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**Summary Sheet**

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### **Abstract**

“Taraxacum” commonly known as dandelion, is a plant native to Eurasia and can now be found worldwide. This plant is easily identifiable by its bright yellow flowers and its unique "bubble" seed head. Each seed on this head is attached to a parachute-like structure, called "Pafoss", which helps the wind to spread. We create a mathematical model that combines the effects of climate, precipitation, temperature, soil and light on dandelion growth to simulate the spread of a dandelion in January, February, March, June and December on a hectare of land. For the modeling of dandelion propagation problem, we use the Markov chain to predict the probability of its propagation range.

The relationship between dandelions, humans, and other flora is complicated. Because of the plant's ability to thrive in diverse environments, some label it a pesky weed or as an invasive species. Conversely, every part of the dandelion is edible, and the plant has a rich history of medicinal and culinary use. We have integrated the variables of multiple invasive species impact factors into four parts: dandelion itself, the relationship with the local ecosystem, time, and the external environment. We have improved these four parts and refined them into. After sorting out. We used both the Malthus model and the Logistic model in this section fuzzy evaluation is carried out by establishing random models and biological system dynamics models.

We should make it clear before studying whether dandelions are invasive species. Non-native species are plants and animals living in areas where they do not naturally exist. “Non-native species” and “invasive species” cannot be used interchangeably. Many commonly grown fruits and vegetables are not native to the U.S. For example, tomatoes and hot peppers originated from South America, while lettuce was first grown in Egypt. Domestic cows are non- native to North America and were introduced as a food source and considered to be a beneficial organism in an agricultural setting.

Because of their love affair with human habitats, dandelions are clearly here to stay. So how can we learn to live happily with this biological superstar? Herbicides may temporarily reduce dandelion numbers, but they can also poison people and other creatures and pollute water - a huge environmental cost. Instead, let's reduce dandelion habitat by maintaining healthy lawns and using alternative landscaping. If we learn to “dig” our dandelions, we should be able to enjoy the few that remain.

**Key Words:** Dandelion, Malthus, Logistic, Differential equations,, Invasive species, Diffusion effects

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# 1 Introduction

## 1.1 Background information

Dandelion (*Taraxacum mongolicum* Hand.-Mazz.) It is a perennial herb of the family and dandelion. The leaves are obovate-lanceolate, inverted or long circular lanceolate, and the petiole and main vein are often red and purple; the flower is yellow, the base of the flower is not green, and the upper part is purple-red; the inner layer is linear lanceolate; the thin fruit is dark brown obovate lanceolate, and the crown hair is white, about 6 mm long ; The flowering period is 4-9 months, and the fruiting period is 5-10 months. Dandelion can be seen in most parts of world widely found in hillside grasslands, roadsides, fields and river beaches in medium and low altitude areas. Dandelion seeds drift away in the wind and are highly fertile.



Figure 1 dandelion

Dandelion is propagated by seeds. Seeds have no dormancy period, and mature and harvested seeds can be sown at any time from spring to autumn. According to the needs of the field, it can also be sown in a greenhouse in winter.

The open-ground live broadcast uses strip sowing, and a shallow horizontal groove is 25-30 cm on the furrow surface, with a sowing width of about 10 cm. After the seeds are sown, 1 cm is covered, and then slightly suppressed. The sowing amount is 0.5~0.75 kg per mu. Spread the flat, 1.5~2.0 kg of seeds per mu. The sowing capacity of high-quality dandelion seeds is only 25-50 grams per side. After sowing, cover the grass to keep warm. When the seedlings come out, remove the cover grass, and the seedlings can come out in about 6 days. According to the results of the test data, it takes 10-12 days to sow the seeds immediately after the seeds are not harvested in May, and it takes 15 days from sowing to seedlings. The sowing amount is generally about 3 to 4 grams per square meter, which can protect 700 to 1000 seedlings.

The main value of dandelion: raw dandelion is rich in vitamin A, vitamin C and potassium, as well as iron, calcium, vitamin B2, vitamin B1, magnesium, vitamin B6, folic acid and copper. The specific element content is mainly water. Every 60 grams of raw dandelion leaves contain 86% water, 1.6 grams of protein, 5.3 grams of carbohydrates, and about 108.8 kilojoules of calories. In 2002, it was classified by the Ministry of Health as homologous substances that can be eaten raw, fried and made into soup. It is a plant for both medicine and food.

## 1.2 Problem Restatement

Invasive plants can reproduce rapidly and spread quickly, taking space, nutrients, water, and light from other plants. If uncontrolled, they can damage parks, streams, and infrastructure.

It has been estimated that only 5–50% of all non indigenous species become widespread and of those, only (approximately) 0.3% of non-native plants and 25% of non-native animals ultimately cause harm (Keller . [2009](#)). But because the harm an invasive species causes can be extreme, the control and eradication of an invasive species requires the well-funded and coordinated efforts of scientists, policy makers, and the general public.

Because of the inherent danger of the spread of an invasive species as well as the large time and distance scales at which an invasion may occur, traditional means of experimentation in the field may not be practical. In this

regard, mathematical models have proven to be helpful.

In the first part, we create a mathematical model to predict the spread of dandelions over the course of 1, 2, 3, 6, and 12 months in a particular initial condition. In the second part, we formulate a mathematical model capable of determining an 'impact factor' for invasive species. This model would integrate multiple variables, including the plant's characteristics and the nature and extent of the harm it inflicts on its environment. Finally, this model is to be used to compute an impact factor for dandelions and validate the other two plant that are often considered invasive.

In order to solve the problem, we divide the model building into the following steps:

1. Calculating the reproduction of dandelion within 1 year is mainly to see whether the population can be established (it will still enter the trend of extinction), and the initial assumption conditions are used for prediction calculation.
2. For the establishment of a population, calculating the diffusion speed and clarifying its invasion impact factors are generally certain that the faster the spread, the greater the impact. This function value is the intrusion factor.
3. For the observation data of invasive species, such as in two adjacent places, at a distance of X kilometers, the first observation of the species's established time, calculate the diffusion speed, and compare with the speed of the "invasion" definition calculated by your model. If it is faster, it means that the model is effective.

### **1.3 Ecology of Dandelions**

Dandelions can bloom for extended periods of time but are most visible in May and June. The florets develop from the center, blooming into a circular flower head. After flowering for a couple of days, the flower head closes and the seeds develop inside the closed head. As the seeds form, the flower stalk extends higher so that it can reach the breeze. The seeds have bristled that function like a parachute and float on the wind.



