PLANETARY WOMBS

THE PROTOPLANETARY DISC AND THE MAGNETIC FLUID IN ASTROPHYSICS

Barshan Ray.

WHAT'S THE FIRST THING THAT COMES TO YOUR MIND WHEN YOU THINK OF THE WORD, "ASTROPHYSICS"?

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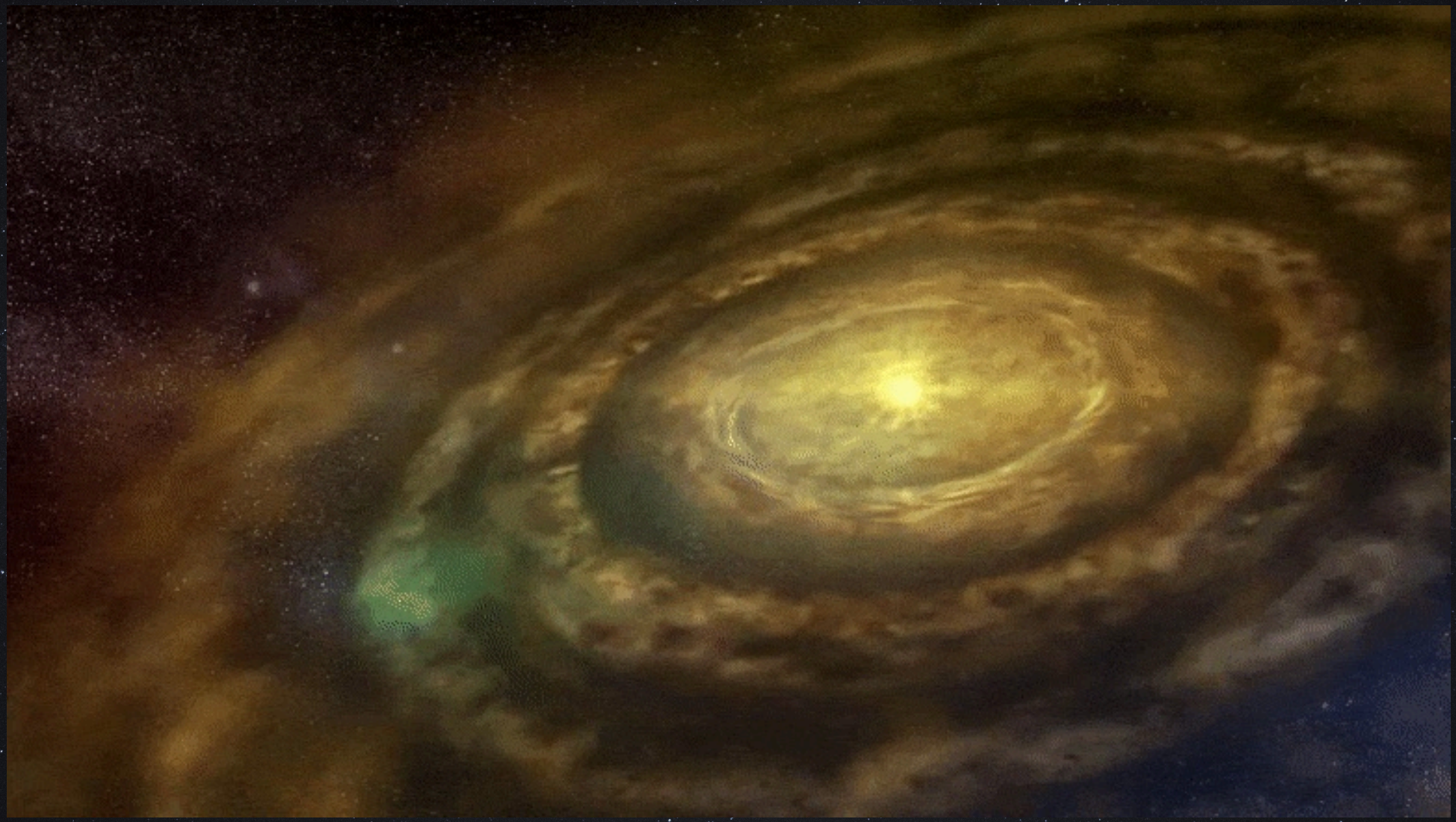
LET US FOCUS OUR ATTENTION TO STARS.

