BAKER CHARTERS SCHOOL

HONORS ALGEBRA 2B

Baker Web Academy School Notes

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Contents

CHAPTER ONE

Graphs and Functions

Unit 1

Feb 21 2022 Mon (11:15:44)

Lesson 1: Writing parallel and perpendicular **Unit 1**

Consider the line: $y = -2x + 1$.

Let's:

- Find the equation of the line that is parallel to this line and passes through the point $(7, -5)$.
- Find the equation of the line that is perpendicular to this line through the point $(7, -5)$.

Let's first go over these Properties:

Property 1. (Parallel Slope Property) Two non-vertical lines are parallel if and only if they have the same slope.

Property 2. (Perpendicular Slope Property) Two non-vertical lines are perpendicular if and only if the product of their slopes is equal to -1.

Now, to equation $y = -2x + 1$ is written in the slope-intercept form: $y = mx + b$. In this form, the slope m is -2.

- We can use the **Parallel Slope Property**. Since the given lines has a slope of -2, a line parallel to it must also have the same slope, which is -2. So, the equation of the parallel line will have the form $y = -2x + b$. The line passes through $(7, -5)$, so we use $x = 7$ and $y = -5$ to solve for b:

$$\begin{aligned} y &= -2x + b \\ -5 &= -2(7) + b \\ -5 &= -14 + b \\ b &= 9 \end{aligned} . \tag{1.1}$$

Now, we know the equation of the parallel line, which is $y = -2x + 9$.

- We use the **Perpendicular Slope Property**. Since the given lines has the slope -2, a line with the slope $\frac{1}{2}$ is perpendicular to it.

  So, the equation of the perpendicular line will have the form $y = \frac{1}{2}x + b$.

  The line passes through $(7, -5)$, so we use $x = 7$ and $y = -5$ to solve for b.

$$\begin{aligned} y &= \frac{1}{2}x + b \\ -5 &= \frac{1}{2}(7) + b \\ -5 &= \frac{7}{2} + b \\ -\frac{10}{2} &= \frac{7}{2} + b \\ b &= -\frac{17}{2} \end{aligned} . \tag{1.2}$$

Equation of the parallel line: $y = -2x + 9$

Equation of the perpendicular line: $y = \frac{1}{2}x - \frac{17}{2}$

Feb 21 2022 Mon (12:24:44)

Lesson 2: Solving a linear equation by graphing **Unit 1**

Let's look at this equation:

$$-4 = 5 - 3x. \tag{1.3}$$

Here's one method to solve it (by graphing):

- First, we write the equation with 0 on one side

$$\begin{aligned} -4 &= 5 - 3x \\ 0 &= 9 - 3x \end{aligned}. \tag{1.4}$$

- Then, we graph the equation and find the **x-intercept**:

x	$y = -3x + 9$	(x, y)
0	$y = -3 \times 0 + 9 = 9$	$(0, 9)$
1	$y = -3 \times 1 + 9 = 6$	$(1, 6)$
2	$y = -3 \times 2 + 9 = 3$	$(2, 3)$

Table 1.1: X-Y Table

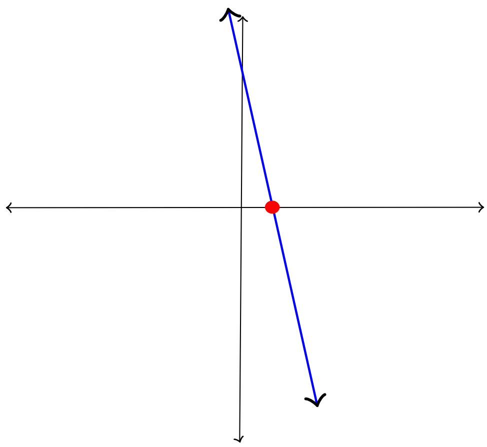


Figure 1.1: $-4 = 5 - 3x$ Graphed

We get that $x = 3$ is a solution to the original equation, which was $-4 = 5 - 3x$

Feb 27 2022 Sun (15:34:39)

Lesson 3: Identify correlation and causation **Unit 1**

Definition 1. (Correlation) A mutual relationship or connection between two or more things.
Two quantities have a *correlation* if they tend to vary together.

Definition 2. (Causation) This indicates a relationship between two events where one event is affected by the other.

