



A Complete Crash Course on Trigonometry for Idiots

A Complete Guide

Hashem A. Damrah



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Contents

I

The Basic Tools

1	What is Trigonometry	11
1.1	How it all Started	11
1.2	Where Ancients Used Trigonometry	12
1.3	Modern Uses of Trigonometry	13
1.4	Trigonometry and Other Mathematics Disciplines	14
1.5	Two Historical Perspectives of Trigonometry	15
2	Geometry Tools Needed to Study Trigonometry	17
2.1	What is the Unit Circle	17
2.1.1	The Pythagorean Theorem	17
2.1.2	Sample Problem 1	17
2.1.3	Sample Problem 2	17
2.2	More on Triangles	18
2.2.1	Area of Triangles	18
2.2.2	Conditions for Congruence	18
2.3	Circles, Areas, and Chords	19
2.3.1	Chords and Arcs	19
2.3.2	Area of Circles	19
3	Algebra Tools Needed to Study Trigonometry	21
3.1	Cartesian Coordinates	21
3.1.1	Writing Coordinates	21
3.1.2	Four Quadrants	21
3.2	What Are Functions	22
3.2.1	The Basics of Functions	22
3.2.2	The Vertical Line Test	22
3.2.3	Inverse Functions and the Horizontal Line Test	22

3.3	Complex Numbers	23
3.3.1	How They Came into Existence	23
3.3.2	Operations with Complex Numbers	23

II

Triangle Trigonometry

4	Trigonometric Functions and Right Triangles	27
4.1	The Tangent Ratio	27
4.1.1	Sample Problem 1	27
4.1.2	Sample Problem 2	27
4.2	Introducing the Sine and Cosine Ratios	28
4.2.1	Cofunction Identity Between the Sine and Cosine	28
4.2.2	Pythagorean Identity with the Sine and Cosine	28
4.2.3	Values of Trig Functions for 0° and 90° Angles	28
4.3	Practical Applications of Trigonometric Ratios	28
4.3.1	Sample Problem 3	28
4.4	Practice Problems	29
5	Relations Among Trigonometric Ratios	31
5.1	Six Trigonometric Relatives	31
5.1.1	Reciprocal Function	31
5.1.2	Three Fundamental Identities	31
5.1.3	Other Pythagorean Relationships	31
5.1.4	Sample Problem 1	31
5.1.5	Sample Problem 2	31
5.2	Determining Values of Trigonometric Functions	32
5.3	Solving Right Triangles	33
5.3.1	Finding the Sides	33
5.3.2	Sample Problem 3	33
5.3.3	Sample Problem 4	33
5.3.4	Finding the Angles	33
5.3.5	Sample Problem 5	33
5.4	Practice Problems	34
6	The Law of Sines	35
6.1	Oblique Triangles	35
6.1.1	The Area of a Triangle	35
6.1.2	The Law of Sines	35
6.1.3	Sample Problem 1	35
6.2	The Sine of an Obtuse Angle	36
6.3	The Ambiguous Case	37
6.3.1	Sample Problem 2	37
6.3.2	Sample Problem 3	37
6.4	Solving Oblique Triangles Using the Law of Sines	38
6.4.1	Practice Problem 4	38
6.5	Practice Problems	39

7	The Law of Cosines	41
7.1	The Law of Cosines	41
7.1.1	Proof of the Law of Cosines	41
7.1.2	The Law of Cosines for an Obtuse Triangle	41
7.1.3	Finding Angles with the Law of Cosines	41
7.2	Two Cases That the Law of Cosines Helps to Solve	42
7.2.1	Sample Problem 1	42
7.2.2	Sample Problem 2	42
7.3	Application of Trigonometry to Navigation and Surveying	43
7.3.1	Sample Problem 3	43
7.3.2	Sample Problem 4	43
7.4	Hero's Formula for the Area of a Triangle	44
7.4.1	Sample Problem 5	44
7.5	Practice Problems	45

