BAKER CHARTERS SCHOOL

HONORS ALGEBRA 2A

Baker Web Academy School Notes

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CHAPTER ONE

Radical and Polynomial Operations

Unit 1

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Lesson 1: Rational Exponents **Unit 1**

Rational Exponents to Radical Expressions

Here's the basic expression:

$$x^{\frac{z}{y}} = \sqrt[y]{x^a}$$

.

Example. Rewrite $(6x)^{\frac{4}{5}}$ as a radical expression. Now, let's just use that expression from above to solve this:

$$(6x)^{\frac{4}{5}} = 6\sqrt[5]{x^4}$$

.

Radical Expressions to Rational Exponents

Now, let's do the reverse. Like before, here's the basic expression:

$$\sqrt[y]{x^z} = x^{\frac{z}{y}}$$

.

Example. Let's rewrite $\sqrt[8]{w^5}$ as a rational expression:

$$\sqrt[8]{x^5} = x^{\frac{5}{8}}$$

.

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Lesson 2: Properties of Rational Exponents **Unit 1**

Dividing Rational Exponents

You already know that when like variables are multiplied, their exponents are added. But what happens when you divide them? Look at this property 1

Property 1. (Quotient of a Power Property) To divide powers of the same base, subtract the exponents:

Example.

$$\begin{aligned} \frac{a^5}{a^3} &= \frac{\cancel{a} \times \cancel{a} \times \cancel{a} \times a \times a}{\cancel{a} \times \cancel{a} \times \cancel{a}} = a^2 \\ &= a^{5-3} \end{aligned}. \tag{1.1}$$

Review 1. (Finding a Common Denominator) Look at the factors: $\frac{1}{2}$ and $\frac{1}{4}$.
First, you start by listing the multiples of each denominator:

$$2 : 2, \boxed{4}, 6, 8, 10, 12, \ldots$$

.

$$4 : \boxed{4}, 8, 12, 16, 20, 24 \ldots$$

.
So, we just found our common denominator: 4. Since the first fraction: $\frac{1}{4}$, already has a denominator of 4, we can just leave it alone. But, the second fraction: $\frac{1}{2}$, needs to have a denominator of 4. So, we just do: $4 \div 2 = 2$. So, we multiply the second fraction by $\frac{2}{2}$.

$$\frac{1}{2} \times \frac{2}{4} = \frac{2}{4}$$

.
Multiplying the numerator and denominator by the same factor is the same as multiplying the entire fraction by 1 since $\frac{2}{2} = 1$. By multiplying $\frac{1}{2}$ by $\frac{2}{2}$, the value of the fraction hasn't changed. In fact, if you simplify the new fraction we got, which was $\frac{2}{4}$, it goes back to the original fraction: $\frac{1}{2}$. But now, it just has a common denominator of 4, which is what we need to add the exponents.

Raising a Power to a Power

Multiplication of powers with the same base involves the addition of exponents. Division of powers with the same base involves the subtraction of exponents. What operation is performed on the exponents of 2 and 3 to result in the exponent of 6? Take a look at this property property **??**.

Property 2. (Power of a Power Property) To raise a power to a power, multiply the exponents:

$$(a^m)^n = a^{m\times n}$$

.

Example.

$$\begin{aligned}\left(c^{\frac{1}{2}}\right)^{\frac{1}{4}} &= c^{\frac{1}{2}\times\frac{1}{4}} \\ &= c^{\frac{1}{8}} \\ &= \sqrt[8]{c}\end{aligned} \quad . \tag{1.2}$$

Negative Rational Exponents

Let's take a look at the pattern that forms as you look at decreasing powers of 4:

$$\begin{aligned}4^4 &= 4\times4\times4\times4 = 256 \\ 4^3 &= 4\times4\times4 = 64 \\ 4^2 &= 4\times4 = 16 \\ 4^1 &= 4 = 4 \\ 4^0 &= 1 \\ 4^{-1} &= \frac{1}{4^1} = \frac{1}{4} \\ 4^{-2} &= \frac{1}{4^2} = \frac{1}{16} \\ 4^{-3} &= \frac{1}{4^3} = \frac{1}{64} \\ 4^{-4} &= \frac{1}{4^4} = \frac{1}{256}\end{aligned} \quad . \tag{1.3}$$

Recall that the inverse of a positive is a negative, and vise versa. Also, the inverse of multiplication is division and the inverse of addition is subtraction. When the exponent is positive, it tells you to multiply the base the number of times, which is indicated by the exponent.

$$4^3 = 4\times4\times4 = 64$$

.

