EE499

MODELLING SLIDING MODE CONTROLLER AND SIMULATION OF VEHICLE SUSPENSION SYSTEM WITH ACTIVE AND SEMI-ACTIVE DAMPERS

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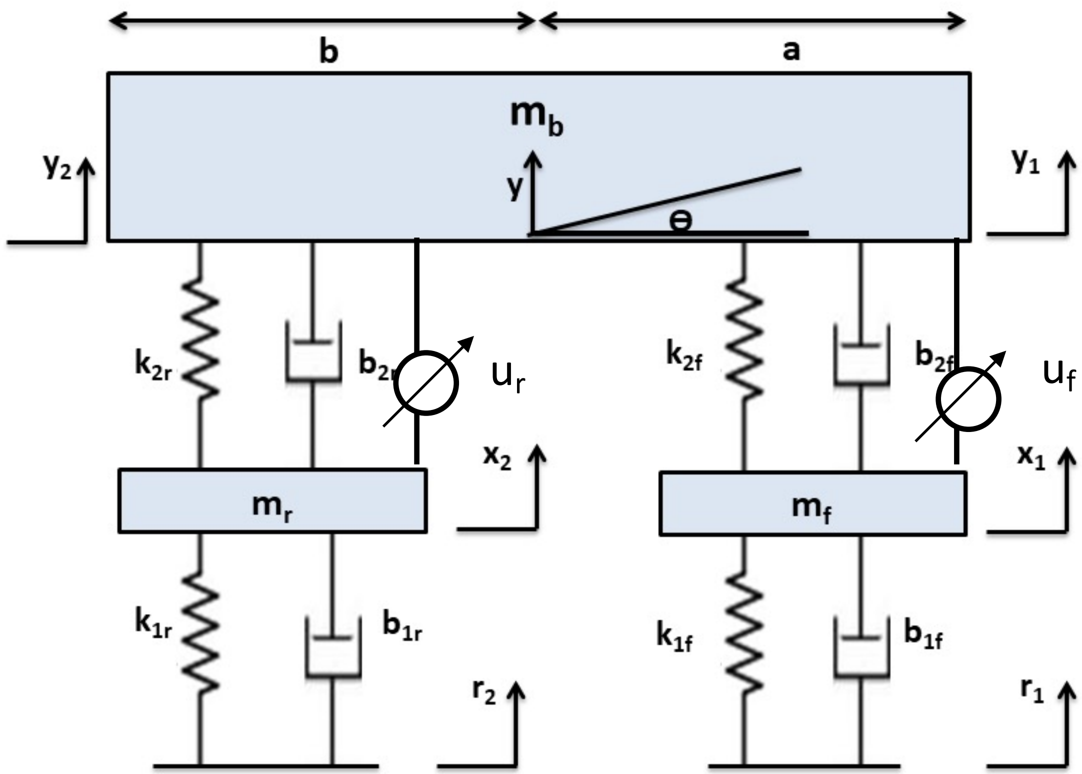
HALF CAR MODEL



FBD OF FRONT WHEEL

Equations from the free-body diagram:

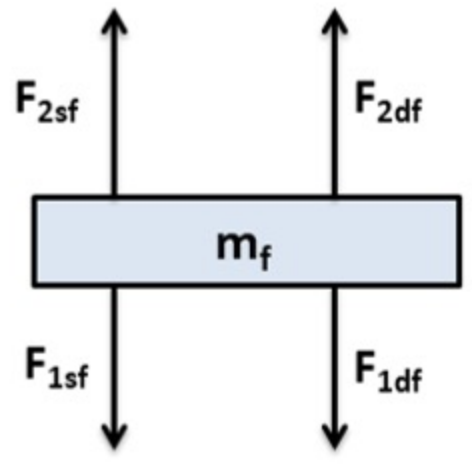
mf x1” = F2sf + F2df - F1sf – F1df

mf x1” = k2f (y1 – x1) + b2f (y1’ – x1’) – k1f (x1 – r1) - b1f (x1’ – r1’)

