Baker Charters School

Honors Algebra 2B

Baker Web Academy School Notes

Author:
Hashem Damrah

Feb. 2 2022

Contents

CHAPTER ONE

Graphs and Functions

Unit 1

Mar 18 2022 Fri (16:25:45)

Lesson 1: Identify parallel and perpendicular lines **Unit 1**

Definition 1. (Parallel) When two lines don't intersect each other.

Definition 2. (Perpendicular) When two lines cross at a right angle (90°).

Definition 3. (Neither) When two lines cross, but not at right angles.

Let's take a look at some important proprieties:

Property 1. (Parallel Slope Property) Two non-vertical lines are parallel if and only if they have the same slope.

Property 2. (Perpendicular Slope Property) Two non-vertical lines are perpendicular if and only if the product of their slopes is equal to -1.

Property 3. (Neither Slope Property) Two non-vertical lines are neither parallel and perpendicular if and only if there slope isn't the same and the products of their slopes aren't equal to -1.

Example. Let's say we have the following system of linear equations:

$$\begin{cases} 3x - 2y = 6 \\ y = \frac{2}{3}x + 10 \end{cases} . \tag{1.1}$$

We need to find if they are **Parallel** (1) **Perpendicular** (2) **Neither** (3). The key to do this re-writing these equations in the form $y = mx + b$ where m is the slope.

Let's re-write the first equation in the form $y = mx + b$:

$$\begin{aligned} 3x - 2y &= 6 && \text{Given} \\ -2y &= -3x + 6 && \text{Subtract } 3x \text{ from both sides} \\ y &= \frac{3}{2}x - 3 && \text{Divide both sides by } -2 \end{aligned} \quad . \tag{1.2}$$

So, our slope (m) is $\frac{3}{2}$. If we look at our second equation in the system of linear equations, we can see that it too has a slope of $\frac{3}{2}$, which makes this system of linear equations **Parallel**.

Example. Let's say we have the following system of linear equations:

$$\begin{cases} 2x - 5y = 20 \\ y = -\frac{5}{2}x - 3 \end{cases} \quad . \tag{1.3}$$

We need to find if they are **Parallel** (1), **Perpendicular** (2), or **Neither** (3).
The key to do this re-writing these equations in the form $y = mx + b$ where m is the slope.
Let's re-write the first equation in the form $y = mx + b$:

$$\begin{aligned} 2x - 5y &= 20 && \text{Given} \\ -2x + 20 &= -5y && \text{Subtract } 2x \text{ from both sides} \\ y &= \frac{2}{5}x - 4 && \text{Divide both sides by } -5 \end{aligned} \quad . \tag{1.4}$$

So, our slope (m) is $\frac{2}{5}$. If we look at our second equation in the system of linear equations, we can see that it has a slope of $-\frac{5}{2}$, which is the reciprocal of $\frac{2}{5}$, which makes this system of linear equations **Perpendicular**

Example. Let's say we have the following system of linear equations:

$$\begin{cases} 3x + 8y = 12 \\ 2x - y = 7 \end{cases} \quad . \tag{1.5}$$

We need to find if they are **Parallel** (1), **Perpendicular** (2), or **Neither** (3).
The key to do this re-writing these equations in the form $y = mx + b$ where m is the slope.
Let's re-write the first equation in the form $y = mx + b$:

$$\begin{aligned} 3x + 8y &= 12 && \text{Given} \\ 8y &= -3x + 12 && \text{Subtract } 3x \text{ from both sides} \\ y &= -\frac{3}{8}x + \frac{3}{2} && \text{Divide both sides by } 8 \end{aligned} \quad . \tag{1.6}$$

So, our slope (m) is $-\frac{3}{8}$.

Let's re-write the second equation in the form $y = mx + b$:

$$\begin{aligned} 2x - y &= 7 && \text{Given} \\ -2x + 7 &= -y && \text{Subtract } 2x \text{ from both sides.} \\ y &= 2x - 7 && \text{Divide both sides by } -1 \end{aligned} \tag{1.7}$$

So, our slope (m) is $\frac{2}{1}$.
Since they aren't the same and they aren't opposite reciprocals, then that makes this system of linear equations **Neither**.

Mar 18 2022 Fri (16:26:55)

Lesson 2: Write parallel and perpendicular line **Unit 1**

Consider the line: $y = -2x + 1$.

Let's:

- Find the equation of the line that is parallel to this line and passes through the point $(7, -5)$.
- Find the equation of the line that is perpendicular to this line through the point $(7, -5)$.

Now, to equation $y = -2x + 1$ is written in the slope-intercept form: $y = mx + b$. In this form, the slope m is -2.

- We can use the **Parallel Slope Property**. Since the given lines has a slope of -2, a line parallel to it must also have the same slope, which is -2. So, the equation of the parallel line will have the form $y = -2x + b$. The line passes through $(7, -5)$, so we use $x = 7$ and $y = -5$ to solve for b:

$$\begin{aligned} y &= -2x + b \\ -5 &= -2(7) + b \\ -5 &= -14 + b \\ b &= 9 \end{aligned} . \tag{1.8}$$

  Now, we know the equation of the parallel line, which is $y = -2x + 9$.

- We use the **Perpendicular Slope Property**. Since the given lines has the slope -2, a line with the slope $\frac{1}{2}$ is perpendicular to it.

  So, the equation of the perpendicular line will have the form $y = \frac{1}{2}x + b$.

  The line passes through $(7, -5)$, so we use $x = 7$ and $y = -5$ to solve for b.

$$\begin{aligned} y &= \frac{1}{2}x + b \\ -5 &= \frac{1}{2}(7) + b \\ -5 &= \frac{7}{2} + b \\ -\frac{10}{2} &= \frac{7}{2} + b \\ b &= -\frac{17}{2} \end{aligned} . \tag{1.9}$$

Equation of the parallel line: $y = -2x + 9$

Equation of the perpendicular line: $y = \frac{1}{2}x - \frac{17}{2}$

Mar 18 2022 Fri (16:27:35)

Lesson 3: Solve linear equation by graphing **Unit 1**

Let's look at this equation:

$$-4 = 5 - 3x. \tag{1.10}$$

Here's one method to solve it (by graphing):

- First, we write the equation with 0 on one side

$$\begin{aligned} -4 &= 5 - 3x \\ 0 &= 9 - 3x \end{aligned}. \tag{1.11}$$

- Then, we graph the equation and find the **x-intercept**:

x	$y = -3x + 9$	(x, y)
0	$y = -3 \times 0 + 9 = 9$	$(0, 9)$
1	$y = -3 \times 1 + 9 = 6$	$(1, 6)$
2	$y = -3 \times 2 + 9 = 3$	$(2, 3)$

Table 1.1: X-Y Table

We get that $x = 3$ is a solution to the original equation, which was $-4 = 5 - 3x$

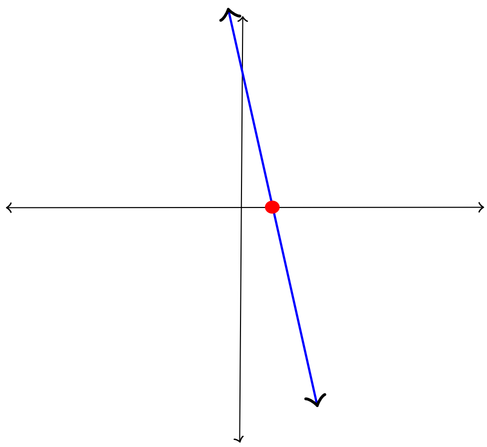


Figure 1.1: $-4 = 5 - 3x$ Graphed

Mar 18 2022 Fri (16:33:35)

Lesson 4: Graph line through given point with slope **Unit 1**

There are three major equations we can use:

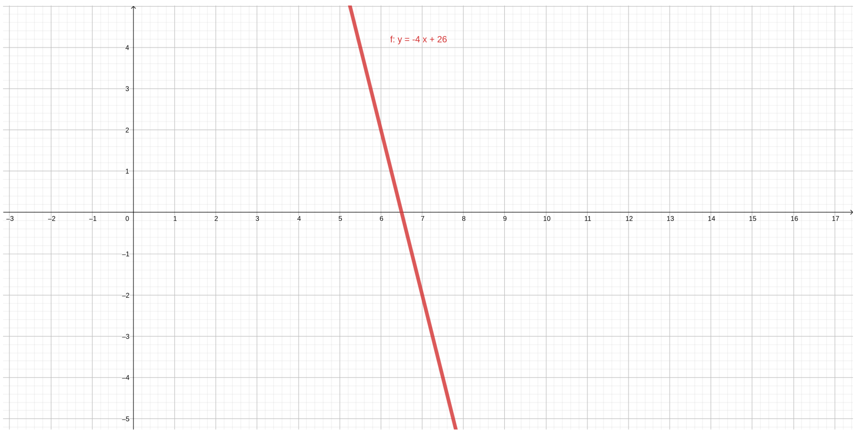
$$\begin{aligned} &y = mx + b \\ &Ax + Bx = C \\ &y_1 - y = m(x_1 - x) \end{aligned} \quad . \tag{1.12}$$

We can use either of these equations. Now, we just plug in the given point and given slope.

