Pair emission from bare magnetized strange stars

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ABSTRACT

The dominant emission from bare strange stars is thought to be electron-positron pairs, produced through spontaneous pair creation (SPC) in a surface layer of electrons tied to the star by a superstrong electric field. The positrons escape freely, but the electrons are directed towards the star and quickly fill all available states, such that their degeneracy suppresses further SPC. An electron must be reflected and gain energy in order to escape, along with the positron. Each escaping electron leaves a hole that is immediately filled by another electron through SPC. We discuss the collisional processes that produce escaping electrons. When the Landau quantization of the motion perpendicular to the magnetic field is taken into account, electron-electron collisions can lead to an escaping electron only through a multi-stage process involving higher Landau levels. Although the available estimates of the collision rate are deficient in several ways, it appears that the rate is too low for electron-electron collisions to be effective. A simple kinetic model for electron-quark collisions leads to an estimate of the rate of pair production that is analogous to thermionic emission, but the work function is poorly determined.

Key words: acceleration of particles — dense matter — plasmas — radiation mechanisms: general — stars: neutron — pulsars

1 INTRODUCTION

It was proposed by Witten (1984) that deconfined, strange quark matter (SQM), consisting of the u-, d- and s-quarks, is an absolutely stable form of matter. A strange star (SS) is a compact object composed of SQM. In a model for a SS (Alcock, Farhi & Olinto 1986), the quarks are bound by the strong force, rather than the gravitational force that binds other stars. At the surface of the star, where the quark density drops abruptly to zero, the electrons extend into a layer of thickness $\Delta z \sim 10^3 \, \mathrm{fm}$ above the surface, where there is a superstrong electric field that ties the electrons to the star. It has been suggested that such a bare SS might be covered by a crust (Alcock, Farhi & Olinto 1986) of ordinary nuclear matter (and neutralizing electrons), but such a crust could be blown away as the SS forms (Usov 1997), or be destroyed by thermal effects (Kettner et al. 1995). Thus, if SSs form, one expects them to be bare, in the sense that the surface is this thin layer of electrons. The estimated plasma frequency in the surface layer, $\omega_p \approx 19 \,\mathrm{MeV}$, precludes emission of photons below this frequency. Concerning the emission from SSs, Alcock, Farhi & Olinto (1986) commented that the luminosity may be very high, but added the proviso that only if heat can be supplied rapidly enough. On the contrary, if heat cannot be supplied rapidly enough, the emission from a SS may be

strongly suppressed. A SS may be a 'silver sphere' rather than black body (BB) (Alcock, Farhi & Olinto 1986). The estimated electric field in the electron layer is $E \approx 5 \times 10^{19} \, \mathrm{V \, m^{-1}}$ and this corresponds to $E \sim 30E_c$, where $E_c = 1.3 \times 10^{18} \, \mathrm{V \, m^{-1}}$ is the Schwinger electric field (Schwinger 1951). According to quantum electrodynamics an electric field is intrinsically unstable to decay due to spontaneous pair creation (SPC) (Schwinger 1951). Although this effect is negligible for $E \ll E_c$, it becomes extremely efficient for superstrong electric fields with $E \gtrsim E_c$. This suggests that SSs may be highly luminous in emission of pairs, with the luminosity not restricted by the Eddington limit (Usov 1998). It might be remarked that at the expected temperature, $T \approx 10^{11} \, \mathrm{K}$, just after a SS forms, BB emission is dominated by pairs, with the power radiated in pairs being 7/4 times the BB power radiated in photons in vacuo (Landau & Lifshitz 1959). The luminosity due to SPC is not BB and once the star reaches thermal equilibrium, the luminosity must be suppressed to no more than the BB level. More recent discussions of the emission spectrum have included thermal effects, emphasizing interactions between escaping pairs and photons (Usov 2001; Aksenov, Milgrom & Usov 2003, 2004).

The original motivation for the investigation reported here was to generalize existing treatments of pair emission from a bare SS (Usov 1998) to include the effects of a magnetic field. However, questions concerning the underlying physics arose, and these are emphasized in the present paper, specifically, self-regulation of SPC, the role of collisions, and the sequence of processes that allows an electron to escape. Usov (1998) noted that unsuppressed SPC would produce pairs at the rate $\approx 4 \times 10^{56} \, \mathrm{s}^{-1}$, but that such

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emission could persist only for a short time. SPC creates electrons propagating towards the star and positrons propagating away from the star, and a steady state escape is possible only if upward propagating electrons are available to accompany the escaping positrons. The copious source of downward propagating electrons from SPC fills any unoccupied electron states, suppressing SPC through the Pauli exclusion principle. No electron can be created by SPC if the state in which it would be created is already occupied. Hence, once all relevant states are filled, SPC is suppressed. In the absence of collisions, SPC completely suppresses itself, so that the SS would be an ideal silver sphere with no pair emission.

Collisions play a central role in allowing electrons (and hence pairs) to escape. For escape to occur, a sequence of collisions must transfer an electron from the region of phase space where it was created by SPC into the region of phase space corresponding to escape, that is, such that it has an upward directed motion at greater than the escape energy. The electron escapes, leaving a hole in the distribution that is immediately filled by SPC, with the newly created positron escaping with this electron. The rate of pair emission is determined by the rate collisions transfer electrons between these two regions of phase space. In existing treatments, this rate is (implicitly) assumed arbitrarily high, so that there is sufficient time for thermalization, allowing the electron distribution to be approximated by a Fermi-Dirac (FD) distribution.

