EE 229 - S2 (19-08-24) Quiz 1

Q1) (a) $y[n] = \text{Even}\{x[n-1]\}$

Let $x_0[n] = x[n-1]$

$y[n] = \text{Even}\{x_0[n]\} = \frac{1}{2}(x_0[n] + x_0[-n])$

$$\boxed{y[n] = \frac{1}{2}(x[n-1] + x[-n-1])}$$

Minus 0.5

(-0.5 for wrong expression & ~~most parts are correct~~)

Linear: $x_3[n] = a x_1[n] + b x_2[n]$

$$y_3[n] = \frac{x_3[n-1]}{2} + \frac{x_3[-n-1]}{2}$$

$$= a\frac{x_1[n-1]}{2} + b\frac{x_2[n-1]}{2} + \frac{a}{2}x_1[-n-1] + \frac{b}{2}x_2[-n-1]$$

$$= \frac{a}{2}y_1[n] + b y_2[n]$$

$y_3[n] = a y_1[n] + b y_2[n] \rightarrow$ Hence, Linear (0.5)

Time Invariancy: Let $x_1[n] = x[n-n_0]$

$$y_1[n] = \frac{1}{2}(x_1[n-1] + x_1[-n-1])$$

$$= \frac{1}{2}(x[n-1-n_0] + x[-n-1-n_0])$$

$$y[n-n_0] = \frac{1}{2}(x[n-n_0-1] + x[-(n-n_0)-1])$$

$$= \frac{1}{2}[x[n-n_0-1] + x[-n+n_0-1]]$$

$y[n-n_0] \neq y_1[n]$ Hence, not time invariant

(0.5)

Causal : not causal as $y[n]$ depends on $x[n-1]$ &

$y[-1] \to x[-2]$ & $x[0]$ $\xrightarrow{x[-n-1]}$ future value.

(0.5 for both

Memoryless $\to$ not memoryless as o/p value depends on past values.

causal & memoryless)
correct.

BIBO stable ; as bounded i/p gives bounded o/p. (0.5)

Note: No marks given if no justification is provided and only yes / no is written. Same for part (b)

(b) $y[n] = x[n] \quad n \geq 1$

$= 0 \quad n = 0$

$= x[n] \quad n \leq -1$

Linear:

Let $x_3[n] = a x_1[n] + b x_2[n]$

$y_3[n] = x_3[n] \quad n \geq 1$

$0 \quad n = 0$

$x_3[n] \quad n \leq -1$

$= a x_1[n] + b x_2[n] \quad , \quad n \geq 1$ & $n \leq -1$

$= a y_1[n] + b y_2[n] \quad \forall \; n \geq 1$ & $n \leq -1$

$= 0 \quad n = 0$

$= a y_1[n] + b y_2[n] \quad , \quad n = 0$

Hence, linear (0.5)

Time Invariant:

Let $x_1[n] = x[n-n_0]$

$$y_1[n] = \begin{cases} x_1[n] & n \geq 1 \\ 0 & n = 0 \\ x_1[n] & n \leq -1 \end{cases}$$

$$= \begin{cases} x[n-n_0] & n \geq 1 \\ 0 & n = 0 \\ x[n-n_0 & n \leq -1 \end{cases} = y_1[n]$$

And,

$$y[n-n_0] = \begin{cases} x[n-n_0] & n-n_0 \geq 1 \\ 0 & n-n_0 = 0 \\ x[n-n_0] & n-n_0 \leq -1 \end{cases}$$

$y_1[n] = y[n-n_0]$ only when $n_0 = 0$

Hence, not time invariant (0.5)

causal: system is causal as o/p value does not depend on future value

memoryless: system is memoryless as it depends only on $x[n]$ and not any past / future values. (0.5)

BIBO stable ✓ bounded $x[n]$ gives bounded $y[n]$ (0.5)

Q 2)

(ii) $y(t) = x(2t)$ As distinct input gives distinct output.

Let $u = 2t$

$t = u/2$

$y(u/2) = x(u)$

Now, $u = t$ $\boxed{x(t) = y(t/2)}$

↳ Invertible

(1)

0 if no justification is given

(iii)

$y[n] = x[2n]$

$y[0] = x[0]$

$y[1] = x[2]$

$y[2] = x[4]$

⋮

~~Odd~~ Input for odd value of n ~~cannot be retrieved~~ doesnt make any difference.

Like $x[1]$, $x[3]$ etc.

	0		2		4					
$x \rightarrow$	1	1	2	1	3	1	$y \rightarrow$	1	2	3
	1	0	2	0	3	0		1	2	3

Not invertible system

Hence, distinct i/p does not give distinct o/p.

Not Invertible

OR
If written how inputs at odd n can't be retrieved.