Substituting for y1 = y + a Θ and rearranging,

- k2f y - b2f y’ - k2f a Θ - b2f a Θ’+ mf x1” + k2f x1 + k1f x1 + b2f x1’ + b1f x1’ =

k1f r1 + b1f r1’



FBD OF SUSPENSION SYSTEM

- Equations from the free-body diagram: mb y” = - k2f (y1 – x1) - b2f (y1’ – x1’) - k2r (y2 – x2) - b2r (y2’ – x2’)
- Substituting for (y1 = y + a ϴ) and (y2 = y - b ϴ) and rearranging,

k2f y + k2r y + b2f y’ + b2r y’ + mb y” + k2f a ϴ - k2r b ϴ + b2f a ϴ’ - b2r b ϴ’ - k2f x1 - b2fx1’ - k2r x2 - b2r x2’ = 0

DYNAMIC MODELLING OF THE HALF CAR

- $M\ddot{x} + Sx + Tx = Du + Er,$

- Where the state, active control and excitation vectors are given by:

$$x = (y_1 \quad x_1 \quad y_2 \quad x_2 \quad \dot{y_1} \quad \dot{x_1} \quad \dot{y_2} \quad \dot{x_2})^T$$
$$u = (u_f \quad u_r)^T$$
$$r = (\dot{r_1} \quad r_1 \quad \dot{r_2} \quad r_2)^T$$

MATRICES

- $M = \begin{pmatrix} (bm_b)/L & 0 & (am_b)/L & 0 \\ I_b/L & 0 & -I_b/L & 0 \\ 0 & m_1 & 0 & 0 \\ 0 & 0 & 0 & m_2 \end{pmatrix}$
- $S = \begin{pmatrix} b_{2f} & -b_{2f} & b_{2r} & b_{2r} \\ a(b_{2f}) & -a(b_{2f}) & -b(b_{2r}) & b(b_{2r}) \\ -b_{2f} & b_{2f} & 0 & 0 \\ 0 & 0 & -b_{2r} & b_{2r} \end{pmatrix}$

Where I_b is the moment of inertia for the vehicle body

MATRICES

- $T = \begin{pmatrix} k_{2f} & -k_{2f} & k_{2r} & -k_{2r} \\ a(k_{2f}) & -a(k_{2f}) & -b(k_{2r}) & -b(k_{2r}) \\ -k_{2f} & (k_{2f} + k_{1f}) & 0 & 0 \\ 0 & 0 & (-k_{2r}) & (k_{2r}+k_{1r}) \end{pmatrix}$

- $E = \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ k_{1f} & 0 & 0 & 0 \\ 0 & 0 & k_{1r} & 0 \end{pmatrix}$

$D = \begin{pmatrix} 1 & 1 \\ a & -b \\ -1 & 0 \\ 0 & -1 \end{pmatrix}$

STATE SPACE EQUATION AND SMC

- $\dot{x(t)} = Ax(t) + B_{in}u(t) + r(t)$

Where $x(t) \in R^n$ is state vector and $u(t) \in R^m$ is the control input and $r(t)$ represents uncertainties with mismatched conditions.

SWITCHING SURFACE AND CONTROLLER DESIGN

- The sliding surface is given as:

$$\sigma(t) = Cx(t)$$

- To enter sliding mode, $\sigma(t) = 0$
- $\boxed{\dot{\sigma(t)} = 0}$
- $\boxed{u_{eq} = (CB_{in})^{-1}(-CAx - Cr(t))}$
-

