Honors Algebra 2

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CHAPTER ONE

Radical and Polynomial Operations

Unit 1

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Lesson 1: Rational Exponents

Unit 1

Here are the basic steps to simplifying a radical expression:

Property 1 (Radical Expressions).

$$t^{\frac{3}{4}} t^{\frac{3}{4}} \times t^{\frac{3}{4}} \times t^{\frac{3}{4}} \tag{1.1}$$

$$\sqrt[4]{t^3} \tag{1.2}$$

$$\sqrt[4]{t} \times \sqrt[4]{t} \times \sqrt[4]{t} \tag{1.3}$$

$$\frac{2}{4}$$
 (1.4)

$$\sqrt[4]{t^3} \tag{1.5}$$

(1.6)

And here are the basic steps to simplifying a rational expression:

Property 2 (Rational Expressions).

$$\sqrt[5]{x^3} \sqrt[5]{x} \times \sqrt[5]{x} \times \sqrt[5]{x} \tag{1.7}$$

$$x^{\frac{3}{5}} \tag{1.8}$$

$$x^{\frac{1}{5}} \times x^{\frac{1}{5}} \times x^{\frac{1}{5}} \tag{1.9}$$

$$x^{\frac{3}{5}} \tag{1.10}$$

$$\sqrt[5]{x^3}$$
 (1.11)

(1.12)

Lesson 2: Properties of Rational Exponents

Unit 1

Now you know that when like variables are multiplied, their exponents are added. But what happens when variables are divided:

Property 3 (Quotient of Power Property). So, when you divide powers of the same base, subtract the exponents:

$$\frac{a^m}{a^n} \ a^{m-n} \tag{1.13}$$

(1.14)

Example (Example 1). Express the quotient of $\frac{r^{\frac{6}{7}}}{r^{\frac{7}{7}}}$ as a radical:

$$\frac{r^{\frac{6}{7}}}{r^{\frac{2}{7}}} r^{\frac{6}{7} - \frac{2}{7}} \tag{1.15}$$

$$r^{\frac{4}{7}} \tag{1.16}$$

$$r^{\frac{4}{7}}$$
 (1.16)

$$\sqrt[7]{r^4} \tag{1.17}$$

Property 4 (Quotient of Power Property). To raise a power to a power, multiply the exponents:

$$(a^m)^n \ a^{m \times n} \tag{1.18}$$

Example (Raising a Power to a Power).

$$\left(c^{\frac{1}{2}}\right)^{\frac{1}{4}} c^{\frac{1}{2} \times \frac{1}{4}} \tag{1.19}$$

$$c^{\frac{1}{8}}$$
 (1.20)

$$\sqrt[8]{c}$$
 (1.21)

Property 5 (Negative Rational Exponents). You have learned all about positive integer exponents and rational, or fractional, exponents. Let's take a look at another type: Negative Integer Exponents.

$$4^{-1} \frac{1}{4^{1}} \frac{1}{4} \tag{1.22}$$

$$4^{-2} \frac{1}{4^2} \frac{1}{16} \tag{1.23}$$

$$4^{-3} \frac{1}{4^3} \frac{1}{64} \tag{1.24}$$

Example (Negative Rational Exponents).

$$9^{-\frac{1}{2}} \frac{1}{0^{\frac{1}{2}}} \tag{1.25}$$

$$\frac{1}{\sqrt{9}} \tag{1.26}$$

$$\frac{1}{3} \tag{1.27}$$

Lesson 3: Solving Radical Equations

Unit 1

Definition 1 (Radical Equations). Here is what a **Radical Equation** looks like:

$$T 2\pi \sqrt{\frac{L}{32}}$$
.

