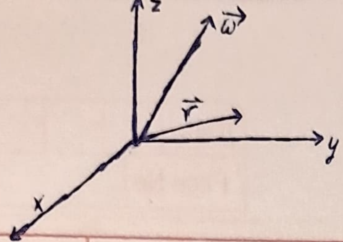


$$\frac{d\vec{r}}{dt}\Big|_s = \frac{d}{dt}\Big|_s \left(r_{x'}\hat{i}' + r_{y'}\hat{j}' + r_{z'}\hat{k}'\right)$$

derivatives of scalars do not depend on frame of reference. Thus for derivatives like of r_x, r_y, r_z, we lose ~~the subscr~~ the subscript

$$\Rightarrow \frac{d\vec{r}}{dt}\Big]_s = \frac{dr_{x'}}{dt}\cdot\hat{i}' + r_{x'}\cdot\frac{d\hat{i}'}{dt}\Big|_s + \frac{dr_{y'}}{dt}\cdot\hat{j}' + r_{y'}\cdot\frac{d\hat{j}'}{dt}\Big]_s + \frac{dr_{z'}}{dt}\cdot\hat{k}' + r_{z'}\cdot\frac{d\hat{k}'}{dt}\Big]_s$$

$$= \left[\left(\frac{dr_{x'}}{dt}\right)\hat{i} + \left(\frac{dr_{y'}}{dt}\right)\hat{j} + \left(\frac{dr_{z'}}{dt}\right)\hat{k}\right] + r_{x'}\cdot\frac{d\hat{i}'}{dt}\Big|_s + r_{y'}\cdot\frac{d\hat{j}'}{dt}\Big|_s + r_{z'}\cdot\frac{d\hat{z}'}{dt}\Big|_s \quad \text{-- (i)}$$

note

Now, $\frac{d\vec{r}}{dt}\Big]_{s'} = \frac{d}{dt}\Big|_{s'}\left(r_{x'}\hat{i}' + r_{y'}\hat{j}' + r_{z'}\hat{k}\right)$

$$= \frac{dr_{x'}}{dt}\hat{i}' + \frac{dr_{y'}}{dt}\cdot\hat{j}' + \frac{dr_{z'}}{dt}\cdot\hat{k}'$$ since as seen from S', $\hat{i}', \hat{j}', \hat{k}'$ are fixed

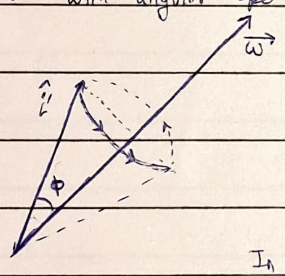
end

Hence, we have, continuing from (i)

$$\frac{d\vec{r}}{dt}\Big]_s = \frac{d\vec{r}}{dt}\Big]_{s'} + r_{x'}\frac{d\hat{i}'}{dt}\Big|_s + r_{y'}\cdot\frac{d\hat{j}'}{dt}\Big]_s + r_{z'}\cdot\frac{d\hat{z}'}{dt}\Big]_s \quad \text{-- (ii)}$$

note

$\frac{d\hat{i}'}{dt}\Big]_s$ is the rate of change of the $\hat{i}'$ vector as seen from a stationary frame. In this frame, $\hat{i}'$ is rotating about $\vec{\omega}$ with angular ~~spe~~ velocity $\vec{\omega}$



The head of the $\hat{i}'$ revolves around $\vec{\omega}$ with a radius $|\hat{i}'|\sin\phi$ and an angular velocity $\vec{\omega}$. Thus speed of head of $\hat{i}'$ is $|\vec{\omega}||\hat{i}'|\sin\phi$ and using the right hand thumb rule, velocity of head of $\hat{i}'$ is $\vec{\omega}\times\hat{i}'$

In time dt, the tail of $\hat{i}'$ stays put while the head moves by $\vec{\omega}\times\hat{i}'\,dt$. Thus,

$$\frac{d\hat{i}'}{dt}\Big]_s = \vec{\omega}\times\hat{i}'$$

end

Hence, we have, continuing from (ii)

$$\frac{d\vec{r}}{dt}\Big]_s = \frac{d\vec{r}}{dt}\Big]_{s'} + r_{x'}\vec{\omega}\times\hat{i}' + r_{y'}\cdot\vec{\omega}\times\hat{j}' + r_{z'}\,\vec{\omega}\times\hat{k}'$$

$$= \frac{d\vec{r}}{dt}\Big]_{s'} + \vec{\omega}\times\vec{r}$$

We thus get the overall result as a relation b/w derivative of a vector $\vec{O}$ as seen from a stationary frame with the derivative seen from a rotating frame

$$\frac{d\vec{O}}{dt}\Big]_s = \frac{d\vec{O}}{dt}\Big]_{s'} + \vec{\omega}\times\vec{O}$$