

BAKER CHARTERS SCHOOL



HONORS ALGEBRA 2B

Baker Web Academy School Notes

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CHAPTER ONE

Graphs and Functions

Unit 1

Feb 21 2022 Mon (11:15:44)

Lesson 1: Writing parallel and perpendicular**Unit 1**

Consider the line: $y = -2x + 1$.

Let's:

- Find the equation of the line that is parallel to this line and passes through the point $(7, -5)$.
- Find the equation of the line that is perpendicular to this line through the point $(7, -5)$.

Let's first go over these Properties:

Property 1. (Parallel Slope Property) Two non-vertical lines are parallel if and only if they have the same slope.

Property 2. (Perpendicular Slope Property) Two non-vertical lines are perpendicular if and only if the product of their slopes is equal to -1 .

Now, to equation $y = -2x + 1$ is written in the slope-intercept form: $y = mx + b$. In this form, the slope m is -2 .

- We can use the **Parallel Slope Property**. Since the given lines has a slope of -2 , a line parallel to it must also have the same slope, which is -2 . So, the equation of the parallel line will have the form $y = -2x + b$. The line passes through $(7, -5)$, so we use $x = 7$ and $y = -5$ to solve for b :

$$\begin{aligned}y &= -2x + b \\-5 &= -2(7) + b \\-5 &= -14 + b \\b &= 9\end{aligned}\tag{1.1}$$

Now, we know the equation of the parallel line, which is $y = -2x + 9$.

- We use the **Perpendicular Slope Property**. Since the given lines has the slope -2 , a line with the slope $\frac{1}{2}$ is perpendicular to it.

So, the equation of the perpendicular line will have the form $y = \frac{1}{2}x + b$.

The line passes through $(7, -5)$, so we use $x = 7$ and $y = -5$ to solve for b .

$$\begin{aligned}y &= \frac{1}{2}x + b \\-5 &= \frac{1}{2}(7) + b \\-5 &= \frac{7}{2} + b \\-\frac{10}{2} &= \frac{7}{2} + b \\b &= -\frac{17}{2}\end{aligned}\tag{1.2}$$

Equation of the parallel line: $y = -2x + 9$

Equation of the perpendicular line: $y = \frac{1}{2}x - \frac{17}{2}$

Feb 21 2022 Mon (12:24:44)

Lesson 2: Solve linear equation by graphing**Unit 1**

Let's look at this equation:

$$-4 = 5 - 3x. \quad (1.3)$$

Here's one method to solve it (by graphing):

- First, we write the equation with 0 on one side

$$\begin{aligned} -4 &= 5 - 3x \\ 0 &= 9 - 3x \end{aligned} \quad (1.4)$$

- Then, we graph the equation and find the **x-intercept**:

x	$y = -3x + 9$	(x, y)
0	$y = -3 \times 0 + 9 = 9$	$(0, 9)$
1	$y = -3 \times 1 + 9 = 6$	$(1, 6)$
2	$y = -3 \times 2 + 9 = 3$	$(2, 3)$

Table 1.1: X-Y Table

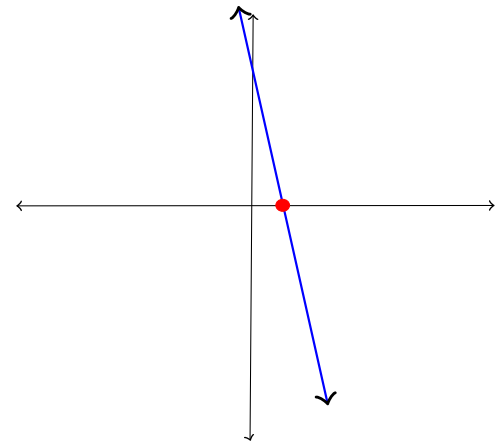


Figure 1.1: $-4 = 5 - 3x$ Graphed

We get that $x = 3$ is a solution to the original equation, which was $-4 = 5 - 3x$

Feb 27 2022 Sun (15:34:39)

Lesson 3: Identify correlation and causation**Unit 1**

Definition 1. (Correlation) A mutual relationship or connection between two or more things.

Two quantities have a *correlation* if they tend to vary together.

Definition 2. (Causation) This indicates a relationship between two events where one event is affected by the other.

Let's take a look at a couple of examples for each definition:

Example. Maria and Andy are high school students in Arizona. Andy always gets an A on his math test when it's sunny outside.

You may think that Andy is acing his test because it's sunny when he takes the test, but that's not true. It just happens that it's sunny when there's a math test.

So, the two events: The sunniness and the acing his test occurred together without one causing the other. In simpler terms, the two events are **Correlated**, but there's no **Causal** relationship between them.

Note. If you have a correlation (1) you don't always have a causation (2).

Mar 03 2022 Thu (17:42:42)

Lesson 4: Domain and range

Unit 1

Definition 3. (Domain) The domain of a function is the set of all possible inputs.

Definition 4. (Range) The range of a function is the set of all possible outputs of the function.

Example. Suppose that the function f is defined by the following table:

x	$f(x)$
1	1
2	8
3	27
4	64

The domain of f is the set of numbers in the left column. The range of the function f is the set of numbers in the right column. That is:

$$\begin{aligned} \text{Domain: } f &= \{1, 2, 3, 4\} \\ \text{Range } f &= \{1, 8, 27, 64\} \end{aligned} \quad (1.5)$$

The function f could also be written as a set of ordered pairs:

$$f(x) = \{(1, 1), (2, 8), (3, 27), (4, 64)\}$$

Here's a cooler way of showing it:

$$f(x) = 3x, x \in [-2, 5]. \quad (1.6)$$

The statement above (1.6) is saying that the domain of the function $f(x)$ is the interval $[-2, 5]$

Note. Note that the domain is the set of all first elements in the ordered pairs, and the range is the set of all second elements in ordered pairs.

Example. A construction crew is lengthening the road. Let L be the total length of the road (in miles). Let D be the number of days the crew has worked.

Suppose that $L = 4D + 400$ gives L as a function of D . The crew can work for at most 70 days.

Identify the correct description of the values in both the domain and range of the function.

Here's how we would approach such a problem:

We are given the function: $L = 4D + 400$. Here, L is the total length of the road and D is the total number of days the crew has worked.

Domain

Description of values for the domain For our function, the input is given by D . So, the values in the domain correspond to the number of days the crew has worked.

Set of values for the domain We are given that the crew can work for at most 70 days.

So, our domain will be $[0, 70]$.

Range

Description of Values for the Range For our function, the output is given by L . So the values in the range correspond to the total length of the road.

Set of Values for the Range To find the range, let's look at the output L for some values of D :

After 0 days, the total length of the road will be: $L = 4(0) + 400 = 400$ miles

After 1 day, the total length of the road will be: $L = 4(1) + 400 = 404$ miles

After 10 day, the total length of the road will be: $L = 4(10) + 400 = 440$ miles

After $20\frac{1}{2}$ day, the total length of the road will be: $L = 4(20\frac{1}{2}) + 400 = 482$ miles

After 70 day, the total length of the road will be: $L = 4(70) + 400 = 680$ miles

Note also that we can get any length between 400 and 680 miles for the road.

So, we choose all of the real numbers between 400 and 680 miles.

Or, we can write it like this:

$$L \in [400, 680]. \quad (1.7)$$

Note. The number of days must also be $x \geq 0$.

The number of days could also be decimals, fractions, etc like:

$$\frac{1}{2} \quad \frac{10}{19} \quad \frac{7}{11} \quad \frac{11}{7}$$

The only restriction we have is that the number of days must be $x \geq 0$

Mar 03 2022 Thu (18:38:15)

Lesson 5: Word problem ARC

Unit 1

Definition 5. (Slope) The slope of a line is a number measuring how **steep** the line is.

The farther the slope is from zero, the steeper the line is. The slope of a vertical line is undefined.

The sign of the slope tells us if the line will go **up** or **down**.

- If the slope is **positive**, then the line goes upward from **left to right**.
- If the slope is **negative**, then the line goes **downward** from **left to right**.
- If the slope is 0, then the line is **horizontal**.

Suppose that a non-vertical line passes through the two points:

$(x_1, y_1), (x_2, y_2)$

The *rise* from the first point to the second is $y_2 - y_1$.

The *run* from the first point to the second is $x_2 - x_1$

The slope formula is:

$$\text{slope} = \frac{\text{rise}}{\text{run}} = \frac{y_2 - y_1}{x_2 - x_1}. \quad (1.8)$$

The number of bacteria in a culture increase rapidly. The table below gives the number $N(t)$ of bacteria at a few times t (in hours) after the moment when $N = 1000$.

Time t hours	Number of bacteria $N(t)$
0	1000
3.4	1510
6.8	2292
10.2	3856
13.6	5080

1. We're going to find the Average Rate of Change (ARC) for the number of bacteria from 0 hours to 6.8 hours.
2. Find the ARC for the number of bacteria from 10.2 hours to 13.6

The ARC for the number of bacteria from $t = a$ to $t = b$ is:

$$\frac{N(b) - N(a)}{b - a}, \quad \text{where } b \neq a. \quad (1.9)$$

Note. Look how similar the equation above (1.9) is similar to the slope (5) formula.

The ARC is the slope of the line passing through: $(a, N(a))$ $(b, N(b))$.

1. For part **A**, we're asked to find the ARC from 0 hours to 6.8 hours.

So we take $a = 0$ $b = 6.8$.

From the table, we get $N(a) = 1000$ $N(b) = 2292$.

Applying the formula for ARC, we get:

$$\begin{aligned}\frac{N(b) - N(a)}{b - a} &= \frac{2292 - 1000}{6.8 - 0} \\ &= \frac{1292}{6.8} \\ &= \boxed{190 \text{ Bacteria per Hour}}\end{aligned}\quad (1.10)$$

2. For part **B**, we're asked to find the ARC from 10.2 hours to 13.6 hours.

So, we take $a = 10.2$ $b = 13.6$.

From the table, we get $N(a) = 3856$ $N(b) = 5080$.

