Baker Charters School

Honors Algebra 2A

Baker Web Academy School Notes

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Sep. 7 2021

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CHAPTER ONE

Radical and Polynomial Operations

Unit 1

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Lesson 1: Rational Exponents **Unit 1**

Rational Exponents to Radical Expressions

Here's the basic expression:

$$x^{\frac{z}{y}} = \sqrt[y]{x^a}$$

.

Example. Rewrite $(6x)^{\frac{4}{5}}$ as a radical expression. Now, let's just use that expression from above to solve this:

$$(6x)^{\frac{4}{5}} = 6\sqrt[5]{x^4}$$

.

Radical Expressions to Rational Exponents

Now, let's do the reverse. Like before, here's the basic expression:

$$\sqrt[y]{x^z} = x^{\frac{z}{y}}$$

.

Example. Let's rewrite $\sqrt[8]{w^5}$ as a rational expression:

$$\sqrt[8]{x^5} = x^{\frac{5}{8}}$$

.

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Lesson 2: Properties of Rational Exponents **Unit 1**

Dividing Rational Exponents

You already know that when like variables are multiplied, their exponents are added. But what happens when you divide them? Look at this property 1

Property 1. (Quotient of a Power Property) To divide powers of the same base, subtract the exponents:

Example.

$$\frac{a^5}{a^3} = \frac{\cancel{a} \times \cancel{a} \times \cancel{a} \times a \times a}{\cancel{a} \times \cancel{a} \times \cancel{a}} = a^2 \\ = a^{5-3} . \tag{1.1}$$

Review 1. (Finding a Common Denominator) Look at the factors: $\frac{1}{2}$ and $\frac{1}{4}$.
First, you start by listing the multiples of each denominator:

$$2 : 2, \boxed{4}, 6, 8, 10, 12, \ldots$$

.

$$4 : \boxed{4}, 8, 12, 16, 20, 24 \ldots$$

.

So, we just found our common denominator: 4. Since the first fraction: $\frac{1}{4}$, already has a denominator of 4, we can just leave it alone. But, the second fraction: $\frac{1}{2}$, needs to have a denominator of 4. So, we just do: $4 \div 2 = 2$. So, we multiply the second fraction by $\frac{2}{2}$.

$$\frac{1}{2} \times \frac{2}{4} = \frac{2}{4}$$

.

Multiplying the numerator and denominator by the same factor is the same as multiplying the entire fraction by 1 since $\frac{2}{2} = 1$. By multiplying $\frac{1}{2}$ by $\frac{2}{2}$, the value of the fraction hasn't changed. In fact, if you simplify the new fraction we got, which was $\frac{2}{4}$, it goes back to the original fraction: $\frac{1}{2}$. But now, it just has a common denominator of 4, which is what we need to add the exponents.

Raising a Power to a Power

Multiplication of powers with the same base involves the addition of exponents. Division of powers with the same base involves the subtraction of exponents. What operation is performed on the exponents of 2 and 3 to result in the exponent of 6? Take a look at this property property **??**.

Property 2. (Power of a Power Property) To raise a power to a power, multiply the exponents:

$$(a^m)^n = a^{m \times n}$$

.

Example.

$$\begin{aligned} \left(c^{\frac{1}{2}}\right)^{\frac{1}{4}} &= c^{\frac{1}{2} \times \frac{1}{4}} \\ &= c^{\frac{1}{8}} \\ &= \sqrt[8]{c} \end{aligned} \quad . \tag{1.2}$$

Negative Rational Exponents

Let's take a look at the pattern that forms as you look at decreasing powers of 4:

$$\begin{aligned} 4^4 &= 4 \times 4 \times 4 \times 4 = 256 \\ 4^3 &= 4 \times 4 \times 4 = 64 \\ 4^2 &= 4 \times 4 = 16 \\ 4^1 &= 4 = 4 \\ 4^0 &= 1 \\ 4^{-1} &= \frac{1}{4^1} = \frac{1}{4} \\ 4^{-2} &= \frac{1}{4^2} = \frac{1}{16} \\ 4^{-3} &= \frac{1}{4^3} = \frac{1}{64} \\ 4^{-4} &= \frac{1}{4^4} = \frac{1}{256} \end{aligned} \quad . \tag{1.3}$$

Recall that the inverse of a positive is a negative, and vise versa. Also, the inverse of multiplication is division and the inverse of addition is subtraction. When the exponent is positive, it tells you to multiply the base the number of times, which is indicated by the exponent.

$$4^3 = 4 \times 4 \times 4 = 64$$

.

