# Data Analysis PI
# Theoretical assignment #6

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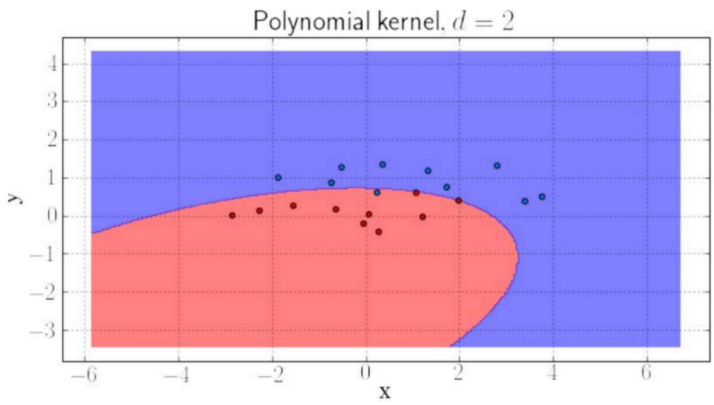
# Task 1.

_Problem:_ Consider a linearly separable dataset and SVM with polynomial kernel $K(x, y) = (x^\top y + 1)^d$. Is that right, that for any $d > 1$ the decision boundary representation in initial feature space will be the **error - free hyperplane** (linear decision boundary).

_Solution:_ The decision boundary just cannot be linear if the kernel is RBF and $d > 1$.

For example,



# Task 2.

<u>*Problem:*</u> Find the computational complexity of the linear and kernel SVM **classification** procedure of a single object (SVM is already trained).

<u>*Solution:*</u> If an input feature vector is a vector of float values, then the result is

$$Y = f(wx) = f\left(\sum_{i=1}^{n} w_i x_i\right)$$

Where $w$ is a weight matrix, $f$ - function to compute the result from dot product.

So, the space complexity of classifying 1 object using a linear classifier is equal to complexity of computing dot product of 2 vectors.

$$O(f) = O(d)$$

,d - number of features.

SVC:

$$a(x) = sign(\sum_{i=1}^{n} \lambda i \cdot ci \cdot xi \cdot x - b)$$

Where:

$x_i$ - a sample object.

$x$ - a classidied object.

Using only those objects, for which $\lambda \neq 0$, let's assume that $m$ is number of objects for which $\lambda \neq 0$. Then, the complexity will be $O(md)$. d - number of features.

So, the linear classifier is $O(d)$, and SVM is $O(md)$

## Task 3.

<u>*Problem:*</u> Consider a linearly separable dataset. Write down ML loss function for logistic regression. Show that the maximum likelihood solution for the logistic regression model is obtained by finding a vector $w$ and $w_0$ with all coefficients tend to infinity. Name a technique that can help to overcome this issue.

<u>*Solution:*</u> The binomial likelihood function looks like follows:

$$L = \sum_i t_i \log(p_i) + (1 - t_i) \log(1 - p_i)$$

In each term in the summation only one of $t_i \log(p_i)$ or $(1 - t_i) \log(1 - p_i)$ is non-zero, with a contribution of $p_i$ for $t_i = 1$ and $1 - p_i$ for $t_i = 0$.

To be more accurate, the following could be written:

$$S(\beta,x) = \frac{1}{1 + \exp(-\beta x)}$$

for the sigmoid function. There are also two limits, each approaching limit monotonically:

$$\lim_{\beta \to \infty} S(\beta,x) = 0 \text{ for } x < 0$$
$$\lim_{\beta \to \infty} S(\beta,x) = 1 \text{ for } x > 0$$

The first limit is decreasing, the second is increasing. Each of these follows easily from the formula for $S$.