# Data Analysis PI
# Theoretical assignment #3

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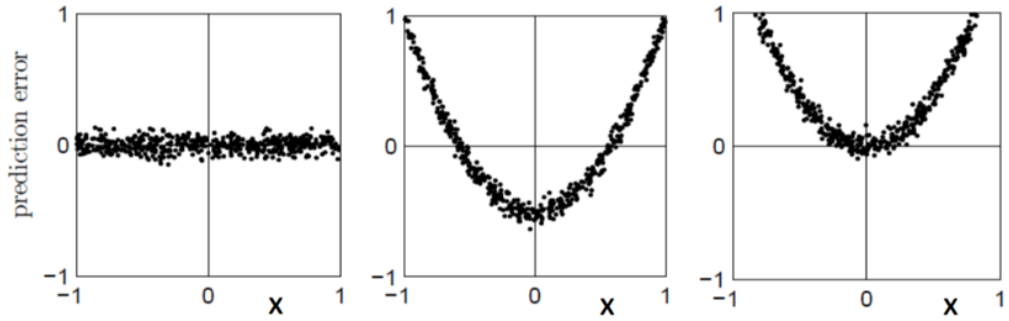
# Task 1.

*Problem:* Consider linear regression task in one-dimensional space. $(x_i, y_i)_{i=1}^N$ is a training dataset and for object $i : x_i \in [-1, 1]$ is a feature, $y_i$ is an answer we want to predict, $y_i^* = kx_i + b$ is our prediction. At the picture below you can see three different plots of the prediction error $(y - y^*)$ against $x$. Which of these plots cannot be obtained if least squares method is used to train a regression model?




*Solution:* The first picture represents how data is spread pretty linearly. Obviously, this situation is valid.
Pictures 2 and 3 are representing non-linear, but but kind of uniformly distributed dependencies of features. Also these distributions could roughly be described with the following equation: $y - \hat{y} = x^2 + b$

$$y - \hat{y} = x^2 + b$$
$$(y - \hat{y})^2 = (x^2 + b^2)^2$$
$$\sum_{i=1}^{N} (y_i - \hat{y}_i)^2 \overset{\text{OLS}}{\to} \min$$
$$\int_{-1}^{1} (y - \hat{y})^2 \to \min$$

Now we have $\int_{-1}^{1} (x^2 + b)^2$

$$F((x^2 + b)^2) = \frac{x^5}{5} + 2b\frac{x^3}{3} + xb^2$$
$$\int_{-1}^{1} (x^2 + b)^2 = \frac{1}{5} + \frac{2b}{3} + b^2 + \frac{1}{5} + \frac{2}{3}b + b^2 = \frac{2}{5} + \frac{4}{3}b + 2b^2$$

Taking derivative, a value of $b$ could be evaluated. With this $b$, integral reaches its minimum ponts.

$$4b + \frac{4}{3} = 0$$
$$b = -\frac{1}{3}$$

So we get $(x^2 - \frac{1}{3})$. Second picture is more likely to be chosen than the trird one.

So the answer is that *third* picture could not be obtained.

# Task 2.

*Problem:* Consider linear regression task in one-dimensional space $y = kx + b$ and two datasets: $(x_1^1, y_1^1), \ldots, (x_n^1, y_n^1)$ and $(x_1^2, y_1^2), \ldots, (x_m^2, y_m^2)$. Assume that the least squares method is used to train a regression models in this task. It turns out, that if we train a regression model on the first dataset we obtain a coefficient $k_1 > 0$. Similarly, if we train a regression model on the second dataset we obtain a coefficient $k_2 > 0$. Is it true that if we train the regression model on both datasets together then the obtained coefficient $k$ will also be positive? Additionally answer that question if we know that $\sum_{i=1}^{n} x_i^1 = 0$ and $\sum_{i=1}^{m} x_i^2 = 0$.

*Solution, first question:* Suppose, the green dots are representing the first dataset. Then, the green line is the solution, which will be given by a regression. For the second dataset there are red dots and red line respectively.

Obviously, $k_1 > 0$ and $k_2 > 0$. The purpule line is the result of fitting using both datasets. As it could be seen, $k_{purple} < 0$.

So, the answer to the first question is *no*.

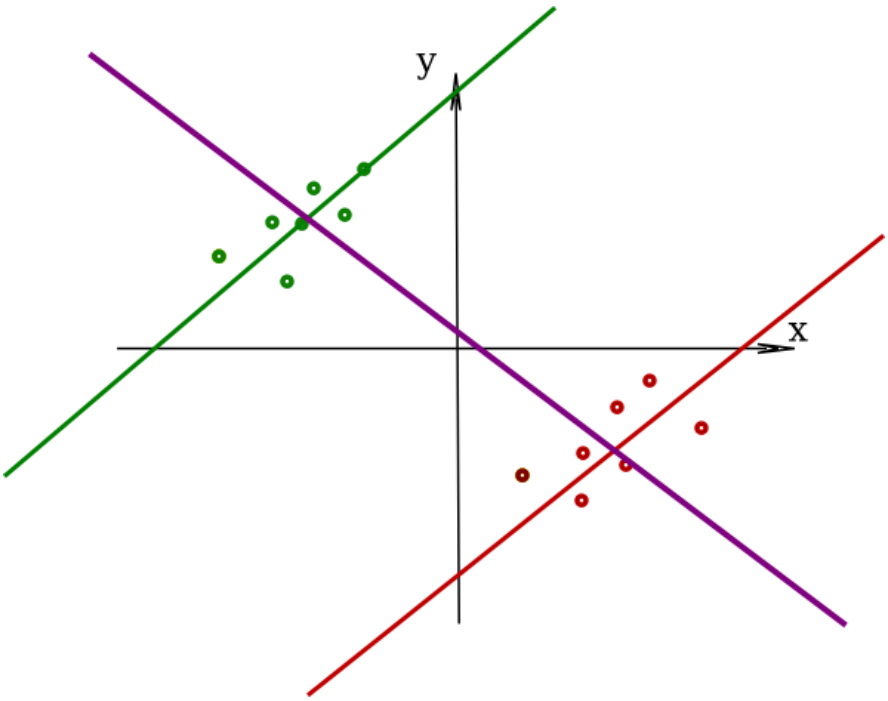



Рис. 1: 2 fitted datasets

*Solution, second question:* There are several possible outcomes.

1. Suppose that for every dataset, $\forall x_i^c \ \exists \ -x_i^c$. Like, for each 5 there is only one $-5$, and for each $-1$ there is only one 1. So this satisfies the condition in the problem statement. Note, that this satisfies condition only for $x^c$ coordinate, not for $y^c$. That's why $k$ in every dataset can be both 1 and $-1$. When fitted of both datasets, the *purple* line would be just a straight vertical line.

2. $\forall a x_i^c \overset{t}{\exists} -\frac{ma}{n} x_i^c$. For example: for every point 2 there is 2 poins of $-1$. Or for every point $-10$, there is 2 points of 5 or 10 points of 1. In this case $k$ also could be anything. So does $k_{purple}$.

So, the answer to the second question, *not always*.