# PHY 240: Basic Electronics
# Homework Problem H1

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1. How Fast Are Those lil' Buggers Moving?

   (a) The first step is to determine the number density of free electrons in a typical wire. I say "free" electrons because most of the electrons in a wire are bound to their atoms and may not move freely through the wire. We will assume that our wire is copper, and here are the only things that you may use while solving this problem:

   - The density of Copper at room temperature (approximately 8.95 g/cm$^3$).
   - The mass of a single Copper atom (63.55 amu)
   - The fact that each copper atom contriubutes *one* free electron.
   - Avagadro's Number.

   Use this information to determine the number density of free electrons (that is, the number of free electrons per volume) in Copper. Explain the reasoning behind each step of your calculation *clearly*.

   (b) Suppose that our wire is an AWG 14 gauge wire1 that has a current of 500 mA flowing through it. Determine the average velocity of the electrons "down the wire" that is necessary to produce this current.

   (c) The answer that you obtained for Part (b) above is called the "electron drift velocity", because it is the velocity at which electrons drift down the wire. Even if you turn the electric current off, however, the electrons are still moving in the wire (they simply lack a tendency to drift in any particular direction). Because of the Pauli exclusion principle, the electrons in copper are stacked up so high that the free electrons in the wire have a kinetic energy of roughly 7 eV.

   Convert this 7 eV to Joules. Look up the mass of an electron. Dust off your Physics I, and use the expression for kinetic energy to determine how fast these free electrons must be moving when the current is off. The velocity that you so determine is called the Fermi velocity, $v_{ferm}$.

   (d) Let us now turn the 500 mA current back on. Compare the drift velocity, $v_{drift}$, to the Fermi velocity, $v_{ferm}$. What fraction of $v_{ferm}$ is $v_{drift}$?

**Solution:**

(a) To determine the number density of free electrons in copper, we need to calculate how many free electrons exist per unit volume of copper. Here's how we can do it:

Given data:

- **Density of Copper ($\rho$)**: 8.95 g/cm$^3$
- **Atomic mass of Copper ($M$)**: 63.55 amu (atomic mass units)
- **Avogadro's number ($N_A$)**: $6.022 \times 10^{23}$ atoms/mol

The atomic mass unit (amu) is defined as 1/12 of the mass of a carbon-12 atom. The molar mass (in grams per mole) of copper is equivalent to its atomic mass in amu. Therefore:

$$M = 63.55\,\text{g/mol}$$

Using the density ($\rho$) and the molar mass ($M$), we can calculate the molar volume of copper:

$$\text{Molar volume} = \frac{M}{\rho} = \frac{63.55\,\text{g/mol}}{8.95\,\text{g/cm}^3} = 7.1\,\text{cm}^3/\text{mol}$$

This is the volume occupied by one mole of copper atoms.

We can now determine the number of atoms per cubic centimeter:

$$\text{Number density of atoms} = \frac{N_A}{\text{Molar volume}} = \frac{6.022 \times 10^{23}\,\text{atoms/mol}}{7.1\,\text{cm}^3/\text{mol}} \approx 8.48 \times 10^{22}\,\text{atoms/cm}^3$$

Since each copper atom contributes one free electron, the number density of free electrons is the same as the number density of copper atoms:

$$\text{Number density of free electrons} \approx 8.48 \times 10^{22}\,\text{electrons/cm}^3$$

**The number density of free electrons in copper is approximately $8.48 \times 10^{22}$ electrons per cubic centimeter. This means that in every cubic centimeter of copper, there are around $8.48 \times 10^{22}$ free electrons available to conduct electricity.**

(b) To determine the average velocity of the electrons, also known as the drift velocity, we can use the following relationship between the current, charge, and drift velocity:

$$I = n \cdot A \cdot v_d \cdot e$$

Where:

- $I$ is the current (500 mA $= 0.5\,\text{A}$)
- $n$ is the number density of free electrons (from part (a))
- $A$ is the cross-sectional area of the wire
- $v_d$ is the drift velocity (what we're trying to find)
- $e$ is the elementary charge ($1.6 \times 10^{-19}\,\text{C}$)

