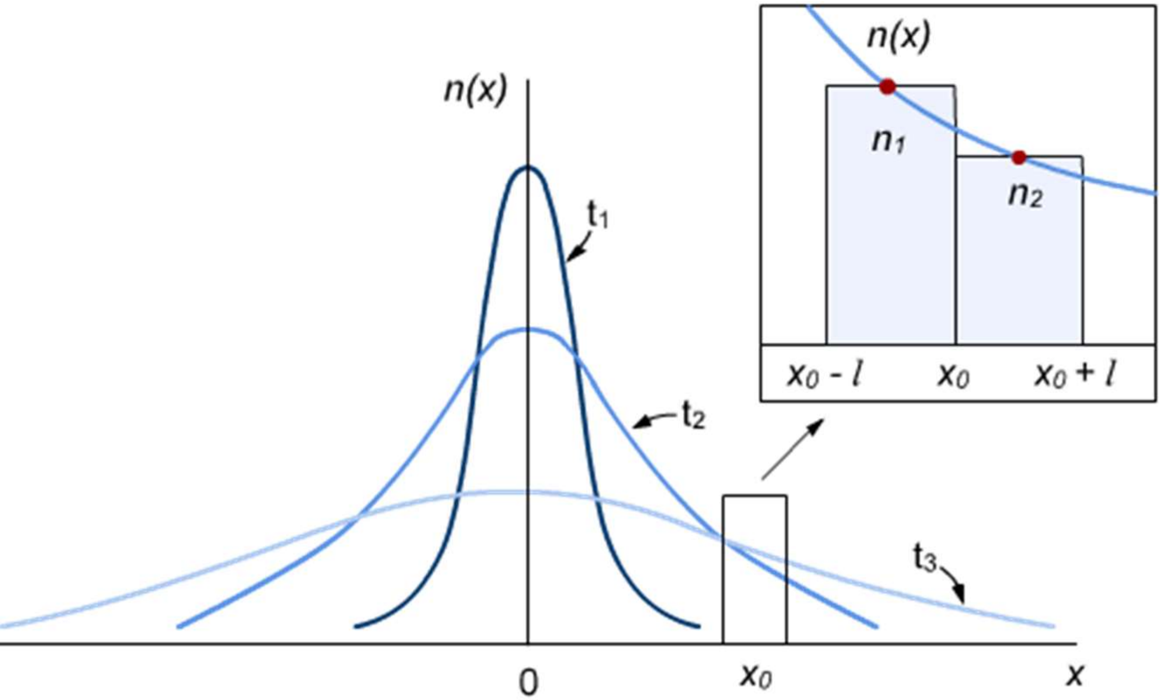
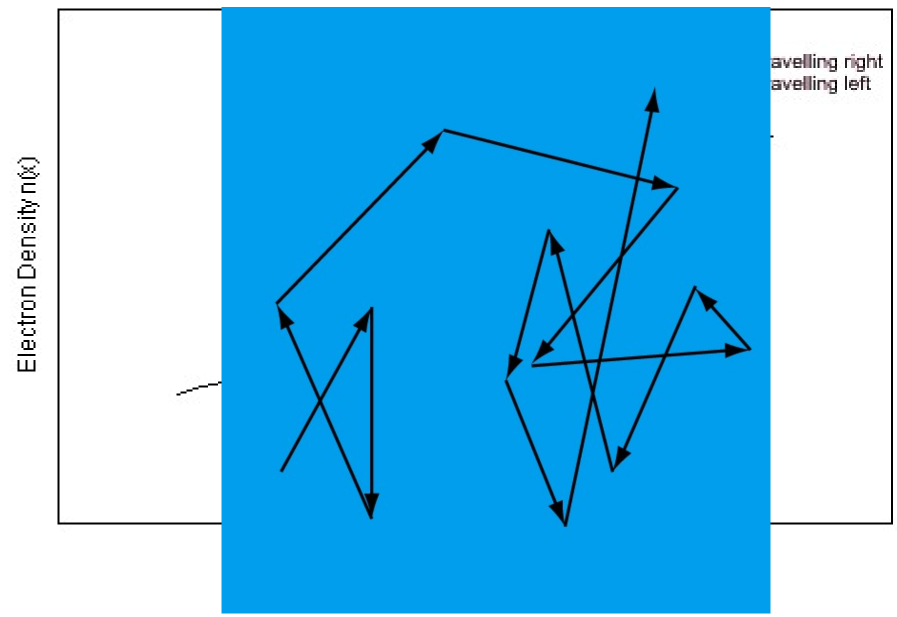
Diffusion