Example (Radical Equations 1). Let's solve $\sqrt{(5x \ 4)} \ 7$:

$$sqrt(5x 4) 7$$
 (1.28)

$$(\sqrt{5x} \ 4)^2 \ (7)^2 \tag{1.29}$$

$$5x \ 4 \ 49$$
 (1.30)

$$5x \ 45$$
 (1.31)

$$\frac{5x}{5} \frac{45}{5}$$
 (1.32)

$$x 9$$
 (1.33)

Now, let's check our work:

$$\sqrt{5x} \stackrel{4}{4} 7 \tag{1.34}$$

$$\sqrt{5(9)} \ 4 \ 7$$
 (1.35)

$$\sqrt{45} \ 4 \ 7$$
 (1.36)

$$\sqrt{49} \ 7$$
 (1.37)

Let's try another one:

Example (Radical Equations 2). Let's solve $\sqrt{x-3} \ 4 \ 1$:

$$\sqrt{x-3} \ 4 \ 1$$
 (1.39)

$$\sqrt{x-3} -3 \tag{1.40}$$

$$(\sqrt{x-3})^2 \ (-3)^2 \tag{1.41}$$

$$x - 39$$
 (1.42)

$$x 12$$
 (1.43)

Now, let's check our work:

$$\sqrt{x-3} \ 4 \ 1$$
 (1.44)

$$\sqrt{12-3} \ 4 \ 1$$
 (1.45)

$$\sqrt{9} \ 4 \ 1$$
 (1.46)

We call this situation extraneous solution.

Quick Review of Factoring

Let's look at factoring by group.

First, you want the split the middle term into factors of 15 that combine to equal $2. \,$

Then, just factor by GCF.

$$x^2 2x - 15 x^2 - 3x 5x - 15 (1.49)$$

$$(x^2 - 3x) (5x - 15) (1.50)$$

$$x(x-3)\ 5(x-3) \tag{1.51}$$

$$(x\ 5)(x-3)$$
 (1.52)

Lesson 4: Complex Numbers

Unit 1

Imaginary Numbers

What pairs of identical factors will produce -1 when multiplied? 1×1 doesn't work since it equals positive 1. The same is true of -1×1 which also equals positive 1.

 1×-1 equals -1, but these are not identical factors.

Definition 2 (*i*). To solve this problem, the concept of the imaginary number i was invented. The imaginary number was defined to be:

$$i\sqrt{-1}$$

With this definition, the square root of negative radicands, in addition to positive radicands, may be simplified.

Example (i). Simplify: $\sqrt{-8}$

$$\sqrt{-1} \times \sqrt{8} \ i \times \sqrt{8} \tag{1.53}$$

$$i\sqrt{8}$$
 (1.54)

$$i\sqrt{4} \times \sqrt{2} \tag{1.55}$$

$$2i\sqrt{2} \tag{1.56}$$

Squaring Imaginary Numbers

You know that $i \sqrt{-1}$

Consequently, $i^2 (\sqrt{-1})^2$

Since squaring a square root will eliminate the square root sign:

$$\sqrt{3^2} \sqrt{3} \times \sqrt{3} \tag{1.57}$$

$$\sqrt{9} \tag{1.58}$$

$$3$$
 (1.59)

Then:

$$i^2 (\sqrt{-1})^2$$
 (1.60)
 i^2 (1.61)

$$^{2} \tag{1.61}$$

$$-1 \tag{1.62}$$

Cubing Imaginary Numbers

What about cubing the imaginary number:

$$i^3 i \times i \times i \tag{1.63}$$

$$i^3$$
 (1.64)

$$(i \times i) \times i \tag{1.65}$$

$$i^3 i^2 \times i \tag{1.66}$$

$$i^3 -1 \times i \tag{1.67}$$

$$i^3$$
 (1.68)

$$-i (1.69)$$

Biquadrate Imaginary Numbers

i to the fourth power can be found in a similar way:

$$i^4 \ i \times i \times i \times i \tag{1.70}$$

$$^{4} \tag{1.71}$$

$$(i \times i) \times (i \times i) \tag{1.72}$$

$$i^4$$
 (1.73)

$$-1 \times -1 \tag{1.74}$$

$$i 1$$
 (1.75)

Simplifying i

Example (Simplifying i). Simplify: i^{27}

$$\begin{array}{ccc} \frac{27}{4} & 6R \longrightarrow 3 \\ & i^{R \longrightarrow 3} \end{array}$$
 (1.76)

$$i^{R\longrightarrow 3}$$
 (1.77)

$$-i ag{1.78}$$

Lesson 7: Operations on Complex Numbers

Unit 1

Multiplying Complex Numbers

While the process of adding and subtracting complex numbers was similar to that of polynomials and radicals, multiplying complex numbers is just a little bit different. Think about in which order this multiplication should take place.

