Baker Charters School

Honors Algebra 2B

Baker Web Academy School Notes

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Contents

CHAPTER ONE

Graphs and Functions

Unit 1

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Lesson 1: Writing parallel and perpendicular **Unit 1**

Consider the line: $y = -2x + 1$.

Let's:

- Find the equation of the line that is parallel to this line and passes through the point $(7, -5)$.
- Find the equation of the line that is perpendicular to this line through the point $(7, -5)$.

Let's first go over these Properties:

> **Property 1.** (Parallel Slope Property) Two non-vertical lines are parallel if and only if they have the same slope.

> **Property 2.** (Perpendicular Slope Property) Two non-vertical lines are perpendicular if and only if the product of their slopes is equal to -1.

Now, to equation $y = -2x + 1$ is written in the slope-intercept form: $y = mx + b$. In this form, the slope m is -2.

- We can use the **Parallel Slope Property**. Since the given lines has a slope of -2, a line parallel to it must also have the same slope, which is -2. So, the equation of the parallel line will have the form $y = -2x + b$. The line passes through $(7, -5)$, so we use $x = 7$ and $y = -5$ to solve for b:

$$\begin{aligned} y &= -2x + b \\ -5 &= -2(7) + b \\ -5 &= -14 + b \\ b &= 9 \end{aligned} . \tag{1.1}$$

Now, we know the equation of the parallel line, which is $y = -2x + 9$.

- We use the **Perpendicular Slope Property**. Since the given lines has the slope -2, a line with the slope $\frac{1}{2}$ is perpendicular to it.

  So, the equation of the perpendicular line will have the form $y = \frac{1}{2}x + b$.

  The line passes through $(7, -5)$, so we use $x = 7$ and $y = -5$ to solve for b.

$$\begin{aligned} y &= \frac{1}{2}x + b \\ -5 &= \frac{1}{2}(7) + b \\ -5 &= \frac{7}{2} + b \\ -\frac{10}{2} &= \frac{7}{2} + b \\ b &= -\frac{17}{2} \end{aligned} . \tag{1.2}$$

Equation of the parallel line: $y = -2x + 9$

Equation of the perpendicular line: $y = \frac{1}{2}x - \frac{17}{2}$

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Lesson 2: Solving a linear equation by graphing **Unit 1**

Let's look at this equation:

$$-4 = 5 - 3x. \tag{1.3}$$

Here's one method to solve it (by graphing):

- First, we write the equation with 0 on one side

$$\begin{aligned} -4 &= 5 - 3x \\ 0 &= 9 - 3x \end{aligned}. \tag{1.4}$$

- Then, we graph the equation and find the **x-intercept**:

x	$y = -3x + 9$	(x, y)
0	$y = -3 \times 0 + 9 = 9$	$(0, 9)$
1	$y = -3 \times 1 + 9 = 6$	$(1, 6)$
2	$y = -3 \times 2 + 9 = 3$	$(2, 3)$

Table 1.1: X-Y Table

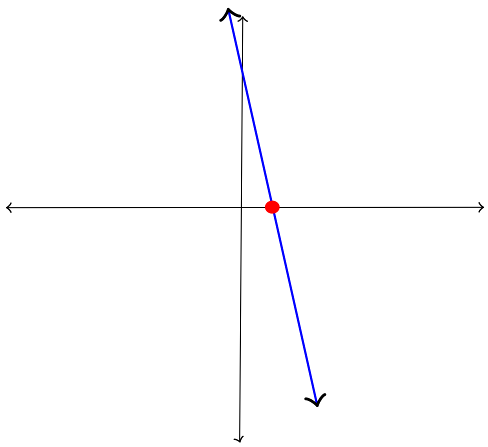

Figure 1.1: $-4 = 5 - 3x$ Graphed

We get that $x = 3$ is a solution to the original equation, which was $-4 = 5 - 3x$

Feb 27 2022 Sun (15:34:39)

Lesson 3: Identify correlation and causation **Unit 1**

Definition 1. (Correlation) A mutual relationship or connection between two or more things.
Two quantities have a *correlation* if they tend to vary together.

Definition 2. (Causation) This indicates a relationship between two events where one event is affected by the other.

Let's take a look at a couple of examples for each definition:

Example. Maria and Andy are high school students in Arizona. Andy always gets an A on his math test when it's sunny outside.
You may think that Andy is acing his test because it's sunny when he takes the test, but that's not true. It just happens that it's sunny when there's a math test.
So, the two events: The sunniness and the acing his test occurred together without one causing the other. In simpler terms, the two events are **Correlated**, but there's no **Causal** relationship between them.

Note. If you have a correlation (1) you don't always have a causation (2).

Mar 03 2022 Thu (17:42:42)

Lesson 4: Domain and range of a linear function **Unit 1**

Definition 3. (Domain) The domain of a function is the set of all possible inputs.

Definition 4. (Range) The domain of a function is the set of all possible inputs of the function.

Example. Suppose that the function f is defined by the following table:

x	$f(x)$
1	1
2	8
3	27
4	64

The domain of f is the set of numbers in the left column. The range of the function f is the set of numbers in the right column. That is:

$$\begin{array}{ll} \text{Domain:} & f = \{1, 2, 3, 4\} \\ \text{Range} & f = \{1, 8, 27, 64\} \end{array}. \tag{1.5}$$

The function f could also be written as a set of ordered pairs:

$$f(x) = \{(1, 1), (2, 8), (3, 27), (4, 64)\}$$

.

Here's a cooler way of showing it:

$$f(x) = 3x, x \in [-2, 5]. \tag{1.6}$$

The statement above (**??**) is saying that the domain of the function $f(x)$ is the interval (**??**) $[-2, 5]$

Note. Note that the domain is the set of all first elements in the ordered pairs, and the range is the set of all second elements in ordered pairs.

Example. A construction crew is lengthening the road. Let L be the total length of the road (in miles). Let D be the number of days the crew has worked.
Suppose that $L = 4D + 400$ gives L as a function of D. The crew can work for at most 70 days.
Identify the correct description of the values in both the domain and range of the function.
Here's how we would approach such a problem:

We are given the function: $L = 4D + 400$. Here, L is the total length of the road and D is the total number of days the crew has worked.

Domain

Description of values for the domain For our function, the input is given by D. So, the values in the domain correspond to the number of days the crew has worked.

Set of values for the domain We are given that the crew can work for at most 70 days.
So, our domain will be $[0, 70]$.

Range

Description of Values for the Range For our function, the output is given by L. So the values in the range correspond to the total length of the road.

Set of Values for the Range To find the range, let's look at the output L for some values of D:
After 0 days, the total length of the road will be: $L = 4(0) + 400 = 400 \quad \text{miles}$
After 1 day, the total length of the road will be: $L = 4(1) + 400 = 401 \quad \text{miles}$
After 10 day, the total length of the road will be: $L = 4(10) + 400 = 440 \quad \text{miles}$
After $20\frac{1}{2}$ day, the total length of the road will be: $L = 4(20\frac{1}{2}) + 400 = 482 \quad \text{miles}$
After 70 day, the total length of the road will be: $L = 4(70) + 400 = 680 \quad \text{miles}$
Note also that we can get any length between 400 and 680 miles for the road.
So, we choose all of the real numbers between 400 and 680 miles.
Or, we can write it like this:

$$L \in [400, 680]. \tag{1.7}$$

Note. The number of days must also be $x \geq 0$.
The number of days could also be decimals, fractions, etc like: $\frac{1}{2}$ $\frac{10}{19}$ $\frac{7}{11}$ $\frac{11}{7}$
The only restriction we have is that the number of days must be $x \geq 0$

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Lesson 5: Word problem involving average rate of change **Unit 1**

Definition 5. (Slope) The slope of a line is a number measuring how **Steep** the line is.
The farther the slope is from zero, the steeper the line is. The slope of a vertical line is undefined.
The sign of the slope tells us if the line will go **up** or **down**.

- If the slope is **positive**, then the line goes upward from **left** to **right**.
- If the slope is **negative**, then the line goes **downward** from **left** to **right**.
- If the slope is 0, then the line is **horizontal**.

Suppose that a non-vertical line passes through the two points: $(x_1, y_1), (x_2, y_2)$
The *rise* from the first point to the second is $y_2 - y_1$.
The *run* from the first point to the second is $x_2 - x_1$
The slope formula is:

$$\text{slope} = \frac{\text{rise}}{\text{run}} = \frac{y_2 - y_1}{x_2 - x_1}. \tag{1.8}$$

The number of bacteria in a culture increase rapidly. The table below gives the number $N(t)$ of bacteria at a few times t (in hours) after the moment when $N = 1000$.