Applying the formula for ARC, we get:

$$\begin{aligned}\frac{N(b) - N(a)}{b - a} &= \frac{5080 - 3856}{13.6 - 10.2} \\ &= \frac{1224}{3.4} \\ &= \boxed{360 \text{ Bacteria per Hour}}\end{aligned}\quad (1.11)$$

So, our final answers are:

The ARC for the number of bacteria from 0 hours to 6.8 hours is:

190 Bacteria per hour

The ARC for the number of bacteria from 10.2 hours to 13.6 hours is:

360 Bacteria per hour

Mar 03 2022 Thu (20:33:31)

Lesson 6: Transforming Graphs**Unit 1****Translating Graphs**

If each of the points of a graph is moved the same distance in the same direction, we say that the graph is **translated** and the resulting graph is a translation of the original.

• Horizontally Translating:

Suppose that f is a function whose domain (3) and range (4) are subsets of the set of real numbers

- The horizontal translation of the graph of f to the right c units is the graph of

$$g(x) = f(x - c)$$

.

- The horizontal translation of the graph of f to the left c units is the graph of

$$g(x) = f(x + c)$$

.

• Vertically Translating:

Suppose that f is a function whose domain (3) and range (4) are subsets of the set of real numbers

- The vertical translation of the graph of f upward c units is the graph of

$$g(x) = f(x) + c$$

.

- The vertical translation of the graph of f downward c units is the graph of

$$g(x) = f(x) - c$$

.

Example. If $f(x) = x^2$, then the graph of $g(x) = x^2 + 2$ is the vertical translation of the graph of f upward 2 units. The graph of $h(x) = x^2 - 1$ is the vertical translation of the graph of f downward 1 unit.

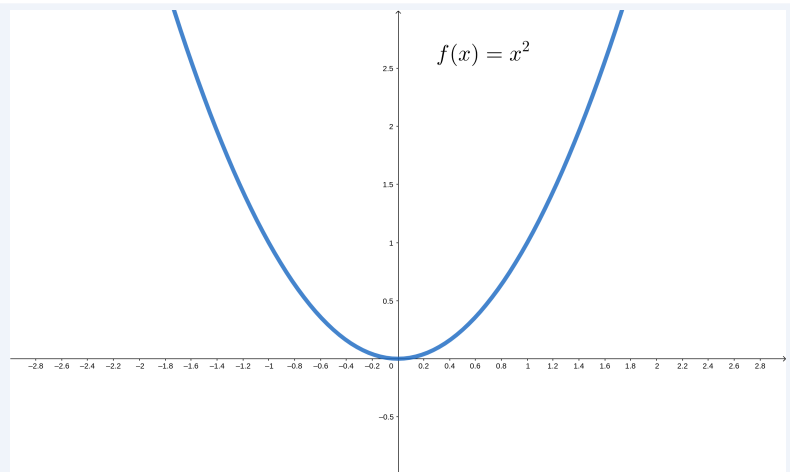


Figure 1.2: The starting graph $f(x) = x^2$

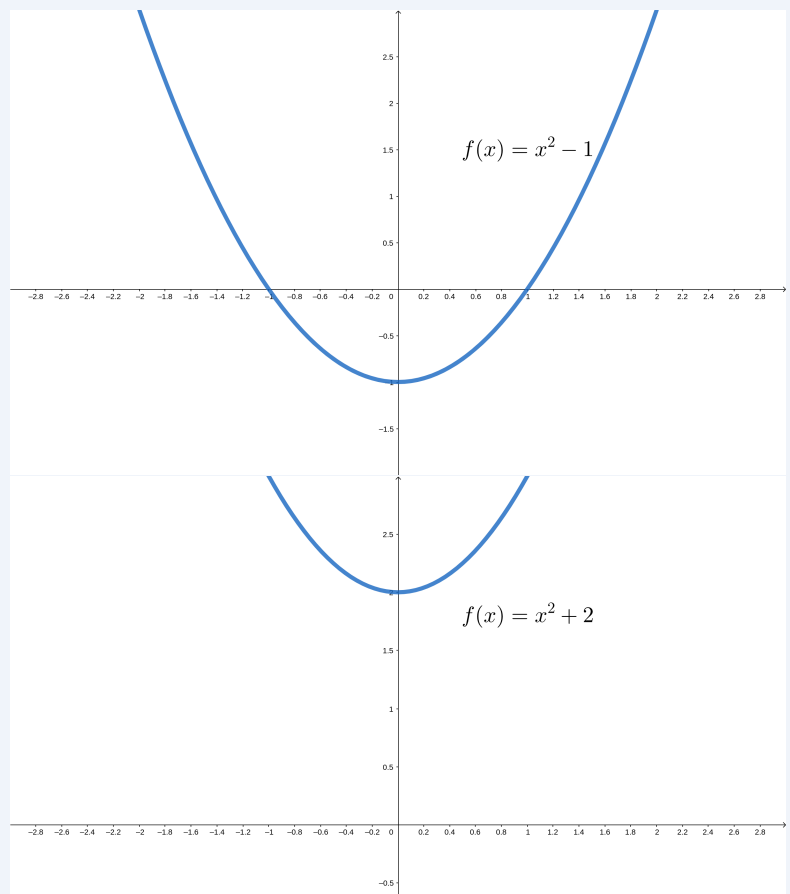


Figure 1.3: Translating Graphs

When the graph of a function f is translated to get the graph of a function g , it is important to

Note. When the graph of a function f is translated to get the graph of a function g , it is important to note that the rigidity of the graph of f is maintained. That is, the graph of g has the same "**size**" and "**shape**" as the graph of f .

Reflecting Graphs

- Reflecting about the y axis.

The **reflection** of the point (a, b) about the y axis is the point $(-a, b)$.

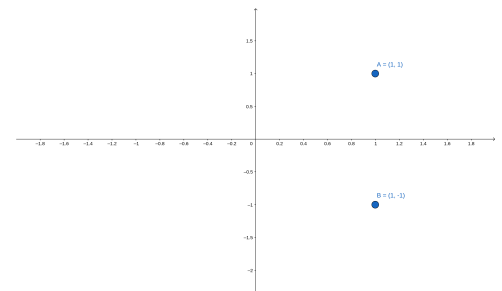
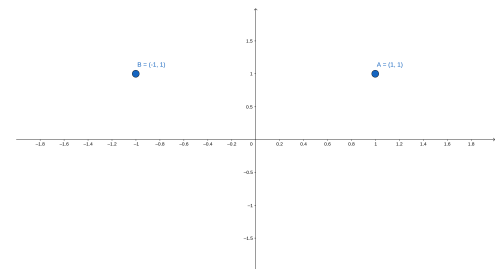
If f is a function whose domain (3) and range (4) are subsets of the set of real numbers, then the reflection of the graph of f about the y axis is the reflection of every point on the graph of f about the y axis. The reflection of the graph of f about the y axis is the graph of the function $g(x) = f(-x)$

- Reflecting about the x axis.

The reflection of the point (a, b) about the x axis is the point $(a, -b)$.

If f is a function whose domain (3) and range (4) are subsets of the set of real numbers, then the reflection of the graph of f about the x axis is the reflection of every point on the graph of f about the x axis. The reflection of the graph of f about the x axis is the graph of the function $g(x) = -f(x)$

Example. The graph of $f(x) = \sqrt{x}$ is shown in blue. The graph of $g(x) = -\sqrt{x}$, which is the reflection of the graph of f about the x axis is shown in red.



Stretching and Shrinking Graphs

- Vertically Stretching and Shrinking.

Suppose that f is a function whose domain (3) and range (4) are subsets of the set of real numbers.

- If c is a number greater than 1, then the graph of

$$g(x) = cf(x)$$

is **Vertical Stretching**, also called **Vertical Expansion**, of the graph of f .

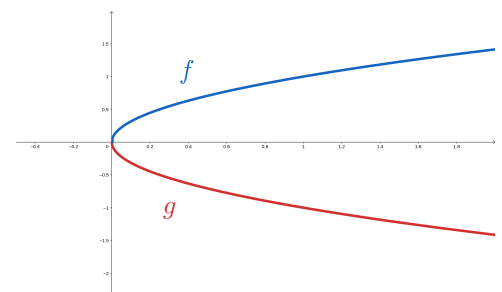
- If c is a number between 0 and 1, then the graph of

$$g(x) = cf(x)$$

is **Vertical Shrinking**, also called a **Vertical Contraction**, of the graph f .

- Horizontally Stretching and Shrinking.

Suppose that f is a function whose domain (3) and range (4) are subsets of the set of real numbers.



- If c is a number greater than 1, then the graph of

$$g(x) = f(cx)$$

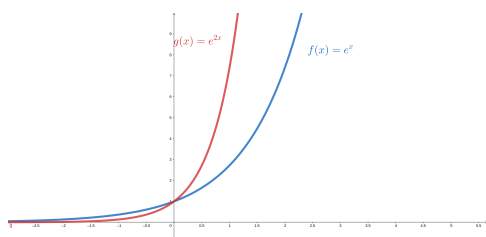
is a **Horizontal Shrinking**, also called a **Horizontal Contraction**, of the graph of f .

- If c is a number between 0 and 1, then the graph of

$$g(x) = f(cx)$$

is a **Horizontal Stretching**, also called a **Horizontal Expansion**, of the graph of f .

In both cases, for a given y coordinate, the x coordinate of the point on the graph of g is $\frac{1}{c}$ times the x coordinate of the point on the graph of f .



Example. The graph of the function $f(x) = e^x$ is shown in blue and the graph of the function $g(x) = e^{2x}$ is shown in red. The graph of g is a horizontal contraction of the graph of f because $g(x) = f(2x)$, that is, for a given y coordinate, the x coordinate of a point on the graph of g is $\frac{1}{2}$ times the x coordinate of the point on the graph of f .

Example. Below is the graph of $y = x^2$. Transform it to make it the graph of $y = -2(x - 4)^2$.

