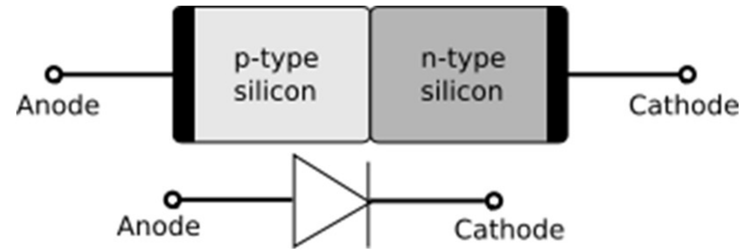
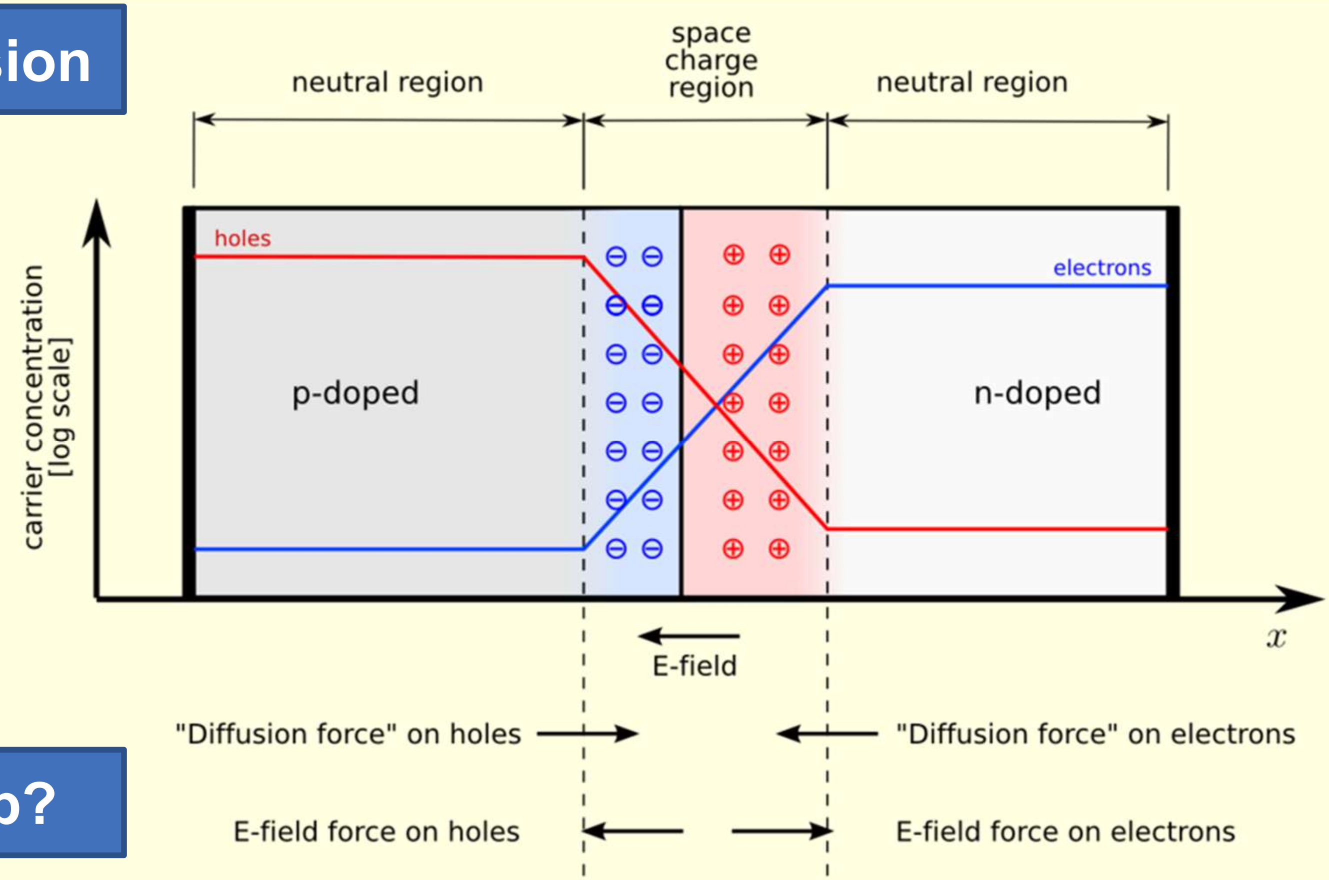
Idealized p-n junction diode

Ideal approximation: 1-D, abrupt

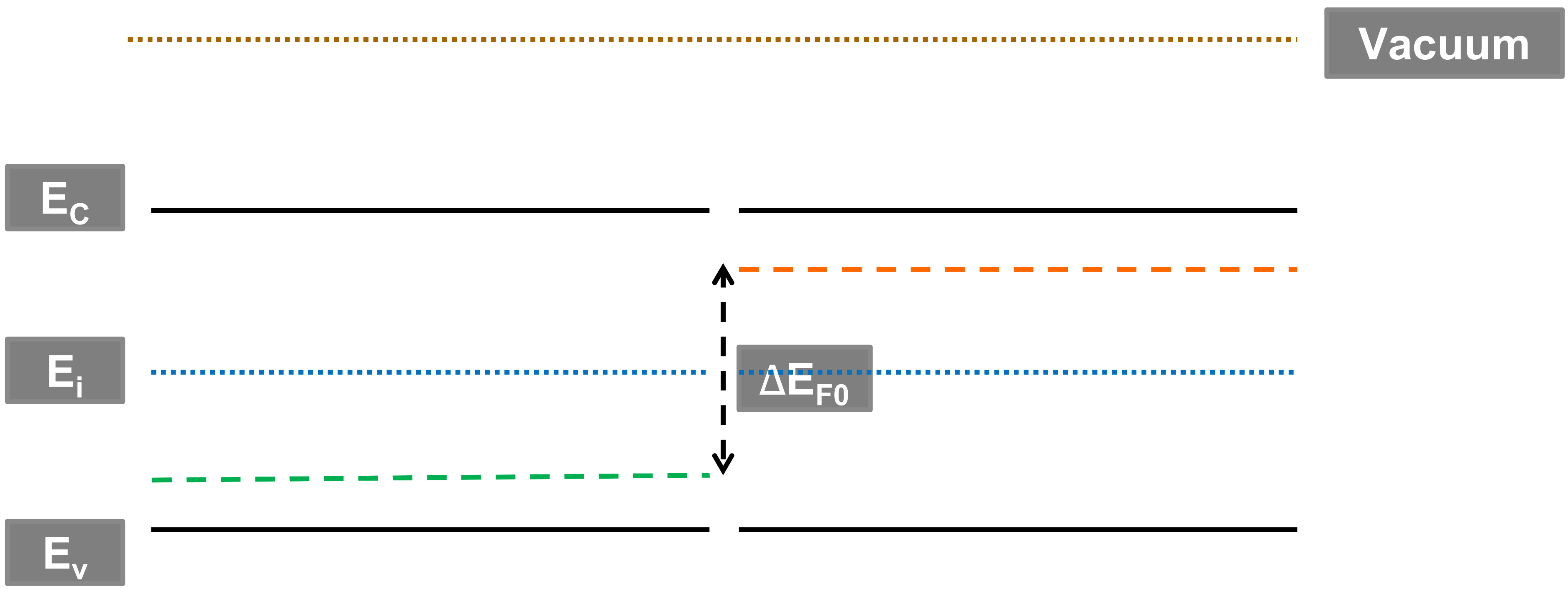


Inhomogeneity → diffusion



Why/where does it stop?