III

Trigonometric Functions and the Unit Circle

8	Angles and Rotations	49
8.1	Measuring Angles	49
8.1.1	Degree Measures	49
8.1.2	Coterminal Angles	49
8.1.3	Radian Measure	49
8.1.4	Sample Problem 1	49
8.2	Measuring Angles	49
8.2.1	Sample Problem 2	49
8.2.2	Sample Problem 3	49
8.2.3	Sample Problem 4	49
8.2.4	Sample Problem 5	49
8.3	Practice Problems	49
9	The Unit Circle Approach	51
9.1	What is the Unit Circle	51
9.2	Placing Points on the Unit Circle	52
9.3	Making Sense of the Unit Circle	52
9.4	Evaluating Trigonometric Functions of Real Numbers	52
9.4.1	Practice Problem 1	52
9.4.2	Practice Problem 2	52
9.4.3	Practice Problem 3	52
9.4.4	Practice Problem 4	52
9.5	Practice Problems	53
10	Algebra Tools Needed to Study Trigonometry	55
10.1	Cartesian Coordinates	55
10.1.1	Writing Coordinates	55
10.1.2	Four Quadrants	55

10.2	What Are Functions	56
10.2.1	The Basics of Functions	56
10.2.2	The Vertical Line Test	56
10.2.3	Inverse Functions and the Horizontal Line Test	56
10.3	Complex Numbers	57
10.3.1	How They Came into Existence	57
10.3.2	Operations with Complex Numbers	57
	Bibliography	59
	Articles	59
	Books	59
	Index	61
	Appendices	61
A	Appendix Chapter Title	61
A.1	Appendix Section Title	61
B	Appendix Chapter Title	63
B.1	Appendix Section Title	63



List of Figures



List of Tables



The Basic Tools

1	What is Trigonometry	11
1.1	How it all Started	11
1.2	Where Ancients Used Trigonometry	12
1.3	Modern Uses of Trigonometry	13
1.4	Trigonometry and Other Mathematics Disciplines	14
1.5	Two Historical Perspectives of Trigonometry	15
2	Geometry Tools Needed to Study Trigonometry	17
2.1	What is the Unit Circle	17
2.2	More on Triangles	18
2.3	Circles, Areas, and Chords	19
3	Algebra Tools Needed to Study Trigonometry	21
3.1	Cartesian Coordinates	21
3.2	What Are Functions	22
3.3	Complex Numbers	23



1. What is Trigonometry

1.1 How it all Started

1.2 Where Ancients Used Trigonometry

1.3 Modern Uses of Trigonometry

1.4 Trigonometry and Other Mathematics Disciplines

1.5 Two Historical Perspectives of Trigonometry



2. Geometry Tools Needed to Study Trigonometry

2.1 What is the Unit Circle

2.1.1 The Pythagorean Theorem

2.1.2 Sample Problem 1

2.1.3 Sample Problem 2

2.2 More on Triangles

2.2.1 Area of Triangles

2.2.2 Conditions for Congruence

2.3 Circles, Areas, and Chords

2.3.1 Chords and Arcs

2.3.2 Area of Circles



3. Algebra Tools Needed to Study Trigonometry

3.1 Cartesian Coordinates

3.1.1 Writing Coordinates

3.1.2 Four Quadrants

3.2 What Are Functions

3.2.1 The Basics of Functions

3.2.2 The Vertical Line Test

3.2.3 Inverse Functions and the Horizontal Line Test

3.3 Complex Numbers

3.3.1 How They Came into Existence

3.3.2 Operations with Complex Numbers

Triangle Trigonometry

4	Trigonometric Functions and Right Triangles	27
4.1	The Tangent Ratio	27
4.2	Introducing the Sine and Cosine Ratios	28
4.3	Practical Applications of Trigonometric Ratios	28
4.4	Practice Problems	29
5	Relations Among Trigonometric Ratios	31
5.1	Six Trigonometric Relatives	31
5.2	Determining Values of Trigonometric Functions	32
5.3	Solving Right Triangles	33
5.4	Practice Problems	34
6	The Law of Sines	35
6.1	Oblique Triangles	35
6.2	The Sine of an Obtuse Angle	36
6.3	The Ambiguous Case	37
6.4	Solving Oblique Triangles Using the Law of Sines	38
6.5	Practice Problems	39
7	The Law of Cosines	41
7.1	The Law of Cosines	41
7.2	Two Cases That the Law of Cosines Helps to Solve	42
7.3	Application of Trigonometry to Navigation and Surveying	43
7.4	Hero's Formula for the Area of a Triangle	44
7.5	Practice Problems	45



