Honors Algebra 2A

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CHAPTER ONE

Radical and Polynomial Operations

Unit 1

Sep 07 2021 Tue (16:52:35)

Lesson 1: Rational Exponents **Unit 1**

Rational Exponents become $\rightarrow$ Radical Expressions

The **numerator** of the **rational exponent** becomes the **exponent** on the **radicand**. The **denominator** of the **rational exponent** becomes the **index**, or **root**, of the **radical**.

$$t^{\frac{3}{4}} = \sqrt[4]{t^3} \tag{1.1}$$

Radical Expressions become $\rightarrow$ Rational Exponents

The **exponent** on the **radicand** becomes the **numerator** of the **rational exponent**. The **index**, or **root**, of the **radical** becomes the **denominator** of the **rational exponent**.

$$\sqrt[8]{w^5} = w^{\frac{5}{8}} \tag{1.2}$$

Sep 14 2021 Tue (09:54:31)

Lesson 2: Properties of Rational Exponents **Unit 1**

Property 1. (Product Property) When multiplying 2 of the same variable, the exponents are added. **Fractions** must have common **denominators** before being **added** or **subtracted**.

$$a^{\frac{2}{5}} \times a^{\frac{1}{2}} = a^{\frac{4}{10}} \times a^{\frac{5}{10}} \tag{1.3}$$
$$= a^{\frac{9}{10}} \tag{1.4}$$

Property 2. (Quotient Property) When dividing 2 of the same variable, the exponents are **subtracted**. **Fractions** must have been common **denominators** before being **added** or **subtracted**.

$$\frac{b^{\frac{5}{6}}}{b^{\frac{1}{4}}} = \frac{b^{\frac{10}{12}}}{b^{\frac{3}{12}}} \tag{1.5}$$
$$= b^{\frac{10}{12}-\frac{3}{12}} \tag{1.6}$$
$$= b^{\frac{7}{12}} \tag{1.7}$$

Property 3. (Negative Exponents) When simplifying **negative exponents**, take the **reciprocal** of the **expression** and make the **exponent positive**.

$$c^{-3} = \frac{1}{c^3} \tag{1.8}$$
$$\frac{1}{d^{-4}} = \frac{1}{\frac{1}{d^4}} \tag{1.9}$$
$$= 1 \div \frac{1}{d^4} \tag{1.10}$$
$$= d^4 \tag{1.11}$$

Sep 14 2021 Tue (16:54:18)

Lesson 3: Solving Radical Equations **Unit 1**

Definition 1. (Radical Equations) Here is what a **Radical Equation** looks like:

$$T = 2\pi\sqrt{\frac{L}{32}}.$$

Example 1. (Radical Equations 1) Let's solve $\sqrt{(5x+4)} = 7$:

$$\begin{aligned}
\sqrt{(5x+4)} &= 7 && (1.12)\\
&= (\sqrt{5x+4})^2 = (7)^2 && (1.13)\\
&= 5x+4 = 49 && (1.14)\\
&= 5x = 45 && (1.15)\\
&= \frac{5x}{5} = \frac{45}{5} && (1.16)\\
&= x = 9 && (1.17)
\end{aligned}$$

Now, let's check our work:

$$\begin{aligned}
\sqrt{5x+4} &= 7 && (1.18)\\
&= \sqrt{5(9)+4} = 7 && (1.19)\\
&= \sqrt{45+4} = 7 && (1.20)\\
&= \sqrt{49} = 7 && (1.21)\\
&= 7 = 7 && (1.22)
\end{aligned}$$

Let's try another one:

Example 2. (Radical Equations 2) Let's solve $\sqrt{x-3}+4 = 1$:

$$\begin{aligned}
\sqrt{x-3}+4 &= 1 && (1.23)\\
&= \sqrt{x-3} = -3 && (1.24)\\
&= (\sqrt{x-3})^2 = (-3)^2 && (1.25)\\
&= x-3 = 9 && (1.26)\\
&= x = 12 && (1.27)
\end{aligned}$$

Now, let's check our work:

$$\sqrt{x-3}+4=1 \tag{1.28}$$
$$=\sqrt{12-3}+4=1 \tag{1.29}$$
$$=\sqrt{9}+4=1 \tag{1.30}$$
$$=3+4=1 \tag{1.31}$$
$$=7\neq 1 \tag{1.32}$$

We call this situation extraneous solution.

Quick Review of Factoring

Let's look at factoring by group.

First, you want the split the middle term into factors of 15 that combine to equal 2.

Then, just factor by GCF.

$$x^2+2x-15=x^2-3x+5x-15 \tag{1.33}$$
$$=(x^2-3x)+(5x-15) \tag{1.34}$$
$$=x(x-3)+5(x-3) \tag{1.35}$$
$$=(x+5)(x-3) \tag{1.36}$$

Sep 22 2021 Wen (12:43:32)

Lesson 4: Complex Numbers **Unit 1**

Imaginary Numbers

What pairs of identical factors will produce -1 when multiplied?
1×1 doesn't work since it equals positive 1. The same is true of -1×1 which also equals positive 1.
1×-1 equals -1, but these are not identical factors.