A star, in all its might, is not immortal.
Each star has a life cycle, often referred to as the **"stellar life cycle"**.

Among all the beautiful, complex, intricate, and involved stages in a star's life, **two stand out.**

THE BLACK HOLE

THE PROTOSTAR



WHAT THIS TALK IS NOT ABOUT

WHAT THIS TALK IS NOT ABOUT

1. Black holes.

WHAT THIS TALK IS NOT ABOUT

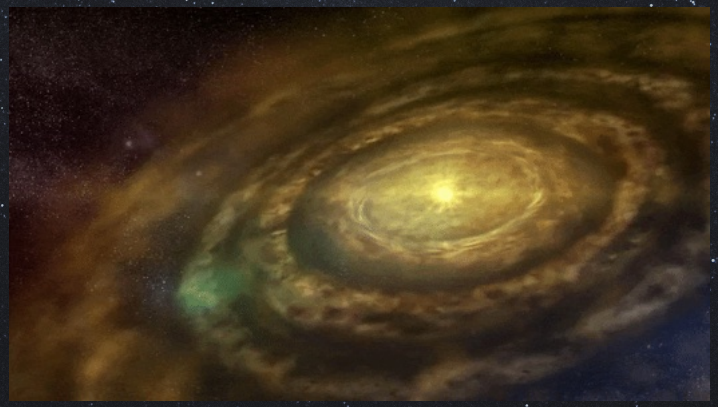
1. Black holes.
2. I really don't understand black holes.

THE PROTOSTAR

A **baby star**. Literally - unlike every Indian parent's dream.

Formed in stellar nurseries, called nebulae, inside molecular clouds.

Let us return to the two pictures.



Do you notice any similarity?

It's a similarity that your crush shares with them too.

They always seem to be surrounded by things revolving around them.

QUESTION

What does the solar system look like?

(a) A flat surface, like a roti perhaps, with the Sun and the planets on the surface? The way we were taught in primary school?

(b) Maybe like an atom, with all the planets in three-dimensional orbits around the Sun?

QUESTION

What does the solar system look like?

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WHY?

Why are all the planets in the same plane (a.k.a. the orbital plane) around the Sun?

WHY?

As the spinning cloud of dust and gas started to collapse, it also flattened. Imagine a pizza maker throwing a spinning slab of dough into the air. As it spins, the dough expands but becomes increasingly thin and flat. That's what happened to the very early solar system.