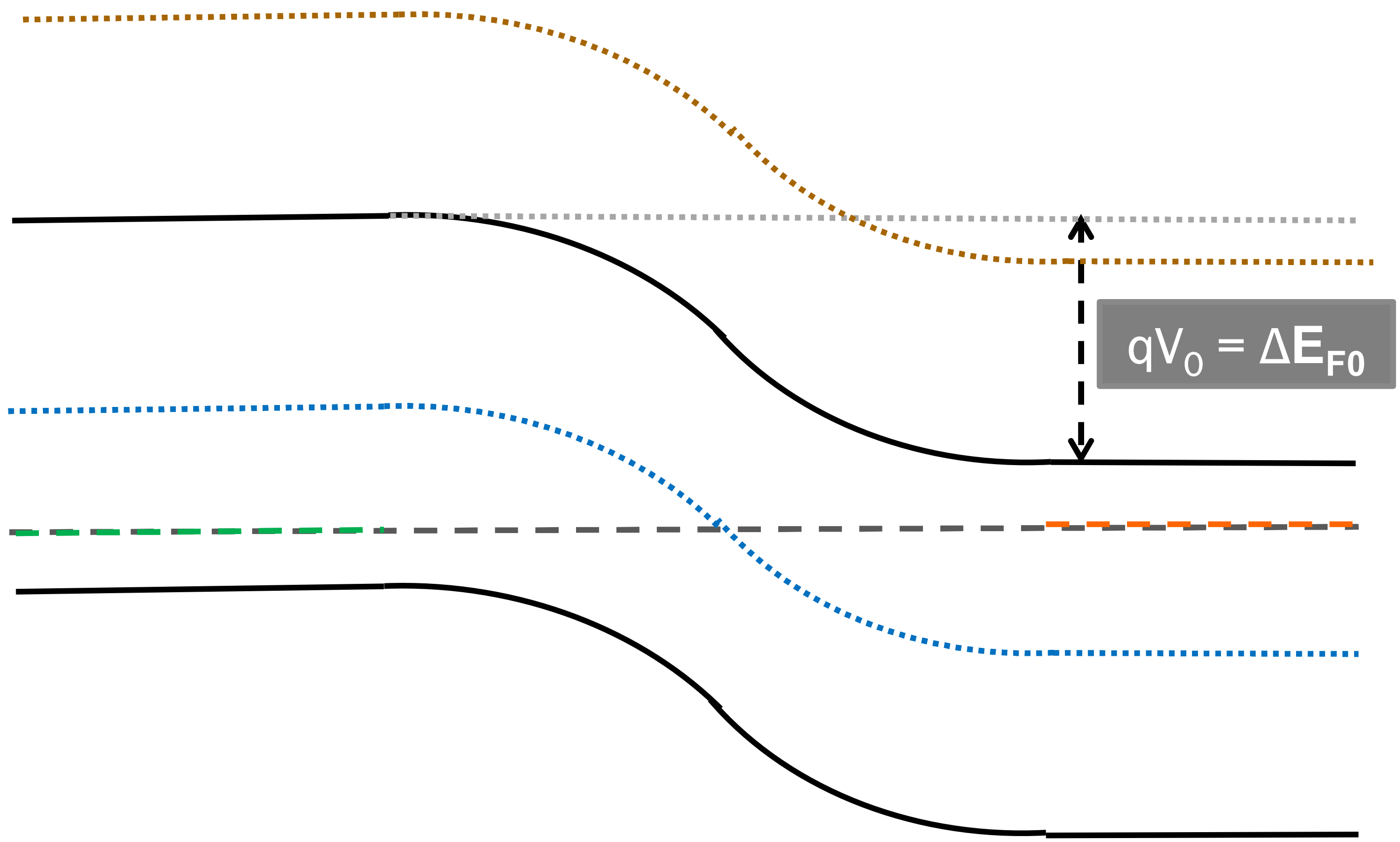
Equilibrium barrier height…



$$E_{Fp} = E_i - \ln\left(\frac{p_{p0}}{n_i}\right) = E_i - k_B T \ln\left(\frac{N_A}{n_i}\right) \quad E_{Fn} = E_i + \ln\left(\frac{n_{n0}}{n_i}\right) = E_i + k_B T \ln\left(\frac{N_D}{n_i}\right)$$

$$\Delta E_{F0} = E_{Fn} - E_{Fp} = k_B T \ln\left(\frac{N_D}{n_i}\right) + k_B T \ln\left(\frac{N_A}{n_i}\right)$$

…from band diagram



$$V_0 = \frac{k_B T}{q} \ln\left(\frac{N_D}{n_i}\right) + \frac{k_B T}{q} \ln\left(\frac{N_A}{n_i}\right)$$

Contact potential / built-in voltage

$$V_0 = \frac{k_B T}{q} \ln\left(\frac{N_A N_D}{n_i^2}\right)$$

Hetero-junction?

Equilibrium with potentials/band-bending

Equilibrium, homogeneous

$$n_0 = n_i \exp\left(\frac{E_F - E_{i0}}{k_B T}\right) \qquad p_0 = n_i \exp\left(\frac{E_{i0} - E_F}{k_B T}\right)$$

