Honors Algebra 2

Hashem A. Damrah

Sep 7 2021

Contents

Unit 1

Sep 07 2021 Tue (16:52:35)

Lesson 1: Rational Exponents **Unit 1**

In this lesson, I go over how you:

- Convert radical expressions to rational exponents.
- Convert rational exponents to radical expressions.

Definition 1 (Simplifying radical expressions). Rational expressions can be written as radical exponents.

$$t^{\frac{3}{4}} = t^{\frac{3}{4}} \times t^{\frac{3}{4}} \times t^{\frac{3}{4}}.$$
$$\sqrt[4]{t^3} = \sqrt[4]{t} \times \sqrt[4]{t} \times \sqrt[4]{t}.$$
$$t^{\frac{3}{4}} = \sqrt[4]{t^3}.$$

Definition 2 (Simplifying radical expressions). Radical expressions can be written as rational exponents.

$$\sqrt[5]{x^3} = \sqrt[5]{x} \times \sqrt[5]{x} \times \sqrt[5]{x}.$$
$$x^{\frac{3}{5}} = x^{\frac{1}{5}} \times x^{\frac{1}{5}} \times x^{\frac{1}{5}}.$$
$$x^{\frac{3}{5}} = \sqrt[5]{x^3}.$$

Sep 14 2021 Tue (09:54:31)

Lesson 2: Properties of Rational Exponents **Unit 1**

Now you know that when like variables are multiplied, their exponents are added. But what happens when variables are divided:

Property 1 (Quotient of Power Property)**.** So, when you divide powers of the same base, subtract the exponents:

$$\frac{a^m}{a^n} = a^{m-n}$$

$$\frac{a^5}{a^3} = \frac{\cancel{a} \times \cancel{a} \times \cancel{a} \times a \times a}{\cancel{a} \times \cancel{a} \times \cancel{a}} = a^2 = a^{5-3}.$$

Example 1 (Example 1)**.** Express the quotient of $\frac{r^{\frac{6}{7}}}{r^{\frac{2}{7}}}$ as a radical:

$$\frac{r^{\frac{6}{7}}}{r^{\frac{2}{7}}} = r^{\frac{6}{7}-\frac{2}{7}} = r^{\frac{4}{7}} = \sqrt[7]{r^4}.$$

Property 2 (Quotient of Power Property)**.** To raise a power to a power, multiply the exponents:

$$(a^m)^n = a^{m \times n}.$$

EXAMPLE:

$$(a^2)^3 = (a \times a) \times (a \times a) \times (a \times a) = a^6.$$

Example 2 (Raising a Power to a Power)**.**

$$\left(c^{\frac{1}{2}}\right)^{\frac{1}{4}} c^{\frac{1}{2} \times \frac{1}{4}} c^{\frac{1}{8}} \sqrt[8]{c}.$$

Property 3 (Negative Rational Exponents)**.** You have learned all about positive integer exponents and rational, or fractional, exponents. Let's take a look at another type: **Negative Integer Exponents**.

$$4^{-1} = \frac{1}{4^1} = \frac{1}{4}$$
$$4^{-2} = \frac{1}{4^2} = \frac{1}{16}$$
$$4^{-3} = \frac{1}{4^3} = \frac{1}{64}.$$

Example 3 (Negative Rational Exponents)**.**

$$9^{-\frac{1}{2}} = \frac{1}{9^{\frac{1}{2}}} = \frac{1}{\sqrt{9}} = \frac{1}{3}.$$

Sep 14 2021 Tue (16:54:18)

Lesson 3: Solving Radical Equations **Unit 1**

Definition 3 (Radical Equations)**.** Here is what a **Radical Equation** looks like:

$$T = 2\pi\sqrt{\frac{L}{32}}.$$

Example 4 (Radical Equations 1)**.** Let's solve $\sqrt{(5x+4)} = 7$:

$$\begin{aligned}
\sqrt{(5x+4)} &= 7 \\
(\sqrt{5x+4})^2 &= (7)^2 \\
5x+4 &= 49 \\
5x &= 45 \\
\frac{5x}{5} &= \frac{45}{5} \\
x &= 9.
\end{aligned}$$

Now, let's check our work:

$$\begin{aligned}
\sqrt{5x+4} &= 7 \\
\sqrt{5(9)+4} &= 7 \\
\sqrt{45+4} &= 7 \\
\sqrt{49} &= 7 \\
7 &= 7.
\end{aligned}$$

