PHY 240: Basic Electronics
Homework Problem H5

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1. When Resistors Play Together.

   (a) When a resistor with resistance R_1 is placed in series with a resis- tor of resistance R_2, prove that the total resistance of the combination is $R_{tot} = R_1 + R_2$. This needs to be an iron-clad proof, with clear steps and justifications for each step.

   (b) When a resistor with resistance R_1 is placed in parallel with a resistor of resistance R_2, prove that the total resistance of the combination is $R_{tot} = \frac{R_1 R_2}{R_1 + R_2}$. Again, this needs to be an iron-clad proof, with clear steps and justifications for each step.

   (c) You are given a black box with three terminals, labeled Rock (R), Paper (P), and Scissors (S). You know that the box contains five 100 Ω resistors connected in a particular way between these terminals. Upon measuring the resistances between various terminals, you find the following:

$$R_{RP} = 300\Omega$$

$$R_{PS} = 250\Omega$$

$$R_{RS} = 150\Omega$$

   Here, R_{XY} represents the resistance between terminals X and Y when the third terminal is not connected to anything. Provide a schematic showing how the five resistors are connected within the box. Be sure to label the box outputs R, P, and S as appropriate.

Solution:

(a) We know that $V = IR$ is true, let that be our guiding light. We can say...

$$V_{total} = IR_{total}$$

So...

$$R_{total} = \frac{V_{total}}{I}$$

By definition:

$$V_{total} = \sum_{i=1}^{n} V_n$$

$$V_{total} = V_1 + V_2$$

But $V = IR$, so naturally...

$$V_1 = IR_1$$

$$V_2 = IR_2$$

$$R_{total} = \frac{IR_1 + IR_2}{I}$$

$$R_{total} = I \times \frac{R_1 + R_2}{I}$$

Hence,

$$R_{total} = R_1 + R_2 \blacksquare$$

(b) Once again, $V = IR$, but since these resistors are in parallel, the voltage drop across them are the same, however their current is different.

$$V = I_{total}R_{total}$$

That is to say...

$$R_{total} = \frac{V}{I_{total}}$$

Once again, by definition:

$$I_{total} = \sum_{i=1}^{n} I_n$$

$$I_{total} = I_1 + I_2$$

But $I = \frac{V}{R}$, so

$$I_1 = \frac{V}{R_1}$$

$$I_2 = \frac{V}{R_2}$$

$$R_{total} = \frac{V}{\frac{V}{R_1} + \frac{V}{R_2}}$$

$$R_{total} = \frac{V}{V \times (\frac{1}{R_1} + \frac{1}{R_2})}$$

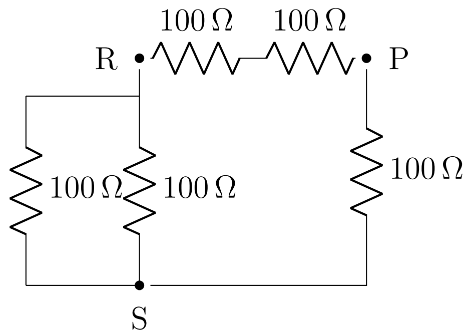
$$R_{total} = \frac{1}{\frac{1}{R_1} + \frac{1}{R_2}}$$

Now we find a common denominator for this fraction

$$R_{total} = \frac{1}{\frac{R_1+R_2}{R_1R_2}}$$

Hense,

$$R_{total} = \frac{R_1R_2}{R_1 + R_2} \blacksquare$$



(c)