Figure 2 The life cycle of the Dandelion

**Temperature:** Dandelions are adaptable to a wide range of temperatures. They can grow in cooler temperatures and continue to thrive in warmer summer conditions. Seeds will germinate at temperatures from 41–95°F, but percentage germination and speed of germination are reduced at temperatures above 68°F.

**Humidity:** Dandelions are not particularly sensitive to humidity. They can grow in both dry and moderately moist conditions.

**Soil pH:** Dandelions exhibit a strong tolerance to soil pH levels, being able to grow in acidic, neutral, and alkaline soils. However, they tend to prefer soils that are neutral to slightly alkaline.

**Lifecycle:** Seeds take 14-21 days to germinate. The flowering process begins 56 to 105 from sowing<sup>1</sup>, and once a dandelion blossom appears, it usually lasts for about 9 to 15 days, eventually transforming into the recognizable, fluffy seed head or “dandelion clock.”

**Bloom cycle:** The peak blooming period may occur during late spring to early summer, but sporadic blossoms are commonly observed throughout the summer months. During their blooming season, a single dandelion plant may produce multiple blossoms, contributing to the perception of a continuous blooming cycle. Dandelions generally stop blooming as late autumn approaches.

**Reproduction:** On average, a single dandelion plant can produce 10 flower heads. Each flower head has 150-200 florets, and each floret produces 1 seed. That means that a single plant can produce up to 2000 seeds<sup>2</sup>.

**Dispersal:** Seeds are wind dispersed by means of umbrella-like fluff. A model of dandelion seed transport determined that 99.5% of seeds would land within 11 yards, 0.05% would travel more than 108 yards, and 0.01% would travel greater than 1,083 yards.

**Seed longevity:** Seeds can last one to five years in the soil, but in normal agricultural conditions only a few survive to the next season. The overwhelming majority of seedlings come from recently dispersed

<sup>1</sup> [Dandelion - SARE](#)

<sup>2</sup> <https://gardens.duke.edu/sites/default/files/Duke%20Gardens%20-%20Meet%20a%20Plant%20Dandelion.pdf>

seeds. ( only 2–13% survived as seedlings)

**Competitive:** Dandelion captures space in forage crops and in no-till systems. It is not competitive for light but captures soil moisture and nutrients.

## 1.4 Variables

Variable	Description
$N$	The population of dandelion
$N_0$	The initial population of dandelion
$N(t)$	The total number of moments $t$
$M(t)$	Density at time $t$
$r$	The growth rate of dandelion population
$k$	Maximum environmental carrying capacity
$t$	Time
$E$	Diffusion quantity
$h(N)$	The function of dandelion growth rate per unit density
$M(p, t)$	Number of dandelions at $p$ position $t$ time
$M(x, y, t)$	$(x, y, t)$ probability density distribution

## 1.5 Other Assumptions

The blooming can continue throughout the summer and into early autumn, depending on various factors like climate, soil quality, and rainfall. In

warmer climates, it is even possible to see dandelions blooming year-round.

1. **Geographical location:** In our research and our paper, we assume that the background environment of our research is a temperate region, such as Tianshui, Gansu, China, with initial temperature and humidity suitable for seed growth and survival.

2. **Growth cycle:** New dandelion seedlings require between 8 and 15 weeks to reach maturity and bloom. The warmer the conditions, the quicker the plant matures and begins flowering. The quick maturation of the dandelion allows several generations of the plant to grow during the same season.

3. **Reproduction:** Under good conditions, a dandelion plant will produce 50-150 seed heads per year with an average of 250 seeds per head (Stewart-Wade et al. 2002).<sup>3</sup> Considering the seed longevity and seedling emergence, it is assumed that 2-5 percent of seeds successfully produce new dandelion plants.

5. **Environment:** According to "...The dandelion is adjacent to one hectare of open land...", assuming that there are no other species (conspecifics or competitors) within the one-hectare area, the dandelion seeds have good spatial conditions for exponential growth and the carrying capacity of the environment is ignored.

6. **Dispersal:** Seeds are wind dispersed by means of umbrella-like fluff. Updrafts are most important for long distance dispersal, and a model of dandelion seed transport determined that 99.5% of seeds would land within 11 yards, 0.05% would travel more than 108 yards, and 0.01% would travel greater than 1,083 yards.

## 2 Analysis of the Problem

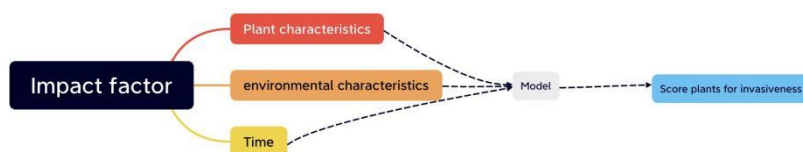


Figure 3

<sup>3</sup> <https://cals.cornell.edu/weed-science/weed-profiles/dandelion>



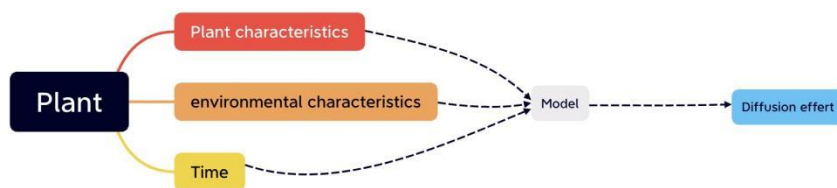


Figure 4

According to the Clarification and Guidance on the Definition of Invasive Species<sup>4</sup>, the "invasiveness" of a species is assessed in the following ways:

- Non-native: Invasive species are usually non-native, i.e. they are not native to the ecosystem under consideration.
- Economic or environmental harm: The introduction of an invasive species may result in economic or environmental harm, including damage to agriculture, forests, wildlife habitat, and threats to human health.
- Survival and dispersal capacity: Invasive species usually have a high capacity for survival and dispersal, and are able to survive in new environments and spread rapidly.

For example, the several barriers must be overcome for a plant to be considered an invasive weed:

- large-scale geographical barriers
- survival barriers
- establishment barriers
- dispersal and spread barriers
- harm and impact

For above, the determination of the "invasiveness" of dandelion is mainly based on 2), 3) and 4). The growth of dandelions in introduced sites is classified as following:

- a) If the dandelion does not survive within a limited period of time, i.e. does not establish a population, it is not considered to be "invasive";
- b) If the dandelion survives and reproduces for a limited period of time and reaches a maximum steady state in a limited area without spreading further, it is not considered "invasive" either;
- c) if the dandelion reproduces and spreads rapidly over a limited period of time before it reaches a peak steady state, and continues to integrate with the surrounding ecosystem, it is considered "invasive";

To understand the 'impact factor' for invasive species, we need to estimate and predict rates of invasive spread from biological parameters such as population growth and dispersal. Generally speaking, the greater the

<sup>4</sup> <https://www.sare.org/publications/manage-weeds-on-your-farm/dandelion/#dandelion-ecology>

spread of dandelion, the greater the 'invasive effect'.