Inversely, a negative exponent tells you to divide the base by the number of times indicated by the exponent.

$$4^{-3} = 4 \div 4 \div 4 = 0.015625$$

.

or

$$4^{-3} = \frac{1}{4^3} = \frac{1}{64} = 0.015625$$

.

Example. Simplify the following expression:

$$\begin{aligned} 9^{-\frac{1}{2}} &= \frac{1}{9^{\frac{1}{1}}} \\ &= \frac{1}{\sqrt{9}}. \\ &= \frac{1}{3} \end{aligned} \tag{1.4}$$

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Lesson 3: Solving Radical Equations **Unit 1**

Solving Radical Equations

Let's go over how to solve this radical equation: $\sqrt{(5x+4)} = 7$:

1. Isolate the Radical: When solving equations, the goal is always to isolate the variable to find its value. The goal is the same with radical equations. So, we must make the radical isolated first. But in this case, we already have the radical isolated.

2. Square both Sides: Do the opposite operation of taking the square root and square both sides of the equation.

$$\begin{aligned} \sqrt{(5x+4)} &= 7 \\ \sqrt{(5x+4)}^2 &= 7^2. \\ 5x+4 &= 49 \end{aligned} \tag{1.5}$$

   Once the radical is gone, we just simply solve for the variable.

3. Solve for the Variable: To solve a two-step equation, remember that you must use the reverse order of operations.
   (SADMEP: **S**ubtraction, **A**ddition, **D**ivision, **M**ultiplication, **E**xponents, **P**arentheses)

$$\begin{aligned} 5x+4 &= 49 \\ 5x &= 45 \\ \frac{5x}{5} &= \frac{45}{5}. \\ \boxed{x=9} \end{aligned} \tag{1.6}$$

4. Check your Work: When solving radical equations, you must always check your work. Even if you do all of the work perfectly, you may not find the correct solution. So, let's plug the value of 9 into the original equation to see if it works.

$$\begin{aligned} \sqrt{5x+4} &= 7 \\ \sqrt{5(9)+4} &= 7 \\ \sqrt{45+4} &= 7. \\ \sqrt{49} &= 7 \\ \boxed{7=7} \end{aligned} \tag{1.7}$$

Let's take a look at a radical equation that doesn't have any solutions:

Example.

$$\begin{aligned} \sqrt{x-3}+4 &= 1 \\ \sqrt{x-3} &= -3 \\ \sqrt{x-3}^2 &= -3^2. \\ x-3 &= 9 \\ \boxed{x=12} \end{aligned} \tag{1.8}$$

Now, let's check our work:

$$\begin{aligned} \sqrt{x-3}+4 &= 1 \\ \sqrt{12-3}+4 &= 1 \\ \sqrt{9}+4 &= 1. \\ 3+4 &= 1 \\ 7 &\neq 1 \end{aligned} \tag{1.9}$$

This statement is false. This is known as a **extraneous solution**. (1)

Definition 1. (Extraneous Solution) An **Extraneous Solution** is a solution to an equation that doesn't fit the requirements of the original equation.

Solving Radical Equations with Variables Outside the Radical

Before we start to solve the equation below (1.10), take a look at the following property (3)

Property 3. (Zero Product Property) If $a \times b = 0$, then $a = 0$ and/or $b = 0$
Let's say we are given the following polynomial: $(x+8)(x-4) = 0$. Here's how we would solve it. First, we take each item within the parentheses and set that equal to 0. For example:

$$(x+8) = 0 \quad (x-4) = 0$$

.
Now, let's find the zeros of the polynomial.

$$x = -8 \quad x = 4$$

.

