

Tutorial-4

Ans 1 $T(n) = 3T(n/2) + n^2$

$a=3, b=2$

$n^{\log_a b} = n^{\log_2 3}$

Comparing $n^{\log_2 3}$ & n^2
 $n^{\log_2 3} < n^2$

(Case 3)

∴ According to master theorem,

$T(n) = \Theta(n^2)$

Ans 2 $T(n) = 4T(n/2) + n^2$

$a=4, b=2$

$n^{\log_2 4} = n^2 = f(n)$

(Case 2)

∴ According to master theorem,

$T(n) = \Theta(n^2 \log n)$

Ans 3 $T(n) = T(n/2) + 2^n$

$a=1, b=2$

$n^{\log_2 1} = n^0 = 1$

∴ $T(n) = \Theta(2^n)$

Ans 4 $T(n) = 2^n T(n/2) + n^n$

Master's theorem is not applicable.

Ans 5 $T(n) = 16 T(n/4) + n$

$a=16, b=4 \quad f(n)=n$

$$n^{\log_4 16} = n^2$$

$$n^2 > f(n)$$

$\therefore T(n) = \Theta(n^2)$

Ans 6 $T(n) = 2 T(n/2) + n \log^n$

$a=2, b=2, f(n) = n \log^n$

$$n^{\log_2 2} = n$$

$$f(n) > n$$

\therefore

$\therefore T(n) = \Theta(n \log n)$

Ans 7 $T(n) = 2 T(n/2) + n/\log n$

$$a=2, b=2, f(n) = n/\log n$$

$$n^{\log_2 2} = n$$

$$n > f(n)$$

$$\therefore T(n) = \Theta(n)$$

$$\text{Ans 8 } T(n) = 2T(n/4) + n^{0.51}$$

$$a=2, b=4, f(n) = n^{0.51}$$

$$n^{\log_4 2} = n^{0.5}$$

$$\therefore T_n = \Theta(n^{0.51})$$

$$\text{Ans 9 } T(n) = 0.5T(n/2) + 1/n$$

Master's Theorem is not applicable as $a < 1$.

$$\text{Ans 10 } T(n) = 16T(n/4) + n!$$

$$a=16, b=4, f(n) = n!$$

$$n \log_4^{16} = n^2$$

$$\therefore T(n) = \Theta(n!)$$

Ans 11 $T(n) = 4T(n/2) + \log n$

$$a=4, b=2, f(n) = \log n$$

$$n^{\log_2 4} = n^2$$

$$\therefore T(n) = \Theta(n^2)$$

Ans 12 $T(n) = \text{sqrt}(n) + T(n/2) + \log n$

Master's Theorem is not applicable.

Ans 13 $T(n) = 3T(n/2) + n$

$$a=3, b=2, f(n) = n$$

$$n \log_2^3 = n^{1.58}$$

$$T(n) = O(n \log_2^3)$$

Ans 14 $T(n) = 3T(n/3) + \sqrt{n}$

$$a=3, b=3, f(n) = \sqrt{n}$$

$$n^{\log_3 3} = n$$

$$T(n) = \Theta(n)$$

Ans 16 $T(n) = 3T(n/4) + n \log n$
 $a=3, b=4, f(n) = n \log n$

$$n^{\log_4 3} = n^{0.79}$$

$$T(n) = \Theta(n \log n)$$

Ans 17 $T(n) = 3T(n/3) + n/2$
 $a=3, b=3, f(n) = n/2$
 $n^{\log_3 3} = n$

$$\Theta(n) = \Theta(n/2)$$

$$T(n) = \Theta(n \log n)$$

Ans 18 $T(n) = 6T(n/3) + n^2 \log n$
 $a=6, b=3, f(n) = n^2 \log n$

$$n^{\log_3 6} = n^{1.63}$$

$$T(n) = \Theta(n^2 \log n)$$

Ans 19 $T(n) = 4T(n/2) + n/\log n$
 $a=4, b=2, f(n) = n/\log n$

$$n^{\log_2 4} = n^2$$

$$\therefore T(n) = \Theta(n^2)$$

Ans 20 $T(n) = 6T(n/8) - n^2 \log n$

Master's Theorem is not applicable.

Ans 21 $T(n) = 7T(n/3) + n^2$

$$a=7, b=3, f(n)=n^2$$

$$n^{\log_3 7} = n^{1.7}$$

$$\therefore T(n) = \Theta(n^2)$$

Ans 22 $T(n) = T(n/2) + n(2 - \cos n)$

Master's Theorem is not applicable.