EE 210 : SIGNALS & SYSTEMS
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2.28	The following are the impulse responses of
	discrete-time LTI systems. Determine whether
	The following are the impulse responses of discrete-time LTI systems. Determine whether each system is causal and for stable. Justify
	you answer.

(a) 
$$h[n] = \left(\frac{1}{5}\right)^n u[n]$$

- · For system to be causal h[n] = 0 for n < 0
- For system to be stable  $\mathbb{I}$ hin $\mathbb{I}$  <  $\infty$   $n=-\infty$   $\rightarrow h[n]=0$  for n<0 .: CAUSAL

continuous - time ITI anothers. Determine

$$\rightarrow \sum_{n=-\infty}^{\infty} h(n) = \sum_{n=0}^{\infty} \left(\frac{1}{5}\right)^n = 1 = 5 < \infty \text{ ... STABLE}$$

ageten so coused (d) h[n] = 5" u[3-n]

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u[3-n] is u[n] fripped at 0 and shifted to right by 3.

5 1 h[n] = 0 n > 3

-> But h[n] to n < 0 : NON-CAUSAL

. STABLE

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	EE 2.10 : SIQNALL & SYSTEMS
	2 2 N = 2
	(f) $h[n] = (-1)^n u[n] + (1.01)^n u[1-n] = 1.85.5$
Justily	for $n < 0$ $\left(-\frac{1}{2}\right)^n u[n] = 0$
	2) . Domario may
	But u[-n]. (1.01)" +0
	> : h[n] \$0 for n<0 : NON-CAUSAL
	> : h[n] to for n < 0 : NON-CAUSAL
	2 h [m] = 5 [(-1, ) n   ( [m] + 5 ( ( ( m) ) ) ( ( [ m ] )
and and	$\rightarrow \sum_{n=-\infty}^{\infty} h(n) = \sum_{n=0}^{\infty} (-1/2)^n u[n] + \sum_{n=-\infty}^{\infty} (1\cdot01)^n u[1-n]$
	s > [m] All about 1 ab at mayor and on . STARIE
	JAZUAD : 0 > M 10 1 10 1 10 1 10 1
	- KMJ -10.1 for M<0 CAUSAL
	N/11 7 - C-2-7 -
	- 2 MW = 5 M
	The following are impulse responses of continuous - time LTI systems. Determine whethere each system is causal and for stable. Justify.
2.29	The following are impulse responses of continuous - time LTI systems. Determine whether each system is causal and for stable. Justify.  (a) h(t) = e^{-4t} u(t-2)
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$$\Rightarrow \int |h(t)| dt = \int e^{-4t} dt = -1 \left[e^{-4t}\right]^{\infty}$$

$$= 1 e^{-8} < \infty : STABLE$$

$$\rightarrow \int |h(t)| dt = \int e^{-6t} dt = \infty :: NOT STABLE$$

(e) 
$$h(t) = e^{-6|t|} = e^{-6t} u[-t] + e^{-6$$

2:30 Consider the first order difference equation

Assuming the condition of initial rest (i.e. x[n] = 0 for n < no, then y[n] = 0 for n < no),

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find the impulse response of a system whose input and output are related by this difference equation. You may solve the problem by rearranging the difference equations so as to express y[n] in terms of y[n-1] and x[n] and generating values of y[o], y[+1], y[+2]... Soln: - Holy 1

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we need to find the output y [n] when the input x[n] = S[n].

Assuming initial rest condition

y[n] = 0 for \*\* n < 0

y[n] = x[n] - 2y(n-1)

12 y[0] = x[0] - 2y[-1] = 1 -0 -1

y[i] = x[i] - 2y[o] = 0-2

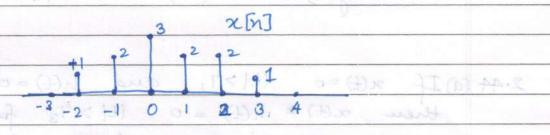
y[2] = -2y[1] = 4

y [3] = -2y [2] = -8

y[n] = (-2) nu[n]

Consider the LTI system initially at rest and described by the difference equation 2.31

Find the response of the system to the input depicted below by solving the difference equation recursively



SolM: Initial inv rest condition implies

y[n] = 0 for n < -2.

