Calculus I TA Session

October, 2023

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1. **(Implicity differentiation)** 108 A1 Midterm Problem 3 (b)
Suppose function g has the following property

$$g(\sin 3x) = 2(g(x) + x)$$

for any real number x and g is differentiable at $x = 0$. Find $g(0)$ and $g'(0)$.

2. **(Linearization)** 11101 (01-05) Midterm Problem 4
Let $f(x) = x + e^{2(x-1)}$. Let $g(x) = f^{-1}(x)$ be the inverse function of $f(x)$.

   (a) Find $g(2)$ and $g'(2)$.

   (b) Prove that $g''(x) < 0$ for all $x \in \mathbb{R}$.

   (c) Write down the linearization $L(x)$ of $g(x)$ at $x = 2$. Hence determine whether $g(2.1)$ or $L(2.1)$ is larger.

3. **(Extreme value)** 110 (01-05) Midterm Problem 2
Suppose that the equation

$$x^2 \cos(xy) + e^{y^2} - 2x + y = 0$$

is satisfied by a differentiable function $y(x)$ defined on an open interval I containing 1 such that $y(1) = 0$. Besides, we assume that y'' exists everywhere on I.

   (a) Compute $y'(1)$.

   (b) Compute $y''(1)$.

   (c) Does $y(x)$ attain a local extremum at $x = 1$? if your answer is YES, tell the type of local extremum (local maximum or local minimum) and give your reason.

4. **(MVT)** 105A Midterm Problem 6
Suppose that f is a differentiable function. If $f'(a) > 0$ and $f'(b) < 0$, explain that there exists $c \in (a, b)$ such that $f'(c) = 0$. (Note that f' may not be continuous.)