- $u = u_{eq} + u_{sw}$

- Thus,

$$u = -(CB_{in})^{-1}((CAx + Cr) + \mathrm{sgn}(\sigma))$$

Simulations

Road Input	Simulation figure
Sine wave (0.1m Amplitude, Freq = 1 Hz) Sine wave (0.1m Amplitude, Freq= 0.5Hz) Sine wave (0.1m Amplitude, Freq= 2 Hz)	..\Downloads\HC_Active\hcsine_0.1_1_f.fig ..\Downloads\HC_Active\hcsine_0.1_1_r.fig ..\Downloads\HC_Active\hcsine_0.1_1_control.fig ..\Downloads\HC_Active\hcsine_0.1_0.5_f.fig ..\Downloads\HC_Active\hcsine_0.1_0.5_r.fig ..\Downloads\HC_Active\hcsine_0.1_2_f.fig ..\Downloads\HC_Active\hcsine_0.1_2_r.fig
Step Input (0.1m Amplitude)	..\Downloads\HC_Active\hcstep_0.1_f.fig ..\Downloads\HC_Active\hcstep_0.1_r.fig
Pulse (0.049m Amp., 0.56s width)	..\Downloads\HC_Active\hcpulse_0.49amp_0.56width_f.fig ..\Downloads\HC_Active\hcpulse_0.49amp_0.56width_r.fig
Random (0.046m Amplitude)	..\Downloads\HC_Active\hcrandom_0.46max_f.fig ..\Downloads\HC_Active\hcrandom_0.46max_r.fig

Semi-active suspension

Advantages

- Cost-effective
- Lower power consumption
- Robust

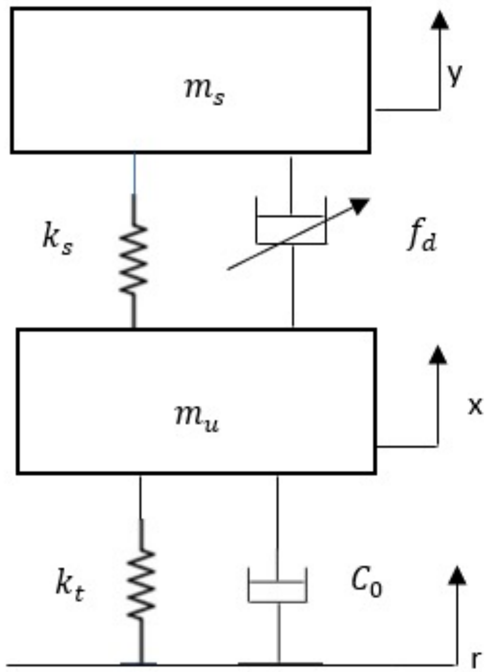
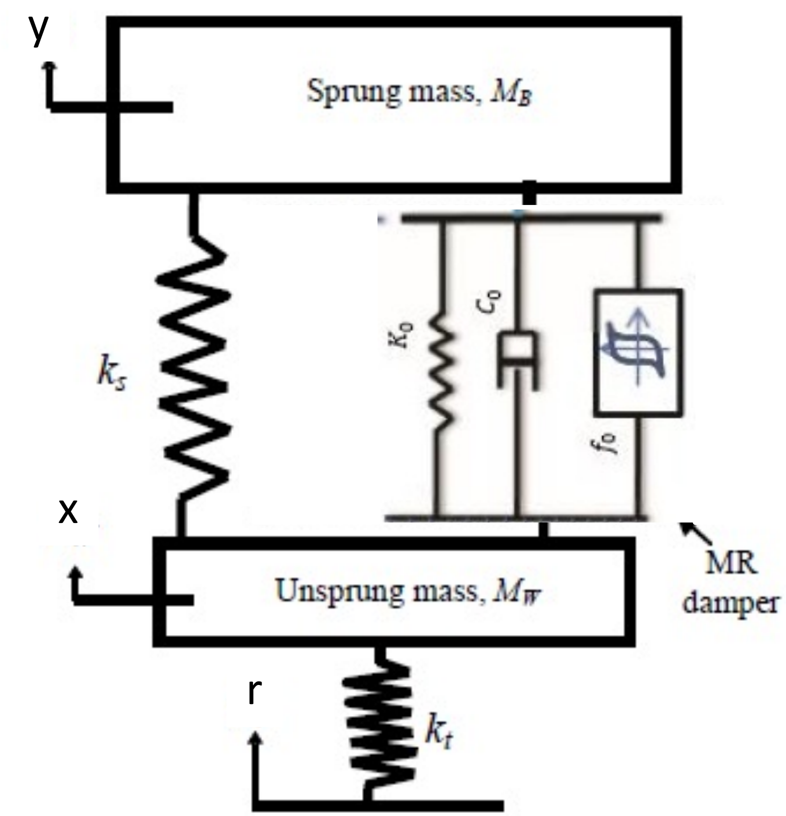