Example. We have the point $(6, 2)$ and the slope of -4. Let's use the three equations above to create an equation and then graph:

$$\begin{aligned} &\text{You can use any of the 3 equations} \\ y &= mx + b \\ 2 &= -4(6) + b && \text{Plugin the } x \text{ and } y \text{ values} \\ &\text{Solve for } b \\ -b &= -4(6) - 2 && \text{Subtract } b \text{ and } 2 \\ b &= 4(6) + 2 && \text{Divide both sides by } -1 \\ b &= 26 && \text{Do basic arithmetic} \\ &\text{Final Equation} \\ y &= -4x + 26 \end{aligned} \quad . \tag{1.13}$$

Now, we just graph the equation:



Mar 18 2022 Fri (16:33:58)

Lesson 5: Transform graph from a function **Unit 1**

Translating Graphs

If each of the points of a graph is moved the same distance in the same direction, we say that the graph is **translated** and the resulting graph is a transition of the original.

- **Horizontally Translating:**

  Suppose that f is a function whose domain (7) and range (8) are subsets of the set of real numbers

  - The horizontal translation of the graph of f to the right c units is the graph of

  $$g(x) = f(x - c)$$

  .

  - The horizontal translation of the graph of f to the left c units is the graph of

  $$g(x) = f(x + c)$$

  .

- **Vertically Translating:**

  Suppose that f is a function whose domain (7) and range (8) are subsets of the set of real numbers

  - The vertical translation of the graph of f upward c units is the graph of

  $$g(x) = f(x) + c$$

  .

  - The vertical translation of the graph of f downward c units is the graph of

  $$g(x) = f(x) - c$$

  .

> **Example.** If $f(x) = x^2$, then the graph of $g(x) = x^2 + 2$ is the vertical translation of the graph of f upward 2 units. The graph of $h(x) = x^2 - 1$ is the vertical translation of the graph of f downward 1 unit.

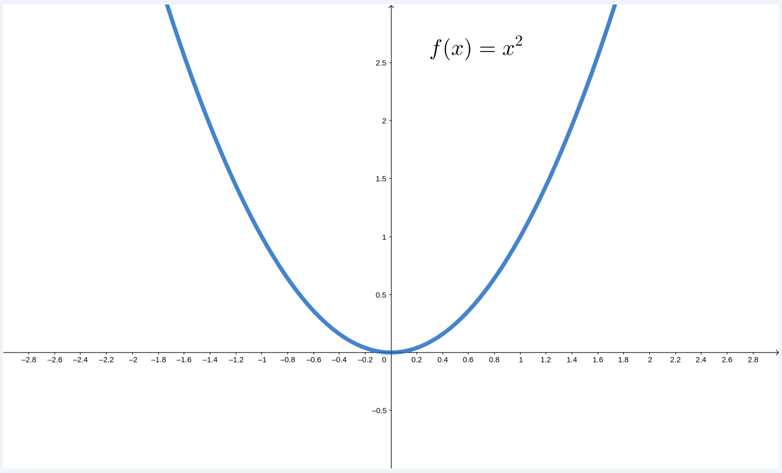
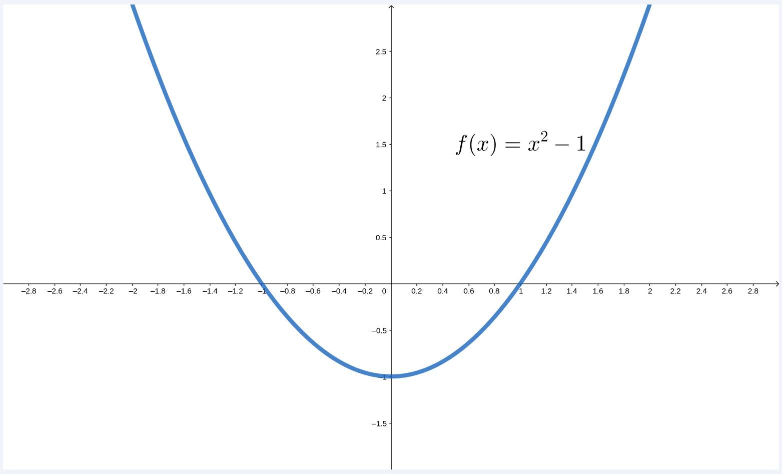
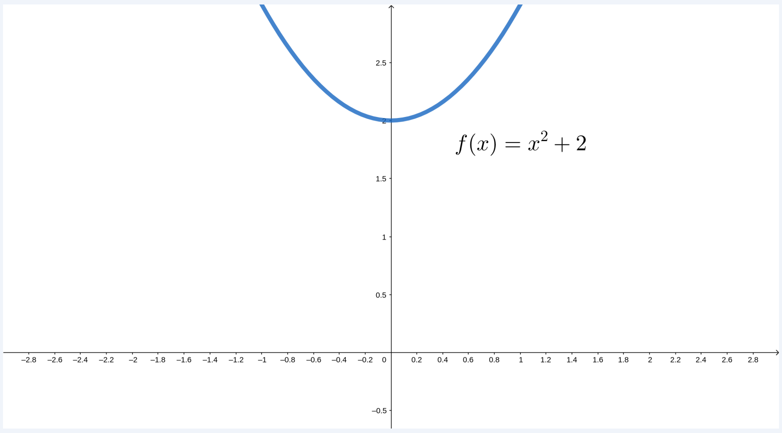


Figure 1.2: The starting graph $f(x) = x^2$



(a) label 1



(b) label 2

Figure 1.3: Translating Graphs

When the graph of a function f is translated to get the graph of a function g, it is important to

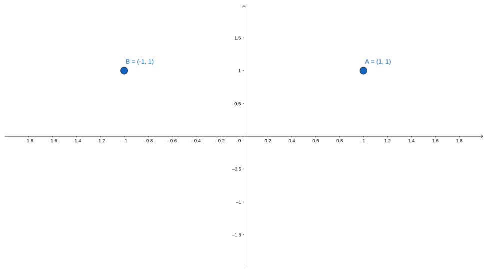
Note. When the graph of a function f is translated to get the graph of a function g, it is important to note that the rigidity of the graph of f is maintained. That is, the graph of g has the same **"size"** and **"shape"** as the graph of f.

Reflecting Graphs

- Reflecting about the y axis.

  The **reflection** of the point (a, b) about the y axis is the point $(-a, b)$.

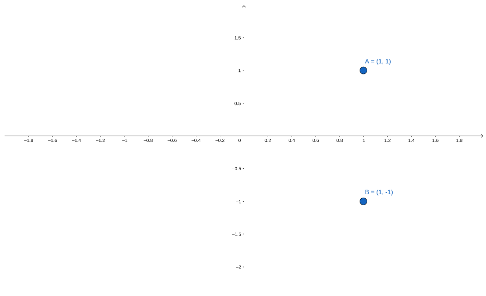
  If f is a function whose domain (7) and range (8) are subsets of the set of real numbers, then the reflection of the graph of f about the y axis is the reflection of every point on the graph of f about the y axis. The reflection of the graph of f about the y axis is the graph of the function $g(x) = f(-x)$



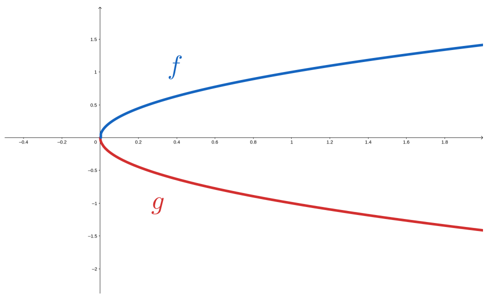
- Reflecting about the x axis.

  The reflection of the point (a, b) about the x axis is the point $(a, -b)$.

  If f is a function whose domain (7) and range (8) are subsets of the set of real numbers, then the reflection of the graph of f about the x axis is the reflection of every point on the graph of f about the x axis. The reflection of the graph of f about the x axis is the graph of the function $g(x) = -f(x)$



Example. The graph of $f(x) = \sqrt{x}$ is shown in blue. The graph of $g(x) = -\sqrt{x}$, which is the reflection of the graph of f about the x axis is shown in red.



Stretching and Shrinking Graphs

- Vertically Stretching and Shrinking.

  Suppose that f is a function whose domain (7) and range (8) are subsets of the set of real numbers.

  - If c is a number greater than 1, then the graph of

    $$g(x) = cf(x)$$

    is **Vertical Stretching**, also called **Vertical Expansion**, of the graph of f.