Inversely, a negative exponent tells you to divide the base by the number of times indicated by the exponent.

$$4^{-3} = 4\div4\div4 = 0.015625$$

.

or

$$4^{-3} = \frac{1}{4^3} = \frac{1}{64} = 0.015625$$

.

Example. Simplify the following expression:

$$\begin{aligned} 9^{-\frac{1}{2}} &= \frac{1}{9^{\frac{1}{1}}} \\ &= \frac{1}{\sqrt{9}}. \\ &= \frac{1}{3} \end{aligned} \tag{1.4}$$

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Lesson 3: Solving Radical Equations **Unit 1**

Solving Radical Equations

Let's go over how to solve this radical equation: $\sqrt{(5x+4)} = 7$:

1. Isolate the Radical: When solving equations, the goal is always to isolate the variable to find its value. The goal is the same with radical equations. So, we must make the radical isolated first. But in this case, we already have the radical isolated.

2. Square both Sides: Do the opposite operation of taking the square root and square both sides of the equation.

$$\begin{aligned} \sqrt{(5x+4)} &= 7 \\ \sqrt{(5x+4)}^2 &= 7^2. \\ 5x+4 &= 49 \end{aligned} \tag{1.5}$$

   Once the radical is gone, we just simply solve for the variable.

3. Solve for the Variable: To solve a two-step equation, remember that you must use the reverse order of operations.
   (SADMEP: **S**ubtraction, **A**ddition, **D**ivision, **M**ultiplication, **E**xponents, **P**arentheses)

$$\begin{aligned} 5x+4 &= 49 \\ 5x &= 45 \\ \frac{5x}{5} &= \frac{45}{5}. \\ \boxed{x=9} \end{aligned} \tag{1.6}$$

4. Check your Work: When solving radical equations, you must always check your work. Even if you do all of the work perfectly, you may not find the correct solution. So, let's plug the value of 9 into the original equation to see if it works.

$$\begin{aligned} \sqrt{5x+4} &= 7 \\ \sqrt{5(9)+4} &= 7 \\ \sqrt{45+4} &= 7. \\ \sqrt{49} &= 7 \\ \boxed{7=7} \end{aligned} \tag{1.7}$$

Let's take a look at a radical equation that doesn't have any solutions:

Example.

$$
\begin{aligned}
\sqrt{x-3}+4 &= 1\\
\sqrt{x-3} &= -3\\
\sqrt{x-3}^2 &= -3^2. \qquad (1.8)\\
x-3 &= 9\\
\boxed{x=12}
\end{aligned}
$$

Now, let's check our work:

$$
\begin{aligned}
\sqrt{x-3}+4 &= 1\\
\sqrt{12-3}+4 &= 1\\
\sqrt{9}+4 &= 1. \qquad (1.9)\\
3+4 &= 1\\
7 &\neq 1
\end{aligned}
$$

This statement is false. This is known as a **extraneous solution**. (1)

Definition 1. (Extraneous Solution) An **Extraneous Solution** is a solution to an equation that doesn't fit the requirements of the original equation.

Solving Radical Equations with Variables Outside the Radical

Before we start to solve the equation below (1.10), take a look at the following property (3)

Property 3. (Zero Product Property) If $a \times b = 0$, then $a = 0$ and/or $b = 0$
Let's say we are given the following polynomial: $(x+8)(x-4) = 0$. Here's how we would solve it. First, we take each item within the parentheses and set that equal to 0. For example:

$$(x+8) = 0 \quad (x-4) = 0$$

.
Now, let's find the zeros of the polynomial.

$$x = -8 \quad x = 4$$

.