Fig: Quarter Car Model with Semi-active suspension

Quarter Car Model with MR Damper



$$f_d = C_0(\dot{a}) + K_0\, a + \alpha(z) + f_0]$$

$$\dot{z} = -\gamma|\dot{a}|z|z|^{n-1} - \beta(\dot{a})|z|^{n} + \delta(\dot{a})$$

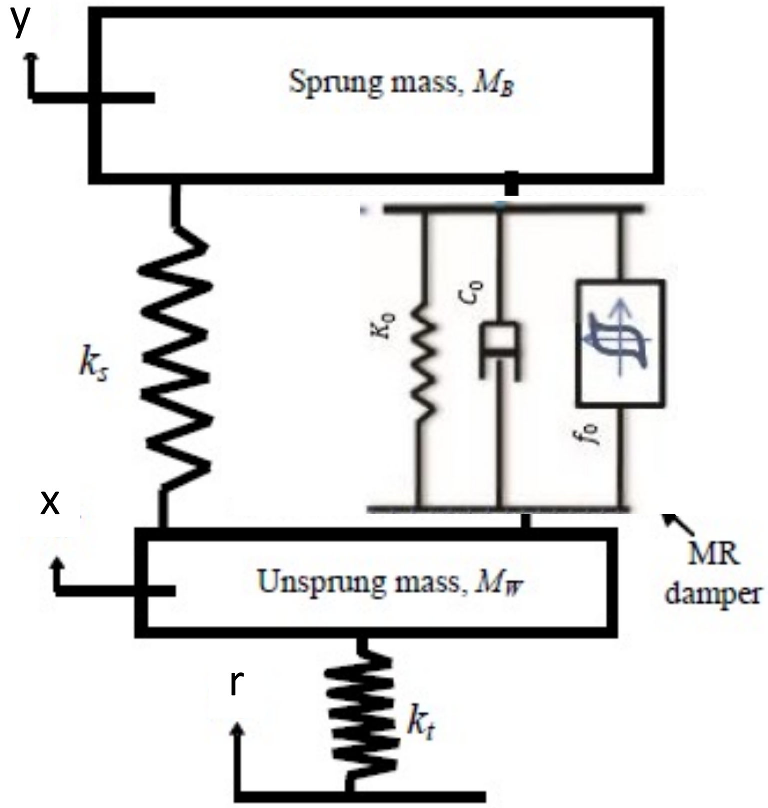
$$\alpha = \alpha_a + \alpha_b\, V$$

$$C_0 = C_{0a} + (C_{0b})V$$

$$\dot{V} = -\eta(V - v)$$

Deriving Controller Output

- $m_s(\ddot{y}) + k_s(y - x) + f_d = 0$
- $m_w\ddot{x} - k_s(y - x) - k_t(r - x) - f_d = 0$



Sliding Surface :
$S = \dot{e} + \lambda e$
$e = y - y_{ref}$

$$\dot{S} = \frac{-K_s}{m_s}(y - x) - \frac{f_d}{m_s} - \ddot{y_{ref}} + \lambda \dot{e}$$

Putting $\dot{S} = 0$,

$$f_{d0} = -K_s(y - x) - m_{s0}(\ddot{yref}) + m_{s0}(\lambda)(\dot{y} - \dot{yref})$$

$$\boxed{\boldsymbol{f_c = f_{d0} - (k_d)sgn(S)}}$$

Deriving k_d

$$\dot{V_{SMC}} = S\dot{S} \leq -\phi|S|$$

$$f_{d0} = -k_s(y-x) - m_{s0}(\ddot{yref}) + m_{s0}(\lambda)(\dot{y} - \dot{yref})$$