Based on the above analysis of the problems, we will first establish a growth model of dandelion to predict the growth and reproduction of its population; secondly, we will establish a growth-dispersal model over the dandelion population to study the dispersal of dandelion under different initial scales.

## 3 Modelling

### 3.1 Population Growth Models

Dandelions has several features:

- Reproduce early, have many offspring
- Offspring are small, mature rapidly, receive little parental care
- Generations are relatively short, large brood size

#### 3.1.1 Malthus model

We have used both Malthusian and Logistic models in this section, and we will start modeling step by step below

The Malthusian model is a mathematical equation that describes population growth in the short run and assumes that the growth rate is proportional to the current population size, which is functionally equivalent to exponential growth. Here the growth rate of dandelion density is set to be proportional to the amount of dandelions. In the following modeling, we use this model to simulate dandelion population dynamics.

$$\frac{dN}{dt} = \lim_{\Delta t \rightarrow 0} \frac{N(t) - N(t + \Delta t)}{\Delta t} = rN(t) \quad (1)$$

From the calculation of  $r$  in the classical Logistics model, we know that

$$r = \frac{1}{t} \ln \frac{N(t)}{N(0)} \quad (2)$$

A logarithmic transformation on both sides of (2) yields

$$e^r = e^{\frac{1}{t} \ln \left( \frac{N(t)}{N(0)} \right)} \quad (3)$$

We can come out that

$$N(t) = N(0)e^{rt} \quad (4)$$

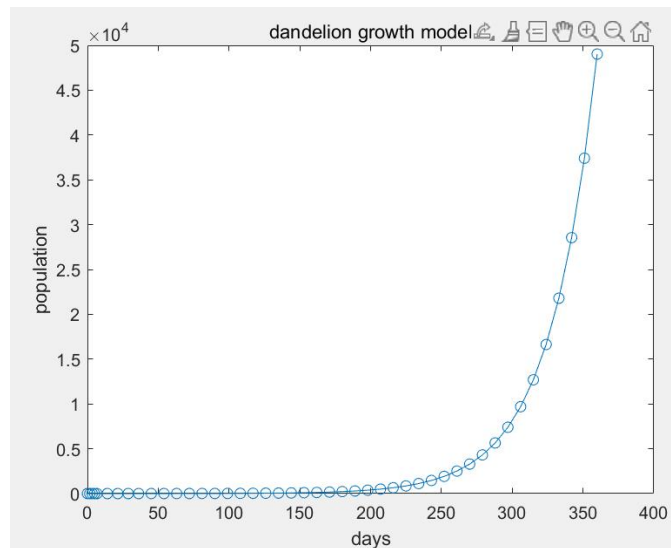


Figure 5 Exponential growth of dandelions over the course of a year with  $M_0 = 1$  and  $r = 0.03$

The per capita growth rate  $r$  is a parameter in the model and is typically be determined experimentally. In general, the per capita growth rate  $r$  may depend on time as, for example, environmental conditions change. In this paper, this parameter is considered to be constants that reflect the type of dandelions being modeled. To estimate the growth rate of dandelions, we can rearrange the formula to solve for the growth rate:

$$r = \ln(N(t)/N_0)/t$$

We know that the warmer the conditions, the faster the dandelions mature and start flowering. the rapid maturation of the dandelion allows several generations of the plant to grow in the same season. We now estimate the growth rate of dandelions under two different life cycles.

#### a) One generation on year

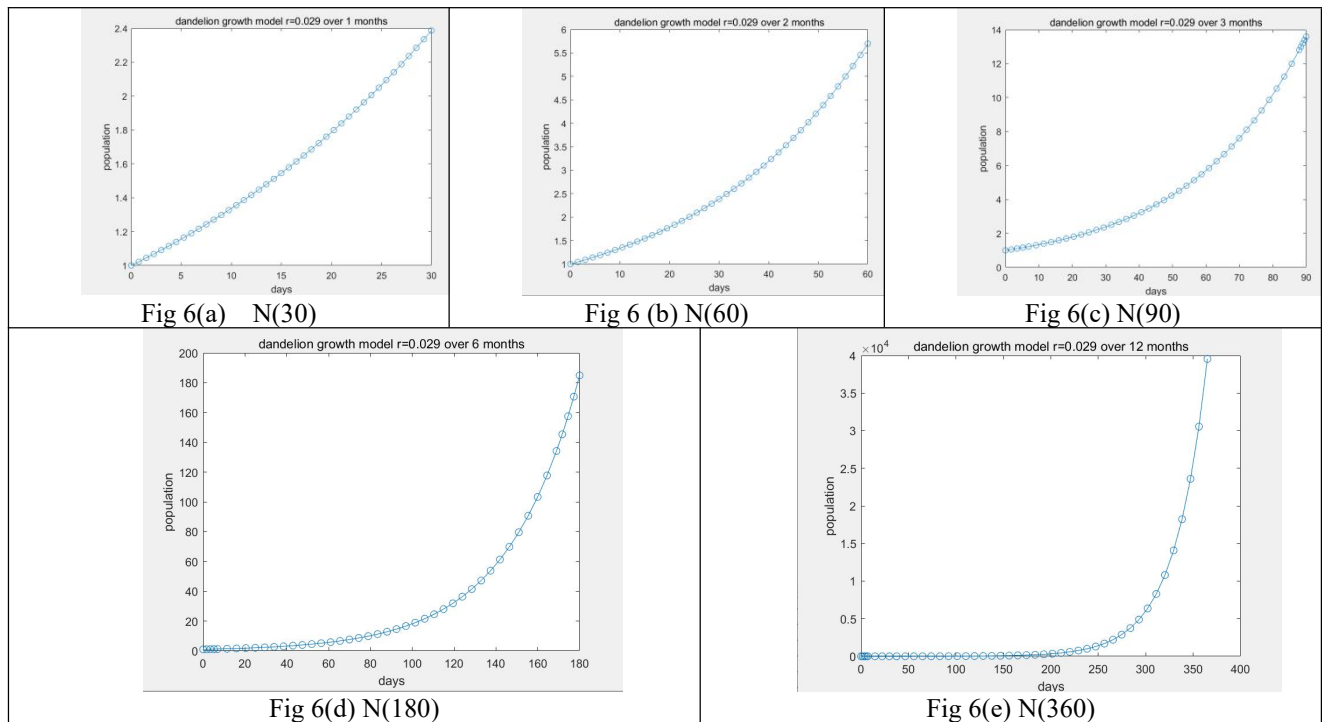
Most dandelion seeds will bloom the first spring and not bloom again until the following spring.