Equilibrium, inhomogeneous

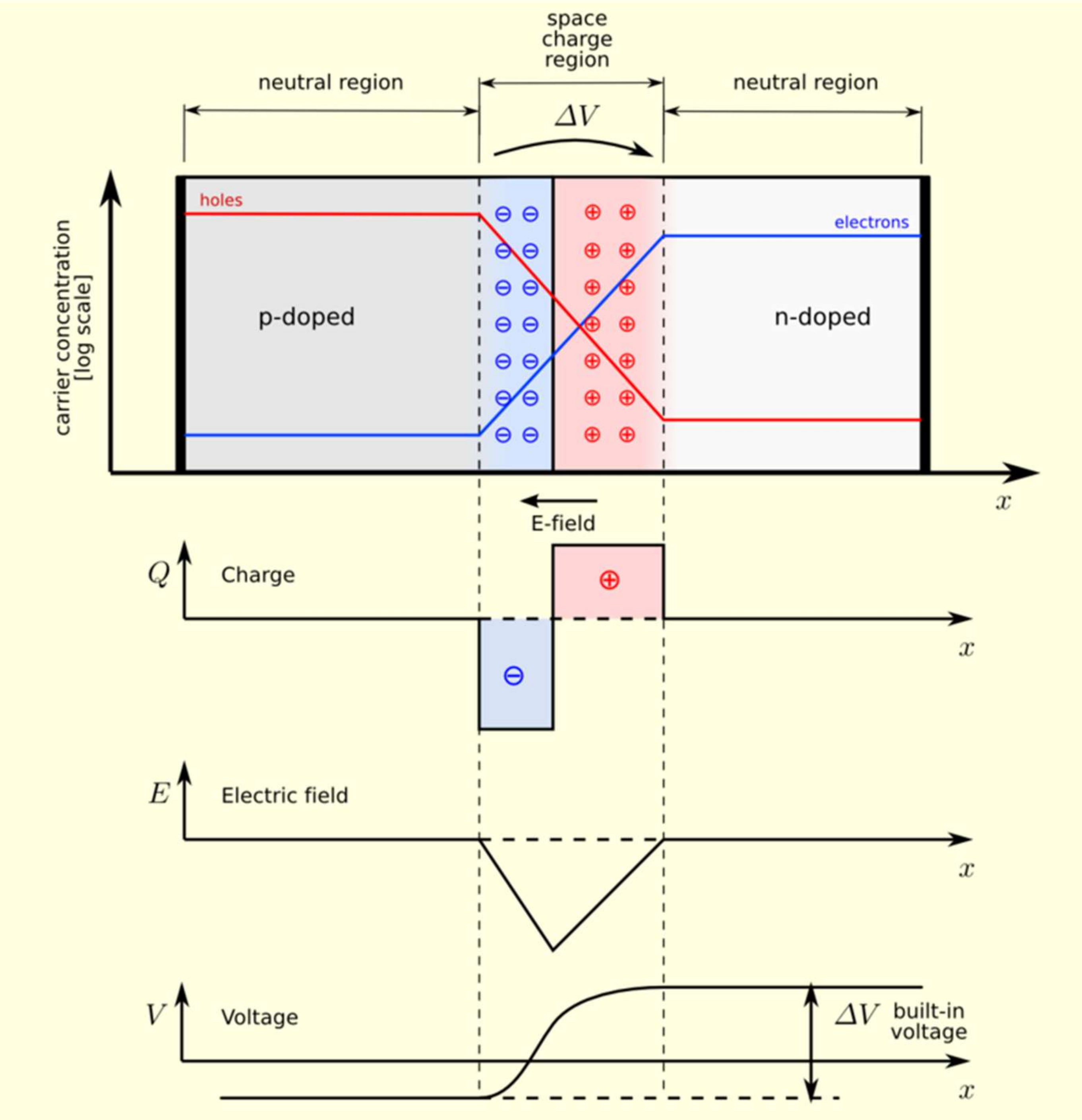
$$n = n_i \exp\left(\frac{E_F - E_{i0} + eV}{k_B T}\right) = n_0 \exp\left(\frac{+eV}{k_B T}\right) \qquad E_i = E_{i0} - eV$$

$$p = n_i \exp\left(\frac{E_{i0} - E_F - eV}{k_B T}\right) = p_0 \exp\left(\frac{-eV}{k_B T}\right)$$

Drift = Diffusion?

Contact potential?

Barrier width – electrostatics



Charge density

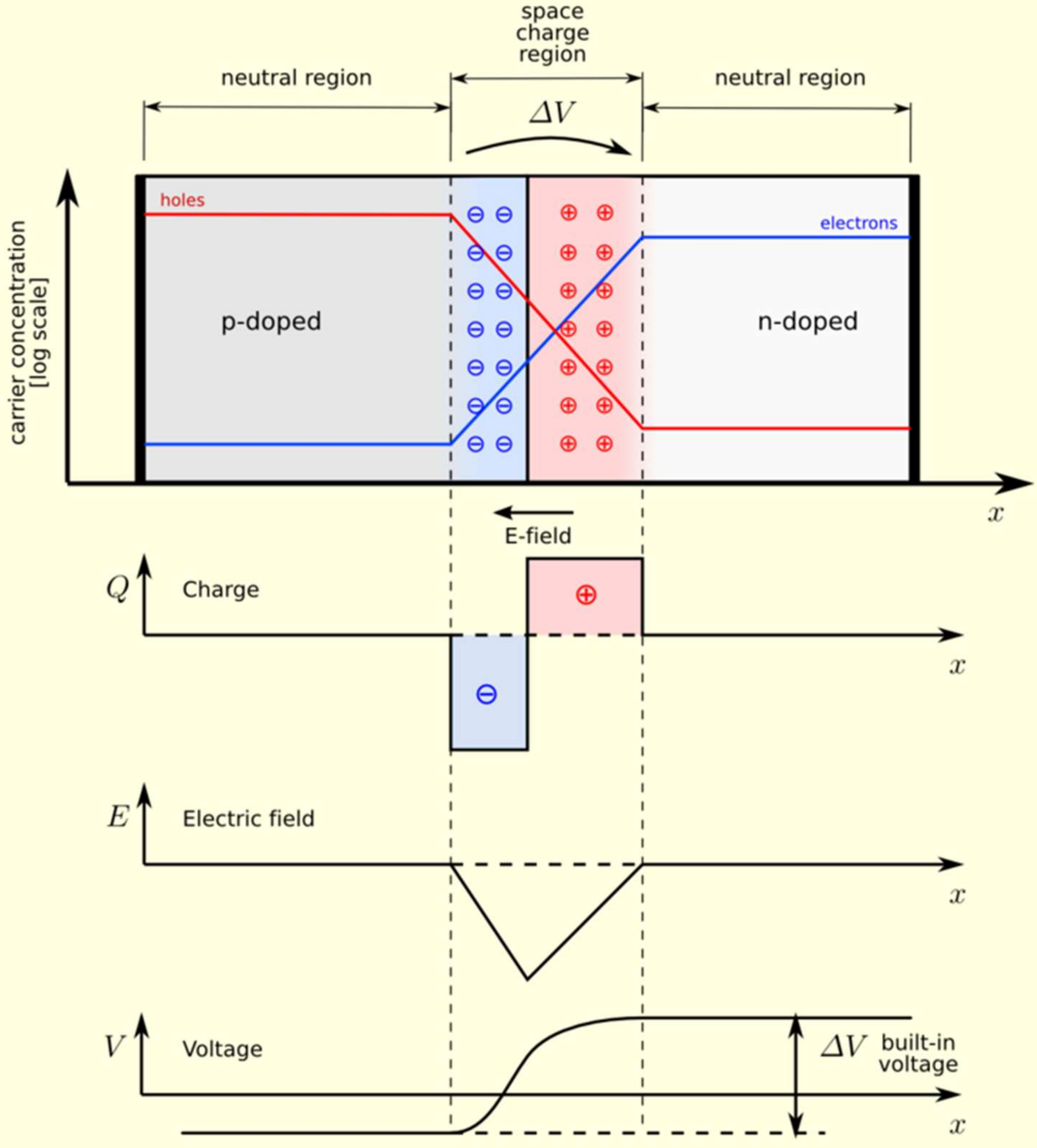
$$\rho(x) = \begin{cases} -eN_A, & -x_p \le x \le 0 \\ +eN_D, & +x_n \ge x \ge 0 \\ 0, & x \le -x_p \cup x \ge x_n \end{cases}$$