Let's solve: $\sqrt{x+9} - 7 = x$:

$$\begin{aligned}
\sqrt{x+9}-7&=x\\
\sqrt{x+9}&=x+7 \quad \text{Isolate the variable}\\
(\sqrt{x+9})^2&=(x+7)^2 \quad \text{Square both sides}\\
x+9&=(x+7)(x+7)\\
x+9&=x^2+14x+49 \quad \text{Solve for the variable}\\
9&=x^2+13x+49\\
0&=x^2+13x+40 \quad \text{Factor}\\
0&=(x+5)(x+8) \quad \text{Apply the zero product property (3)}.\\
x+5=0 &\quad \text{AND} \quad x+8=0
\end{aligned}$$

$\boxed{x=-5,-8}$ Check your Work

$$\begin{aligned}
\sqrt{x+9}-7&=x \quad \sqrt{x+9}-7=x\\
\sqrt{-5+9}-7&=-5 \quad \sqrt{-8+9}-7=-8\\
\sqrt{4}-7&=-5 \quad \sqrt{1}-7=-8\\
2-7&=-5 \quad 1-7=-8\\
-5&=-5 \quad -6\neq-8
\end{aligned} \tag{1.10}$$

$x=-5$ is the only solution and $x=8$ is extraneous.

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Lesson 4: Complex Numbers **Unit 1**

Definition 2. (Imaginary Numbers) Here's a quick question. Solve this equation: $a \times b = -1$. They must not be identical factors, meaning it cannot be $1 \times -1 = -1$.
To solve this problem, we came up with the imaginary number i. It's defined to:

$$i = \sqrt{-1}$$

.
With this definition, the square root of a negative radicand, in addition to positive radicands can be simplified.

History of the Imaginary Numbers

Thought history, there have been times when a mathematical construct was invented before its purpose was discovered. For example, early man knew the concept of 1 before knowing 0. Numbers, at that time, were used to count sheep and other animals as a source for food. There was no need to define the concept of 0. But, when fractions were defined, they were thought of useless. What purpose could there be to have a number between 0 and 1. But today, fractions become part of our daily life. Imaginary numbers have the same history. It's use will come out eventually, we just don't know when.

Solving an equation such as: $x^2 - 9 = 0$ should be already familiar. The goal is to isolate the variable on one side by first adding 9 to both sides.

$$\begin{aligned} x^2 &= 9 \\ \sqrt{x^2} &= \sqrt{9}. \\ \boxed{x = \pm 3} & \end{aligned} \tag{1.11}$$

Notice that the $\sqrt{9}$ is indicated as ± 3. The reason this is because there are two pairs of identical factors that produce 9 when multiplied: $3 \times 3 = 9$ and $-3 \times -3 = 9$.

But, how do you solve an equation such as: $x^2 + 1 = 0$? You begin just as in the last equation. Isolate the variable, but this time, subtract 1 from both sides of the equal sign.

$$\begin{aligned} x^2 + 1 &= 0 \\ x^2 &= -1 \quad . \\ \sqrt{x^2} &= \sqrt{-1} \end{aligned} \tag{1.12}$$

This is what troubled mathematicians for centuries. All of the radicans they have been working with have all been positive. So, to overcome this obstacle, they invented i.

Example. Let's try and simplify the following expression: $\sqrt{-4}$

$$\begin{aligned} \sqrt{-4} &= \sqrt{-1} \times \sqrt{4} \\ &= i \times \sqrt{4} \\ &= i\sqrt{4} \\ &= \boxed{2i} \end{aligned} \quad . \tag{1.13}$$

Using i to help solve equations

You already know that $i = \sqrt{-1}$, but what about the different powers of i. Take a look at the table below (1.1)

Table 1.1: Different Powers of i

$i = i$	$i^5 = i$	$i^9 = i$	$i^{13} = i$
$i^2 = -1$	$i^6 = -1$	$i^{10} = -1$	$i^{14} = -1$
$i^3 = -i$	$i^7 = -i$	$i^{11} = -i$	$i^{15} = -i$
$i^4 = 1$	$i^8 = 1$	$i^{12} = 1$	$i^{16} = 1$

Example. Simplify: i^{27}:
First, you take the exponent (27) and divide it by 4. The reason we divide it by 4 is because that's how many version of i there are.

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