Let's try another one:

Example 5 (Radical Equations 2)**.** Let's solve $\sqrt{x-3}+4 = 1$:

$$\begin{aligned}
\sqrt{x-3}+4 &= 1 \\
\sqrt{x-3} &= -3 \\
(\sqrt{x-3})^2 &= (-3)^2 \\
x-3 &= 9 \\
x &= 12.
\end{aligned}$$

Now, let's check our work:

$$\begin{aligned}
\sqrt{x-3}+4 &= 1 \\
\sqrt{12-3}+4 &= 1 \\
\sqrt{9}+4 &= 1 \\
3+4 &= 1 \\
7 = 1 \text{ No Solution :(.}
\end{aligned}$$

We call this situation extraneous solution.

Quick Review of Factoring

Let's look at factoring by group.

First, you want the split the middle term into factors of 15 that combine to equal 2.

Then, just factor by GCF.

$$\begin{aligned}
x^2+2x-15 \\
x^2-3x+5x-15 \\
(x^2-3x)+(5x-15) \\
x(x-3)+5(x-3) \\
(x+5)(x-3).
\end{aligned}$$

Sep 22 2021 Wen (12:43:32)

Lesson 6: Complex Numbers **Unit 1**

Imaginary Numbers

What pairs of identical factors will produce -1 when multiplied?
1×1 doesn't work since it equals positive 1. The same is true of -1×1 which also equals positive 1.
1×-1 equals -1, but these are not identical factors.

Definition 4 (i)**.** To solve this problem, the concept of the imaginary number i was invented. The imaginary number was defined to be:

$$i = \sqrt{-1}$$

With this definition, the square root of negative radicands, in addition to positive radicands, may be simplified.

Example 6 (i)**.** Simplify: $\sqrt{-8}$

$$\begin{aligned}\sqrt{-1}\times\sqrt{8}\\ i\times\sqrt{8}\\ i\sqrt{8}\\ i\sqrt{4}\times\sqrt{2}\\ 2i\sqrt{2}\end{aligned}$$

Squaring Imaginary Numbers

You know that $i=\sqrt{-1}$
Consequently, $i^2=(\sqrt{-1})^2$
Since squaring a square root will eliminate the square root sign:

$$\begin{aligned}\sqrt{3^2}&=\sqrt{3}\times\sqrt{3}\\ &=\sqrt{9}\\ &=3\end{aligned}$$

Then:

$$\begin{aligned}i^2&=(\sqrt{-1})^2\\ i^2&=-1\end{aligned}$$

Cubing Imaginary Numbers

What about cubing the imaginary number:

$$\begin{aligned}i^3&=i\times i\times i\\ i^3&=(i\times i)\times i\\ i^3&=i^2\times i\\ i^3&=-1\times i\\ i^3&=-i\end{aligned}$$

Biquadrate Imaginary Numbers

i to the fourth power can be found in a similar way:

$$\begin{aligned} i^4 &= i \times i \times i \times i \\ i^4 &= (i \times i) \times (i \times i) \\ i^4 &= -1 \times -1 \\ i &= 1 \end{aligned}$$

Simplifying i

Example 7 (Simplifying i)**.** Simplify: i^{27}

$$\begin{aligned} 27 \div 4 &= 6R \longrightarrow 3 \\ i^{R \longrightarrow 3} &= -i \end{aligned}$$

Sep 27 2021 Mon (07:10:04)

Lesson 7: Operations on Complex Numbers **Unit 1**

Multiplying Complex Numbers

While the process of adding and subtracting complex numbers was similar to that of polynomials and radicals, multiplying complex numbers is just a little bit different. Think about in which order this multiplication should take place. Should you factor the imaginary number first or multiply the radicands first? Let's see what happens when the imaginary number is factored first.

$$\begin{aligned} \sqrt{-12} \times \sqrt{-5} \\ (\sqrt{-1} \times \sqrt{12}) \times (\sqrt{-1} \times \sqrt{5}) \\ i\sqrt{12} \times i\sqrt{5} \end{aligned}$$

Once the imaginary numbers have been factored, multiply the imaginary numbers together and multiply the radical factors together.

$$\begin{aligned} i\sqrt{12} \times i\sqrt{5} \\ i^2\sqrt{60} \end{aligned}$$

Now that the two factors have been multiplied, simplify the remaining radicand by either rewriting it as a product of its prime factors or by finding the perfect square factor.