Definition 2. (i) To solve this problem, the concept of the imaginary number i was invented. The imaginary number was defined to be:

$$i = \sqrt{-1}$$

With this definition, the square root of negative radicands, in addition to positive radicands, may be simplified.

Example 3. (i) Simplify: $\sqrt{-8}$

$$\sqrt{-1} \times \sqrt{8} = i \times \sqrt{8} \tag{1.37}$$
$$= i\sqrt{8} \tag{1.38}$$
$$= i\sqrt{4} \times \sqrt{2} \tag{1.39}$$
$$= 2i\sqrt{2} \tag{1.40}$$

Squaring Imaginary Numbers

You know that $i = \sqrt{-1}$
Consequently, $i^2 = (\sqrt{-1})^2$
Since squaring a square root will eliminate the square root sign:

$$\sqrt{3^2} = \sqrt{3} \times \sqrt{3} \tag{1.41}$$
$$= \sqrt{9} \tag{1.42}$$
$$= 3 \tag{1.43}$$

Then:

$$i^2 = (\sqrt{-1})^2 \tag{1.44}$$
$$= i^2 \tag{1.45}$$
$$= -1 \tag{1.46}$$

Cubing Imaginary Numbers

What about cubing the imaginary number:

$$
\begin{aligned}
i^3 &= i \times i \times i && (1.47)\\
&= i^3 && (1.48)\\
&= (i \times i) \times i && (1.49)\\
&= i^3 = i^2 \times i && (1.50)\\
&= i^3 = -1 \times i && (1.51)\\
&= i^3 && (1.52)\\
&= -i && (1.53)
\end{aligned}
$$

Biquadrate Imaginary Numbers

i to the fourth power can be found in a similar way:

$$
\begin{aligned}
i^4 &= i \times i \times i \times i && (1.54)\\
&= i^4 && (1.55)\\
&= (i \times i) \times (i \times i) && (1.56)\\
&= i^4 && (1.57)\\
&= -1 \times -1 && (1.58)\\
&= i = 1 && (1.59)
\end{aligned}
$$

Simplifying i

Example 4. (Simplifying i) Simplify: i^{27}

$$
\begin{aligned}
\frac{27}{4} &= 6R \longrightarrow 3 && (1.60)\\
&= i^{R \longrightarrow 3} && (1.61)\\
&= -i && (1.62)
\end{aligned}
$$

Sep 27 2021 Mon (07:10:04)

Lesson 5: Operations on Complex Numbers **Unit 1**

Multiplying Complex Numbers

While the process of adding and subtracting complex numbers was similar to that of polynomials and radicals, multiplying complex numbers is just a little bit different. Think about in which order this multiplication should take place.

Let's see what happens when the imaginary number is factored first.

$$\begin{aligned}\sqrt{-12} \times \sqrt{-5} &= (\sqrt{-1} \times \sqrt{12}) \times (\sqrt{-1} \times \sqrt{5}) && (1.63)\\ &= i\sqrt{12} \times i\sqrt{5} && (1.64)\end{aligned}$$

Once the imaginary numbers have been factored, multiply the imaginary numbers together and multiply the radical factors together.

$$i\sqrt{12} \times i\sqrt{5} = i^2\sqrt{60} \quad (1.65)$$

Now that the two factors have been multiplied, simplify the remaining radicand by either rewriting it as a product of its prime factors or by finding the perfect square factor.

Sep 27 2021 Mon (07:30:12)

Lesson 6: Polynomial Operations **Unit 1**

Adding Polynomials

The most important part of adding or subtracting polynomials is identifying like terms. Like terms are terms containing the exact same "variable part." Exponents for the variables must be exactly the same. Coefficients can, and probably will, be different. Identify the like terms in the following matching exercise.

Example 5. (Adding Polynomials) Add: $(2x^3 + 4x^2 - x + 7) + (3x^2 + 6x + 10)$

1. Again, there is an understood 1 in front of each set of parentheses. As you've seen, distributing this 1 will not change the expression. Therefore, the parentheses may simply be removed.
2. Highlight each pair of terms containing the same variable part. Combine the like terms.

Note: Since there is no other term with a variable part of x^3, the term $2x^3$ stays the same in the final answer.

$$(2x^3 + 4x^2 - x + 7) + (3x^2 + 6x + 10) = 2x^3 + 4x^2 - x + 7 + 3x^2 + 6x + 10 \tag{1.66}$$
$$= 2x^3 + 7x^2 + 5x + 17 \tag{1.67}$$

Subtracting Polynomials

Subtracting polynomials is almost exactly like adding polynomials. The only "difference" is that now a negative one must be distributed.

