Probability Assignment

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A black and a red dice are rolled.

1. Find the conditional probability of obtaining a sum greater than 9, given that the black die resulted in a 5.
2. Find the conditional probability of obtaining the sum 8, given that the red die resulted in a number less than 4.

Solution

Let X_1 be a random variable representing the outcome of black die and X_2 be a random variable representing the outcome of red die.

1. Given,
   Black die resulted in a 5.

   The probability of obtaining a sum greater than 9 is given by,

$$\begin{aligned}
\Pr\left((X_2 > 4), (X_1 = 5)\right) &= \Pr\left(X_1 + X_2 > 9\right) && (1)\\
&= \Pr\left(X_2 > 9 - X_1\right) && (2)\\
&= \Pr\left(X_2 > 9 - k | X_1 = k\right) p_{X_1}(k) && (3)\\
&= \frac{1}{6}\Pr\left(X_2 > 9 - 5 | X_1 = 5\right) && (4)\\
&= \frac{1}{6}\Pr\left(X_2 > 4\right) && (5)\\
&= \frac{1}{6}(\Pr\left(X_2 = 5\right) + \Pr\left(X_2 = 6\right)) && (6)\\
&= \frac{2}{36} && (7)
\end{aligned}$$

   Conditional probability of event $(X_2 > 4)$ given that $(X_1 = 5)$ has occurred is,

$$\begin{aligned}
\Pr\left((X_2 > 4) \,|\, (X_1 = 5)\right) &= \frac{\Pr\left((X_2 > 4), (X_1 = 5)\right)}{\Pr\left(X_1 = 5\right)} && (8)\\
&= \frac{\frac{2}{36}}{\frac{1}{6}} && (9)\\
&= \frac{1}{3} && (10)
\end{aligned}$$

   Hence the conditional probability of obtaining a sum greater than 9, when black die resulted in a 5 is $\frac{1}{3}$.

2. Given,
   Red die resulted in a number less than 4.

   The probability of obtaining the sum 8 is given by,

$$\begin{aligned}
\Pr\left((X_1 + X_2 = 8), (X_2 < 4)\right) &= \Pr\left(X_1 + X_2 = 8\right) && (11)\\
&= \Pr\left(X_1 = 8 - X_2\right) && (12)\\
&= \Pr\left(X_1 = 8 - k | X_2 < k\right) p_{X_2}(k) && (13)\\
&= \frac{1}{6}\Pr\left(X_1 = 8 - k | X_2 < 4\right) && (14)\\
&= \frac{1}{6}(\Pr\left(X_1 = 5\right) + \Pr\left(X_1 = 6\right)) && (15)\\
&= \frac{2}{36} && (16)
\end{aligned}$$

   Conditional probability of event $(X_1 + X_2 = 8)$ given that $(X_2 < 4)$ has occurred is,

$$\begin{aligned}
\Pr\left((X_1 + X_2 = 8) \,|\, (X_2 < 4)\right) &= \frac{\Pr\left((X_1 + X_2 = 8), (X_2 < 4)\right)}{\Pr\left(X_2 < 4\right)} && (17)\\
&= \frac{\frac{2}{36}}{\frac{3}{6}} && (18)\\
&= \frac{1}{9} && (19)
\end{aligned}$$

   Hence the probability of obtaining the sum 8 when a number is less than 4 is $\frac{1}{9}$.