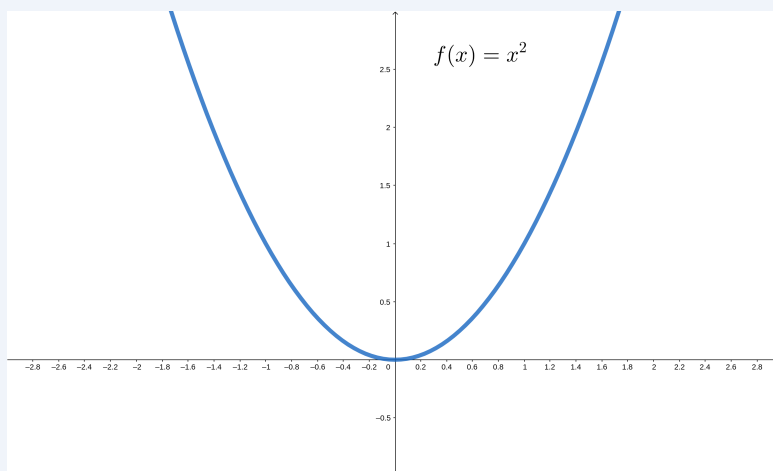


Figure 1.4: $y = x^2$

We are given the graph of $y = x^2$ and asked to graph $y = -2(x - 4)^2$. Starting with the graph of $y = x^2$, we'll first graph $y = (x - 4)^2$. To do this, we translate (1) the graph of $y = x^2$ to the right 4 units.

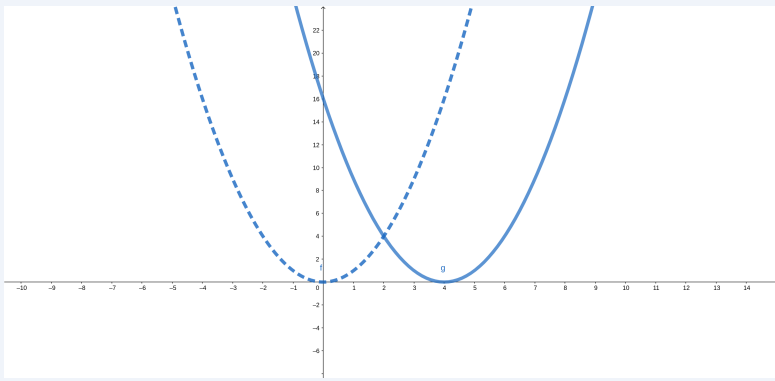


Figure 1.5: Moved to the right by 4 units.

Finally, we'll use the graph of $y = 2(x - 4)^2$ to get the graph of $y = -2(x - 4)^2$. To do this, we reflect the graph of $y = 2(x - 4)^2$ across the x axis.

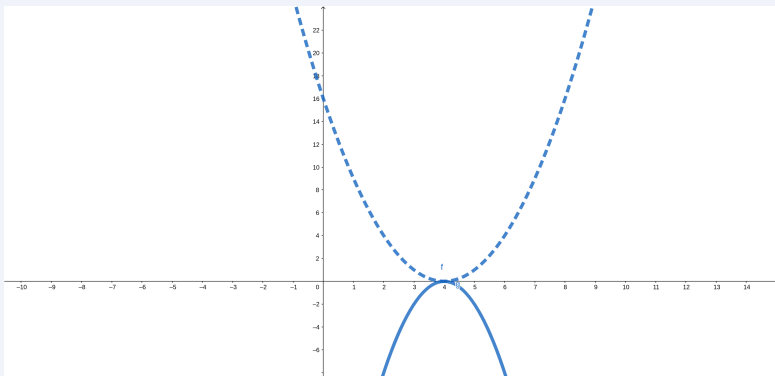


Figure 1.6: Reflected across the x axis.

Mar 05 2022 Sat (02:34:51)

Lesson 7: Equation after Transformation

Unit 1

CHAPTER TWO

Linear Systems

Unit 2

CHAPTER THREE

Exponents and Polynomials

Unit 3

Mar 05 2022 Sat (02:40:16)

Lesson 1: Power, product, and quotient rules

Unit 3

Property 3. (Properties of Exponents) The following are rules for exponents.

For these, rules, we assume that the denominators involved aren't zero.

Name	Rule	Example
Product	$x^r \times x^s = x^{r+s}$	$x^3 \times x^5 = x^8$
Quotient	$\frac{x^r}{x^s} = x^{r-s}$	$\frac{a^6}{a^2} = a^4$
Power of a Power	$(x^r)^s = x^{rs}$	$(z^2)^5 = z^{10}$

Simplify the following:

$$\begin{aligned}
 \left(\frac{x^3 - u^{-1}}{3z^3} \right)^3 (xz^5) &= \frac{x^{-9}u^3}{3^{-3}z^{-9}} \times \frac{xz^5}{1} \\
 &= \frac{x^{-8}u^3z^{14}}{3^{-3}} \\
 &= \frac{3^3u^3z^{14}}{x^8} \\
 &= \frac{27u^3z^{14}}{x^8}
 \end{aligned} \tag{3.1}$$

CHAPTER FOUR

Quadratic and Polynomial Functions

Unit 4

Mar 04 2022 Fri (23:40:39)**Lesson 1: The Factor Theorem****Unit 4**

Theorem 1. (Factor) A polynomial $P(x)$ has a factor $x - c \Leftrightarrow P(c) = 0$.

Example. We must determine whether $x - 2$ is a factor of $P(x) = 2x^3 - 2x^2 - 8$.

So, to use the **Factor Theorem** (1), we must evaluate $P(2)$.

$$\begin{aligned} P(2) &= 2(2)^3 - 2(2)^2 - 8 \\ &= 16 - 8 - 8 \\ &= 0 \end{aligned} \quad (4.1)$$

So, $P(2) = 0$. By the **Theorem Factor** (1), we know that $x - 2$ is a factor of $P(x)$

Mar 16 2022 Wed (10:40:43)

Lesson 2: Find all possible rational zeros: zeros theorem

Unit 4

Theorem 2. (Rational Zero Theorem) Let $f(x) = a_n x^n + a_{n-1} x^{n-1} + \dots + a_1 x + a_0$ be a polynomial with integer coefficients and $a_n \neq 0$. If $\frac{p}{q}$ is a zero of $f(x)$ is written in lowest terms, then p is a factor of a_0 , and q is a factor of a_n .
In other words, the only possibilities for rational zeros of $f(x)$ are numbers of the form $\frac{p}{q}$ (in lowest terms), where p is a factor of a_0 , and q is a factor of a_n .

Example. Let's list all of the possible rational zeros for the following polynomial: $f(x) = x^3 + 2x^2 - 5x - 6$

$$\begin{aligned} \frac{p}{q} &= \frac{\pm 1, \pm 2, \pm 3, \pm 6}{\pm 1} \\ &= \pm 1, \pm 2, \pm 3, \pm 6 \end{aligned} \quad (4.2)$$

Here, p is factors of the constant term, which in this case is -6 and q is the leading coefficient.

Now, let's start to check:

$$\begin{aligned} f(1) &= (1)^3 + 2(1)^2 - 5(1) - 6 \\ &= 1 + 2 - 5 - 6 \\ &= 3 - 11 = -8 \end{aligned} \quad (4.3)$$

Notice that 1 doesn't give you 0. Therefore, 1 isn't a zero of the polynomial $f(x)$.

$$\begin{aligned} f(2) &= (2)^3 + 2(2)^2 - 5(2) - 6 \\ &= 8 + 8 - 10 - 6 \\ &= 16 - 16 = 0 \end{aligned} \quad (4.4)$$

Now, instead of just brute forcing to check all of the zeros, let's use one of the zeros we found, which was 2. For this, we have to use synthetic division (??):

$$\begin{array}{r|rrrr} 2 & 1 & 2 & -5 & -6 \\ & & 2 & 8 & 6 \\ \hline & 1 & 4 & 3 & 0 \end{array}$$

That gives us:

$$\begin{aligned} x^2 + 4x + 3 &= 0 \\ (x + 3)(x + 1) &= 0 \\ x + 3 &= 0 & x + 1 &= 0 \\ x &= -3, -1 \end{aligned} \quad (4.5)$$

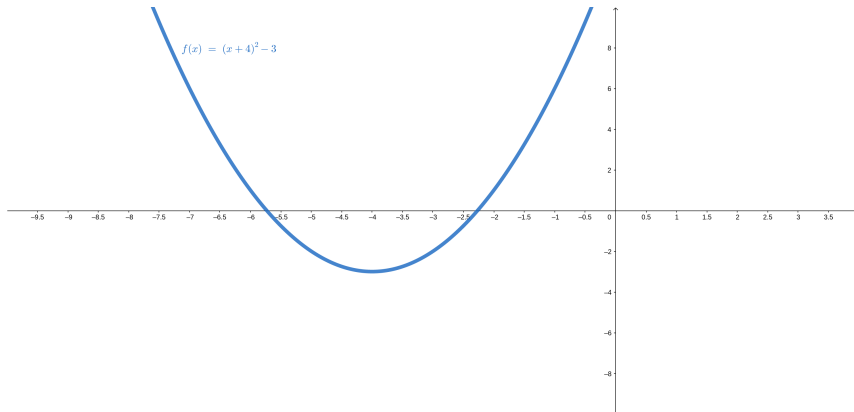
So, here are our final zeros: $x = 2, -3, -1$.

Mar 04 2022 Fri (18:07:42)

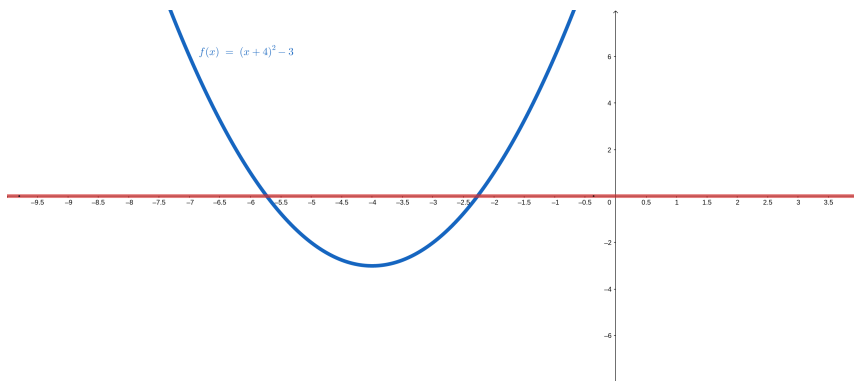
Lesson 3: Domain and range from parabola

Unit 4

Let's try to find the domain (3) and range (4) of this parabola:

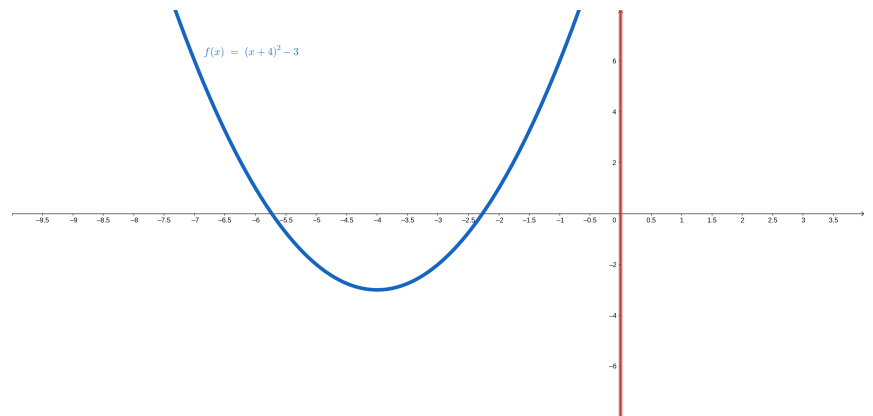


- To find the domain, we project the graph onto the x axis.