Example 6. (Subtracting Polynomials) Subtract: $(4x^2 - 5) - (x^3 + 2x^2 + 7)$

The understood 1 in front of the first set of parentheses will not change the binomial within. However, the trinomial in the second set of parentheses will change because a negative one (-1) must be distributed! In effect, it will change the signs of those three terms.

1. Highlight each pair of terms containing the same variable part.
2. Combine the like terms. Since $-x^3$ does not have a like term, it is written down as is. Arrange terms in descending order, from greatest exponent to least.

$$1(4x^2 - 5) - 1(x^3 + 2x^2 + 7) = 4x^2 - 5 - x^3 - 2x^2 - 7 \tag{1.68}$$
$$= 4x^2 - 5 - x^3 - 2x^2 - 7 \tag{1.69}$$
$$= -x^3 + 2x^2 - 12 \tag{1.70}$$

Inverses of Functions

Finding the inverse of an integer means performing an operation that will cancel that number. For example, if you are given the number 7, you could add -7 to it to cancel it.

Let's look at an example:

Example 7. (Inverse of Functions)

$$f(x) = 3x - 5 \tag{1.71}$$
$$= y = 3x - 5 \tag{1.72}$$
$$= x + 5 = 3y \tag{1.73}$$
$$= \frac{x+5}{3} = y \tag{1.74}$$
$$= f^{-1}x = \frac{x+5}{3} \tag{1.75}$$

Operations on Functions

Addition The addition of two functions $f(x)$ and $g(x)$ is represented using the notation $f(x) + g(x)$.

Let $f(x) = 4x - 7$ and $g(x) = 10x - 3$. To add $f(x)$ and $g(x)$, the expressions $4x - 7$ and $10x - 3$ need to be added.

Therefore, this could be written as $f(x) + g(x) - (4x - 7) + (10x - 3)$.

1. Simplify the right side by first distributing any coefficients outside the parentheses. If no number or variable appears before the parentheses, an understood 1 exists.
2. When 1 is distributed to each term within each set of parentheses, the expression remains unchanged.
3. Identify and combine like terms.

$$f(x) + g(x) = (4x - 7) + (10x - 3) \tag{1.76}$$
$$= 1(4x - 7) + 1(10x - 3) \tag{1.77}$$
$$= 4x - 7 + 10x - 3 \tag{1.78}$$
$$= 14x - 10 \tag{1.79}$$
$$= f(x) + g(x) = 14x - 10 \tag{1.80}$$

Subtraction Subtraction of functions is similar to addition. The only difference is that you must be very careful of sign changes!

Let $f(x) = 4x - 7$ and $g(x) = 10x - 3$. To subtract $f(x)$ and $g(x)$, the expressions $4x - 7$ and $10x - 3$ need to be subtracted.

Therefore, this could be written as $f(x) - g(x) = (4x - 7) - (10x - 3)$.

1. Simplify the right side by first distributing any coefficients outside the parentheses. If no number or variable appears before the parentheses, an understood 1 exists.
2. Be very careful not to forget to distribute the -1 to each term in the second set of parentheses. Identify and combine like terms.

$$\begin{aligned} f(x) - g(x) &= (4x-7) - (10x-3) && (1.81)\\ &= 1(4x-7) - 1(10x-3) && (1.82)\\ &= 4x - 7 - 10x - 3 && (1.83)\\ &= -6x - 4 && (1.84)\\ &= f(x) - g(x) = -6x - 4 && (1.85) \end{aligned}$$

Lesson 8: Polynomial Operations **Unit 1**

CHAPTER TWO

Factoring and Quadratics

Unit 2

Oct 11 2021 Mon (10:42:23)

Lesson 1: GCF and SP **Unit 2**

Greatest Common Factor

A greatest common factor of two or more terms is the largest factor that all terms have in common. The greatest common factor of a polynomial should be factored out first before any further factoring is completed.

Example 8. (GCF)

$$3r^6 + 27r^4 + 15r^2 = 3r^2(r^4 + 9r^2 + 5) \tag{2.1}$$
$$\tag{2.2}$$

When you multiply ($\times$) variables, add the exponents:

$$r^2 \times r^4 = rr \times rrrr \tag{2.3}$$
$$= r^6 \tag{2.4}$$

When factoring a **GCF**, subtract the exponents:

$$\text{To factor } r^2 \text{ from } r^6\text{: } r^{6-2} = r^4 \tag{2.5}$$
$$rrrrrr = (rr)(rrrr) = r^2(r^4) \tag{2.6}$$

Difference of Squares Binomials

A difference of **squares binomial** includes a **perfect square** term subtracted by another **perfect square term**.

Example 9. (Difference of Squares Binomials)

$$a^2 - b^2 = (a+b)(a-b) \tag{2.7}$$
$$r^2 - 4 = (r+2)(r-2) \tag{2.8}$$

Perfect Square Trinomials

A **perfect square trinomial** is a polynomial of **three terms** where the **first** and *last* terms are **perfect squares** and the **middle term** is **twice the product of the square roots** of those terms.