Depletion approximation

$$\boxed{N_A x_p = N_D x_n}$$

Charge neutrality

Barrier width – electrostatics



Electric field

$$\frac{dE}{dx} = \frac{\rho(x)}{\varepsilon}$$

Gauss' Law

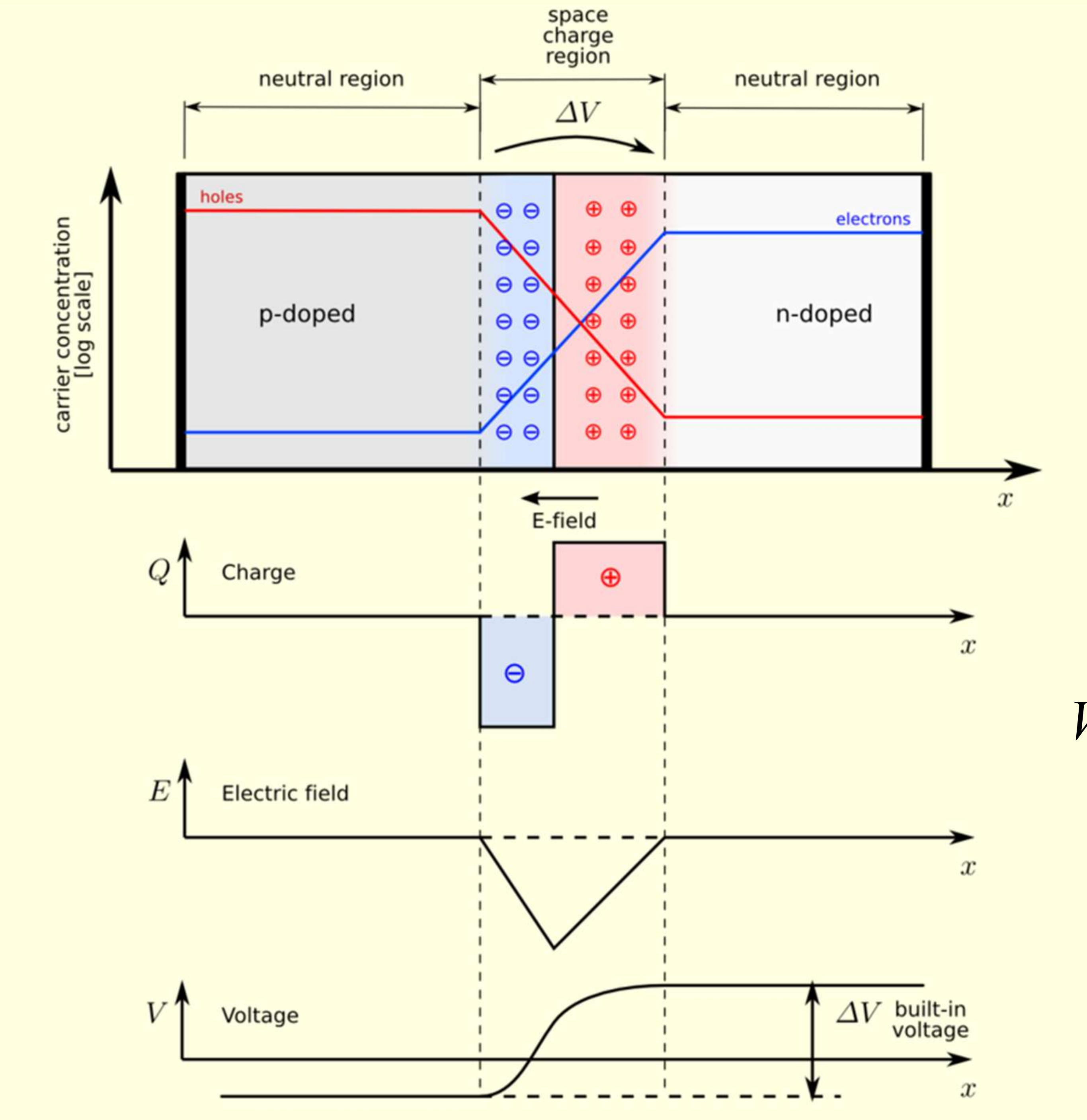
Boundary condition?

$$\rho(x) = \begin{matrix} -eN_A, -x_p \le x \le 0 \\ +eN_D, \ +x_n \ge x \ge 0 \end{matrix}$$

$$E = \begin{matrix} -eN_A\left(x + x_p\right)/\varepsilon \\ eN_D\left(x - x_n\right)/\varepsilon \end{matrix}$$

$$\boxed{E_m = \frac{-eN_A x_p}{\varepsilon} = \frac{-eN_D x_n}{\varepsilon}}$$

Barrier width – electrostatics



Electrostatic potential

$$E = -\frac{dV}{dx}$$

$$E_m = \frac{-eN_A x_p}{\varepsilon} = \frac{-eN_D x_n}{\varepsilon}$$

$$x_p = \frac{-\varepsilon E_m}{eN_A}; \; x_n = \frac{-\varepsilon E_m}{eN_D}$$

$$V_0 = \frac{(-E_m)\cdot W}{2} = \frac{(-E_m)\cdot(x_p + x_n)}{2}$$

$$\boxed{W = \sqrt{\frac{2\varepsilon V_0}{e}\left(\frac{1}{N_A} + \frac{1}{N_D}\right)}}$$

Depletion layer width

Depletion layer width

Equilibrium

$$W = \sqrt{\frac{2\varepsilon V_0}{e}\left(\frac{1}{N_A}+\frac{1}{N_D}\right)}$$

One-sided junction?

Voltage drop?