Suppose the dandelion started as 1 plant with 5000 seeds, assuming the survival rate of the seeds is 4%, then 200 seeds can successfully emerge in the first year, these 200 new plants can grow 200\*5000 seeds, still according to the survival rate of 4%, these seeds can grow 40000 new plants in the next spring.

$$N(365 \times 24) = 2002, N(0) = 1, t = 365 \times 24 \text{ (units converted to hours)}$$

$$R = 0.03 / \text{day}$$

Bring  $r$  into the growth model and get the growth data of 1, 2, 3, 6 and 12 months, as shown in the figure below:



## b) Two generation on year

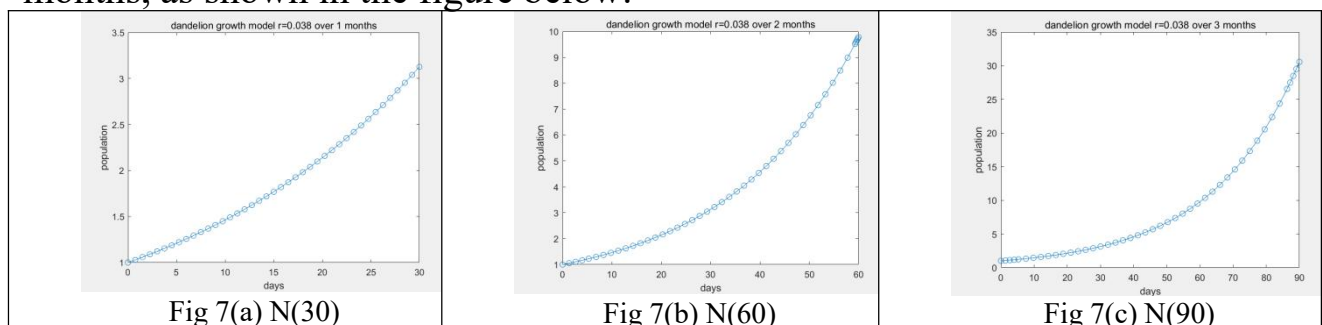
Seedlings that emerge in spring may flower in their first year. Established plants that bloom in spring can flower again in autumn.

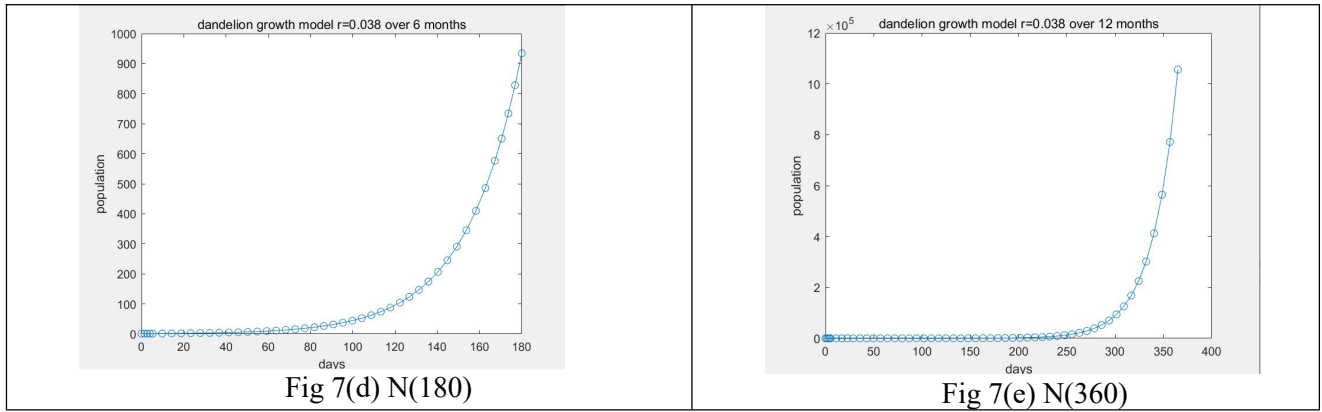
Supposing the survival rate of the seeds is 4%, then 200 seeds can successfully emerge in the summer, and the 200 new seedlings reproduce  $2000 \times 5000$  seeds, still according to the survival rate of 4% seeds, and produce 40000 new seedlings in the autumn, bearing  $40000 \times 5000$  seeds; With a 4% survival rate, these seeds can grow  $40000 \times 200$  new plants the following spring.

$$y(365 \times 24) = 2003, y(0) = 1, t = 365 \times 24 \text{ (units converted to hours)}$$

$$R = 0.04 / \text{day}$$

Bring  $r$  into the growth model and get the growth data of 1, 2, 3, 6 and 12 months, as shown in the figure below:





### 3.1.2 Logistic model

We found that the issue of environmental carrying capacity was not taken into account here and introduced a Logistics model, which includes a new parameter  $K$  - maximum environmental carrying capacity. It is capable of modeling dynamic trends over a long period of time. According to the Logistics model, there are

$$h(N) = r \cdot \left(1 - \frac{N}{k}\right) \quad (5)$$

where  $h(M)$  is the unit density bushel growth rate function thereby obtaining

$$\frac{dN}{dt} = N \cdot h(N) \quad (6)$$

Substituting (5) into (6) yields

$$\frac{dN}{dt} = N \cdot r \left(1 - \frac{N}{k}\right) \quad (7)$$

## 3.2 Diffusion model

### 3.2.1 Differential equation

The diffusion equation is considered, which is a partial differential equation often used to describe the spatial spread of a species. In particular, the rate of spread of an invading species can be calculated by mathematical analysis of different forms of the diffusion equation. Knowing the rate of

spread has important implications for controlling a biological invasion.

