CALCULUS TA SESSION FOR GROUP 1 OCTOBER 7 (VERSION 1)

(1) | Differentiable and Continuous |

$$f(x) = \begin{cases} \frac{\sin^2(ax)}{x}, & x > 0 \\ |2x+1| - |2x-1| + b\cos x, & x \leq 0 \end{cases}$$

(a) For what values of a and b will $f(x)$ be continuous at $x = 0$?

(b) For what values of a and b will $f(x)$ be differentiable at $x = 0$?

(2) | Differentiable and Continuous | *1041 A1 Midterm Problem 1*

Let a function

$$f(x) = \begin{cases} x^\alpha \sin\left(\frac{1}{x^\beta}\right), & x > 0 \\ 0, & x = 0 \\ \frac{\sin\left(x^\beta\right)}{1-\cos x}, & x < 0 \end{cases}$$

(a) For what values of α and β will $f(x)$ be continuous at $x = 0$?

(b) For what values of α and β will $f(x)$ be differentiable at $x = 0$?

Hint: (a) $\alpha > 0$ and $\beta > 0$ (b) $\alpha > 1$ and $\beta > 3$

(3) [Find limit]

Determine α, β such that

$$\lim_{x\to\infty} \sqrt{4x^2 - 3x + 2} - \alpha x + \beta = 0$$

(4) [Find limit]

Compute the following limit

$$\lim_{\to\infty} x(\sqrt{x^6 - 3x^5 + 1} - x^3) \tan\left(\frac{1}{x^3}\right)$$

(5) [Find inverse function]

Find $\sin \sec^{-1} x$ for $x \leq -1$.

(6) [Find inverse function]

Find $\sin \sec^{-1} x$.

Hint: $\frac{\sqrt{x^2-1}}{|x|}$. Note that range of the $\cos^{-1}$ is $[0, \pi]$ so $\sin([0, \pi]) \geq 0$.