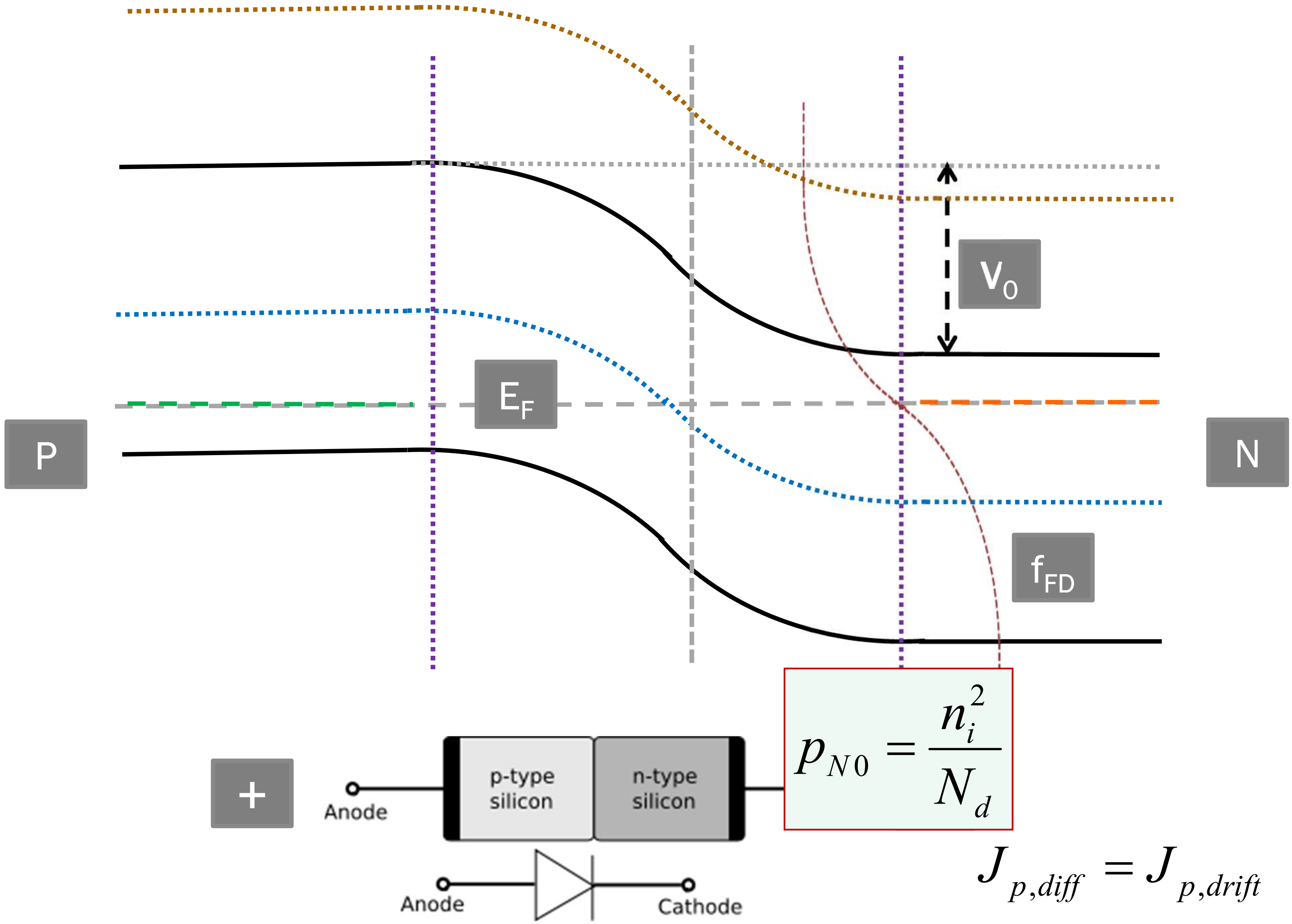
With bias

$$W = \sqrt{\frac{2\varepsilon\left(V_0 - V\right)}{e}\left(\frac{1}{N_A}+\frac{1}{N_D}\right)}$$

Forward/Reverse?

Fermi level?

P-N junction in equilibrium

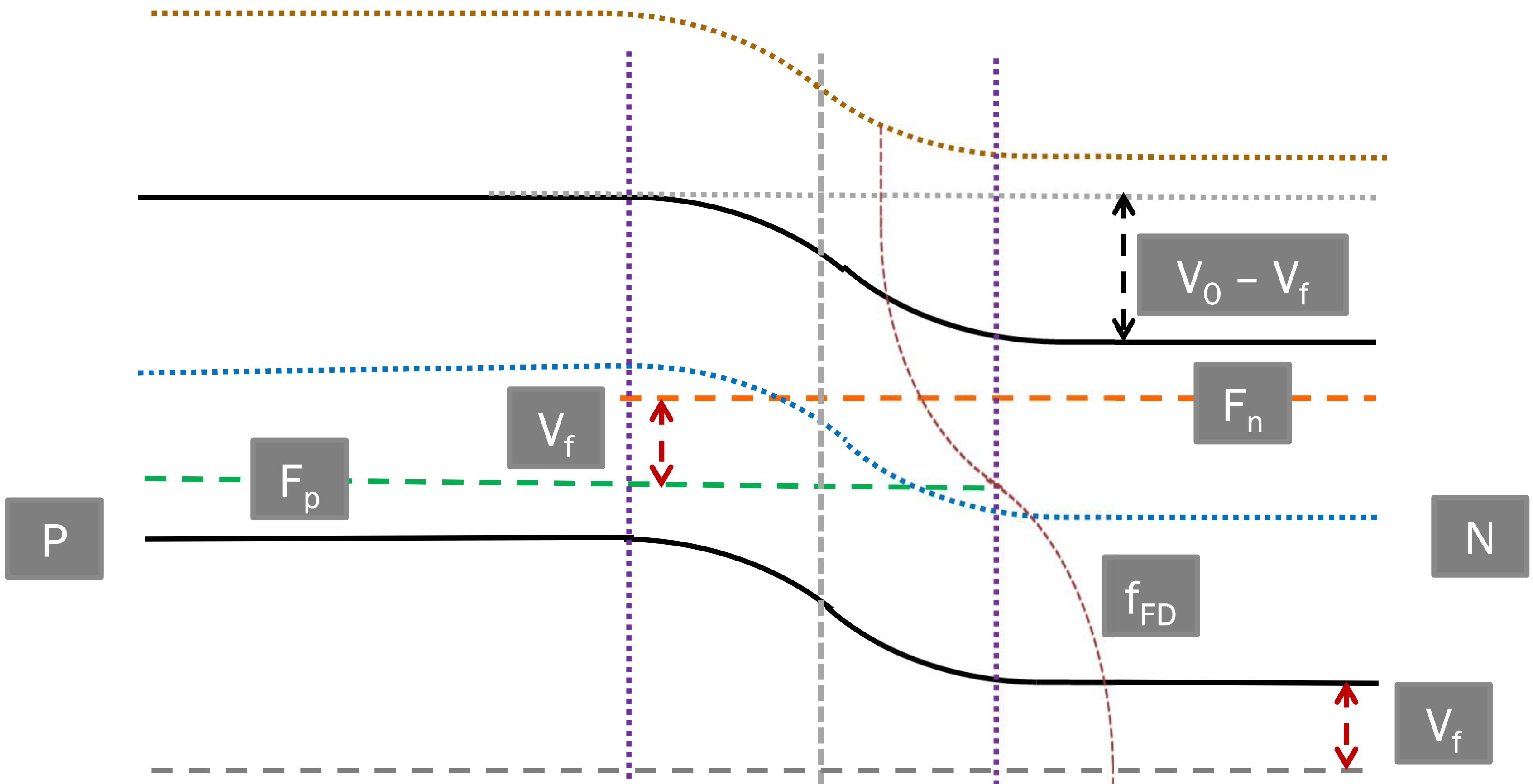


$$p_{N0} = \frac{n_i^2}{N_d}$$

$$J_{p,diff} = J_{p,drift}$$

What happens under forward bias?

