10/1/2015

EE 210 : SIGNALS AND SYSTEMS

WEEK 1 - HOMEWORK EXERCISES

1.3 Determine values of P_∞ and E_∞ for each of the following signals

(a) $x_1(t) = e^{-2t}\, u(t)$

$$E_\infty \triangleq \lim_{T\to\infty} \int_{-T}^{T} |x(t)|^2\, dt \qquad (1.6)$$

$$P_\infty \triangleq \lim_{T\to\infty} \frac{1}{2T} \int_{-T}^{T} |x(t)|^2\, dt \qquad (1.8)$$

$$\therefore\ E_\infty = \int_0^\infty e^{-4t}\, dt = \frac{1}{4}$$

$$P_\infty = \lim_{T\to\infty} \frac{1}{2T} \int_0^T e^{-4t}\, dt = \lim_{T\to\infty} \frac{1-e^{-4T}}{8T} = 0$$

1.3 (b) $x_2(t) = e^{j(2t+\pi/4)}$

$$|x_2(t)| = 1$$

$$E_\infty = \int_{-\infty}^{\infty} |x_2(t)|^2 dt = \int_{-\infty}^{\infty} 1\, dt = \infty$$

$$P_\infty = \lim_{T\to\infty} \frac{1}{2T} \int_{-T}^{T} 1\, dt = 1$$

1.3 (c) $x_3(t) = \cos(t)$

$$E_\infty = \int_{-\infty}^{\infty} |x_3(t)|^2 dt = \int_{-\infty}^{\infty} \cos^2(t)\, dt = \infty$$

$$P_\infty = \lim_{T \to \infty} \frac{1}{2T} \int_{-T}^{T} \cos^2(t)\, dt$$

$$= \frac{1}{2T} \int_{-T}^{T} \frac{1 + \cos(2t)}{2} dt = \lim_{T \to \infty} \frac{1}{2} + \frac{1}{2T} \int_{-T}^{T} \frac{\cos(2t)}{2}$$

$$P_\infty = \frac{1}{2}$$

1.3 (d) $x_1[n] = \left(\frac{1}{2}\right)^n u[n]$

$$E_\infty \triangleq \lim_{N \to \infty} \sum_{n=-N}^{N} |x[n]|^2 = \sum_{n=-\infty}^{+\infty} |x[n]|^2 \quad (1.7)$$

$$P_\infty \triangleq \lim_{N \to \infty} \frac{1}{2N+1} \sum_{n=-N}^{N} |x[n]|^2$$

$$E_\infty = \sum_{n=0}^{\infty} \left(\frac{1}{2}\right)^{2n} = \sum_{n=0}^{\infty} \left(\frac{1}{4}\right)^n = \frac{1}{1-\left(\frac{1}{4}\right)} = \frac{4}{3}$$

$P_\infty = 0$ since $E_\infty < \infty$

(average power)

1.3 (e) $x_2[n] = e^{j(\pi n/2 + \pi/8)}$

$$|x_2[n]| = 1 \quad \forall\; n \in (-\infty, \infty)$$

$$E_\infty = \sum_{n=-\infty}^{\infty} 1 = \infty$$

$$P_\infty = \lim_{N \to \infty} \frac{1}{2N+1} \sum_{n=N}^{N} 1 = \lim_{N \to \infty} \frac{2N+1}{2N+1} = 1$$

1.3 (f) $x_3[n] = \cos\left(\frac{\pi n}{4}\right)$

$$|x_3[n]|^2 = \cos^2\left(\frac{\pi n}{4}\right)$$

$$E_\infty = \sum_{n=-\infty}^{\infty} |x_3[n]|^2 = \sum_{n=-\infty}^{\infty} \cos^2\left(\frac{\pi n}{4}\right) = \infty$$

$$P_\infty = \lim_{N \to \infty} \frac{1}{2N+1} \sum_{n=-N}^{N} \cos^2\left(\frac{\pi n}{4}\right)$$

$$= \lim_{N \to \infty} \frac{1}{2N+1} \sum_{n=-N}^{N} \frac{1}{2}\left(1 + \cos\left(\frac{\pi}{2} n\right)\right)$$

$$= \frac{1}{2}$$

1.4 Let $x[n]$ be a signal with $x[n] = 0$ for $n < -2$ and $n > 4$.
For each signal below determine the values of n for which it is guaranteed to be zero.