Because the graph extends to the left and right forever, all real numbers are in the domain.

- To find the range, we project the graph onto the y axis.



We see that the minimum value in the range is -3 .

All values greater than -3 are also part of the range. This is because the graph extends upward forever.

Therefore, the range of our function is given by $y \geq -3$.

Mar 04 2022 Fri (19:03:22)

Lesson 4: Create QE from roots and LC

Unit 4

Theorem 3. (Fundamental Theorem of Algebra) Any polynomial of degree $n > 0$ has at least one zero (real or non-real).

Theorem 4. (Factor Theorem) If r is a zero of the polynomial $P(x)$, then $x - r$ is a factor of $P(x)$. Conversely, if $x - r$ is a factor of $P(x)$, then r is a zero of $P(x)$.

Example. Consider the polynomial $P(x) = x^2 + 3x + 2$.
Because

$$P(-1) = (-1)^2 + 3(-1) + 2 = 1 - 3 + 2 = 0. \quad (4.6)$$

we have that -1 is a zero of the polynomial $P(x)$.

Hence, the **Factor Theorem** (4) guarantees that $x - (-1) = x + 1$ is a factor of $P(x)$.

That is, the factor theorem guarantees that there exists a polynomial $Q(x)$ such that

$$P(x) = (x + 1)Q(x). \quad (4.7)$$

That can be confirmed directly. Since $P(x) = x^2 + 3x + 2 = (x + 1)(x + 2)$, we have that $Q(x) = x + 2$

Remark. Given a polynomial $P(x) = a_n x^n + a_{n-1} x^{n-1} + \dots + a_1 x + a_0$ of degree $n > 0$, we can apply the **Fundamental Theorem of Algebra** (3) and factor theorem n times to obtain what is commonly referred to as the complete factorization of $P(x)$. Namely, we can write $P(x)$ as follows:

$$P(x) = a_n (x - r_1)(x - r_2) \cdots (x - r_n). \quad (4.8)$$

where a_n is the leading coefficient of $P(x)$ and r_1, r_2, \dots, r_n are the n zeroes of $P(x)$. This fact is sometimes referred to as the **Linear Factors Theorem**

Let's write a quadratic equation whose roots are -1 and -3 and whose leading coefficient is 2.

We use the **Factor Theorem** (4), which states that if k is a root of the polynomial equation $P(x) = 0$, then $x - k$ is a factor of the polynomial $P(x)$.

In our problem, we have $P(-1) = 0$ and $P(-3) = 0$. Thus, $x + 1$ and $x + 3$ are

both factors of the quadratic polynomial. Since the coefficient is 2, we can write the polynomial as:

$$2(x + 1)(x + 3) = 2x^2 + 8x + 6. \quad (4.9)$$

Mar 04 2022 Fri (19:33:18)

Lesson 5: Vertex, Intercept, Axis Symmetry**Unit 4****Find the Axis of Symmetry**

The axis of symmetry is the line that divides the parabola into two mirror images.

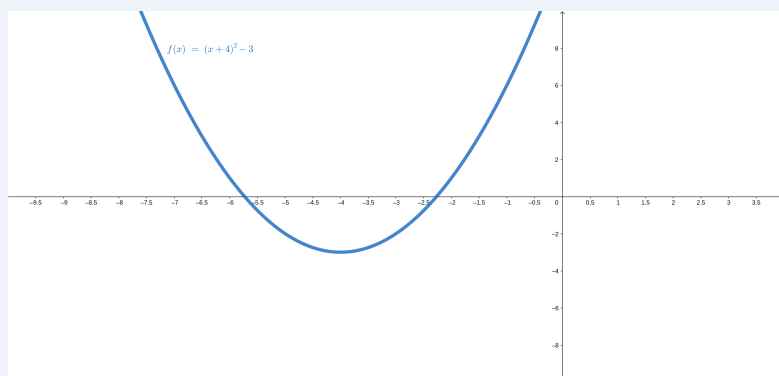
Find the Vertex

If the parabola opens upward, the vertex will be the lowest point.

If the parabola opens downward, the vertex will be the highest point.

Note. The vertex always lies on the axis of symmetry.

Example. Use the graph of the parabola to answer the following questions:



1. Does the parabola open upward or downward?
The parabola opens upward.
2. Find the equation of the axis of symmetry.
The equation of the axis of symmetry is $x = -4$.
3. Find the coordinates of the vertex.
The vertex is at $(-4, -3)$.
4. Find the intercept(s).
The x intercepts are $(-5, 0)$ $(-3, 0)$.
There are no y intercepts.

Mar 04 2022 Fri (19:51:45)

Lesson 6: Solve quadratic using square root**Unit 4**

Consider this equation: $x^2 = a$

- When a is positive, the equation has two solutions: $x = \sqrt{a}$ $x = -\sqrt{a}$
- When a is negative, the equation has no solution. It only has imaginary solutions because the square root of a negative number and is not a real number. It has the variable i , which is an imaginary number.

Example. Solve $x^2 = 50$, where $x \in \mathbb{R}$.

So, the equation $x^2 = 50$ has two solutions: $x = \sqrt{50}$ $x = -\sqrt{50}$

We can simplify:

$$\sqrt{50} = \sqrt{25 \times 2} = 5\sqrt{2}. \quad (4.10)$$

So, our final answer is: $x = 5\sqrt{2}$ $x = -5\sqrt{2}$.

Mar 04 2022 Fri (20:56:39)

Lesson 7: Graphing a quadratic inequality

Unit 4

To graph an inequality, we first graph the associated parabola.

Example. Graph the inequality $y > -x^2 + 7$. First you graph $y = -x^2 + 7$.

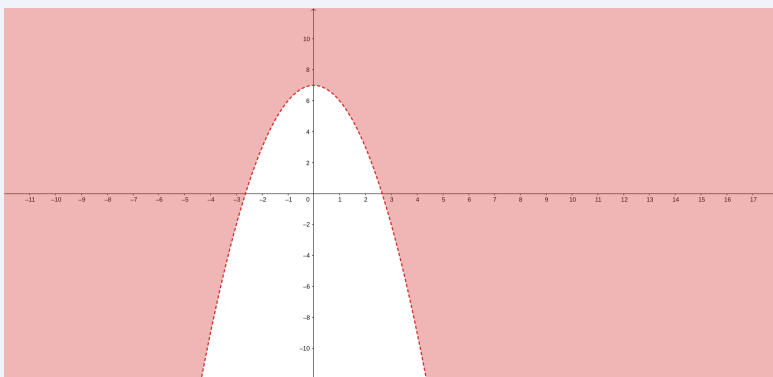
The inequality has **y greater than** $-x^2 + 7$, so we shade **above** the curve.

If it was **y less than**, we shade **below** the curve.

Since the inequality is only $>$, that means the curve itself is not part of the graph. We use a dashed curve to show this.

If the inequality was \geq , then we would use a solid curve.

So, here's the inequality $y > -x^2 + 7$ graphed.



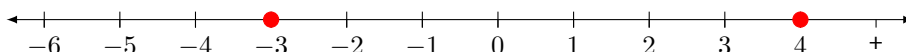
Mar 04 2022 Fri (21:13:37)

Lesson 8: Graph a quadratic inequality: Problem Type 1**Unit 4**

Let's say we have $x^2 - x \geq 12$. When dealing with an equation where $x = a$, x has a set value. But if you're dealing with an inequality, x has a range of values.

$$\begin{array}{ll} x^2 - x - 12 \geq 12 & \text{Set everything to 0} \\ (x - 4)(x + 3) \geq 12 & \text{Factor the quadratic.} \\ x = 4, -3 & \text{Find the roots} \end{array} \quad (4.11)$$

Since the solution includes 4 and -3 , we're going to put filled circles at those points on a number line.



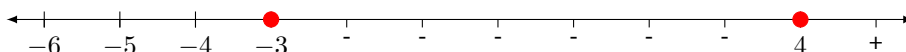
Since it's ≥ 0 , we need to check the 3 regions that's in between the points to see if it's positive. So, we got to plugin numbers and test the signs.

Let's plug-in 5 into the expression: $(x - 4)(x + 3) \geq 0$:

$$\begin{array}{ll} (x - 4)(x + 3) \geq 0 & \\ 1(8) = 8 & \text{Which is } > 0 \end{array} \quad (4.12)$$

Now, we need to check a number that's between -3 and 4 . Let's go with 0 .

$$\begin{array}{ll} (x - 4)(x + 3) \geq 0 & \\ -4(3) = -12 & \text{Which is } < 0 \end{array} \quad (4.13)$$



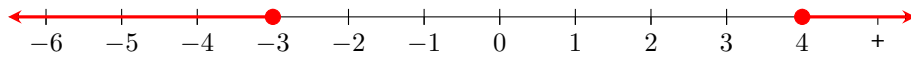
Now, we need to check a number that's greater than -3 . Let's go with -4 .

$$\begin{array}{ll} (x - 4)(x + 3) \geq 0 & \\ -8(-1) = 8 & \text{Which is } > 0 \end{array} \quad (4.14)$$



Here's a quicker method. Take a look at the multiplicity of the factored quadratic: $(x - 4)^1(x + 3)^1$. When the multiplicity is odd, the sign will change at that point. So, if you know the first sign, you can easily determine the rest. So, for the zero 4 , the multiplicity is odd, so, it's going to switch from $+$ to $-$. And for the zero 3 , the multiplicity is odd, so it's going to switch from $-$ to $+$.