Time t hours	Number of bacteria $N(t)$
0	1000
3.4	1510
6.8	2292
10.2	3856
13.6	5080

1. We're going to find the Average Rate of Change (ARC) for the number of bacteria from 0 hours to 6.8 hours.
2. Find the ARC for the number of bacteria from 10.2 hours to 13.6

The ARC for the number of bacteria from $t = a$ to $t = b$ is:

$$\frac{N(b) - N(a)}{b - a}, \quad \text{where} \quad b \neq a. \tag{1.9}$$

Note. Look how similar the equation above (1.9) is similar to the slope (5) formula.

The ARC is the slope of the line passing through: $(a, N(a)) \quad (b, N(b))$.

1. For part **A**, we're asked to find the ARC from 0 hours to 6.8 hours.

   So we take $a = 0 \quad b = 6.8$.

   From the table, we get $N(a) = 1000 \quad N(b) = 2292$.

   Applying the formula for ARC, we get:

$$\begin{aligned} \frac{N(b) - N(a)}{b - a} &= \frac{2292 - 1000}{6.8 - 0} \\ &= \frac{1292}{6.8} \\ &= \boxed{190 \quad \text{Bacteria per Hour}} \end{aligned} \quad . \tag{1.10}$$

2. For part **B**, we're asked to find the ARC from 10.2 hours to 13.6 hours.

   So, we take $a = 10.2 \quad b = 13.6$.

   From the table, we get $N(a) = 3856 \quad N(b) = 5080$.

   Applying the formula for ARC, we get:

$$\begin{aligned} \frac{N(b) - N(a)}{b - a} &= \frac{5080 - 3856}{13.6 - 10.2} \\ &= \frac{1224}{3.4} \\ &= \boxed{360 \quad \text{Bacteria per Hour}} \end{aligned} \quad . \tag{1.11}$$

So, our final answers are:

The ARC for the number of bacteria from 0 **hours to** 6.8 **hours is**:
$\boxed{190 \quad \textbf{Bacteria per hour}}$

The ARC for the number of bacteria from 10.2 **hours to** 13.6 **hours is:**
$\boxed{360 \quad \textbf{Bacteria per hour}}$

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Lesson 6: Transforming Graphs **Unit 1**

Translating Graphs

If each of the points of a graph is moved the same distance in the same direction, we say that the graph is **translated** and the resulting graph is a transition of the original.

- **Horizontally Translating:**

  Suppose that f is a function whose domain (3) and range (4) are subsets of the set of real numbers

  - The horizontal translation of the graph of f to the right c units is the graph of

    $$g(x) = f(x - c)$$

    .

  - The horizontal translation of the graph of f to the left c units is the graph of

    $$g(x) = f(x + c)$$

    .

- **Vertically Translating:**

  Suppose that f is a function whose domain (3) and range (4) are subsets of the set of real numbers

  - The vertical translation of the graph of f upward c units is the graph of

    $$g(x) = f(x) + c$$

    .

  - The vertical translation of the graph of f downward c units is the graph of

    $$g(x) = f(x) - c$$

    .

> **Example.** If $f(x) = x^2$, then the graph of $g(x) = x^2 + 2$ is the vertical translation of the graph of f upward 2 units. The graph of $h(x) = x^2 - 1$ is the vertical translation of the graph of f downward 1 unit.

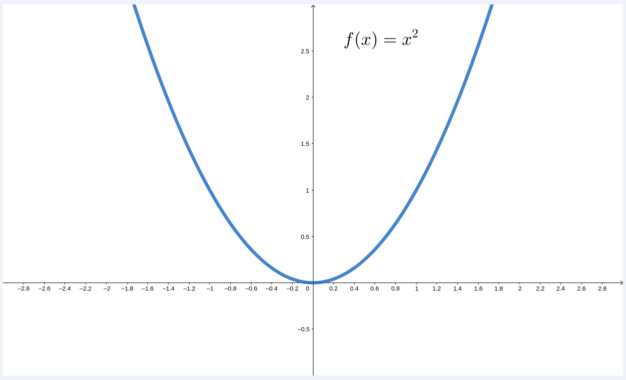
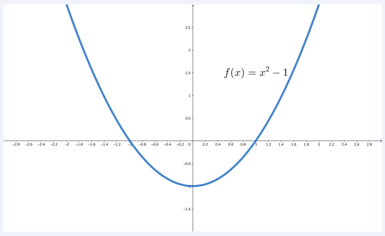
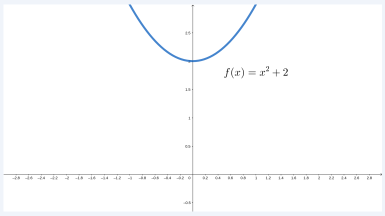


Figure 1.2: The starting graph $f(x) = x^2$



(a) label 1



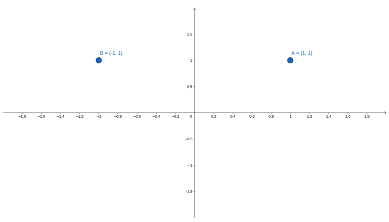
(b) label 2

Figure 1.3: Translating Graphs

When the graph of a function f is translated to get the graph of a function g, it is important to

Note. When the graph of a function f is translated to get the graph of a function g, it is important to note that the rigidity of the graph of f is maintained. That is, the graph of g has the same **"size"** and **"shape"** as the graph of f.

Reflecting Graphs



- Reflecting about the y axis.

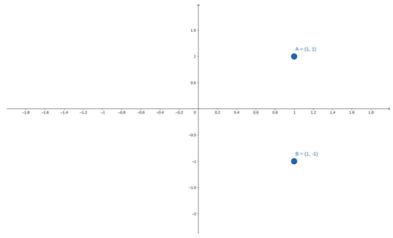
  The **reflection** of the point (a, b) about the y axis is the point $(-a, b)$.

  If f is a function whose domain (3) and range (4) are subsets of the set of real numbers, then the reflection of the graph of f about the y axis is the reflection of every point on the graph of f about the y axis. The reflection of the graph of f about the y axis is the graph of the function $g(x) = f(-x)$

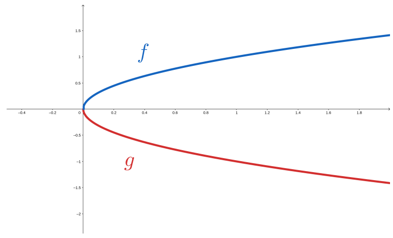
- Reflecting about the x axis.

  The reflection of the point (a, b) about the x axis is the point $(a, -b)$.

  If f is a function whose domain (3) and range (4) are subsets of the set of real numbers, then the reflection of the graph of f about the x axis is the reflection of every point on the graph of f about the x axis. The reflection of the graph of f about the x axis is the graph of the function $g(x) = -f(x)$



> **Example.** The graph of $f(x) = \sqrt{x}$ is shown in blue. The graph of $g(x) = -\sqrt{x}$, which is the reflection of the graph of f about the x axis is shown in red.



Stretching and Shrinking Graphs

- Vertically Stretching and Shrinking.

  Suppose that f is a function whose domain (3) and range (4) are subsets of the set of real numbers.

  - If c is a number greater than 1, then the graph of

    $$g(x) = cf(x)$$

    is **Vertical Stretching**, also called **Vertical Expansion**, of the graph of f.

  - If c is a number between 0 and 1, then the graph of

    $$g(x) = cf(x)$$

    is **Vertical Shrinking**, also called a **Vertical Contraction**, of the graph f.

- Horizontally Stretching and Shrinking.

  Suppose that f is a function whose domain (3) and range (4) are subsets of the set of real numbers.

  - If c is a number greater than 1, then the graph of

    $$g(x) = f(cx)$$

    is a **Horizontal Shrinking**, also called a **Horizontal Contraction**, of the graph of f.

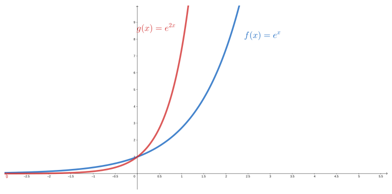
  - If c is a number between 0 and 1, then the graph of

    $$g(x) = f(cx)$$

    is a **Horizontal Stretching**, also called a **Horizontal Expansion**, of the graph of f.