Let's take a look at a couple of examples for each definition:

Example. Maria and Andy are high school students in Arizona. Andy always gets an A on his math test when it's sunny outside.
You may think that Andy is acing his test because it's sunny when he takes the test, but that's not true. It just happens that it's sunny when there's a math test.
So, the two events: The sunniness and the acing his test occurred together without one causing the other. In simpler terms, the two events are **Correlated**, but there's no **Causal** relationship between them.

Note. If you have a correlation (1) you don't always have a causation (2).

Mar 03 2022 Thu (17:42:42)

Lesson 4: Domain and range of a linear function **Unit 1**

Definition 3. (Domain) The domain of a function is the set of all possible inputs.

Definition 4. (Range) The domain of a function is the set of all possible inputs of the function.

Example. Suppose that the function f is defined by the following table:

x	$f(x)$
1	1
2	8
3	27
4	64

The domain of f is the set of numbers in the left column. The range of the function f is the set of numbers in the right column. That is:

$$\begin{array}{c} \text{Domain:} \quad f = \{1,2,3,4\} \\ \text{Range} \quad f = \{1,8,27,64\} \end{array}. \tag{1.5}$$

The function f could also be written as a set of ordered pairs:

$$f(x) = \{(1,1),(2,8),(3,27),(4,64)\}$$

.

Here's a cooler way of showing it:

$$f(x) = 3x, x \in [-2,5]. \tag{1.6}$$

The statement above (**??**) is saying that the domain of the function $f(x)$ is the interval (**??**) $[-2,5]$

Note. Note that the domain is the set of all first elements in the ordered pairs, and the range is the set of all second elements in ordered pairs.

Example. A construction crew is lengthening the road. Let L be the total length of the road (in miles). Let D be the number of days the crew has worked.
Suppose that $L = 4D + 400$ gives L as a function of D. The crew can work for at most 70 days.
Identify the correct description of the values in both the domain and range of the function.
Here's how we would approach such a problem:

We are given the function: $L = 4D + 400$. Here, L is the total length of the road and D is the total number of days the crew has worked.

Domain

Description of values for the domain For our function, the input is given by D. So, the values in the domain correspond to the number of days the crew has worked.

Set of values for the domain We are given that the crew can work for at most 70 days.
So, our domain will be $[0, 70]$.

Range

Description of Values for the Range For our function, the output is given by L. So the values in the range correspond to the total length of the road.

Set of Values for the Range To find the range, let's look at the output L for some values of D:
After 0 days, the total length of the road will be: $L = 4(0) + 400 = 400 \quad \text{miles}$
After 1 day, the total length of the road will be: $L = 4(1) + 400 = 401 \quad \text{miles}$
After 10 day, the total length of the road will be: $L = 4(10) + 400 = 440 \quad \text{miles}$
After $20\frac{1}{2}$ day, the total length of the road will be: $L = 4(20\frac{1}{2}) + 400 = 482 \quad \text{miles}$
After 70 day, the total length of the road will be: $L = 4(70) + 400 = 680 \quad \text{miles}$
Note also that we can get any length between 400 and 680 miles for the road.
So, we choose all of the real numbers between 400 and 680 miles.
Or, we can write it like this:

$$L \in [400, 680]. \tag{1.7}$$

Note. The number of days must also be $x \geq 0$.
The number of days could also be decimals, fractions, etc like: $\frac{1}{2}$ $\frac{10}{19}$ $\frac{7}{11}$ $\frac{11}{7}$
The only restriction we have is that the number of days must be $x \geq 0$

Mar 03 2022 Thu (18:38:15)

Lesson 5: Word problem involving average rate of change **Unit 1**

Definition 5. (Slope) The slope of a line is a number measuring how **Steep** the line is.
The farther the slope is from zero, the steeper the line is. The slope of a vertical line is undefined.
The sign of the slope tells us if the line will go **up** or **down**.

- If the slope is **positive**, then the line goes upward from **left** to **right**.
- If the slope is **negative**, then the line goes **downward** from **left** to **right**.
- If the slope is 0, then the line is **horizontal**.

Suppose that a non-vertical line passes through the two points: $(x_1, y_1), (x_2, y_2)$
The *rise* from the first point to the second is $y_2 - y_1$.
The *run* from the first point to the second is $x_2 - x_1$
The slope formula is:

$$\text{slope} = \frac{\text{rise}}{\text{run}} = \frac{y_2 - y_1}{x_2 - x_1}. \tag{1.8}$$

The number of bacteria in a culture increase rapidly. The table below gives the number $N(t)$ of bacteria at a few times t (in hours) after the moment when $N = 1000$.