Thermal energy

$$\frac{1}{2}m^{*}v_{th}^{2} = \frac{3k_{B}T}{2}$$

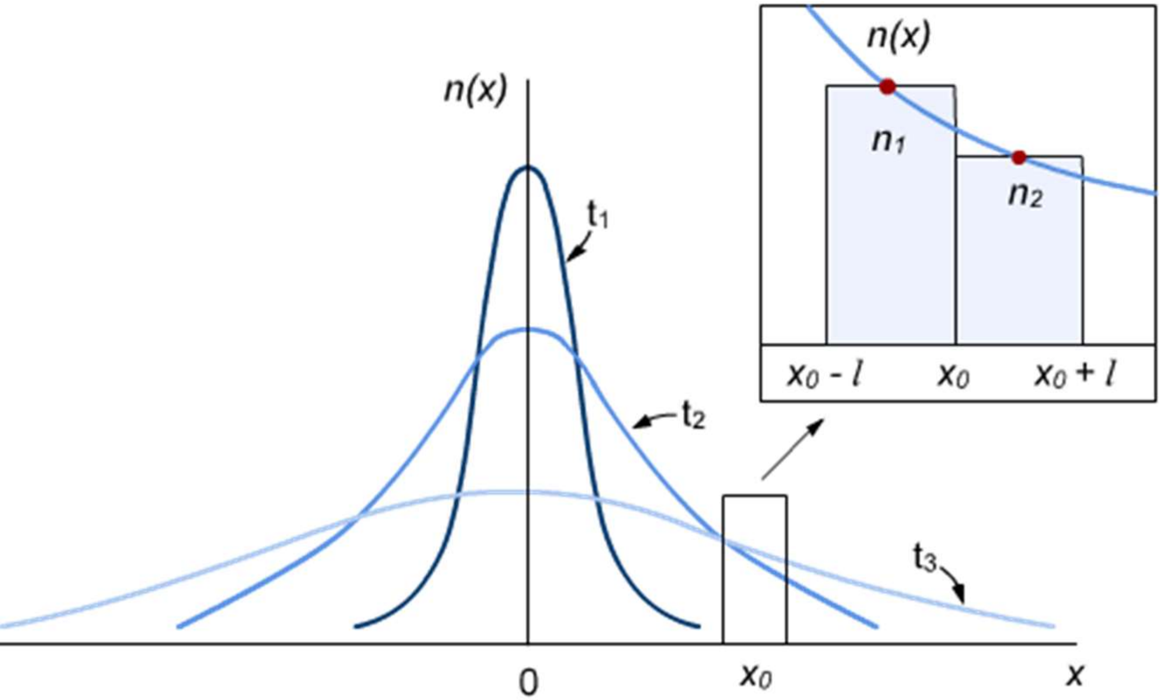


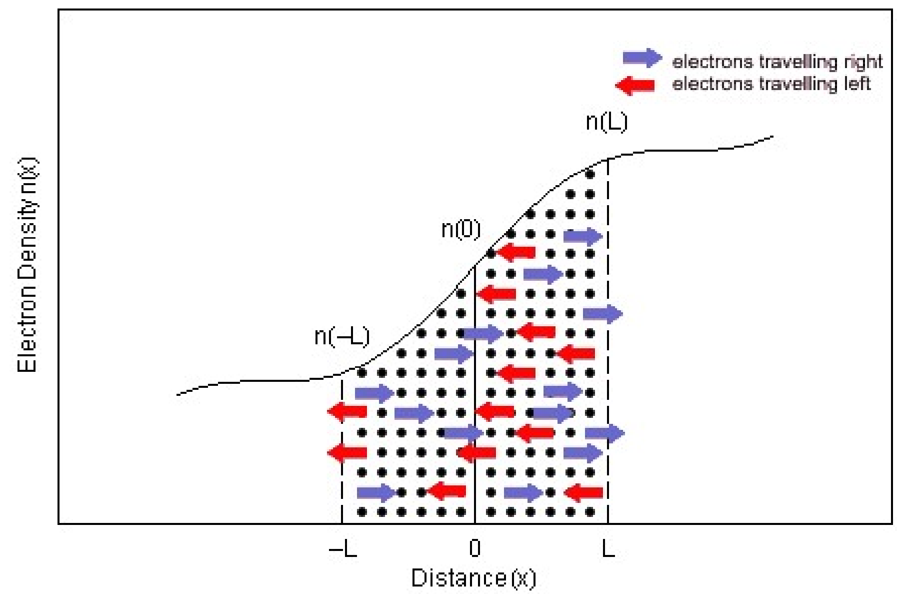
Transport of particles (electrons here) due to a concentration gradient

