PHY 240: Basic Electronics
Homework Problem H1

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1. How Fast Are Those lil' Buggers Moving?

   (a) The first step is to determine the number density of free electrons in a typical wire. I say "free" electrons because most of the electrons in a wire are bound to their atoms and may not move freely through the wire. We will assume that our wire is copper, and here are the only things that you may use while solving this problem:

   - The density of Copper at room temperature (approximately 8.95 g/cm^3).
   - The mass of a single Copper atom (63.55 amu)
   - The fact that each copper atom contriubutes *one* free electron.
   - Avagadro's Number.

   Use this information to determine the number density of free electrons (that is, the number of free electrons per volume) in Copper. Explain the reasoning behind each step of your calculation *clearly*.

   (b) Suppose that our wire is an AWG 14 gauge wire1 that has a current of 500 mA flowing through it. Determine the average velocity of the electrons "down the wire" that is necessary to produce this current.

   (c) The answer that you obtained for Part (b) above is called the "electron drift velocity", because it is the velocity at which electrons drift down the wire. Even if you turn the electric current off, however, the electrons are still moving in the wire (they simply lack a tendency to drift in any particular direction). Because of the Pauli exclusion principle, the electrons in copper are stacked up so high that the free electrons in the wire have a kinetic energy of roughly 7 eV.

   Convert this 7 eV to Joules. Look up the mass of an electron. Dust off your Physics I, and use the expression for kinetic energy to determine how fast these free electrons must be moving when the current is off. The velocity that you so determine is called the Fermi velocity, v_{ferm}.

   (d) Let us now turn the 500 mA current back on. Compare the drift velocity, v_{drift}, to the Fermi velocity, v_{ferm}. What fraction of v_{ferm} is v_{drift}?

Solution:

(a) To determine the number density of free electrons in copper, we need to calculate how many free electrons exist per unit volume of copper. Given data:

- **Density of Copper** (ρ): 8.95 g/cm^3
- **Atomic mass of Copper** (M): 63.55 amu (atomic mass units)
- **Avogadro's number** (N_A): 6.022×10^{23} atoms/mol

The molar mass (in grams per mole) of copper is equivalent to its atomic mass in amu. Therefore:

$$M = 63.55 \text{ g/mol}$$

Using the density (ρ) and the molar mass (M), we can calculate the molar volume of copper:

$$\text{Molar volume} = \frac{M}{\rho} = \frac{63.55 \text{ g/mol}}{8.95 \text{ g/cm}^3} = 7.1 \text{ cm}^3/\text{mol}$$

This is the volume occupied by one mole of copper atoms. We can now determine the number of atoms per cubic centimeter:

$$\text{Number density of atoms} = \frac{N_A}{\text{Molar volume}} =$$

$$\frac{6.022 \times 10^{23} \text{ atoms/mol}}{7.1 \text{ cm}^3/\text{mol}} \approx 8.48 \times 10^{22} \text{ atoms/cm}^3$$

Since each copper atom contributes one free electron, the number density of free electrons is the same as the number density of copper atoms:

$$\text{Number density of free electrons} \approx 8.48 \times 10^{22} \text{ electrons/cm}^3$$

The number density of free electrons in copper is approximately 8.48×10^{22} electrons per cubic centimeter. This means that in every cubic centimeter of copper, there are around 8.48×10^{22} free electrons available to conduct electricity.

(b) To determine the average velocity of the electrons, also known as the drift velocity, we can use the following relationship between the current, charge, and drift velocity:

$$I = n \cdot A \cdot v_d \cdot e$$

Where the:

- **Current** (I): 0.5 A
- **Number density of free electrons** (n): 8.48×10^{28} electrons/m^3
- **Cross-sectional area of the wire** (A)
- **Drift velocity** (v_d)
- **Elementary charge** (e): 1.6×10^{-19} C

Using the formula given for the diameter of an AWG wire, the diameter of the wire is:

$$d = 0.127\text{mm} \cdot 92^{\frac{22}{39}} \approx 1.628\text{mm}$$

The cross-sectional area A of the wire is given by:

$$A = \pi \left(\frac{d}{2} \right)^2 = \pi \left(\frac{1.628 \times 10^{-3} \text{ m}}{2} \right)^2$$

$$A \approx 2.08 \times 10^{-6} \text{ m}^2$$

Rearrange the equation $I = n \cdot A \cdot v_d \cdot e$ to solve for v_d:

$$v_d = \frac{I}{n \cdot A \cdot e}$$

Now, substitute the known values into the equation:

$$v_d = \frac{0.5 \, \text{A}}{(8.48 \times 10^{28} \, \text{electrons/m}^3) \cdot (2.08 \times 10^{-6} \, \text{m}^2) \cdot (1.6 \times 10^{-19} \, \text{C})}$$

$$v_d \approx \frac{0.5}{8.48 \times 10^{28} \cdot 2.08 \times 10^{-6} \cdot 1.6 \times 10^{-19}}$$

$$v_d \approx 1.77 \times 10^{-5} \text{ m/s}$$

The average drift velocity of the electrons down the wire necessary to produce a current of 500 mA is approximately 0.177×10^{-5} **m/s**

(c)
$$7 \text{ eV} = 1.122 \times 10^{-18} \text{ J}$$

The kinetic energy (K) of an electron can be given by the classical expression:
$$K = \frac{1}{2} m_e v_{\text{Fermi}}^2$$

Where:

- **Kinetic energy (K):** 1.122×10^{-18} J
- **Mass of an electron m_e:** 9.109×10^{-31} kg
- **Fermi velocity $v_{\textbf{Fermi}}$**

Rearrange the kinetic energy equation to solve for v_{Fermi}:

$$v_{\text{Fermi}} = \sqrt{\frac{2K}{m_e}}$$

Substituting the known values:

$$v_{\text{Fermi}} = \sqrt{\frac{2 \cdot 1.122 \times 10^{-18} \text{ J}}{9.109 \times 10^{-31} \text{ kg}}}$$

$$v_{\text{Fermi}} = \sqrt{\frac{2.244 \times 10^{-18}}{9.109 \times 10^{-31}}}$$

$$v_{\text{Fermi}} \approx \sqrt{2.463 \times 10^{12}} \text{ m/s}$$

$$v_{\text{Fermi}} \approx 1.57 \times 10^{6} \text{ m/s}$$

The Fermi velocity ($v_{\textbf{Fermi}}$) of the electrons in copper, when the current is off, is approximately 1.57×10^{6} m/s.

(d) Given:

- $v_{\text{drift}} \approx 1.77 \times 10^{-5}$ m/s
- $v_{\text{Fermi}} \approx 1.57 \times 10^{6}$ m/s

The fraction of v_{Fermi} that is v_{drift} is:

$$\text{Fraction} = \frac{v_{\text{drift}}}{v_{\text{Fermi}}}$$

$$\text{Fraction} = \frac{1.77 \times 10^{-5} \text{ m/s}}{1.57 \times 10^{6} \text{ m/s}} \approx 1.13 \times 10^{-11}$$

The drift velocity v_{drift} is approximately 1.13×10^{-11} times the Fermi velocity v_{Fermi}