  - If c is a number between 0 and 1, then the graph of

    $$g(x) = cf(x)$$

    is **Vertical Shrinking**, also called a **Vertical Contraction**, of the graph f.

- Horizontally Stretching and Shrinking.

  Suppose that f is a function whose domain (7) and range (8) are subsets of the set of real numbers.

  – If c is a number greater than 1, then the graph of

  $$g(x) = f(cx)$$

  is a **Horizontal Shrinking**, also called a **Horizontal Contraction**, of the graph of f.

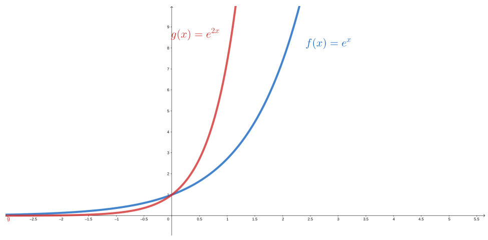
  – If c is a number between 0 and 1, then the graph of

  $$g(x) = f(cx)$$

  is a **Horizontal Stretching**, also called a **Horizontal Expansion**, of the graph of f.

  In both cases, for a given y coordinate, the x coordinate of the point on the graph of g is $\frac{1}{c}$ times the x coordinate of the point on the graph of f.

Example. The graph of the function $f(x) = e^x$ is shown in blue and the graph of the function $g(x) = e^{2x}$ is shown in red. The graph of g is a horizontal contraction of the graph of f because $g(x) = f(2x)$, that is, for a given y coordinate, the x coordinate of a point on the graph of g is $\frac{1}{2}$ times the x coordinate of the point on the graph of f.



Example. Below is the graph of $y = x^2$.
Transform it to make it the graph of $y = -2(x-4)^2$.

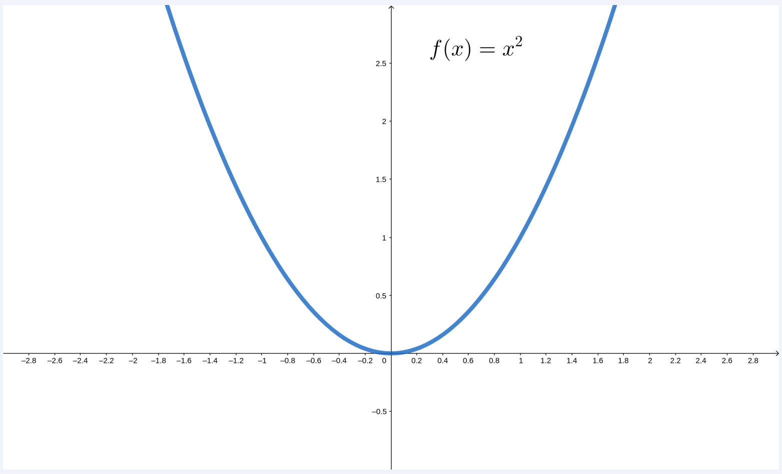


Figure 1.4: $y = x^2$

We are given the graph of $y = x^2$ and asked to graph $y = -2(x-4)^2$. Starting with the graph of $y = x^2$, we'll first graph $y = (x-4)^2$. To do this, we translate (1) the graph of $y = x^2$ to the right 4 units.

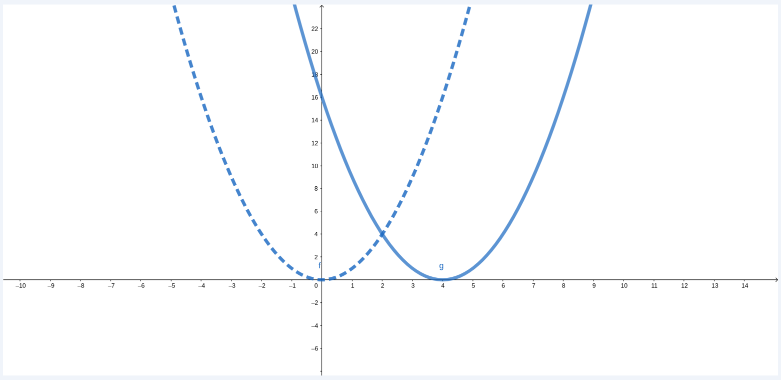


Figure 1.5: Moved to the right by 4 units.

Finally, we'll use the graph of $y = 2(x-4)^2$ to get the graph of $y = -2(x-4)^2$. To do this, we reflect the graph of $y = 2(x-4^2)$ across the x axis.

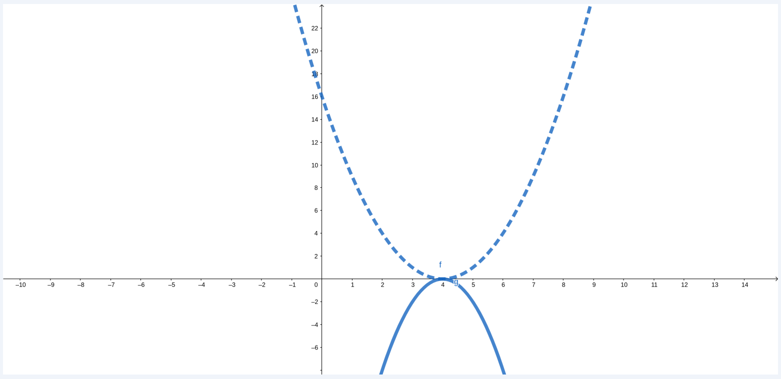


Figure 1.6: Reflected across the x axis.

Mar 18 2022 Fri (16:34:31)

Lesson 6: Word problems with average rate of change **Unit 1**

Definition 4. (Slope) The slope of a line is a number measuring how **Steep** the line is.
The farther the slope is from zero, the steeper the line is. The slope of a vertical line is undefined.
The sign of the slope tells us if the line will go **up** or **down**.

- If the slope is **positive**, then the line goes upward from **left** to **right**.
- If the slope is **negative**, then the line goes **downward** from **left** to **right**.
- If the slope is 0, then the line is **horizontal**.

Suppose that a non-vertical line passes through the two points: $(x_1, y_1), (x_2, y_2)$
The *rise* from the first point to the second is $y_2 - y_1$.
The *run* from the first point to the second is $x_2 - x_1$
The slope formula is:

$$\text{slope} = \frac{\text{rise}}{\text{run}} = \frac{y_2 - y_1}{x_2 - x_1}. \tag{1.14}$$

The number of bacteria in a culture increase rapidly. The table below gives the number $N(t)$ of bacteria at a few times t (in hours) after the moment when $N = 1000$.

Time t hours	Number of bacteria $N(t)$
0	1000
3.4	1510
6.8	2292
10.2	3856
13.6	5080

1. We're going to find the Average Rate of Change (ARC) for the number of bacteria from 0 hours to 6.8 hours.
2. Find the ARC for the number of bacteria from 10.2 hours to 13.6

The ARC for the number of bacteria from $t = a$ to $t = b$ is:

$$\frac{N(b) - N(a)}{b - a}, \quad \text{where} \quad b \neq a. \tag{1.15}$$

Note. Look how similar the equation above (1.15) is similar to the slope (4) formula.

The ARC is the slope of the line passing through: $(a, N(a)) \quad (b, N(b))$.

1. For part **A**, we're asked to find the ARC from 0 hours to 6.8 hours.

   So we take $a = 0 \quad b = 6.8$.

   From the table, we get $N(a) = 1000 \quad N(b) = 2292$.

   Applying the formula for ARC, we get:

$$\begin{aligned} \frac{N(b) - N(a)}{b - a} &= \frac{2292 - 1000}{6.8 - 0} \\ &= \frac{1292}{6.8} \\ &= \boxed{190 \quad \text{Bacteria per Hour}} \end{aligned} \qquad (1.16)$$

2. For part **B**, we're asked to find the ARC from 10.2 hours to 13.6 hours.

   So, we take $a = 10.2 \quad b = 13.6$.

   From the table, we get $N(a) = 3856 \quad N(b) = 5080$.

   Applying the formula for ARC, we get:

$$\begin{aligned} \frac{N(b) - N(a)}{b - a} &= \frac{5080 - 3856}{13.6 - 10.2} \\ &= \frac{1224}{3.4} \\ &= \boxed{360 \quad \text{Bacteria per Hour}} \end{aligned} \qquad (1.17)$$

So, our final answers are:

The ARC for the number of bacteria from 0 hours to 6.8 hours is:
$\boxed{190 \quad \textbf{Bacteria per hour}}$

The ARC for the number of bacteria from 10.2 hours to 13.6 hours is:
$\boxed{360 \quad \textbf{Bacteria per hour}}$

Mar 18 2022 Fri (16:34:47)

Lesson 7: Identify correlation and causation **Unit 1**

> **Definition 5.** (Correlation) A mutual relationship or connection between two or more things.
> Two quantities have a *correlation* if they tend to vary together.

> **Definition 6.** (Causation) This indicates a relationship between two events where one event is affected by the other.