Example 10. (Perfect Square Trinomials) Pattern:

$$a^2 + 2ab + b^2 = (a+b)^2 \text{ OR } a^2 - 2ab + b^2 = (a-b)^2 \tag{2.9}$$

Example:

$$r^2 + 12r + 36 = (r+6)^2 \text{ OR } r^2 - 12r + 36 = (r-6)^2 \tag{2.10}$$

Oct 12 2021 Tue (09:23:42)

Lesson 2: Factoring by Grouping **Unit 2**

Factoring by Grouping (Four-Term Polynomials)

Let's just jump into an example:

Example 11. (Factor $30c^3 - 20c^2 - 15c + 10$)

$$\begin{aligned} 30c^2 - 20c^2 - 15c + 10 &= 5(6c^3 - 4c^2 - 3c + 2) && (2.11)\\ &= 5[2c^2(3c-2) - 1(3c-2)] && (2.12)\\ &= 5[2c^2(3c-2) - 1 = (3c-2)] && (2.13)\\ &= 5(3c-2)(2c^2-1) && (2.14) \end{aligned}$$

Oct 12 2021 Tue (10:03:31)

Lesson 3: Sum and Difference of Cubes **Unit 2**

Factoring the Sums and Differences of Cubes

Example 12. (Factor $30c^3 - 20c^2 - 15c + 10$)

$$2z^3 + 250 = 2(z^3 + 125) \tag{2.15}$$
$$= z^3 + 5^3 = (z+5)(z^2 - z \times 5 + 5^2) \tag{2.16}$$
$$= 2(z+5)(z^2 - z \times 5 + 5^2) \tag{2.17}$$
$$\tag{2.18}$$

Oct 18 2021 Mon (10:55:28)

Lesson 4: Graphing Quadratics **Unit 2**

Parts of a Parabola

Parabolas consist of a:

- Vertex
- Axis of Symmetry
- Domain
- Range
- X-Y Intercepts (Sometimes though)

Vertex Here is how you find the vertex of a parabola:

- **Step One: x-Coordinate** Set the expression inside the parentheses equal to 0 and solve for the value of x
- **Step Two: y-Coordinate** Substitute the value of x from step one into the original equation and solve for y

Let's try an example:

Example 13. (Given the equation $f(x) = -2(x-5)^2+8$, find the (x, y) coordinate of the vertex)

$$\begin{aligned} f(x) &= -2(x-5)^2+8 && && (2.19)\\ &= -2([x-5=0])^2+8 && = 5-5=0 && (2.20)\\ &= x=5 && && (2.21) \end{aligned}$$

Now, we're going to solve for the y coordinate

$$\begin{aligned} f(x) &= -2(5)(5-5)^2+8 && (2.22)\\ &= -2(5-5)^2+8 && (2.23)\\ &= -2(0)^2+8 && (2.24)\\ &= -2(0)+8 && (2.25)\\ &= y=8 && (2.26)\\ & && (2.27) \end{aligned}$$

Now, we know that the vertex of the parabola is $(5, 8)$.

Elements of the Parabola

Line of Symmetry The **axis of symmetry** is in the line that divides the parabola into two equal parts where each part is a mirror reflection of the other.

As you can see, the axis of symmetry is along the line $x = -3$.

To find the equation of the axis of symmetry algebraically, set the expression inside the parentheses equal to 0 and solve for x.

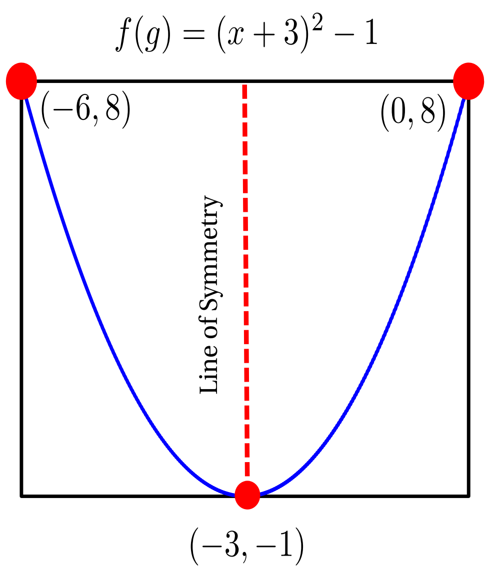

Figure 2.1: Here is an example of what a parabola looks like.

Domain and Range Any number can be substituted for the variable x to produce a unique y value. Therefore, the domain is **"All Real Numbers**.

