Optimization Assignment - 1

Sinkona Chinthamalla

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***Problem Statement* - Find the absolute maximum and minimum values of the function *f* given by**

$$f(x) = \cos^2 x + \sin x, \quad x \in [0, \pi]$$

1 Solution

Given function is,

$$f(x) = \cos^2 x + \sin x \tag{1}$$

1.1 Calculation using normal differentiation

Differentiating (1) yields,

$$\nabla f(x) = \cos x - 2\sin x \cos x \tag{2}$$

Calculating the critical points: $\nabla f(x) = 0$

$$\implies \cos x = 0 \tag{3}$$

$$\implies -2\sin x + 1 = 0 \tag{4}$$

Therefore, the critical points are

$$\frac{\pi}{6}, \quad \frac{5\pi}{6}, \quad \frac{\pi}{2} \tag{5}$$

1.1.1 Finding absolute maximum and minimum

Since given interval is $x \in [0, \pi]$

value of x	value of
At x $=0$	1
At x $=\frac{\pi}{6}$	$\frac{5}{4}$
At x $=\frac{\pi}{2}$	1
At x $=\frac{5\pi}{6}$	$\frac{5}{4}$
at x $=\pi$	1

Hence,

$$\text{absolute maximum} = \frac{5}{4} \tag{6}$$

$$\text{absolute minimum} = 1 \tag{7}$$

1.2 Calculation of Maxima using gradient ascent algorithm

Maxima of eq(1) is calculated by using gradient ascent method

$$x_{n+1} = x_n + \alpha \nabla f(x_n) \tag{8}$$

$$x_{n+1} = x_n + \alpha\left(cosx_n - 2sinx_n cosx_n\right) \tag{9}$$

where
1)$x_0 = 0.5$
2)$\alpha = 0.001$
3)precision $= 0.00000001$
values obtained using python are:

$$\boxed{\text{Maxima} = 1.25} \tag{10}$$

$$\boxed{\text{Maxima Point} = 0.52} \tag{11}$$

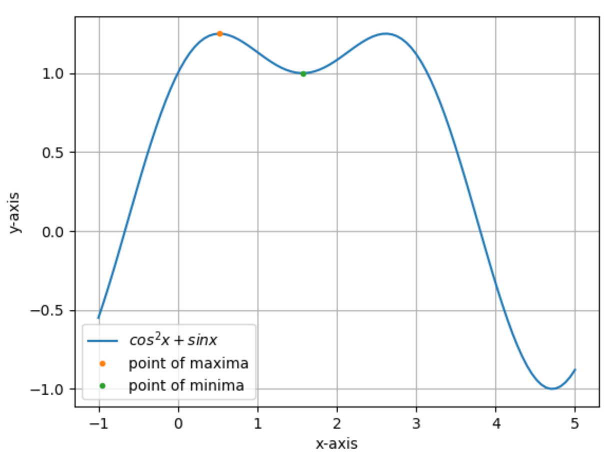

Figure 1: The function f(x) with maxima and minima points

1.3 Calculation of Minima using gradient descent algorithm

To find:

$$\min_x f(x) \tag{12}$$

Given:

$$f(x) = \cos^2 x + \sin x, \quad x \in [0, \pi] \tag{13}$$

Minima of eq(1) is found by using gradient descent method

$$x_{n+1} = x_n - \alpha \nabla f(x_n) \tag{14}$$

$$x_{n+1} = x_n - \alpha \left(cosx_n - 2sinx_n cosx_n \right) \tag{15}$$

where
1)$x_0 = 0.5$
2)$\alpha = 0.001$
3)precision = 0.00000001
values obtained using python are:

$$\boxed{\text{Minima} = 1} \tag{16}$$

$$\boxed{\text{Minima Point} = 1.57} \tag{17}$$