  In both cases, for a given y coordinate, the x coordinate of the point on the graph of g is $\frac{1}{c}$ times the x coordinate of the point on the graph of f.

Example. The graph of the function $f(x) = e^x$ is shown in blue and the graph of the function $g(x) = e^{2x}$ is shown in red. The graph of g is a horizontal contraction of the graph of f because $g(x) = f(2x)$, that is, for a given y coordinate, the x coordinate of a point on the graph of g is $\frac{1}{2}$ times the x coordinate of the point on the graph of f.



Example. Below is the graph of $y = x^2$.
Transform it to make it the graph of $y = -2(x-4)^2$.

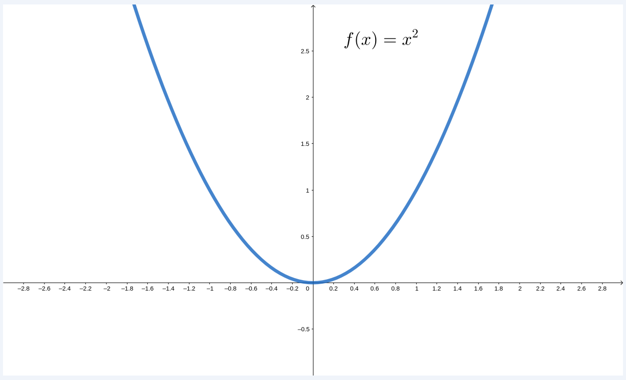


Figure 1.4: $y = x^2$

We are given the graph of $y = x^2$ and asked to graph $y = -2(x-4)^2$. Starting with the graph of $y = x^2$, we'll first graph $y = (x-4)^2$. To do this, we translate (1) the graph of $y = x^2$ to the right 4 units.

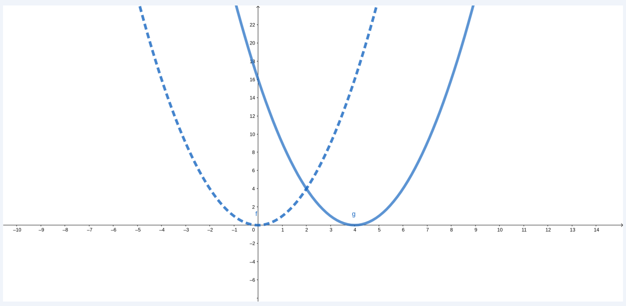


Figure 1.5: Moved to the right by 4 units.

Finally, we'll use the graph of $y = 2(x-4)^2$ to get the graph of $y = -2(x-4)^2$. To do this, we reflect the graph of $y = 2(x-4^2)$ across the x axis.

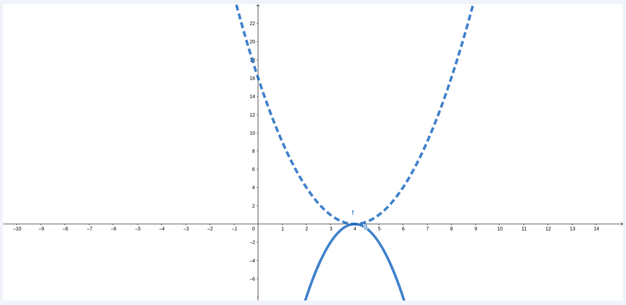


Figure 1.6: Reflected across the x axis.

CHAPTER TWO

Linear Systems

Unit 2

CHAPTER THREE

Exponents and Polynomials

Unit 3

CHAPTER FOUR

Quadratic and Polynomial Functions

Unit 4

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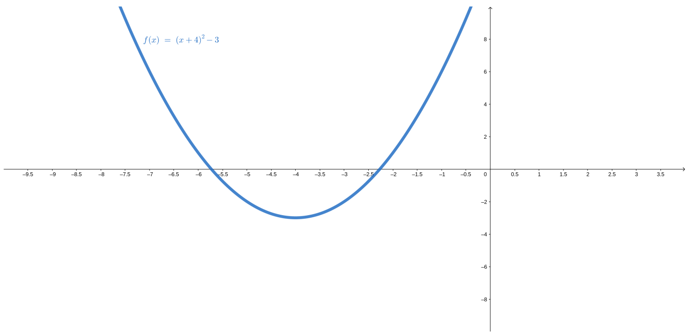
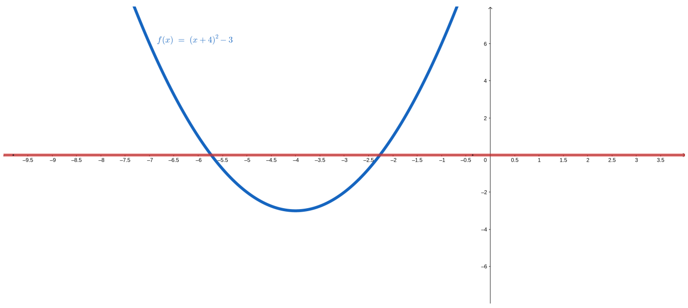
Lesson 1: Domain and range from parabola **Unit 4**

Let's try to find the domain (3) and range (4) of this parabola:



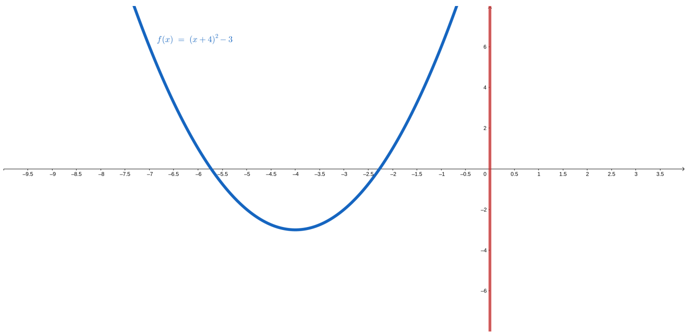
- To find the domain, we project the graph onto the x axis.



  Because the graph extends to the left and right forever, all real numbers are in the domain.

- To find the range, we project the graph onto the y axis.

  We see that the minimum value in the range is -3.



All values greater than -3 are also part of the range. This is because the graph extends upward forever.

Therefore, the range of our function is given by $y \geq -3$.

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Lesson 2: Create QE from roots and LC **Unit 4**

Theorem 1. (Fundamental Theorem of Algebra) Any polynomial of degree $n > 0$ has at least one zero (real or non-real).

Theorem 2. (Factor Theorem) If r is a zero of the polynomial $P(x)$, then $x - r$ is a factor of $P(x)$. Conversely, if $x - r$ is a factor of $P(x)$, then r is a zero of $P(x)$.

Example. Consider the polynomial $P(x) = x^2 + 3x + 2$.
Because

$$P(-1) = (-1)^2 + 3(-1) + 2 = 1 - 3 + 2 = 0. \tag{4.1}$$

we have that -1 is a zero of the polynomial $P(x)$.
Hence, the **Factor Theorem** (2) guarantees that $x - (-1) = x + 1$ is a factor of $P(x)$.
That is, the factor theorem guarantees that there exists a polynomial $Q(x)$ such that

$$P(x) = (x + 1)Q(x). \tag{4.2}$$

That can be confirmed directly. Since $P(x) = x^2 + 3x + 2 = (x + 1)(x + 2)$, we have that $Q(x) = x + 2$

Remark. Given a polynomial $P(x) = a_n x^n + a_{n-1}x^{n-1} + \ldots + a_1 x + a_0$ of degree $n > 0$, we can apply the **Fundamental Theorem of Algebra** (1) and facto theorem n times to obtain what is commonly referred to as the complete factorization of $P(x)$. Namely, we can write $P(x)$ as follows:

$$P(x) = a_n(x - r_1)(x - r_2)\cdots(x - r_n). \tag{4.3}$$

where a_n is the leading coefficient of $P(x)$ and $r_1, r_2, \ldots r_n$ are the n zeroes of $P(x)$. This fact is sometimes referred to as the **Linear Factors Theorem**

Let's write a quadratic equation whose roots are $-1 \quad -3$ and whose leading coefficient is 2.

We use the **Factor Theorem** (2), which states that if k is a root of the polynomial equation $P(x) = 0$, then $x - k$ is a factor of the polynomial $P(x)$.

