

Probability Assignment

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A black and a red dice are rolled.

1. Find the conditional probability of obtaining a sum greater than 9, given that the black die resulted in a 5.
2. Find the conditional probability of obtaining the sum 8, given that the red die resulted in a number less than 4.

Solution

Random Variable	Description
X_1	Outcome of Black die
X_2	Outcome of Red die

1. Given,
Black die resulted in a 5.

The probability of obtaining a sum greater than 9 is given by,

$$\Pr((X_2 > 4), (X_1 = 5)) = \Pr(X_1 + X_2 > 9) \quad (1)$$

$$= \Pr(X_2 > 9 - X_1) \quad (2)$$

$$= \Pr(X_2 > 9 - k | X_1 = k) p_{X_1}(k) \quad (3)$$

$$= \frac{1}{6} \Pr(X_2 > 9 - 5 | X_1 = 5) \quad (4)$$

$$= \frac{1}{6} \Pr(X_2 > 4) \quad (5)$$

$$= \frac{1}{6} (\Pr(X_2 = 5) + \Pr(X_2 = 6)) \quad (6)$$

$$= \frac{2}{36} \quad (7)$$

Conditional probability of event $(X_2 > 4)$ given that $(X_1 = 5)$ has occurred is,

$$\Pr((X_2 > 4) | (X_1 = 5)) = \frac{\Pr((X_2 > 4), (X_1 = 5))}{\Pr(X_1 = 5)} \quad (8)$$

$$= \frac{\frac{2}{36}}{\frac{1}{6}} \quad (9)$$

$$= \frac{1}{3} \quad (10)$$

Hence the conditional probability of obtaining a sum greater than 9, when black die resulted in a 5 is $\frac{1}{3}$.

2. Given,

Red die resulted in a number less than 4.

The probability of obtaining the sum 8 is given by,

$$\Pr((X_1 + X_2 = 8), (X_2 < 4)) = \Pr(X_1 + X_2 = 8) \quad (11)$$

$$= \Pr(X_1 = 8 - X_2) \quad (12)$$

$$= \Pr(X_1 = 8 - k | X_2 < k) p_{X_2}(k) \quad (13)$$

$$= \frac{1}{6} \Pr(X_1 = 8 - k | X_2 < 4) \quad (14)$$

$$= \frac{1}{6} (\Pr(X_1 = 5) + \Pr(X_1 = 6)) \quad (15)$$

$$= \frac{2}{36} \quad (16)$$

Conditional probability of event $(X_1 + X_2 = 8)$ given that $(X_2 < 4)$ has occurred is,

$$\Pr((X_1 + X_2 = 8) | (X_2 < 4)) = \frac{\Pr((X_1 + X_2 = 8), (X_2 < 4))}{\Pr(X_2 < 4)} \quad (17)$$

$$= \frac{\frac{2}{36}}{\frac{3}{6}} \quad (18)$$

$$= \frac{1}{9} \quad (19)$$

Hence the probability of obtaining the sum 8 when a number is less than 4 is $\frac{1}{9}$.