Let's take a look at a couple of examples for each definition:

> **Example.** Maria and Andy are high school students in Arizona. Andy always gets an A on his math test when it's sunny outside.
> You may think that Andy is acing his test because it's sunny when he takes the test, but that's not true. It just happens that it's sunny when there's a math test.
> So, the two events: The sunniness and the acing his test occurred together without one causing the other. In simpler terms, the two events are **Correlated**, but there's no **Causal** relationship between them.

> **Note.** If you have a correlation (5) you don't always have a causation (6).

Mar 18 2022 Fri (16:35:01)

Lesson 8: Domain and range of a linear function **Unit 1**

Definition 7. (Domain) The domain of a function is the set of all possible inputs.

Definition 8. (Range) The domain of a function is the set of all possible inputs of the function.

Example. Suppose that the function f is defined by the following table:

x	$f(x)$
1	1
2	8
3	27
4	64

The domain of f is the set of numbers in the left column. The range of the function f is the set of numbers in the right column. That is:

$$\begin{array}{cc} \text{Domain:} & f = \{1,2,3,4\} \\ \text{Range} & f = \{1,8,27,64\} \end{array}. \tag{1.18}$$

The function f could also be written as a set of ordered pairs:

$$f(x) = \{(1,1),(2,8),(3,27),(4,64)\}$$

.

Here's a cooler way of showing it:

$$f(x) = 3x, x \in [-2,5]. \tag{1.19}$$

The statement above (**??**) is saying that the domain of the function $f(x)$ is the interval (**??**) $[-2,5]$

Note. Note that the domain is the set of all first elements in the ordered pairs, and the range is the set of all second elements in ordered pairs.

Example. A construction crew is lengthening the road. Let L be the total length of the road (in miles). Let D be the number of days the crew has worked.

Suppose that $L = 4D + 400$ gives L as a function of D. The crew can work for at most 70 days.
Identify the correct description of the values in both the domain and range of the function.
Here's how we would approach such a problem:
We are given the function: $L = 4D + 400$. Here, L is the total length of the road and D is the total number of days the crew has worked.

Domain

Description of values for the domain For our function, the input is given by D. So, the values in the domain correspond to the number of days the crew has worked.

Set of values for the domain We are given that the crew can work for at most 70 days.
So, our domain will be $[0, 70]$.

Range

Description of Values for the Range For our function, the output is given by L. So the values in the range correspond to the total length of the road.

Set of Values for the Range To find the range, let's look at the output L for some values of D:
After 0 days, the total length of the road will be: $L = 4(0) + 400 = 400 \quad \text{miles}$
After 1 day, the total length of the road will be: $L = 4(1) + 400 = 401 \quad \text{miles}$
After 10 day, the total length of the road will be: $L = 4(10) + 400 = 440 \quad \text{miles}$
After $20\frac{1}{2}$ day, the total length of the road will be: $L = 4(20\frac{1}{2}) + 400 = 482 \quad \text{miles}$
After 70 day, the total length of the road will be: $L = 4(70) + 400 = 680 \quad \text{miles}$
Note also that we can get any length between 400 and 680 miles for the road.
So, we choose all of the real numbers between 400 and 680 miles.
Or, we can write it like this:

$$L \in [400, 680]. \tag{1.20}$$

Note. The number of days must also be $x \geq 0$.
The number of days could also be decimals, fractions, etc like: $\frac{1}{2}$ $\frac{10}{19}$ $\frac{7}{11}$ $\frac{11}{7}$
The only restriction we have is that the number of days must be $x \geq 0$

CHAPTER TWO

Linear Systems

Unit 2

Mar 18 2022 Fri (16:40:24)

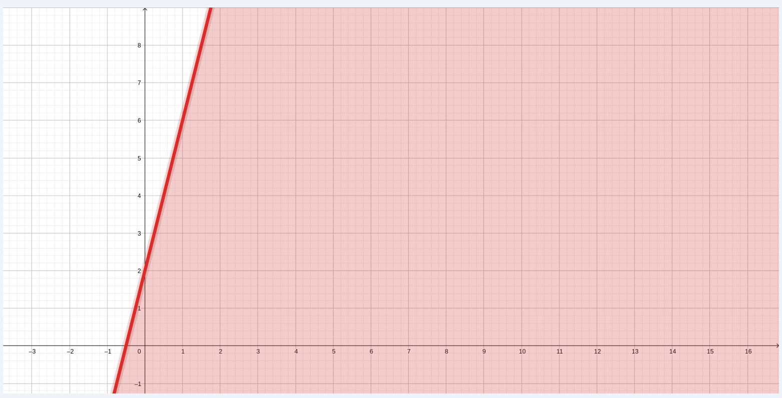
Lesson 1: Write an inequality from Graph **Unit 2**

To find the inequality of a graph is very similar to finding the equation to a linear graph. You would just find out the y-intercept and the slope (4). The difference is that if the line is dashed, then instead of an $=$ symbol, it's going to be either $<$ or $>$. But, if it's a straight line, you would use $\leq$ or $\geq$ in replace for the $=$ symbol.

The $>$ and $\geq$ means the shaded area is below the line because it's saying that the y value is bigger than the x value.

The $<$ and $\leq$ means the shaded area is above the line because it's saying that the y value is less than the x value.

Example. Let's say we have the following inequality graph:



We already know that b is 2, because that's the y-intercept. To find the slope, let's just pick two random points. I'm going with: $(0, 2)$ $(1, 6)$.

Let's use the slope (4) formula to find the slope of this line.

$$\begin{aligned}\frac{y_2 - y_1}{x_2 - x_1} &= \frac{6-2}{1-0} \\ &= \frac{4}{1} \\ &= 4\end{aligned} \quad . \tag{2.1}$$

So, $m = 4$, which is our slope. So, here's our final equation:

$$y = 4x + 2$$

.
To find the inequality symbol, we look at the shaded area. Since the shaded area is below the line, then that means the symbol will either be: $<$ or $\leq$. Since the line is straight (meaning, not dashed), that means that it also includes all of the numbers that the line is on. For that, we use the $\leq$ symbol.
Here's our final answer:

$$\boxed{y \leq 4x + 2}$$

.

Mar 18 2022 Fri (16:48:56)

Lesson 2: Solve system of equations **Unit 2**

Let's just start with a simple example.

Example.

$$\begin{cases} 2x + y = 5 \\ 3x - y = 5 \end{cases}. \tag{2.2}$$

This is a rather straight forward answer. You just add the two equations together, which gets you: $5x = 10$. Now, just simply divide both sides by 5 to get the x-value, which is 2.
Now, just plug the x-value into either of the two equations to get your y-value. I'm going to use the first equation:

$$\begin{aligned} 2x + y &= 5 \\ 2(2) + y &= 5 \\ 4 + y &= 5 \\ y &= 1 \end{aligned}. \tag{2.3}$$

Now, let's do a more complicated example:

Example.

$$\begin{cases} 5x + 4y = 22 \\ 7x + 6y = 32 \end{cases}. \tag{2.4}$$

We need to manipulate these equations such that when we add them, either the x or y variables will cancel.
So, we need to find the LCM of 4 and 6, which is 12.
Now, we're going to multiply the first equation by 3, so the $4y$ will turn into $12y$. Then, multiply the second equation by -2 to bring $6y$ to $-12y$ so the y-values will cancel.
So, here's our result:

$$\begin{cases} 15x + 12y = 66 \\ -14x - 12y = -64 \end{cases}. \tag{2.5}$$

Let's go ahead and add the two equations, which leaves us with: $x = 2$.
Now, plugin the x-value into any of the two original equations (2.4).
I'm going to use the first one.

$$\begin{aligned} 5x + 4y &= 22 \\ 5(2) + 4y &= 22 \\ 10 + 4y &= 22. \\ 4y &= 12 \\ y &= 3 \end{aligned} \tag{2.6}$$

Mar 18 2022 Fri (16:49:12)

Lesson 3: Identify how make equal System of Equation **Unit 2**

Let's just start with an example:

Example.

$$\begin{cases} -4x + 3y = 4 & \text{[A1]} \\ -5x + 6y = 14 & \text{[A2]} \end{cases} \quad \text{System } A. \tag{2.7}$$

$$\begin{cases} -4x + 3y = 4 & \text{[B1]} \\ 3x = 6 & \text{[B2]} \end{cases} \quad \text{System } B. \tag{2.8}$$

$$\begin{cases} -4x + 3y = 4 & \text{[C1]} \\ x = 2 & \text{[C2]} \end{cases} \quad \text{System } C. \tag{2.9}$$

There are 3 operations we can do to transform a system into another:

1. Multiply an equation by a constant
2. Add 2 equations together
3. Add a multiple of one equation to another

Let's see how **System A** turned into **System B**:
We can see that the first equation in both systems stayed the same, which tells us that there weren't any operations done on either of them. However, equation 2 clearly changed.
Our first option is that **[A2]** was multiplied by a constant to get **[B2]**. But there's no number that we can multiply the first equation by to get the second equation. So, it's not the first option.
Let's try our second option. If the two systems were added, it wouldn't equal equation **[B2]**. So, it's not the second option.
That leaves us with our final option. Notice how in the equation **[B2]**, there isn't a y variable, which means when we multiply the first equation (**[A1]**) and then add it to the second equation (**[A2]**), it should cancel out the y variable. Now, the fist equation (**[A1]**) has $+3y$ and the second equation (**[A2]**) has $+6y$. So, to make those cancel, we should multiply the first equation (**[A1]**) by -2 then add it to the second equation (**[A2]**).
Let's try it out:

$$\begin{gathered} -2(-4x + 3y) = -2(4) \\ 8x - 6y = -8 \\ (-5x + 6y = 14) + (8x - 6y = -8) \\ 3x = 6 \end{gathered}. \tag{2.10}$$

So, we were correct. To get from **System A** to **System B**, we leave the first equation alone, but we multiply the second equation by -2, then add the first equation to it.
Now, how would we get from **System B** to **System C**? We leave the first equation alone, but you divide the second equation by 3.