Let's solve: $\sqrt{x+9} - 7 = x$:

$$
\begin{aligned}
\sqrt{x+9}-7&=x\\
\sqrt{x+9}&=x+7 \quad \text{Isolate the variable}\\
(\sqrt{x+9})^2&=(x+7)^2 \quad \text{Square both sides}\\
x+9&=(x+7)(x+7)\\
x+9&=x^2+14x+49 \quad \text{Solve for the variable}\\
9&=x^2+13x+49\\
0&=x^2+13x+40 \quad \text{Factor}\\
0&=(x+5)(x+8) \quad \text{Apply the zero product property (3).}\\
x+5=0 \quad &\text{AND} \quad x+8=0
\end{aligned}
$$

$\boxed{x=-5,-8}$ Check your Work

$$
\begin{aligned}
\sqrt{x+9}-7&=x \quad \sqrt{x+9}-7=x\\
\sqrt{-5+9}-7&=-5 \quad \sqrt{-8+9}-7=-8\\
\sqrt{4}-7&=-5 \quad \sqrt{1}-7=-8\\
2-7&=-5 \quad 1-7=-8\\
-5&=-5 \quad -6\neq-8
\end{aligned}
\tag{1.10}
$$

$x=-5$ is the only solution and $x=8$ is extraneous.

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Lesson 4: Complex Numbers **Unit 1**

Definition 2. (Imaginary Numbers) Here's a quick question. Solve this equation: $a \times b = -1$. They must not be identical factors, meaning it cannot be $1 \times -1 = -1$.
To solve this problem, we came up with the imaginary number i. It's defined to:

$$i = \sqrt{-1}$$

.
With this definition, the square root of a negative radicand, in addition to positive radicands can be simplified.

History of the Imaginary Numbers

Thought history, there have been times when a mathematical construct was invented before its purpose was discovered. For example, early man knew the concept of 1 before knowing 0. Numbers, at that time, were used to count sheep and other animals as a source for food. There was no need to define the concept of 0. But, when fractions were defined, they were thought of useless. What purpose could there be to have a number between 0 and 1. But today, fractions become part of our daily life. Imaginary numbers have the same history. It's use will come out eventually, we just don't know when.

Solving an equation such as: $x^2 - 9 = 0$ should be already familiar. The goal is to isolate the variable on one side by first adding 9 to both sides.

$$\begin{aligned} x^2 &= 9 \\ \sqrt{x^2} &= \sqrt{9}. \\ \boxed{x = \pm 3} \end{aligned} \tag{1.11}$$

Notice that the $\sqrt{9}$ is indicated as ± 3. The reason this is because there are two pairs of identical factors that produce 9 when multiplied: $3 \times 3 = 9$ and $-3 \times -3 = 9$.

But, how do you solve an equation such as: $x^2 + 1 = 0$? You begin just as in the last equation. Isolate the variable, but this time, subtract 1 from both sides of the equal sign.

$$\begin{aligned} x^2 + 1 &= 0 \\ x^2 &= -1 \quad . \\ \sqrt{x^2} &= \sqrt{-1} \end{aligned} \tag{1.12}$$

This is what troubled mathematicians for centuries. All of the radicans they have been working with have all been positive. So, to overcome this obstacle, they invented i.

Example. Let's try and simplify the following expression: $\sqrt{-4}$

$$\begin{aligned} \sqrt{-4} &= \sqrt{-1} \times \sqrt{4} \\ &= i \times \sqrt{4} \\ &= i\sqrt{4} \\ &= \boxed{2i} \end{aligned} \quad . \tag{1.13}$$

Using i to help solve equations

You already know that $i = \sqrt{-1}$, but what about the different powers of i. Take a look at the table below (1.1)

$i = i$	$i^5 = i$	$i^9 = i$	$i^{13} = i$
$i^2 = -1$	$i^6 = -1$	$i^{10} = -1$	$i^{14} = -1$
$i^3 = -i$	$i^7 = -i$	$i^{11} = -i$	$i^{15} = -i$
$i^4 = 1$	$i^8 = 1$	$i^{12} = 1$	$i^{16} = 1$

Table 1.1: Different Powers of i

Example. Simplify: i^{27}:
First, you take the exponent (27) and divide it by 4 and keep the remainder. The reason we divide it by 4 is because that's how many version of i there are.

$$\begin{array}{r} 6 \\ 4\,\overline{)\,27} \\ \underline{24} \\ 3 \end{array}$$

We raise i to the power of 3, which is the remainder of $27 \div 4$. If we look at the different powers of i table (1.1), we will see that $i^3 = -i$.

