Obj $\max \left[\sum_{i \in M} \sum_{j \in J} c_{ij} x_{ij} \right]$

- M = Machines $M = \{1, 2, \ldots, 8\}$
- J = Jobs $J = \{1, 2, 3, \ldots, 24\}$

Constraints

① $\sum_{i \in M} x_{ij} = 1 \quad \forall j \in J$ (MC)

② $\sum_{j} a_{ij} x_{ij} \le b_i \quad \forall i \in M$

⇓

If machine 'i' is doing multiple Jobs, Total Time spent by should be less than b_i

$x_{ij} = 1$

Machine i ⇒ Does Job j

we want: Each Job Done by 1 Machine

$\Rightarrow \boxed{\sum_i x_{ij} = 1 \quad \forall j \in J}$

So Job 1

$\boxed{x_{11} + x_{21} + \ldots x_{8,1} = 1}$

only 1 machine doing Job 1

NOW Ⓐ Relaxing w.r.t "MC"

$$\text{obj} = \max \left[\sum_{i,j} c_{ij} x_{ij} + \sum_{j \in J} \lambda_j \left(1 - \sum_i x_{ij} \right) \right]$$

s.t. $\sum_j a_{ij} x_{ij} \le b_i \quad \forall i$

$$\Rightarrow \max \left[\sum_j \sum_i c_{ij} x_{ij} + \sum \lambda_j - \sum_j \sum_i \lambda_j x_{ij} \right]$$

$$= \max \left[\sum_i \left[\sum_j c_{ij} x_{ij} - \sum_j \lambda_j x_{ij} \right] + \sum \lambda_j \right]$$

$$= \sum \lambda_j + \max \left[\sum_i \left[\sum_j c_{ij} x_{ij} - \sum_j \lambda_j x_{ij} \right] \right]$$

$\sum_j a_{ij} x_{ij} \le b_i$

finally

$$\sum \lambda_j + \max \sum_i \left[\sum_j (c_{ij} - \lambda_j) x_{ij} \right] \quad \text{LR}_\lambda$$

$\sum_j a_{ij} x_{ij} \le b_i \quad \forall i$

@ λ^k

$v(LR_\lambda^k)$ = optimal soln with $\lambda = \lambda^k$

$$v(LR_\lambda^k) = \sum_j \lambda_j + \sum_i \max \left[\sum_j (c_{ij} - \lambda_j) x_{ij} \right]$$

s.t. $\sum a_{ij} x_{ij} \le b_i \quad \forall i$

$x_{ij} \in \{0,1\}$

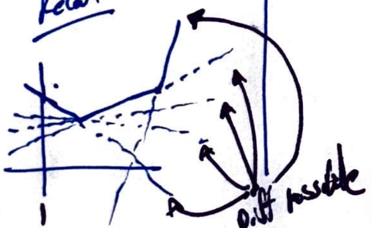
Breaks into i-s sub-problems.

let : optimal soln be $x(k)$

$$\Rightarrow \eta^k = \sum_j \lambda_j^k + \sum_i \sum_j (c_{ij} - \lambda_j) x_{ij}^k$$] max objective

$a_{ij} x_{ij}^k \le b_i$

Recall



$LR_\lambda = \max(x, \lambda)$

Linear func in λ

$\Rightarrow$ upper envelope

$\boxed{\text{i.e. Convex}}$

$$obj \equiv \max \left[\sum_j \sum_i c_{ij} x_{ij} + \sum_j \lambda_j \left(1 - \sum_i x_{ij}\right) \right]$$

$$\begin{bmatrix} \frac{\partial obj}{\partial \lambda_1} \\ \frac{\partial (\,)}{\partial \lambda_2} \\ \vdots \\ \frac{\partial (\,)}{\partial \lambda_j} \end{bmatrix} = \begin{bmatrix} 1 - \sum_i x_{i1} \\ 1 - \sum_i x_{i2} \\ \vdots \\ 1 - \sum_i x_{ij} \end{bmatrix}_{j \times 1} = S^{(k)}$$