Mar 18 2022 Fri (16:49:49)

Lesson 4: Solve tax rate/interest using system of equations **Unit 2**

A theater group made appearances in two cities. The hotel charge before tax in the second city was $\$1500$ lower than in the first. The tax in the first city was 3.5%, and the tax in the second city was 6%. The hotel tax paid for the two cities was $\$385$. How much was the hotel charge in each city before tax?

Let's first create our needed variables:

x = the hotel charge (in dollars) in the first city before tax y = the hotel charge (in dollars) in the second city before tax

Now, let's create a table to help organize all of our data:

	Charge before tax	Tax rate	Tax paid
First city	x	$3.5\% = 0.035$	$0.035x$
Second city	y	$6\% = 0.06$	$0.06y$
Total			385

Using this information, we can create our system of equations.

$$\begin{cases} y = x - 1500 \\ 0.035x + 0.06y = 385 \end{cases} \quad . \tag{2.11}$$

Before we go any further, we need to make the second equation into the form of $y = mx + b$:

$$\begin{aligned} 0.035x + 0.06y &= 385 \\ \frac{385 - 0.035x}{0.06} &= y \end{aligned} \quad . \tag{2.12}$$

Now, we just need to graph this system of equations and find the point of intersection, which gives us the following solution:

$x = 5000 \qquad y = 35000$

CHAPTER THREE

Exponents and Polynomials

Unit 3

Mar 18 2022 Fri (16:53:02)

Lesson 1: Power/quotient rules with exponents **Unit 3**

Property 4. (Divide Powers) To divide a power by another power, subtract exponents.

Proof. If you have $\frac{n^5}{n^2}$, you can expand each of the variables, which make: $\frac{n \times n \times n \times n \times n}{n \times n}$. From that, we can cancel the two n on the bottom, and two n from the top, which leaves us with n^3, which is the same thing as $n^{5-2} = n^3$

Property 5. (Raise Powers) To raise a power to another power, multiply exponents

Proof. If you have $(n^3)^2$, you can expand the inside of the parenthesis: $(n \times n \times n)^2$.
Then, you can expand the outside as well: $(n \times n \times n) \times (n \times n \times n)$
We can see we have 6 n's, which is the same thing as n^6, which is equivalent to $n^{3\times 2} = n^6$.

Let's simplify the following item. It has all of the rules we will go over:

$$\begin{aligned} \left(\frac{3m^4n^7}{6n^5}\right)^4 &= \left(\frac{m^4n^7}{2n^5}\right)^4 \\ &= \left(\frac{m^4n^2}{2}\right)^4. \\ &= \frac{m^{16}n^8}{16} \end{aligned} \tag{3.1}$$

Mar 18 2022 Fri (16:53:18)

Lesson 2: Power/product/quotient rules **Unit 3**

Let's simplify the following item. It has all of the rules we will go over:

$$\begin{aligned}
(-2u^3v^2w^{-3})^{-3}\left(\frac{u^{-1}}{w^3}\right) &= \frac{1}{(-2u^3v^2w^{-3})}\frac{1}{u^1w^3} \\
&= \frac{1}{(-2)^3u^9v^6w^3}\times\frac{1}{u^1w^3}. \\
&= \frac{1}{-8u^{10}v^6w^6} \\
&= -\frac{1}{8u^{10}v^6w^6}
\end{aligned} \tag{3.2}$$

Mar 18 2022 Fri (16:53:32)

Lesson 3: Factor sum/difference of two cubes **Unit 3**

Formula 1. (Sum of Perfect Cubes)

$$(a^3 + b^3) = (a + b)(a^2 - ab + b^2). \tag{3.3}$$

Formula 2. (Difference of Perfect Cubes)

$$(a^3 - b^3) = (a - b)(a^2 + ab + b^2). \tag{3.4}$$

Info

It's always helpful if you memorize the table for all of the cubes:

$1^3 = 1$	$7^3 = 343$
$2^2 = 8$	$8^3 = 512$
$3^8 = 27$	$9^3 = 729$
$4^8 = 64$	$10^3 = 1000$
$5^8 = 125$	
$6^8 = 216$	

It will help you recognize when you have a sum/difference of two cubes.

Example. $x^3 - 27$. We can clearly see that this is a difference of two cubes. Now, we need to get the a and b values. To do this, take the cube root of each of the variables/numbers, and assign them respectfully.
So, $a = \sqrt[3]{x^3} = x \qquad b = \sqrt[3]{27} = 3$
Then, plug-in play: $(x - 3)(a^2 + 3x + 9)$

Example. $8y^3 + 1$. We can clearly see that this is a sum of two cubes. Now, let's assign the a and b variables. So, $a = \sqrt[3]{8y^3} = 2y \qquad b = \sqrt[3]{1} = 1$
Then, plug-in: $(2y + 1)(4y^2 - 2y + 1)$

Mar 18 2022 Fri (16:53:47)

Lesson 4: Factor out a monomial from a polynomial **Unit 3**

Let's factor the following expression:

$$26u^2v^4w^3 - 16u^9v^9. \tag{3.5}$$

To get started, we're going to get the coefficients, which are: $26 \quad 16$

Now, we're going to get the greatest common factor from both of those numbers, which is a 2. Then, you notice that both of the expressions has at least a u^2 and a v^4. The reason we're getting this information is because we're trying to factor out as much as we can from both of the expressions at the same time.

Now that we've identified the monomial that we're factoring out, we just need to find out what's left of the terms. To find out what's left, we can just divide the monomial by the expression that we're going to factor:

$$\begin{gathered} \frac{26u^2v^4w^3}{2u^2v^4} = 13w^3 \\ \frac{-16u^9v^9}{2u^2v^4} = -8u^7v^5 \end{gathered}. \tag{3.6}$$

So, here's our final result: $2u^2v^4(13w^3 - 8u^7v^5)$

List of Theorems

List of Postulates

List of Conjectures

List of Corollaries

List of Definitions

List of Reviews

List of Properties

List of Propositions

Essential Formulas for Algebra 2 Final Exam

Laws of Exponents

Multiply Powers of the Same Base = Adding Exponents	$(a^m)(a^n) = a^{m+n}$
Divide Powers of the Same Base = Subtracting Exponents	$\frac{a^m}{a^n} = a^{m-n}$
Power Rule = Multiplying Exponents	$(a^m)^n = a^{m \times n}$
Zero Exponent = 1	$a^0 = 1$
Distribution of Exponent with Multiple Bases	$(ab)^n = a^n b^n$ $\left(\frac{a}{b}\right)^n = \frac{a^n}{b^n}$
Negative Exponent = Reciprocal	$a^{-n} = \frac{1}{a^n}$ $\frac{a^{-m}}{b^{-n}} = \frac{b^n}{a^m}$
Distribution of Negative Exponent with Multiple Bases	$(ab)^{-n} = a^{-n}b^{-n} = \frac{1}{a^n b^n}$ $\left(\frac{a}{b}\right)^{-n} = \left(\frac{b}{a}\right)^n = \frac{b^n}{a^n}$

$\sqrt{a+b} \neq \sqrt{a} + \sqrt{b}$	$\sqrt{a \times b} = \sqrt{a} \times \sqrt{b}$
$\sqrt{a-b} \neq \sqrt{a} - \sqrt{b}$	$\sqrt{a \div b} = \sqrt{a} \div \sqrt{b}$

Properties of Radicals

Distribution of Radicals of the Same Index (where $a \geq 0$ and $b \geq 0$ if n is even)	$\sqrt[n]{ab} = \left(\sqrt[n]{a}\right)\left(\sqrt[n]{b}\right)$ $\sqrt[n]{\frac{a}{b}} = \frac{\sqrt[n]{a}}{\sqrt[n]{b}}$
Power Rule of Radicals = Multiplying Exponents	$\sqrt[m]{\sqrt[n]{a}} = \sqrt[(m \times n)]{a}$
Reverse Operations of Radicals and Exponents	$\sqrt[n]{a^n} = a$ (if n is odd) $\sqrt[n]{a^n} = \lvert a \rvert$ (if n is even)

$$a^{\frac{m}{n}} = \sqrt[n]{a^m}$$

The index of the radical is the denominator of the fractional exponent.

