



Find out the following
where S_j is length of
jth panel

$$x_j(s_j) = \frac{xb_{j+1} - xb_j}{S_j} s_j + xb_j = \cos \theta_j s_j + xb_j$$

$$y_j(s_j) = \frac{yb_{j+1} - yb_j}{S_j} s_j + yb_j = \sin \theta_j s_j + yb_j$$

The total potential
becomes as
following

$$\phi(x, y) = U(x \cos \alpha + y \sin \alpha) + \sum_{j=1}^m \frac{\sigma_j}{2\pi} \int_0^{S_j} \ln \sqrt{(x - xb_j - \cos \theta_j s_j)^2 + (y - yb_j - \sin \theta_j s_j)^2} ds_j$$

Boundary
Conditions
are applied

$$V_{\infty, n} + V_n = 0$$

Condition is
applied on
control points
and we get following

$$\frac{\partial \phi(xc_i, yc_i)}{\partial n_i} = U \left(\frac{\partial x}{\partial n_i} \cos \alpha + \frac{\partial y}{\partial n_i} \sin \alpha \right) + \sum_{j=1}^m \frac{\sigma_j}{2\pi} \int_0^{S_j} \frac{\partial}{\partial n_i} \left(\ln \sqrt{(xc_i - xb_j - \cos \theta_j s_j)^2 + (yc_i - yb_j - \sin \theta_j s_j)^2} \right) ds_j$$

Since we know

$$\frac{\partial x}{\partial n_i} = \sin \theta_i, \frac{\partial y}{\partial n_i} = -\cos \theta_i$$

Thus, we obtain
following

$$\sum_{j=1}^m \frac{\sigma_j}{2\pi} \int_0^{s_j} \frac{\partial}{\partial n_i} \left(\ln \sqrt{(xc_i - xb_j - \cos \theta_j s_j)^2 + (yc_i - yb_j - \sin \theta_j s_j)^2} \right) ds_j = U \sin(\theta_i - \alpha)$$

We will carry out the
integration such
that

The boundary condition becomes:

$$\sum_{j=1}^m \frac{\sigma_j}{2\pi} \left(-\frac{C_{ij} F_{ij}}{2} + D_{ij} G_{ij} \right) = U \sin(\theta_i - \alpha)$$

Where: $A_{ij} = -(xc_i - xb_j) \cos \theta_j - (yc_i - yb_j) \sin \theta_j$

$$B_{ij} = (xc_i - xb_j)^2 + (yc_i - yb_j)^2$$

$$C_{ij} = \sin(\theta_i - \theta_j), \quad D_{ij} = \cos(\theta_i - \theta_j), \quad F = \ln \left(1 + \frac{S_j^2 + 2A_{ij}S_j}{B_{ij}} \right)$$

$$E_{ij} = (xc_i - xb_j) \sin \theta_j - (yc_i - yb_j) \cos \theta_j, \quad G_{ij} = \tan^{-1} \left(\frac{E_{ij} S_j}{A_{ij} S_j + B_{ij}} \right)$$

$$-\frac{C_{ii} F_{ii}}{2} + D_{ii} G_{ii} = -\pi$$

The tangential
velocity is
given as

$$v_{t_i} = \frac{\partial \phi(xc_i, yc_i)}{\partial t_i}$$

Thus, coefficient of
pressure is obtained
as follows

$$c_{p_i} = 1 - \left(\frac{v_{t_i}}{U} \right)^2$$

Code for Airfoil
NACA 4 digit

1st digit = maximum camber as a percentage of chord
2nd digit = distance of maximum camber from the airfoil leading edge in tenths of the chord
3rd & 4th digits: maximum thickness of airfoil as percent of chord

Symmetric
airfoil

$$y_t = 5t \left[0.2969\sqrt{x} - 0.1260x - 0.3516x^2 + 0.2843x^3 - 0.1015x^4 \right]$$

where:

x is the position along the chord from 0 to 1.00 (0 to 100%),

y_t is the half thickness at a given value of x (centerline to surface),

t is the maximum thickness as a fraction of the chord

$$x_U = x_L = x$$

$$y_U = +y_t$$

$$y_L = -y_t$$

(x_U, y_U) and (x_L, y_L)
are coordinates of
upper and lower airfoil
surface

Cambered
airfoil

$$y_c = \begin{cases} \frac{m}{p^2} \left(2p \left(\frac{x}{c} \right) - \left(\frac{x}{c} \right)^2 \right), & 0 \leq x \leq pc, \\ \frac{m}{(1-p)^2} \left((1-2p) + 2p \left(\frac{x}{c} \right) - \left(\frac{x}{c} \right)^2 \right), & pc \leq x \leq c, \end{cases}$$

m is the maximum camber

p is the location of maximum camber

$$x_U = x - y_t \sin \theta, \quad y_U = y_c + y_t \cos \theta.$$

$$x_L = x + y_t \sin \theta, \quad y_L = y_c - y_t \cos \theta.$$