1.4 (a) $x[n-3]$

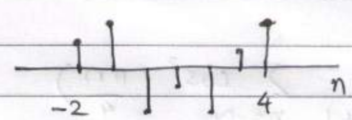
$x[n] = 0 \qquad n < -2 \quad n > 4$

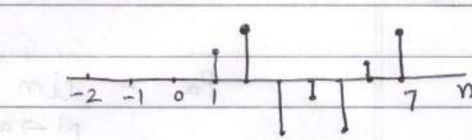
$x[n-3] = 0 \qquad n-3 < -2 \quad n-3 > 4$

$\Rightarrow n < 1 \quad n > 7$

Aliter: signal $x[n]$ is shifted by 3 to the right.

$\therefore$ shifted signal zero for $n < 1$; $n > 7$





1.4 (b) $x[n+4]$

$x[n]$ shifted to left by 4

$\therefore$ Shifted signal zero for $n < -6$
$n > 0$

1.4 (e) $x[-n-2]$

$$x[n] = 0 \quad n < -2 \ , \ n > 4$$
$$x[-n-2] = 0 \quad -n-2 < -2 \ , \ -n-2 > 4$$
$$\Rightarrow \quad n > 0 \ , \ n < -6$$

Aliter:

signal $x[n]$ flipped and flipped signal is shifted to left by 2.

New signal zero for $n < -6$, $n > 0$

1.5 Let $x(t)$ be a signal with $x(t) = 0$ for $t < 3$.
For signals given below
Determine the values for which it is guaranted to be zero.

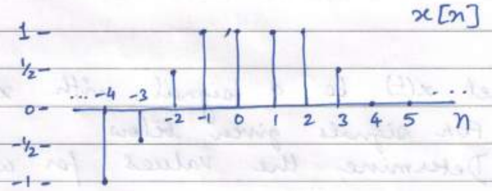
(e) $x(t/3)$

$$x(t) = 0 \quad \text{for} \quad t < 3$$
$$x(t/3) = 0 \quad \text{for} \quad t/3 < 3$$
$$\therefore \ t < 9$$

Aliter: $x(t/3)$ obtained by linearly stretching $x(t)$ b
a factor of 3.

$$\therefore x(t/3) = 0 \ \text{for} \ t < 9$$

1.22 $x[n]$ Figure P1.22



Sketch the following signals

(e) $x[n]\,u[3-n]$

$\rightarrow u[3-n] = u[-(n-3)]$

(shift by 3 to right
then flip around 3)

$\therefore u[3-n] = 0$ for $n > 3$; 1 for $n \leq 3$

$x[n] = 0$ for $n > 3$

$\therefore x[n]\,u[3-n] = x[n]$

EE 210 - SIGNALS AND SYSTEMS

WEEK 2 → HOMEWORK EXERCISES

1.9 Determine whether or not each of the following signals is periodic. If a signal is periodic, specify its fundamental period.

(a) $$x_1(t) = j e^{j10t}$$

Periodic continuous time signals follow the property

$$x(t) = x(t+T) \qquad (1.11)$$

where T is the period. Fundamental period T_0 is the smallest possible T for (1.11) to hold.

$$x_1(t) = je^{j10t} = e^{j(10t+\frac{\pi}{2})}$$

for $x_1(t)$ to be periodic

$$e^{j(10t+\frac{\pi}{2})} = e^{j(10(t+T_0)+\frac{\pi}{2})}$$

$$\Rightarrow \quad 1 = e^{j10T_0}$$

$$\Rightarrow \quad T_0 = \frac{2\pi}{10} \qquad (\text{as } e^{j2\pi} = 1)$$

Hence, $x_1(t)$ is periodic with fundamental period $\frac{2\pi}{10}$

(b) $$x_2(t) = e^{(-1+j)t}$$

$$x_2(t) = e^{-t} \cdot e^{jt}$$

e^{-t} → Decaying exponential (Non-periodic)
e^{jt} → Complex exponential (Periodic)

$\because$ a non-periodic decaying exponential is multiplied to the complex exponential,
$\therefore$ $x_2(t)$ is not-periodic.

(c) $x_3[n] = e^{j7\pi n}$

for discrete time signal, periodicity implies

$$x[n] = x[n+N] \qquad (1.12)$$

fundamental period N_0 is the smallest possible N for (1.12) to hold

Note $\rightarrow$ N is a positive integer.

for periodicity $x_3[n] = e^{j7\pi n} = e^{j7\pi(n+N_0)}$

$$\Rightarrow \quad 1 = e^{j7\pi N_0}$$
$$\Rightarrow \quad 1 = e^{j6\pi N_0} e^{j\pi N_0}$$
$$\Rightarrow \quad 1 = e^{j\pi N_0} \quad \text{(as } N_0 \text{ is integer)}$$
$$\Rightarrow \quad N_0 = 2$$

