Probability Assignment

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A black and a red dice are rolled.

1. Find the conditional probability of obtaining a sum greater than 9, given that the black die resulted in a 5.

2. Find the conditional probability of obtaining the sum 8, given that the red die resulted in a number less than 4.

Solution

1. *The Uniform Distribution:* Let $X_i \in \{1,2,3,4,5,6\}, i = 1, 2,$ be the random variables representing the outcome for each die. Assuming the dice to be fair, the probability mass function (pmf) is expressed as

$$p_{X_i}(n) = \Pr(X_i = n) = \begin{cases} \frac{1}{6} & 1 \le n \le 6 \\ 0 & otherwise \end{cases} \tag{1}$$

The desired outcome is

$$X = X_1 + X_2, \tag{2}$$
$$\implies X \in \{1, 2, \dots, 12\} \tag{3}$$

Convolution:
From (2)

$$p_X(n) = \Pr(X_1 + X_2 > n) = \Pr(X_1 > n - X_2) \tag{4}$$
$$= \sum_k \Pr(X_1 > n - k | X_2 = k) \, p_{X_2}(k) \tag{5}$$

after unconditioning. $\because$ X_1 and X_2 are independent. Then,

$$p_X(n) = \Pr(X_1 + X_2 > 9) = \Pr(X_1 > 9 - X_2) \tag{6}$$
$$= \Pr(X_1 > 9 - k | X_2 = k) \, p_{X_2}(k) \tag{7}$$
$$= \frac{1}{6} \Pr(X_1 > 9 - 5 | X_2 = 5) \tag{8}$$
$$= \frac{1}{6} \Pr(X_1 > 4) \tag{9}$$
$$= \frac{1}{6} (\Pr(X_1 = 5) + \Pr(X_1 = 6)) \tag{10}$$
$$= \frac{2}{36} \tag{11}$$

From (1) and (11), Conditional probability of event $(X_1 + X_2 > 9)$ given that $(X_2 = 5)$ has occurred is, $\Pr((X_1 + X_2 > 9) \,|\, (X_2 = 5))$

$$= \frac{\Pr((X_1 + X_2 > 9)(X_2 = 5))}{\Pr(X_2 = 5)} \tag{12}$$
$$= \frac{\frac{2}{36}}{\frac{1}{6}} \tag{13}$$
$$= \frac{1}{3} \tag{14}$$

Hence the probability of obtaining a sum greater than 9, when black die resulted in a 5 is $\frac{1}{3}$.

2. *The Uniform Distribution:* Let $X_i \in \{1,2,3,4,5,6\}, i = 1, 2,$ be the random variables representing the outcome for each die.
Convolution:
From (2)

$$p_X(n) = \Pr(X_1 + X_2 = n) = \Pr(X_1 = n - X_2) \tag{15}$$
$$= \sum_k \Pr(X_1 = n - k | X_2 = k) \, p_{X_2}(k) \tag{16}$$

after unconditioning. $\because$ X_1 and X_2 are independent. Then,

$$p_X(n) = \Pr(X_1 + X_2 = 8) = \Pr(X_1 = 8 - X_2) \tag{17}$$
$$= \Pr(X_1 = 8 - k | X_2 < k) \, p_{X_2}(k) \tag{18}$$
$$= \frac{1}{6} \Pr(X_1 = 8 - k | X_2 < 4) \tag{19}$$
$$= \frac{1}{6} (\Pr(X_1 = 5) + \Pr(X_1 = 6)) \tag{20}$$
$$= \frac{2}{36} \tag{21}$$

From (1) and (21), Conditional probability of event $(X_1 + X_2 = 8)$ given that $(X_2 < 4)$ has occurred is, $\Pr((X_1 + X_2 = 8) \,|\, (X_2 < 4))$

$$= \frac{\Pr((X_1 + X_2 = 8)(X_2 < 4))}{\Pr(X_2 < 4)} \tag{22}$$
$$= \frac{\frac{2}{36}}{\frac{3}{6}} \tag{23}$$
$$= \frac{1}{9} \tag{24}$$

Hence the probability of obtaining the sum 8 when a number is less than 4 is $\frac{1}{9}$.