P-N junction in forward-bias



$$\frac{p(x_N)}{p_{N0}} = \exp\left(qV_f / k_B T\right)$$

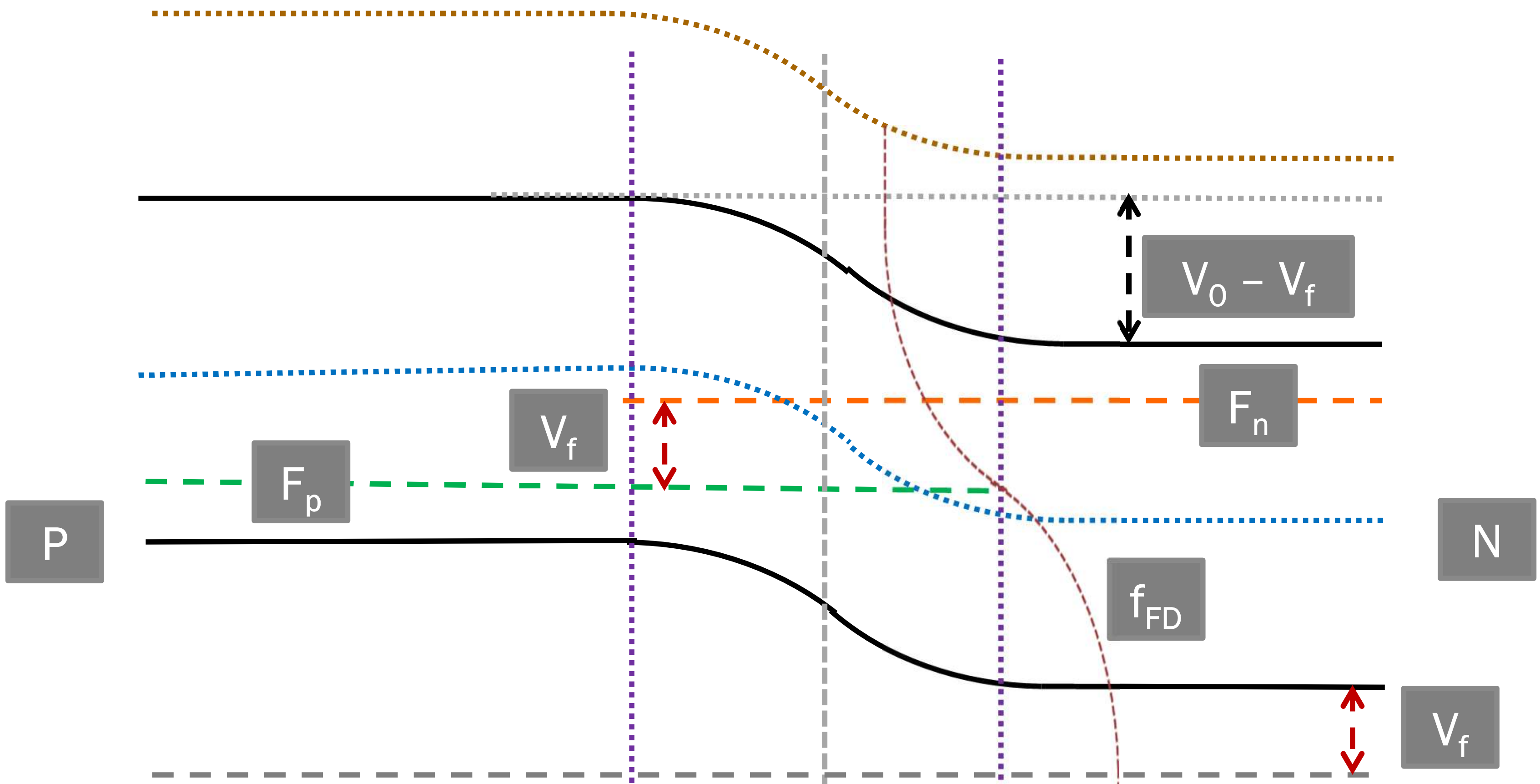
$$J_{p,diff} \gg J_{p,drift}$$

$$p(x_N) = N_v \exp\left[\left(E_v(x_N) - F_p\right)/k_B T\right]$$

$$p(x_N) = N_v \exp\left[\left(E_{v0} + qV_f - F_p\right)/k_B T\right]$$

$$p(x_N) = N_v \exp\left[\left(E_{v0} - E_F\right)/k_B T\right] \cdot \exp\left(+qV_f / k_B T\right)$$

P-N junction in forward-bias



$$\frac{n(x_P)}{n_{P0}} = \exp\left(qV_f / k_B T\right) = \frac{p(x_N)}{p_{N0}}$$

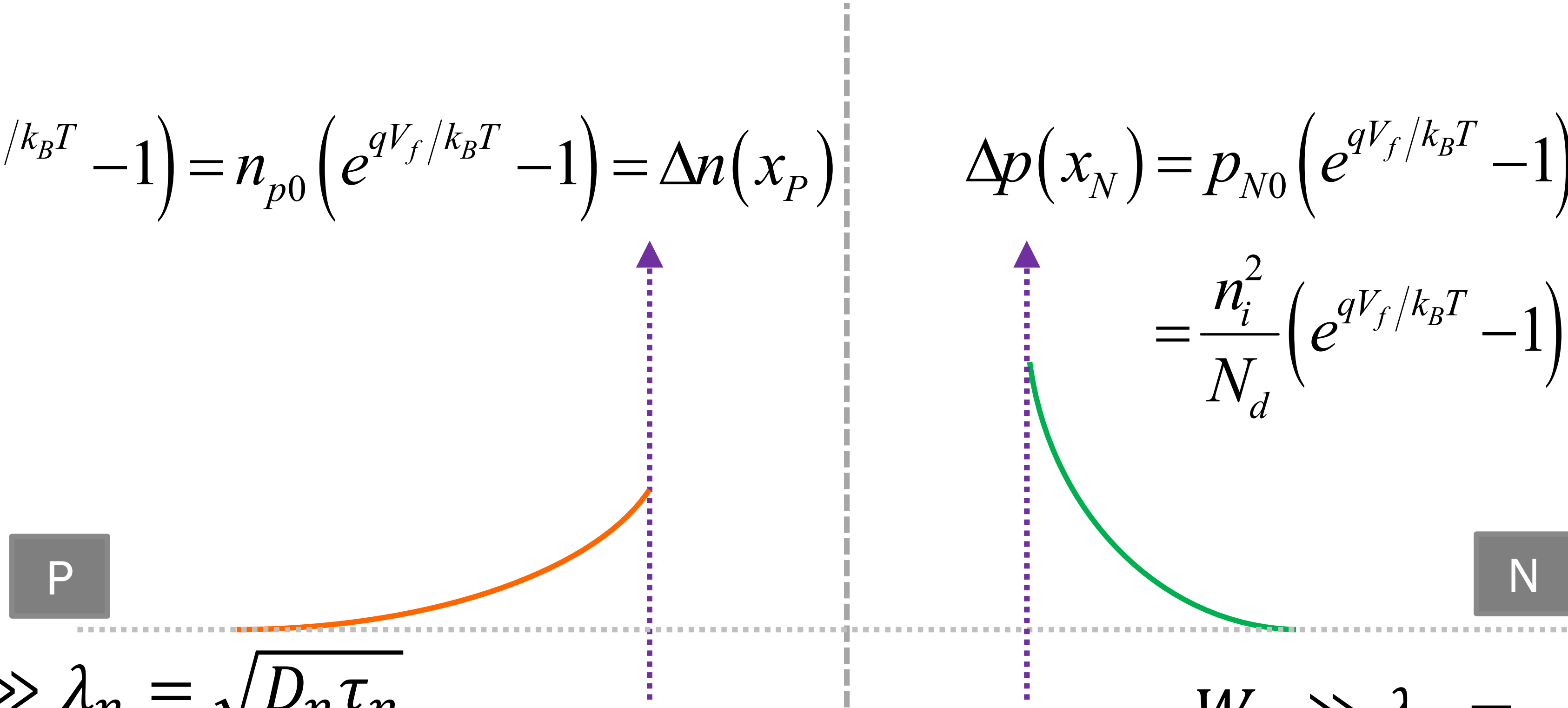
$$\Delta p(x_N) = p_{N0}\left[\exp\left(qV_f / k_B T\right) - 1\right]$$

