Pre-calculus 2

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Lecture 1: Sets and Numbers

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Definition 1 (Set). A set is a collection of objects specified in a manner that enables one to determine if a given object is or isn't in the set.

Exercise 1. Which of the following represent a set?

1 The students registered for MTH 112 at PCC this quarter.

2 The good students registered for MTH 112 at PCC this quarter.

Notation. Roster Notation involves listing the elements in a set within curly brackets: "{} "

Definition 2 (Element). An object in a set is called an **element** of the set. (symbol: "$\in$")

Example. 5 is an element of the set $\{4, 5, 6, 7, 8, 9\}$. We can express this symbolically:

$$5 \in \{4, 5, 6, 7, 8, 9\}$$

.

Definition 3 (Subset). A set S of a set T, denoted $S \subseteq T$, if all elements of S are also elements of T.

If S and T are sets and $S = T$, then $S \subseteq T$. Sometimes it's useful to consider a subset S of a set T that isn't equal to T. In such case, we write $S \subset T$ and say that S is a proper subset of T.

Example. $\{4, 7, 8\}$ is a subset of the set $\{4, 5, 6, 7, 8, 9\}$.

We can express this fact symbolically by $\{4, 7, 8\} \subseteq \{4, 5, 6, 7, 8, 9\}$

Since these two sets aren't equal, $\{4, 7, 8\}$ is a proper subset of $\{4, 5, 6, 7, 8, 9\}$, so can write:

$$\{4, 7, 8\} \subset \{4, 5, 6, 7, 8, 9\}$$

.

Definition 4 (Empty Set). The empty set, denoted $\emptyset$, is the set with no elements

$$\emptyset = \{\}$$

.

Definition 5 (Union)**.** The union of two sets A and B, denoted $A \cup B$, is the set containing all of the elements in either A or B (or both A and B).

Example. Consider the sets $\{4, 7, 8\}$, $\{0, 2, 4, 6, 8\}$, and $\{1, 3, 5, 7\}$. Then ...

- $\{4, 7, 8\} \cup \{1, 3, 5, 7\} = \{1, 3, 4, 5, 6, 8\}$
- $\{4, 7, 8\} \cup \{0, 2, 4, 6, 8\} = \{0, 2, 4, 6, 7, 8\}$
- $\{0, 2, 4, 6, 8\} \cup \{1, 3, 5, 7\} = \{0, 1, 2, 3, 4, 5, 6, 7, 8\}$

Definition 6 (Intersection)**.** The intersection of two sets A and B, denoted $A \cap B$, is the set containing all the elements in both A and B.

Example. Consider the sets $\{4, 7, 8\}$, $\{0, 2, 4, 6, 8\}$, and $\{1, 3, 5, 7\}$. Then ...

- $\{4, 7, 8\} \cap \{0, 2, 4, 6, 8\} = \{4, 8\}$
- $\{4, 7, 8\} \cap \{1, 3, 5, 7\} = \{7\}$
- $\{0, 2, 4, 6, 8\} \cap \{1, 3, 5, 7\} = \emptyset$

All the whole numbers form a set. This set is called the integers, and is represented by the symbol $\mathbb{Z}$. We can express the set of integers in roster notation:

$$\mathbb{Z} = \{\ldots, -3, -2, -1, 0, 1, 2, 3, \ldots\}$$

.

Now, we can use set builder notation to create a set with a range of numbers.

Notation. Set Builder Notation.
"All the whole numbers between 3 and 10" $= \{x | x \in \mathbb{Z} \text{ and } 3 < x < 10\}$

Definition 7 (Important Sets of Numbers)**.** The set of natural numbers:

$$\mathbb{N} = \{1, 2, 3, 4, 5, \ldots\}$$

.

The set of integers:

$$\mathbb{Z} = \{\ldots, -3, -2, -1, 0, 1, 2, 3, \ldots\}$$

.