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Lesson 5: Operations on Complex Numbers **Unit 1**

Multiplying Complex Numbers

Multiplying complex numbers is a little bit different than adding and subtracting complex numbers.

Example. Let's simplify the following expression: $\sqrt{-12} \times \sqrt{-5}$

$$\begin{gathered}\sqrt{-12} \times \sqrt{-5} \\ (\sqrt{-1} \times \sqrt{12}) \times (\sqrt{-1} \times \sqrt{5}). \\ i\sqrt{12} \times i\sqrt{5}\end{gathered} \tag{1.14}$$

Once the imaginary numbers have been factored, multiply the imaginary numbers together and multiply the radical factors together.

$$\begin{gathered}i\sqrt{12} \times i\sqrt{5} \\ i^2\sqrt{60} \\ -2\sqrt{15}\end{gathered}\ . \tag{1.15}$$

There is another method for multiplying complex numbers. You first multiply the square roots instead of factoring. Here's how that would look like:

$$\begin{gathered}\sqrt{-12} \times \sqrt{-5} \\ \sqrt{60} \\ \sqrt{4} \times \sqrt{15} \\ 2\sqrt{15}\end{gathered}\ . \tag{1.16}$$

You can see here (1.16) that the negative was lost when the radicands were multiplied before factoring the imaginary number. This shows why the imaginary number must be factored first when multiplying radicals.

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Lesson 6: Polynomial Operations **Unit 1**

Adding Polynomials

Definition 3. (Like Terms) Like terms are terms that have the same **variable** and **exponent**.
Ex: $5y^2$ and $5909y^2$.
Ex: $9zsa^d$ and $8sa^dz$.

The most important part of adding or subtracting polynomials is identifying **like terms** (3).

Example. Let's add the following polynomials: $(7x^2+3x-5)+(2x^2-4x+9)$

$$\begin{aligned} &\text{Step 1:}\quad \text{Distribute any Coefficient} \\ 1(7x^2+3x-5)+1(2x^2-4x+9) &= 7x^2+3x-5+2x^2-4x+9 \\ &\text{Step 2:}\quad \text{Identify the Like Terms} \\ 7x^2+3x-5+2x^2-4x+9 &= \boxed{9x^2-x+4} \end{aligned} \tag{1.17}$$

The reason there's a 1 in front of each polynomial is because we're adding, and when you add, you distribute a positive 1. Take a look at this polynomial: $5(3x^2+3x-1)+4(x^2+2)$. You distribute a positive 5 to the first polynomial and a positive 4 to the second polynomial.

Subtracting Polynomials

Review 2. (Distributive Property)

$$\begin{aligned} a(b+c) &= a(b)+a(c) \\ -a(b+c) &= -(a(b))+-(a(c)) \end{aligned}. \tag{1.18}$$

Subtracting polynomials is almost exactly like adding polynomials. The only difference is you distribute a negative coefficient instead of a positive one.

Example. Let's subtract the following polynomials: $(9x^2+x+2)-(7x^2+3x-4)$.

$$\begin{aligned} &\text{Step 1:}\quad \text{Distribute any Coefficient} \\ (9x^2+x+2)-(7x^2+3x-4) &= 9x^2+x+2-7x^2-3x+4 \\ &\text{Step 2:}\quad \text{Identify the Like Terms} \\ 9x^2+x+2-7x^2-3x+4 &= \boxed{2x^2-2x+6} \end{aligned} \tag{1.19}$$

This time, instead of distributing a positive 1, we distribute a negative 1 because we're subtracting. Take a look at the **Distributive Property** review (2) if you need a refresher.