Driven by thermal motion of particles (electrons)

Flux (particle current) is proportional to concentration gradient

Diffusion current





Flux to the right $\frac{1}{2}v_{th}\cdot n(-l/2)$

$\frac{1}{2}v_{th}\cdot n(+l/2)$ **Flux to the left**

$$\Phi = \frac{1}{2}v_{th}\cdot n\left(-\frac{l}{2}\right) - \frac{1}{2}v_{th}\cdot n\left(+\frac{l}{2}\right)$$

Net flux

Diffusion coefficient $D = \frac{lv_{th}}{2} = \frac{l^2}{2\tau}$

$$\Phi = \frac{lv_{th}}{2}\frac{n\left(-\frac{l}{2}\right) - n\left(+\frac{l}{2}\right)}{l} \simeq -D\frac{dn}{dx}$$

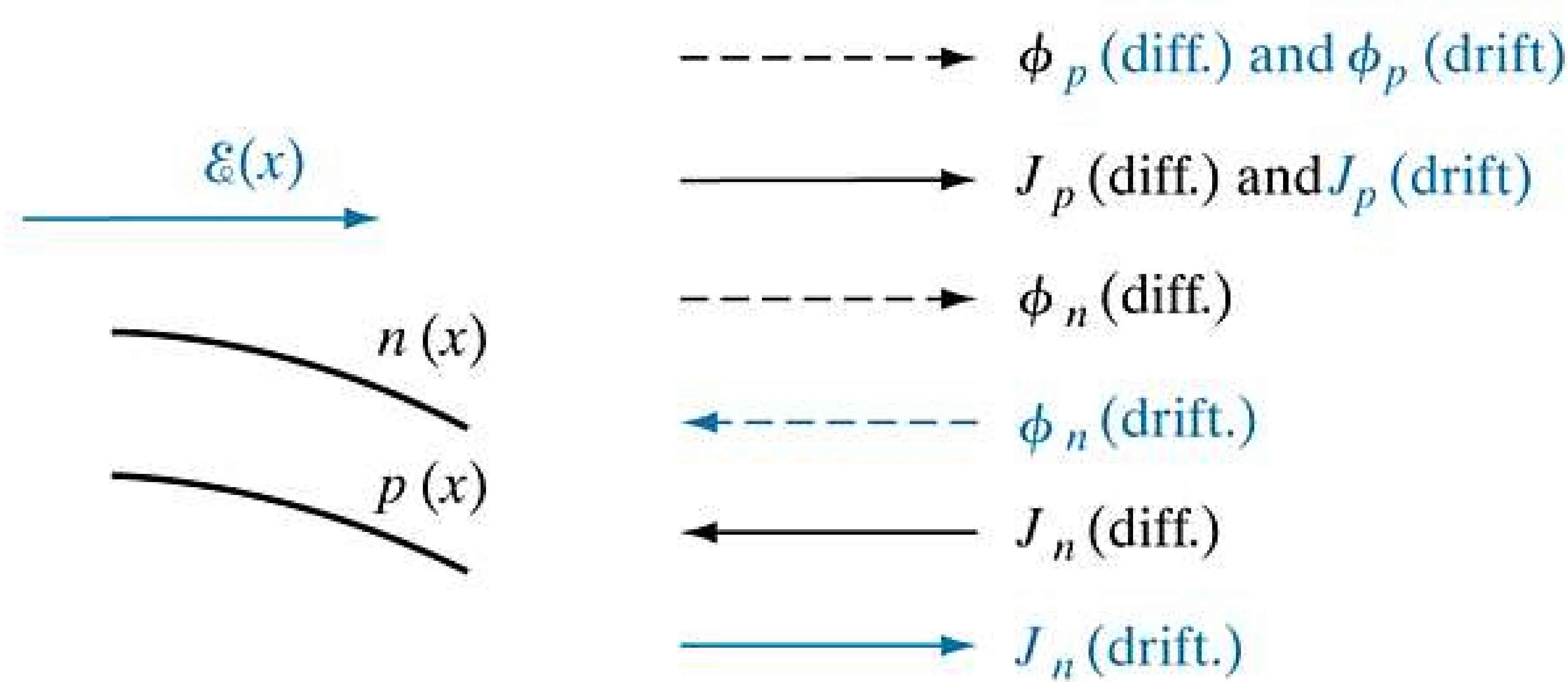
$J = eD\frac{dn}{dx}$ **Diffusion current**