The set of rational numbers:

$$\mathbb{Q} = \left\{ x | x = \frac{p}{q} \text{ and } p, q \in \mathbb{Z} \text{ and } q \neq 0 \right\}$$

.

The set of real numbers: $\mathbb{R}$

(All the numbers on the number line)

.

The set of complex numbers:

$$\mathbb{C} = \left\{ x | x = a + bi \text{ and } a, b \in \mathbb{R} \text{ and } i = \sqrt{-1} \right\}$$

.

Note. $\mathbb{N} \subset \mathbb{Z} \subset \mathbb{Q} \subset \mathbb{R} \subset \mathbb{C}$, the set of natural numbers ($\mathbb{N}$) is a subset of the set of integers ($\mathbb{Z}$) which is a subset of the set of rational numbers ($\mathbb{Q}$) which is a subset of the set of real numbers ($\mathbb{R}$) which is a subset of the set of complex numbers ($\mathbb{C}$).

Notation. Since we use the real numbers so often, we have special notation for subsets of the real numbers. **Interval Notation**. Interval Notation involves square or round brackets.

Example. Quick demo of Interval Notation

- $\{x | x \in \mathbb{R} \text{ and } -2 \leq x \leq 3\} = [-2, 3]$
- $\{x | x \in \mathbb{R} \text{ and } -2 < x < 3\} = (-2, 3)$
- $\{x | x \in \mathbb{R} \text{ and } -2 < x \leq 3\} = (-2, 3]$
- $\{x | x \in \mathbb{R} \text{ and } -2 \leq x < 3\} = [-2, 3)$

When the interval has no upper or lower bound, we use the infinity symbol (∞ or $-\infty$)

- $\{x | x \in \mathbb{R} \text{ and } x \leq 4\} = (-\infty, 4]$
- $\{x | x \in \mathbb{R} \text{ and } x \geq 4\} = [4, -\infty)$

Exercise 2. Simplify the following expressions:

- $(-4, \infty) \cup [-8, 3]$
- $(-4, \infty) \cup (-\infty, 2]$
- $(-4, \infty) \cap (-\infty, 2]$
- $(-4, \infty) \cap [-10, -5]$

Lecture 2: Angles and Arc-Length

Apr 06 2022 (07:25:34)

