

$$\begin{aligned}
\langle v|w\rangle &= \langle v|\left(\sum_{i\in\mathbb{N}}|e_i\rangle\langle e_i|\right)|w\rangle \\
&= \langle v|\left(\sum_{i\in\mathbb{N}}|e_i\rangle\langle e_i|\right)\left(\sum_{j\in\mathbb{N}}|e_j\rangle\langle e_j|\right)|w\rangle \\
&= \langle v|e_i\rangle\langle e_i|e_j\rangle\langle e_j|w\rangle
\end{aligned}$$