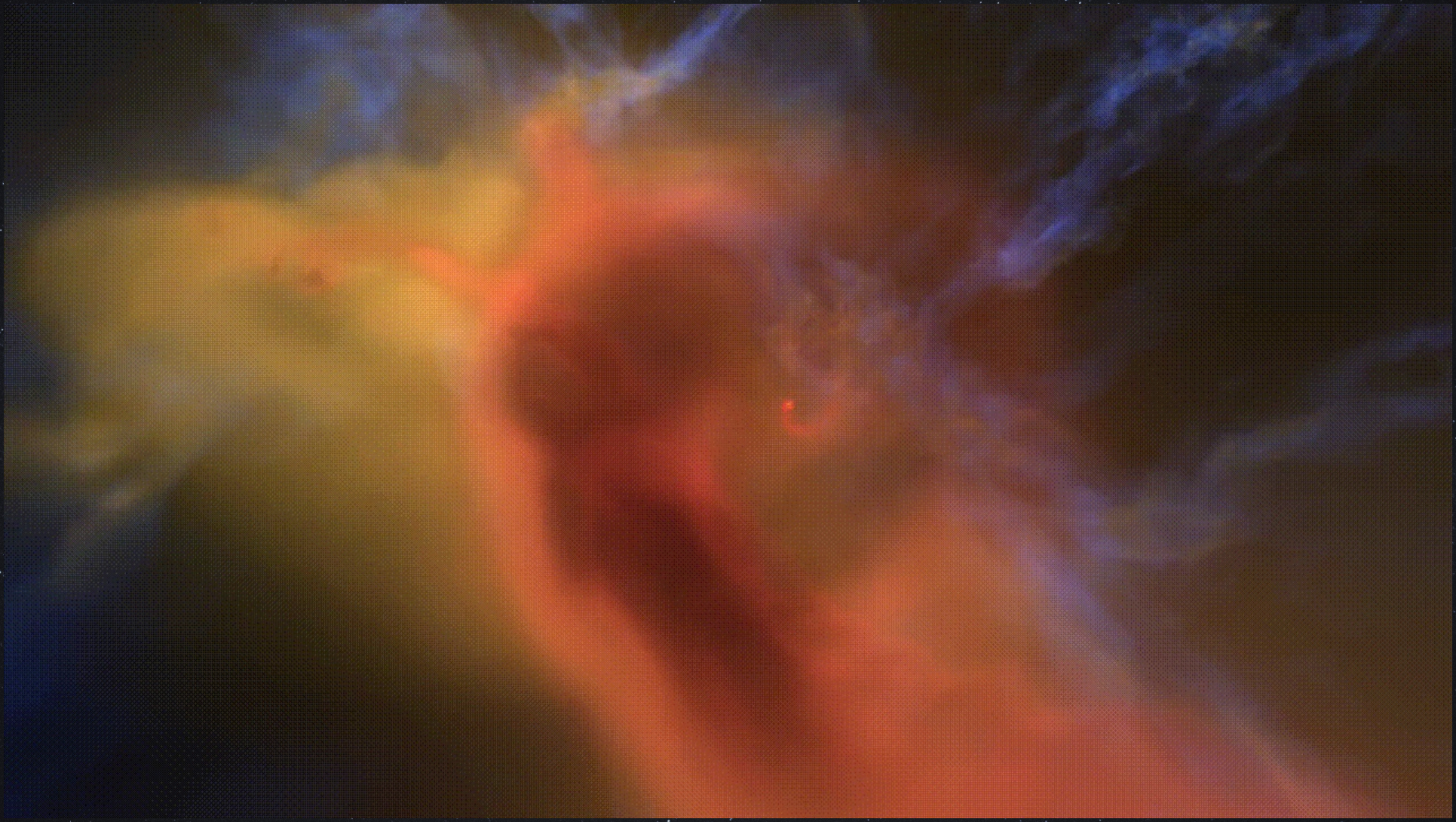
https://www.livescience.com/planets-orbit-same-plane

WHY?

In short, angular momentum.

> Rings and disks are common in astronomy. When a cloud collapses, the conservation of angular momentum amplifies any initial tiny spin of the cloud. As the cloud spins faster and faster, it collapses into a disk, which is the maximal balance between gravitational collapse and centrifugal force created by rapid spin.
>
> The result is the coplanar planets, the thin disks of spiral galaxies, and the accretion disks around black holes.

https://earthsky.org/space/planets-single-plane/



THE PROTOPLANETARY DISC

This flattened out disc around a protostar - the birthplace for planets and planetesimals, "proto-planets".

This is made up of the "thing revolving around [the star]".

WHY DO WE CARE?

1. Birthplace of planets.
2. Provides insight into our own solar system.

WHY DO WE CARE?

3. Looks cool as hell.

Image credit: ESO
(http://www.eso.org/public/images/eso1301a/)

Enough introduction. Let's take a deep dive into this object.

Before that,

How is physics different from the usual idea of "mathematics"?

How is physics different from maths?

Application of a mathematical framework to systems observed, to obtain insights into it.

1. Insights into time evolution of system - dynamics.
    1. Involves mechanics.
2. Insights into equilibria - statics.

How is physics different from maths?

"MODELLING."

MODELLING

What is astrophysical modelling?

- Stars posing as models?

MODELLING

What is astrophysical modelling?

- ~~Stars posing as models?~~
- Essentially solving equations to obtain information about a system.
- Results must match observed data.

MODELLING

Let us try our hand at modelling our current system, and see if we can establish some information about it, given certain initial/boundary conditions.

Let's start by breaking down the system into smaller components.

What are we looking at?

- The protostar.
- The disc.
- An infalling envelope of gas.
- Jet outflows.