(1)

0 marks if no justification is given

For correct justificⁿ along with Invertible / Non Invertible

EE229 Quiz 1 (19/8/24) Solutions

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उत्तर पुस्तिका/ Answer Book-4

रोल नं./Roll No. पाठ्यक्रम नाम/Course Name

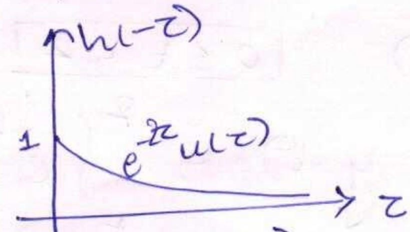
शाखा/प्रभाग/Branch/Div. शैक्षणिक बैच /Tutorial Batch अनुभाग/Section

[Maximum Marks = 3]

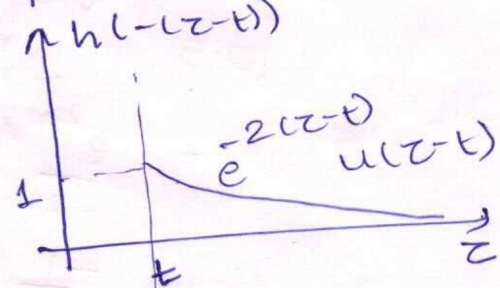
Q3] $h(\tau) = e^{2\tau} u(-\tau)$

$h(t-\tau) = h(-(-t+\tau)) = h(-\tau+t)$

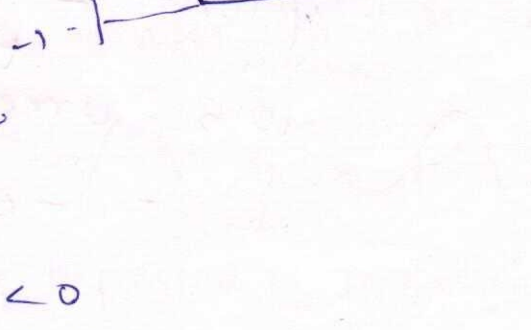
$h(-\tau) = e^{-2\tau} u(\tau)$



$h(-(\tau-t)) = h(-\tau+t)$



$x(\tau) = u(\tau) - 2u(\tau-2) + u(\tau-5)$



$y(t) = x(t) * h(t)$

$$y(t) = \int_{-\infty}^{\infty} x(\tau) h(t-\tau) d\tau$$

$$= \int_{-\infty}^{\infty} x(\tau) h(-(\tau-t)) d\tau$$

Thus, we get 4 cases
(i) $t < 0$
(ii) $0 \le t < 2$
(iii) $2 \le t < 5$
(iv) $t \geq 5$

(i) $\boxed{\text{For } t<0}$

$$y(t) = \int_0^2 e^{-2(\tau-t)}\,d\tau - \int_2^5 e^{-2(\tau-t)}\,d\tau$$

[0.5 - correct integrand and limits]

$$= \frac{e^{2t}}{-2}\left[e^{-2\tau}\right]_0^2 - \frac{e^{2t}}{-2}\left[e^{-2\tau}\right]_2^5$$

$$= -\frac{1}{2}e^{2t}\left[e^{-4} - 1 - e^{-10} + e^{-4}\right]$$

$$y(t) = \frac{e^{2t}}{2}\left[1 + e^{-10} - 2e^{-4}\right] \quad \text{for } t<0$$

(ii) $\boxed{0 \le t < 2}$

$$y(t) = \int_t^2 e^{-2(\tau-t)}\,d\tau - \int_2^5 e^{-2(\tau-t)}\,d\tau$$

[0.5 - correct integrand and limits]

$$= \frac{e^{2t}}{-2}\left[e^{-2\tau}\right]_t^2 - \frac{e^{2t}}{2}\left[e^{-2\tau}\right]_2^5$$

$$= -\frac{1}{2}e^{2t}\left[e^{-4} - e^{-2t} - e^{-10} + e^{-4}\right]$$

$$y(t) = \frac{e^{2t}}{2}\left[e^{-2t} + e^{-10} - 2e^{-4}\right] \quad \text{for } 0 \le t < 2$$

(iii) $\boxed{2 \le t < 5}$

$$y(t) = -\int_t^5 e^{-2(\tau-t)}\,d\tau$$

[0.5 - correct integrand and limits]

$$= -e^{2t}\int_t^5 e^{-2\tau}\,d\tau = \frac{e^{2t}}{2}\left[e^{-2\tau}\right]_t^5$$

