Indian Institute of Technology Mandi
भारतीय प्रौद्योगिकी संस्थान मण्डी

IC-252

Theory Assignment - 2 (B)

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1. Suppose X has a continuous uniform distribution over the interval $[-1, 1]$.
   (a) Determine the mean, variance, and standard deviation of X.
   (b) Determine the value for x such that $P(-x < X < x) = 0.90$.

2. The probability density function of the time required to complete an assembly operation is $f(x) = 0.1$ for $30 < x < 40$ seconds.
   (a) Determine the proportion of assemblies that requires more than 35 seconds to complete.
   (b) What time is exceeded by 90% of the assemblies?
   (c) Determine the mean and variance of time of assembly.

3. If X is uniformly distributed with mean 2 and variance 12, find $P[X < 3]$.

4. Assume X is normally distributed with a mean of 10 and a standard deviation of 2. Determine the value of x that solve each of the following...
   (a) $P(X > x) = 0.5$
   (b) $P(x < X < 10) = 0.2$
   (c) $P(-x < X - 10 < x) = 0.99$

5. In a distribution exactly normal 7% of the items are under 35 and 89% are under 63. What are the mean and standard deviation of the distribution. What percentage of items lies between 40 and 70.

6. Your friend challenges you to a coin flipping contest. However you know this friend to be of questionable moral character. In fact you know that he usually carries a weighted coin that comes up heads with probability 0.55, along with a fair coin. You demand that he flip one coin 1000 times, and if it comes up heads more than 525 times, then you will play using the other coin. Assuming he uses the fair coin, find an approximation for the probability that you will think it is the biased coin.

7. A certain type of light bulb has an output that is normally distributed with mean $2,000$ end foot candles and standard deviation 85 end foot candles. Determine a lower specification limit L so that only 5 percent of the light bulbs produced will be defective. (That is, determine L so that $P\{X \geq L\} = 0.95$, where X is the output of a bulb.)

8. In an examination it is laid down that a. student passes if he secures 30% or more marks. He is placed in the first, second or third division according as he secures 60% or more marks, between 45% and 60% marks and marks between 30% and 45% respectively. He gets distinction in case he secures 80% or more marks. It is noticed from the result that 10% of the student failed in the examination, whereas 5% of them obtained distinction. Calculate the percentage of students placed in the second division. (Assume normal distribution of marks.)

9. Lifetimes of VLSI chips manufactured by a semiconductor manufacturer are approximately normally distributed with $\mu = 5 \times 10^6$ h and $\sigma = 5 \times 10^5$ h. A computer manufacturer requires that at least 95% of a batch should have a lifetime greater than 4×10^6 h. Will the deal be made?

10. A process is said to be of six-sigma quality if the process mean is at least six standard deviations from the nearest specification. Assume a normally distributed measurement.
    (a) If a process mean is centered between the upper and lower specifications at a distance of six standard deviations from each, what is the probability that a product does not meet specifications? Using the result that 0.000001 equals one part per million, express the answer in parts per million.
    (b) Because it is difficult to maintain a process mean centered between the specifications, the probability of a product not meeting specifications is often calculated after assuming the process shifts. If the process mean positioned as in part (a) shifts upward by 1.5 standard deviations, what is the probability that a product does not meet specifications? Express the answer in parts per million.

(c) Rework part (a). Assume that the process mean is at a distance of three standard deviations.

(d) Rework part (b). Assume that the process mean is at a distance of three standard deviations and then shifts upward by 1.5 standard deviations.

(e) Compare the results in parts (b) and (d) and comment.

11. The lifetime of an electronic amplifier is modeled as an exponential random variable. If 10% of the amplifiers have a mean of $20,000$ hours and the remaining amplifiers have a mean of $50,000$ hours, what proportion of the amplifiers fail before $60,000$ hours?

12. You are visiting the rain-forest, but unfortunately your insect repellent has run out. Imagine the rain-forest had only one mosquito, which arrived in the following way: the time of arrival is exponentially distributed with $\lambda = 0.2$.

    (a) What's the expected time until the first mosquito lands on you?

    (b) If you weren't bitten for the first ten seconds, what would be the expected time until the first mosquito lands on you (from time $t = 10$) ?

13. The number of years a radio functions is exponentially distributed with parameter $\lambda = \frac{1}{8}$. If Jones buys a used radio, what is the probability that it will be working after an additional 10 years?

14. Telephone calls arrive at a switchboard following an exponential distribution with parameter $\lambda = 12$ per hour. If we are at the switchboard, what is the probability that the waiting time for a call is

    (a) at least 15 minutes

    (b) not more than 10 minutes.