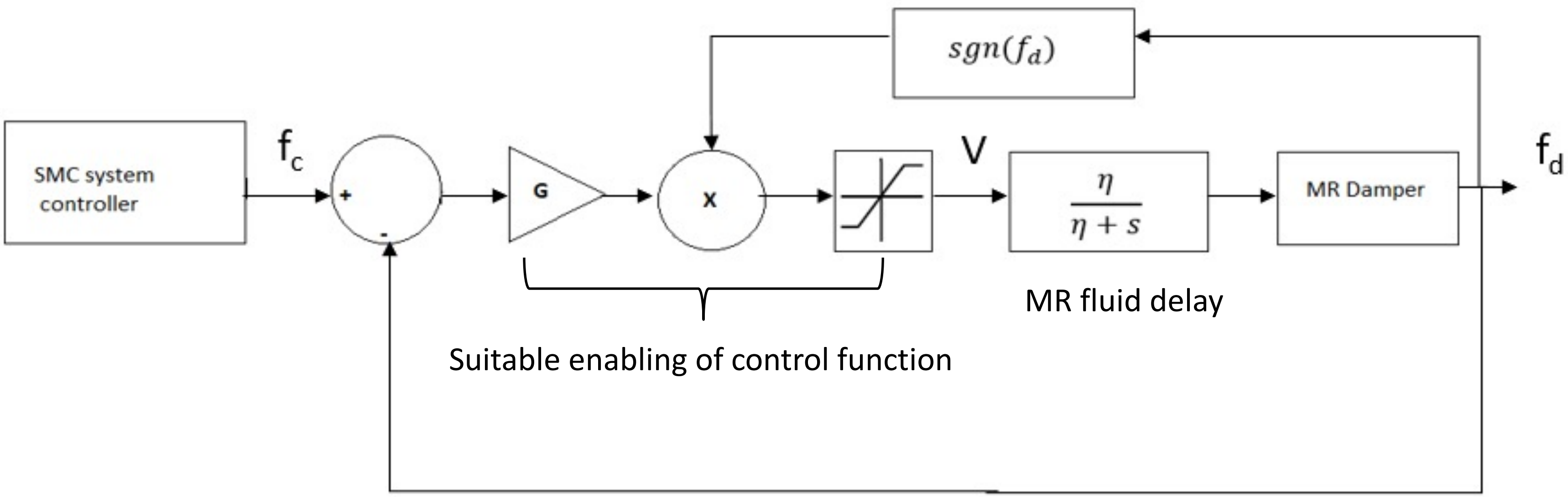
$$\dot{S} = \frac{1}{m_s}[(-k_s)(y-x) - f_d] - \ddot{y_{ref}} + \lambda\dot{e}$$

$$\dot{V_{SMC}} = [(\frac{1}{m_s} - \frac{1}{m_{s0}})(-k_s y + k_s x - f_{d0} + \frac{k_d}{m_s}sgn(s)]s$$

$$\left[\left(\frac{1}{m_s} - \frac{1}{m_{s0}}\right)(-K_s y + K_s x - f_{d0}) + \frac{k}{m_s}sgn(s)\right]s \leq -\phi.s.sgn(s)$$

