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Page No.
Date: / /20

Tutorial - 4:

1. $T(n) = 3T(n/2) + n^2$

$a = 3, \quad b = 2, \quad f(n) = n^2$

$c = \log_b a = \log_2 3$

$\log_2 3$

$n^c < n^2 \quad (f(n) > n^c)$

Case III

$T(n) = \Theta(n^2)$

2. $T(n) = 4T(n/2) + n^2$

$a = 4, \quad b = 2, \quad f(n) = n^2$

$k = \log_2 4 = 2$

$n^2 = n^2 \quad (f(n) = n^c)$

Case II:

$T(n) = \Theta(n^2 \log n)$

3. $T(n) = T(n/2) + 2^n$

$a = 1, \quad b = 2, \quad f(n) = 2^n$

$c = \log_2 1 = 0$

$n^0 = 1$

$1 < 2^n \quad (f(n) > n^c)$

$T(n) = \Theta(2^n)$

4. $T(n) = 2^n T(n/2) + n^n$

$a = 2^n$ α → Master theorem does not apply as 'a' is not a constant

5. $T(n) = 16T(n/4) + n$

$a = 16, \quad b = 4, \quad f(n) = n$

$c = \log_4 16 = 2$

$n^2 > n \quad (n^c > f(n))$

$T(n) = \Theta(n^2)$

6. $T(n) = 2T(n/2) + n \log n$

$a = 2, \quad b = 2, \quad f(n) = n \log n$

$c = \log_2 2 = 1$

$n < n \log n \quad (n^c < f(n))$

$T(n) = \Theta(n \log n)$

7. $T(n) = 2T(n/2) + n/\log n$

$a = 2, \quad b = 2$

$c = \log_2 2 = 1$

$n > \frac{n}{\log n} \quad (n^c > f(n))$

$T(n) = \Theta(n)$

Page No.
Date : / /20

8. $T(n) = 2T(n/4) + n^{0.51}$

$a = 2, \; b = 4, \; f(n) = n^{0.51}$

$c = \log_4 2 = 0.5$

$n^{0.5} < n^{0.51}$ ($n^c < f(n)$)

$T = \Theta(n^{0.51})$

9. $T(n) = 0.5\,T(n/2) + \frac{1}{n}$

$a = 0.5$ ✗ → a must be equal to or greater 1.

master theorem does not apply.

10. $T(n) = 16\,T(n/4) + n!$

$a = 16, \; b = 4, \; f(n) = n!$

$c = \log_4 16 = 2$

$n^2 < n!$ ($f(n) > n^c$)

$T(n) = \Theta(n!)$

11. $T(n) = 4\,T(n/2) + \log n$

$a = 4, \; b = 2, \; c = \log n$

$c = \log_2 4 = 2$

$n^2 > \log n$, $T(n) = \underline{\Theta(n^2)}$

12. $T(n) = \sqrt{n}\, T(n/2) + \log n$

$a = \sqrt{n}$ & (a must be a constant)

Master theorem does not apply.

13. $T(n) = 3T(n/2) + n$

$a = 3, \; b = 2, \; f(n) = n$

$c = \log_2 3 =$

$n^{\log_2 3} > n$ $\quad (f(n) < n^c)$

$T(n) = \Theta(n^{\log_2 3})$

14. $T(n) = 3T(n/3) + \sqrt{n}$

$a = 3, \; b = 3, \; f(n) = \sqrt{n}$

$c = \log_3 3 = 1$

$n > \sqrt{n}$ $\quad (n^c > f(n))$

$T = \Theta(n)$

15. $T(n) = 4T(n/2) + cn$

$a = 4, \; b = 2 \Rightarrow f(n) = cn$

$c = \log_2 4 = 2$

$n^2 > cn$ $\quad (n^c > f(n))$

$T(n) = \underline{\Theta(n^2)}$

16. $T(n) = 3T(n/4) + n \log n$

$a = 3, \quad b = 4, \quad f(n) = n \log n$

$c = \log_4 3 =$

$n \log n > n^{\log_4 3} \qquad (f(n) > n^c)$

$T(n) = \Theta(n \log n)$

17. $T(n) = 3t(n/3) + n/2$

$a = 3, \quad b = 3, \quad f(n) = n/2$

$c = \log_3 3 = 1$

$n = \frac{n}{2}$

$T(n) = \Theta(n \log n)$

18. $T(n) = 6T(n/3) + n^2 \log n$

$a = 6, \quad b = 3, \quad c = n^2 \log n$

$c = \log_3 6$

$n^2 \log n > n^{\log_3 6} \qquad (f(n) > \Theta n^c)$

$T(n) = \Theta(n^2 \log n)$

19. $T(n) = 4T(n/2) + n/\log n$

$a = 4, \quad b = 2 \quad , \quad f(n) = n/\log n$

$c = \log_2 4 = 2$

$n^2 > \frac{n}{\log n}$

$T(n) = \Theta(n^2)$

20. $T(n) = 64T(n/8) - n^2 \log n$

$f(n) = -n^2 \log n \quad \times$

Should be a positive f(n)

Master theorem doesn't apply.

21. $T(n) = 7T(n/3) + n^2$

$a = 3, \quad b = 7 \quad , \quad f(n) = n^2$

$c = \log_3 7 =$

$n^2 > n^{\log_3 7}$

$T(n) = \Theta(n^2)$

Page No.
Date : / /20

22. $T(n) = T(n/2) + n(2 - \cos n)$

$a = 1, \quad b = 2, \quad f(n) = n(2 - \cos n)$

$c = \log_2 1 = 0$

$1 < n(2 - \cos n)$

$T(n) = \Theta(n)$ ✓