As a species spreads to neighboring regions, its population density may vary over space and time. Consider a one-dimensional model with  $p$  as the spatial coordinate, and denote by  $M(p,t)$  the number of dandelion populations at location  $p$ , at time  $t$ . The classical modeling equation for animal dispersal is used here, and the dispersal equation is

$$\frac{\partial M}{\partial t} = E \cdot \frac{\partial^2 M}{\partial p^2} \quad (8)$$

where  $E$  is the diffusion coefficient. Consider further the two-dimensional model,  $x, y$  are spatial coordinates, and denote the number of dandelion populations at position  $(x,y)$ , time  $t$  by  $M(x,y,t)$ , the diffusion equation is

$$\frac{\partial M}{\partial t} = E \cdot \left( \frac{\partial^2 M}{\partial x^2} + \frac{\partial^2 M}{\partial y^2} \right) \quad (9)$$

The number of dandelions grows exponentially, the sample size is large, and the spreading process is random and affected by many factors. Therefore we will use the normal distribution to calculate the probability density distribution of dandelions. Use  $M(x,y,t)$  function to represent the probability density distribution of dandelions at  $(x,y)$  position, time  $t$ . Because the function  $M(x,y,t)$  involves three variables, we are going to use a three-dimensional normal distribution function, and the general form of a three-dimensional normal distribution function is shown here.

$$f(x, y, z) = \frac{1}{2\pi\sigma_x\sigma_y\sigma_z} e^{-\frac{1}{2} \left( \frac{(x-\mu_x)^2}{\sigma_x^2} + \frac{(y-\mu_y)^2}{\sigma_y^2} + \frac{(z-\mu_z)^2}{\sigma_z^2} \right)} \quad (10)$$

Since the above equation in  $\mu, \sigma$  is related to  $M_0, E$ , considering this information, we can get the probability density distribution function of dandelion

$$M(x, y, z) = \frac{M_0}{4\pi Et} e^{-\frac{x^2+y^2}{4Et}} \quad (11)$$

If the growth of dandelions is also taken into account, incorporating the  $e^{rt}$  into the model, one gets

$$M(x, y, z) = \frac{M_0}{4\pi Et} e^{\left(r t - \frac{x^2 + y^2}{4Et}\right)} \quad (12)$$

Fig 8 shows a plot of this function at  $t = 365$ ,  $E = 2\text{m}^2/\text{day}$  and  $r = 0.038/\text{day}$ .

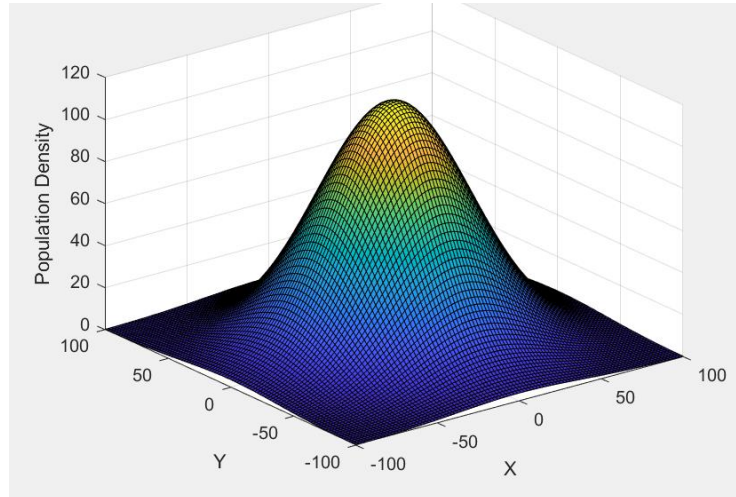


Fig 8 shows the spatial distribution of dandelion growth dispersal over  $t=360$ ,  $E = 2$ ,  $r = 0.038$

$t = 365$ ,  $E = 2$

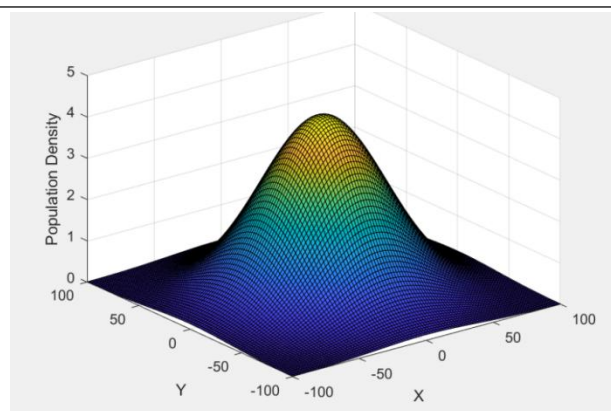


Fig 9(a) shows the spatial distribution of dandelion growth dispersal over  $t=365$ ,  $E = 2$ ,  $r = 0.029$ .

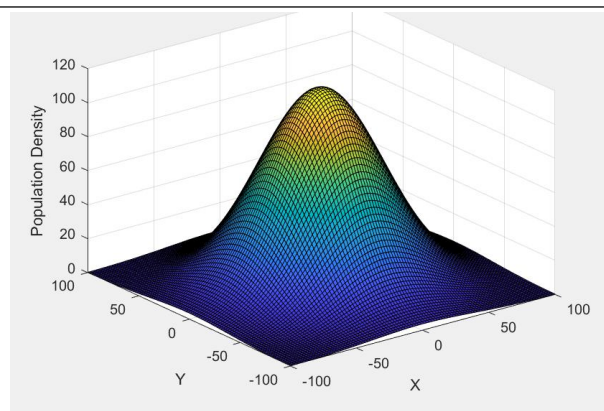


Fig 9(b) shows the spatial distribution of dandelion growth dispersal over  $t=365$ ,  $E = 2$ ,  $r = 0.038$ .

Fig 10 shows that for the same diffusion coefficient, the larger the  $t$  value, the greater the spatial extent covered by growth diffusion. The situation is the same as our intuition.

$$r = 0.038, E = 2$$

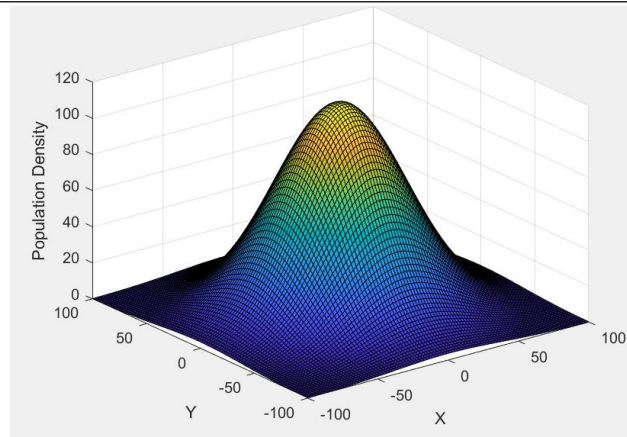


Fig 10(a) shows the spatial distribution of dandelion growth dispersal over  $t=1$  year,  $D = 2$ ,  $r = 0.038$

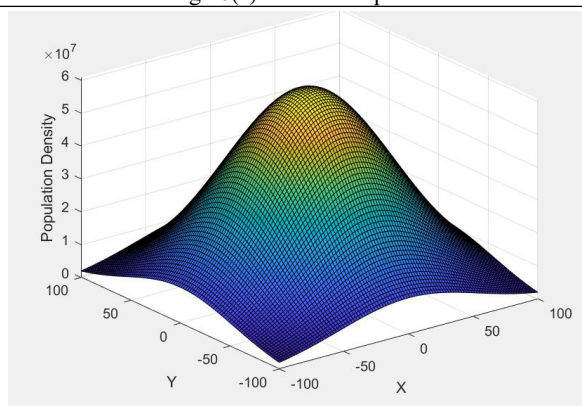


Fig 10(b) shows the spatial distribution of dandelion growth dispersal over  $t=2$  years,  $E = 2$ ,  $r = 0.038$ .