In our problem, we have $P(-1) = 0 \quad P(-3) = 0$. Thus, $x + 1 \quad x + 3$ are both factors of the quadratic polynomial. Since the coefficient is 2, we can write the polynomial as:

$$2(x+1)(x+3) = 2x^2 + 8x + 6. \tag{4.4}$$

Mar 04 2022 Fri (19:33:18)

Lesson 3: Find vertex, intercept, axis symmetry **Unit 4**

Find the Axis of Symmetry

The axis of symmetry is the line that divides the parabola into two mirror images.

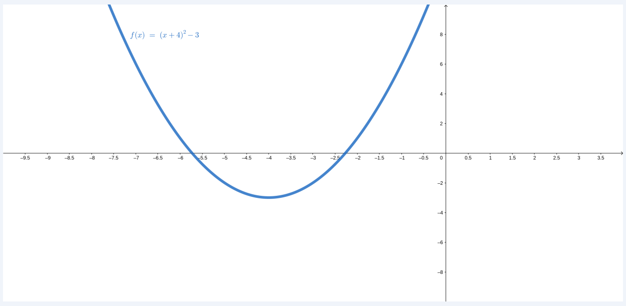
Find the Vertex

If the parabola opens upward, the vertex will be the lowest point.

If the parabola opens downward, the vertex will be the highest point.

> **Note.** The vertex always lies on the axis of symmetry.

Example. Use the graph of the parabola to answer the following questions:



1. Does the parabola open upward or downward?
   The parabola opens upward.
2. Find the equation of the axis of symmetry.
   The equation of the axis of symmetry is $x = -4$.
3. Find the coordinates of the vertex.
   The vertex is at $(-2, -3)$.
4. Find the intercept(s).
   The x intercepts are $(-5, 0) \quad (-2, 0)$.
   There are no y intercepts.

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Lesson 4: Solve quadratic equation using square root **Unit 4**

Consider this equation: $x^2 = a$

- When a is positive, the equation has two solutions: $x = \sqrt{a} \quad x = -\sqrt{a}$
- When a is negative, the equation has no solution. It only has imaginary solutions because the square root of a negative number and is not a real number. It has the variable i, which is an imaginary number.

Example. Solve $x^2 = 50$, where $x \in \mathbb{R}$.
So, the equation $x^2 = 50$ has two solutions: $x = \sqrt{50} \quad x = -\sqrt{50}$
We can simplify:

$$\sqrt{50} = \sqrt{25 \times 2} = 5\sqrt{2}. \tag{4.5}$$

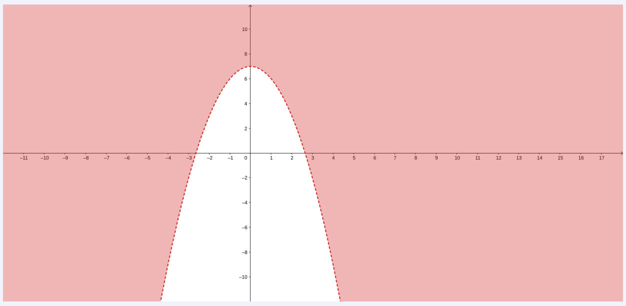
So, our final answer is: $x = 5\sqrt{2} \quad x = -5\sqrt{2}$.

Mar 04 2022 Fri (20:56:39)

Lesson 5: Graphing a quadratic inequality **Unit 4**

To graph an inequality, we first graph the associated parabola.

Example. Graph the inequality $y > -x^2 + 7$. First you graph $y = -x^2 + 7$.
The inequality has **y greater than** $-x^2 + 7$, so we shade **above** the curve.
If it was **y less than**, we shade **below** the curve.
Since the inequality is only $>$, that means the curve itself is not part of the graph. We use a dashed curve to show this.
If the inequality was $\geq$, then we would use a solid curve.
So, here's the inequality $y > -x^2 + 7$ graphed.



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Lesson 6: Solve quadratic inequality that's in factored form **Unit 4**

We need to graph the solution to the following inequality on the number line:
$(x+7)(x-2) < 0$

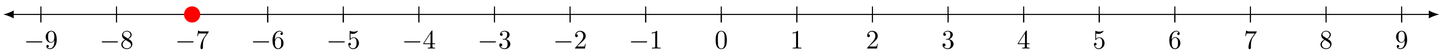
We need to find all values of x that make the product $(x+7)(x-2)$ negative.

To find these values, we do a sign analysis.

We first look at the sign of $(x+7)$.

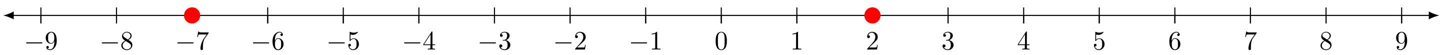
- If $x = -7$, then $(x+7) = 0$.
- If $x < -7$, then $(x+7)$ is negative.
- If $x > -7$, then $(x+7)$ is positive.

We show this on the number line.



Then we look at the sign of $(x-2)$.

- If $x = 2$, then $(x-2) = 0$.
- If $x < 2$, then $(x-2)$ is negative.
- If $x > 2$, then $(x-2)$ is positive.

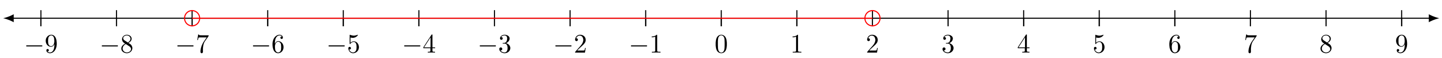


Finally, to find the sign of $(x+7)(x-2)$, we use the rules for the sign of a product.

So, we have $(x+7)(x-2) < 0$ when $-7 < x$ $x < 2$.

> **Note.** For $x = -7$ $x = 2$, the product $(x+7)(x-2)$ is 0 and so these values are not part of the solution.

We graph the solution on the line as follows



Mar 04 2022 Fri (21:47:06)

Lesson 7: Polynomial from given degree and zeroes **Unit 4**

Find a polynomial $f(x)$ of degree 5 that has the following zeroes:

$$-4, 1, 3, 8, 0$$

.

The **Factor Theorem** (5) tells us the following:

A number c is a zero of a polynomial $f(x)$ if and only if $x - c$ is a factor of $f(x)$.

We also get that, if c is a zero of multiplicity k, then $(x - c)^k$ is a factor.

We're told that $f(x)$ has the zeros shown below:

Zero	Factor
-4	$x - (-4) = x + 4$
1	$x - 1$
3	$x - 3$
8	$x - 8$
0	$x - 8 = x$

Table 4.1: Zeros of the polynomial $f(x)$

These factors give the following polynomial:

$$f(x) = x(x + 4)(x - 1)(x - 3)(x - 8). \tag{4.6}$$

Since this polynomial has degree 5, it's a possible answer.

Mar 04 2022 Fri (22:30:44)

Lesson 8: Using the remainder theorem to evaluate a polynomial Unit 4

Theorem 3. (Division Algorithm) Let $P(x)$ be any polynomial and $d(x)$ any non constant polynomial or degree less than or equal to $P(x)$. Then, there exist unique polynomials $Q(x)$ and $R(x)$ such that:

$$P(x) = d(x)Q(x) + R(x). \tag{4.7}$$

where either $R(x) = 0$ or the degree of $R(x)$ is less than the degree of $d(x)$.

Note. Note that we can divide both sides of the equation above by $d(x) \neq 0$ to obtain:

$$\frac{P(x)}{d(x)} = Q(x) + \frac{R(x)}{d(x)}. \tag{4.8}$$

This shows the connection with division: when $P(x)$ is divided by $d(x)$, the result is $Q(x)$ and the remainder is $R(x)$.

Remark.
- In the statement of the division algorithm, the polynomial $d(x)$ must be **"Non constant"**
  A constant polynomial $d(x)$ is one of the form $d(x) = c$ for some value c. Thus, a non constant polynomial is one that is not of this form, that is, one whose degree is at least 1.
- A **Similar Division Algorithm** exists for integers.
- If $R(x) = 0$, then $d(x)$ divided $P(x)$ or divided evenly into $P(x)$, or is a factor of $P(x)$

Theorem 4. (Remainder Theorem) If a number c is substituted for x in the polynomial $P(x)$, then the result, $P(c)$ is the remainder obtained when $P(x)$ is divided by $x - c$.