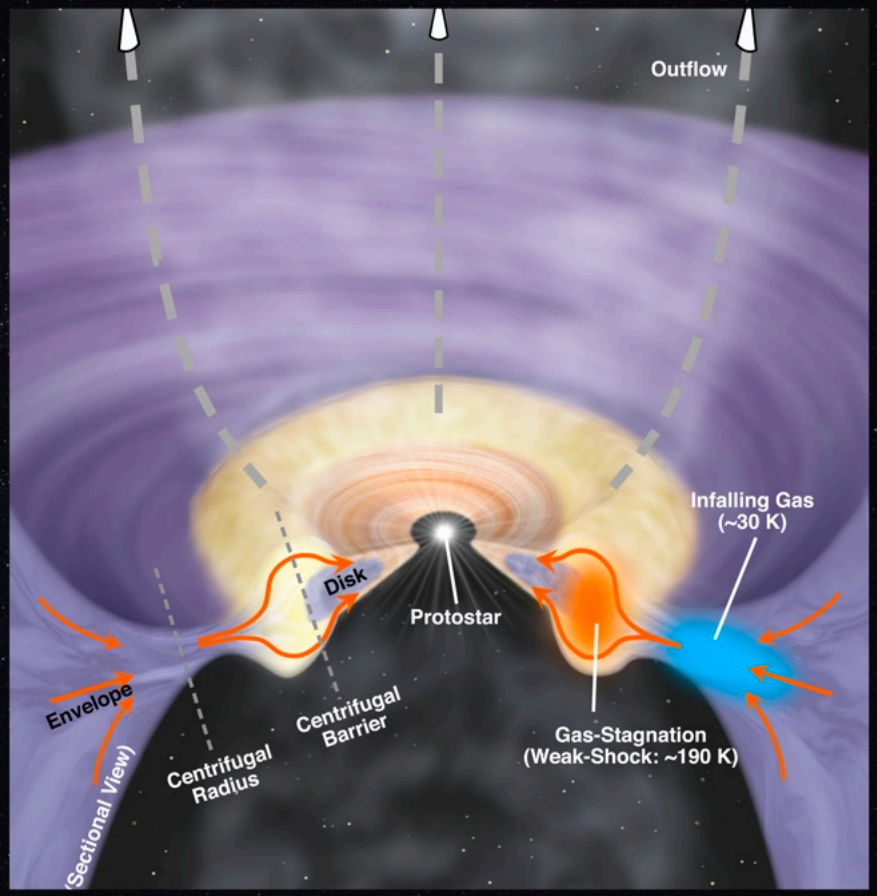


Image credit:
https://www.irap.omp.eu/en/2017/02/alma-reveals-the-curious-geometry-of-a-proto-star/

HOW DO WE DESCRIBE IT?

Now that we have a system to work with, we need to choose an appropriate prescription to apply, that we may figure out what governs the system.

In this particular talk, we shall try to infer about the radius of a stable disc formed.

HOW DO WE DESCRIBE IT?

1. We know that the matter surrounding the protostar is fluid.
2. Observations have shown that the underlying magnetic field plays an important role in determining the dynamics of the system. (Why? Ask me.)
3. Fluid and magnetic fields unite - do we have a prescription for this?

MAGNETOHYDRODYNAMICS.

IDEAL MHD

Let us quickly go through an overview of the MHD equations.

1. The equation of motion (momentum conservation equation):

$$\frac{\partial \rho \vec{u}}{\partial t} = -\vec{\nabla} \cdot \left[\rho \vec{u}\vec{u} + \left(P + \frac{B^2}{2} \right) \mathbb{I} - \vec{B}\vec{B} \right] - \rho \vec{\nabla}$$

2. The induction equation:

$$\frac{\partial \vec{B}}{\partial t} = \vec{\nabla} \times \left(\vec{u} \times \vec{B} \right)$$

They look scary, but don't have to be!

Looking at the <u>momentum conservation</u> equation,

$$\underbrace{\frac{\partial \rho \vec{u}}{\partial t}}_{\text{Rate of change of mass flux}} = [\underbrace{\rho \vec{u}\vec{u}}_{\text{Convective term}} + \underbrace{\left(P + \frac{B^2}{2}\right)}_{\text{The Pressure term}} \mathbb{I} - \vec{B}\vec{B}] - \underbrace{\rho \vec{\nabla}}_{\text{The potent}}$$

essentially relates how the mass flux in the system change

epending on different mechanical forces (including pressur

potentials such as gravitational potentials, etc.)

As for the induction equation,

$$\frac{\partial \vec{B}}{\partial t} = \vec{\nabla} \times \left(\vec{u} \times \vec{B} \right)$$

it describes the advection of magnetic field by the fluid flow. Definitely not as scary.

Does using ideal MHD help us describe the system any better than a simple fluid dynamics model?

Well, scientists went ahead and did the math so we don't have to, and found

NO.