An important qualitative point is that the dominant role of SPC implies that the electrons in the electron layer cannot be in thermal equilibrium. The difference, δn say, between the electron occupation number and unity regulates SPC, such that SPC is reduced from its value in vacuo by the factor δn . This suppression was discussed by Usov (1998) assuming a FD distribution for the electrons. However, a FD distribution applies only in thermal equilibrium, and the presence of SPC invalidates this assumption. Granted that the suppression of SPC must be self-regulatory, the value of δn must adjust to allow pair creation at the rate required to replace pairs that escape. The time scale for escape is much shorter than the time scale for thermal equilibrium to be reached in the electron layer, and the steady state distribution is determined by collisions, SPC and escape, and not by thermal equilibrium. In principle, escape can result from a sequence of electron-electron collisions in the electron layer, or from electron-quark collisions below this layer. Both are discussed in this paper.

The presence of a magnetic field affects pair emission from a bare SS in several ways. Typical surface magnetic fields in ordinary pulsars are of order $0.1B_c$, where $B_c = m^2c^2/e\hbar = 4.4 \times 10^9 \,\mathrm{T}$ is the so-called critical magnetic field. The surface field of a SS is thought to be considerably higher (Aksenov, Milgrom & Usov 2004), of order $10B_c$, as in magnetars (Thompson & Duncan 1995). Such a magnetic field affects the rate of SPC in vacuo. However, granted that SPC is regulated through the Pauli exculsion principle, this effect is relatively unimportant. A magnetic field also affects the electron energy states: free motion is restricted to one dimension (1D) along the magnetic field lines, with the perpendicular motion replaced by a set of discrete Landau levels. An electron in the nth Landau level acts like a 1D particle with an effective mass, m_n , that depends on the Landau level. This poses a problem if all electrons are in their lowest Landau level: collisions in 1D between particles with the same rest mass cannot change the energies of the particles, and hence electron-electron collisions at fixed n, specifically both initial and both final particles with the same n, cannot transfer electrons from one region of momentum space to another, and so cannot contribute to the escape of pairs. In order for an electron to escape as a result of electron-electron collisions,

collision-induced jumps in Landau level are essential. There is no corresponding constraint on electron-quark collisions.

In section 2 we discuss how SPC and the escape of electrons are regulated by collisions. In section 3 we identify the sequence of electron-electron collisions that allows escape. In section 4 we consider the effects of electron-quark collisions. The results are summarized and discussed in section 5.

2 REGULATION OF SPC THROUGH DEGENERACY

The nonthermal nature of pair emission by a SS raises some unfamiliar problems that are discussed from a qualitative viewpoint in this section. For the purpose of discussion, the magnetic field is assumed to be vertical, and the perpendicular motion of the electrons is assumed to be quantized into the Landau levels, $n=0,1,2,\ldots$ SPC is assumed to generate only downward propagating electrons with $\varepsilon < eV$ in the ground level, n=0.

2.1 Suppression of SPC

The decay of an electric field $E \gg E_c$ in vacuo produces pairs at the Schwinger rate $a_{\rm SPC}(E/E_c)^2$ with $a_{\rm SPC}=1.7$ × $10^{56}\,\mathrm{m}^{-3}\,\mathrm{s}^{-1}$ (Schwinger 1951). The generalization to an electric field along a magnetic field was given by Daugherty & Lerche (1976), and is written down in the Appendix A. The pair creation produces electrons only in a restricted region of momentum space, centered on momenta anti-parallel to the electric field, with energy $\varepsilon \leqslant eV$. It is convenient to introduce the density of states factor $D_{\rm SPC} = \int d^3 \mathbf{p}/(2\pi\hbar)^3$, where the integral is over the region of momentum space to which SPC is restricted. The quantity D_{SPC} is estimated below, cf. (12), for the magnetized case. The pair creation rate in a degenerate electron gas is suppressed by a factor $1 - n(\mathbf{p})$, where $n(\mathbf{p})$ is the occupation number of the created electron. In the absence of such suppression, SPC would lead to a SS luminosity in pairs of $\approx 3 \times 10^{44}$ W; this may occur just after the formation of a SS when its surface is very hot, but could persist only for a very brief period (Usov 1998). In treating this suppression, Usov (1998) assumed that $n(\mathbf{p}) = n_{\text{FD}}(\mathbf{p})$ has its thermal (FD) value. The value of $n(\mathbf{p})$ in the region of momentum space where the electrons are created results from a balance between gains and losses, and a FD distribution results only if both gains and losses are dominated by collisions. Creation of electrons implies $n(\mathbf{p}) > n_{\rm FD}(\mathbf{p})$ in this momentum range, and the suppression factor, $\delta n(\mathbf{p}) = 1 - n(\mathbf{p})$, is then smaller than the thermal value, $1 - n_{\rm FD}(\mathbf{p})$.

The kinetic equation for electrons in the region of momentum space where SPC operates is

$$\frac{dn(\mathbf{p})}{dt} = \nu_{\text{SPC}}(\mathbf{p})\delta n(\mathbf{p}) - \nu_{\text{loss}}(\mathbf{p}) n(\mathbf{p}), \tag{1}$$

with $\nu_{\rm SPC}(\mathbf{p}) = a_{\rm SPC}(E/E_c)^2/D_{\rm SPC}$ and where $\nu_{\rm loss}(\mathbf{p})$ includes collisional losses and escape of electrons. When the suppression is strong, one has $n(\mathbf{p}) \approx 1$, $\delta n(\mathbf{p}) \approx \nu_{\rm loss}(\mathbf{p})/\nu_{\rm SPC}(\mathbf{p})$. SPC is then adjusted to the value required to balance the escape of pairs from the star.