Proof. By the division algorithm (3), when we divided $P(x)$ by $x - c$, we get a quotient $Q(x)$ and a remainder $R(x)$. We can write this as follows:

$$P(x) = (x - c)Q(x) + R(x). \tag{4.9}$$

The degree of $R(x)$ must be less than the degree of $x - c$, so $R(x)$ must be constant. This means that $R(x) = r$ for some number r.

$$P(x) = (x - c)Q(x) + r. \tag{4.10}$$

We see that $P(c) = r$:

$$\begin{aligned} P(c) &= (c-c)Q(c) + r \\ &= 0 \times Q(c) + r \quad . \\ &= r \end{aligned} \tag{4.11}$$

Thus, $P(c)$ is equal to the remainder when $P(x)$ is divided by $x - c$.

Synthetic Division

I'll illustrate these steps by using them to divide $P(x) = 4x^2 - x^4 - 7 - 2x$ by $x + 1$

1. Arrange the coefficients of $P(x)$ in order of descending powers of x. Write 0 as the coefficient for each missing power of x.

$$p(x) = -x^4 + 0x^3 - 4x^2 - 3x - 7. \tag{4.12}$$

2. Insert r (which is any number) to the left of the division symbol.

   Then, bring the leading coefficient of the dividend down.

$$\begin{array}{r|rrrrr} & -4 & 0 & -4 & -3 & -7 \\ -1 & & & & & \\ \hline & -4 & & & & \end{array}$$

3. Generate the second and third rows of numbers as follows.

   (a) After bringing down the first coefficient of the dividend, multiply it by r and place the resulting product under the second coefficient.

$$\begin{array}{r|rrrrr} & -4 & 0 & -4 & -3 & -7 \\ -1 & & 4 & & & \\ \hline & -4 & & & & \end{array}$$

   (b) Add the new number to the second coefficient, and place the sum below.

$$\begin{array}{r|rrrrr} & -4 & 0 & -4 & -3 & -7 \\ -1 & & 4 & & & \\ \hline & -4 & 4 & & & \end{array}$$

   (c) Repeat until you've completed all of the coefficients. Here's how it should look like after you've completed all steps properly:

$$\begin{array}{r|rrrrr} & -4 & 0 & -4 & -3 & -7 \\ -1 & & 4 & -4 & 8 & -5 \\ \hline & -4 & 4 & -8 & 5 & -12 \end{array}$$

4. The final entry in the last row is the remainder R, and the other entries in the row are the coefficients (in descending order) of a polynomial whose degree is 1 less than that of the dividend; This polynomial is the quotient $Q(x)$

$$\begin{aligned} Q(x) &= -x^3 + x^2 + 3x - 5 \\ R &= -2 \end{aligned} . \tag{4.13}$$

Example. Use the remainder theorem (4) to find $P(2)$ for $P(x) = 2x^3 - 4x^2 - 9$.
Specifically, give the quotient and the remainder for the associated division and the values of $P(2)$.
So, if $P(x) = 2x^3 - 4x^2 - 9$ is divided by $x - 2$, then the remainder is $P(2)$.
We first do the division, rewriting $P(x) = 2x^3 - 4x^2 - 9$ as $P(x) = 2x^3 - 4x^2 + 0x - 9$:

$$\begin{array}{r|rrrr} & 2 & -4 & 0 & -9 \\ -2 & & -4 & 16 & -32 \\ \hline & 2 & -8 & 16 & -41 \end{array}$$

So, here's our final result:

$$\begin{aligned} Q(x) &= 2x^3 - 8x^2 + 16x \\ R &= -41 \end{aligned} . \tag{4.14}$$

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Lesson 9: The Factor Theorem **Unit 4**

Theorem 5. (Factor) A polynomial $P(x)$ has a factor $x - c \Leftrightarrow P(c) = 0$.

Example. We must determine whether $x - 2$ is a factor of $P(x) = 2x^3 - 2x^2 - 8$.
So, to use the **Factor Theorem** (5), we must evaluate $P(2)$.

$$\begin{aligned} P(2) &= 2(2)^3 - 2(2)^2 - 8 \\ &= 16 - 8 - 8 \\ &= 0 \end{aligned} \quad . \tag{4.15}$$

So, $P(2) = 0$. By the **Theorem Factor** (5), we know that $x - 2$ is a factor of $P(x)$

Mar 04 2022 Fri (23:55:49)

Lesson 10: Solve quadratic equation using square root **Unit 4**

Example. Solve $(w-9)^2-36=0$, where w is a real number.
Simplify your answer as much as possible.
We first rewrite the equation with only the squared expression on the left.

$$(w-9)^2=36. \tag{4.16}$$

For the current problem, we have $w-9=\sqrt{36} \quad w-9=-\sqrt{36}$
Solving and simplifying, we get the following:

$$\begin{aligned} w-9&=\sqrt{36} & w-9&=-\sqrt{36} \\ w-9&=6 & w-9&=-6 \\ &\boxed{w=15} & &\boxed{w=3} \end{aligned} \quad . \tag{4.17}$$

Mar 05 2022 Sat (01:03:47)

Lesson 11: Discriminant of a quadratic equation **Unit 4**

The solutions of the quadratic equation $ax^2 + bx + c = 0$ are given by the quadratic formula:

CHAPTER FIVE

Rational Expressions

Unit 5

CHAPTER SIX

Exponential and Logarithmic Functions

Unit 6

CHAPTER SEVEN

Statistics and Probability

Unit 7

CHAPTER EIGHT

Sequences and Conics

Unit 8

CHAPTER NINE

Trigonometry

Unit 9

List of Theorems

List of Postulates

List of Conjectures

List of Corollaries

List of Definitions

List of Reviews

List of Properties

List of Propositions

Essential Formulas for Algebra 2 Final Exam

Laws of Exponents

Multiply Powers of the Same Base = Adding Exponents	$(a^m)(a^n) = a^{m+n}$
Divide Powers of the Same Base = Subtracting Exponents	$\dfrac{a^m}{a^n} = a^{m-n}$
Power Rule = Multiplying Exponents	$(a^m)^n = a^{m \times n}$
Zero Exponent = 1	$a^0 = 1$
Distribution of Exponent with Multiple Bases	$(ab)^n = a^n b^n$ $\left(\dfrac{a}{b}\right)^n = \dfrac{a^n}{b^n}$
Negative Exponent = Reciprocal	$a^{-n} = \dfrac{1}{a^n}$ $\dfrac{a^{-m}}{b^{-n}} = \dfrac{b^n}{a^m}$
Distribution of Negative Exponent with Multiple Bases	$(ab)^{-n} = a^{-n}b^{-n} = \dfrac{1}{a^n b^n}$ $\left(\dfrac{a}{b}\right)^{-n} = \left(\dfrac{b}{a}\right)^n = \dfrac{b^n}{a^n}$

$$\sqrt{a+b} \neq \sqrt{a} + \sqrt{b} \qquad \sqrt{a \times b} = \sqrt{a} \times \sqrt{b}$$
$$\sqrt{a-b} \neq \sqrt{a} - \sqrt{b} \qquad \sqrt{a \div b} = \sqrt{a} \div \sqrt{b}$$

Properties of Radicals

Distribution of Radicals of the Same Index (where $a \geq 0$ and $b \geq 0$ if n is even)	$\sqrt[n]{ab} = \left(\sqrt[n]{a}\right)\left(\sqrt[n]{b}\right)$ $\sqrt[n]{\dfrac{a}{b}} = \dfrac{\sqrt[n]{a}}{\sqrt[n]{b}}$
Power Rule of Radicals = Multiplying Exponents	$\sqrt[m]{\sqrt[n]{a}} = \sqrt[(m \times n)]{a}$
Reverse Operations of Radicals and Exponents	$\sqrt[n]{a^n} = a$ (if n is odd) $\sqrt[n]{a^n} = \lvert a \rvert$ (if n is even)

$$a^{\frac{m}{n}} = \sqrt[n]{a^m}$$

The index of the radical is the denominator of the fractional exponent.