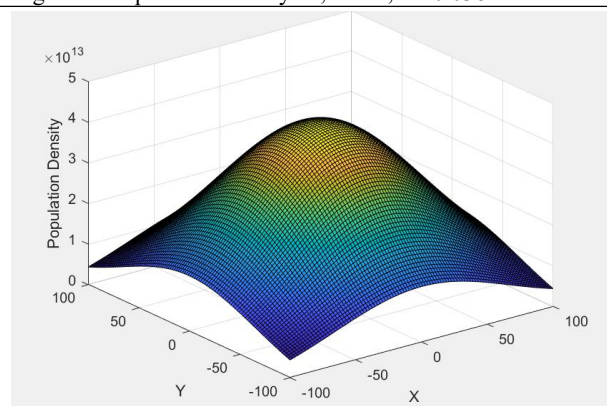


Fig 10(b) shows the spatial distribution of dandelion growth dispersal over  $t=3$  years,  $E = 2$ ,  $r = 0.038$ .



### 3.2.2 Calculation and Results

As shown above, reaction-diffusion equations are able to describe the densities of dandelion populations and the extent of dispersal at different times. We then use Matlab modelling and adjust the time  $t$ , diffusion coefficient  $D$  and growth rate  $r$  to intuitively understand the effect of different parameter variations on the 'invasiveness' of dandelion from the graphical changes.

According to “Seeds are wind dispersed by means of umbrella-like fluff. A model of dandelion seed transport determined that 99.5% of seeds would land within 11 yards, 0.05% would travel more than 108 yards, and 0.01% would travel greater than 1,083 yards.”

Figure 10a shows, the first time a new dandelion plant is seen is about a week, We calculate  $E$  under the following assumptions:

$$E = 3.14 \times 100 \text{m}^2 / 3.14 \times 7 = 14.28 \text{m}^2/\text{day}$$

Fig8 shows the spatial distribution of the growth spread of dandelion over  $t=50$  on a hectare of open land for a dispersal coefficient of  $E = 14.28 \text{m}^2/\text{day}$ .

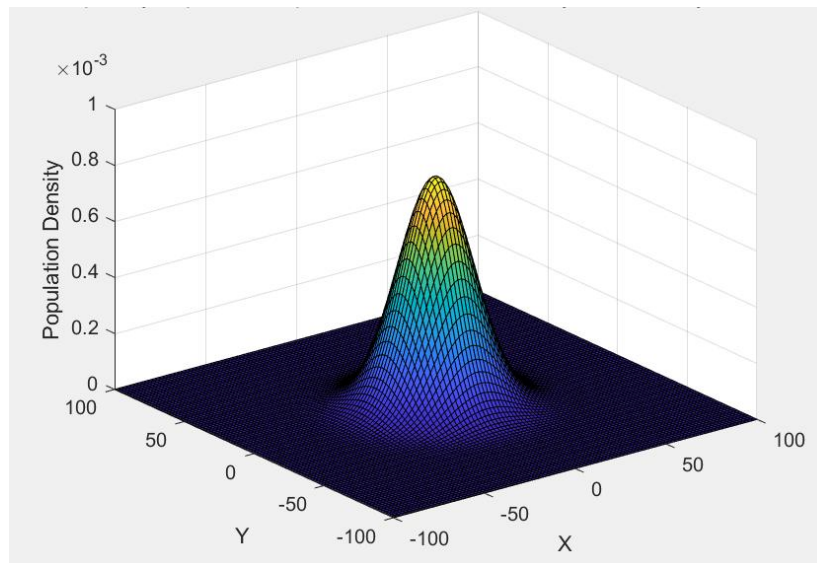


Fig 11 shows the spatial distribution of dandelion growth dispersal over  $t=50$

## 4 Validating the Model(Evaluation of invasiveness)

In order to verify that the response diffusion model is indeed helpful to judge the diffusion of species, and that  $E$  is the main factor to determine the impact of species "invasiveness", we will infer its diffusion coefficient  $E$  according to the observation data of specific invasive species provided by the government, and plug it into the model for verification.

### 4.1.1 Experiment data 1

Apalachicola, *Eichhornia crassipes*<sup>5</sup> (RGR 0.07/day)<sup>6</sup>

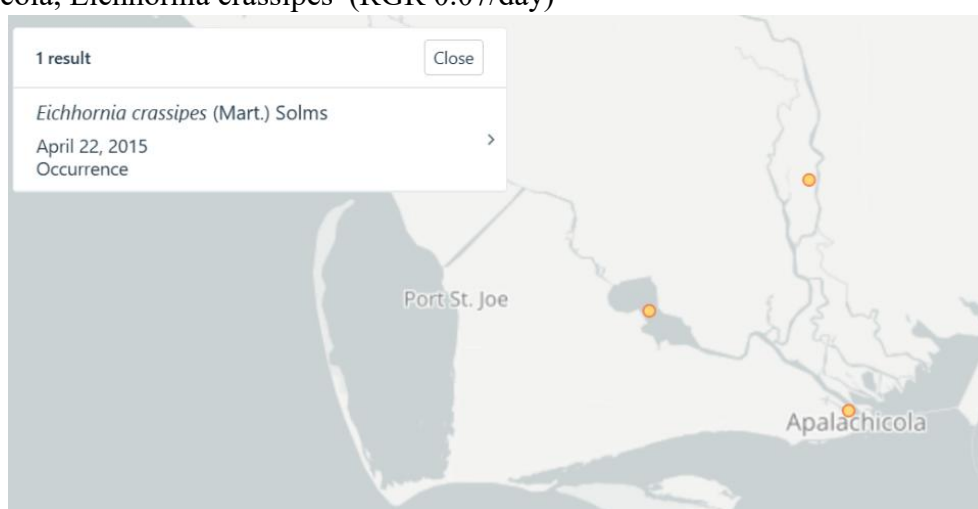


Figure 12

Observation point A April 22, 2015 Occurrence	Observation point B September 02, 2015 Occurrence	Observation point C August 27, 2015 Occurrence
Latitude 29.7323 Longitude -84.9887	Latitude 29.80134 Longitude -85.14753	Latitude 29.89237 Longitude -85.01978

diffusion from A to B: 14 km for 150 days

diffusion from B to C: 18 km for 125 days

According to the formula for solving  $E = \text{MSD}/(\pi t)$ , We can roughly get:

$E_1 = 0.33 \text{ km}^2/\text{day}$

$E_2 = 0.65 \text{ km}^2/\text{day}$

$E = (E_1 + E_2)/2 = 0.49 \text{ km}^2/\text{day} = 490000 \text{ m}^2/\text{day}$

Substituting  $E$  into the reaction-diffusion model gives the following Fig13

<sup>5</sup> [Data - GBIF.us](http://data.gbif.us)

<sup>6</sup>

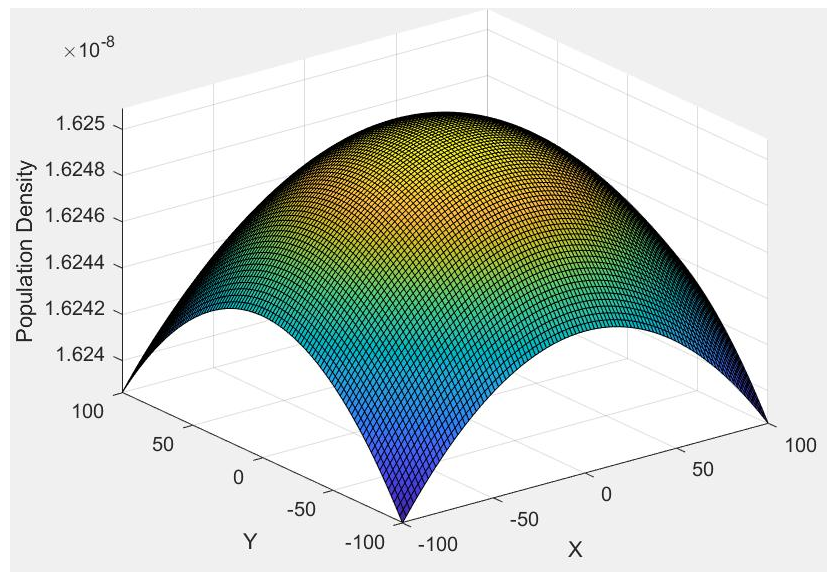


Fig 13 Eichhornia crassipes diffusion over t=50

#### 4.1.2 Experiment data 2:

Florida, Washington, Pueraria montana var. lobata<sup>7</sup> (RGR = 3%/day)

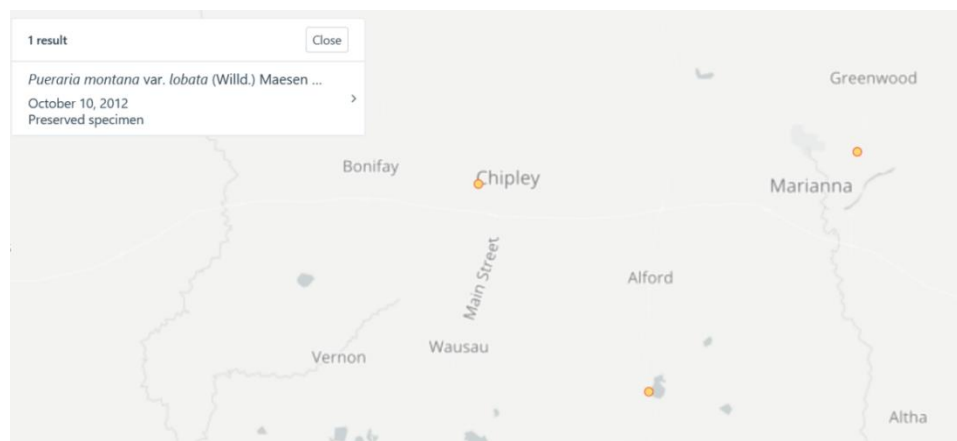


Figure 14

Observation point A October 10, 2012	Observation point B August 01, 2013	Observation point C June 13, 2014
Latitude 30.80388, Longitude -85.17847	Latitude 30.59256 Longitude -85.39321	Latitude 30.77574 Longitude -85.56786

diffusion A to B: 27.22 km for 290 days

diffusion A to B: 29.15 km for 310 days

According to the formula for solving  $E = \text{MSD}/(\pi t)$ , We can roughly get:

$E_1 = 0.638 \text{ km}^2/\text{day}$

$E_2 = 0.685 \text{ km}^2/\text{day}$

<sup>7</sup> [Data - GBIF.us](http://Data-GBIF.us)

$$E=(E_1+E_2)/2=0.661 \text{ km}^2/\text{day} = 661000\text{m}^2/\text{day}$$

Substituting D into the reaction-diffusion model gives the following Fig10

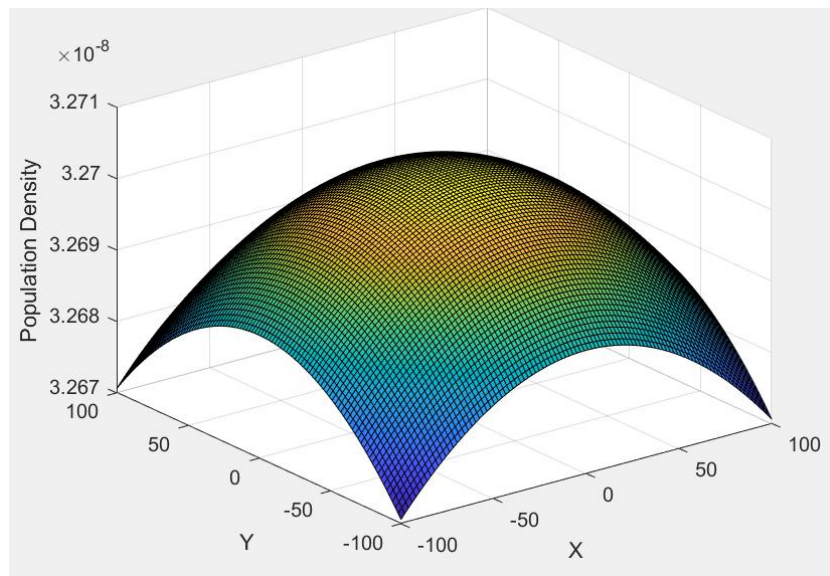


Fig 15 *Pueraria montana* diffusion over  $t=50$

## 5 Summarize

### 5.1 Conclusion

Apalachicola, Eichhornia crassipes and Florida, Washington, Pueraria montana var. lobata are clearly invasive plants, From the Fig16, we can see that their growth spreads to a much greater extent than that of the dandelions, and that at the same  $t=50$  the former spread over a much larger area than the dandelions.