Well of course, considering there's still so much time left in my talk, it'd be a shame if this is all there was to it.

THE MAGNETIC CATASTROPHE

THE MAGNETIC CATASTROPHE

Attempting to describe this system using ideal MHD leads to a fatal problem.

1. NO DISK is formed.
2. Catastrophic magnetic braking. What is it? Glad you asked.

THE WAY OUT?

~~Cry and give up? The system is complicated enough as it is. All this for nothing?~~

An ongoing discussion in the astrophysical community. Several approaches have been proposed, but one is widely accepted and pursued as of now.

The goal? Inhibit the magnetic effects to some extent.

One thing is for certain - all of these approaches complicate things more.

"THE WORLD IS NON-IDEAL."

It's not really a quote. I just kind of made that up.

NON-IDEAL MHD

Ideal MHD assumes perfect conductivity in the fluid. It isn't hard to convince oneself that dissipative effects are not negligible in a system where collisions are important. The fluid no longer remains adiabatic.

NON-IDEAL MHD

Three effects primarily add to non-ideal effects in magnetohydrodynamic systems.

1. Ambipolar diffusion.
2. Ohmic dissipation.
3. Hall effect.

NON-IDEAL MHD

Three effects primarily add to non-ideal effects in magnetohydrodynamic systems.

1. Ambipolar diffusion. Separation between charged and uncharged particles.
2. Ohmic dissipation. Finite resistivity; dissipation of magnetic field into heat.
3. Hall effect. More complicated. Coupling between magnetic field and electric field due to particle motion.

THE TERMS

1. Ambipolar diffusion.

$$\eta_A \frac{\vec{B}}{||\vec{B}||} \times \left(\vec{\nabla} \times \vec{B}\right) \times \frac{\vec{B}}{||\vec{B}||}$$

THE TERMS

2. Ohmic dissipation.

$$\eta_O \vec{\nabla} \times \vec{B}$$

THE TERMS

3. Hall effect.

$$\eta_H \left(\vec{\nabla} \times \vec{B} \right) \times \frac{\vec{B}}{\|\vec{B}\|}$$

THE INDUCTION EQUATION

FINALLY, and I promise, finally, we have the non-ideal magnetohydrodynamic induction equation:

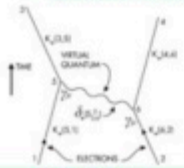
$$= \vec{\nabla} \times \left(\vec{u} \times \vec{B}\right) - \vec{\nabla} \times \left\{ \eta_A \frac{\vec{B}}{\|\vec{B}\|} \times \left(\vec{\nabla} \times \vec{B}\right) \times \right.$$

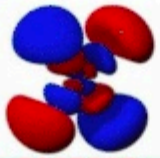
$$\left. + \eta_H \left(\vec{\nabla} \times \vec{B}\right) \times \frac{\vec{B}}{\|\vec{B}\|} + \eta_O \vec{\nabla} \times \vec{B} \right\}$$

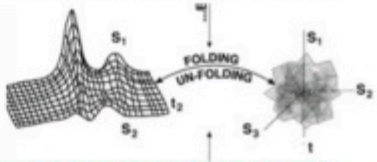
- PARTICLES WERE NOT SUPPOSED TO BE LOOKED AT
- YEARS OF BONKERS THEORIES yet NO REAL-WORLD USE FOUND for looking beyond the FIRST ORDER DERIVATIVES
- Wanted to build a weirder model for a laugh? We had a tool for that: It was called "EXPERIMENTAL CONJECTURES"
- "Nice throw Jimmy, the ball really went $H(t)|\psi(t)\rangle = i\hbar\frac{d}{dt}|\psi(t)\rangle$! I'm feeling very $\hat{H} = \sum_{n=1}^{N}\frac{\hat{p}_n^2}{2m_n} + V(x_1, x_2, \cdots x_N)$ today" - Statements dreamed up by the utterly Deranged

LOOK at what Physicists have been demanding your Respect for all this time, with all the labs & computer programs we built for them

(This is REAL Physics, done by REAL Phyisicists):







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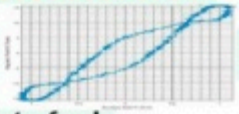
"""""The uncertainty in is  """"""

They have played us for absolute fools

|-|-

Life of a "theoretical astrophysicist".