Special Products

$(A+B)^2 = A^2 + 2AB + B^2$

$(A-B)^2 = A^2 - 2AB + B^2$

$(A+B)(A-B) = A^2 - B^2$

$(A+B)^3 = A^3 + 3A^2B + 3AB^2 + B^3$

$(A-B)^3 = A^3 - 3A^2B + 3AB^2 - B^3$

Special Expressions

Difference of Squares	$A^2 - B^2 = (A+B)(A-B)$
Perfect Trinomial Squares	$A^2 + 2AB + B^2 = (A+B)^2$
Perfect Trinomial Squares	$A^2 - 2AB + B^2 = (A-B)^2$
Sum of Cubes	$A^3 + B^3 = (A+B)(A^2 - AB + B^2)$
Difference of Cubes	$A^3 - B^3 = (A-B)(A^2 + AB + B^2)$

Quadratic Formula: $x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

Discriminant $= b^2 - 4ac$

When Discriminant is *Positive*, $b^2 - 4ac > 0 \rightarrow$ ***Two Distinct Real Roots***

When Discriminant is *Zero*, $b^2 - 4ac = 0 \rightarrow$ ***One Distinct Real Root (or Two Equal Real Roots)***

When Discriminant is *Negative*, $b^2 - 4ac < 0 \rightarrow$ ***No Real Roots***

Note the pattern:

$i^1 = i \quad i^2 = -1 \quad i^3 = -i \quad i^4 = 1$

$i^5 = i \quad i^6 = -1 \quad i^7 = -i \quad i^8 = 1$

$i^9 = i \quad i^{10} = -1 \quad \ldots..$

Pattern repeats every 4th power of *i*.

Product of Conjugate Complex Numbers

$(a+bi)(a-bi) = a^2 - b^2i^2 = a^2 - b^2(-1)$

$(a+bi)(a-bi) = a^2 + b^2$

Midpoint of a Line Segment

$$M = \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$$

Distance of a Line Segement

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

Slope

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

Standard Equation for Circles

$$(x-h)^2 + (y-k)^2 = r^2$$

$P\ (x, y)$ = any point on the path of the circle

$C\ (h, k)$ = centre of the circle

r = length of the radius

Point-Slope form: - a form of a linear equation when given a slope (m) and a point (x_1, y_1) on the line

$\frac{y - y_1}{x - x_1} = m$ (slope formula) $\qquad$ $y - y_1 = m\,(x - x_1)$ **(Point-Slope form)**

If we rearrange the equations so that all terms are on one side, it will be in **standard (general) form**:

$Ax + By + C = 0$ $\qquad$ **(Standard or General form)**

($A \geq 0$, the leading coefficient for the x term must be positive)

When given a slope (m) and the y-intercept (0, b) of the line, we can find the equation of the line using the **slope and y-intercept form**:

$$y = mx + b$$

where m = slope and b = y-intercept

Parallel Lines

slope of line 1 = slope of line 2

$m_1 = m_2$

Perpendicular Lines

slope of line 1 = negative reciprocal slope of line 2

$$m_{l_1} = \frac{-1}{m_{l_2}}$$

$y \propto x$ (y is directly proportional to x)

$$y = kx$$

where k = constant of variation (constant of proportionality – rate of change)

$y \propto \frac{xz}{w}$ (y is jointly proportional to x, z and w)

$$y = k\frac{xz}{w}$$

where k = constant of variation (constant of proportionality)

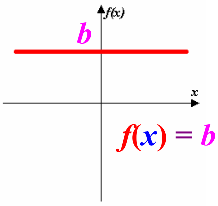
Average Rate of Change = m = $\frac{\Delta y}{\Delta x}$

Average Rate of Change = $\frac{f(b) - f(a)}{b - a}$

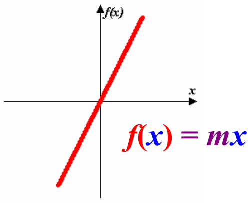
It is the slope of the secant line between the points (a, $f(a)$) and (b, $f(b)$)

Summary of Types of Functions: (see page 226 of textbook)

Linear Functions $f(x) = mx + b$



Domain: $x \in R$
Range: $f(x) \in R$



Domain: $x \in R$
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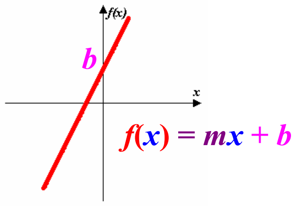
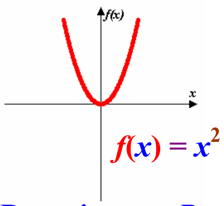
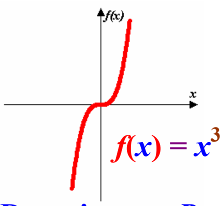


Domain: $x \in R$
Range: $f(x) \in R$

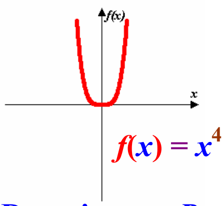
Power Functions $f(x) = x^n$ where $n > 1$ and $n \in N$



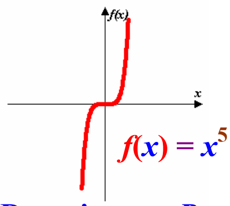
Domain: $x \in R$
Range: $f(x) \geq 0$



Domain: $x \in R$
Range: $f(x) \in R$

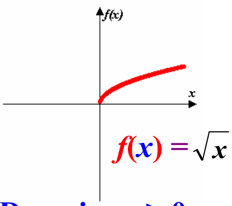


Domain: $x \in R$
Range: $f(x) \geq 0$

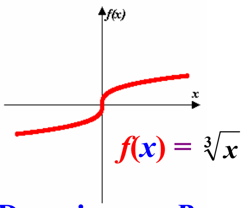


Domain: $x \in R$
Range: $f(x) \in R$

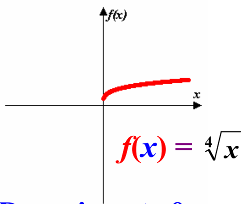
Root Functions $f(x) = \sqrt[n]{x}$ where $n \geq 2$ and $n \in N$



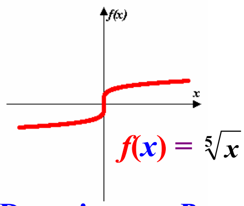
Domain: $x \geq 0$
Range: $f(x) \geq 0$



Domain: $x \in R$
Range: $f(x) \in R$

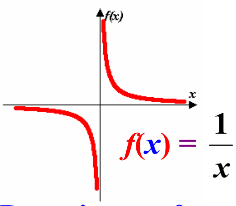


Domain: $x \geq 0$
Range: $f(x) \geq 0$

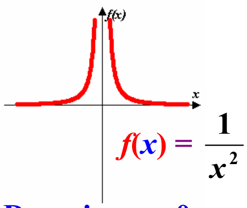


Domain: $x \in R$
Range: $f(x) \in R$

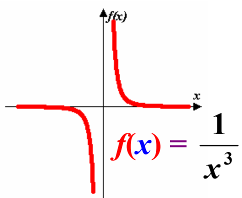
Reciprocal Functions $f(x) = \dfrac{1}{x^n}$ where $n \in N$



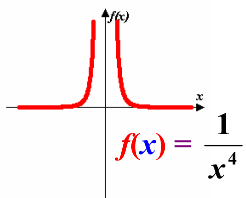
Domain: $x \neq 0$
Range: $f(x) \neq 0$



Domain: $x \neq 0$
Range: $f(x) > 0$

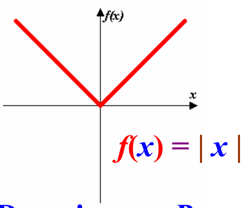


Domain: $x \neq 0$
Range: $f(x) \neq 0$



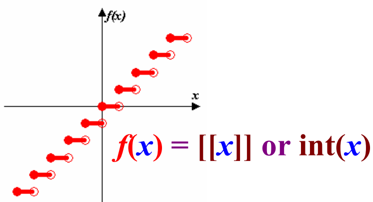
Domain: $x \neq 0$
Range: $f(x) > 0$

Absolute Value Functions



Domain: $x \in R$
Range: $f(x) \geq 0$

Greatest Integer Functions



Domain: $x \in R$
Range: $f(x) \in I$

$g(x) = f(x + h) + k$

h = amount of horizontal movement — $h > 0$ (move left); $h < 0$ (move right)

k = amount of vertical movement — $k > 0$ (move up); $k < 0$ (move down)

Reflection off the x-axis

$g(x) = -f(x)$

All values of y has to switch signs but all values of x remain unchanged.

Reflection off the y-axis

$g(x) = f(-x)$

All values of x has to switch signs but all values of y remain unchanged.