Sep 27 2021 Mon (07:30:12)

Lesson 8: Polynomial Operations **Unit 1**

Adding Polynomials

The most important part of adding or subtracting polynomials is identifying like terms. Like terms are terms containing the exact same "variable part." Exponents for the variables must be exactly the same. Coefficients can, and probably will, be different. Identify the like terms in the following matching exercise.

Example 8 (Adding Polynomials)**.** Add: $(2x^3+4x^2-x+7)+(3x^2+6x+10)$

1. Again, there is an understood 1 in front of each set of parentheses. As you've seen, distributing this 1 will not change the expression. Therefore, the parentheses may simply be removed.
2. Highlight each pair of terms containing the same variable part. Combine the like terms.

Note: Since there is no other term with a variable part of x^3, the term $2x^3$ stays the same in the final answer.

$$\begin{aligned}(2x^3+4x^2-x+7)+(3x^2+6x+10)\\ 2x^3+4x^2-x+7+3x^2+6x+10\\ 2x^3+7x^2+5x+17\end{aligned}$$

Subtracting Polynomials

Subtracting polynomials is almost exactly like adding polynomials. The only "difference" is that now a negative one must be distributed.

Example 9 (Subtracting Polynomials)**.** Subtract: $(4x^2-5)-(x^3+2x^2+7)$

The understood 1 in front of the first set of parentheses will not change the binomial within. However, the trinomial in the second set of parentheses will change because a negative one (-1) must be distributed! In effect, it will change the signs of those three terms.

1. Highlight each pair of terms containing the same variable part.
2. Combine the like terms. Since $-x^3$ does not have a like term, it is written down as is. Arrange terms in descending order, from greatest exponent to least.

$$\begin{aligned} 1(4x^2-5)-1(x^3+2x^2+7) \\ 4x^2-5-x^3-2x^2-7 \\ 4x^2-5-x^3-2x^2-7 \\ -x^3+2x^2-12 \end{aligned}$$

Inverses of Functions

Finding the inverse of an integer means performing an operation that will cancel that number. For example, if you are given the number 7, you could add -7 to it to cancel it.

Let's look at an example:

Example 10 (Inverse of Functions)**.**

$$\begin{aligned} f(x) &= 3x-5 \\ y &= 3x-5 \\ x+5 &= 3y \\ \frac{x+5}{3} &= y \\ f^{-1}x &= \frac{x+5}{3} \end{aligned}$$

Operations on Functions

Addition The addition of two functions $f(x)$ and $g(x)$ is represented using the notation $f(x)+g(x)$.

Let $f(x) = 4x-7$ and $g(x) = 10x-3$. To add $f(x)$ and $g(x)$, the expressions $4x-7$ and $10x-3$ need to be added.

Therefore, this could be written as $f(x)+g(x)-(4x-7)+(10x-3)$.

1. Simplify the right side by first distributing any coefficients outside the parentheses. If no number or variable appears before the parentheses, an understood 1 exists.

2. When 1 is distributed to each term within each set of parentheses, the expression remains unchanged.

3. Identify and combine like terms.

$$
\begin{aligned}
f(x) + g(x) &= (4x - 7) + (10x - 3) \\
f(x) + g(x) &= 1(4x - 7) + 1(10x - 3) \\
f(x) + g(x) &= 4x - 7 + 10x - 3 \\
f(x) + g(x) &= 14x - 10
\end{aligned}
$$

Subtraction Subtraction of functions is similar to addition. The only difference is that you must be very careful of sign changes!

Let $f(x) = 4x-7$ and $g(x) = 10x-3$. To subtract $f(x)$ and $g(x)$, the expressions $4x-7$ and $10x-3$ need to be subtracted.

Therefore, this could be written as $f(x) - g(x) = (4x-7) - (10x-3)$.

1. Simplify the right side by first distributing any coefficients outside the parentheses. If no number or variable appears before the parentheses, an understood 1 exists.
2. Be very careful not to forget to distribute the -1 to each term in the second set of parentheses. Identify and combine like terms.

$$
\begin{aligned}
f(x) - g(x) &= (4x - 7) - (10x - 3) \\
f(x) - g(x) &= 1(4x - 7) - 1(10x - 3) \\
f(x) - g(x) &= 4x - 7 - 10x - 3 \\
f(x) - g(x) &= -6x - 4
\end{aligned}
$$