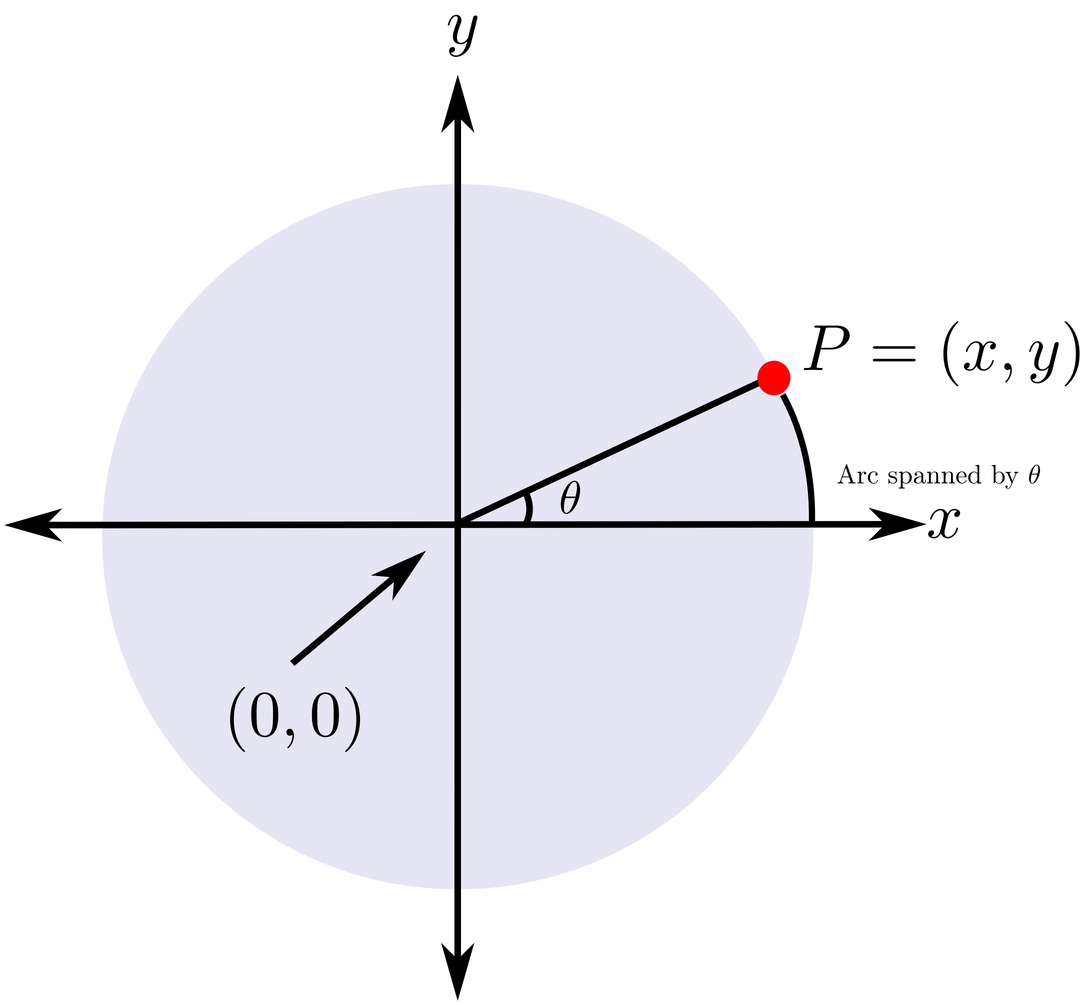

Figure 1

- The standard way that we put an angle in a circle we start the angle at the positive x-axis and go **counterclockwise**. We measure negative angles **clockwise**.
- When put an angle in standard position and you rotate, it ends someplace. Where it ends is called the **Terminal Side**.
- The point P on the circumference of the circle is **specified by the angle** θ.

- Angle θ corresponds with a portion of the circumference of the circle called the **arc spanned by** θ
- Two angles with the same terminal side are called **co-terminal angles**.

Three hundred and sixty degrees ($360°$) represents a complete trip around a circle, which is a full rotation. So, $1°$ corresponds to $\frac{1}{360°}$ of a full rotation. Degrees are more like percentages, where they represent concepts, and not numbers. But, just like how you can transform a percentage to a number ($10\% = \frac{10}{100}$), you can do the same with degrees ($10° = \frac{10}{360}$).

Since $360°$ represents a full rotation around the circle, if we add any integer multiple of $360°$ to an angle θ_1, we'll obtain an angle co-terminal to θ_1.

Example. So $45°$ and $405°$ are co-terminal.

Definition 8 (π)**.** The number π represents the ratio of the circumference of a circle to the diameter of the circle. $\pi = \frac{c}{d}$, where c is the circumference and d is the diameter.

$\pi \approx 3.14$, but π is an **"irrational"** number.

Definition 9 (Radian)**.** The **radian** measure of an angle is the ratio of the length of the arc on the circumference of the circle spanned by the angle, s, and the radius, r, of the circle.

