Probability Assignment

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A black and a red dice are rolled.

1. Find the conditional probability of obtaining a sum greater than 9, given that the black die resulted in a 5.

2. Find the conditional probability of obtaining the sum 8, given that the red die resulted in a number less than 4.

Solution

Let $X_i \in \{1,2,3,4,5,6\}, i = 1,2$, be the random variables representing the outcome for each die.

1. Since X_1 and X_2 are independent,

$$\Pr((X_1 > 4),(X_2 = 5)) = \Pr(X_1 + X_2 > 9) \tag{1}$$
$$= \Pr(X_1 > 9 - X_2) \tag{2}$$
$$= \Pr(X_1 > 9 - k|X_2 = k)\, p_{X_2}(k) \tag{3}$$
$$= \frac{1}{6}\Pr(X_1 > 9 - 5|X_2 = 5) \tag{4}$$
$$= \frac{1}{6}\Pr(X_1 > 4) \tag{5}$$
$$= \frac{1}{6}(\Pr(X_1 = 5) + \Pr(X_1 = 6)) \tag{6}$$
$$= \frac{2}{36} \tag{7}$$

Conditional probability of event $(X_1 > 4)$ given that $(X_2 = 5)$ has occurred is,

$$\Pr((X_1 > 4)\,|\,(X_2 = 5)) = \frac{\Pr((X_1 > 4),(X_2 = 5))}{\Pr(X_2 = 5)} \tag{8}$$
$$= \frac{\frac{2}{36}}{\frac{1}{6}} \tag{9}$$
$$= \frac{1}{3} \tag{10}$$

Hence the probability of obtaining a sum greater than 9, when black die resulted in a 5 is $\frac{1}{3}$.

2. Since X_1 and X_2 are independent,

$$\Pr((X_1 + X_2 = 8),(X_2 < 4)) = \Pr(X_1 + X_2 = 8) \tag{11}$$
$$= \Pr(X_1 = 8 - X_2) \tag{12}$$
$$= \Pr(X_1 = 8 - k|X_2 < k)\, p_{X_2}(k) \tag{13}$$
$$= \frac{1}{6}\Pr(X_1 = 8 - k|X_2 < 4) \tag{14}$$
$$= \frac{1}{6}(\Pr(X_1 = 5) + \Pr(X_1 = 6)) \tag{15}$$
$$= \frac{2}{36} \tag{16}$$

Conditional probability of event $(X_1 + X_2 = 8)$ given that $(X_2 < 4)$ has occurred is,

$$\Pr((X_1 + X_2 = 8)\,|\,(X_2 < 4)) = \frac{\Pr((X_1 + X_2 = 8),(X_2 < 4))}{\Pr(X_2 < 4)} \tag{17}$$
$$= \frac{\frac{2}{36}}{\frac{3}{6}} \tag{18}$$
$$= \frac{1}{9} \tag{19}$$

Hence the probability of obtaining the sum 8 when a number is less than 4 is $\frac{1}{9}$.