Let's see what happens when the imaginary number is factored first.

$$\sqrt{-12} \times \sqrt{-5} \ (\sqrt{-1} \times \sqrt{12}) \times (\sqrt{-1} \times \sqrt{5}) \tag{1.79}$$

$$i\sqrt{12} \times i\sqrt{5} \tag{1.80}$$

Once the imaginary numbers have been factored, multiply the imaginary numbers together and multiply the radical factors together.

$$i\sqrt{12} \times i\sqrt{5} \ i^2\sqrt{60} \tag{1.81}$$

Now that the two factors have been multiplied, simplify the remaining radicand by either rewriting it as a product of its prime factors or by finding the perfect square factor.

Sep 27 2021 Mon (07:30:12)

Lesson 8: Polynomial Operations

Unit 1

Adding Polynomials

The most important part of adding or subtracting polynomials is identifying like terms. Like terms are terms containing the exact same "variable part." Exponents for the variables must be exactly the same. Coefficients can, and probably will, be different. Identify the like terms in the following matching exercise.

Example (Adding Polynomials). Add: $(2x^3 4x^2 - x 7) (3x^2 6x 10)$

- 1. Again, there is an understood 1 in front of each set of parentheses. As you've seen, distributing this 1 will not change the expression. Therefore, the parentheses may simply be removed.
- 2. Highlight each pair of terms containing the same variable part. Combine the like terms.

Note: Since there is no other term with a variable part of x^3 , the term $2x^3$ stays the same in the final answer.

$$(2x^3 4x^2 - x 7) (3x^2 6x 10) 2x^3 4x^2 - x 7 3x^2 6x 10$$
 (1.82)

$$2x^3 \ 7x^2 \ 5x \ 17 \tag{1.83}$$

Subtracting Polynomials

Subtracting polynomials is almost exactly like adding polynomials. The only "difference" is that now a negative one must be distributed.

Example (Subtracting Polynomials). Subtract: $(4x^2 - 5) - (x^3 2x^2 7)$

The understood 1 in front of the first set of parentheses will not change the binomial within. However, the trinomial in the second set of parentheses will change because a negative one (-1) must be distributed! In effect, it will change the signs of those three terms.

- 1. Highlight each pair of terms containing the same variable part.
- 2. Combine the like terms. Since $-x^3$ does not have a like term, it is written down as is. Arrange terms in descending order, from greatest exponent to least.

$$1(4x^2-5)-1(x^3\ 2x^2\ 7)\ 4x^2-5-x^3-2x^2-7$$
 (1.84)

$$4x^2 - 5 - x^3 - 2x^2 - 7 \tag{1.85}$$

$$-x^3 2x^2 - 12 \tag{1.86}$$

Inverses of Functions

Finding the inverse of an integer means performing an operation that will cancel that number. For example, if you are given the number 7, you could add -7 to it to cancel it.

Let's look at an example:

Example (Inverse of Functions).

$$f(x) \ 3x - 5 \tag{1.87}$$

$$y \ 3x - 5$$
 (1.88)

$$x \ 5 \ 3y$$
 (1.89)

$$\frac{x \, 5}{3} \, y \tag{1.90}$$

$$f^{-1}x \, \frac{x \, 5}{3} \tag{1.91}$$

$$f^{-1}x \frac{x \, 5}{3} \tag{1.91}$$

Operations on Functions

Addition The addition of two functions f(x) and g(x) is represented using the notation f(x) g(x).

Let f(x) 4x-7 and g(x) 10x-3. To add f(x) and g(x), the expressions 4x-7and 10x - 3 need to be added.

Therefore, this could be written as f(x) g(x) - (4x - 7) (10x - 3).