Time t hours	Number of bacteria $N(t)$
0	1000
3.4	1510
6.8	2292
10.2	3856
13.6	5080

1. We're going to find the Average Rate of Change (ARC) for the number of bacteria from 0 hours to 6.8 hours.
2. Find the ARC for the number of bacteria from 10.2 hours to 13.6

The ARC for the number of bacteria from $t = a$ to $t = b$ is:

$$\frac{N(b) - N(a)}{b - a}, \quad \text{where} \quad b \neq a. \tag{1.9}$$

Note. Look how similar the equation above (1.9) is similar to the slope (5) formula.

The ARC is the slope of the line passing through: $(a, N(a)) \quad (b, N(b))$.

1. For part **A**, we're asked to find the ARC from 0 hours to 6.8 hours.

   So we take $a = 0 \quad b = 6.8$.

   From the table, we get $N(a) = 1000 \quad N(b) = 2292$.

   Applying the formula for ARC, we get:

$$\begin{aligned} \frac{N(b) - N(a)}{b - a} &= \frac{2292 - 1000}{6.8 - 0} \\ &= \frac{1292}{6.8} \\ &= \boxed{190 \quad \text{Bacteria per Hour}} \end{aligned} \quad . \tag{1.10}$$

2. For part **B**, we're asked to find the ARC from 10.2 hours to 13.6 hours.

   So, we take $a = 10.2 \quad b = 13.6$.

   From the table, we get $N(a) = 3856 \quad N(b) = 5080$.

   Applying the formula for ARC, we get:

$$\begin{aligned} \frac{N(b) - N(a)}{b - a} &= \frac{5080 - 3856}{13.6 - 10.2} \\ &= \frac{1224}{3.4} \\ &= \boxed{360 \quad \text{Bacteria per Hour}} \end{aligned} \quad . \tag{1.11}$$

So, our final answers are:

The ARC for the number of bacteria from 0 **hours to** 6.8 **hours is**:
$\boxed{190 \quad \textbf{Bacteria per hour}}$

The ARC for the number of bacteria from 10.2 **hours to** 13.6 **hours is:**
$\boxed{360 \quad \textbf{Bacteria per hour}}$

Mar 03 2022 Thu (20:33:31)

Lesson 6: Transforming Graphs **Unit 1**

Translating Graphs

If each of the points of a graph is moved the same distance in the same direction, we say that the graph is **translated** and the resulting graph is a transition of the original.

- **Horizontally Translating:**

  Suppose that f is a function whose domain (3) and range (4) are subsets of the set of real numbers

  - The horizontal translation of the graph of f to the right c units is the graph of

    $$g(x) = f(x - c)$$

    .

  - The horizontal translation of the graph of f to the left c units is the graph of

    $$g(x) = f(x + c)$$

    .

- **Vertically Translating:**

  Suppose that f is a function whose domain (3) and range (4) are subsets of the set of real numbers

  - The vertical translation of the graph of f upward c units is the graph of

    $$g(x) = f(x) + c$$

    .

  - The vertical translation of the graph of f downward c units is the graph of

    $$g(x) = f(x) - c$$

    .

> **Example.** If $f(x) = x^2$, then the graph of $g(x) = x^2 + 2$ is the vertical translation of the graph of f upward 2 units. The graph of $h(x) = x^2 - 1$ is the vertical translation of the graph of f downward 1 unit.

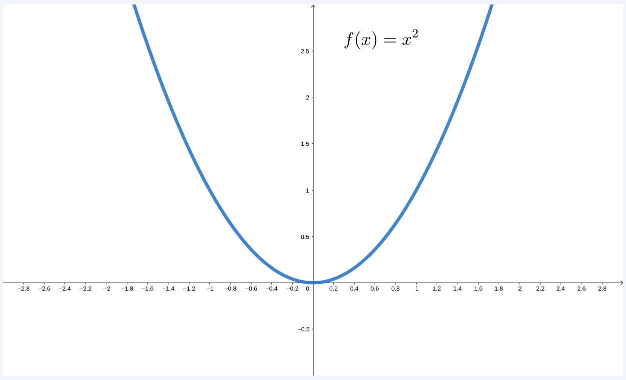
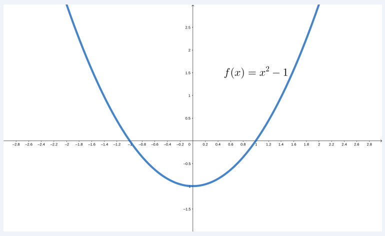
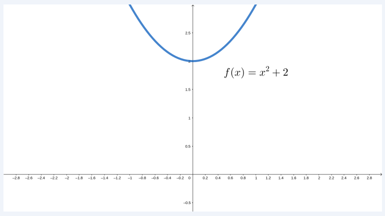