2.2 Role of collisions

When SPC is the dominant effect on the distribution of electrons in the electron layer, the distribution function may be determined by first neglecting collisions and thermal motions, to find a zeroth order distribution, and then including collisions and thermal motions as perturbations.

In the absence of collisions and thermal motions, SPC in the electron layer fills all the electron states corresponding to downward motion with $\varepsilon < eV, n = 0$. Each electron propagates downward until it encounters a quark and is reflected. In the absence of thermal motions, the quark is stationary, and due to the large mass ratio, the electron is reflected with no change in energy, resulting in all the upward electrons states with $\varepsilon < eV, n = 0$ also being filled. Electrons oscillate across the electron layer, with downgoing electrons reflected off quarks below the electron layer and with upward electrons reflected at the top of the layer by the electric field. Thus, the zeroth order distribution of electrons in the electron layer corresponds to occupation number equal to unity for $\varepsilon < eV$ and zero for $\varepsilon > eV$, with no electrons in Landau levels with n > 0.

Now consider the effect of collisions, which tend to thermalize the distribution. A thermal distribution is a FD distribution, $n(p) = 1/(\exp\{[\varepsilon - \mu_e]/T\} + 1)$, where μ_e is the chemical potential, and T is the temperature in energy units. Provided the Fermi energy satisfies $\varepsilon_F\gg T$, one has $\mu_e\approx\varepsilon_F$. A FD distribution has a Boltzmann-like tail such that there is a probability $\propto \exp(-\varepsilon/T)$ of finding an electron with $\varepsilon\gg\varepsilon_F$. Because upgoing electrons with $\varepsilon>\varepsilon_{\rm esc}$ escape directly, even if collisions produce a thermal distribution at lower energies, this escape leaves a depleted high-energy tail. Collisions tend to restore a thermal distribution by feeding electrons into the depleted tail. The rate at which escaping electrons are produced is determined by the rate collisions feed electrons into the tail (Gurevich 1960). The relevant collisions can be either electron-electron collisions in the electron layer, or electron-quark collisions below this layer.

This discussion presupposes that there is a magnetic field present so that the perpendicular energy is quantized. In the absence of a magnetic field, SPC produces electrons only parallel to E. Collisions tend to scatter the electrons into other directions, so that they diffuse in angle, leading to an increase in the angular spread. Discussion of this effect is more complicated that discussion in the case where the perpendicular energy is quantized case, and we restrict our discussion to the quantized case.

2.3 Transitions between Landau levels

The zeroth order distribution function, generated by SPC, is one dimensional (1D), along B. Electron-electron collisions in 1D do not change the energies of the electrons. Collisions between electrons in different energy levels, that remain in those levels, can redistribute parallel momentum. This may be understood by noting that the electrons in different Landau levels act like particles with different rest masses.

Let p be the component of the momentum along the magnetic field. The energy of an electron in the nth Landau level is

$$\varepsilon_n(p) = (m_n^2 c^4 + p^2 c^2)^{1/2}, \qquad m_n = m(1 + 2nB/B_c)^{1/2}.$$
(2)

The form (2) implies that electrons act like 1D particles with a rest mass, m_n . The presence of an electric field along the magnetic field does not affect the Landau levels.

2.4 Electron distribution in a magnetized electron plasma

The zeroth order distribution function, identified above, has occupation number equal to unity in the ground state for $\varepsilon < eV$,

with no electrons with $\varepsilon > eV$, and none in higher Landau levels. This corresponds formally to a FD distribution in the limit $T \to 0$, $B \to \infty$. Thermal contact between the electron layer and the quark layer below it tends to drive this zeroth order distribution towards a FD distribution at the temperature of the quarks. Thus collisions tend to transfer electrons from $\varepsilon < \varepsilon_{\rm F}$, where the zeroth order distribution is overpopulated compared with a FD distribution, to $\varepsilon > \varepsilon_{\rm F}$, where it is underpopulated. Similarly, any tendency to thermalization causes the occupation number of the higher Landau levels, $n \geqslant 1$, to be nonzero.

A FD distribution, $n_n(p)$, for electrons in the nth Landau level is $n_n(p)=1/(\exp\{[\varepsilon_n(p)-\mu_e]/T\}+1)$. The Fermi energy, $\varepsilon_{\rm F}$, is the same for all Landau levels, but the Fermi momentum, $p_{{\rm F}n}$, depends on n, with $n_n(p)$ non-negligible only for $-p_{{\rm F}n}\lesssim p\lesssim p_{{\rm F}n}$. One has

$$p_{Fn} = (\varepsilon_F^2/c^2 - m_n^2 c^2)^{1/2}. (3)$$

There is a maximum Landau level, $n=n_{\rm max}$, such that $p_{\rm F}n$, as defined by (3), is real for $n< n_{\rm max}$ and imaginary for $n>n_{\rm max}$. Assuming $\varepsilon_{\rm F}=eV\gg mc^2$, one finds

$$2n_{\text{max}}\frac{B}{B_c} \approx \left(\frac{eV}{mc^2}\right)^2$$
. (4)

The number density of electrons, N_e , assuming that all Landau levels up to $n_{\rm max}$ are filled is then

$$N_e = \sum_{n=0}^{n_{\text{max}}} 4 \frac{eB}{2\pi\hbar} \frac{p_{\text{F}n}}{2\pi\hbar} \approx 8\pi \left(\frac{eV}{2\pi\hbar c}\right)^3.$$
 (5)

The final expression in (5) is the same as for a degenerate FD distribution in the absence of a magnetic field.