$$y[n] = x[n] + 2x[n-2] - 2y[n-1]$$

for 
$$n \ge 5$$
  $y[n] = -2y[n-1]$   
 $y[n] = -114(-2)^{n-5}$   $n > 5$ 



2.49	y(n) = 1 $n = -2$	
acres	Consider the 4I eyeten initially of near	18-12
	traviled by other difference equations	
	-4	
	[E-17] = x (16) x = [1-12] = x (17) + 2x [71-2]	
	-27 3	
suff of	Find the properse 88 the sometime of	
difference	and depicted belattle by solving the	
	equation reconstinely	
	$y[n] = -114(-2)^{n-5}$ $n > 5$	
	IMLX 8	
	5, 5, 5, 15	
2.44	(a) If x(t)=0  t >T, and h(t)=0  t >T	2
	then x(t) * h(t) = 0 (t) > T3 for some	T3.
	Express To in terms of To and To	
	Sol's Tritial for next condition ingolier	
	$\rightarrow$ x(t) * h(t) = [x(t) h(t-T)dt= [x(t) sh(t-	T) at
	- <del> </del>	
	CON CONTRACTOR OF THE PROPERTY	
	Note that h(-t) = 0 for It1> T2	
	: h(t-t) = 0 for T > t+T2 and.	t < -T2+t
	1=[3-] [-2]	
	: Integral evaluates to zero either	if
	: Integral evaluates to zero either $T_1 < -T_2 + t$ or $T_2 - t < -T_1$	V
	14 - E CONTRACTOR OF STOLE OF THE STOLE OF T	
	A : Integral = 0 for Itl	> T1+T2
	E-124E+13	
	1. B [N] = +114 (-=) 4-5 N > 5	- 7
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(b) A discrete - time LTI system has input x[n], impulse response h[n] and output y[n]. If h[n] is known to be zero everywhere outside No S n S N, and x [n] = 0 only for N S N S then output y[n] =0 everywhere except Nq = n < N5 i) Determine N4 and N5 in terms of No, N, N2, N3 ii) If No ≤n ≤ N, has length Mn and N2 ≤ n ≤ N3 has length Mx & N4 ≤ n ≤ N5 has length My Find My in terms of Mx and Mn. → i) y[n] = 2[n] \* n[n] = [ 2[n] h[n-k] = I h[K] x[n-K]  $z[-k] \neq 0$  for  $-N_3 \leq n \leq -N_2$  $\therefore x[n+k] \neq 0 \quad \text{for} \quad -N_3+n \leq k \leq -N_2+kn$ :. Sum non zero if -N3+n \in N1 and -N2 +n >No ... y[n] non-zero for n < N, + N3 and 0 M > No + N2 As we can see My = N1+N3 - (N0+N2) +1  $= (N_1 - N_0) + 1 + (N_3 - N_2) + 1 - 1$  $My = M_x + M_n - 1$ 

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2.44	(C) Consider a discrete time LTI system with the
	property that if input x(n) = 0 for all n = 10
بعدف	then output y [n] = 0 for all n = 15.
NEWZNS	what condition must h[n] satisfy for this?
Mes na Me	their output you so everywhere energy
Ma, Maskly all	h[n] is convolved with x[n] to get y[n]
	11) It NO SM S Made length the and
	:. y[n] = 0 + n = 15 given
	2(n)=0 + n=10
	M was an I amen't we am the sound
	A HERTER
	y[16] h $y[15] = x[9] h[6] + x[8] h[7]$
	(Expanding
	convolution)
	clearly h[n] = 0 + n>5
Va. + ke n	- = = 1 + 01 - 1 + 0+ ph(t) x =
2.44	(d) Given h(t) 1 1
and	M = N + M = D
	all x N + all -
Less been	over what range should we know x(t) to
	know y(b) completely ?
	~
1+ (=14+0	-> y(t) = h(t) * x(t) = h(t) x(t-t) dt =   x(t-t)
	- 60 -2
1-17	(=M-=M) + 1+ (=M-14) = +2(+-6)
	.: y(0) = x(-t)dt+x(-6)
	-2 : $t=-6$ , $(1,2)$
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2.49 Consider an LTI system with impulse response h[n] that is not absolutely summable i.e. enter (c) For a continuous (MI) (A I) is necessary that absolute integrability (A I) is necessary We will show that absolutely summability is a necessary condition for stability. Sufficiency of A.S. for stability is known and used in solving problem 2.28. (a) suppose  $x[n] = \begin{cases} 0 & \text{if } h[-n] = 0 \\ h[-n], & 0.\omega. \end{cases}$ [h[-n]] Is x[n] Bounded. Find the upper bound on 1x[n]1 Bound d.  $|x[n]| = \begin{cases} 6 & \text{if } h[-n] = 0 \\ 1 & \text{of } h[-n] \neq 0 \end{cases}$  $|x[n]| \leq 1$ (b) Calculate the output at N=0 for this particular input. Does the result prove  $y[0] = \sum_{k=-\infty}^{\infty} x[-k] h[k] = \sum_{k=-\infty}^{\infty} \frac{h[k]}{|h[k]|} = \sum_{k=-\infty}^{\infty} \frac{h[k]}{|h[k]|}$ .. Output is not Bounded . . . System is

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not stable

	:. Failure of A.S. causes instability.
وبدوو	-> Clearly this proper that A.S. is a Pt.
٠.	-> Clearly this proves that A.S. is a necessary condition for stability.
2.49	(c) For a continuous-time LTI system show
	that absolute integrability (A.I.) is necessary
0.79	for stability of system.
\ \	for stability of system.  Consider a LTI system with $\int  \mathbf{h}(t)  = \infty$
N CHAR	man a francourse wat . C. H To bought than
	Consider a particular input
	0=[N]N 2(t) = 50 Mx if h(-t)=0(x)
	(n(-t)) (n(-t))
_	(h(-t))
[[0]35]	The state of the s
12,32,41	Clearly  x(t)  = 1 Input to system is
	Boundea,
	Consider y(0) = 1 x (-t) h(t) dt
	$= \int h(t)^{2} = \int  h(t)  dt = \infty$
	(b) Calculate the output of N=0 for thi
we amplicing.	Output is unformed
15 7 15	1500 7 - 5000 15 do 2 :500 metall
LE   11 (E)	[Catal Constable.
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