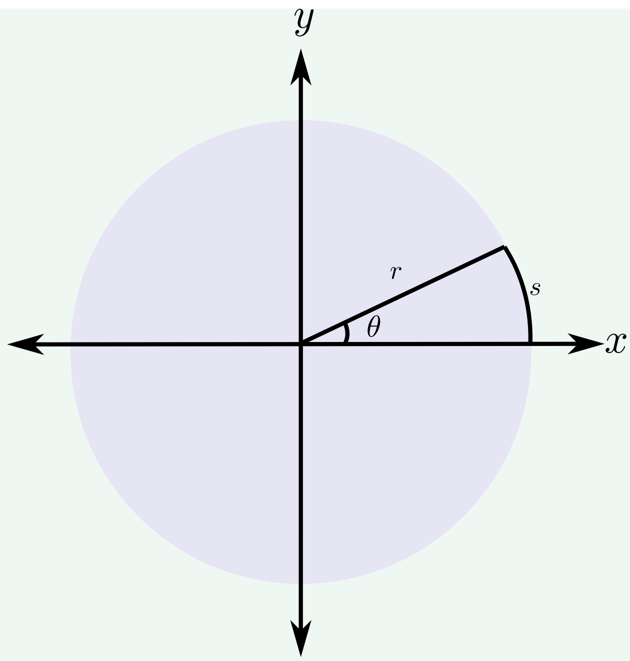

Figure 2: θ measures $\frac{s}{r}$ radian.

$$\begin{aligned} \pi &= \frac{c}{d} \\ &= \frac{2}{2r} \qquad d = 2r \qquad . \qquad (1) \\ 2\pi &= \frac{c}{r} \qquad \text{Multiply both sides by 2} \end{aligned}$$

From this, we can conclude that: $2\pi = 360°$. From that, we can get $\pi = 180°$ since we just divided both sides by 2. From that, we get $\frac{\pi}{2} = 90°$. So on, and so forth.

Here's a table with all of the radians:

θ (degrees)	$0°$	$30°$	$45°$	$60°$	$90°$	$180°$	$270°$	$360°$
θ (radians)	0	$\frac{\pi}{6}$	$\frac{\pi}{4}$	$\frac{\pi}{3}$	$\frac{\pi}{2}$	π	$\frac{3\pi}{2}$	2π

Table 1: Degrees into Radians

Exercise 3. Convert 8 radians into degrees and 8 degrees into radians:

Definition 10 (Arc Length)**.** The **arc length** s spanned in a circle of radius

r by an angle θ measured in radians is given by:

$$s = r \times \mid \theta \mid$$

.

Note. This formula only applies if θ is measured in radians.

Exercise 4. What is the arc length spanned by a $40°$ angle on a circle radius 30 meters?

Lecture 3: Introduction to Periodic Functions

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Any activity that repeats on a regular time interval can be described as *periodic*.

Definition 11 (Periodic Function)**.** A **periodic function** whose values repeat on regular intervals. Hence, f is a periodic if there exists some constant c such that:

$$f(x+c) = f(x)$$

.

for all x in the domain of f such that $f(x+c)$ is defined.

Recall that this means that if the graph $y = f(x)$ is shifted horizontally c units then it will appear unaffected.

Definition 12 (Period)**.** The **period** of a function f is the smallest value $\mid c \mid$ such that $f(x+t)$ for all x in the domain of f such that $f(x+c)$ is defined.

Definition 13 (Midline)**.** The **midline** of a periodic function is the horizontal line midway between the function's minimum and maximum values.

Example. We know that it's a periodic since an interval of the graph repeats over and over and over.