3 ELECTRON-ELECTRON COLLISIONS

In this section we assume that the production of escaping pairs is determined by electron-electron collisions in the electron layer. However, there are sufficient uncertainties that it is unclear whether or not the rate of electron-electron collisions is high enough for this to be the case.

3.1 Kinematics of electron collisions in a magnetic field

Conservation of energy and momentum in a collision places a severe restriction on collisions that can lead to an electron escaping. Let the initial and final values of p be p_i , with i=1,2 for the initial electrons, and i=3,4 for the final electrons. The ith electron is assumed to be in the Landau level n_i . Electrons in different Landau levels effectively have different rest masses, with the effective rest mass for the Landau level n_i being m_{n_i} , cf. (2). To simplify the notation we write the effective mass of the ith particle as M_i , with $M_i=m_{n_i}$. Qualitatively, the kinematics are equivalent to those for collisions between (relativistic) particles of different mass constrained to move in 1D. Conservation of momentum implies $P=p_1+p_2=p_3+p_4$, and conservation of energy implies $E=\varepsilon_{n_1}(p_1)+\varepsilon_{n_2}(p_2)=\varepsilon_{n_3}(p_3)+\varepsilon_{n_4}(p_4)$. There are two solutions for the final variables, and we solve for p_3 and ε_3 , with $p_4=P-p_3$ and $\varepsilon_4=E-\varepsilon_3$. The solutions are

$$p_3 = \frac{D_1 P \pm D_2 E/c}{2M^2}, \quad \varepsilon_3 = \frac{D_1 E \pm D_2 Pc}{2M^2},$$
 (6)

with $M^2 = (E/c^2)^2 - (P/c)^2$ and with

$$D_1 = M^2 + M_3^2 - M_4^2$$
, $D_2 = [D_1^2 - 4M^2M_3^2]^{1/2}$. (7)

3.2 Collision leading to escape

For an electron to escape as the result of a sequence of collisions, its energy must increase from below the escape threshold to above it. The first step in the sequence must be a collision in which one or both initial electrons are in the lowest Landau level, with both final electrons in higher Landau levels, n > 0. The electron-electron collision rate (Langer 1981; Storey & Melrose 1987) is a rapidly decreasing function of the differences between the Landau levels of the initial and final electrons. This implies that the changes in Landau level occur preferentially in single steps, with say $|n_3|$ $|n_1| = 1$ and $|n_4| = n_2$. Using (6) and (7), one finds that collisions in which the Landau level increases, specifically $n_3 = n_1 + 1$ in the present case, lead to a decrease in the kinetic energy, which may be attributed to a conversion of kinetic energy into rest energy. Such scattering impedes rather than facilitates electrons escaping. On the other hand, scattering from higher to lower Landau levels, $n_3 = n_1 - 1$ in the present case, effectively converts rest energy into kinetic energy and can result in an electron with sufficient energy

A sequence of collisions that leads to escape is as follows. First, an initial electron with $n_1=0, -eV/c < p_1 < eV/c$ is scattered to $n_3=1$ by colliding with an electron with $n_2=n_4\geqslant 1$. Second, collisions between electrons with $n_1, n_2 \neq 0, n_1 \neq n_2$ allow the energies of the electrons to change, feeding electrons into the higher energy states. Third, the final collision is between an electron with $n_1=1, \, \varepsilon_1\approx eV$ and an electron with $n_2>1, \, \varepsilon_2\approx eV$ that results in an electron with $n_3=0, \, \varepsilon_3>eV$ that can escape.

In a steady state the SPC rate must be equal to the escape rate of pairs. The transfer rate of electrons from the state in which they are created to the final state from which they escape is regulated by the value of δn in the intermediate states. The following semi-quantitative argument indicates how this regulation occurs.

3.3 Steady state solution for higher Landau levels

Consider a Landau level n_1 . Collisions transfer electrons into this level from other levels, and out of this level to other levels. Ignoring SPC and escape, the net rate at which electrons enter the n_1 th level with momentum p_1 is

$$\frac{dn_{n_1}(p_1)}{dt} = \int \frac{dp_2}{A} \sum_{n_2, n_3, n_4} [G_{34}^{12} - G_{12}^{34}], \tag{8}$$

with

$$G_{34}^{12} - G_{12}^{34} = r_{34}^{12} \{ n_{n_3}(p_3) n_{n_4}(p_4) \delta n_{n_1}(p_1) \delta n_{n_2}(p_2) - n_{n_1}(p_1) n_{n_2}(p_2) \delta n_{n_3}(p_3) \delta n_{n_4}(p_4) \},$$
(9)

with $\delta n_k = 1 - n_k$, and where A is a normalization constant; r_{34}^{12} is the collision rate between electrons in initial and final Landau levels n_1, n_2 and n_3, n_4 . The collision rate is symmetric under the interchanges $1 \leftrightarrow 2$, $3 \leftrightarrow 4$ and $12 \leftrightarrow 34$. In a steady state one has $dn_{n_1}(p_1)/dt = 0$ for all n_1 and p_1 . The kinetic equation (8) needs to be modified for the ground state, n=0, to take into account SPC and escape of electrons. For the present we ignore the ground state.

We consider only transitions that involve one electron changing Landau level by unity. For simplicity, we assume that all transitions occur at the same rate, R say, and that all the occupation numbers are nearly equal to unity, $\delta n_{n_i} = 1 - n_{n_i} \ll 1$ for energies below the Fermi energy. Then, for $n_1 > 0$, (8) is replaced by

$$\frac{dn_{n_1}}{dt} = R[\delta n_{n_1} \, \delta n_{n_2} - \delta n_{n_3} \, \delta n_{n_4}],\tag{10}$$

where the arguments p_i are redundant in this simplified model. A steady state requires $dn_{n_1}/dt=0$, and in this simplified model this condition is satisfied if all the occupation numbers are equal, $\delta n_{n_i}=\delta n_1$, below the Fermi energy.