Drift and diffusion currents

$$J_n = ne\mu_n\mathcal{E} + eD_n\frac{dn}{dx}$$

Drift-diffusion

$$J_p = pe\mu_p\mathcal{E} - eD_p\frac{dp}{dx}$$



Equilibrium Fermi level

Rate of transfer of electrons from one spatial point to another

$$R_{1\to 2} \propto N_1(E) f_1(E) \cdot N_2(E)\left[1 - f_2(E)\right]$$

$$R_{2\to 1} \propto N_2(E) f_2(E) \cdot N_1(E)\left[1 - f_1(E)\right]$$

Equilibrium → Detailed Balance $R_{1\to 2} = R_{2\to 1} \Rightarrow f_1 = f_2$

$$\left[1 + e^{(E-E_{F_1})/kT}\right]^{-1} = \left[1 + e^{(E-E_{F_2})/kT}\right]^{-1} \Rightarrow E_{F_1} = E_{F_2}$$

Fermi level is flat in equilibrium

Einstein Relation

Drift-diffusion

$$J_p = pe\mu_p \mathcal{E} - eD_p \frac{dp}{dx}$$

Equilibrium

$$J_p = 0 \Rightarrow \mathcal{E} = \frac{D_p}{p\mu_p}\frac{dp}{dx}$$

$$E_i = E_{i0} - qV$$

$$p_0 = n_i e^{(E_i - E_F)/kT}$$

Carrier statistics

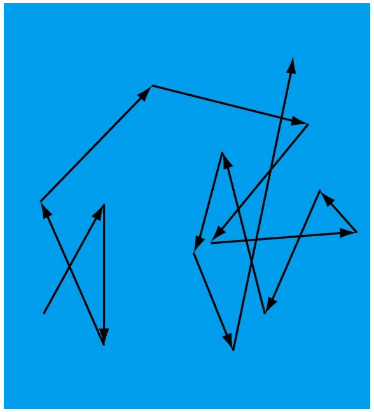
$$\frac{D_p}{\mu_p}\frac{1}{k_B T}\left[\frac{dE_i}{dx} - \frac{dE_F}{dx}\right] = \mathcal{E} \cong -\frac{dV}{dx} = \frac{1}{e}\frac{dE_i}{dx}$$