MC Heuristic

Direction of Descent

$$\boxed{S^k = \frac{\partial obj}{\partial \lambda}}$$

GAP Problem

LD MC Dualsel

TASK: optimizing Lagrangian

$\min_\lambda g(\lambda)$

$\boxed{\text{I Cutting Plane Pseudo Code}}$

Problem

$$LR_\lambda \equiv \text{Max} \left[\sum_j \sum_i c_{ij} x_{ij} + \sum \lambda_j \left(1 - \sum_i x_{ij}\right) \right] = obj$$

s.t.

$$\sum_j a_{ij} x_{ij} \le b_i \quad \forall i$$

$\boxed{\text{II Cutting Plane}}$

$$v(LR) = \min_\lambda \left[v(LR_\lambda) \right]$$

Iteration 1

- add Constraint $\Rightarrow$ $\overbrace{\sum_j a_{ij} x_{ij} \le b_i}^{i=1}$

if (it = 1): Do NOT Solve LR

- add Const. to M.P. (LP)

$\eta \ge obj_i \quad \{i=1\}$.

Solve $\min(\eta) \to \eta^1$

To get λ^2

also subject to:$\Rightarrow$

$\sum_j a_{ij} x_{ij} \le b_i \quad i = \{1\}$

Iteration 2

- add Constraint $\Rightarrow$ $\overbrace{\sum_j a_{ij} x_{ij} \le b_i}^{i=2}$

Solve LR_λ $(\lambda = \lambda^2)$

$\max \sum\sum c_{ij} x_{ij} + \sum_j \lambda_j^{(2)} (1 - \sum_i x_{ij})$

s.t.

$\sum_j a_{ij} x_{ij} \le b_i \quad i = \{1,2\}$

$opt = z_{opt}^{(2)}$

if ($z_{opt}^2 == \eta^1$)] we found $V(LR)$

break

if NOT:

- add Constraint to Master (LP) P.

$\eta \ge obj_i \quad \{i = 1,2\}$

also constraints $\sum_j a_{ij} x_{ij} \le b_i \quad i = \{1,2\}$

Solve $\min(\eta) \longrightarrow \eta^2$

To get λ^3

NOT WORKING

~~[illegible]~~

we NEED CONVEX HULL

DEFINITE POINTS TO SOLVE FOR η

Iteration 3

- add Constraint $\Rightarrow$ $\overbrace{\sum_j a_{ij} x_{ij} \le b_i}^{i=3}$

Solve LR_λ $(\lambda = \lambda^3)$

$\text{Max} \left[\sum_j \sum_i c_{ij} x_{ij} + \sum_j \lambda_j^{(3)} (1 - \sum_i x_{ij}) \right]$

s.t. $\sum_j a_{ij} x_{ij} \le b_i \quad i = \{1,2,3\}$

$opt = z_{opt}^{(3)}$

Itr 3 --- Contd.

if ($3_{opt}^{3} = \eta^{2}$)

break

ELSE : Add Constraint to Master LP

$\eta \geq obj_i \quad \{i = 1,2,3\}$

also Constraints

$\sum_j a_{ij} x_{ij} \leq b_i \quad i = \{1,2,3\}$

Solve $\min(\eta) \to \eta^{3}$

also get $\lambda^{(4)}$.

⋮ So on

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iteration = (M - 1)

{ USE λ^{M-1}

get $3_{opt}^{(M-1)}$

if ($3_{opt}^{M-1} = \eta^{M-2}$)

break

If NOT : SOLVE MASTER LP.

- η^{M-1}
- λ^{M}

FINAL ITERATION ⟹

Iteration M

. add Final Constraint ($i = M$)

$\sum_j a_{ij} x_{ij} \leq b_i$

Solve ZLP { use λ^{M}

opt $= 3_{opt}^{(M)}$

{ Since Last iteration, We Should find }

$3_{opt}^{(M)} = \eta^{(M-1)}$.