Here's the final graphed answer:



And here's the final answer in interval notation:

$$(-\infty, -3] \cup [4, \infty) \quad \text{or} \quad x \leq -3 \quad \text{or} \quad x \geq 4. \quad (4.15)$$

Mar 16 2022 Wed (11:07:03)

Lesson 9: Graph a quadratic inequality: Problem Type 2**Unit 4**

To graph the inequality $y \leq x^2 - 10x - 23$, we first graph the associated parabola $y = x^2 - 10x - 23$.

A parabola in the form $y = a(x - h)^2 + k$ has a vertex (h, k) .

We'll rewrite $y = -10x^2 = 10x - 23$ in this form. We start with the first two terms:

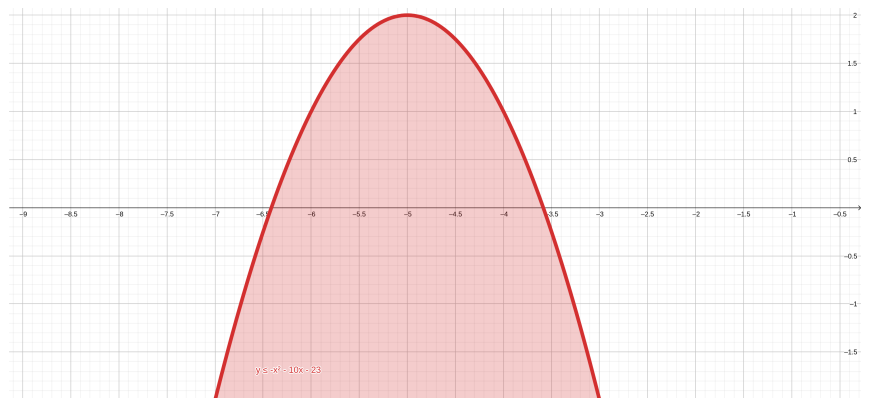
$$\begin{aligned} y &= -x^2 - 10x - 23 \\ &= -(x^2 + 10x) - 23 \end{aligned} \quad (4.16)$$

Now, we need to complete the square of the expression $x^2 + 10x$.

Since we have $x^2 + 10x$, we must add $\left(\frac{10}{2}\right)^2 = 25$ inside and outside the parenthesis. So, our final answer is:

$$y = -(x + 5)^2 + 2. \quad (4.17)$$

So, now we know that the vertex is at $(-5, 2)$. And we graph the quadratic. Since we have $y \leq$, we shade below the curve since everything in the curve is less than y .



Mar 04 2022 Fri (21:47:06)

Lesson 10: Polynomial from degree and zeroes**Unit 4**

Find a polynomial $f(x)$ of degree 5 that has the following zeroes:

$$-4, 1, 3, 8, 0$$

.

The **Factor Theorem** (1) tells us the following:

A number c is a zero of a polynomial $f(x)$ if and only if $x - c$ is a factor of $f(x)$.

We also get that, if c is a zero of multiplicity k , then $(x - c)^k$ is a factor.

We're told that $f(x)$ has the zeros shown below:

Zero	Factor
-4	$x - (-4) = x + 4$
1	$x - 1$
3	$x - 3$
8	$x - 8$
0	$x - 8 = x$

Table 4.1: Zeros of the polynomial $f(x)$

These factors give the following polynomial:

$$f(x) = x(x + 4)(x - 1)(x - 3)(x - 8). \quad (4.18)$$

Since this polynomial has degree 5, it's a possible answer.

Mar 04 2022 Fri (23:55:49)

Lesson 11: Solve quadratic using square root

Unit 4

Example. Solve $(w - 9)^2 - 36 = 0$, where w is a real number.

Simplify your answer as much as possible.

We first rewrite the equation with only the squared expression on the left.

$$(w - 9)^2 = 36. \quad (4.19)$$

For the current problem, we have $w - 9 = \sqrt{36}$ $w - 9 = -\sqrt{36}$

Solving and simplifying, we get the following:

$$\begin{array}{ll} w - 9 = \sqrt{36} & w - 9 = -\sqrt{36} \\ w - 9 = 6 & w - 9 = -6 \\ \boxed{w = 15} & \boxed{w = 3} \end{array} \quad (4.20)$$

Mar 05 2022 Sat (01:03:47)

Lesson 12: Discriminant of a quadratic

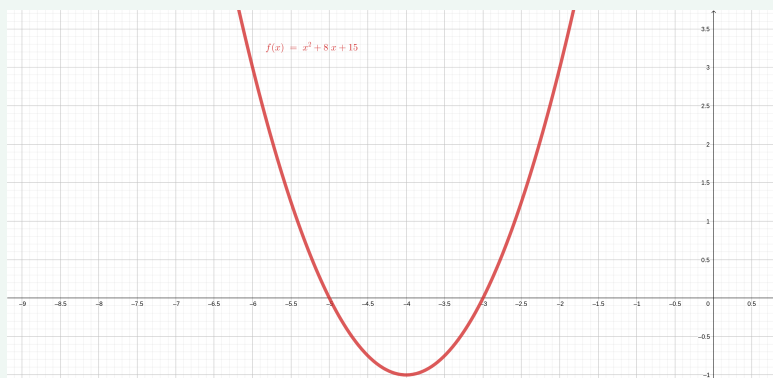
Unit 4

Review 1. (Quadratic Equation) Introduction

A **Quadratic Equation** is when the variable is squared. So we can have something called a **Quadratic Expression**, which looks something like this: $x^2 + 8x + 15$. Most of the time, quadratic equations are written in the form of: $x^2 \pm ax \pm b = 0$. But, you may see them sometimes written in this form: $ax^2 + bx + c = 0$. That form is called the **Standard Form**.

We can do a lot of things with **Quadratic Expressions**. For one thing, we can:

1. Factor them (most of the time). The equation above will be factored to $(x + 3)(x + 5)$.
2. Graph them. We take the expression and re-write it as a function.
 $f(x) = x^2 + 8x + 15$.

**Solving a Quadratic Equation**

Let's solve for x : $2x^2 + 3x - 2 = 0$.

For this, we need the quadratic equation: $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$$\begin{aligned}
 x &= \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \\
 2x^2 + 3x - 2 = 0 &\Rightarrow x = \frac{-3 \pm \sqrt{3^2 - 4(2)(-2)}}{2(2)} \\
 &= \frac{-3 \pm \sqrt{9 + 16}}{4} \quad (4.21) \\
 &= \frac{-3 \pm \sqrt{25}}{4} \\
 &= \frac{-3 + 5}{4} \\
 x &= \frac{1}{2} \qquad x = -2
 \end{aligned}$$

Definition 6. (Discriminant) The expression $b^2 - 4ac$ that appears under the square root sign in the quadratic equation (1) is called the **Discriminant** of the quadratic equation.

The solutions of the quadratic equation $ax^2 + bx + c = 0$ are given by the quadratic formula:

The expression under the square root, $b^2 - 4ac$, is the **Discriminant** (6) of the quadratic equation.

The value of the discriminant determines the number of real solutions for $ax^2 + bx + c = 0$.

1. if $b^2 - 4ac > 0$, there are 2 real solutions to $ax^2 + bx + c = 0$
2. if $b^2 - 4ac = 0$, there is 1 real solution to $ax^2 + bx + c = 0$
3. if $b^2 - 4ac < 0$, there are no real solution to $ax^2 + bx + c = 0$

Example. For the quadratic equation $5x^2 - 9x + 1 = 0$, we have $a = 5$ $b = -9$ $c = 1$. So, the value of the discriminant is as the follows:

$$b^2 - 4ac = (-9)^2 - 4(5)(1) = 81 - 21 = 61. \quad (4.22)$$

Since the discriminant is > 0 , $5x^2 - 9x + 1 = 0$ has 2 real solutions.

Mar 15 2022 Tue (21:49:41)

Lesson 13: Solve quadratic equation with roots**Unit 4**

To find all complex solutions to a quadratic equation, we use the quadratic equation (1).

According to the quadratic equation, the solutions of the equation $ax^2 + bx + c = 0$ are as follows:

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}. \quad (4.23)$$

Example. We must find the solutions of $2x^2 - x + 6 = 0$. For this problem, we have the following information:

$$\begin{aligned} a &= 2 \\ b &= -1. \\ c &= 6 \end{aligned} \quad (4.24)$$

We use these values in the quadratic equation:

$$\begin{aligned} x &= \frac{-(-1) \pm \sqrt{(-1)^2 - 4 \times 2 \times 6}}{2 \times 2} \\ &= \frac{1 \pm \sqrt{-47}}{4} \end{aligned} \quad (4.25)$$

Note. The discriminant (6), $b^2 - 4ac = -47$, is negative. So, our solutions are not real numbers. We get the following:

$$\begin{aligned} x &= \frac{1 \pm \sqrt{47}i}{4} \\ &= \frac{1}{4} + \frac{\sqrt{47}}{4}i \end{aligned} \quad (4.26)$$

So, our final answer is:

$$x = \frac{1}{4} + \frac{\sqrt{47}}{4}i \quad \frac{1}{4} - \frac{\sqrt{47}}{4}i. \quad (4.27)$$

 Mar 16 2022 Wed (12:40:50)

Lesson 14: Solve word problem using quadratic equation**Unit 4**

A ball is thrown from an initial height of 5 feet with an initial upward velocity of $36 \frac{ft}{s}$. The ball's height h (in feet) after t seconds is given by the following quadratic equation: $h = 5 + 36t - 16t^2$.

Find all values of t for which the ball's height is 21 feet.

To find when the ball's height is 21 feet, we substitute 21 for h and solve for t . $21 = 5 + 36t - 16t^2$. Then, we set the whole thing equal to 0. Finally, we use the quadratic formula to solve for t :

$$\begin{aligned}
 a &= 16 \\
 b &= -36 \\
 c &= 5 \\
 t &= \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \\
 &= \frac{-(-36) \pm \sqrt{(-36)^2 - 4 \times 16 \times 5}}{2 \times 16} \\
 &= \frac{36 \pm \sqrt{272}}{32}
 \end{aligned} \tag{4.28}$$

0.6096 1.6403

So, the ball's height is 21 at approximately 0.61 seconds (on its way up) or 1.64 seconds (on its way down).