Figure 1.2: The starting graph $f(x) = x^2$



(a) label 1



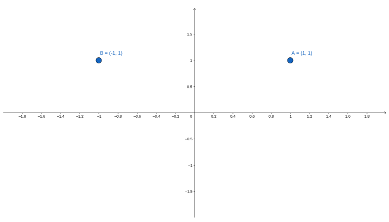
(b) label 2

Figure 1.3: Translating Graphs

When the graph of a function f is translated to get the graph of a function g, it is important to

Note. When the graph of a function f is translated to get the graph of a function g, it is important to note that the rigidity of the graph of f is maintained. That is, the graph of g has the same **"size"** and **"shape"** as the graph of f.

Reflecting Graphs



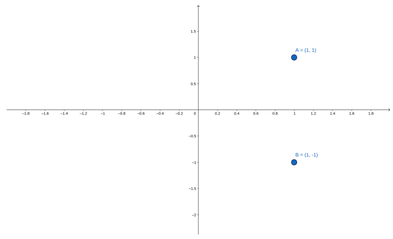
- Reflecting about the y axis.

  The **reflection** of the point (a, b) about the y axis is the point $(-a, b)$.

  If f is a function whose domain (3) and range (4) are subsets of the set of real numbers, then the reflection of the graph of f about the y axis is the reflection of every point on the graph of f about the y axis. The reflection of the graph of f about the y axis is the graph of the function $g(x) = f(-x)$

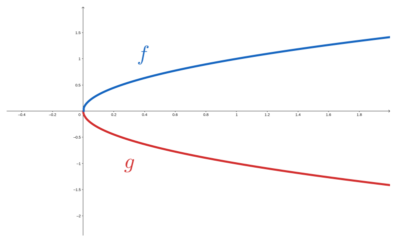
- Reflecting about the x axis.

  The reflection of the point (a, b) about the x axis is the point $(a, -b)$.

  If f is a function whose domain (3) and range (4) are subsets of the set of real numbers, then the reflection of the graph of f about the x axis is the reflection of every point on the graph of f about the x axis. The reflection of the graph of f about the x axis is the graph of the function $g(x) = -f(x)$

> **Example.** The graph of $f(x) = \sqrt{x}$ is shown in blue. The graph of $g(x) = -\sqrt{x}$, which is the reflection of the graph of f about the x axis is shown in red.



Stretching and Shrinking Graphs

- Vertically Stretching and Shrinking.

  Suppose that f is a function whose domain (3) and range (4) are subsets of the set of real numbers.

  - If c is a number greater than 1, then the graph of

    $$g(x) = cf(x)$$

    is **Vertical Stretching**, also called **Vertical Expansion**, of the graph of f.

  - If c is a number between 0 and 1, then the graph of

    $$g(x) = cf(x)$$

    is **Vertical Shrinking**, also called a **Vertical Contraction**, of the graph f.

- Horizontally Stretching and Shrinking.

  Suppose that f is a function whose domain (3) and range (4) are subsets of the set of real numbers.

  - If c is a number greater than 1, then the graph of

    $$g(x) = f(cx)$$

    is a **Horizontal Shrinking**, also called a **Horizontal Contraction**, of the graph of f.

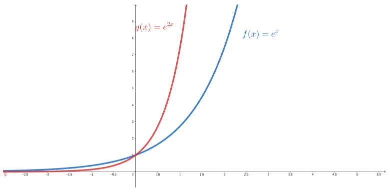
  - If c is a number between 0 and 1, then the graph of

    $$g(x) = f(cx)$$

    is a **Horizontal Stretching**, also called a **Horizontal Expansion**, of the graph of f.

  In both cases, for a given y coordinate, the x coordinate of the point on the graph of g is $\frac{1}{c}$ times the x coordinate of the point on the graph of f.