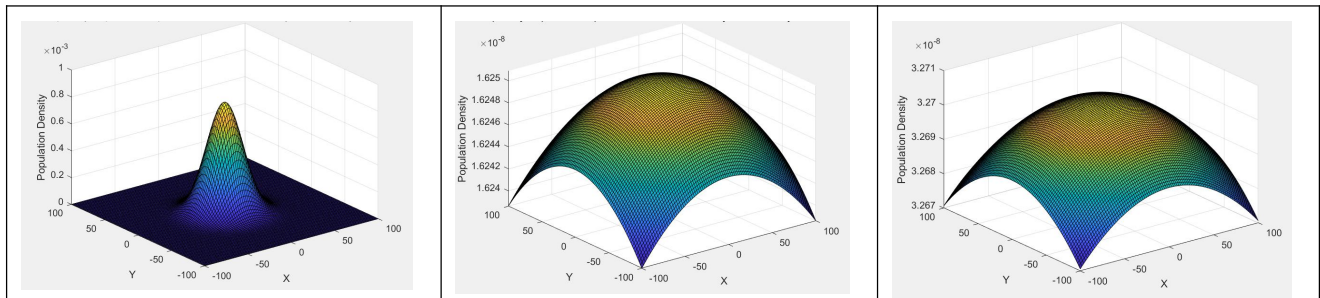


Figure 16

If the extent of spread of plant growth in the same amount of time is the main factor to look at in terms of "invasive" impacts, we believe that dandelions are benign compared to most truly invasive species. Considering that dandelions also have many benefits, such as providing food for bees with their flowers and improving soil quality with their roots, we believe that humanity should coexist peacefully with the dandelion and celebrate its tenacious and resilient life force.


The growth and spread of dandelions is a very complex affair involving many variables. In this paper, we use Logistic model and Malthus model for growth prediction, and combine differential equation for diffusion prediction. Then, each plant was scored for invasiveness based on the characteristics of diffusion and plant characteristics, and this system was applied to evaluate plants other than dandelion. In the whole paper, we construct a prediction class model and an evaluation class model.

We are aware that there is still room for improvement and complexity in our model, and we will continue to improve it in future research and exploration.

We are willing to make "useful" models. As George E.P. Bull said, "All models are wrong, but some are useful."



## 5.2 Poster

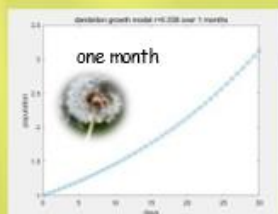


### Dandelions & Humans: Living Together.

If the extent of spread of plant growth in the same amount of time is the main factor to look at in terms of "invasive" impacts, we believe that dandelions are benign compared to most truly invasive species. Considering that dandelions also have many benefits, such as providing food for bees with their flowers and improving soil quality with their roots, We believe that humanity should coexist peacefully with the dandelion.

### Exponential Growth

- A species of flowering plant in the family Asteraceae.
- Have compound, bright yellow flowers
- 150-200 seeds per flower and up to 10 flowers per plant
- Blooms repeatedly through summer
- Up to three generations in a 30 -week growing season
- An average of 15,000 seeds is produced per dandelion plant

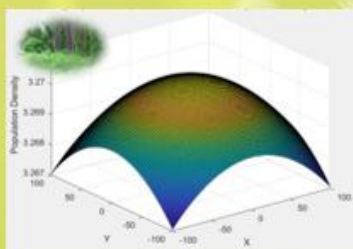


#### exponential growth

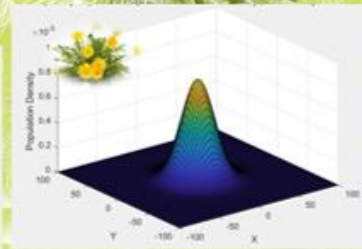
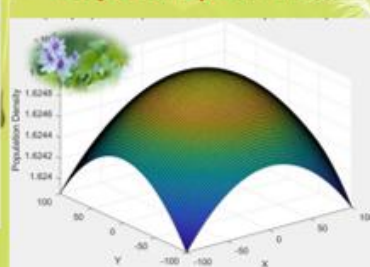


### Dispersal by Reaction-Diffusion

- 99.5 per cent of dandelion seeds land within 10 meters of their parent plant.
- Dispersed dandelion seeds were among the most likely to be intercepted by standing vegetation
- the spread of dandelion is related to the density of plants in the area



#### Dispersal by Diffusion



## 5.3 References

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## 5.4 Appendices

### Appendix A MATLAB code to Growth Model

Here are simulation programmes we used in exponential growth model as follow.

Input matlab source:

---

**% Spatially dependent exponential growth modeling in MATLAB**

**% r-selected spices Exponential growth modeling**

**% parms**

**r = 0.03; % Growth rate**

**% initial value**

**N0 = 1; % Initial population**

**% time span**

**tspan = [0 120];**

**% calculating**

**[t, N] = ode45(@(t, N) r \* N, tspan, N0);**

**% display**

**figure;**

**plot(t, N, '-o');**

**xlabel('days');**

**ylabel('population');**

**title('dandelion growth model');**

**zlabel('Population Density');**

---

### Appendix B MATLAB code to Diffusion

Here are simulation programmes we used in Reaction-Diffusion model as follow.

Input matlab source:

---

**% Spatially dependent exponential growth modeling in MATLAB**

**% Define parameters**

**N0 = 200; % Initial population**

**D = 0.1; % Diffusion coefficient**

**r = 0.03; % Growth rate**

**% Define spatial coordinates (assuming a 2D space)**

**x = linspace(-10, 10, 100);**



```
y = linspace(-10, 10, 100);

% Create a grid of spatial coordinates
[X, Y] = meshgrid(x, y);

% Define time values
t = 1;

% Calculate population density using the given formula
N = N0 / (4 * pi * D * t) * exp(r * t - (X.^2 + Y.^2) / (4 * D * t));

% Plot the results
figure;
surf(X, Y, N);
title('Spatially Dependent Exponential Growth Model');
xlabel('X');
ylabel('Y');
zlabel('Population Density');
```

---