4. Trigonometric Functions and Right Triangles

4.1 The Tangent Ratio

4.1.1 Sample Problem 1

4.1.2 Sample Problem 2

4.2 Introducing the Sine and Cosine Ratios

4.2.1 Cofunction Identity Between the Sine and Cosine

4.2.2 Pythagorean Identity with the Sine and Cosine

4.2.3 Values of Trig Functions for 0° and 90° Angles

4.3 Practical Applications of Trigonometric Ratios

4.3.1 Sample Problem 3

4.4 Practice Problems



5. Relations Among Trigonometric Ratios

5.1 Six Trigonometric Relatives

5.1.1 Reciprocal Function

5.1.2 Three Fundamental Identities

5.1.3 Other Pythagorean Relationships

5.1.4 Sample Problem 1

5.1.5 Sample Problem 2

5.2 Determining Values of Trigonometric Functions

5.3 Solving Right Triangles

5.3.1 Finding the Sides

5.3.2 Sample Problem 3

5.3.3 Sample Problem 4

5.3.4 Finding the Angles

5.3.5 Sample Problem 5

5.4 Practice Problems



6. The Law of Sines

6.1 Oblique Triangles

6.1.1 The Area of a Triangle

6.1.2 The Law of Sines

6.1.3 Sample Problem 1

6.2 The Sine of an Obtuse Angle

6.3 The Ambiguous Case

6.3.1 Sample Problem 2

6.3.2 Sample Problem 3

6.4 Solving Oblique Triangles Using the Law of Sines

6.4.1 Practice Problem 4

6.5 Practice Problems



7. The Law of Cosines

7.1 The Law of Cosines

7.1.1 Proof of the Law of Cosines

7.1.2 The Law of Cosines for an Obtuse Triangle

7.1.3 Finding Angles with the Law of Cosines

7.2 Two Cases That the Law of Cosines Helps to Solve

7.2.1 Sample Problem 1

7.2.2 Sample Problem 2

7.3 Application of Trigonometry to Navigation and Surveying

7.3.1 Sample Problem 3

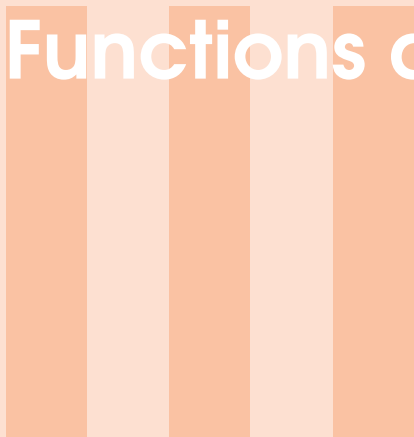
7.3.2 Sample Problem 4

7.4 Hero's Formula for the Area of a Triangle

7.4.1 Sample Problem 5

7.5 Practice Problems

Trigonometric Functions and the Unit Circle



8	Angles and Rotations	49
8.1	Measuring Angles	49
8.2	Measuring Angles	49
8.3	Practice Problems	49
9	The Unit Circle Approach	51
9.1	What is the Unit Circle	51
9.2	Placing Points on the Unit Circle	52
9.3	Making Sense of the Unit Circle	52
9.4	Evaluating Trigonometric Functions of Real Numbers	52
9.5	Practice Problems	53
10	Algebra Tools Needed to Study Trigonometry	55
10.1	Cartesian Coordinates	55
10.2	What Are Functions	56
10.3	Complex Numbers	57



8. Angles and Rotations

8.1 Measuring Angles

- 8.1.1 Degree Measures
- 8.1.2 Coterminal Angles
- 8.1.3 Radian Measure
- 8.1.4 Sample Problem 1

8.2 Measuring Angles

- 8.2.1 Sample Problem 2
- 8.2.2 Sample Problem 3
- 8.2.3 Sample Problem 4
- 8.2.4 Sample Problem 5

8.3 Practice Problems



9. The Unit Circle Approach

9.1 What is the Unit Circle

9.2 Placing Points on the Unit Circle**9.3 Making Sense of the Unit Circle****9.4 Evaluating Trigonometric Functions of Real Numbers****9.4.1 Practice Problem 1****9.4.2 Practice Problem 2****9.4.3 Practice Problem 3****9.4.4 Practice Problem 4**