- "What do you do?"
- "Oh actually, I'm a tHeOrETicaL aStrOpHySiCisT working with nOn-iDEaL mAgNeTohYdRodYnAmiCs."
- "Oh my god really? I'm a Libra!"

Okay, okay, let's take a step back.

We're not solving that equation, thank you very much.

Let us make some simplifying assumptions.

1. No jet outflows.
2. Let's ignore ohmic dissipation.
3. Let's ignore Hall effect.

Let's set up the problem.

Recall, we had three parts:

1. The Protostar
2. The Infalling Envelope of Gas
3. The Disc

As for the protostar,

We choose cylindrical coordinates,

$$\vec{x} = (r, \phi, z)$$

with the protostar at the origin.

Now about the envelope, we choose a widely accepted density profile of

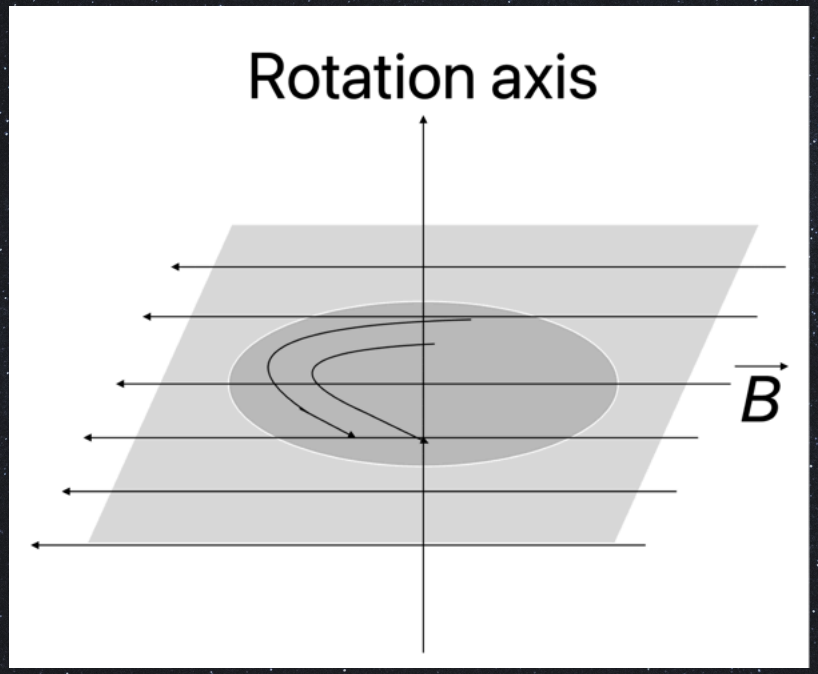
$$\rho_e(r) = \frac{\delta_\rho C_s^2}{2\pi G r^2}$$

called the Singular Isothermal Sphere (SIS) profile. Source? Trust me bro.

About the disc, we assume vertical hydrostatic equilibrium, i.e.

$$h(r) \simeq C_s \sqrt{\frac{r^3}{GM}}$$

Let us deal with a specific geometry of the system.



where we set $B_z = 0$.

Remember that awful evil induction equation? Yes, the one that takes away your lunch and makes you cry?

Well, we don't need it... entirely. We focus our attention to the azimuthal ϕ component of the equation, consider only ambipolar diffusion, and substitute $B_z = 0$, along with an assumption of uniform approach of the field lines.

From the induction equation, we obtain the timescales

$$\tau_{\rm ind} = \frac{rB_\phi}{u_\phi B_r}, \qquad \tau_{\rm diff,H} = \frac{r^2}{\eta_A}, \qquad \tau_{\rm diff,V} = \frac{z^2}{\eta_A}$$

For the strong field case, horizontal diffusion dominates, while vertical diffusion dominates for the weak field case.

Similarly, from the momentum conservation equation, we obtain the timescales

$$\tau_{\mathrm{adv}} = \frac{r}{u_r}, \qquad \tau_{\mathrm{br}} = \frac{r\rho u_\phi}{B_r B_\phi}$$

Where are we headed with this?

THINK...

What was our goal? To obtain an expression for the radius of a stable disc.

At the outer edge of the disc, the magnetic braking timescale and the advective timescale must be comparable.

Similarly, the induction and diffusion timescale must be comparable.