Equilibrium

$$\frac{dE_F}{dx} = 0$$

$$D_p = \frac{\mu_p k_B T}{e}$$

$$D_n = \frac{\mu_n k_B T}{e}$$

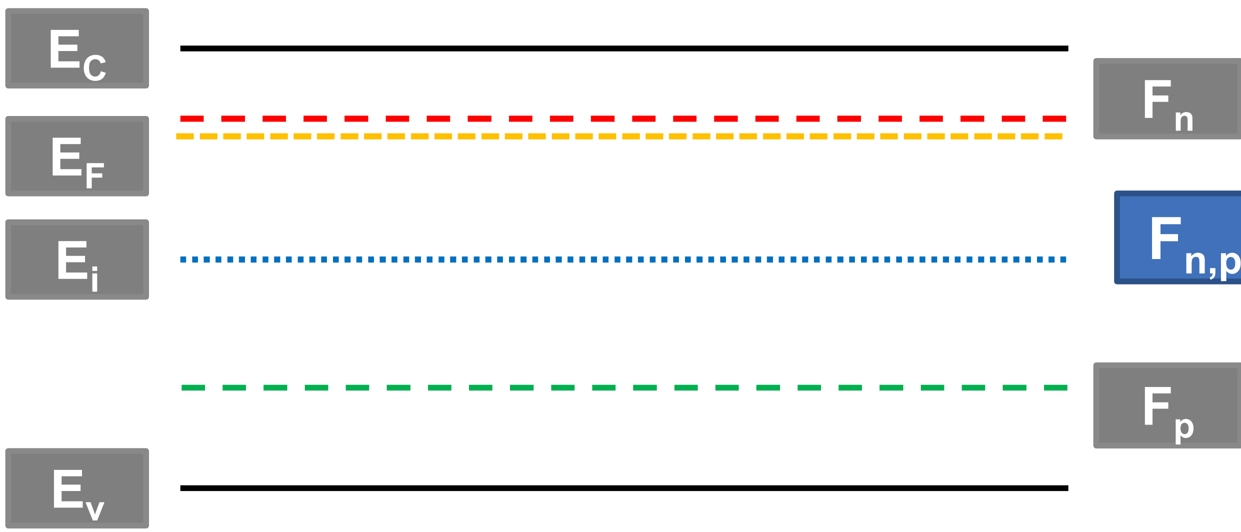


Carrier-statistics dependent?

Fluctuation-Dissipation

Non-equilibrium: quasi-Fermi levels

Bulk semiconductor under steady-state illumination
➔ excess electrons and holes



$$n = n_i e^{(F_n - E_i)/kT}$$

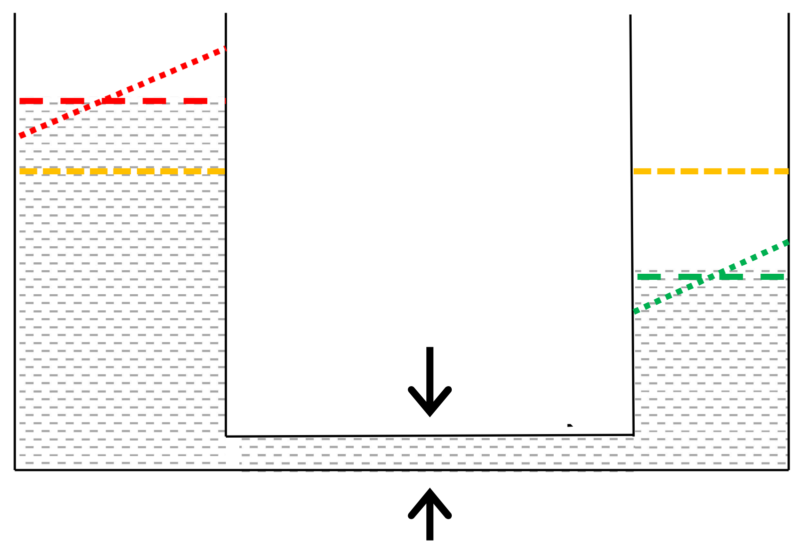
$F_{n,p}$: quasi-Fermi level / Imref

$$p = n_i e^{(E_i - F_p)/kT}$$

Non-equilibrium ≈ Local equilibrium

Electrons (holes) in local equilibrium within conduction (valence) band

Other instances of local equilibrium?



Drift-diffusion with quasi-Fermi levels

Drift-diffusion

$$J_n = ne\mu_n \mathcal{E} + eD_n \frac{dn}{dx}$$

Carrier statistics

$$n = n_i e^{(F_n - E_i)/kT}$$

$$J_n = ne\mu_n \left(\frac{1}{e} \frac{dE_i}{dx} \right) + eD_n \frac{n}{k_B T} \left(\frac{dF_n}{dx} - \frac{dE_i}{dx} \right)$$

Einstein relation

$$D_n = \mu_n k_B T / e$$

$$J_n = n\mu_n \frac{dE_i}{dx} + e\left(\frac{\mu_n k_B T}{e} \right) \frac{n}{k_B T} \left(\frac{dF_n}{dx} - \frac{dE_i}{dx} \right)$$

First and third terms cancel out; significance?

Drift-diffusion

$$\boxed{J_n = n\mu_n \frac{dF_n}{dx}}$$

Holes?

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Artwork Sources:

1. Prof. Sanjay Banerjee
2. www.pveducation.org
3. britneyspears.ac