Indian Institute of Technology Mandi
भारतीय प्रौद्योगिकी संस्थान मण्डी

IC-252

Theory Assignment - 2 (A)

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1. Suppose that X is uniformly distributed over $(-a, a)$. Determine a so that:
   (a) $P[X > 4] = \frac{1}{3}$
   (b) $P[X < 1] = \frac{3}{4}$
   (c) $P[\,|X| < 2] = P[\,|X| > 2]$.
2. Metro trains are scheduled every 5 minutes at a certain station. A person comes to the station at a random time. Let the random variable X count the number of minutes he/she has to wait for the next train. Assume X has a uniform distribution over the interval $(0, 5)$. Find the probability that he/she has to wait at least 3 minutes for the train.
3. The line width for semiconductor manufacturing is assumed to be normally distributed with a mean of 0.5 micrometer and a standard deviation of 0.05 micrometer.
   (a) What is the probability that a line width is greater than 0.62 micrometer?
   (b) What is the probability that a line width is between 0.47 and 0.63 micrometer?
   (c) The line width of 90% of samples is below what value?
4. A grinding machine is set so that its production of shaft has an average diameter of 10.10 cm and a standard deviation of 0.20 cm. The product specialization call for shaft diameter between 10.05 cm and 10.20 cm. What percentage of output meets the specialization assuming normal distribution.
5. Assume the mean height of soldiers to be 68.22 inches with a variance of 10.8 sq. inches. How many soldiers in a regiment of 1000 would you expect to be over 6 feet tall.
6. An automated manufacturing process produces a component with an average width of 7.55 cm, with a standard deviation of 0.02 cm. All components deviating by more than 0.05 cm from the mean must be rejected. What percent of the parts must be rejected, on the average? Assume a normal distribution.
7. An IQ test produces scores that are normally distributed with mean value 100 and standard deviation 14.2. The top 1 percent of all scores are in what range?
8. If X, Y are independent normal variables with means $6, 7$ and variances $9, 16$ respectively, determine λ such that
$$P(2X + Y \leq \lambda) = P(4X - 3Y \geq 4\lambda).$$
9. A supplier ships a lot of 1000 electrical connectors. A sample of 25 is selected at random, without replacement. Assume the lot contains 100 defective connectors.
   (a) Using a binomial approximation, what is the probability that there are no defective connectors in the sample?
   (b) Use the normal approximation to answer the result in part(a). Is the approximation satisfactory?
   (c) Redo parts (a) and (b) assuming the lot size is 500. Is the normal approximation to the probability that there are no defective connectors in the sample satisfactory in this case?
10. The time between calls to a plumbing supply business is exponential distributed with a mean time between calls of 15 minutes.
   (a) What is the probability that there are no calls within a 30-minutes interval?
   (b) What is the probability that at least one call arrives within a 10-minutes interval?
   (c) What is the probability that the first call arrives within 5 and 10 minutes after opening?
   (d) Determine the length if an interval of time such that the probability of at least one call in the interval is 0.90.

11. A doctor has scheduled two appointments, one at 1 P.M. and the other at $1:30$ P.M.The amounts of time that appointments last are independent exponential random variables with mean 30 minutes. Assuming that both patients are on time, find the expected amount of time that the $1:30$ appointment spends at the doctor's office.

12. The time (in hours) required to repair a machine is an exponentially distributed random variable with parameter $\lambda = \frac{1}{2}$. What is

    (a) the probability that a repair time exceeds 2 hours?

    (b) the conditional probability that a repair takes at least 10 hours, given that its duration exceeds 9 hours?

13. Jobs arriving to a compute server have been found to require CPU time that can be modeled by an exponential distribution with parameter $1/140\ ms^{-1}$. The CPU scheduling discipline is quantum-oriented so that a job not completing within a quantum of 100 ms will be routed back to the tail of the queue of waiting jobs. Find the probability that an arriving job is forced to wait for a second quantum. Of the 800 jobs coming in during a day, how many are expected to finish within the first quantum?

14. Suppose that accidents occur in a factory at a rate of $\lambda = \dfrac{1}{20}$ per working day. Suppose in the factory six days (from Monday to Saturday) are working. Suppose we begin observing the occurrence of accidents at the starting of work on Monday. Let X be the number of days until the first accident occurs. Find the probability that

    (a) first week is accident free

    (b) first accident occurs any time from starting of working day on Tuesday in second week till end of working day on Wednesday in the same week.