- 1. Simplify the right side by first distributing any coefficients outside the parentheses. If no number or variable appears before the parentheses, an understood 1 exists.
- 2. When 1 is distributed to each term within each set of parentheses, the expression remains unchanged.
- 3. Identify and combine like terms.

$$f(x) g(x) (4x-7) (10x-3)$$
 (1.92)

$$1(4x-7)\ 1(10x-3)$$
 (1.93)

$$4x - 7\ 10x - 3$$
 (1.94)

$$14x - 10$$
 (1.95)

$$f(x) g(x) 14x - 10 (1.96)$$

Subtraction Subtraction of functions is similar to addition. The only difference is that you must be very careful of sign changes!

Let f(x) 4x-7 and g(x) 10x-3. To subtract f(x) and g(x), the expressions 4x - 7 and 10x - 3 need to be subtracted.

Therefore, this could be written as f(x) - g(x) (4x - 7) - (10x - 3).

- 1. Simplify the right side by first distributing any coefficients outside the parentheses. If no number or variable appears before the parentheses, an understood 1 exists.
- 2. Be very careful not to forget to distribute the -1 to each term in the second set of parentheses. Identify and combine like terms.

$$f(x) - g(x) (4x - 7) - (10x - 3)$$
 (1.97)

$$1(4x-7) - 1(10x-3) (1.98)$$

$$4x - 7 - 10x - 3 \tag{1.99}$$

$$-6x - 4$$
 (1.100)

$$f(x) - g(x) - 6x - 4$$
 (1.101)

CHAPTER TWO

Factoring and Quadratics

Unit 2

Oct 11 2021 Mon (10:42:23)

Lesson 1: Greatest Common Factors and Special Products

Unit 2

Difference of Squares Binomials

One of the special products in this lesson is the difference of squares binomial. Before discovering how to identify and factor these special products, it is important to review how to multiply binomials using the Distributive Property.

Question. What do you notice about the first terms in each of the prdocuts of $q^2 - 16$, $g^2 - 81$, and $25g^2 - 64$?

Answer. The terms g^2 and $25g^2$ are perfect squares. Remeber, prefect squares are numbers whose square root is a whole number, or variables with an even exponent.

Question. What do you notice about the last terms in each of the products $g^2 - 16$, $g^2 - 81$, and $225g^2 - 64$?

Answer. They, too, are all perfect squares.

Question. What sign separates the first and last terms in each of the products $g^2 - 16$, $g^2 - 81$, and $225g^2 - 64$?

Answer. They are all subtraction signs.

Question. Now try working backwards. Find the two sets of binomials that multiply together to produce g^2-49 . What must you do to each of the terms in g^2-49 to come up with the factors?

Answer. The two binomial factors of $g^2 - 49$ are g 7 and g - 7. Take the square root of g^2 to come up with g and 7.

Question. Why do you think the signs must be different in each of the factors?

Answer. The signs are different so that, when the two sets of parentheses are multiplied, the last term will be negative and the middle terms will cancel.

Question. Will $25g^2\ 64$ factor into two sets of parentheses like the examples above?

Answer. No, to fit the pattern, there must be a subtraction sign between the two terms. $25g\ 64$ is considered prime because it cannot be factored any further.

Example.

$$(g 6)(g 6) g^2 6g 6g 36$$
 (2.1)

$$g^2 \ 2(6g) \ 36$$
 (2.2)

$$g^2 \ 12g \ 36$$
 (2.3)

When you see a trinomial where the first and last terms are perfect squares and the middle term is twice the product of the square roots of those terms, you have a perfect square trinomial that can be factored using the following pattern:

$$a^2 \ 2ab \ b^2 \ (a \ b)2$$
 (2.4)

OR

$$a^2 - 2ab \ b^2 \ (a - b)2$$
 (2.5)

Oct 12 2021 Tue (09:23:42)

Lesson 2: Factoring by Grouping

Unit 2

Factoring by Grouping (Four-Term Polynomials)

Let's just jump into an example:

Example (Factor $30c^3 - 20c^2 - 15c \ 10$).

