Probability Assignment

Sinkona Chinthamalla

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12.13.1.10
A black and a red dice are rolled.

1. Find the conditional probability of obtaining a sum greater than 9, given that the black die resulted in a 5.
2. Find the conditional probability of obtaining the sum 8, given that the red die resulted in a number less than 4.

Solution

Random Variable	Description
X_1	Outcome of Black die
X_2	Outcome of Red die

1. Given,
   Black die resulted in a 5.
   Conditional probability of event $(X_1 + X_2 > 9)$ given that $(X_1 = 5)$ has occurred is given by,

$$\Pr\left((X_1 + X_2 > 9) \,|\, (X_1 = 5)\right) = \frac{\Pr\left((X_1 + X_2 > 9), (X_1 = 5)\right)}{\Pr\left(X_1 = 5\right)} \tag{1}$$

   The probability of obtaining a sum greater than 9 is given by,

$$\begin{aligned}
\Pr\left((X_1 + X_2 > 9), (X_1 = 5)\right) &= \Pr\left((X_2 > 9 - X_1), (X_1 = 5)\right) && (2)\\
&= \Pr\left(X_2 > 9 - k | X_1 = k\right) p_{X_1}(k) && (3)\\
&= \frac{1}{6} \Pr\left(X_2 > 9 - 5 | X_1 = 5\right) && (4)\\
&= \frac{1}{6} \Pr\left(X_2 > 4\right) && (5)\\
&= \frac{1}{6} (\Pr\left(X_2 = 5\right) + \Pr\left(X_2 = 6\right)) && (6)\\
&= \frac{2}{36} && (7)
\end{aligned}$$

From (1) and (7),
Conditional probability of event $(X_2 > 4)$ given that $(X_1 = 5)$ has occurred is,

$$\Pr\left((X_2 > 4) \,|\, (X_1 = 5)\right) = \frac{\Pr\left((X_1 + X_2 > 9), (X_1 = 5)\right)}{\Pr\left(X_1 = 5\right)} \tag{8}$$

$$= \frac{\frac{2}{36}}{\frac{1}{6}} \tag{9}$$

$$= \frac{1}{3} \tag{10}$$

Hence the conditional probability of obtaining a sum greater than 9, when black die resulted in a 5 is $\frac{1}{3}$.

2. Given,
Red die resulted in a number less than 4.

Conditional probability of event $(X_1 + X_2 = 8)$ given that $(X_2 < 4)$ has occurred is given by,

$$\Pr\left((X_1 + X_2 = 8) \,|\, (X_2 < 4)\right) = \frac{\Pr\left((X_1 + X_2 = 8), (X_2 < 4)\right)}{\Pr\left(X_2 < 4\right)} \tag{11}$$

The probability of obtaining the sum 8 is given by,

$$\Pr\left((X_1 + X_2 = 8), (X_2 < 4)\right) = \Pr\left((X_1 = 8 - X_2), (X_2 < 4)\right) \tag{12}$$

$$= \Pr\left(X_1 = 8 - k | X_2 < k\right) p_{X_2}(k) \tag{13}$$

$$= \frac{1}{6} \Pr\left(X_1 = 8 - k | X_2 < 4\right) \tag{14}$$

$$= \frac{1}{6} (\Pr\left(X_1 = 5\right) + \Pr\left(X_1 = 6\right)) \tag{15}$$

$$= \frac{2}{36} \tag{16}$$

From (11) and (16),
Conditional probability of event $(X_1 + X_2 = 8)$ given that $(X_2 < 4)$ has occurred is,

$$\Pr\left((X_1 + X_2 = 8) \,|\, (X_2 < 4)\right) = \frac{\Pr\left((X_1 + X_2 = 8), (X_2 < 4)\right)}{\Pr\left(X_2 < 4\right)} \tag{17}$$

$$= \frac{\frac{2}{36}}{\frac{3}{6}} \tag{18}$$

$$= \frac{1}{9} \tag{19}$$

Hence the probability of obtaining the sum 8 when a number is less than 4 is $\frac{1}{9}$.