Using the Distributive Property

Multiplying by a Monomial

Definition 4. (Monomial) A monomial is a polynomial with only one term. Ex: $7 \quad 4x \quad 19z^{10}$.

Example. Let's try to multiply the following monomial by the following polynomial: $3x^2(5x^2 - 7x + 8)$

$$\begin{aligned} 3x^2(5x^2 - 7x + 8) &= 3x^2 \times 5x^2 \\ &= 15x^4 \\ 3x^2(5x^2 - 7x + 8) &= 3x^2 \times (-7x) \\ &= -21x^3 \\ 3x^2(5x^2 - 7x + 8) &= 3x^2 \times 8 \\ &= 24x^2 \\ &= \boxed{15x^4 - 21x^3 + 24x^2} \end{aligned} \quad . \tag{1.20}$$

Multiplying by a Binomial

Definition 5. (Binomial) A binomial is a polynomial with only two term. Ex: $7 + 9x \quad 4y^3 - 4x \quad 19z^{10} \times 8z$.

Example. Let's try to multiply the following monomial by the following polynomial: $(x - 7)^2$. The following polynomial can also be written as: $(x - 7)(x - 7)$

$$\begin{aligned} (x-7)(x-7) &= x \times x \\ &= x^2 \\ (x-7)(x-7) &= x \times (-7) \\ &= -7x \\ (x-7)(x-7) &= (-7) \times x \\ &= -7x \\ (x-7)(x-7) &= (-7) \times (-7) \\ &= 49 = x^2 - 7x - 7x + 49 \\ &= \boxed{x^2 - 14x + 49} \end{aligned} \quad . \tag{1.21}$$

Multiplying by a Trinomial

Definition 6. (Trinomial) A trinomial is a polynomial with three terms. Ex: $8x + 1 - y \quad u + 8x - 1 \quad 1 \times 14x - z$

Example. Let's try to multiply the following polynomial by the following binomial $(4x^2 - 5x + 9)(x - 5)$

$$
\begin{aligned}
(4x^2 - 5x + 9)(x - 5) &= 4x^2 \times x \\
&= 4x^3 \\
(4x^2 - 5x + 9)(x - 5) &= 4x^2 \times (-5) \\
&= -20x^2 \\
(4x^2 - 5x + 9)(x - 5) &= (-5x) \times x \\
&= -5x^2 \\
(4x^2 - 5x + 9)(x - 5) &= (-5x) \times (-5) \\
&= 25x \\
(4x^2 - 5x + 9)(x - 5) &= 9 \times x \\
&= 9x \\
(4x^2 - 5x + 9)(x - 5) &= 9 \times (-5) \\
&= -45 = 4x^3 - 20x^2 - 5x + 25x + 9x - 45 \\
&= \boxed{4x^3 - 25x^2 + 34x - 45}
\end{aligned}
\tag{1.22}
$$

Equations vs Functions

Definition 7. (Equation) An equation is usually defined as the state of being equal and is often shown as a math expression with equal values on either side, or refers to a problem where many things need to be taken into account. Ex: $2 + 2 = 4 + 2 - 1(2)$

Definition 8. (Function) A function in mathematics is an expression, rule, or law that defines a relationship between one variable and another variable. Ex: $f(x) = x^2 + x$
In order for an equation to be a function, there must be only one y value for each x value. The function $f(x) = x^2$ isn't a function. If you input $f(2) = 2^2 = 4$ and $f(-2) = -2^2 = 4$. You get the same y value for two different x values. So, it doesn't meet the requirements to be a function.

The mathematical term function (8) applies to some equations (7) in math. But, not all equations are functions.

Example. $y = 2x + 3$
Let's take a look at the equations x/y table:

x	y
-2	-1
-1	1
0	3
1	5
2	7

Since this equation doesn't have any y values for multiple x values, then it's considered a function.

Vertical Line Test

Definition 9. (Vertical Line Test) The **Vertical Line Test** is a visual method to determine if an equation is a function or not. You first must graph the equation. Then, if you two or more y intercepts, then it's not a function because it has the same y value for multiple x values. Likewise, if it has 1 or 0 y intercepts, then it's a function.