This parabola opens up since the value is positive $(+)$. Because the parabola opens up, the vertex represents the minimum point on the graph. All of the y values on the parabola must be above the minimum $y - coordinate$ of -1, which makes the range of the parabola this:

$$y \geq -1 \tag{2.28}$$

Intercepts Substituting 0 for x and solving for y will result in the $y-intercept$:

$$\begin{aligned} f(g) &= (x+3)^2 - 1 && (2.29)\\ &= (0+3)^2 - 1 && (2.30)\\ &= (3)^2 - 1 && (2.31)\\ &= 9 - 1 && (2.32)\\ &= 8 && (2.33) \end{aligned}$$

There you go, you have your $y - intercept$. To determine the $x - intercept$, you just plug in the $y - intercept$:

$$\begin{aligned} f(9) &= (9+3)^2 - 1 && (2.34)\\ &= (3)^2 - 1 && (2.35)\\ &= 9 - 1 && (2.36)\\ &= 8 && (2.37) \end{aligned}$$

Different Quadratic Equation forms

While identifying the:

1. Vertex
2. Axis of Symmetry
3. Domain
4. Range

5. Intercepts

is most easily done using the vertex form of a quadratic equation, most of the time, quadratic equations are written in standard form.

Here is the standard form of a quadratic equation:

$$f(x) = ax^2 + bx + c \tag{2.38}$$

Now, we can move that into vertex form, here is an example:

$$\begin{align} f(x) &= -2x(x-5)^2 + 8 \tag{2.39}\\ &= -2x(x-5)(x-5) + 8 \tag{2.40}\\ &= -2x(x^2 - 5x - 5x + 25) + 8 \tag{2.41}\\ &= -2x(x^2 - 10x + 25) + 8 \tag{2.42}\\ &= -2x^2 + 20x - 50 + 8 \tag{2.43}\\ &= -2x^2 + 20x - 42 \tag{2.44}\\ & \tag{2.45} \end{align}$$

But, it's not that easy to identify the vertex and axis, so let's us **Axis of Symmetry** equation, which is:

$$f(x) = -\frac{b}{2a} \tag{2.46}$$

Now, let's try to convert an equation in the standard form to this form:

$$\begin{align} f(x) &= -2x^2 + 20x - 42 \tag{2.47}\\ &= -\frac{20}{2(-2)} \tag{2.48}\\ &= -\frac{20}{-4} \tag{2.49}\\ &= x = 5 \tag{2.50} \end{align}$$

which gives you the axis of symmetry for the $x - axis$. Now, just substitute 5 for x to find the $y - coordinate$:

$$\begin{align} f(x) &= -2x^2 + 20x - 42 \tag{2.51}\\ &= -2(5)^2 + 20(5) - 42 \tag{2.52}\\ &= -2(25) + 100 - 42 \tag{2.53}\\ &= -50 + 100 - 42 \tag{2.54}\\ &= 8 \tag{2.55}\\ &= (5, 8) \tag{2.56} \end{align}$$

Working our way up

Now, I have discussed how you can find key features within the equation, but let's try and make our equation from the graph. Taking the image of the parabola, let's try and see if we can create the equation from the graph:

Let's take the vertex $(-3, -1)$ and a random point, $(0, 8)$. Here is how it works:

$$
\begin{aligned}
f(x) &= a(x-h)^2 + k && (2.57)\\
&= a(x+3)^2 - 1 && (2.58)\\
&= 8 = a(0+3)^2 - 1 && (2.59)\\
8 &= a(0+3)^2 - 1 && (2.60)\\
&= a(3)^2 - 1 && (2.61)\\
&= 9a - 1 && (2.62)\\
&= 9 = 9a && (2.63)\\
&= \frac{9}{9} = \frac{9}{9a} && (2.64)\\
&= 1 && (2.65)
\end{aligned}
$$

Now, let's put it all together:

$$
\begin{aligned}
a &= 1, h = -3, k = -1 && (2.66)\\
f(x) &= a(x-h)^2 + k && (2.67)\\
&= (x+3)^2 - 1 && (2.68)\\
&= (x+3)(x+3) - 1 && (2.69)\\
&= (x^2 + 3x + 3x + 9) - 1 && (2.70)\\
&= (x^2 + 6x + 9) - 1 && (2.71)\\
&= x^2 + 6x + 8 && (2.72)\\
& && (2.73)
\end{aligned}
$$

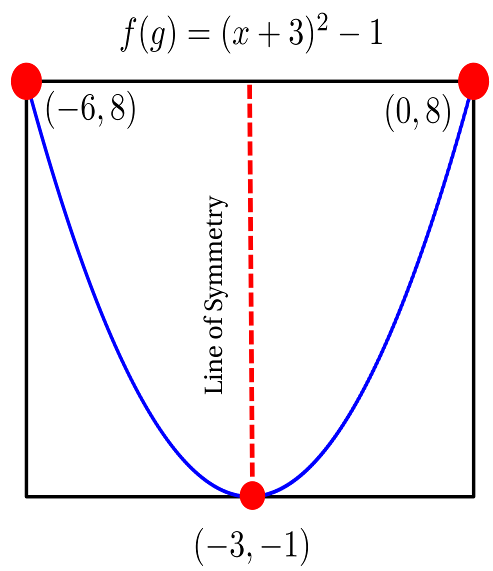

Figure 2.2: Here is an example of what a parabola looks like.

