Line Assignment

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I. Question

Class 11, Exercise 10.1, Q(1): **Draw a quadrilateral in the Cartesian plane, whose vertices are (-4,5), (0,7), (5,-5), (-4,-2). Also, find its area.**

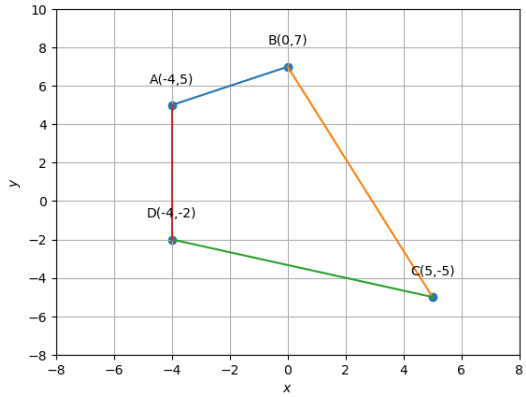

Figure 1: Quadrilateral ABCD

II. Construction

Symbol	Value	Description
A	$\begin{pmatrix}-4\\5\end{pmatrix}$	Vertex A
B	$\begin{pmatrix}0\\7\end{pmatrix}$	Vertex B
C	$\begin{pmatrix}5\\-5\end{pmatrix}$	Vertex C
D	$\begin{pmatrix}-4\\-2\end{pmatrix}$	Vertex D

III. Solution

We can divide the quadrilateral into two triangles, one with sides **AB**, **BC**, and **AC**, and the other with sides **AC**, **CD**, and **AD**.

1. Finding area using Matrices

Consider $\triangle ABC$,

$$Ar(\triangle \mathbf{ABC}) = \frac{1}{2}\begin{vmatrix}\mathbf{x_1} & \mathbf{y_1} & 1\\ \mathbf{x_2} & \mathbf{y_2} & 1\\ \mathbf{x_3} & \mathbf{y_3} & 1\end{vmatrix} \tag{1}$$

$$= \frac{1}{2}\begin{vmatrix}-4 & 5 & 1\\ 0 & 7 & 1\\ 5 & -5 & 1\end{vmatrix}$$

$$Ar(\triangle ABC) = 29 \quad sq.units \tag{2}$$

(Since area cannot be negative)

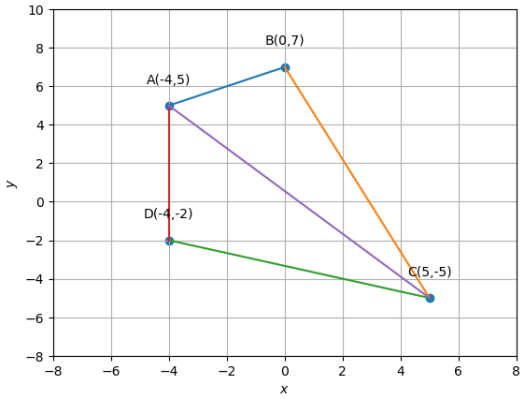

Figure 2: Quadrilateral ABCD with diagonal AC

Now consider $\triangle ADC$,

$$Ar(\triangle \mathbf{ADC}) = \frac{1}{2}\begin{vmatrix}\mathbf{x_1} & \mathbf{y_1} & 1\\ \mathbf{x_2} & \mathbf{y_2} & 1\\ \mathbf{x_3} & \mathbf{y_3} & 1\end{vmatrix} \tag{3}$$

$$= \frac{1}{2}\begin{vmatrix}-4 & 5 & 1\\ -4 & -2 & 1\\ 5 & -5 & 1\end{vmatrix}$$

$$Ar(\triangle ADC) = 31.5 \quad sq.units \tag{4}$$

Area of Quadrilateral ABCD

$$= \boldsymbol{Ar(\triangle ABC) + Ar(\triangle ADC)} \tag{5}$$

= 60.5 sq.units

2. Finding area using Cross product

Consider $\triangle \boldsymbol{ABC}$,

$$\boldsymbol{Ar}(\triangle \boldsymbol{ABC}) = \frac{\mathbf{1}}{\mathbf{2}}|\mathbf{B}-\mathbf{A} \quad \mathbf{B}-\mathbf{C}| \tag{6}$$

$$= \frac{1}{2}\begin{vmatrix} 4 & 2 \\ -5 & 12 \end{vmatrix} \tag{7}$$

$Ar(\triangle ABC) = 29$ sq.units

Now consider $\triangle ADC$,

$$Ar(\triangle ADC) = \frac{1}{2}|\mathbf{D}-\mathbf{A} \quad \mathbf{D}-\mathbf{C}| \tag{8}$$

$$= \frac{1}{2}\begin{vmatrix} 0 & -7 \\ -9 & 3 \end{vmatrix} \tag{9}$$

$Ar(\triangle ADC) = 31.5$ sq.units

Area of Quadrilateral ABCD
$= \boldsymbol{Ar}(\triangle \boldsymbol{ABC}) + \boldsymbol{Ar}(\triangle \boldsymbol{ADC})$

$= 60.5$ sq.units

Get the python code from

https://github.com/SinkonaChinthamalla/fwc/blob/main/matrix/line/codes