For an American Wire Gauge (AWG) 14 wire, the diameter is approximately 1.628 mm. The cross-sectional area $A$ of the wire is given by:

$$A = \pi \left(\frac{d}{2}\right)^2 = \pi \left(\frac{1.628 \times 10^{-3}\,\text{m}}{2}\right)^2$$

$$A \approx 2.08 \times 10^{-6}\,\text{m}^2$$

Rearrange the equation $I = n \cdot A \cdot v_d \cdot e$ to solve for $v_d$:

$$v_d = \frac{I}{n \cdot A \cdot e}$$

Now, substitute the known values into the equation:

$$v_d = \frac{0.5\,\text{A}}{(8.48 \times 10^{22}\,\text{electrons/m}^3) \cdot (2.08 \times 10^{-6}\,\text{m}^2) \cdot (1.6 \times 10^{-19}\,\text{C})}$$

Now we calculate the Drift Velocity:

$$v_d \approx \frac{0.5}{8.48 \times 10^{22} \times 2.08 \times 10^{-6} \times 1.6 \times 10^{-19}}$$

$$v_d \approx 1.78 \times 10^{-4}\,\text{m/s}$$

**The average drift velocity of the electrons down the wire necessary to produce a current of 500 mA is approximately $1.78 \times 10^{-4}$ m/s, or 0.178 mm/s.**

(c) To determine the Fermi velocity ($v_{\text{Fermi}}$) of the electrons, we can follow these steps:

First, we need to convert the energy from electron volts (eV) to joules (J).

$$1\,\text{eV} = 1.602 \times 10^{-19}\,\text{J}$$

So,

$$7\,\text{eV} = 7 \times 1.602 \times 10^{-19}\,\text{J} = 1.1214 \times 10^{-18}\,\text{J}$$

The kinetic energy ($K$) of an electron can be given by the classical expression:

$$K = \frac{1}{2} m_e v_{\text{Fermi}}^2$$

Where:

- $K$ is the kinetic energy ($1.1214 \times 10^{-18}\,\text{J}$)
- $m_e$ is the mass of an electron ($9.109 \times 10^{-31}\,\text{kg}$)
- $v_{\text{Fermi}}$ is the Fermi velocity (what we are solving for)

Rearrange the kinetic energy equation to solve for $v_{\text{Fermi}}$:

$$v_{\text{Fermi}} = \sqrt{\frac{2K}{m_e}}$$

Substituting the known values:

$$v_{\text{Fermi}} = \sqrt{\frac{2 \times 1.1214 \times 10^{-18}\,\text{J}}{9.109 \times 10^{-31}\,\text{kg}}}$$

$$v_{\text{Fermi}} = \sqrt{\frac{2.2428 \times 10^{-18}}{9.109 \times 10^{-31}}}$$

$$v_{\text{Fermi}} \approx \sqrt{2.462 \times 10^{12}}\,\text{m/s}$$

$$v_{\text{Fermi}} \approx 1.57 \times 10^{6}\,\text{m/s}$$

**The Fermi velocity ($v_{\text{Fermi}}$) of the electrons in copper, when the current is off, is approximately $1.57 \times 10^{6}$ m/s.**

(d) To compare the drift velocity $v_{\text{drift}}$ to the Fermi velocity $v_{\text{Fermi}}$ and determine what fraction of $v_{\text{Fermi}}$ is $v_{\text{drift}}$, we can follow these steps:

Given:

- $v_{\text{drift}} \approx 1.78 \times 10^{-4}\,\text{m/s}$ (from part (b))
- $v_{\text{Fermi}} \approx 1.57 \times 10^{6}\,\text{m/s}$ (from part (c))

$$\text{Fraction} = \frac{v_{\text{drift}}}{v_{\text{Fermi}}}$$

Substituting the given values:

$$\text{Fraction} = \frac{1.78 \times 10^{-4}\,\text{m/s}}{1.57 \times 10^{6}\,\text{m/s}}$$

$$\text{Fraction} \approx \frac{1.78 \times 10^{-4}}{1.57 \times 10^{6}} = 1.13 \times 10^{-10}$$

**The drift velocity $v_{\text{drift}}$ is approximately $1.13 \times 10^{-10}$ times the Fermi velocity $v_{\text{Fermi}}$. This shows that the drift velocity is an extremely small fraction of the Fermi velocity, indicating that even when a current is flowing, the electrons move very slowly on average compared to their random motion due to thermal energy.**