Oct 18 2021 Mon (11:31:19)

Lesson 5: Completing the Square **Unit 2**

Intro to the Discriminant

The formula for the **discriminant** is:

$$b^2 - 4ac \tag{2.74}$$

Let's try a quick example:

Example 14.

$$
\begin{align}
f(x) &= x^2 + 12x + 26 \tag{2.75}\\
&= b^2 - 4ac \tag{2.76}\\
&= (12)^2 - 4(1)(26) \tag{2.77}\\
&= 144 - 104 \tag{2.78}\\
&= 40 \tag{2.79}
\end{align}
$$

So, the discriminant isn't a perfect square. This tells us that there will be two irrational numbers.

Using Vertex Form: Minimum or Maximum

Example 15.

$$f(x) = (x+6)^2 - 10 \tag{2.80}$$

From this equation, we get the vertex of the parabola, which is $(-6, -10)$ But, it raises the question. Is it the vertex minimum, or the vertex maximum?
To solve this, you must look at the value or the leading coefficient to make this determination.
In this case, it would be $a = 1$, which is positive, meaning it goes upward.

Oct 25 2021 Mon (09:11:53)

Lesson 6: Solving Quadratic Equations **Unit 2**

Lesson 7: Solving QEs with Complex Solutions **Unit 2**

Lesson 8: Investigating Quadratics **Unit 2**

CHAPTER THREE

Solving Polynomials

Unit 3

Nov 15 2021 Mon (08:01:30)

Lesson 1: Polynomial Long Division **Unit 3**

Steps to Dividing Polynomials

1. Divide
2. Multiply
3. Subtract
4. Bring down
5. Repeat

Missing terms

When either polynomial does not have all terms according to the descending powers on the variable, it is necessary to add in terms with 0 coefficients.

Example 16. (Dividing Polynomials)

$$\begin{array}{r|l} & x+2 \\ \hline x+1 & x^2+3x+2 \\ & -x^2 - x \\ \hline & 2x+2 \\ & -2x-2 \\ \hline & 0 \end{array}$$

Let me quickly go over everything:

1. First part, I'm multiplying x by x to get x^2.
2. Second part, I'm also multiplying 1 by 2 to get 2 because every time we do something to one, we must do it to all.
3. Third part, I'm moving the answer we get (x^2+2) down. Now, I also distributed a negative $(-)$, because I'm subtracting.

4. Then, I repeat. I move the 2 down next to the result I got, which was $2x$.
5. Then, I multiply x by 2, which gets me $2x$.
6. Since I multiplied x by 2, I have to do the same on the 1. In the end, it brought me to $2x + 1$. But, I wanted to subtract the entire thing from the other answer, so I negated the whole equation.
7. After subtracting, you are left with a 0, meaning there is no remainder.

Remainders

If the quotient has no remainder (a remainder of 0), then the divisor is said to be a factor of the dividend.

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Lesson 2: Polynomial Transformations **Unit 3**

Definition 3. (Theorems) A theorem is a proven statement based on experimentation. I will go over these three **Theorems**:
- Fundamental Theorem of Algebra
- Factor Theorem
- Remainder Theorem

Theorem 1. (Fundamental Theorem of Algebra) The Fundamental Theorem of Algebra generally states that the degree of a polynomial is equivalent to the number of zeros (both real and complex) of a function. By the **Fundamental Theorem of Algebra**, the polynomial function $f(x) = x^2 - 3x - 28$ has two zeros since the degree of the function is two. To determine these zeros, replace the function notation of $f(x)$ with 0 and solve by factoring.
To factor, let's use the **Quadratic Equation**:

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \tag{3.1}$$
$$f(x) = x^2 - 3x - 28 \tag{3.2}$$
$$a = 1, b = -3, c = -28 \tag{3.3}$$
$$x = \frac{-(-3) \pm \sqrt{(-3)^2 - 4 \times 1(-28)}}{2 \times 1} \tag{3.4}$$
$$x = \frac{3 + 11}{2} \text{ AND } x = \frac{3 - 11}{2} \tag{3.5}$$
$$x = 7 \text{ AND } -4 \rightarrow (x+4)(x-7) \tag{3.6}$$

The zero product property tells us that for these factors to result in a product of 0, one or both of them must equal 0.

	0	=	x	+	4
-	4	=		-	4
	x	=		-	4

	0	=	x	-	7
+	7	=		+	7
	x	=		+	7

The **Zeros** of $f(x) = x^2 - 3x - 28$ are: $x_1 = -4, x_2 = 7$.