3.4 Collisions involving the ground state

Now consider the kinetic equation for the ground state. In the region, $-eV/c , where SPC occurs, <math>\delta n_0 = 1 - n_0 \ll 1$ is determined by balancing the rate of increase due to SPC with the rate of decrease due to collisions. According to the foregoing model, collisions that transfer electrons from $n_1 = 0$ to $n_3 = 1$ cause losses to the ground state at the rate $-R(\delta n_1)^2$. The rate of increase in n_0 due to SPC is $\nu_{\rm SPC}\delta n_0$ where $\nu_{\rm SPC}$ is the intrinsic rate of SPC, given by

$$\nu_{\rm SPC} = \frac{w_{\parallel}}{D_{\rm SPC}},\tag{11}$$

with w_{\parallel} given by (A1), and the density of states factor by

$$D_{\rm SPC} = \frac{eB}{2\pi\hbar} \int_{-eV/c}^{0} \frac{dp}{2\pi\hbar} = 2\pi \frac{B}{B_c} \frac{eV}{mc^2} \left(\frac{mc}{2\pi\hbar}\right)^3. \quad (12)$$

The model is valid only for $\delta n_0 \ll \delta n_1$, and then one has $\delta n_0 = R(\delta n_1)^2/\nu_{\rm SPC} \ll 1$. In the opposite limit, collisional effects dominate and lead to a FD distribution.

The collisions that lead to escape involve an electron with n=1, $\varepsilon\lesssim \varepsilon_{\rm F}$ being scattered to n=0, $\varepsilon>\varepsilon_{\rm F}$. The most favorable case is for $p_1=p_2=p_{\rm F1}$, when one has $P=2p_{\rm F1}$, $E=2\varepsilon_{\rm F}$. According to (6) and (7), with $n_1=n_2=n_4=1$, $n_3=0$, the +-solutions has $p_3>p_{\rm F}$, $\varepsilon_3>\varepsilon_{\rm F}$, implying that the electron can escape. There is a small range of p_1 near $p_{\rm F1}$ that allows $p_3>p_{\rm F}$, and this implies that there is a fraction of collisions that result in an escaping electron, with this fraction, Δ_e say, being of order the ratio of this small momentum range to $p_{\rm F1}$. The rate escaping electrons are produced is then $\Delta_e R \delta n_1$. This rate must be equal to the rate, $\nu_{\rm SPC} \delta n_0$, of SPC. Equating the rates then gives $\delta n_1=\Delta_e$, $\delta n_0=R(\Delta_e)^2/\nu_{\rm SPC}$.

This simple model establishes in principle how the system can regulate SPC to balance the rate of escape of pairs. The important qualitative point is that the pair creation rate and the escape rate are determined purely by collisional effects. These rates are equal to $R(\Delta_e)^2$ in this simple model.

3.5 Scattering rate

Collisions in the electron layer determine the net rate of SPC only if a typical electron has many collision before reaching the lower surface of the electron layer. The collision rate was estimated by Usov (2001) using the results of (Potekhin et al. 1999):

$$\nu_{ee} \approx A \frac{\varepsilon_{\rm F}}{\hbar} h(\zeta), \quad A \approx 1.3 \times 10^{-5},$$
 (13)

with $\zeta \approx 0.1\varepsilon_{\rm F}/T$, and $h(\zeta) \approx 51/\zeta^2$ for $\zeta \gg 20$ and $h(\zeta) \approx (\zeta/3) \ln(2/\zeta)$ for $\zeta \lesssim 1$, with interpolation formulas at intermediate values. The maximum value of ν_{ee} is of order $A\varepsilon_{\rm F}/\hbar \approx 3 \times 10^{17}~{\rm s}^{-1}$. An electron propagates through the electron layer in a time $\approx 3 \times 10^{-21}~{\rm s}$, suggesting that a typical electron experiences a probability of a collision of order 10^{-3} in one traversal of the electron layer. We conclude that electron-electron collisions in

the electron layer can govern the rate that escaping pairs are produced only if (13) seriously underestimates the collision rate.

The rate (13) neglects several important effects, and needs to be modified to take them into account. We comment on three separate effects that might lead to (13) being a serious underestimate of the actual rate. The three effects are: electromagnetic rather than electrostatic interactions for relativistic particles, the nonthermal form of the distribution function, and the quantization of the perpendicular energy for a magnetized electron.

The collision rate (13) is calculated assuming that the interaction between the two electrons is longitudinal, with the interaction cut off at distances greater than the Debye length. However, for highly relativistic particles, the dominant interaction is transverse rather than longitudinal, and this is not affected by Debye screening (Heiselberg & Pethick 1993). As a result the effective electron-electron collision rate is higher than estimated by Usov (2001). Heiselberg & Pethick (1993) estimated that the contribution from the transverse response exceeds that from the longitudinal response by a factor of $0.70(\hbar cq_D/kT)^{1/3}$ for $kT \ll \hbar cq_D$, where q_D is the Debye wave number. With $q_D =$ $(\omega_p/c)(kT/mc^2)^{-1/2}$, this factor becomes $0.70(\hbar cq_D/kT)^{1/3} \approx 1.7(\hbar\omega_p/20\,\mathrm{MeV})^{1/3}(kT/1\,\mathrm{MeV})^{-1/2}$ for $kT \ll \hbar\omega_p$. This factor is large only for $kT \ll 1 \, \mathrm{MeV}$, whereas the relativistic assumption requires $kT\gg 1\,\mathrm{MeV}.$ Hence, although the neglect of the transverse response made lead to (13) being an underestimate, the underestimattion is probably not by a very large factor.