That's a lead! Let us try equating them!

$$\tau_{\mathbf{adv}} \simeq \tau_{\mathbf{br}}$$

$$\frac{r}{u_r} \simeq \frac{r\rho u_\phi}{B_r B_\phi}$$

$$B_r = \frac{u_r \rho u_\phi}{B_\phi}$$

FOR STRONG FIELDS,

$$\tau_{\text{ind}} \simeq \tau_{\text{diff,H}}$$

$$\frac{rB_\phi}{u_\phi B_r} \simeq \frac{r^2}{\eta_A}$$

$$r = \frac{\eta_A B_\phi}{u_\phi B_r}$$

Performing substitutions to basic parameters, we obtain

$$r = \frac{\eta_{\mathrm{A}}^{\frac{2}{9}} \delta_{\rho}^{\frac{2}{9}} \delta_{r}^{\frac{2}{3}} G^{\frac{1}{9}} M^{\frac{1}{3}}}{(2\pi)^{\frac{2}{9}} B_{r}^{\frac{4}{9}}}$$

$$19.23 \text{ au} \left[\delta_{\rho}^{\frac{2}{9}} \delta_{r}^{\frac{2}{3}}\right] \left[\frac{\eta_{\mathrm{A}}}{10^{19} \text{ cm}^2\text{s}^{-1}}\right]^{\frac{2}{9}} \left[\frac{M}{0.1\mathrm{M}_{\odot}}\right]^{\frac{1}{3}} \left[\frac{B_r}{0.1 \text{ G}}\right]^{-}$$

FOR WEAK FIELDS,

$$\tau_{\text{ind}} \simeq \tau_{\text{diff,V}}$$

$$\frac{rB_\phi}{u_\phi B_r} \simeq \frac{z^2}{\eta_A}$$

$$r^2 = \frac{\eta_A GMB_\phi}{u_\phi B_r C_s^2}$$

Performing substitutions to basic parameters, we obtain

$$r = \frac{\eta_{\mathrm{A}}^{\frac{2}{11}} M^{\frac{5}{11}} \delta_{\rho}^{\frac{2}{11}} \delta_{r}^{\frac{6}{11}} G^{\frac{3}{11}}}{(2\pi)^{\frac{2}{11}} B_{r}^{\frac{4}{11}} C_{s}^{\frac{4}{11}}}$$

$$= 5.53 \text{ au} \left[\delta_{\rho}^{\frac{2}{11}} \delta_{r}^{\frac{6}{11}}\right] \left[\frac{\eta_{\mathrm{A}}}{10^{19} \text{ cm}^2\text{s}^{-1}}\right]^{\frac{2}{11}} \left[\frac{M}{0.1\mathrm{M}_{\odot}}\right]^{\frac{5}{11}} \left[\frac{B_r}{0.1 \text{ G}}\right]$$

Phew!

Okay, that was a lot of math. Are the results any good?

YES!

Obtained radii agree well with the observed fact that radius of the disc is greater when the field is misaligned with the rotation axis.

RESULTS AND DISCUSSION

- Similar work done in case of a vertical field has yielded radii of $\simeq 10\ \mathrm{au}$.
- Agreeing with expectations (Hirano, 2020), radii in a misaligned field are greater.

RESULTS AND DISCUSSION

We also note, at the boundary,

$$\tau_{\mathrm{diff,H}} \simeq \frac{r^2}{\eta_A} \gg \frac{z^2}{\eta_A} \simeq \tau_{\mathrm{diff,V}}$$

Thus, we're almost always in a weak-field regime.

WE HAVE A DISC!

For further details (e.g. regarding a generalised case with any angle of inclination i between the angular momentum vector and the field, regarding impact of the Hall effect, etc.), please consult:

THE ASTROPHYSICAL JOURNAL LETTERS

OPEN ACCESS

Protoplanetary Disk Size under Nonideal Magnetohydrodynamics: A General Formalism with Inclined Magnetic Field

Yueh-Ning Lee (李悅寧)[1,2,3], Barshan Ray[1,4], Pierre Marchand[5], and Patrick Hennebelle[6,7,8]

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THANK YOU!

EGG ME IF I BORED YOU.

QUESTIONS?

किशोर वैज्ञानिक प्रोत्साहन योजना
KVPY
National Fellowships For Students Interested In Research Careers

IISER KOLKATA

NCTS
National Center for Theoretical Sciences

NSTC | International Internship Pilot Program