$$30c^2 - 20c^2 - 15c \ 10 \ 5(6c^3 - 4c^2 - 3c \ 2) \tag{2.6}$$

$$5[2c^2(3c-2) - 1(3c-2)] (2.7)$$

$$5[2c^2(3c-2)-1 (3c-2)]$$
 (2.8)

$$5(3c-2)(2c^2-1) (2.9)$$

Oct 12 2021 Tue (10:03:31)

Lesson 3: Sum and Difference of Cubes

Unit 2

Factoring the Sums and Differences of Cubes

Example (Factor $30c^3 - 20c^2 - 15c \ 10$).

$$2z^3 \ 250 \ 2(z^3 \ 125)$$
 (2.10)

$$z^3 5^3 (z 5)(z^2 - z \times 5 5^2)$$
 (2.11)

$$2(z\ 5)(z^2 - z \times 5\ 5^2) \tag{2.12}$$

(2.13)

Oct 18 2021 Mon (10:55:28)

Lesson 4: Graphing Quadratics

Unit 2

Parts of a Parabola

Parabolas consist of a:

- Vertex
- · Axis of Symmetry
- Domain
- Range
- X-Y Intercepts (Sometimes though)

Vertex Here is how you find the vertex of a parabola:

- Step One: x-Coordinate Set the expression inside the parentheses equal to 0 and solve for the value of x
- Step Two: y-Coordinate Substitute the value of x from step one into the original equation and solve for y

Let's try an example:

Example (Given the equation $f(x) - 2(x-5)^2$ 8, find the (x,y) coordinate of the vertex).

$$f(x) -2(x-5)^2 8 (2.14)$$

$$-2([x-5 \ 0])^2 \ 8 \qquad \qquad 5-5 \ 0 \tag{2.15}$$

$$x 5$$
 (2.16)

Now, we're going to solve for the y coordinate

$$f(x) -2(5)(5-5)^2 8$$
 (2.17)

$$-2(5-5)^2 8 (2.18)$$

$$-2(0)^2 8 ag{2.19}$$

$$-2(0)$$
 8 (2.20)

$$y 8$$
 (2.21)

(2.22)

Now, we know that the vertex of the parabola is (5,8).

Elements of the Parabola

Line of Symmetry The **axis of symmetry** is in the line that divides the parabola into two equal parts where each part is a mirror reflection of the other.

As you can see, the axis of symmetry is along the line x-3.

To find the equation of the axis of symmetry algebraically, set the expression inside the parentheses equal to 0 and solve for x.

Domain and Range Any number can be substituted for the variable x to produce a unique y value. Therefore, the domain is "All Real Numbers.

This parabola opens up since the value is positive (). Because the parabola opens up, the vertex represents the minimum point on the graph. All of the y values on the parabola must be above the minimum y-coordinate of -1, which makes the range of the parabola this:

$$y \ge -1 \tag{2.23}$$

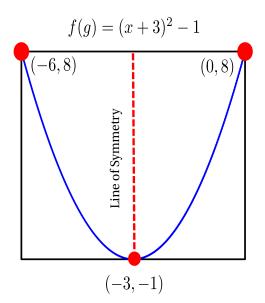


Figure 2.1: Here is an example of what a parabola looks like

Intercepts Substituting 0 for x and solving for y will result in the y-intercept:

$$f(g) (x 3)^2 - 1$$
 (2.24)

$$(0\ 3)^2 - 1$$
 (2.25)

$$(3)^2 - 1$$
 (2.26)

$$9-1$$
 (2.27)

There you go, you have your y-intercept. To determine the x-intercept, you just plug in the y-intercept:

8

$$f(9) (9 3)^2 - 1$$
 (2.29)

$$(3)^2 - 1 \tag{2.30}$$

$$9-1$$
 (2.31)

Different Quadratic Equation forms

While identifying the:

- 1. Vertex
- 2. Axis of Symmetry
- 3. Domain
- 4. Range

5. Intercepts

is most easily done using the vertex form of a quadratic equation, most of the time, quadratic equations are written in standard form.