Hence, $x_3[n]$ is periodic with fundamental period $= 2$

(d) $x_4[n] = 3e^{j3\pi(n+\frac{1}{2})/5}$

$x_4[n] = x_4[n + N_0]$

$$x_4[n] = 3e^{j\frac{3\pi n}{5}} e^{j\frac{3\pi}{10}} = 3e^{j\frac{3\pi(n+\cancel{m}N_0)}{5}} e^{j\frac{3\pi}{10}}$$

$$\Rightarrow \quad 1 = e^{j\frac{3\pi \cancel{m} N_0}{5}}$$

$$\Rightarrow \quad 2\pi m = \frac{3\pi N_0}{5}$$ (as $e^{j2\pi m} = 1$ if m is integer)

$$\Rightarrow \quad N_0 = m\left(\frac{10}{3}\right)$$

for N_0 to be integer $m = 3$

$$\Rightarrow \quad N_0 = 10$$

Hence, $x_4[n]$ is periodic with fundamental period 10.

(e) $$x_5[n] = 3e^{j3\left(\frac{n}{5}+\frac{1}{2}\right)}$$

$x_5[n] = x_5[n+N_0]$ for periodicity & N_0 integer

$$\Rightarrow 3e^{j\frac{3n}{5}} e^{j\frac{3}{10}} = 3e^{j3\frac{(n+N_0)}{5}} e^{3j/10}$$

$$\Rightarrow \quad 1 = e^{j\frac{3N_0}{5}}$$

$$\Rightarrow \quad 2\pi m = \frac{3N_0}{5}$$ (where m is an integer)

$$\Rightarrow \quad N_0 = \frac{10\pi}{3} m$$

$\because$ no integer N_0 exists for any possible integer m,

$\therefore$ $x_5[n]$ is not periodic.

1.10 Determine the fundamental period of the signal

$$x(t) = 2\cos(10t+1) - \sin(4t-1)$$

Sol. for periodicity $x(t) = x(t+T)$

Now, $x_1(t) = 2\cos(10t+1) = 2\cos(10(t+T_1)+1)$

$\Rightarrow T_1 = \frac{2\pi}{10} m$ (m is integer)

Also, $x_2(t) = -\sin(4t-1) = -\sin(4(t+T_2)-1)$

$\Rightarrow T_2 = \frac{2\pi}{4} n$ (n is integer)

Now since $x(t) = x_1(t) + x_2(t)$

fundamental period $T = T_1 = T_2$ for minimum possible values of m and n

i.e for $m = 5$ & $n = 2$

we have $T = T_1 = T_2 = \pi$

In other words, overall period is the least common multiple of the individual periods of added signals.

Hence, fundamental period $= \pi$

1.11 Determine the fundamental period of

$$x[n] = 1 + e^{j4\frac{\pi n}{7}} - e^{j2\frac{\pi n}{5}}$$

Sol $x[n] = x[n + N_0]$ where N_0 is an integer

$$1 + e^{j4\frac{\pi n}{7}} - e^{j2\frac{\pi n}{5}} = 1 + e^{j4\pi\left(\frac{n}{7}+N_0\right)} - e^{j2\frac{\pi}{5}(n+N_0)}$$

$\Rightarrow$ $N_1 = \dfrac{2\pi m_1}{4\pi/7}$ (using the second term m_1 is an integer)

Also $N_2 = \dfrac{2\pi m_2}{2\pi/5}$ (using the third term m_2 is an integer)

fundamental period $N_0 = N_1 = N_2$ for smallest possible integers m_1 & m_2

~~Hence~~ $N_1 = 7\dfrac{m_1}{2}$ $N_2 = 5m_2$

$N_0 = N_1 = N_2 = 35$ for $m_1 = 10$ & $m_2 = 7$

In other words, Period of second term = 7

Period of third term = 5

Overall period is the LCM of the 2 periods.