Special Products

$(A+B)^2 = A^2 + 2AB + B^2$

$(A-B)^2 = A^2 - 2AB + B^2$

$(A+B)(A-B) = A^2 - B^2$

$(A+B)^3 = A^3 + 3A^2B + 3AB^2 + B^3$

$(A-B)^3 = A^3 - 3A^2B + 3AB^2 - B^3$

Special Expressions

Difference of Squares	$A^2 - B^2 = (A+B)(A-B)$
Perfect Trinomial Squares	$A^2 + 2AB + B^2 = (A+B)^2$
Perfect Trinomial Squares	$A^2 - 2AB + B^2 = (A-B)^2$
Sum of Cubes	$A^3 + B^3 = (A+B)(A^2 - AB + B^2)$
Difference of Cubes	$A^3 - B^3 = (A-B)(A^2 + AB + B^2)$

Quadratic Formula: $x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

Discriminant $= b^2 - 4ac$

When Discriminant is *Positive*, $b^2 - 4ac > 0 \rightarrow$ *Two Distinct Real Roots*

When Discriminant is *Zero*, $b^2 - 4ac = 0 \rightarrow$ *One Distinct Real Root (or Two Equal Real Roots)*

When Discriminant is *Negative*, $b^2 - 4ac < 0 \rightarrow$ *No Real Roots*

Note the pattern:

$i^1 = i \quad i^2 = -1 \quad i^3 = -i \quad i^4 = 1$

$i^5 = i \quad i^6 = -1 \quad i^7 = -i \quad i^8 = 1$

$i^9 = i \quad i^{10} = -1 \quad \ldots..$

Pattern repeats every 4th power of i.

Product of Conjugate Complex Numbers

$(a+bi)(a-bi) = a^2 - b^2i^2 = a^2 - b^2(-1)$

$(a+bi)(a-bi) = a^2 + b^2$

Midpoint of a Line Segment

$$M = \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$$

Distance of a Line Segement

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

Slope

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

Standard Equation for Circles

$$(x-h)^2 + (y-k)^2 = r^2$$

$P(x, y)$ = any point on the path of the circle

$C(h, k)$ = centre of the circle

r = length of the radius

Point-Slope form: - a form of a linear equation when given a slope (m) and a point (x_1, y_1) on the line

$$\frac{y-y_1}{x-x_1}=m \text{ (slope formula)} \qquad y - y_1 = m\,(x - x_1) \quad \text{(Point-Slope form)}$$

If we rearrange the equations so that all terms are on one side, it will be in **standard (general) form**:

$$Ax + By + C = 0 \qquad \text{(Standard or General form)}$$

($A \geq 0$, the leading coefficient for the x term must be positive)

When given a slope (m) and the y-intercept (0, b) of the line, we can find the equation of the line using the **slope and y-intercept form**:

$$y = mx + b$$ where m = slope and b = y-intercept

Parallel Lines	**Perpendicular Lines**
slope of line 1 = slope of line 2	slope of line 1 = negative reciprocal slope of line 2
$m_1 = m_2$	$m_{l_1} = \frac{-1}{m_{l_2}}$

$y \propto x$ (y is directly proportional to x)

$$y = kx$$

where k = constant of variation (constant of proportionality – rate of change)

$y \propto \frac{xz}{w}$ (y is jointly proportional to x, z and w)

$$y = k\frac{xz}{w}$$

where k = constant of variation (constant of proportionality)

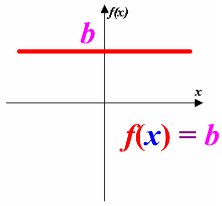
Average Rate of Change = m = $\frac{\Delta y}{\Delta x}$

Average Rate of Change = $\frac{f(b)-f(a)}{b-a}$

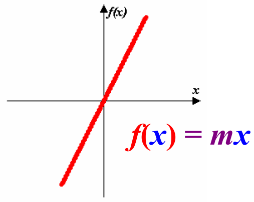
It is the slope of the secant line between the points (a, $f(a)$) and (b, $f(b)$)

Summary of Types of Functions: (see page 226 of textbook)

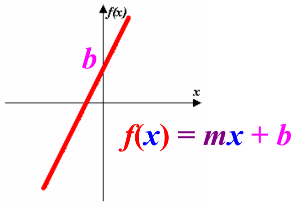
Linear Functions $f(x) = mx + b$



Domain: $x \in R$
Range: $f(x) \in R$

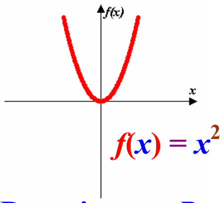


Domain: $x \in R$
Range: $f(x) \in R$

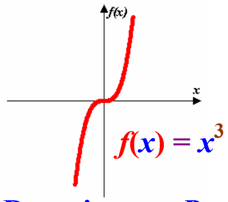


Domain: $x \in R$
Range: $f(x) \in R$

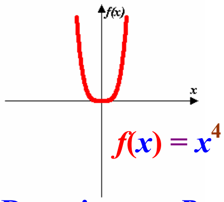
Power Functions $f(x) = x^n$ where $n > 1$ and $n \in N$



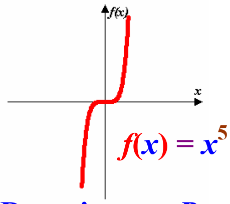
Domain: $x \in R$
Range: $f(x) \geq 0$



Domain: $x \in R$
Range: $f(x) \in R$

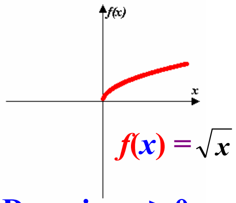


Domain: $x \in R$
Range: $f(x) \geq 0$

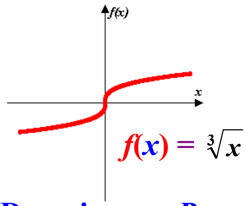


Domain: $x \in R$
Range: $f(x) \in R$

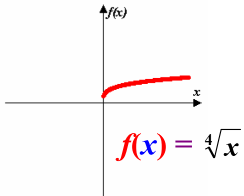
Root Functions $f(x) = \sqrt[n]{x}$ where $n \geq 2$ and $n \in N$



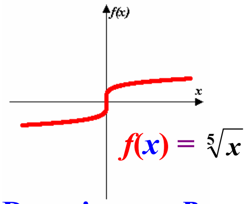
Domain: $x \geq 0$
Range: $f(x) \geq 0$



Domain: $x \in R$
Range: $f(x) \in R$

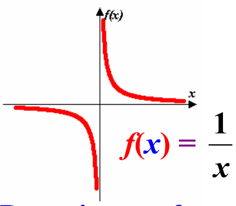


Domain: $x \geq 0$
Range: $f(x) \geq 0$

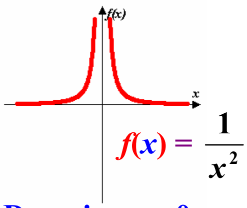


Domain: $x \in R$
Range: $f(x) \in R$

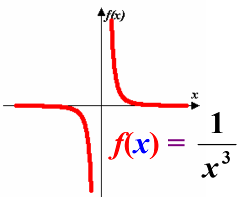
Reciprocal Functions $f(x) = \dfrac{1}{x^n}$ where $n \in N$



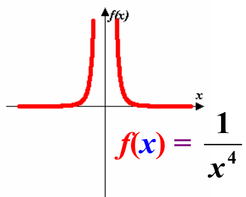
Domain: $x \neq 0$
Range: $f(x) \neq 0$



Domain: $x \neq 0$
Range: $f(x) > 0$

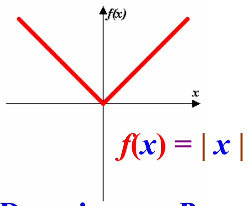


Domain: $x \neq 0$
Range: $f(x) \neq 0$



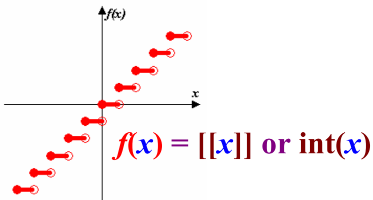
Domain: $x \neq 0$
Range: $f(x) > 0$

Absolute Value Functions



Domain: $x \in R$
Range: $f(x) \geq 0$

Greatest Integer Functions



Domain: $x \in R$
Range: $f(x) \in I$

$g(x) = f(x + h) + k$

h = amount of horizontal movement — $h > 0$ (move left); $h < 0$ (move right)
k = amount of vertical movement — $k > 0$ (move up); $k < 0$ (move down)

Reflection off the x-axis

$g(x) = -f(x)$

All values of y has to switch signs but all values of x remain unchanged.