Another way to determine of an equation is a function is to graph the equation and perform the **Vertical Line Test** (9).

Example. Let's graph the equation: $y = 2x + 3$:

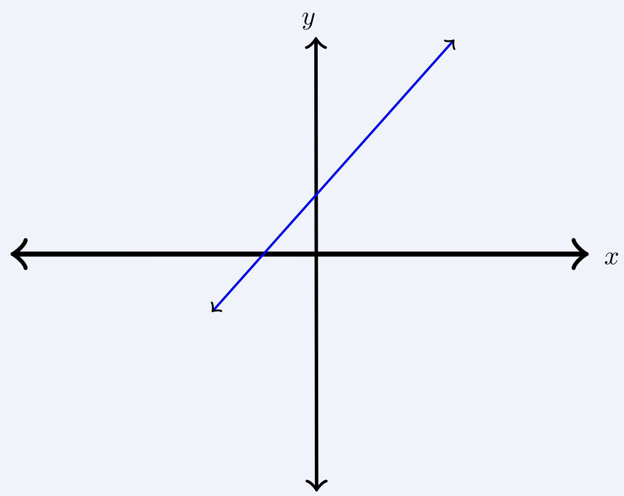

Figure 1.1: Vertical Line Test

Since there aren't any multiple y intercepts, then it's a function.

Inverses of Functions

Finding the inverse of an integer means performing an operating that will cancel that number. For example, if you are given the number 7, you could add -7 to it to cancel it: $7 + (-7) = 0$. Or you could multiply it by it's reciprocal: $7 \times \frac{1}{7} = 1$.

With equations and functions, finding the inverse requires reversing the variables and solving for y.

Example. Let $f(x) = 3x - 5$. Find its inverse represented by the function notation: $f^{-1}(x)$

$$\begin{aligned} f(x) &= 3x - 5 \\ y &= 3x - 5 \\ x &= 3y - 5 \\ x + 5 &= 3y \\ \frac{x+5}{3} &= \frac{3y}{3} \\ \frac{x+5}{3} &= y \\ f^{-1}(x) &= \frac{x+5}{3} \end{aligned} \quad . \tag{1.23}$$

Operations on Functions

Addition The addition of two functions $f(x)$ and $g(x)$ is represented using the following notation: $f(x) + g(x)$

Example. Let $f(x) = 4x - 7$
Let $g(x) = 10x - 3$

$$\begin{aligned} f(x) &= 4x - 7 \\ g(x) &= 10x - 3 \\ f(x) + g(x) &= (4x - 7) + (10x - 3). \\ &= 4x - 7 + 10x - 3 \\ &= \boxed{14x - 10} \end{aligned} \tag{1.24}$$

Subtraction Subtracting functions is similar to adding functions. The only difference is that you need to be very caucus with the signs.

Example. Let $f(x) = 4x - 7$
Let $g(x) = 10x - 3$

$$\begin{aligned} f(x) &= 4x - 7 \\ g(x) &= 10x - 3 \\ f(x) - g(x) &= (4x - 7) - (10x - 3). \\ &= 4x - 7 - 10x + 3 \\ &= \boxed{-6x - 4} \end{aligned} \tag{1.25}$$

Multiplication The multiplication of functions is usually just an extension of the distribute property (2). The notation used to multiply functions is $f(x) \times g(x)$.

Example. Let $f(x) = 4x - 7$
Let $g(x) = 10x - 3$

$$\begin{aligned} f(x) \times g(x) &= (4x-7)(10x-3) \\ &= 4x \times 10x = 40x^2 \\ &= 4x \times (-3) = -12x \\ &= (-7) \times 10x = -70x \\ &= (-7) \times (-3) = 21 \\ &= 40x^2 - 12x - 70x + 21 \\ &= \boxed{40x^2 - 82x + 21} \end{aligned} \quad . \tag{1.26}$$

Function Composition of Polynomials

Definition 10. (Function Composition) **Function Composition** is applying the function to the result of another.
There are multiple ways for writing function compositions. Here's one way: $(f \circ g)(x)$. This is the same thing as: $f(g(x))$.
What this means is we solve for $g(x)$, then whatever the output of that is, we plug that into $f(x)$.