Minority carrier injection ← Boltzmann statistics

Forward-bias current (steady-state)

$$\frac{n_i^2}{N_a}\left(e^{qV_f/k_BT}-1\right)=n_{p0}\left(e^{qV_f/k_BT}-1\right)=\Delta n\left(x_P\right)$$

$$\Delta p\left(x_N\right)=p_{N0}\left(e^{qV_f/k_BT}-1\right)$$
$$=\frac{n_i^2}{N_d}\left(e^{qV_f/k_BT}-1\right)$$



$$W_P \gg \lambda_n = \sqrt{D_n \tau_n}$$

$$\boxed{J_{n,diff}\left(x_P\right)=\frac{qD_n}{\lambda_n}\frac{n_i^2}{N_a}\left(e^{qV_f/k_BT}-1\right)}$$

$$W_N \gg \lambda_p = \sqrt{D_p \tau_p}$$

$$\Delta p\left(x\right)=\Delta p\left(x_N\right)e^{-\left(x-x_N\right)/\lambda_p}$$

$$J_{p,diff}\left(x_N\right)=-qD_p\left.\frac{dp}{dx}\right|_{x=x_N}=\frac{qD_p}{\lambda_p}\Delta p\left(x_N\right)$$

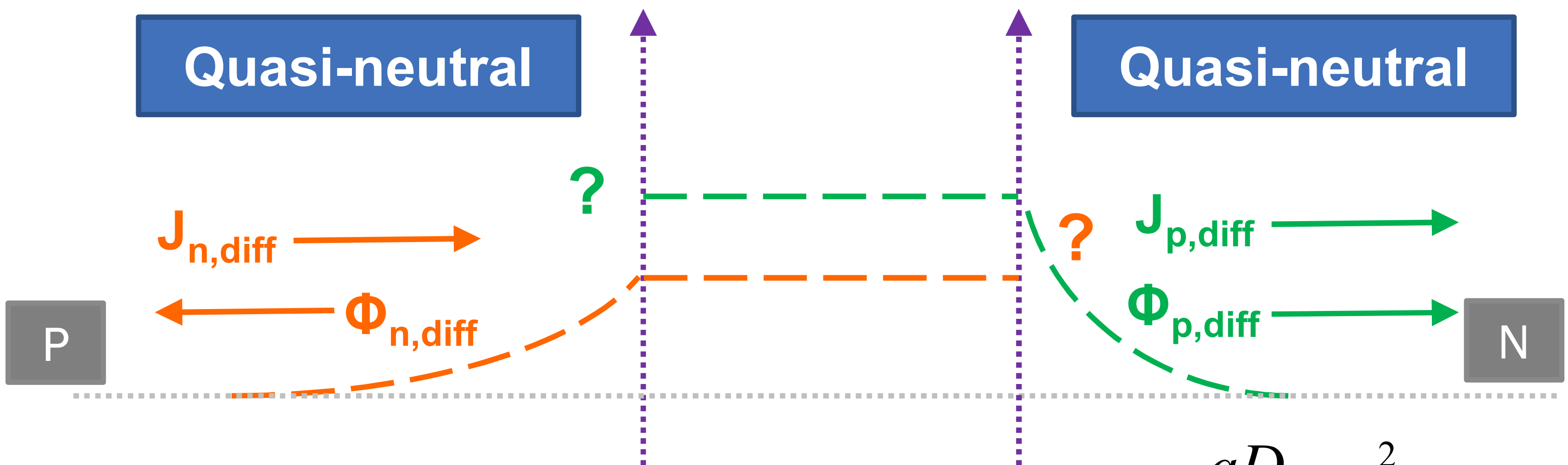
$$\boxed{J_{p,diff}\left(x_N\right)=\frac{qD_p}{\lambda_p}\frac{n_i^2}{N_d}\left(e^{qV_f/k_BT}-1\right)}$$

Continuity → J is constant (?)

How to sum up its components?

Forward-bias current (steady-state)

$$\frac{\partial p}{\partial t} = -\frac{1}{q}\vec{\nabla}\cdot\vec{J}_p + \left(G_p - R_p\right) \qquad \frac{\partial n}{\partial t} = +\frac{1}{q}\vec{\nabla}\cdot\vec{J}_n + \left(G_n - R_n\right)$$



Minority carriers: $J_{diff} \gg J_{drift}$

$$J_{p,diff}\left(x_N\right) = \frac{qD_p}{\lambda_p}\frac{n_i^2}{N_d}\left(e^{qV/k_BT} - 1\right)$$

Total current

$$J_{p,diff}\left(x\right) = \frac{qD_p}{\lambda_p}\frac{n_i^2}{N_d}\left(e^{qV/k_BT} - 1\right)e^{-\left(x - x_N\right)/\lambda_p}$$

$$J = J_{p,diff}\left(x_N\right) + J_{n,diff}\left(x_P\right)$$

$$\boxed{J = \left(\frac{qD_p}{\lambda_p}\frac{n_i^2}{N_d} + \frac{qD_n}{\lambda_n}\frac{n_i^2}{N_a}\right)\left(e^{qV/k_BT} - 1\right) = J_0\left(e^{qV/k_BT} - 1\right)}$$

Narrow-base?

Reverse-bias current (steady-state)

Reverse-bias band-diagram?

$$J = J_0\left(e^{qV/k_BT} - 1\right) \xrightarrow{V<0} -J_0$$

Why is the reverse current bias-independent?

Small-signal response

Charge stored in (nonlinear) element as function of voltage

Taylor Series

$$Q(V) = Q(V_0 + v) = Q(V_0) + v \cdot \left.\frac{dQ}{dV}\right|_{V=V_0} + ..$$

DC bias + AC *small-signal*

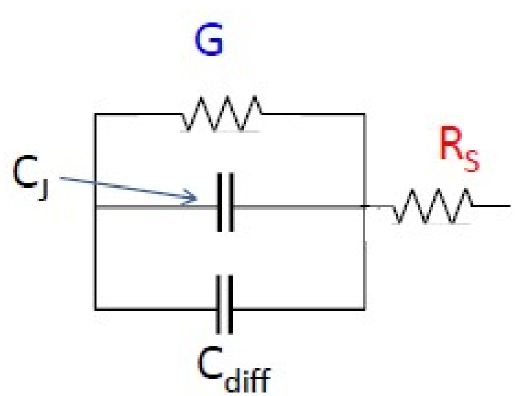
$$q = Q(V_0 + v) - Q(V_0) = v \cdot \left.\frac{dQ}{dV}\right|_{V=V_0} = C(V_0) \cdot v$$

Small-signal response: linearization; DC property

Small-signal capacitance

What about current as a function of voltage?

