Indian Institute of Technology Mandi
भारतीय प्रौद्योगिकी संस्थान मण्डी

IC-252

Theory Assignment 1

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1. Five men and 5 women are ranked according to their scores on an examination. Assume that no two scores are alike and all 10! possible rankings are equally likely. Let X denote the highest ranking achieved by a woman (for instance, $X = 2$ if the top-ranked person was male and the next-ranked person was female). Find $P\{X = i\}, i = 1, 2, 3, ..., 8, 9, 10$.

2. The distribution function of the random variable X is given by:

$$F(x) = \begin{cases} 0 & x < 0 \\ \frac{x}{2} & 0 \le x < 1 \\ \frac{2}{3} & 1 \le x < 2 \\ \frac{11}{12} & 2 \le x < 3 \\ 1 & 3 \le x \end{cases}$$

   (a) Plot this distribution function.
   (b) What is $P\{X > \frac{1}{2}\}$?
   (c) What is $P\{2 < X \le 4\}$?
   (d) What is $P\{X < 3\}$?
   (e) What is $P\{X = 1\}$?

3. The amount of time, in hours, that a computer functions before breaking down is a continuous random variable with probability density function given by

$$f(x) = \begin{cases} \lambda e^{\frac{-x}{100}} & x \ge 0 \\ 0 & x < 0 \end{cases}$$

   What is the probability that a computer will function between 50 and 150 hours before breaking down? What is the probability that it will function less than 100 hours?

4. An insurance company writes a policy to the effect that an amount of money A must be paid if some event E occurs within a year. If the company estimates that E will occur within a year with probability p, what should it charge the customer so that its expected profit will be 10 percent of A?

5. The density function of X is given by

$$f(x) = \begin{cases} a + bx^2 & 0 \le x \le 1 \\ 0 & \text{otherwise} \end{cases}$$

   If $E[X] = \frac{3}{5}$, find a, b.

6. If X is a continuous random variable having distribution function F, then its median is defined as that value of m for which

$$F(m) = \frac{1}{2}$$

   Find the median of the random variables with density function

   (a) $f(x) = e^{-x}; \quad x \ge 0$
   (b) $f(x) = 1; \quad 0 \le x \le 1.$

7. Compute the mean and variance of the number of heads that appear in 3 flips of a fair coin.

8. A random variable X, which represents the weight (in ounces) of an article, has density function given by $f(z)$,

$$f(z) = \begin{cases} (z - 8) & 8 \le x \le 9 \\ (10 - z) & 9 < x \le 10 \\ 0 & \text{otherwise} \end{cases}$$

(a) Calculate the mean and variance of the random variable X.

(b) The manufacturer sells the article for a fixed price of \$2.00. He guarantees to refund the purchase money to any customer who finds the weight of his article to be less than 8.25 oz. His cost of production is related to the weight of the article by the relation $\frac{x}{15} + 0.35$. Find the expected profit per article.

9. A satellite system consists of 4 components and can function adequately if at least 2 of the 4 components are in working condition. If each component is, independently, in working condition with probability 0.6, what is the probability that the system functions adequately?

10. Let X be a binomial random variable with $E[X] = 7$ and $Var(X) = 2.1$. Find

    (a) $P\{X = 4\}$

    (b) $P\{X > 12\}$.

11. If you buy a lottery ticket in 50 lotteries, in each of which your chance of winning a prize is $\frac{1}{100}$, what is the probability that you will win a prize

    (a) at least once

    (b) exactly once

    (c) at least twice?

12. In the 1980s, an average of 121.95 workers left their job each week. Give estimates of the following quantities:

    (a) the proportion of weeks having 130 attrition or more

    (b) the proportion of weeks having 100 attrition or less.