Reflection off the y-axis

$g(x) = f(-x)$

All values of x has to switch signs but all values of y remain unchanged.

Vertical Stretching and Shrinking

$g(x) = af(x)$

a is the Vertical Stretch Factor
$a > 1$ (Stretches Vertically by a factor of a)
$0 < a < 1$ (Shrinks Vertically by a factor of a)

Horizontal Stretching and Shrinking

$g(x) = f(bx)$

b is the Horizontal Stretch Factor
$0 < b < 1$ (Stretches Horizontally by a factor of $1/b$)
$b > 1$ (Shrinks Horizontally by a factor of $1/b$)

For Quadratic Functions in Standard Form of $f(x) = a(x - h)^2 + k$

Vertex at (h, k) — Axis of Symmetry at $x = h$ — Domain: $x \in R$

a = Vertical Stretch Factor

$a > 0$ Vertex at Minimum (Parabola opens UP) — Range: $y \geq k$ (Minimum)
$a < 0$ Vertex at Maximum (Parabola opens DOWN) — Range: $y \leq k$ (Maximum)
$|a| > 1$ Stretched out Vertically — $|a| < 1$ Shrunken in Vertically

h = Horizontal Translation (Note the standard form has $x - h$ in the bracket!)

$h > 0$ Translated Right — $h < 0$ Translated Left

k = Vertical Translation

$k > 0$ Translated Up — $k < 0$ Translated Down

For Quadratic Functions in General Form: $f(x) = ax^2 + bx + c$

y-intercept at $(0, c)$ by letting $x = 0$ (Note: Complete the Square to change to *Standard Form*)

x-intercepts at $\left(\dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}, 0\right)$ if $b^2 - 4ac \geq 0$. No x-intercepts when $b^2 - 4ac < 0$

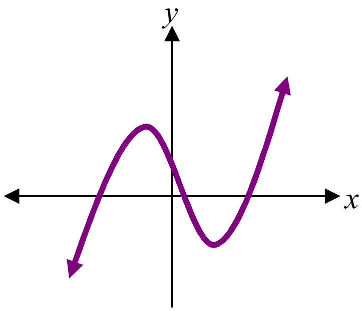
Vertex locates at $x = -\dfrac{b}{2a}$ $\quad y = f\left(-\dfrac{b}{2a}\right)$ Minimum when $a > 0$; Maximum when $a < 0$

$f(x)$ = One-to-One Function — $f^{-1}(x)$ = Inverse Function
(x, y) — (y, x)

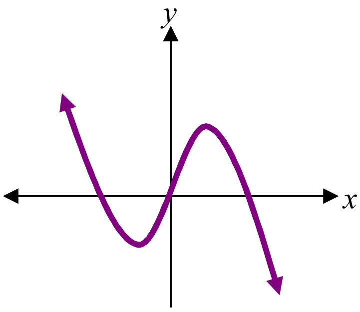
Domain of $f(x) \rightarrow$ Range of $f^{-1}(x)$
Range of $f(x) \rightarrow$ Domain of $f^{-1}(x)$

Note: $f^{-1}(x) \neq \dfrac{1}{f(x)}$ (Inverse is DIFFERENT than Reciprocal)

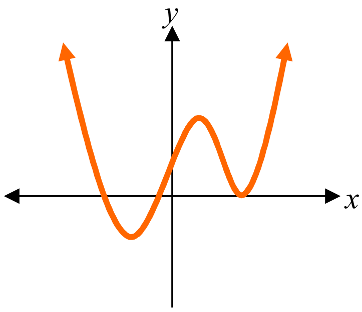
End Behaviours and Leading Terms



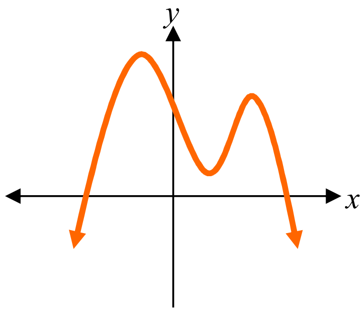
Odd Degree Polynomial Function and Positive Leading Coefficient, $a > 0$



Odd Degree Polynomial Function and Negative Leading Coefficient, $a < 0$



Even Degree Polynomial Function and Positive Leading Coefficient, $a > 0$



Even Degree Polynomial Function and Negative Leading Coefficient, $a < 0$

<u>Odd Degree Polynomial Functions</u>

When $a > 0$, Left is Downward ($y \to -\infty$ as $x \to -\infty$) and Right is Upward ($y \to \infty$ as $x \to \infty$).

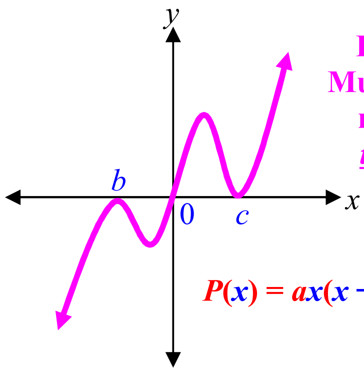
When $a < 0$, Left is Upward ($y \to \infty$ as $x \to -\infty$) and Right is Downward ($y \to -\infty$ as $x \to \infty$).

<u>Even Degree Polynomial Functions</u>

When $a > 0$, Left is Upward ($y \to \infty$ as $x \to -\infty$) and Right is Upward ($y \to \infty$ as $x \to \infty$).

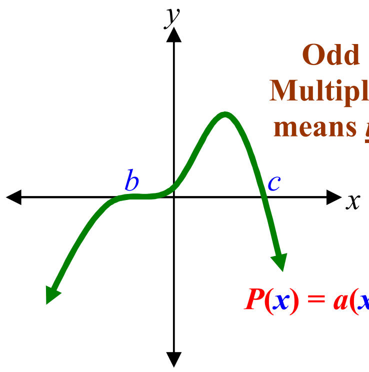
When $a < 0$, Left is Downward ($y \to -\infty$ as $x \to -\infty$) and Right is Downward ($y \to -\infty$ as $x \to \infty$).

<u>Multiplicity</u>: - when a factored polynomial expression has exponents on the factor that is greater than 1.



Even Multiplicity or Multiplicities of 2, 4, 6… means *x*-intercept is *<u>tangent to the x-axis</u>*

$P(x) = ax(x-b)^{2,\,4,\,6\,\ldots}(x-c)^{2,\,4,\,6,\ldots}$



Odd Multiplicity or Multiplicities of 3, 5, 7… means *<u>inflection</u>* at *x*-int

$P(x) = a(x-b)^{3,\,5,\,7\,\ldots}(x-c)$

Polynomial Function **Divisor Function**

In general, for $P(x) \div D(x)$, we can write

$$\frac{P(x)}{D(x)} = Q(x) + \frac{R}{D(x)} \quad \text{or} \quad P(x) = D(x)\,Q(x) + R$$

Restriction: $D(x) \neq 0$

Quotient Function **Remainder**

If $R = 0$ when $\frac{P(x)}{(x-b)}$, then $(x - b)$ is a factor of $P(x)$ and $P(b) = 0$.

$$P(x) = D(x) \times Q(x)$$

$P(x)$ = Original Polynomial $D(x)$ = Divisor (Factor) $Q(x)$ = Quotient

If $R \neq 0$ when $\frac{P(x)}{(x-b)}$, then $(x - b)$ is NOT a factor of $P(x)$.

$$P(x) = D(x) \times Q(x) + R(x)$$

The Remainder Theorem:

To find the remainder of $\frac{P(x)}{x-b}$: Substitute b from the Divisor, $(x - b)$, into the Polynomial, $P(x)$.

In general, when $\frac{P(x)}{x-b}$, $P(b)$ = Remainder.

To find the remainder of $\frac{P(x)}{ax-b}$: Substitute $\left(\frac{b}{a}\right)$ from the Divisor, $(ax - b)$, into the Polynomial, $P(x)$.

In general, when $\frac{P(x)}{ax-b}$, $P\left(\frac{b}{a}\right)$ = Remainder.

The Factor Theorem:

1. If $\frac{P(x)}{x-b}$ gives a Remainder of 0, then $(x - b)$ is the Factor of $P(x)$.

   OR

   If $P(b) = 0$, then $(x - b)$ is the Factor of $P(x)$.

2. If $\frac{P(x)}{ax-b}$ gives a Remainder of 0, then $(ax - b)$ is the Factor of $P(x)$.

   OR

   If $P\left(\frac{b}{a}\right) = 0$, then $(ax - b)$ is the Factor of $P(x)$.