Mar 16 2022 Wed (12:51:03)

Lesson 15: Find x, y-intercepts given a polynomial**Unit 4**

Find all x -intercepts and y -intercepts of the graph of the function: $f(x) = x^3 + 5x^2 - 4x - 20$

1. To find the x -intercepts, we set $f(x) = 0$ and solve for x :

$$\begin{aligned}
 0 &= x^3 + 5x^2 - 4x - 20 && \text{Given} \\
 &= (x^3 + 5x^2) + (-4x - 20) && \text{Gathering terms} \\
 &= x^2(x + 5) - 4(x + 5) && \text{Factoring out } x^2 \text{ and } -4 \\
 &= (x + 5)(x^2 - 4) && \text{Factor } x + 5 \text{ out} \\
 &= (x + 5)(x - 2)(x + 2) && \text{Final Factored Form} \\
 x &= -5, 2, -2 && (4.29)
 \end{aligned}$$

So, the x -intercepts are $-5, 2, -2$.

2. To find the y -intercept, we $x = 0$.

$$\begin{aligned}
 f(x) &= x^3 + 5x^2 - 4x - 20 && \text{Given} \\
 f(0) &= (0)^3 + 5(0)^2 - 4(0) - 20 && \text{Set } x = 0. \\
 &= f(0) = -20 && (4.30)
 \end{aligned}$$

So, our x -intercepts are $(-5, 0), (2, 0), (-2, 0)$ and our y -intercept is $(0, -20)$.

Mar 16 2022 Wed (13:04:37)

Lesson 16: Solve quadratic equation by complete the square

Unit 4

List of Theorems

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List of Conjectures

List of Corollaries

List of Definitions

1	Definition (Correlation)	Page: 4
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List of Properties

- 1 Property (Parallel Slope Property) Page: 1
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List of Propositions

Essential Formulas for Algebra 2 Final Exam

Laws of Exponents

Multiply Powers of the Same Base = Adding Exponents	$(a^m)(a^n) = a^{m+n}$
Divide Powers of the Same Base = Subtracting Exponents	$\frac{a^m}{a^n} = a^{m-n}$
Power Rule = Multiplying Exponents	$(a^m)^n = a^{m \times n}$
Zero Exponent = 1	$a^0 = 1$
Distribution of Exponent with Multiple Bases	$(ab)^n = a^n b^n$ $\left(\frac{a}{b}\right)^n = \frac{a^n}{b^n}$
Negative Exponent = Reciprocal	$a^{-n} = \frac{1}{a^n}$ $\frac{a^{-m}}{b^{-n}} = \frac{b^n}{a^m}$
Distribution of Negative Exponent with Multiple Bases	$(ab)^{-n} = a^{-n} b^{-n} = \frac{1}{a^n b^n}$ $\left(\frac{a}{b}\right)^{-n} = \left(\frac{b}{a}\right)^n = \frac{b^n}{a^n}$

$\sqrt{a+b} \neq \sqrt{a} + \sqrt{b}$	$\sqrt{a \times b} = \sqrt{a} \times \sqrt{b}$
$\sqrt{a-b} \neq \sqrt{a} - \sqrt{b}$	$\sqrt{a \div b} = \sqrt{a} \div \sqrt{b}$

Properties of Radicals

Distribution of Radicals of the Same Index (where $a \geq 0$ and $b \geq 0$ if n is even)	$\sqrt[n]{ab} = (\sqrt[n]{a})(\sqrt[n]{b})$ $\sqrt[n]{\frac{a}{b}} = \frac{\sqrt[n]{a}}{\sqrt[n]{b}}$
Power Rule of Radicals = Multiplying Exponents	$\sqrt[m]{\sqrt[n]{a}} = \sqrt[m \times n]{a}$
Reverse Operations of Radicals and Exponents	$\sqrt[n]{a^n} = a \quad (\text{if } n \text{ is odd})$ $\sqrt[n]{a^n} = a \quad (\text{if } n \text{ is even})$

$$a^{\frac{m}{n}} = \sqrt[n]{a^m}$$

The index of the radical is the denominator of the fractional exponent.

Special Products

$$(A + B)^2 = A^2 + 2AB + B^2$$

$$(A - B)^2 = A^2 - 2AB + B^2$$

$$(A + B)(A - B) = A^2 - B^2$$

$$(A + B)^3 = A^3 + 3A^2B + 3AB^2 + B^3$$

$$(A - B)^3 = A^3 - 3A^2B + 3AB^2 - B^3$$

Special Expressions

Difference of Squares

$$A^2 - B^2 = (A + B)(A - B)$$

Perfect Trinomial Squares

$$A^2 + 2AB + B^2 = (A + B)^2$$

Perfect Trinomial Squares

$$A^2 - 2AB + B^2 = (A - B)^2$$

Sum of Cubes

$$A^3 + B^3 = (A + B)(A^2 - AB + B^2)$$

Difference of Cubes

$$A^3 - B^3 = (A - B)(A^2 + AB + B^2)$$

$$\text{Quadratic Formula: } x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$\text{Discriminant} = b^2 - 4ac$$

When Discriminant is Positive, $b^2 - 4ac > 0 \rightarrow$ Two Distinct Real Roots

When Discriminant is Zero, $b^2 - 4ac = 0 \rightarrow$ One Distinct Real Root
(or Two Equal Real Roots)

When Discriminant is Negative, $b^2 - 4ac < 0 \rightarrow$ No Real Roots

Note the pattern:

$$i^1 = i \quad i^2 = -1 \quad i^3 = -i \quad i^4 = 1$$

$$i^5 = i \quad i^6 = -1 \quad i^7 = -i \quad i^8 = 1$$

$$i^9 = i \quad i^{10} = -1 \quad \dots$$

Pattern repeats every 4th power of i .

Product of Conjugate Complex Numbers

$$(a + bi)(a - bi) = a^2 - b^2i^2 = a^2 - b^2(-1)$$

$$(a + bi)(a - bi) = a^2 + b^2$$

Midpoint of a Line Segment

$$M = \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2} \right)$$

Distance of a Line Segment

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

Slope

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

Standard Equation for Circles

$$(x - h)^2 + (y - k)^2 = r^2$$

$P(x, y)$ = any point on the path of the circle

$C(h, k)$ = centre of the circle

r = length of the radius

Point-Slope form: - a form of a linear equation when given a slope (m) and a point (x_1, y_1) on the line

$$\frac{y - y_1}{x - x_1} = m \text{ (slope formula)} \quad y - y_1 = m(x - x_1) \quad \text{(Point-Slope form)}$$

If we rearrange the equations so that all terms are on one side, it will be in **standard (general) form**:

$$Ax + By + C = 0 \quad \text{(Standard or General form)}$$

($A \geq 0$, the leading coefficient for the x term must be positive)

When given a slope (m) and the y -intercept $(0, b)$ of the line, we can find the equation of the line using the **slope and y -intercept form**:

$$y = mx + b \quad \text{where } m = \text{slope and } b = y\text{-intercept}$$

Parallel Lines

slope of line 1 = slope of line 2

$$m_1 = m_2$$

Perpendicular Lines

slope of line 1 = negative reciprocal slope of line 2

$$m_{l_1} = \frac{-1}{m_{l_2}}$$

$y \propto x$ (y is directly proportional to x)

$$y = kx$$

where k = constant of variation (constant of proportionality – rate of change)

$y \propto \frac{xz}{w}$ (y is jointly proportional to x , z and w)

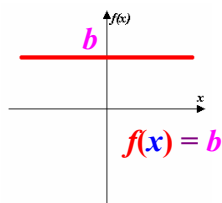
$$y = k \frac{xz}{w}$$

where k = constant of variation (constant of proportionality)

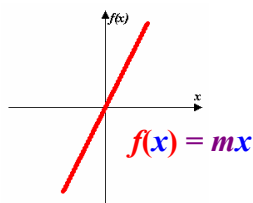
$$\text{Average Rate of Change} = m = \frac{\Delta y}{\Delta x}$$

$$\text{Average Rate of Change} = \frac{f(b) - f(a)}{b - a}$$

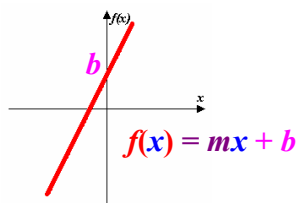
It is the slope of the secant line between the points $(a, f(a))$ and $(b, f(b))$

Summary of Types of Functions: (see page 226 of textbook)**Linear Functions** $f(x) = mx + b$ 