A second effect that needs to be taken into account in revising (13) for the present application is that the occupation numbers are determined primarily by the collisions themselves, and not by thermal effects. In a degenerate thermal electron gas, the combination of occupation numbers in (9) effectively restricts the momentum integral to a small fraction $\sim kT/\varepsilon_{\rm F}$ where the occupation number is not close to either unity or zero, so that the collision rate is suppressed except for electrons around the top of the Fermi sea. When the occupation numbers are determined by the collisions themselves, they approach thermal values only if the transfer between states is dominated by collisions, and this is not the case when the dominant effect is a flow from a source (SPC) in one region of momentum space to a sink (escape) in another. This should also cause (13) to be an underestimate of the actual collision rate. However, to estimate the magnitude of this effect requires a detailed analysis that is not attempted here.

The magnetic field affects the scattering in that the perpendicular energy must change by discrete amounts, associated with transitions between Landau levels. In (13), the perpendicular energy levels are assumed continuous, and an integration is performed over relevant angular variables. Although the relativistic quantum theory of electron-electron scattering in a magnetized plasma is available (Langer 1981; Storey & Melrose 1987), only a few special cases have been calculated (Langer 1981) in detail. With B/B_c of order unity or greater, the transition rate between Landau levels decreases rapidly with the net change in the Landau quantum numbers, and changes by more than unity become increasingly unfavorable with increasing B/B_c .

We conclude that although existing estimates of the collision rate suggest that it is too low for collisions within the electron layer to affect the distribution function substantially, these estimates are for the unmagnetized thermal case, and the thermal assumption in particular can lead to a serious underestimate of the effective rate. A detailed analysis is required to determine whether collisions in the electron layer alone can determine the rate of production of escaping pairs.

4 ELECTRON-QUARK COLLISIONS

In this section we ignore collisions in the electron layer and consider the effect of collisions between quarks and electrons below the electron layer.

4.1 Reflected nonthermal electrons

The downward propagating electrons, with a nonthermal distribution due to SPC, are scattered by quarks and electrons below the surface. As in the electron layer, electron-electron collisions that do not change the Landau level do not change the occupation number. However, a collision with a quark can change the electron energy without changing its Landau level. We concentrate on such collisions, which reflect electrons (along the magnetic field) with small changes in energy. The fractional change in electron energy is of order the ratio of the thermal speed of the quark to the speed of light. A collision is only possible if there is an unoccupied state available to the reflected electron. This restricts effective collisions to near the maximum energy, eV, of the electrons due to SPC. The distribution function is modified from a step function at $\varepsilon = eV$ through the formation of a high energy tail. Collisions cause electrons to diffuse in energy from $\varepsilon < eV$ to $\varepsilon > eV$ to populate this tail. This results in escaping electrons when the tail extends to the escape energy, $\varepsilon_{\rm esc}$.

4.2 Energy flux into the tail

The electron distribution formed by SPC alone has the form of a completely degenerate distribution with Fermi energy eV. In the presence of thermal quarks, at a temperature T, this distribution may be regarded as having a depleted (actually absent) high-energy tail. The argument given by Gurevich (1960) implies that collisions tend to feed electrons into the tail to establish a thermal distribution, in this case a FD distribution at temperature T (and chemical potential eV). In the case considered by Gurevich (1960) the tail is depleted by an acceleration mechanism transferring the particles to higher energy, whereas in the present case the depletion is due to electrons with $\varepsilon>arepsilon_{\mathrm{esc}}$ escaping. A modification of the method to treat this case is outlined in Appendix B. It implies that there is a flux in energy space that corresponds to a flow from an unspecified source at low energy to the sink at $\varepsilon = \varepsilon_{\rm esc}$. The collisions described by a collision frequency, ν_c , and a momentum-dependent diffusion coefficient, d(p), with $p = \varepsilon/c$ in the ultrarelativistic case assumed here. The analysis in Appendix B implies that the rate per unit volume and per unit time that electrons are fed into the range $arepsilon = arepsilon_{
m esc}$ where they escape is $u_c N_e$ times a dimensionless factor that involves an integral over 1/d(p). If the collisions are treated as 1D reflections off hard spheres, this factor becomes $\exp(-W/kT)$, with

$$W = \varepsilon_{\rm esc} - eV \tag{14}$$

the energy gap between the top of the Fermi sea and the escape energy. The model has no spatial dependence, and is concerned only with the flow of electrons in energy space to restore a depleted high-energy tail.

The assumption that the collisions of electrons with quarks may be treated as 1D reflections off hard spheres leads to a Fermilike acceleration of the electrons. The electrons diffuse in p, and tend to gain energy because head-on, energy-increasing collisions are slightly more frequent than overtaking, energy-decreasing col-

lisions. The diffusion coefficient is independent of p, which leads to (14).

