Calculus Ⅱ TA Session

November 23, 2023

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1. **(Riemannian Sum)** 1011 A1 Final Problem 2
Please answer the following questions.

   (a) Evaluate $\int \ln x dx$.

   (b) Show that the function $f(x) = \ln x$ is increasing in $x > 0$.

   (c) Consider the definite integrals of $f(x) = \ln x$ on $[1, n]$ and $[1, n+1]$. By comparing the upper sum and the lower sum for $f(x) = \ln x$ with regular partition of length $\triangle x = 1$, derive the inequalities

$$\int_1^n \ln x dx < \ln 1 + \ln 2 + \cdots + \ln n < \int_1^{n+1} \ln x dx.$$

   (d) Prove that $\left(\frac{n}{e}\right)^n < \frac{n!}{e} < \left(\frac{n+1}{e}\right)^{n+1}$.

2. **(IBP)** 1111 M2 Final Problem 1
Let $h(u)$ be a continuous function such that $h(u) > 0$ for $u \in \mathbb{R}$. Define

$$g(t) = t \int_t^1 h(u)\mathrm{d}u \text{ and } \quad f(x) = \int_0^{x^2} g(t)\mathrm{d}t.$$

   (a) Find $f'(x)$. Express your answer in terms of h.

   (b) Find the interval(s) on which $f(x)$ is increasing and the interval(s) on which $f(x)$ is decreasing.

   (c) Use integration by parts to write $f(1) = \int_0^1 t \left(\int_t^1 h(u)\mathrm{d}u\right) \mathrm{d}t$ as $\int_0^1 p(t)h(t)\mathrm{d}t$. Find $p(t)$.

3. **(FTOC)** 1081 A2 Final Problem 1
Find $f'(2)$ if $f(x) = e^{g(x)}$ and

$$g(x) = \int_4^{x^2} \frac{t}{1+t^4} dt$$

4. **MVT** 103 A1 Final Problem 1
Evaluate the following limit,

$$\lim_{x \to 0} \frac{\int_x^{\tan x} \sqrt{1+t^3} dt}{x^3}$$