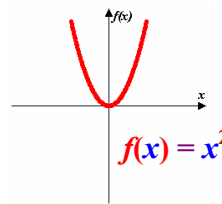
Domain: $x \in R$
Range: $f(x) \in R$



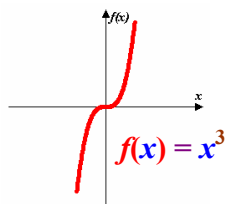
Domain: $x \in R$
Range: $f(x) \in R$



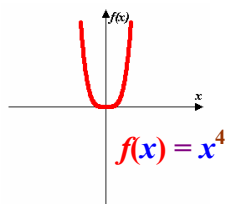
Domain: $x \in R$
Range: $f(x) \in R$

Power Functions $f(x) = x^n$ where $n > 1$ and $n \in N$ 

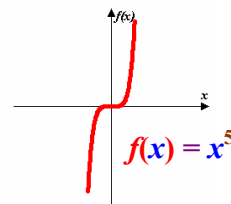
Domain: $x \in R$
Range: $f(x) \geq 0$



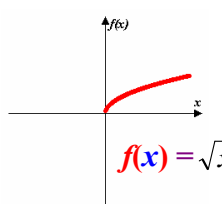
Domain: $x \in R$
Range: $f(x) \in R$



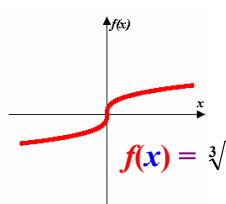
Domain: $x \in R$
Range: $f(x) \geq 0$



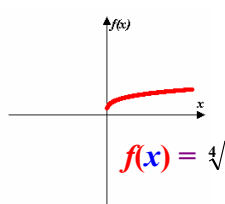
Domain: $x \in R$
Range: $f(x) \in R$

Root Functions $f(x) = \sqrt[n]{x}$ where $n \geq 2$ and $n \in N$ 

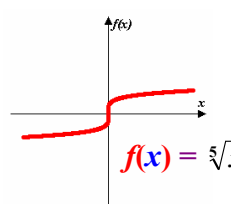
Domain: $x \geq 0$
Range: $f(x) \geq 0$



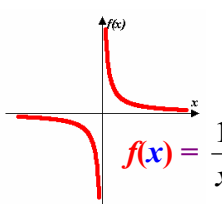
Domain: $x \in R$
Range: $f(x) \in R$



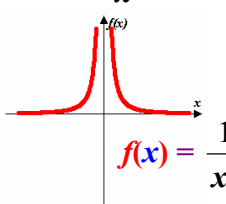
Domain: $x \geq 0$
Range: $f(x) \geq 0$



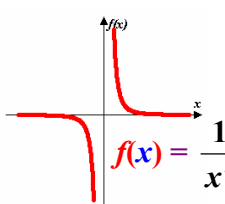
Domain: $x \in R$
Range: $f(x) \in R$

Reciprocal Functions $f(x) = \frac{1}{x^n}$ where $n \in N$ 

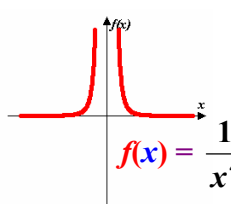
Domain: $x \neq 0$
Range: $f(x) \neq 0$



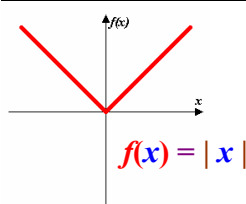
Domain: $x \neq 0$
Range: $f(x) > 0$



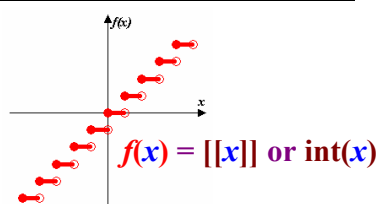
Domain: $x \neq 0$
Range: $f(x) \neq 0$



Domain: $x \neq 0$
Range: $f(x) > 0$

Absolute Value Functions

Domain: $x \in R$
Range: $f(x) \geq 0$

Greatest Integer Functions

Domain: $x \in R$
Range: $f(x) \in I$

$$g(x) = f(x + h) + k$$

h = amount of horizontal movement

$h > 0$ (move left); $h < 0$ (move right)

k = amount of vertical movement

$k > 0$ (move up); $k < 0$ (move down)

Reflection off the x-axis

$$g(x) = -f(x)$$

All values of y has to switch signs but all values of x remain unchanged.

Reflection off the y-axis

$$g(x) = f(-x)$$

All values of x has to switch signs but all values of y remain unchanged.

Vertical Stretching and Shrinking

$$g(x) = af(x)$$

a is the Vertical Stretch Factor

$a > 1$ (Stretches Vertically by a factor of a)

$0 < a < 1$ (Shrinks Vertically by a factor of a)

Horizontal Stretching and Shrinking

$$g(x) = f(bx)$$

b is the Horizontal Stretch Factor

$0 < b < 1$ (Stretches Horizontally by a factor of $1/b$)

$b > 1$ (Shrinks Horizontally by a factor of $1/b$)

For Quadratic Functions in Standard Form of $f(x) = a(x - h)^2 + k$

Vertex at (h, k)

Axis of Symmetry at $x = h$

Domain: $x \in R$

a = Vertical Stretch Factor

$a > 0$ Vertex at Minimum (Parabola opens UP)

Range: $y \geq k$ (Minimum)

$a < 0$ Vertex at Maximum (Parabola opens DOWN)

Range: $y \leq k$ (Maximum)

$|a| > 1$ Stretched out Vertically

$|a| < 1$ Shrunk in Vertically

h = Horizontal Translation (Note the standard form has $x - h$ in the bracket!)

$h > 0$ Translated Right

$h < 0$ Translated Left

k = Vertical Translation

$k > 0$ Translated Up

$k < 0$ Translated Down

For Quadratic Functions in General Form: $f(x) = ax^2 + bx + c$

y -intercept at $(0, c)$ by letting $x = 0$ (Note: Complete the Square to change to *Standard Form*)

x -intercepts at $\left(\frac{-b \pm \sqrt{b^2 - 4ac}}{2a}, 0 \right)$ if $b^2 - 4ac \geq 0$. No x -intercepts when $b^2 - 4ac < 0$

Vertex locates at $x = -\frac{b}{2a}$ $y = f\left(-\frac{b}{2a}\right)$ Minimum when $a > 0$; Maximum when $a < 0$

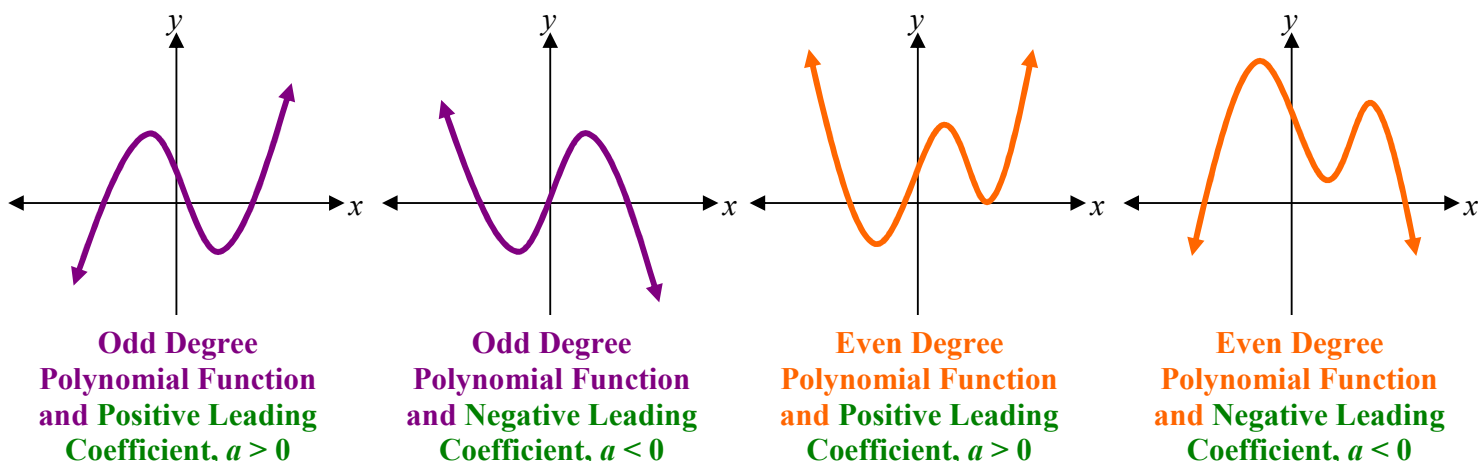
$f(x)$ = One-to-One Function
(x, y)

$f^{-1}(x)$ = Inverse Function
(y, x)

Domain of $f(x) \rightarrow$ Range of $f^{-1}(x)$

Range of $f(x) \rightarrow$ Domain of $f^{-1}(x)$

Note: $f^{-1}(x) \neq \frac{1}{f(x)}$ (Inverse is DIFFERENT than Reciprocal)

End Behaviours and Leading Terms**Odd Degree Polynomial Functions**

When $a > 0$, Left is Downward ($y \rightarrow -\infty$ as $x \rightarrow -\infty$) and Right is Upward ($y \rightarrow \infty$ as $x \rightarrow \infty$).

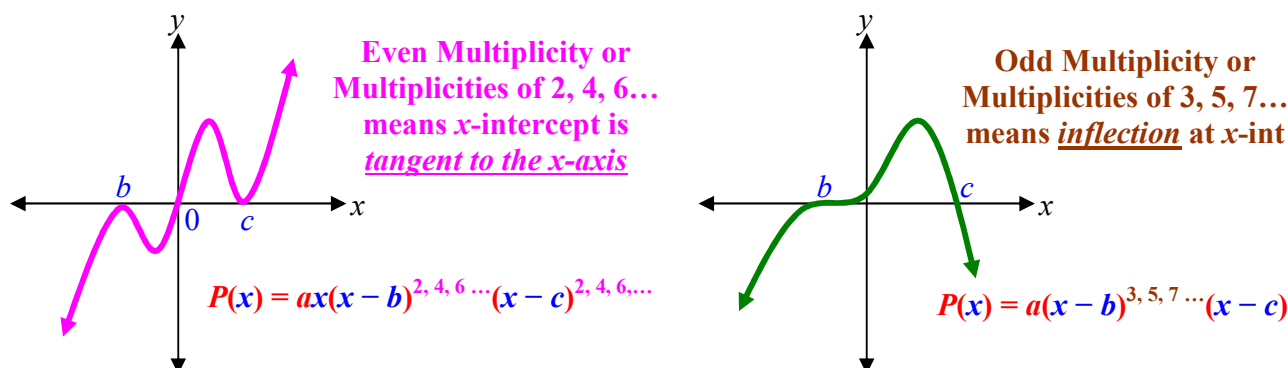
When $a < 0$, Left is Upward ($y \rightarrow \infty$ as $x \rightarrow -\infty$) and Right is Downward ($y \rightarrow -\infty$ as $x \rightarrow \infty$).

Even Degree Polynomial Functions

When $a > 0$, Left is Upward ($y \rightarrow \infty$ as $x \rightarrow -\infty$) and Right is Upward ($y \rightarrow \infty$ as $x \rightarrow \infty$).

When $a < 0$, Left is Downward ($y \rightarrow -\infty$ as $x \rightarrow -\infty$) and Right is Downward ($y \rightarrow -\infty$ as $x \rightarrow \infty$).

Multiplicity: - when a factored polynomial expression has exponents on the factor that is greater than 1.