Collisions cause both diffusion in energy space and diffusion in coordinate space. In a collision time an electron propagates a mean free path, c/ν_c , and in a typical collision, the energy of an electron changes by $\delta \varepsilon \sim \varepsilon_{\rm F} (kT/m_qc^2)^{1/2}$, with $\varepsilon_{\rm F} = eV$, and where m_q is the mass of a quark. An electron diffuses $W/\delta \varepsilon$ mean free paths in the time it take to diffuse through a range W in energy space. This provides an estimate of the thickness of the layer from which electrons escape: $(c/\nu_c)/(W/\delta \varepsilon)$. The rate per unit time and per unit area that electrons escape from the surface of the star is given by multiplying $\nu_c N_e \exp(-W/kT)$ by this thickness. The result is independent of the collision frequency: the thickness of the layer increases with decreasing ν_c and compensates for the reduced rate electrons are fed into the depleted tail.

4.3 Thermionic-like emission

The escape of electrons from the surface of a SS due to collisions with thermal quarks is somewhat analogous to thermionic emission from a metal surface, sometimes called the Richardson effect. Analogous features are that both involve work functions, W, with W given by (14) in the present case, and both correspond to escaping electrons in a high-energy tail, where a FD distribution may be approximated by a Boltzmann distribution. Also in both cases, collisions are required to replenish the depleted high-energy tail, but the rate of escape does not depend explicitly on the collision frequency. A notable difference is that, in the case of SSs, the electrons are highly relativistic.

The estimated rate at which electrons escape from the surface of the SS is

$$R_{\rm Th} = 4\pi R_*^2 c N_e \frac{W}{eV} \left(\frac{kT}{m_q c^2}\right)^{-1/2} e^{-W/kT},$$
 (15)

where R_* is the radius of the star, and with W given by (14). However, the actual value of W is not well-determined due to uncertainties in the estimate of the difference between the electric potential energy eV across the electron layer and the escape energy $\varepsilon_{\rm esc}$.

An upper bound on the power radiated in pairs is the BB limit. BB emission at $T\gg 10^9~\rm K$ includes a luminosity in pairs that is 7/4 times the photon luminosity in vacuo. Thus the thermal rate is $7\pi R_*^2\sigma_{\rm SB}T^4$, where $\sigma_{\rm SB}$ is the Stefan-Boltzmann constant. The estimate (15) applies only if $R_{\rm Th}$ is much less than this thermal rate, in which case the star acts like a silver sphere rather than a BB.

5 DISCUSSION AND CONCLUSIONS

Our initial motivation for this investigation was to generalize the model of Usov (1998) for the emission of pairs from SSs to include the effect of a magnetic field. The most important qualitative effect of the magnetic field is that it leads to quantization of the electron motion perpendicular to the magnetic field into discrete Landau levels, and SPC creates electrons only in the lowest Landau level. However, the investigation raised questions relating to the role of collisions and of thermalization that are the main topics discussed in this paper.

An important qualitative point is whether the electrons distribution in the surface layers of a SS can be treated as a thermal distribution. There are two processes that drive the electrons away from a thermal distribution: SPC and escape of electrons. SPC creates

electrons in one region of phase space, and electrons can escape only from another region of phase space. Collisions must drive the flux (in energy space) of electrons between this source and sink. The electron distribution can be assumed approximately thermal only if these effects can be treated as perturbations. We argue that the reverse is the case: the dominant effects are SPC and escape, and the tendency of collisions to thermalize the distribution can be treated as a perturbation. In the absence of collisions, the electron states available to SPC are all filled, SPC is completely suppressed and the SS is an ideal silver sphere. We discuss two types of collision process that allows SPC to occur and electrons to escape: electron-electron collisions and electron-quark collisions.

Electron-electron collisions can lead to an escaping electron through a three-stage process. The first stage involves excitation of an electron from the lowest Landau level at $\varepsilon < eV$, where SPC can operate, to a higher Landau level. The vacated state is immediately filled by SPC, with the positron escaping. The second stage involves collisions between electrons in different Landau states $n \ge 1$ that redistribute the parallel energy amongst the electrons in each of these levels. The third stage involves a collision that transfers the electron back to the ground state, n = 0, above the escape energy. Scattering to higher Landau levels effectively converts kinetic energy into rest energy, and scattering from higher to lower Landau levels effectively converts rest energy into kinetic energy. Only the latter allows the kinetic energy to increase, as required to produce an escaping electron. This sequence of electron-electron collisions competes with collisions between quarks and electrons below the electron layer in producing escaping electrons, and it can be the dominant effect only if the electron-electron collision rate is high enough. With the collision rate given by Usov (2001), this is not the case, with a typical electron traversing the electron layer many times before experiencing an electron-electron collision. However, the estimate used by Usov (2001) needs revision due to at least three effects. First, it assumes a Debye screened longitudinal interaction, whereas for relativistic electrons the dominant interaction is transverse and not subject to Debye screening (Heiselberg & Pethick 1993). Second, the estimate assumes a thermal (FD) distribution of electrons, which suppresses collisions except near the top of the Fermi sea; however, the distribution is nonthermal and the thermal assumption is inappropriate. Third, the quantization of the perpendicular motion is not taken into account. The first two effects tend to underestimate the actual collision rate, but it seems unlikely that the underestimate is large enough for electron-electron collisions to dominate over electron-quark collisions. A more quantitative treatment of each of these effects is needed in a more detailed discussion.

Collisions with quarks below the electron layer tend to thermalize the electrons, in particular, forming a high-energy thermal tail that extends to the escape energy. We develop a simple model for the production of escaping electrons due to collisions with quarks; the model involves a non-zero flux in energy space feeding electrons into the energy range where they can escape. We interpret the resulting escape in terms of thermionic-like emission. The rate of pair production is given by (15), and is very sensitive to the work function, W, which is identified as the difference between the escape energy and the maximum energy, eV, of the electrons produced by SPC. Although W is poorly determined by models for SSs, it seems likely that the rate (15) is well below the BB emission rate (which it cannot exceed), supporting the suggestion that SSs may be better described as 'silver spheres' than as black bodies (Alcock, Farhi & Olinto 1986).