Vertical Stretching and Shrinking

$g(x) = af(x)$

a is the Vertical Stretch Factor

$a > 1$ (Stretches Vertically by a factor of a)

$0 < a < 1$ (Shrinks Vertically by a factor of a)

Horizontal Stretching and Shrinking

$g(x) = f(bx)$

b is the Horizontal Stretch Factor

$0 < b < 1$ (Stretches Horizontally by a factor of $1/b$)

$b > 1$ (Shrinks Horizontally by a factor of $1/b$)

For Quadratic Functions in Standard Form of $f(x) = a(x - h)^2 + k$

Vertex at (h, k) — Axis of Symmetry at $x = h$ — Domain: $x \in R$

a = Vertical Stretch Factor

$a > 0$ — Vertex at Minimum (Parabola opens UP) — Range: $y \geq k$ (Minimum)

$a < 0$ — Vertex at Maximum (Parabola opens DOWN) — Range: $y \leq k$ (Maximum)

$|a| > 1$ — Stretched out Vertically — $|a| < 1$ — Shrunken in Vertically

h = Horizontal Translation (Note the standard form has $x - h$ in the bracket!)

$h > 0$ — Translated Right — $h < 0$ — Translated Left

k = Vertical Translation

$k > 0$ — Translated Up — $k < 0$ — Translated Down

For Quadratic Functions in General Form: $f(x) = ax^2 + bx + c$

y-intercept at $(0, c)$ by letting $x = 0$ (Note: Complete the Square to change to *Standard Form*)

x-intercepts at $\left(\dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}, 0\right)$ if $b^2 - 4ac \geq 0$. No x-intercepts when $b^2 - 4ac < 0$

Vertex locates at $x = -\dfrac{b}{2a}$ — $y = f\left(-\dfrac{b}{2a}\right)$ — Minimum when $a > 0$; Maximum when $a < 0$

$f(x)$ = One-to-One Function — $f^{-1}(x)$ = Inverse Function

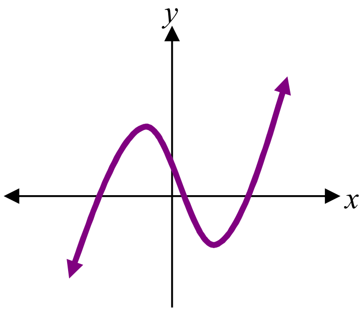
(x, y) — (y, x)

Domain of $f(x) \rightarrow$ Range of $f^{-1}(x)$

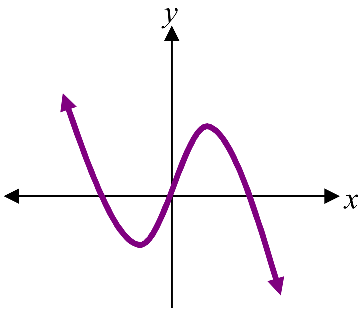
Range of $f(x) \rightarrow$ Domain of $f^{-1}(x)$

Note: $f^{-1}(x) \neq \dfrac{1}{f(x)}$ (Inverse is DIFFERENT than Reciprocal)

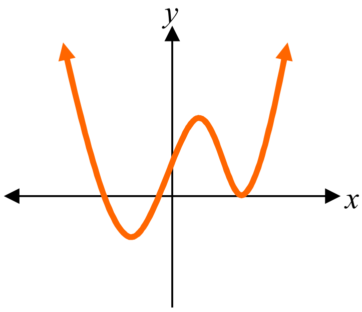
End Behaviours and Leading Terms



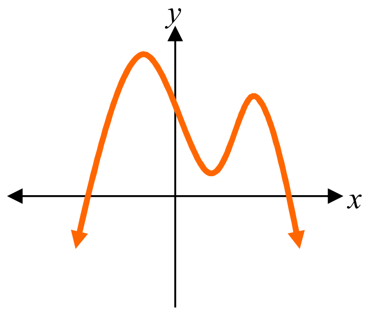
Odd Degree Polynomial Function and Positive Leading Coefficient, $a > 0$



Odd Degree Polynomial Function and Negative Leading Coefficient, $a < 0$



Even Degree Polynomial Function and Positive Leading Coefficient, $a > 0$



Even Degree Polynomial Function and Negative Leading Coefficient, $a < 0$

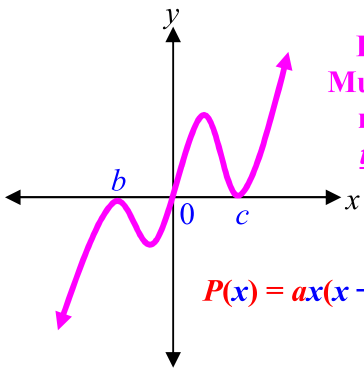
Odd Degree Polynomial Functions

When $a > 0$, Left is Downward ($y \to -\infty$ as $x \to -\infty$) and Right is Upward ($y \to \infty$ as $x \to \infty$).
When $a < 0$, Left is Upward ($y \to \infty$ as $x \to -\infty$) and Right is Downward ($y \to -\infty$ as $x \to \infty$).

Even Degree Polynomial Functions

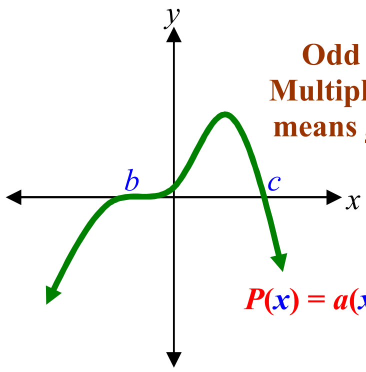
When $a > 0$, Left is Upward ($y \to \infty$ as $x \to -\infty$) and Right is Upward ($y \to \infty$ as $x \to \infty$).
When $a < 0$, Left is Downward ($y \to -\infty$ as $x \to -\infty$) and Right is Downward ($y \to -\infty$ as $x \to \infty$).

Multiplicity: - when a factored polynomial expression has exponents on the factor that is greater than 1.



Even Multiplicity or Multiplicities of 2, 4, 6… means x-intercept is *tangent to the x-axis*

$P(x) = ax(x - b)^{2,\,4,\,6\,\ldots}(x - c)^{2,\,4,\,6,\ldots}$



Odd Multiplicity or Multiplicities of 3, 5, 7… means *inflection* at x-int

$P(x) = a(x - b)^{3,\,5,\,7\,\ldots}(x - c)$

Polynomial Function **Divisor Function**

In general, for $P(x) \div D(x)$, we can write

$$\frac{P(x)}{D(x)} = Q(x) + \frac{R}{D(x)} \quad \text{or} \quad P(x) = D(x)\,Q(x) + R$$

Restriction: $D(x) \neq 0$

Quotient Function **Remainder**

If $R = 0$ when $\frac{P(x)}{(x-b)}$, then $(x - b)$ is a factor of $P(x)$ and $P(b) = 0$.

$$P(x) = D(x) \times Q(x)$$

$P(x)$ = Original Polynomial $D(x)$ = Divisor (Factor) $Q(x)$ = Quotient

If $R \neq 0$ when $\frac{P(x)}{(x-b)}$, then $(x - b)$ is NOT a factor of $P(x)$.

$$P(x) = D(x) \times Q(x) + R(x)$$

The Remainder Theorem:

To find the remainder of $\frac{P(x)}{x-b}$: Substitute b from the Divisor, $(x - b)$, into the Polynomial, $P(x)$.

In general, when $\frac{P(x)}{x-b}$, $P(b)$ = Remainder.

To find the remainder of $\frac{P(x)}{ax-b}$: Substitute $\left(\frac{b}{a}\right)$ from the Divisor, $(ax - b)$, into the Polynomial, $P(x)$.

In general, when $\frac{P(x)}{ax-b}$, $P\left(\frac{b}{a}\right)$ = Remainder.

The Factor Theorem:

1. If $\frac{P(x)}{x-b}$ gives a Remainder of 0, then $(x - b)$ is the Factor of $P(x)$.

   OR

   If $P(b) = 0$, then $(x - b)$ is the Factor of $P(x)$.

2. If $\frac{P(x)}{ax-b}$ gives a Remainder of 0, then $(ax - b)$ is the Factor of $P(x)$.

   OR

   If $P\left(\frac{b}{a}\right) = 0$, then $(ax - b)$ is the Factor of $P(x)$.