Polynomial Function **Divisor Function**

In general, for $P(x) \div D(x)$, we can write

$$\frac{P(x)}{D(x)} = Q(x) + \frac{R}{D(x)} \quad \text{or} \quad P(x) = D(x)Q(x) + R$$

Restriction: $D(x) \neq 0$

Quotient Function **Remainder**

If $R = 0$ when $\frac{P(x)}{(x-b)}$, then $(x-b)$ is a factor of $P(x)$ and $P(b) = 0$.

$$P(x) = D(x) \times Q(x)$$

$P(x)$ = Original Polynomial $D(x)$ = Divisor (Factor) $Q(x)$ = Quotient

If $R \neq 0$ when $\frac{P(x)}{(x-b)}$, then $(x-b)$ is NOT a factor of $P(x)$.

$$P(x) = D(x) \times Q(x) + R(x)$$

The Remainder Theorem:

To find the remainder of $\frac{P(x)}{x-b}$: Substitute b from the Divisor, $(x-b)$, into the Polynomial, $P(x)$.

In general, when $\frac{P(x)}{x-b}$, $P(b)$ = Remainder.

To find the remainder of $\frac{P(x)}{ax-b}$: Substitute $\left(\frac{b}{a}\right)$ from the Divisor, $(ax-b)$, into the Polynomial, $P(x)$.

In general, when $\frac{P(x)}{ax-b}$, $P\left(\frac{b}{a}\right)$ = Remainder.

The Factor Theorem:

1. If $\frac{P(x)}{x-b}$ gives a Remainder of 0, then $(x-b)$ is the Factor of $P(x)$.

OR

If $P(b) = 0$, then $(x-b)$ is the Factor of $P(x)$.

2. If $\frac{P(x)}{ax-b}$ gives a Remainder of 0, then $(ax-b)$ is the Factor of $P(x)$.

OR

If $P\left(\frac{b}{a}\right) = 0$, then $(ax-b)$ is the Factor of $P(x)$.

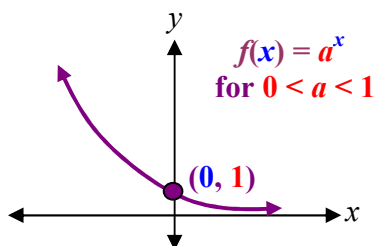
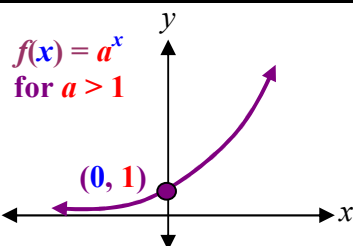
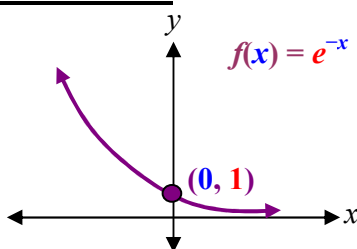
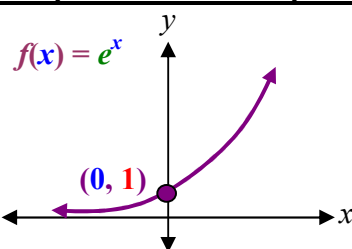
Rational Roots Theorem:

For a polynomial $P(x)$, a List of POTENTIAL Rational Roots can be generated by Dividing ALL the Factors of its Constant Term by ALL the Factors of its Leading Coefficient.

$$\text{Potential Rational Zeros of } P(x) = \frac{\text{ALL Factors of the Constant Term}}{\text{ALL Factors of the Leading Coefficient}}$$

The Zero Theorem

There are n number of solutions (complex, real or both) for any n^{th} degree polynomial function accounting that that a zero with multiplicity of k is counted k times.

Graphs of Exponential Functions**Graphs of Natural Exponential Functions**

$$y = a^x \longleftrightarrow x = \log_a y$$

Simple Properties of Logarithms

$$\log_a 1 = 0$$

because $a^0 = 1$

$$\log_a a = 1$$

because $a^1 = a$

$$a^{\log_a x} = x$$

because **exponent** and **logarithm** are inverse of one another

$$\log_a a^x = x$$

because **logarithm** and **exponent** are inverse of one another

Common and Natural Logarithm

Common Logarithm: $\log x = y \longleftrightarrow 10^y = x$

Natural Logarithm: $\ln x = y \longleftrightarrow e^y = x$

Exponential Laws

$$(a^m)(a^n) = a^{m+n}$$

$$\frac{a^m}{a^n} = a^{m-n}$$

$$(a^m)^n = a^{m \times n}$$

$$a^0 = 1$$

Logarithmic Laws

$$\log_a x + \log_a y = \log_a (xy)$$

$$\log_a x - \log_a y = \log_a \left(\frac{x}{y} \right)$$

$$\log_a x^y = y \log_a x$$

$$\log_a 1 = 0$$

Common Logarithm Mistakes

$$\log_a(x + y) \neq \log_a x + \log_a y$$

Example: $\log(2 + 8) \neq \log 2 + \log 8$
 $1 \neq 0.3010 + 0.9031$

$$\log_a\left(\frac{x}{y}\right) \neq \frac{\log_a x}{\log_a y}$$

Example: $\log\left(\frac{1}{10}\right) \neq \frac{\log 1}{\log 10}$
 $-1 \neq \frac{0}{1}$

$$\log_a(x - y) \neq \log_a x - \log_a y$$

Example: $\log(120 - 20) \neq \log 120 + \log 20$
 $2 \neq 2.0792 + 1.3010$

$$(\log_a x)^y \neq y \log_a x$$

Example: $(\log 100)^3 \neq 3 \log 100$
 $2^3 \neq 3(2)$

$$a^x = y \quad x = \frac{\log y}{\log a}$$

$$A = P\left(1 + \frac{r}{n}\right)^{nt}$$

A = Final Amount after t years
 r = Interest Rate per year

P = Principal
 n = Number of Terms per year

$$A(t) = A_0\left(1 + \frac{r}{n}\right)^{nt} \xrightarrow{n \rightarrow \infty} A(t) = A_0 e^{rt}$$

$A(t)$ = Final Amount after t years

A_0 = Initial Amount

r = Rate of Increase (+ r) / Decrease ($-r$) per year

$$A(t) = A_0 e^{rt}$$

$A(t)$ = Final Amount after t years

A_0 = Initial Amount

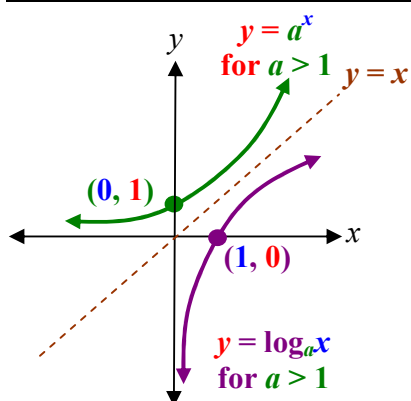
r = Rate of Increase (+ r) / Decrease ($-r$) per year

$$N(t) = N_0 e^{rt}$$

$N(t)$ = Final Population after t years, hours, minutes, or seconds

N_0 = Initial Population

r = Rate of Increase per year, hour, minute, or second

Graphs of Exponential and Logarithmic Functions**Exponential Function**

y -int = 1 No x -intercept

Domain $x \in \mathbb{R}$; Range $y > 0$

Logarithmic Function

x -int = 1 No y -intercept

Domain $x > 0$; Range $y \in \mathbb{R}$

To obtain equation for the inverse of an exponential function, we start with

$$y = a^x$$

$$x = a^y \quad (\text{switch } x \text{ and } y \text{ for inverse})$$

$$y = \log_a x \quad (\text{rearrange to solve for } y)$$

$$\pi \text{ rad} = 180^\circ \quad \text{OR} \quad \frac{\pi}{180} \text{ rad} = 1^\circ$$

$$y = a \sin k(x + b) + c$$

$$y = a \cos k(x + b) + c$$

$|a|$ = Amplitude c = Vertical Displacement (how far away from the x -axis)

b = Horizontal Displacement (Phase Shift) $b > 0$ (shifted left) $b < 0$ (shifted right)

k = number of complete cycles in 2π

$$\text{Period} = \frac{2\pi}{k} = \frac{360^\circ}{k}$$

Range = Minimum $\leq y \leq$ Maximum

$$y = a \sin [\omega(t + b)] + c$$

$$y = a \cos [\omega(t + b)] + c$$

$|a|$ = Amplitude c = Vertical Displacement (distance between *mid-line* and t -axis)

b = Horizontal Displacement (Phase Shift) $b > 0$ (shifted left) $b < 0$ (shifted right)

ω = number of complete cycles in 2π

$$\text{Period} = \frac{2\pi}{\omega}$$

$$\text{Frequency} = \frac{\omega}{2\pi}$$

Range = Minimum $\leq y \leq$ Maximum

Note: $\sin^{-1}(x) \neq \frac{1}{\sin(x)}$ $\sin^{-1}(x) \neq (\sin x)^{-1}$ $(\sin x)^{-1} = \frac{1}{\sin(x)} = \csc x$

$$y = \sin^{-1} x$$

Domain: $-1 \leq x \leq 1$ Range: $-\frac{\pi}{2} \leq y \leq \frac{\pi}{2}$

$$y = \cos^{-1} x$$

Domain: $-1 \leq x \leq 1$ Range: $0 \leq x \leq \pi$

$$\sin(\sin^{-1} x) = x \quad \text{for } -1 \leq x \leq 1$$

$$\sin^{-1}(\sin x) = x \quad \text{for } -\frac{\pi}{2} \leq x \leq \frac{\pi}{2}$$

$$\cos(\cos^{-1} x) = x \quad \text{for } -1 \leq x \leq 1$$

$$\cos^{-1}(\cos x) = x \quad \text{for } 0 \leq x \leq \pi$$

$$y = \tan^{-1} x$$

Domain: $x \in R$ Range: $-\frac{\pi}{2} < y < \frac{\pi}{2}$

$$\tan(\tan^{-1} x) = x \quad \text{for } x \in R$$

$$\tan^{-1}(\tan x) = x \quad \text{for } -\frac{\pi}{2} < x < \frac{\pi}{2}$$

Some Basic Trigonometric Definitions and Identities (proven equations)

$$\tan \theta = \frac{\sin \theta}{\cos \theta} \quad \csc \theta = \frac{1}{\sin \theta} \quad \sec \theta = \frac{1}{\cos \theta} \quad \cot \theta = \frac{1}{\tan \theta} = \frac{\cos \theta}{\sin \theta}$$

$$\cos^2 \theta + \sin^2 \theta = 1$$