The direct effect of the magnetic field on the pair emission is

relatively minor. It affects the rate of SPC in vacuo, but the rate is suppressed by degeneracy and regulated by the collisions such that the net rate of SPC is unrelated to the intrinsic rate in vacuo. The most important consequence of including the magnetic field is the quantization of the electron states into discrete Landau levels. Although a formal theory exists for electron-electron collisions in the magnetized case (Langer 1981; Storey & Melrose 1987), semi-quantitative estimates that would be useful in the present context are not available.

We conclude that treating pair emission from unmagnetized SSs assuming a FD distribution of electrons underestimates the suppression of SPC due to degeneracy, and overestimates the luminosity in pairs. The suppression of SPC is self-regulatory, and it adjusts so that SPC occurs at just the rate needed to replace electrons that escape. The simplest useful model for pair emission from SSs involves treating it as thermionic emission, but a quantitative estimate requires an estimate of the work function, which is poorly determined.

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APPENDIX A: PAIR CREATION IN AN ELECTROMAGNETIC WRENCH

The Lorentz invariants associated with a static electromagnetic field are $\mathbf{E} \cdot \mathbf{B}$ and $|\mathbf{E}|^2 - |\mathbf{B}|^2$. An electromagnetic field can be an electromagnetic wrench $(\mathbf{E} \cdot \mathbf{B} \neq 0)$, an electrostatic field $(\mathbf{E} \cdot \mathbf{B} = 0, |\mathbf{E}| > |\mathbf{B}|)$ or a magnetostatic field $(\mathbf{E} \cdot \mathbf{B} = 0, |\mathbf{B}| > |\mathbf{E}|)$. For a magnetostatic field, there exist frames in which only a magnetic field is present, and such a field is strictly stable against SPC. For

an electrostatic field there exist frames in which there is no magnetic field. An electrostatic field is intrinsically unstable to decay due to SPC (Schwinger 1951). A similar effect occurs for an electromagnetic wrench, for which there exists a frame where $\mathbf{E} \parallel \mathbf{B}$. In this frame the electric field decays, creating pairs at the rate per unit volume per unit time (Daugherty & Lerche 1976)

$$w_{\parallel} = \frac{e^2 |EB|}{4\pi^2 \hbar^2} \sum_{k=1}^{\infty} \frac{1}{k} \coth\left[k\pi \frac{|cB|}{|E|}\right] \exp\left[-k\pi \frac{E_c}{|E|}\right], \quad (A1)$$

with $E_c = m^2 c^3 / e \hbar$.

APPENDIX B: ESCAPING FLUX OF ELECTRONS

Writing $\tau = \nu_c t$, $\tilde{p} = pc/kT$, the effect of collisions between quarks and electrons in the nondegenerate tail of the electron distribution at $pc > \varepsilon_F$ in the ground state, can be approximated by

$$\frac{\partial \tilde{f}(\tilde{p})}{\partial \tau} = \frac{\partial}{\partial \tilde{p}} \left\{ d(\tilde{p}) \left[\frac{\partial \tilde{f}(\tilde{p})}{\partial \tilde{p}} + \tilde{f}(\tilde{p}) \right] \right\}, \tag{B1}$$

where \tilde{f} is the distribution function, and $d(\tilde{p})$ is determined by the p-dependence of the collisions. The two independent stationary solutions of (B1) are

$$\tilde{f}_1(\tilde{p}) = e^{-\tilde{p}}, \qquad \tilde{f}_2(\tilde{p}) = e^{-\tilde{p}} \int^{\tilde{p}} \frac{d\tilde{p}' e^{\tilde{p}'}}{d(\tilde{p}')},$$
 (B2)

and a general solution is $\tilde{f}(\tilde{p}) = A_1 \tilde{f}_1(\tilde{p}) + A_2 \tilde{f}_2(\tilde{p})$, where $A_{1,2}$ are constants. The solution $\tilde{f}_1(\tilde{p})$ contains no flux in momentum space, and the solution $\tilde{f}_2(\tilde{p})$ contains a constant flux, from $\tilde{p}=0$ to an unspecified upper momentum. One can include escape of electrons at $p \geqslant p_0$, or $\tilde{p} \geqslant \tilde{p}_0$, by requiring $\tilde{f}(\tilde{p}_0)=0$. The rate electrons escape is then proportional to the flux in momentum space, determined by

$$-\int^{\tilde{p}_0} d\tilde{p} \frac{\partial \tilde{f}(\tilde{p})}{\partial \tau} = -A_2.$$
 (B3)

(The flux is from $\tilde{p}=0$ to $\tilde{p}=\tilde{p}_0$.) The constants $A_{1,2}$ are determined by $\tilde{f}(\tilde{p}_0)=0$ and say $\tilde{f}(\tilde{p}_{\rm F})=1$, where $p_{\rm F}$ is the Fermi momentum. This gives

$$A_2 = -\left[e^{-\tilde{p}_F} \int_{\tilde{p}_F}^{\tilde{p}_0} \frac{d\tilde{p} \, e^{\tilde{p}}}{d(\tilde{p})}\right]^{-1}. \tag{B4}$$

In the special case in which $d(\tilde{p})$ does not depend on momentum, $d(\tilde{p}) \to 1$, corresponding to reflection off hard spheres, (B4) gives $A_2 \approx e^{-W/kT}$, where $W = (p_0 - p_{\rm F})c$ is assumed much larger than kT