Tutorial-2

Ans 1

```
void function(int n)
{
  int j=1, i=0;
  while(i<n)
  {
    i=i+j;
    j++;
  }
}
```

$j=1 \ , \ i=0+1$

$j=2 \ , \ i=0+1+2$

$\vdots$

Loop ends when $i >= n$

$\frac{k(k+1)}{2} > n$

$k^2 > n$

$k > \sqrt{n}$

$O(\sqrt{n})$

Ans 2 Recurrence Relation for Fibonacci Series:

$T(n) = T(n+1) + T(n-2)$

$T(0) = T(1) = 1$

- if $T(n-1) \approx T(n-2)$

$T(n) = 2\,T(n-2)$

$= \{2\,T(n-4)\} = 4\,T(n-4)$

$= 8\,T(n-6)$

$= 16\,T(n-8)$

$\vdots$

$T(n) = 2^k\,T(n-2k)$

- if $T(n-2) \approx T(n-1)$

$$T(n) = 2T(n-1)$$
$$= 2(2T(n-2)) = 4T(n-2)$$
$$= 8T(n-3)$$
$$\vdots$$
$$T(k) = 2^k T(n-k)$$

$n-k=0$

$k=n$

$T(n) = 2^k \times T(0) = 2^n$

$\Rightarrow T(n) = O(2^n)$

Ans 3. $O(n(\log n)) \Rightarrow$

```
for (int i=0; i<n; i++)
{
    for (int j=1; j<n; j=j*2)
    {
        // Some O(1)
    }
}
```

- $O(n^3) \Rightarrow$

```
for (int i=0; i<n; i++)
{
    for (int j=0; j<n; j++)
    {
        for (int k=0; k<n; k++)
        { // Some O(1) }
    }
}
```

- $O(\log(\log n)) \Rightarrow$

```
for (int i=1; i<=n; i=i*2)
{
    for (int j=1; j<=n; j=j*2)
    {
        // Some O(1)
    }
}
```

Ans 4 $T(n) = T(n/4) + T(n/2) + Cn^2$

- Assume $T(n/2) >= T(n/4)$

So, $T(n) = 2T(n/2) + Cn^2$

Applying master's theorem $(T(n)) = T(n/b) + f(n))$

$a=2, b=2$ $f(n) = n^2$

$c = \log_b a = \log_2 2 = 1$

$n^c = n$

Compare n^c & $f(n) = n^2$,

$f(n) = n^c$

So, $T(n) = \theta(n^2)$

Ans6
```
for (int i=2; i<=n; i= Pow(i,k))
{
    // Some (1)
}
```

Complexity of Pow(i,k) $\rightarrow O(\log N)$
$= \log(k)$

$i=2$
$i=2^k$
$i=2^{k^2}$
$\vdots$
$i=2^{k^m}$

Loop ends when $i>n$

$2^{k^m} > n$
$\log(2^{k^m}) > \log^n$
$k^m \log^2 > \log^n$
$k^m > \log^n$
$\log k^m > \log(\log^n)$
$M \log k > \log(\log^n)$
$M > \log(\log^n) / \log(k)$

$T(c) = O(\log(\log n))$

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Ans 8

a $100 < \log n < \sqrt{n} < n < \log(\log^n) < n\log n < \log n! < n!$
$< n^2 < \log^{2n} < 2^n < 2^{2n} < 4^n$

b $1 < \sqrt{\log n} < \log n < 2\log n < \log 2n < n < 2n < 4n < \log(\log n)$
$< n\log n < \log n! < n! < n^2 < 2 \times 2^n$

c $\log_8 N < \log_2 N < n\log_6 N < n\log_n N < \log n! < N! < 5N < 8N^2 < 7N^3$
$< 8^{2N}$

Mohan

Teacher's Signature