$$\boxed{\boldsymbol{k_d = -m_{s0}q\phi - (q-1)(|f_{d0}| + k_s|y| + k_s|x|)}}$$

Damper Control Strategy

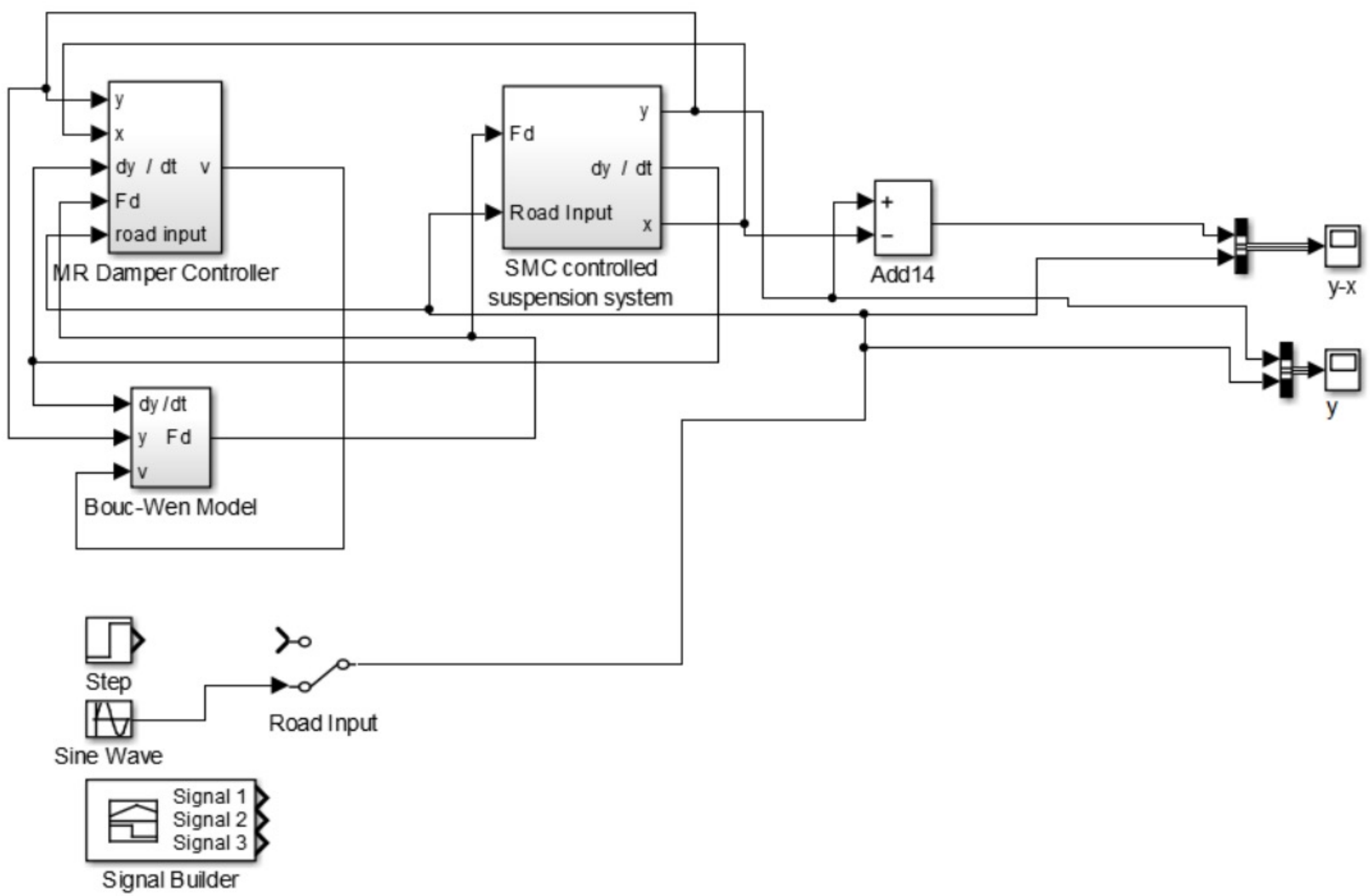


$$v = G(f_c - f_d)sgn(f_d)$$

If $G(f_c - f_d)sgn(f_d) > V_{max}, then\ v = V_{max}$

If $G(f_c - f_d)sgn(f_d) < V_{min}, then\ v = V_{min}$

Simulink Model



Simulations

Road Input	Simulation figure
Sine wave (0.1m Amplitude, Freq = 1 Hz)	..\Downloads\QC_Semiactive_MR\qcsine_0.1_1.fig
Sine wave (0.1m Amplitude, Freq= 0.5Hz)	..\Downloads\QC_Semiactive_MR\qcsine_0.1_0.5.fig
Sine wave (0.1m Amplitude, Freq= 2 Hz)	..\Downloads\QC_Semiactive_MR\qcsine_0.1_2.fig
Step Input (0.1m Amplitude)	..\Downloads\QC_Semiactive_MR\qcstep_0.1.fig
Pulse (0.049m Amp., 0.56s width)	..\Downloads\QC_Semiactive_MR\qcpulse_0.49amp_0.56width.fig
Random (0.046m Amplitude)	..\Downloads\QC_Semiactive_MR\qcrandom_0.46maxamp.fig