Probability Assignment

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12.13.1.10
A black and a red dice are rolled.

1. Find the conditional probability of obtaining a sum greater than 9, given that the black die resulted in a 5.
2. Find the conditional probability of obtaining the sum 8, given that the red die resulted in a number less than 4.

Solution

Let $X_i \in \{1,2,3,4,5,6\}, i = 1,2$, be the random variables representing the outcome for each die.

1. Since X_1 and X_2 are independent,

$$\begin{aligned} p_X(n) &= \Pr(X_1 + X_2 > 9) = \Pr(X_1 > 9 - X_2) && (1)\\ &= \Pr(X_1 > 9 - k | X_2 = k)\, p_{X_2}(k) && (2)\\ &= \frac{1}{6}\Pr(X_1 > 9 - 5 | X_2 = 5) && (3)\\ &= \frac{1}{6}\Pr(X_1 > 4) && (4)\\ &= \frac{1}{6}(\Pr(X_1 = 5) + \Pr(X_1 = 6)) && (5)\\ &= \frac{2}{36} && (6) \end{aligned}$$

   Conditional probability of event $(X_1 > 4)$ given that $(X_2 = 5)$ has occurred is,
   $\Pr((X_1 > 4) \,|\, (X_2 = 5))$

$$\begin{aligned} &= \frac{\Pr((X_1 > 4), (X_2 = 5))}{\Pr(X_2 = 5)} && (7)\\ &= \frac{\frac{2}{36}}{\frac{1}{6}} && (8)\\ &= \frac{1}{3} && (9) \end{aligned}$$

   Hence the probability of obtaining a sum greater than 9, when black die resulted in a 5 is $\frac{1}{3}$.

2. Since X_1 and X_2 are independent,

$$\begin{aligned} p_X(n) &= \Pr(X_1 + X_2 = 8) = \Pr(X_1 = 8 - X_2) && (10)\\ &= \Pr(X_1 = 8 - k | X_2 < k)\, p_{X_2}(k) && (11)\\ &= \frac{1}{6}\Pr(X_1 = 8 - k | X_2 < 4) && (12)\\ &= \frac{1}{6}(\Pr(X_1 = 5) + \Pr(X_1 = 6)) && (13)\\ &= \frac{2}{36} && (14) \end{aligned}$$

   Conditional probability of event $((X_1 = 5) + (X_1 = 6))$ given that $(X_2 < 4)$ has occurred is,
   $\Pr(((X_1 = 5) + (X_1 = 6)) \,|\, (X_2 < 4))$

$$\begin{aligned} &= \frac{\Pr(((X_1 = 5) + (X_1 = 6)), (X_2 < 4))}{\Pr(X_2 < 4)} && (15)\\ &= \frac{\frac{2}{36}}{\frac{3}{6}} && (16)\\ &= \frac{1}{9} && (17) \end{aligned}$$

   Hence the probability of obtaining the sum 8 when a number is less than 4 is $\frac{1}{9}$.