$\therefore$ overall period = $\underline{\underline{35}}$

1.13 Consider the continuous time signal

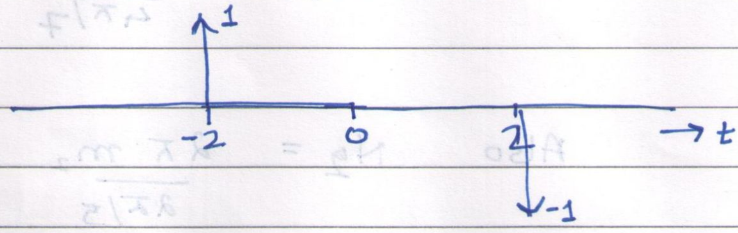
$$x(t) = \delta(t+2) - \delta(t-2)$$

Calculate the value of E_∞ for the signal

$$y(t) = \int_{-\infty}^{t} x(\tau)\, d\tau$$

Sol

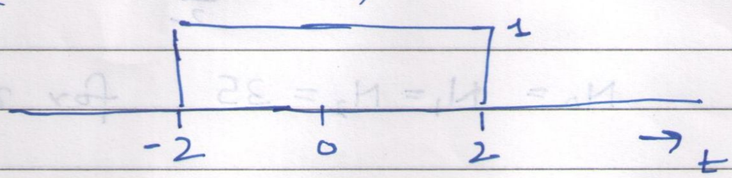
$x(t) =$



$y(t) =$ cummulative fn of $x(t)$

$$= \int_{-\infty}^{t} \left(\delta(\tau+2) - \delta(\tau-2)\right) d\tau$$

$=$



$$= \begin{cases} 0 & \cancel{t \le -2} \quad t < -2 \\ 1 & \cancel{-2 \le t \le 2} \quad -2 \le t < 2 \\ 0 & \cancel{t \ge 2} \quad t \ge 2 \end{cases}$$

$$E_\infty \triangleq \lim_{T \to \infty} \int_{-T}^{T} |x(t)|^2\, dt \qquad (1\text{-}6)$$

$$= \int_{-2}^{2} 1 \cdot dt = \underline{\underline{4}}$$

1.32 Let $x(t)$ be a continuous time signal and let

$$y_1(t) = x(2t) \quad \text{and} \quad y_2(t) = x(t/2)$$

The signal $y_1(t)$ represents a speeded up version of $x(t)$ in the sense that duration of the signal is cut in half. Similarly $y_2(t)$ represents a slowed down version of $x(t)$ in the sense that the duration of the signal is doubled. Consider the following statements

(1) If $x(t)$ is periodic, then $y_1(t)$ is periodic

(2) If $y_1(t)$ is periodic, then $x(t)$ is periodic

(3) If $x(t)$ is periodic, then $y_2(t)$ is periodic

(4) if $y_2(t)$ is periodic, then $x(t)$ is periodic

for each of these statements, determine whether it is true, and if so, determine the relationship between the fundamental periods of the 2 signals considered. If not true produce a counterexample to it.

Sol All statements are true

(1) $x(t)$ periodic with period T; $y_1(t)$ periodic, period $\frac{T}{2}$

(2) $y_1(t)$ periodic with period T; $x(t)$ periodic, period $2T$

(3) $x(t)$ periodic with period T; $y_2(t)$ periodic, period $2T$

(4) $y_2(t)$ periodic with period T; $x(t)$ periodic, period $\frac{T}{2}$

1.33 Let $x[n]$ be a discrete time signal and let

$$y_1[n] = x[2n] \quad \text{and} \quad y_2[n] = \begin{cases} x[n/2] & n \text{ even} \\ 0 & n \text{ odd} \end{cases}$$

Consider the statements →

(1) If $x[n]$ is periodic, then $y_1[n]$ is periodic

(2) if $y_1[n]$ is periodic, then $x[n]$ is periodic

(3) if $x[n]$ is periodic, then $y_2[n]$ is periodic

(4) if $y_2[n]$ is periodic, then $x[n]$ is periodic.

Sol (1) True. $x[n] = x[n+N]$

$y_1[n] = y_1[n+N_0]$ i.e periodic with

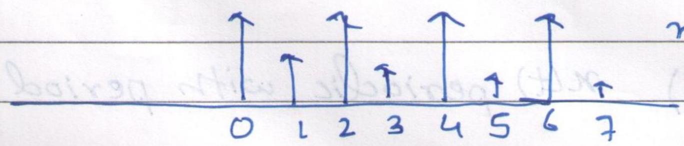
$$N_0 = \begin{cases} N/2 & \text{if } N \text{ is even} \\ N & \text{if } N \text{ is odd} \end{cases}$$

(2) false. $y_1[n]$ is periodic does not imply $x[n]$ is periodic

Let $x[n] = g[n] + h[n]$ where

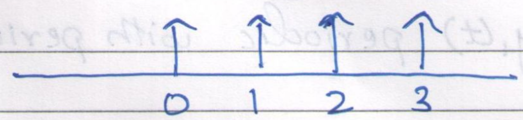
$$g[n] = \begin{cases} 1 & n \text{ even} \\ 0 & n \text{ odd} \end{cases} \qquad h[n] = \begin{cases} 0 & n \text{ even} \\ (1/2)^n & n \text{ odd} \end{cases}$$

$x[n] =$



not-periodic

$y[n] =$



periodic

(3) True. $x[n+N] = x[n]$

$\Rightarrow y_2[n+N_0] = y_2[n]$ where $N_0 = 2N$

(4) True. $y_2[n+N] = y_2[n]$

$\Rightarrow x[n+N_0] = x[n]$ where $N_0 = \frac{N}{2}$