9.5 Practice Problems



10. Algebra Tools Needed to Study Trigonometry

10.1 Cartesian Coordinates

10.1.1 Writing Coordinates

10.1.2 Four Quadrants

10.2 What Are Functions

10.2.1 The Basics of Functions

10.2.2 The Vertical Line Test

10.2.3 Inverse Functions and the Horizontal Line Test

10.3 Complex Numbers

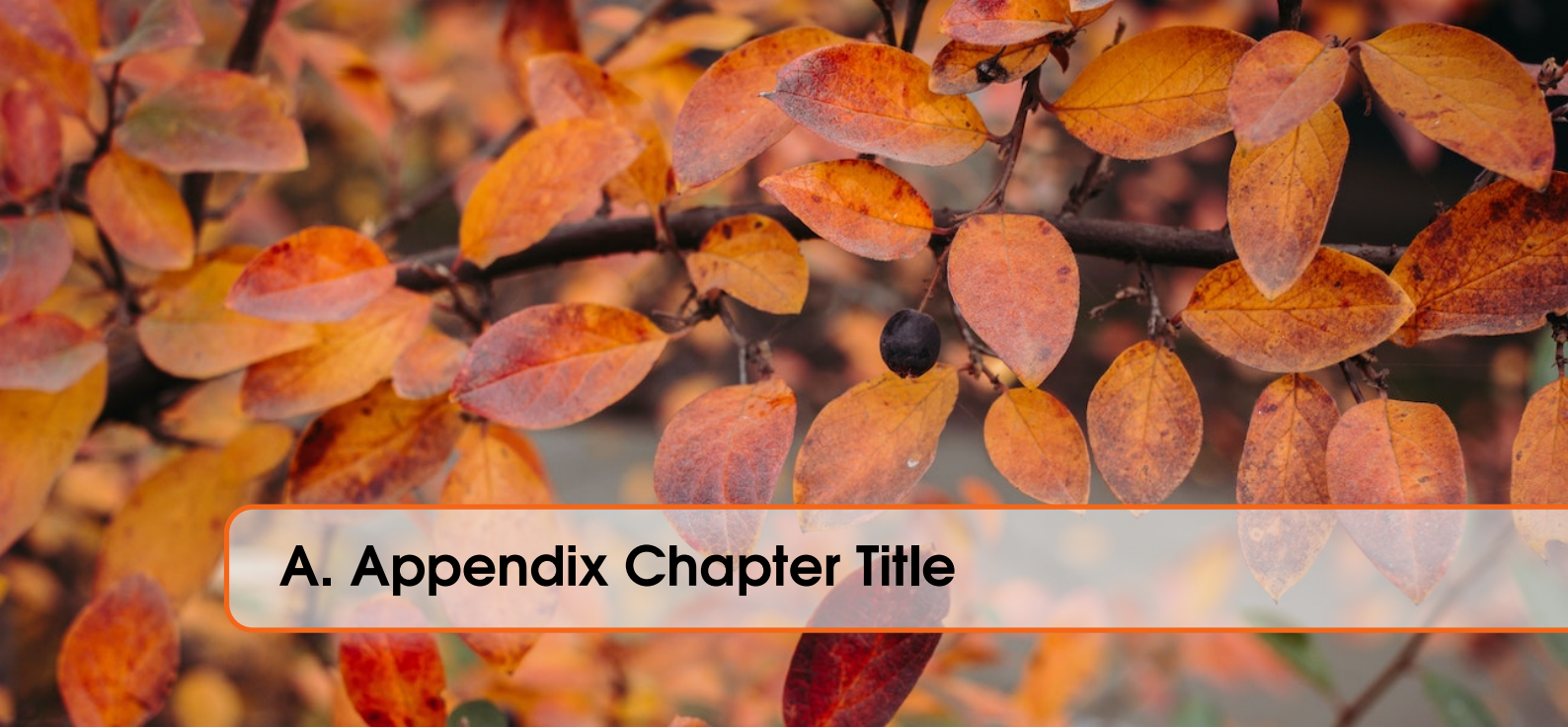
10.3.1 How They Came into Existence

10.3.2 Operations with Complex Numbers

Bibliography

Articles

Books



A. Appendix Chapter Title

A.1 Appendix Section Title

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