Example. Given the functions $f(x) = 2x^2 - 4x + 1$ and $g(x) = x^2 + 3$, find $(f \circ g)(x)$.

$$\begin{aligned} \text{Step 1:} &\quad \text{Substitute expression} \\ f(x) = 2x^2 - 4x + 1 &\quad g(x) = x^2 + 3 \\ f(g(x)) &= 2(x^2+3)^2 - 4(x^2+3) + 1 \\ \text{Step 2:} &\quad \text{Simplify} \\ &\text{Rewrite } (x^2+3)^2 \text{ as } (x^2+3)(x^2+3) \\ &= 2(x^2+3)(x^2+3) - 4(x^2+3) + 1 \\ &\text{Multiply } (x^2+3)(x^2+3) \\ &= 2(x^4 + 3x^2 + 3x^2 + 9) - 4(x^2+3) + 1 \\ &= 2(x^4 + 6x^2 + 9) - 4(x^2+3) + 1 \\ &\text{Distribute the 2 and } -4 \\ &= 2x^4 + 12x^2 + 18 - 4x^2 - 12 + 1 \\ &\text{Combine like terms} \\ &= \boxed{2x^4 + 8x^2 + 7} \end{aligned} \quad . \tag{1.27}$$

Function composition (10) is different from function multiplication. By substituting the function $g(x)$ into the function $f(x)$, you are evaluating $f(x)$ at $g(x)$. As $g(x)$ changes for each x value, the composition of $f(g(x))$ will also change because you will be evaluating $f(x)$ at the new $g(x)$ value each time.

If you're given a specific value for $g(x)$, then you could simply substitute that value into $f(x)$ to solve for the composition, $f(x)$.

Example. If you are given $g(4) = 10$ and you're asked to find $f(g(4))$, you would substitute 10 into $f(x)$. You're evaluating $f(x)$ at the answer for $g(4)$. Most often, you will need to write a new function, $f(g(x))$, using the original functions so that you can have a new function that will represent all x values.

Paying attention to the units that each function, $f(x)$ and $g(x)$, is measured in will help you figure out what represent. If $f(x)$ is measured in $\frac{\text{dollars}}{\text{bagel}}$ and $g(x)$ is measured in $\frac{\text{bagels}}{\text{hour}}$, then the composition of $f(g(x))$ is taking a function measured in $\frac{\text{bagels}}{\text{hour}}$ and substituting that into a function being measured in $\frac{\text{dollars}}{\text{bagel}}$. This means the composition $f(g(x))$ will be measured in $\frac{\text{dollars}}{\text{hour}}$

Verifying Inverses

Function composition is often used to verify inverse functions. If $f(g(x)) = g(f(x)) = x$, then the functions are inverses of each other.

Example. Verify that $f(x) = \frac{x+5}{3}$ is the inverse of $g(x) = 3x - 5$: Use function composition to evaluate $(f \circ g)(x)$:

$$(f \circ g)(x) = \frac{(3x-5)+5}{3}$$

$$= \frac{3x}{3} = \boxed{x}$$

Step 1: Now Simplify

$$\begin{aligned} = 3\left(\frac{x+5}{3}\right) - 5 &= \frac{3(x) + 3(5)}{3} - 5 \\ &= \frac{3x+15}{3} - 5 \\ &= \frac{3x}{3} + 5 - 5 \\ &= \frac{3x}{3} \\ &= \boxed{x} \end{aligned} \tag{1.28}$$

Because $(f \circ g)(x) = (g \circ f)(x) = x$, then $f(x)$ and $g(x)$ are inverses.

CHAPTER TWO

Factoring and Quadratics

Unit 2

Oct 11 2021 Mon (10:42:23)

Lesson 1: GCF and SP **Unit 2**

Difference of Squares Binomials

Definition 11. (Difference of Squares Binomials) A Difference of Squares Binomial (DSB) is when both terms in a binomial are perfect squares **AND** they are being subtracted. A DSB can be factored using the following patter:

$$a^2 - b^2 = (a+b)(a-b). \tag{2.1}$$

Note. The square root of each perfect square $(a^2 \quad b^2)$ is $a \quad b$. These terms are placed inside two sets of parentheses where one is addition and one is subtraction. Multiplying the two factors $(a+b)(a-b)$ results in the following:

$$\begin{gathered}(a+b)(a-b)\\ a^2+ab-ab-b^2\\ a^2+0ab-b^2\\ a^2-b^2\end{gathered}. \tag{2.2}$$

Here's how you identify a DSB.