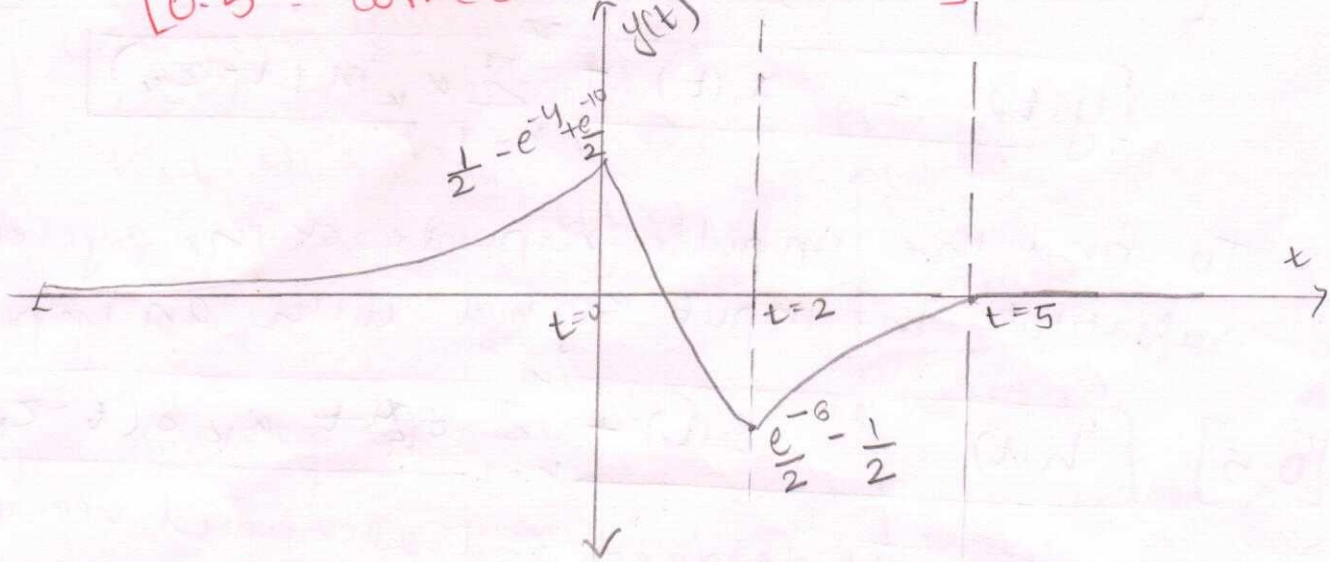
$$y(t) = \frac{e^{2t}}{2}\left[e^{-10} - e^{-2t}\right] \quad \text{for } 2 \le t < 5$$

(iv) $y(t) = 0$ $\boxed{t \ge 5}$

[0.5]

$$y(t) = \begin{cases} \frac{e^{2t}}{2}\left[1+e^{-10}-2e^{-4}\right] & t<0 \\ \frac{e^{2t}}{2}\left[e^{-2t}+e^{-10}-2e^{-4}\right] & 0\le t<2 \\ \frac{e^{2t}}{2}\left[e^{-10}-e^{-2t}\right] & 2\le t<5 \\ 0 & t\geq 5 \end{cases}$$

[0.5 - correct final answer]



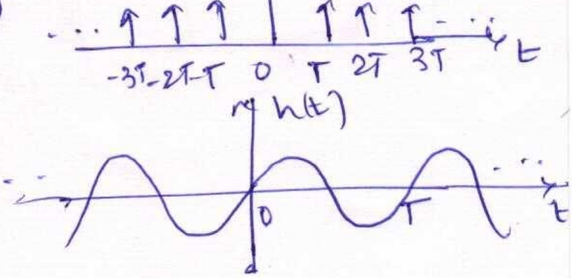
[0.5 - Sketch]

[Maximum marks = 1]

Q4] Any periodic signal with a period of T when convolved with a finite duration input signal will give an output which is periodic with period T.

eg: $h(t) = \sum_{k=-\infty}^{\infty} \delta(t-kT)$

<u>OR</u> $h(t) = \sin\left(\frac{2\pi}{T}t\right)$



[1 - If a periodic signal with a period T is mentioned]

<u>Note</u>: T needs to be a variable in the expression for the signal. A constant T eg: $T=2\pi$ in $\sin(t)$ is not sufficient

[Maximum Marks = 2]

Q5] Let the original signal be $x(t)$

Let the attenuation factor be α_k [0.5]

Let the delay be τ_k [0.5]

The output signal will be the original signal + the delayed and attenuated versions

[0.5 - for adding the original signal $x(t)$]

$$\boxed{y(t) = x(t) + \sum_{k=1}^{\infty} \alpha_k \, x(t-\tau_k)}$$

To find the impulse response of the system, substitute the input signal with an impulse

[0.5] $$\boxed{h(t) = \delta(t) + \sum_{k=1}^{\infty} \cancel{\delta_k(t} \; \alpha_k \, \delta(t-\tau_k)}$$

Note: A single delayed and attenuated version is also fine.