Rational Roots Theorem:

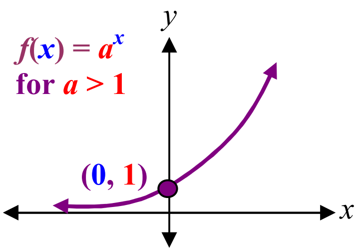
For a polynomial $P(x)$, a List of POTENTIAL Rational Roots can be generated by Dividing ALL the Factors of its Constant Term by ALL the Factors of its Leading Coefficient.

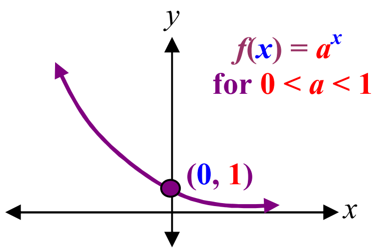
$$\text{Potential Rational Zeros of } P(x) = \frac{\text{ALL Factors of the Constant Term}}{\text{ALL Factors of the Leading Coefficient}}$$

The Zero Theorem

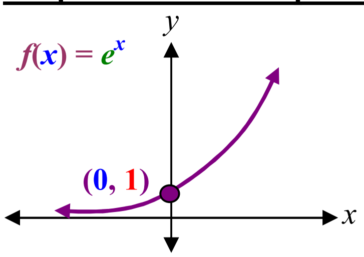
There are n number of solutions (complex, real or both) for any n^{th} degree polynomial function accounting that that a zero with multiplicity of k is counted k times.

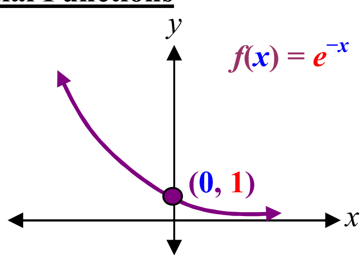
Graphs of Exponential Functions





Graphs of Natural Exponential Functions





$$y = a^x \longleftrightarrow x = \log_a y$$

Simple Properties of Logarithms

$\log_a 1 = 0$	because $a^0 = 1$
$\log_a a = 1$	because $a^1 = a$
$a^{\log_a x} = x$	because **exponent and logarithm are inverse of one another**
$\log_a a^x = x$	because **logarithm and exponent are inverse of one another**

Common and Natural Logarithm

Common Logarithm:	$\log x = y$	$\longleftrightarrow$	$10^y = x$
Natural Logarithm:	$\ln x = y$	$\longleftrightarrow$	$e^y = x$

Exponential Laws	Logarithmic Laws
$(a^m)(a^n) = a^{m+n}$	$\log_a x + \log_a y = \log_a(xy)$
$\dfrac{a^m}{a^n} = a^{m-n}$	$\log_a x - \log_a y = \log_a\left(\dfrac{x}{y}\right)$
$(a^m)^n = a^{m \times n}$	$\log_a x^y = y \log_a x$
$a^0 = 1$	$\log_a 1 = 0$

Common Logarithm Mistakes

$\log_a(x+y) \neq \log_a x + \log_a y$

Example: $\log(2+8) \neq \log 2 + \log 8$

$1 \neq 0.3010 + 0.9031$

$\log_a(x-y) \neq \log_a x - \log_a y$

Example: $\log(120-20) \neq \log 120 + \log 20$

$2 \neq 2.0792 + 1.3010$

$$\log_a\left(\frac{x}{y}\right) \neq \frac{\log_a x}{\log_a y}$$

Example: $\log\left(\frac{1}{10}\right) \neq \frac{\log 1}{\log 10}$

$-1 \neq \frac{0}{1}$

$$(\log_a x)^y \neq y\log_a x$$

Example: $(\log 100)^3 \neq 3 \log 100$

$2^3 \neq 3(2)$

$a^x = y \qquad x = \dfrac{\log y}{\log a}$

$$A = P\left(1+\frac{r}{n}\right)^{nt}$$

A = Final Amount after t years

P = Principal

r = Interest Rate per year

n = Number of Terms per year

$$A(t) = A_0\left(1+\frac{r}{n}\right)^{nt} \xrightarrow{n \to \infty} A(t) = A_0 e^{rt}$$

$A(t)$ = Final Amount after t years

A_0 = Initial Amount

r = Rate of Increase ($+r$) / Decrease ($-r$) per year

$$A(t) = A_0 e^{rt}$$

$A(t)$ = Final Amount after t years

A_0 = Initial Amount

r = Rate of Increase ($+r$) / Decrease ($-r$) per year

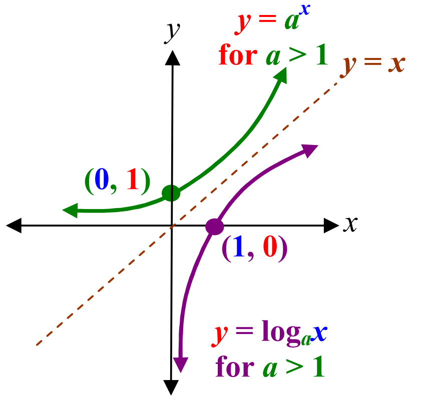
$$N(t) = N_0 e^{rt}$$

$N(t)$ = Final Population after t years, hours, minutes, or seconds

N_0 = Initial Population

r = Rate of Increase per year, hour, minute, or second

Graphs of Exponential and Logarithmic Functions



Exponential Function

y-int = 1 No x-intercept

Domain $x \in R$; Range $y > 0$

Logarithmic Function

x-int = 1 No y-intercept

Domain $x > 0$; Range $y \in R$

To obtain equation for the inverse of an exponential function, we start with

$y = a^x$

$x = a^y$ (switch x and y for inverse)

$y = \log_a x$ (rearrange to solve for y)

π rad = 180^{o} OR $\dfrac{\pi}{180}$ rad = 1^{o}

$y = a \sin k(x+b) + c$ $\qquad$ $y = a \cos k(x+b) + c$

$|a|$ = Amplitude $\qquad$ c = Vertical Displacement (how far away from the x–axis)

b = Horizontal Displacement (Phase Shift) $\qquad$ $b > 0$ (shifted left) $\qquad$ $b < 0$ (shifted right)

k = number of complete cycles in 2π $\qquad$ Period = $\dfrac{2\pi}{k} = \dfrac{360^{o}}{k}$

Range = Minimum $\leq y \leq$ Maximum

$y = a \sin [\omega(t+b)] + c$ $\qquad$ $y = a \cos [\omega(t+b)] + c$

$|a|$ = Amplitude $\qquad$ c = Vertical Displacement (distance between *mid-line* and *t*-axis)

b = Horizontal Displacement (Phase Shift) $\qquad$ $b > 0$ (shifted left) $\qquad$ $b < 0$ (shifted right)

ω = number of complete cycles in 2π $\qquad$ Period = $\dfrac{2\pi}{\omega}$ $\qquad$ Frequency = $\dfrac{\omega}{2\pi}$

Range = Minimum $\leq y \leq$ Maximum

Note: $\sin^{-1}(x) \neq \dfrac{1}{\sin(x)}$ $\qquad$ $\sin^{-1}(x) \neq (\sin x)^{-1}$ $\qquad$ $(\sin x)^{-1} = \dfrac{1}{\sin(x)} = \csc x$

$y = \sin^{-1} x$

Domain: $-1 \leq x \leq 1$ $\qquad$ Range: $-\dfrac{\pi}{2} \leq y \leq \dfrac{\pi}{2}$

$y = \cos^{-1} x$

Domain: $-1 \leq x \leq 1$ $\qquad$ Range: $0 \leq x \leq \pi$

$\sin(\sin^{-1} x) = x$ $\qquad$ for $-1 \leq x \leq 1$

$\sin^{-1}(\sin x) = x$ $\qquad$ for $-\dfrac{\pi}{2} \leq x \leq \dfrac{\pi}{2}$

$\cos(\cos^{-1} x) = x$ $\qquad$ for $-1 \leq x \leq 1$

$\cos^{-1}(\cos x) = x$ $\qquad$ for $0 \leq x \leq \pi$

$y = \tan^{-1} x$

Domain: $x \in R$ $\qquad$ Range: $-\dfrac{\pi}{2} \leq y \leq \dfrac{\pi}{2}$

$\tan(\tan^{-1} x) = x$ $\qquad$ for $x \in R$

$\tan^{-1}(\tan x) = x$ $\qquad$ for $-\dfrac{\pi}{2} \leq x \leq \dfrac{\pi}{2}$

Some Basic Trigonometric Definitions and Identities (proven equations)

$\tan\theta = \dfrac{\sin\theta}{\cos\theta}$ $\qquad$ $\csc\theta = \dfrac{1}{\sin\theta}$ $\qquad$ $\sec\theta = \dfrac{1}{\cos\theta}$ $\qquad$ $\cot\theta = \dfrac{1}{\tan\theta} = \dfrac{\cos\theta}{\sin\theta}$

$\cos^2\theta + \sin^2\theta = 1$