Diode small-signal model



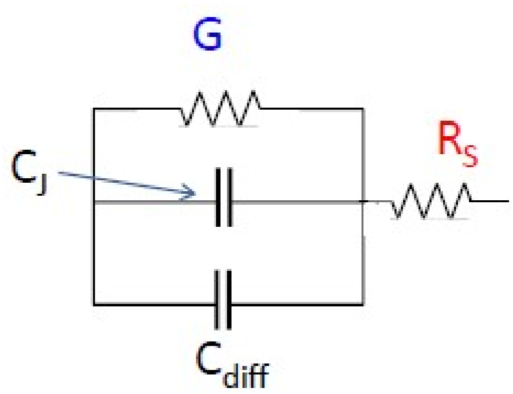
Forward-bias conductance

Diode current

$$I = I_0\left[e^{\beta q(V - R_S I)/m} - 1\right]$$

m: ideality factor (G-R, high-injection)

$$V = \frac{m}{\beta q}\ln\left(\frac{I + I_0}{I_0}\right) + R_S I$$

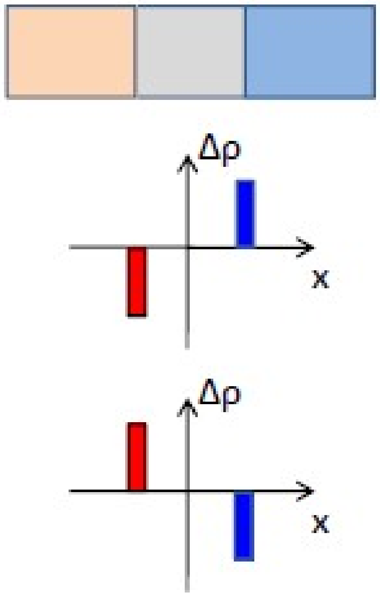


Small-signal resistance

$$\frac{dV}{dI} = \frac{m}{\beta q\left(I + I_0\right)} + R_S = \frac{1}{G} + R_S$$

Where is the bias-dependence?

Junction capacitance



Reverse-bias depletion width

$$\delta V = \frac{q}{\varepsilon}\frac{N_a N_d}{N_a + N_d} W\,\delta W \quad \Leftarrow \left(V_0 + V\right) = \frac{q}{2\varepsilon}\frac{N_a N_d}{N_a + N_d} W^2$$

$$\frac{N_d \delta W}{N_a + N_d} = \frac{N_d\left(\delta x_n + \delta x_p\right)}{N_a + N_d} = \frac{N_a \delta x_p + N_d \delta x_p}{N_a + N_d} = \delta x_p$$

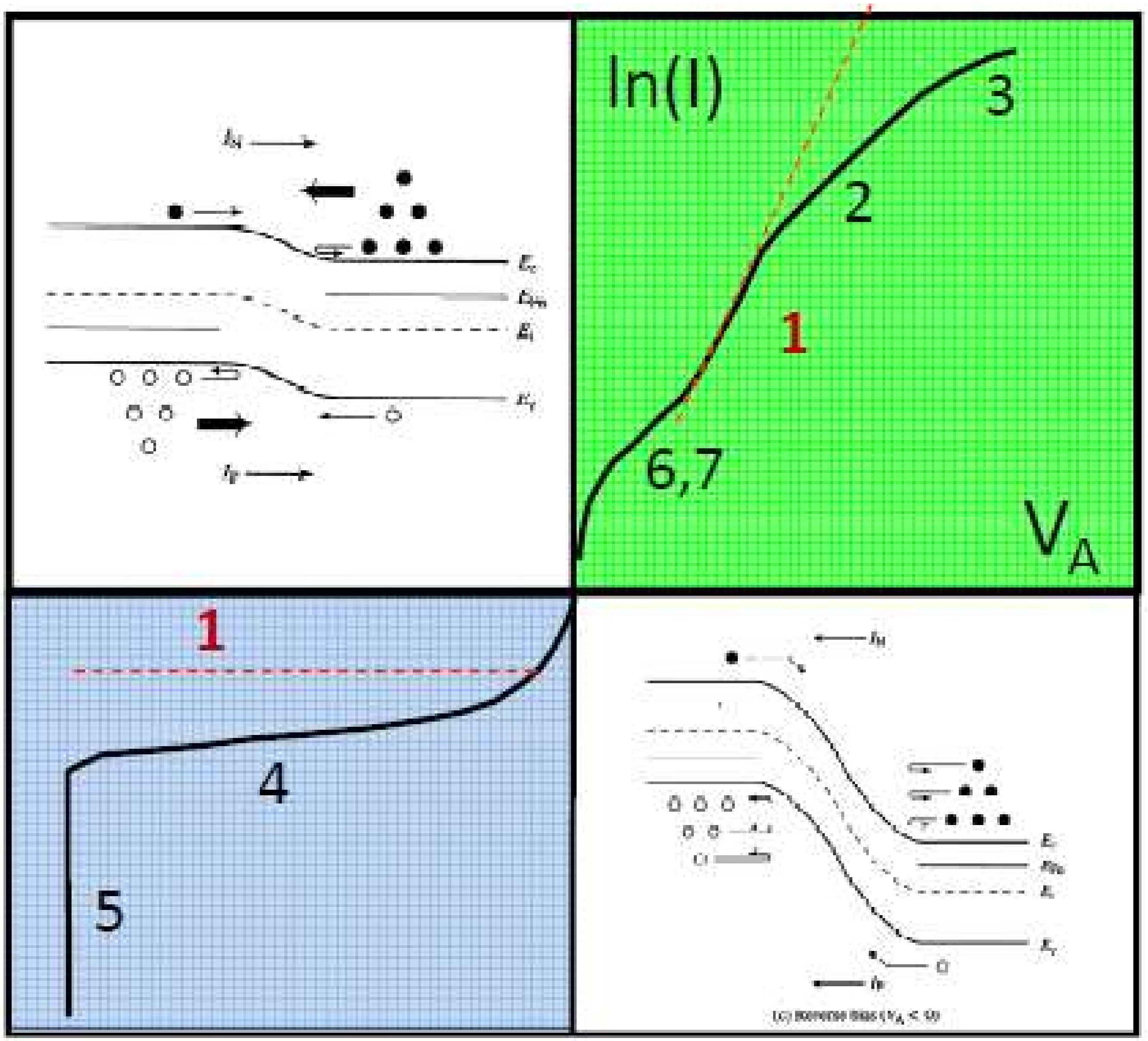
$$\delta V = \frac{W}{\varepsilon} q N_a \delta x_p = \frac{W}{\varepsilon}\delta Q \qquad C_j = \frac{\varepsilon}{W}$$

Bias-dependent/non-linear

Majority or minority response?

Parallel-plate! Why/how?

P-N diode: regimes of I-V



1. Ideal
2. High-injection
3. Ohmic
4. G-R in depletion
5. Breakdown
6. G-R in depletion
7. Inter-band tunneling

Finis

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1. Saumitra Mehrotra & Prof. Gerhard Klimeck, commons.wikimedia.org
2. TheNoise, commons.wikimedia.org
3. Raffamaiden, commons.wikimedia.org
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