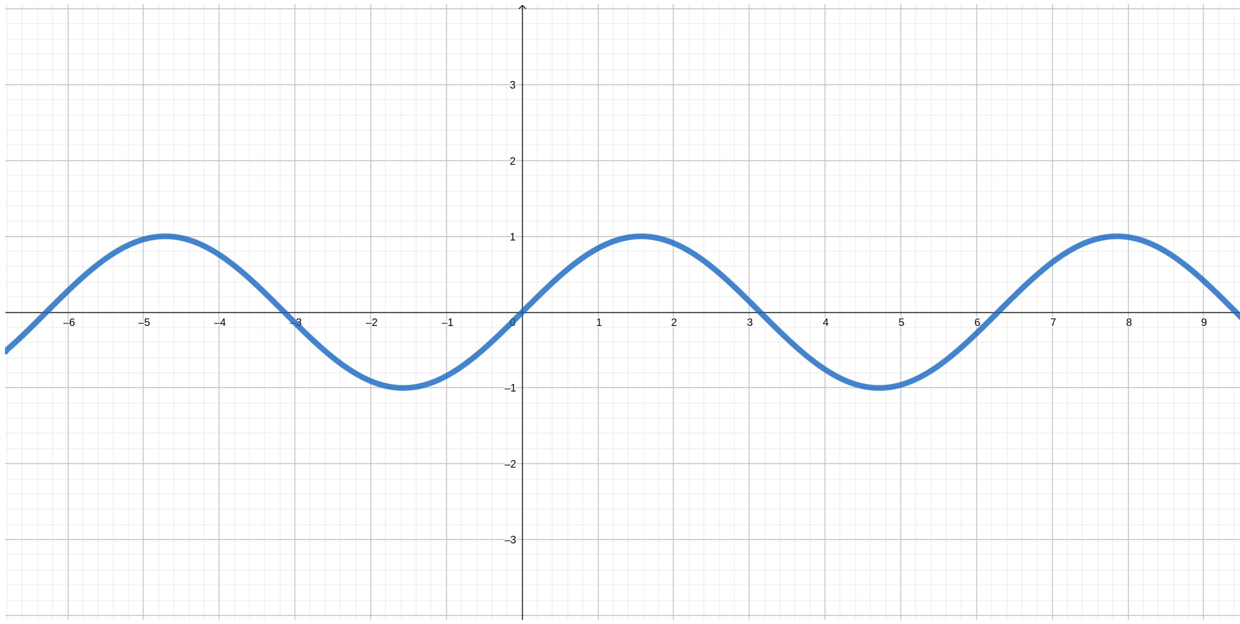

Figure 3: Periodic Function: $\sin(x)$

The period for this function is 2π.

Solutions

Solution. for **Exercise** 1

1. This represents a set since it's well defined. We all know what it means to be registered for a class.
2. This does not represent a set since it's not well defined. There are many different interpretations of what it means to be a good student (get an **A** or pass the class or attend class or avoid falling asleep in class or don't cause trouble in class)

Solution. for **Exercise** 2

- $(-4,\infty)\cup[-8,3]=[-8,\infty)$
- $(-4,\infty)\cup(-\infty,2]=(-\infty,\infty)=\mathbb{R}$
- $(-4,\infty)\cap(-\infty,2]=(-4,2]$
- $(-4,\infty)\cap[-10,-5]=\emptyset$

Solution. for **Exercise** 3

Since $2\pi=360°$ $\frac{2\pi}{360°}=1=\frac{360°}{2\pi}$. We're trying to cancel out the radians. So, to do this, we will need to have the radians on the bottom, and the degrees on the top. After we do the multiplication, we will be left with the degrees.

$$\begin{aligned} 8\,\mathrm{rad}\times\left(\frac{360°}{2\pi\,\mathrm{rad}}\right) &= \frac{8\times 360°}{2\pi} \\ &= \frac{1440°}{\pi} \\ &\approx 458.4. \end{aligned}$$

Now, let's convert 8° into 8 radians.

$$\begin{aligned} 8°\left(\frac{\pi\,\mathrm{rad}}{180°}\right) &= \frac{8\pi}{180}\,\mathrm{rad} \\ &= \frac{\pi}{45}\,\mathrm{rad} \end{aligned}$$

.

Solution. for **Exercise** 4

Convert $40°$ into radians:

$$\begin{aligned} 40° \times \left(\frac{\pi}{180°}\right) &= \frac{40\pi}{180}\,\mathrm{rad} \\ &= \frac{4\pi}{18} \\ &= \frac{2\pi}{9} \\ &\approx 0.7 \end{aligned}$$

.

Now we're ready to compute the arc length:

$$\begin{aligned} s &= r \times \mid \theta \mid \\ &= (30m) \times (\frac{2\pi}{9}) \\ &= \frac{20\pi}{3} m. \end{aligned}$$