Here is the standard form of a quadratic equation:

$$f(x) ax^2 bx c (2.33)$$

Now, we can move that into vertex form, here is an example:

$$f(x) -2x(x-5)^2 8 (2.34)$$

$$-2x(x-5)(x-5) 8 (2.35)$$

$$-2x(x^2 - 5x - 5x \ 25) \ 8 \tag{2.36}$$

$$-2x(x^2 - 10x \ 25) \ 8 \tag{2.37}$$

$$-2x^2 \ 20x - 50 \ 8 \tag{2.38}$$

$$-2x^2 \ 20x - 42 \tag{2.39}$$

(2.40)

But, it's not that easy to identify the vertex and axis, so let's us Axis of Symmetry equation, which is:

$$f(x) - \frac{b}{2a} \tag{2.41}$$

Now, let's try to convert an equation in the standard form to this form:

$$f(x) -2x^2 20x - 42 (2.42)$$

$$-\frac{20}{2(-2)}\tag{2.43}$$

$$-\frac{20}{2(-2)}$$

$$-\frac{20}{-4}$$
(2.43)
(2.44)

$$x 5 (2.45)$$

which gives you the axis of symmetry for the x-axis. Now, just substitute 5for x to find the y-coordinate:

$$f(x) -2x^2 20x - 42 (2.46)$$

$$-2(5)^2 \ 20(5) - 42 \tag{2.47}$$

$$-2(25)\ 100 - 42 \tag{2.48}$$

$$-50\ 100 - 42$$
 (2.49)

$$8 (2.50)$$

$$(5,8)$$
 (2.51)

(2.52)

(2.60)

Working our way up

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Now, I have discussed how you can find key features within the equation, but let's try and make our equation from the graph. Taking the image of the parabola, let's try and see if we can create the equation from the graph:

Let's take the vertex (-3, -1) and a random point, (0, 8). Here is how it works:

 $f(x) a(x-h)^2 k$

$$a(x 3)^{2} - 1$$

$$8 a(0 3)^{2} - 1$$

$$8 a(0 3)^{2} - 1$$

$$(2.54)$$

$$8 a(0 3)^{2} - 1$$

$$(2.55)$$

$$a(3)^{2} - 1$$

$$(2.56)$$

$$9a - 1$$

$$9 9a$$

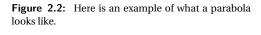
$$(2.57)$$

$$9 9a$$

$$(2.58)$$

$$\frac{9}{9} \frac{9}{9a}$$

$$(2.59)$$



(-3, -1)

 $f(g) = (x+3)^2 - 1$

Line of Symmetry

(0, 8)

-6, 8)

Now, let's put it all together:

$$a 1,h -3,k -1$$
 (2.61)

$$f(x) a(x-h)^{2} k$$
 (2.62)

$$(x 3)^{2} -1$$
 (2.63)

$$(x 3)(x 3) -1$$
 (2.64)

$$(x^{2} 3x 3x 9) -1$$
 (2.65)

$$(x^{2} 6x 9) -1$$
 (2.66)

$$x^{2} 6x 8$$
 (2.67)
(2.68)

Oct 18 2021 Mon (11:31:19)

Lesson 5: Completing the Square

Unit 2

Intro to the Discriminant

The formula for the **discriminant** is:

$$b^2 - 4ac$$
 (2.69)

Let's try a quick example:

Example.

$$f(x) x^2 12x 26 (2.70)$$

$$b^2 - 4ac \tag{2.71}$$

$$(12)^2 - 4(1)(26) \tag{2.72}$$

$$144 - 104$$
 (2.73)

So, the discriminant isn't a perfect square. This tells us that there will be two irrational numbers.

Using Vertex Form: Minimum or Maximum

Example.

$$f(x) (x 6)^2 - 10 (2.75)$$

From this equation, we get the vertex of the parabola, which is (-6, -10) But, it raises the question. Is it the vertex minimum, or the vertex maximum?

To solve this, you must look at the value or the leading coefficient to make this determination.

In this case, it would be a 1, which is positive, meaning it goes upward.

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Lesson 6: Solving Quadratic Equations

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Unit 2

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