Example. The graph of the function $f(x) = e^x$ is shown in blue and the graph of the function $g(x) = e^{2x}$ is shown in red. The graph of g is a horizontal contraction of the graph of f because $g(x) = f(2x)$, that is, for a given y coordinate, the x coordinate of a point on the graph of g is $\frac{1}{2}$ times the x coordinate of the point on the graph of f.



Example. Below is the graph of $y = x^2$.
Transform it to make it the graph of $y = -2(x-4)^2$.

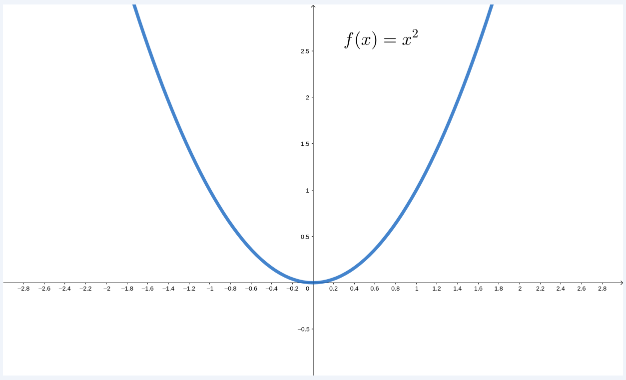


Figure 1.4: $y = x^2$

We are given the graph of $y = x^2$ and asked to graph $y = -2(x-4)^2$. Starting with the graph of $y = x^2$, we'll first graph $y = (x-4)^2$. To do this, we translate (1) the graph of $y = x^2$ to the right 4 units.

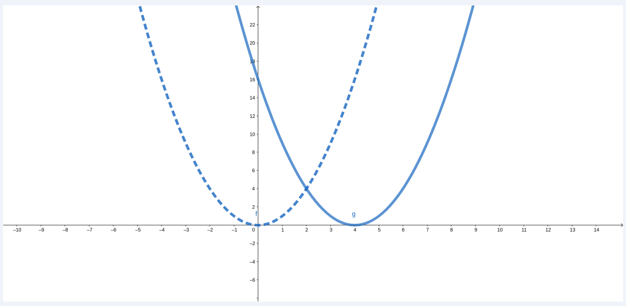


Figure 1.5: Moved to the right by 4 units.

Finally, we'll use the graph of $y = 2(x-4)^2$ to get the graph of $y = -2(x-4)^2$. To do this, we reflect the graph of $y = 2(x-4^2)$ across the x axis.

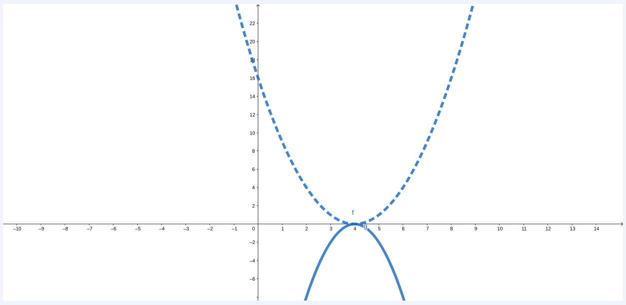


Figure 1.6: Reflected across the x axis.

CHAPTER TWO

Linear Systems

Unit 2

CHAPTER THREE

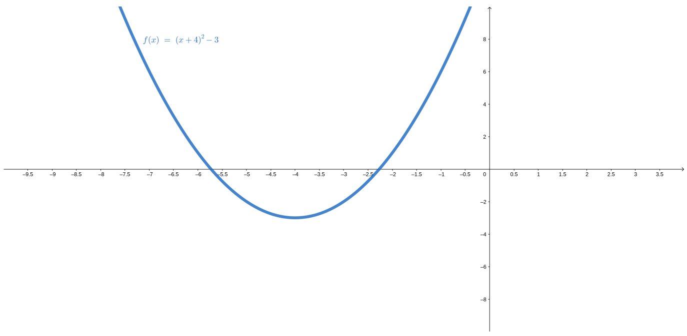
Quadratic and Polynomial Functions

Unit 4

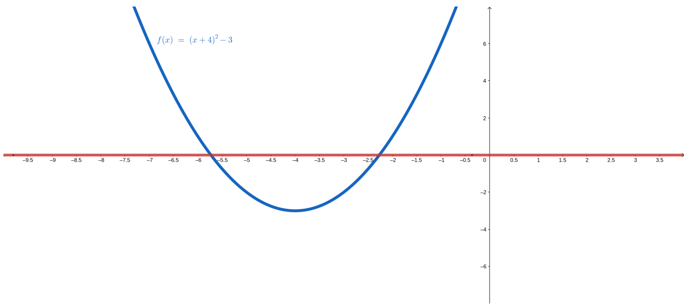
Mar 04 2022 Fri (18:07:42)

