 but within twenty-five percent of the actual value, and 'Grade 4' over twenty-five percent of the actual value.

Figure 17 shows the table in distribution form of the prediction grades for the in-samples training data and the out-of-sample test data. Columns that have ".train.result" are the results from in-sample training data and columns that have ".test.results" are the results from out-of-sample test data.

Based on the results in Figure 17, we see that both in-sample and out-sample pass the policy of our business rule. We see that our models i.e. forward, backward, and stepwise selection, are accurate within ten percent more than fifty percent of the time. Hence, we see the majority falls under Grade 1. When comparing the results to our predictive accuracy results in section 3, we see the trend remains the same. The models predicted out-of-sample results lower than the in-sample results. However, our models passed the underwriting quality in out-of-sample test data.

Figure 17: Prediction Grades

| | forward.train.result | forward.test.result | backward.train.result | backward.test.result | stepwise.train.result | stepwise.test.result | junk.train.result | junk.test.result |
|----------------------|----------------------|---------------------|-----------------------|----------------------|-----------------------|----------------------|-------------------|------------------|
| Grade 1: [0,0.10] | 0.53865337 | 0.5181818 | 0.53865337 | 0.5181818 | 0.53865337 | 0.5181818 | 0.55112219 | 0.58787879 |
| Grade 2: (0.10,0.15] | 0.19825436 | 0.1969697 | 0.19825436 | 0.1969697 | 0.19825436 | 0.1969697 | 0.19576060 | 0.21515152 |
| Grade 3: (0.15,0.25] | 0.17331671 | 0.1848485 | 0.17331671 | 0.1848485 | 0.17331671 | 0.1848485 | 0.17082294 | 0.13636364 |
| Grade 4: (0.25+] | 0.08977556 | 0.1000000 | 0.08977556 | 0.1000000 | 0.08977556 | 0.1000000 | 0.08229426 | 0.06060606 |

Conclusion:

In conclusion, I would like to state that even the junk model here outperformed all the other three models by every measure. However, there's no guarantee that the junk model will be predictive. It did not pass the multicollinearity test. Even though multicollinearity does not affect prediction by much, it is always good to check for it.

Our three models i.e. forward, backward, and stepwise selections, selected the same predictor variables. Hence, the metrics were the same for each of these. When tested these models for predictive accuracy, we came to know that these models were overfitting. However, the difference of approx. \$2,433 was not that significant in terms of the sale price of the house.

Lastly, all three of my automated variable selection models are accurate to within ten percent more than fifty percent of the time. Hence, models are of 'underwriting quality' as per the GSEs rating.