Pre-calculus 2

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Contents

Lecture 1: Sets and Numbers

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> **Definition 1** (Set). A set is a collection of objects specified in a manner that enables one to determine if a given object is or isn't in the set.

Problem. Which of the following represent a set?

1. The students registered for MTH 112 at PCC this quarter.
2. The good students registered for MTH 112 at PCC this quarter.

Solution. The following do represent a set:

1. This represents a set since it's well defined. We all know what it means to be registered for a class.
2. This does not represent a set since it's not well defined. There are many different interpretations of what it means to be a good student (get an $\mathbb{A}$ or pass the class or attend class or avoid falling asleep in class or don't cause trouble in class)

Notation. Roster Notation involves listing the elements in a set within curly brackets: "$\{\}$ "

> **Definition 2** (Element). An object in a set is called an **element** of the set. (symbol: "$\in$")

> **Example.** 5 is an element of the set $\{4, 5, 6, 7, 8, 9\}$. We can express this symbolically:
>
> $$5 \in \{4, 5, 6, 7, 8, 9\}$$
>
> .

Definition 3 (Subset). A set S of a set T, denoted $S \subseteq T$, if all elements of S are also elements of T.

If S and T are sets and $S = T$, then $S \subseteq T$. Sometimes it's useful to consider a subset S of a set T that isn't equal to T. In such case, we write $S \subset T$ and say that S is a proper subset of T.

Example. $\{4, 7, 8\}$ is a subset of the set $\{4, 5, 6, 7, 8, 9\}$.

We can express this fact symbolically by $\{4, 7, 8\} \subseteq \{4, 5, 6, 7, 8, 9\}$

Since these two sets aren't equal, $\{4, 7, 8\}$ is a proper subset of $\{4, 5, 6, 7, 8, 9\}$, so can write:

$$\{4, 7, 8\} \subset \{4, 5, 6, 7, 8, 9\}$$

.

Definition 4 (Empty Set). The empty set, denoted $\emptyset$, is the set with no elements

$$\emptyset = \{\}$$

.

Definition 5 (Union). The union of two sets A and B, denoted $A \cup B$, is the set containing all of the elements in either A or B (or both A and B).

Example. Consider the sets $\{4, 7, 8\}$, $\{0, 2, 4, 6, 8\}$, and $\{1, 3, 5, 7\}$. Then ...

a. $\{4, 7, 8\} \cup \{1, 3, 5, 7\} = \{1, 3, 4, 5, 6, 8\}$

b. $\{4, 7, 8\} \cup \{0, 2, 4, 6, 8\} = \{0, 2, 4, 6, 7, 8\}$

c. $\{0, 2, 4, 6, 8\} \cup \{1, 3, 5, 7\} = \{0, 1, 2, 3, 4, 5, 6, 7, 8\}$

Definition 6 (Intersection). The intersection of two sets A and B, denoted $A \cap B$, is the set containing all the elements in both A and B.

Example. Consider the sets $\{4, 7, 8\}$, $\{0, 2, 4, 6, 8\}$, and $\{1, 3, 5, 7\}$. Then ...

a. $\{4, 7, 8\} \cap \{0, 2, 4, 6, 8\} = \{4, 8\}$

b. $\{4, 7, 8\} \cap \{1, 3, 5, 7\} = \{7\}$

c. $\{0, 2, 4, 6, 8\} \cap \{1, 3, 5, 7\} = \emptyset$

Example. All the whole numbers form a set. This set is called the integers, and is represented by the symbol $\mathbb{Z}$. We can express the set of integers in roster notation:

$$\mathbb{Z} = \{\dots, -3, -2, -1, 0, 1, 2, 3, \dots\}$$

.

Now, we can use set builder notation to create a set with a range of numbers.

Notation. Set Builder Notation.
"All the whole numbers between 3 and 10" $= \{x | x \in \mathbb{Z} \text{ and } 3 < x < 10\}$

Definition 7 (Important Sets of Numbers)**.** The set of natural numbers:

$$\mathbb{N} = \{1, 2, 3, 4, 5, \dots\}$$

.

The set of integers:

$$\mathbb{Z} = \{\dots, -3, -2, -1, 0, 1, 2, 3, \dots\}$$

.

The set of rational numbers:

$$\mathbb{Q} = \left\{ x | x = \frac{p}{q} \text{ and } p, q \in \mathbb{Z} \text{ and } q \neq 0 \right\}$$

.

The set of real numbers: $\mathbb{R}$

(All the numbers on the number line)

.

The set of complex numbers:

$$\mathbb{C} = \left\{ x | x = a + bi \text{ and } a, b \in \mathbb{R} \text{ and } i = \sqrt{-1} \right\}$$

.

Note. $\mathbb{N} \subset \mathbb{Z} \subset \mathbb{Q} \subset \mathbb{R} \subset \mathbb{C}$, the set of natural numbers ($\mathbb{N}$) is a subset of the set of integers ($\mathbb{Z}$) which is a subset of the set of rational numbers ($\mathbb{Q}$) which is a subset of the set of real numbers ($\mathbb{R}$) which is a subset of the set of complex numbers ($\mathbb{C}$).

Notation. Since we use the real numbers so often, we have special notation for subsets of the real numbers. **Interval Notation**. Interval Notation involves square or round brackets.

Example. Quick demo of Interval Notation

a. $\{x|x \in \mathbb{R} \text{ and } -2 \leq x \leq 3\} = [-2, 3]$

b. $\{x|x \in \mathbb{R} \text{ and } -2 < x < 3\} = (-2, 3)$

c. $\{x|x \in \mathbb{R} \text{ and } -2 < x \leq 3\} = (-2, 3]$

d. $\{x|x \in \mathbb{R} \text{ and } -2 \leq x < 3\} = [-2, 3)$

When the interval has no upper or lower bound, we use the infinity symbol (∞ or $-\infty$)

a. $\{x|x \in \mathbb{R} \text{ and } x \leq 4\} = (-\infty, 4]$

b. $\{x|x \in \mathbb{R} \text{ and } x \geq 4\} = [4, -\infty)$

Problem. Simplify the following expressions:

a. $(-4, \infty) \cup [-8, 3]$

b. $(-4, \infty) \cup (-\infty, 2]$

b. $(-4, \infty) \cap (-\infty, 2]$

a. $(-4, \infty) \cap [-10, -5]$

Solution. .

a. $(-4, \infty) \cup [-8, 3] = [-8, \infty)$

b. $(-4, \infty) \cup (-\infty, 2] = (-\infty, \infty) = \mathbb{R}$

b. $(-4, \infty) \cap (-\infty, 2] = (-4, 2]$

a. $(-4, \infty) \cap [-10, -5] = \emptyset$