Lesson 1: Domain and range from parabola **Unit 4**

Let's try to find the domain (3) and range (4) of this parabola:



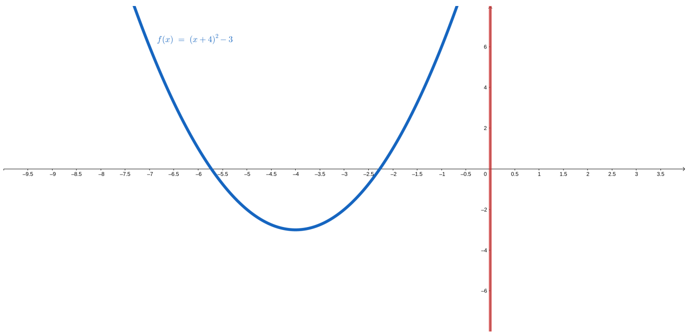
- To find the domain, we project the graph onto the x axis.



  Because the graph extends to the left and right forever, all real numbers are in the domain.

- To find the range, we project the graph onto the y axis.

  We see that the minimum value in the range is -3.



All values greater than -3 are also part of the range. This is because the graph extends upward forever.

Therefore, the range of our function is given by $y \geq -3$.

Mar 04 2022 Fri (19:03:22)

Lesson 2: Create QE from roots and LC **Unit 4**

Theorem 1. (Fundamental Theorem of Algebra) Any polynomial of degree $n > 0$ has at least one zero (real or non-real).

Theorem 2. (Factor Theorem) If r is a zero of the polynomial $P(x)$, then $x - r$ is a factor of $P(x)$. Conversely, if $x - r$ is a factor of $P(x)$, then r is a zero of $P(x)$.

Example. Consider the polynomial $P(x) = x^2 + 3x + 2$.
Because

$$P(-1) = (-1)^2 + 3(-1) + 2 = 1 - 3 + 2 = 0. \tag{3.1}$$

we have that -1 is a zero of the polynomial $P(x)$.
Hence, the **Factor Theorem** (2) guarantees that $x - (-1) = x + 1$ is a factor of $P(x)$.
That is, the factor theorem guarantees that there exists a polynomial $Q(x)$ such that

$$P(x) = (x + 1)Q(x). \tag{3.2}$$

That can be confirmed directly. Since $P(x) = x^2 + 3x + 2 = (x + 1)(x + 2)$, we have that $Q(x) = x + 2$

Remark. Given a polynomial $P(x) = a_n x^n + a_{n-1}x^{n-1} + \ldots + a_1 x + a_0$ of degree $n > 0$, we can apply the **Fundamental Theorem of Algebra** (1) and facto theorem n times to obtain what is commonly referred to as the complete factorization of $P(x)$. Namely, we can write $P(x)$ as follows:

$$P(x) = a_n(x - r_1)(x - r_2)\cdots(x - r_n). \tag{3.3}$$

where a_n is the leading coefficient of $P(x)$ and $r_1, r_2, \ldots r_n$ are the n zeroes of $P(x)$. This fact is sometimes referred to as the **Linear Factors Theorem**

Let's write a quadratic equation whose roots are -1 -3 and whose leading coefficient is 2.

We use the **Factor Theorem** (2), which states that if k is a root of the polynomial equation $P(x) = 0$, then $x - k$ is a factor of the polynomial $P(x)$.

In our problem, we have $P(-1) = 0$ $P(-3) = 0$. Thus, $x + 1$ $x + 3$ are both factors of the quadratic polynomial. Since the coefficient is 2, we can write the polynomial as:

$$2(x+1)(x+3) = 2x^2 + 8x + 6. \tag{3.4}$$

Mar 04 2022 Fri (19:33:18)

Lesson 3: Find VR, IN, AS from parabola **Unit 4**

List of Theorems

List of Postulates

List of Conjectures

List of Corollaries

List of Definitions

List of Reviews

List of Properties

List of Propositions