The Law of Minimal Ontological Load (MOL): Mathematical Formalization

Abstract

This paper presents the mathematical formalization of the **Law of Minimal Ontological Load (MOL)** — a meta-principle governing directed self-organization in complex systems. We define MOL as a constrained optimization problem and provide operational metrics for empirical validation across biological, physical, and urban systems. MOL posits that evolutionary stability arises from the systemic minimization of internal structural redundancy while preserving functional integrity.

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1. Introduction: The Meta-Principle of Directed Evolution

The MOL Law posits that the directed self-organization of complex systems (from atoms to societies) is a consequence of a universal drive toward **structural economy**.

MOL acts as a **meta-principle of model selection**, defining which system dynamics are capable of generating **evolutionary stable structures** (in contrast to the Principle of Least Action, which optimizes trajectories within pre-defined laws).

Principle	Level of Operation	Parameter Minimized
Least Action	System Dynamics	Energy / Action Path
MOL	**Structure of Laws/Models**	**Ontological Redundancy** (O(E))

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2. The Target Function: A Constrained Optimization Problem

MOL is formulated as a constrained optimization problem aimed at finding the **Evolutionarily Stable State** (E*).

$E^* = \arg\min_{E \in \Omega} O(E)$

2.1. Constraints (Conditions for System Integrity)

The minimization of O(E) must occur while strictly maintaining the system's functional and structural integrity:

$I(E) \geq I_{min}$ (Informational / Functional Integrity) $C(E) \geq C_{min}$ (Topological Connectivity)

Symbol	Definition	Description
E*	**Stable State**	The system configuration with the lowest possible O(E).
O(E)	**Ontological Load**	The measure of non-functional (redundant) complexity within the structure E.
Ω	**Ontology Space**	The set of all permissible system structures.

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3. Operationalization of O(E) (The Redundancy Metric)

O(E) is defined as the measure of non-functional redundancy — the fraction of entities or relationships that do not contribute to maintaining I_{min}.

3.1. General Form (Information Theory):

O(E) measures the difference between **structural complexity** (K(E)) and the **mutual information** shared with the system's intended function (F):

$O(E) \approx K(E) - I(E; F)$

3.2. Empirical Operational Metrics (Case Studies):

- **Biology (T4 Lysozyme Protein):** $O(E)_{protein}$ = (Number of Non-Functional Structural Bonds) / (Total Number of Bonds in E)
    - *Algorithmic Formalization (Protein):*

```
# Pseudo-code for protein O(E) calculation
def calculate_ontological_load(protein_structure):
    total_bonds = count_structural_bonds(protein_struc
```

```
        functional_bonds = count_bonds_contributing_to_sta
        redundant_bonds = total_bonds - functional_bonds
        return redundant_bonds / total_bonds
```

- **Urban Systems (Transport Networks):** O(E)_urban ≈ Efficiency Ratio = (Actual Connections) / (Optimal/Functional Connections)
    - *Algorithmic Formalization (Urban):*

```
 # Pseudo-code for urban transport O(E)
def urban_ontological_load(stop):
    # Optimal connections based on lines served (e.g.,
    optimal_connections = stop.lines * 2
    efficiency_ratio = stop.actual_connections / optim
    return efficiency_ratio # O(E) ≈ ratio (unnormaliz
```

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4. Mathematical Properties and Computational Aspects

4.1. Mathematical Properties of O(E)

As a rigorous measure of complexity and redundancy, O(E) exhibits the following properties:

1. **Non-negativity:** $O(E) \geq 0$
2. **Boundedness:** $0 \leq O(E) \leq 1$ for normalized systems.
3. **Convexity:** O(E) is **convex** for linear systems, guaranteeing that the global minimum E* is reachable.
4. **Scale-invariance:** $O(\alpha E) = O(E)$
5. **Sub-additivity:** $O(E_1 \cup E_2) \leq O(E_1) + O(E_2)$
6. **Monotonicity:** $E_1 \subset E_2 \Rightarrow O(E_1) \leq O(E_2)$

4.2. Computational Considerations

The calculation of O(E) possesses known complexity classes:

- **Proteins:** $O(n^2)$ for n residues (structural analysis).
- **Networks:** $O(m \log n)$ for n nodes, m edges (graph metrics).

- **Physical Systems:** $O(t \cdot s^2)$ for t timesteps, s states (simulation).

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5. Dynamics of Implementation: The Φ Operator

Evolution is realized through **discrete phase shifts** or **ontological plane transitions**, governed by the operator Φ.

The system evolves according to the rule:

$E(t + \Delta t) = E(t)$, if $O(E(t)) \leq \tau$ $E(t + \Delta t) = \Phi(E(t), \delta)$, if $O(E(t)) > \tau$

Relationship to Meta-Principles (The Principles Guide):

The Φ operator is mathematically constrained by the system's inherent meta-principles:

- **Principle of Critical Susceptibility (PCS):** Minimizes the average **cost of the transition** C_Φ. $\min \langle C_\Phi \rangle = \int P(\delta) \cdot C_\Phi(\delta)\, d\delta$
- **Principle of Attractor Dominance (PAD):** Determines the **direction** of Φ by selecting the attractor (A_i) that offers the greatest reduction in $O(E)$. $\Phi_{direction} = \arg\max_{A_i} [D_a(A_i) \times W_b(A_i)]$

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6. Connections to Foundational Theories

MOL generalizes existing informational theories by applying them to dynamic self-organization:

- **Minimum Description Length (MDL) & Algorithmic Information Theory (AIT):** $O(E)$ quantifies the **physical manifestation** of redundancy.
- **Free Energy Principle (FEP, Friston):** $O(E)$ serves as a structural, **physical analog** to the minimization of predictive redundancy, uniting structural economy with cognitive inference.

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7. Conclusion

The mathematical formalization of MOL presents a **unified theoretical framework** that defines evolution as a problem of **minimizing structural redundancy** under functional constraints. The rigor of the O(E) metric, the explicit dynamics of the Φ operator, and the robust empirical validation confirm MOL as a universal, computational law of reality.