$f(x) = x^2 - 3x - 28$

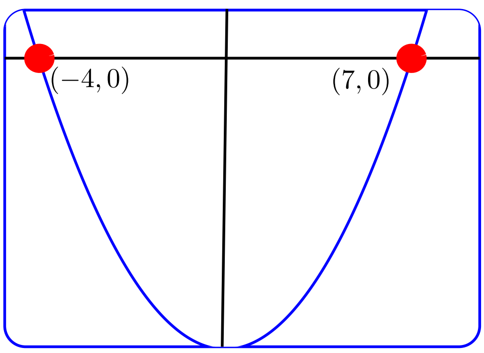

Figure 3.1: $f(x) = x^2 - 3x - 28$ Graphed.

Theorem 2. (Factor Theorem) The **Factor Theorem** states that a first degree binomial is a factor of a polynomial function if the remainder, when the polynomial is divided by the binomial, is 0.
To determine whether $x - 5$ is a factor of the function $f(x) = -4x^3 + 21x^2 - 25$, set up a division problem whereby $-4x^3 + 21x^2 - 25$ is divided by $x - 5$.

$$
\begin{array}{r|l}
 & -4x^2 + x + 5 \\
\hline
x-5 & -4x^3 + 21x^2 \qquad - 25 \\
 & \underline{4x^3 - 20x^2} \\
 & \qquad x^2 \\
 & \qquad \underline{-x^2 + 5x} \\
 & \qquad\qquad 5x - 25 \\
 & \qquad\qquad \underline{-5x + 25} \\
 & \qquad\qquad\qquad 0
\end{array}
$$

When the function $f(x) = -4x^3 + 21x^2 - 25$ is divided by the binomial $x - 5$, the remainder is 0. So, $x - 5$ is a factor of the function $f(x) = -4x^3 + 21x^2 - 25$.

Theorem 3. (Remainder Theorem) The **Remainder Theorem** states that when the opposite of the constant from the binomial divisor is substituted into a function for x, the result is the remainder.
When the polynomial function $f(x) = x^4 + 11x^3 + 26x^2 + 15x - 17$ is divided by $x + 8$ using division, the remainder is the last integer on the bottom row.

$$
\begin{array}{r|l}
 & \qquad x^3 + 3x^2 + 2x - 1 \\
\hline
x+8 & x^4 + 11x^3 + 26x^2 + 15x - 17 \\
 & \underline{-x^4 - 8x^3} \\
 & \qquad 3x^3 + 26x^2 \\
 & \quad \underline{-3x^3 - 24x^2} \\
 & \qquad\qquad 2x^2 + 15x \\
 & \qquad\quad \underline{-2x^2 - 16x} \\
 & \qquad\qquad\qquad -x - 17 \\
 & \qquad\qquad\qquad \underline{x + 8} \\
 & \qquad\qquad\qquad\quad -9
\end{array}
$$

When the opposite constant in the divisor is substituted into the function, the result will be the same as the remainder in the division process.

$$
\begin{aligned}
f(x) &= x^4 + 11x^3 + 26x^2 + 15x - 17 && (3.7)\\
f(-8) &= (-8)^4 + 11(-8)^3 + 26(-8)^2 + 15(-8) - 17 && (3.8)\\
&= 4096 + 11(-512) + 26(64) + 15(-8) - 17 && (3.9)\\
&= 4096 - 5632 + 1664 - 120 - 17 && (3.10)\\
&= -9 \leftarrow \text{Remainder} && (3.11)
\end{aligned}
$$

Using these Theorems

Given a polynomial function $f(x)$ and a number a, if $(x - a)$ is a factor of $f(x)$, then a is a 0 of the polynomial.

The binomial $(x - a)$ can be proved as a factor of $f(x)$ by:

- Using **Long Division** with $(x - a)$ as the divisor.
- Using factoring methods when appropriate (grouping, completing the square, ...).

If a is a 0 of the polynomial function $f(x)$, then:

- The graph of $f(x)$ crosses the x axis at $(a, 0)$.
- Substituting a into $(x - a)$ will equal 0
- Substituting a into $f(x)$ will result in $f(a) = 0$.

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Lesson 3: Polynomial Transformations **Unit 3**

Definition 4. (Polynomial Function) A **Polynomial Function** is a function composed of 1 or more terms, at 1 of which contains a variable. A polynomial function whose degree is 3 is called a **Cubic Function**. A **Polynomial Function** whose degree is 4 is called a **Quartic Function**. Simple polynomial functions may be graphed by hand by substituting x-coordinates and solving for the corresponding y-coordinates.
The real number solutions of a polynomial equation can be found by identifying the x-intercepts from the graph of the function. Graph the function, and find the point(s) of intersection between the graph and the x-axis.

Proof. Fusce mauris. Vestibulum luctus nibh at lectus. Sed bibendum, nulla a faucibus semper, leo velit ultricies tellus, ac venenatis arcu wisi vel nisl. Vestibulum diam. Aliquam pellentesque, augue quis sagittis posuere, turpis lacus congue quam, in hendrerit risus eros eget felis. Maecenas eget erat in sapien mattis porttitor. Vestibulum porttitor. Nulla facilisi. Sed a turpis eu lacus commodo facilisis. Morbi fringilla, wisi in dignissim interdum, justo lectus sagittis dui, et vehicula libero dui cursus dui. Mauris tempor ligula sed lacus. Duis cursus enim ut augue. Cras ac magna. Cras nulla. Nulla egestas. Curabitur a leo. Quisque egestas wisi eget nunc. Nam feugiat lacus vel est. Curabitur consectetuer. Q.E.D.