1. What do you notice about the first terms in each of the products: $g^2 - 16 \quad 25g^2 - 64$?

   The terms g^2 and $25g^2$ are perfect squares.

2. What do you notice about the last terms in each of the products: $g^2 - 16 \quad 25g^2 - 64$?

   The terms 16 and 64 are also perfect squares.

3. What sign separates the first and last terms in each of the products: $g^2 - 16 \quad 25g^2 - 64$?

   They are all subtraction signs.

4. Now try working backwards. What two sets of binomials that multiply together to produce $g^2 - 16$?

   The two binomials are: $g + 4 \quad g - 4$.

5. Why are the signs different for each of the factors?

   The signs are different so that, when the two sets of parentheses are multiplied, the last term will be negative and the middle terms will cancel.

Example. Check if $64d^4 - 361d^2$ is a DSB:

$$\begin{aligned}
\text{Step 1:} &\quad \text{Look at the first term} \\
64d^4 &\quad \text{Is it a perfect square? Yes!} \quad \sqrt{64} = 8 \\
\text{Step 2:} &\quad \text{Look at the last term} \\
361d^4 &\quad \text{Is it a perfect square? Yes!} \quad \sqrt{361} = 19 \\
\text{Step 3:} &\quad \text{What sign separates the first and last term} \\
64d^4 - 361d^4 &\quad \text{The } - \text{ sign, which is what's needed.} \\
\text{Step 4:} &\quad \text{Let's find the two sets of binomials!} \\
&a = \sqrt{64d^4} = 8d^2 \\
&b = \sqrt{361d^2} = 19d \\
&(8d^2 + 19d)(8d^2 - 19d) \\
&\boxed{d^2(8d + 19d)(8d - 19d)} \quad \text{Factor the } d^2
\end{aligned} \tag{2.3}$$

Perfect Square Trinomials

Definition 12. (Difference of Squares Trinomials) A Perfect Square Trinomials (PST) is when first and last term of the PST are both perfect squares **AND** the middle term is made from two numbers that when you multiply you get the last term and when you add them, you get the middle term.
Here's the pattern that a PST follows:

$$a^2 - 2ab + b^2 = (a - b)^2. \tag{2.4}$$

Here's how you identify a PST.

1. What do you notice about the first terms in each of the products: $g^2 + 12g + 36 \quad g^2 - 6g + 9$

   They are all perfect squares.

2. What do you notice about the last terms in each of the products: $g^2 + 12g + 36 \quad g^2 - 6g + 9$

   They too, are perfect squares.

3. What signs do the first and last terms have?

   They are all positive.

4. What about the sign on the middle term in each product?

   The sign of the middle term is identical to the sign in the factors (squared binomials).

5. Now try working backwards. What two sets of binomials that multiply together to produce $g^2 - 16g + 9$?

   The binomials are: $(g-8)(g-8)$ or $(g-8)^2$.

6. How is the middle term of $-16g$ used to find the factors?

   The middle term is used to determine the operation inside each set of parentheses and to verify this polynomial is a PST. -8 and g are multiplied to and doubled to arrive at $-16g$.

Example. Let's find the factors of the following PST: $x^2 - 5x - 24$:

$$\begin{aligned} & x^2 - 5x - 24 \\ & 3 \times -8 = -24 \\ & 3 + -8 = -5 \\ & (x+a)(x+b) \\ & \boxed{(x+3)(x-8)} \end{aligned} \quad . \tag{2.5}$$

I didn't just pluck 3 and -8 out of nowhere. I tried a bunch of numbers by multiplying them together and seeing if they return -24. Then, I add them to see if they return -5. If so, those are the numbers that are needed.

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