Rational Roots Theorem:

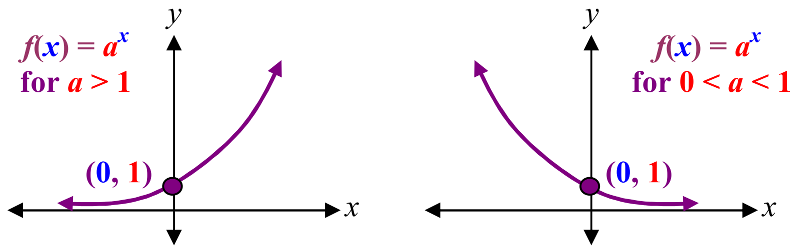
For a polynomial $P(x)$, a List of POTENTIAL Rational Roots can be generated by Dividing ALL the Factors of its Constant Term by ALL the Factors of its Leading Coefficient.

$$\text{Potential Rational Zeros of } P(x) = \frac{\text{ALL Factors of the Constant Term}}{\text{ALL Factors of the Leading Coefficient}}$$

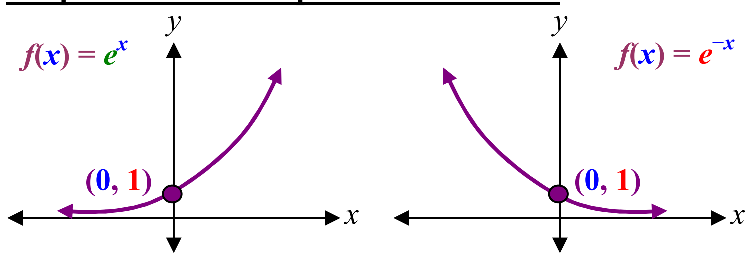
The Zero Theorem

There are n number of solutions (complex, real or both) for any n^{th} degree polynomial function accounting that that a zero with multiplicity of k is counted k times.

Graphs of Exponential Functions



Graphs of Natural Exponential Functions



$$y = a^x \longleftrightarrow x = \log_a y$$

Simple Properties of Logarithms

$\log_a 1 = 0$ because $a^0 = 1$

$\log_a a = 1$ because $a^1 = a$

$a^{\log_a x} = x$ because **exponent and logarithm are inverse of one another**

$\log_a a^x = x$ because **logarithm and exponent are inverse of one another**

Common and Natural Logarithm

Common Logarithm: $\log x = y \longleftrightarrow 10^y = x$

Natural Logarithm: $\ln x = y \longleftrightarrow e^y = x$

Exponential Laws	Logarithmic Laws
$(a^m)(a^n) = a^{m+n}$	$\log_a x + \log_a y = \log_a(xy)$
$\dfrac{a^m}{a^n} = a^{m-n}$	$\log_a x - \log_a y = \log_a\left(\dfrac{x}{y}\right)$
$(a^m)^n = a^{m \times n}$	$\log_a x^y = y \log_a x$
$a^0 = 1$	$\log_a 1 = 0$

Common Logarithm Mistakes

$\log_a(x + y) \neq \log_a x + \log_a y$

Example: $\log(2 + 8) \neq \log 2 + \log 8$

$1 \neq 0.3010 + 0.9031$

$\log_a(x - y) \neq \log_a x - \log_a y$

Example: $\log(120 - 20) \neq \log 120 + \log 20$

$2 \neq 2.0792 + 1.3010$

$$\log_a\left(\frac{x}{y}\right) \neq \frac{\log_a x}{\log_a y}$$

Example: $\log\left(\frac{1}{10}\right) \neq \frac{\log 1}{\log 10}$

$-1 \neq \frac{0}{1}$

$$(\log_a x)^y \neq y \log_a x$$

Example: $(\log 100)^3 \neq 3 \log 100$

$2^3 \neq 3(2)$

$$a^x = y \qquad x = \frac{\log y}{\log a}$$

$$A = P\left(1 + \frac{r}{n}\right)^{nt}$$

A = Final Amount after t years
r = Interest Rate per year
P = Principal
n = Number of Terms per year

$$A(t) = A_0\left(1 + \frac{r}{n}\right)^{nt} \xrightarrow{n \to \infty} A(t) = A_0 e^{rt}$$

$A(t)$ = Final Amount after t years
A_0 = Initial Amount
r = Rate of Increase ($+r$) / Decrease ($-r$) per year

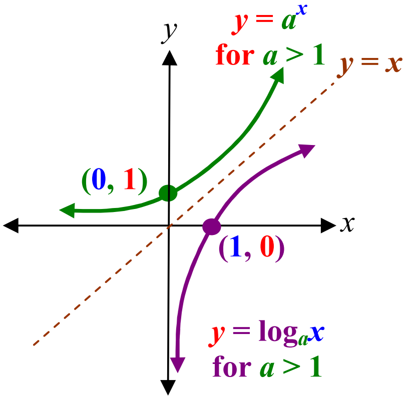
$$A(t) = A_0 e^{rt}$$

$A(t)$ = Final Amount after t years
A_0 = Initial Amount
r = Rate of Increase ($+r$) / Decrease ($-r$) per year

$$N(t) = N_0 e^{rt}$$

$N(t)$ = Final Population after t years, hours, minutes, or seconds
N_0 = Initial Population
r = Rate of Increase per year, hour, minute, or second

Graphs of Exponential and Logarithmic Functions



Exponential Function

y-int = 1 No x-intercept

Domain $x \in R$; Range $y > 0$

Logarithmic Function

x-int = 1 No y-intercept

Domain $x > 0$; Range $y \in R$

To obtain equation for the inverse of an exponential function, we start with

$y = a^x$
$x = a^y$ (switch x and y for inverse)
$y = \log_a x$ (rearrange to solve for y)

$$\pi \text{ rad} = 180^{\circ} \qquad \text{OR} \qquad \frac{\pi}{180} \text{ rad} = 1^{\circ}$$

$$y = a \sin k(x+b) + c \qquad\qquad y = a \cos k(x+b) + c$$

$|a|$ = Amplitude $\qquad$ c = Vertical Displacement (how far away from the x–axis)

b = Horizontal Displacement (Phase Shift) $\qquad$ $b > 0$ (shifted left) $\qquad$ $b < 0$ (shifted right)

k = number of complete cycles in 2π $\qquad$ Period $= \dfrac{2\pi}{k} = \dfrac{360^{\circ}}{k}$

Range = Minimum $\le y \le$ Maximum

$$y = a \sin [\omega(t+b)] + c \qquad\qquad y = a \cos [\omega(t+b)] + c$$

$|a|$ = Amplitude $\qquad$ c = Vertical Displacement (distance between *mid-line* and *t*-axis)

b = Horizontal Displacement (Phase Shift) $\qquad$ $b > 0$ (shifted left) $\qquad$ $b < 0$ (shifted right)

ω = number of complete cycles in 2π $\qquad$ Period $= \dfrac{2\pi}{\omega}$ $\qquad$ Frequency $= \dfrac{\omega}{2\pi}$

Range = Minimum $\le y \le$ Maximum

Note: $\sin^{-1}(x) \ne \dfrac{1}{\sin(x)}$ $\qquad$ $\sin^{-1}(x) \ne (\sin x)^{-1}$ $\qquad$ $(\sin x)^{-1} = \dfrac{1}{\sin(x)} = \csc x$

$$y = \sin^{-1} x$$

Domain: $-1 \le x \le 1$ $\qquad$ Range: $-\dfrac{\pi}{2} \le y \le \dfrac{\pi}{2}$

$$y = \cos^{-1} x$$

Domain: $-1 \le x \le 1$ $\qquad$ Range: $0 \le x \le \pi$

$\sin(\sin^{-1} x) = x$ $\qquad$ for $-1 \le x \le 1$

$\sin^{-1}(\sin x) = x$ $\qquad$ for $-\dfrac{\pi}{2} \le x \le \dfrac{\pi}{2}$

$\cos(\cos^{-1} x) = x$ $\qquad$ for $-1 \le x \le 1$

$\cos^{-1}(\cos x) = x$ $\qquad$ for $0 \le x \le \pi$

$$y = \tan^{-1} x$$

Domain: $x \in R$ $\qquad$ Range: $-\dfrac{\pi}{2} \le y \le \dfrac{\pi}{2}$

$\tan(\tan^{-1} x) = x$ $\qquad$ for $x \in R$

$\tan^{-1}(\tan x) = x$ $\qquad$ for $-\dfrac{\pi}{2} \le x \le \dfrac{\pi}{2}$

Some Basic Trigonometric Definitions and Identities (proven equations)

$\tan\theta = \dfrac{\sin\theta}{\cos\theta}$ $\qquad$ $\csc\theta = \dfrac{1}{\sin\theta}$ $\qquad$ $\sec\theta = \dfrac{1}{\cos\theta}$ $\qquad$ $\cot\theta = \dfrac{1}{\tan\theta} = \dfrac{\cos\theta}{\sin\theta}$

$$\cos^2\theta + \sin^2\theta = 1$$