Odd vs Even vs Neither Polynomial Functions

Result	Function Type	Symmetry
$f(-x) = f(x)$	Even function	y-axis
$f(-x) = -f(x)$	Odd function	Origin
$f(-x) \neq f(x)$ or $f(-x)$	Neither even and odd	

Table 3.1: Odd vs Even vs Neither Polynomial Functions

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Lesson 4: Solving and Graphing PF **Unit 3**

A function written in factored form gives us the roots of the equation. The roots and the end behavior rules below can be used to create a graph for the function.

Some **Polynomial Functions** are far more complicated and should be graphed using graphing technology such as graphing software or a graphing calculator. The real number solutions of a **Polynomial Equation** can be found by identifying the **x-intercepts** from the graph of the function. Graph the function, using graphing technology and find the point(s) of intersection between the graph and the x-axis.

$0.8x^4 \cdot 8x^3 + 2x^2 - 7x - 1$

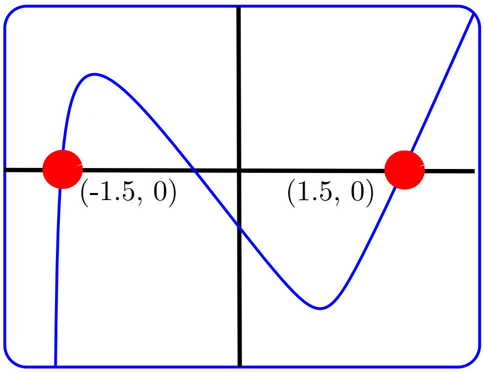

Figure 3.2: $0.8x^4 \times 8x^3 + 2x^2 - 7x - 1$ Graphed.

Definition 5. (Average Rate of Change)

$$m = \frac{y_2 - y_1}{x_2 - x_1} \tag{3.12}$$

Increasing versus Decreasing

- If the **Average Rate of Change** is **Positive**, then the function is considered to be **Increasing**.
- If the **Average Rate of Change** is **Negative**, then the function is considered to be **Decreasing**.

Minimums and Maximums

-
- **Minimums** are located on the **Lowest Point of a Graph**. A **Local Minimum** is the bottom of a **Valley** or **Turning Point** where the graph goes from **Decreasing** to **Increasing**.
- **Maximums** are located on the **Highest Point of a Graph**. A **Local Maximum** is the top of a **Hill** or **Turning Point** where the graph goes from **Increasing** to **Decreasing**.

Example 17. (Example 1) Determine the zeros of the function $f(x) = (x-2)(x-5)(x+1)$, and describe the end behavior of the graph. The zeros of the function represent the values of x that make the function equal to 0. Replace the function notation with 0 to determine the zeros of the function.

$$f(x) = (x-2)(x-5)(x+1) \tag{3.13}$$
$$0 = (x-2)(x-5)(x+1) \tag{3.14}$$
$$0 = x^3 - 6 \times x^2 + 3x + 10 \tag{3.15}$$
$$0 = x^3 - 6 \times x^2 + 3x + 10 \tag{3.16}$$
$$x = 0, x = 5, x = -1 \tag{3.17}$$
$$\tag{3.18}$$

So, from this, we know that the zeros function from this function are:

$0, 5, -1$, which means that's where they will intersect the $x - axis$. The function has an **Odd Degree**, so the ends will travel in the opposite directions. The leading coefficient is positive, so the left side continues down and the right side continues up.

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Lesson 5: Polynomial Identities and Proofs **Unit 3**

Definition 6. (Algebraic Proofs) • **Polynomial Identities** can be proven to be true by simplifying the identity through application of **Algebraic Theorems** and **Principles**.

- Start with the side of the identity that can be simplified the easiest.
- Sometimes, following a *"clue"* will lead to a dead-end in your **Proof**. Do not give up. Just follow a different *"clue"*. The more practice you have with proofs, the more you will be able to predict the dead-ends.

Example 18.

Application to Numerical Relationships

Polynomial identities apply to more than just polynomials. Replacing the variables with numbers can help prove numerical relationships as well.

CHAPTER FOUR

Rational Equations

Unit 4

Jan 13 2022 Thu (14:15:27)

Lesson 2: Simplifying Rational Expressions **Unit 4**

Definition 7. (Rational Expression) **Rational Expressions** are just expressions with a polynomial in the numerator and the denominator. Here are some examples:

$$\frac{5x+35}{5x} \tag{4.1}$$

$$\frac{x^2-6x-27}{x-9}. \tag{4.2}$$