Noida Institute of Engineering and Technology, Greater Noida

Expectations and Probability Distribution

Unit: IV

Subject Name: Eng. Mathematics IV

Subject code: AAS0402

B Tech 4^{th} Sem

Dr. Kunti Mishra

Department of Mathematics

Brief Introduction of Faculty

Dr. Kunti Mishra
Assistant Professor
Department of Mathematics

Qualifications :
M.Sc.(Maths), M. Tech.(Gold Medalist) in Applied and Computational Mathematics, Ph.D

Ph.D. Thesis : Some Investigations in Fractal Theory
Total Number of Research Papers:15
Area of Interests: Fixed Point Theory, Fractals
Teaching Experience: 9 years

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AN AUTONOMOUS INSTITUTE

Evaluation Scheme

NOIDA INSTITUTE OF ENGINEERING & TECHNOLOGY, GREATER NOIDA
(An Autonomous Institute)

B. TECH (CSE)
EVALUATION SCHEME
SEMESTER-IV

Sl. No.	Subject Codes	Subject Name	Periods			Evaluation Scheme				End Semester		Total	Credit
			L	T	P	CT	TA	TOTAL	PS	TE	PE		
1	AAS0402	Engineering Mathematics-IV	3	1	0	30	20	50		100		150	4
2	AASL0401	Technical Communication	2	1	0	30	20	50		100		150	3
3	ACSE0405	Microprocessor	3	0	0	30	20	50		100		150	3
4	ACSE0403A	Operating Systems	3	0	0	30	20	50		100		150	3
5	ACSE0404	Theory of Automata and Formal Languages	3	0	0	30	20	50		100		150	3
6	ACSE0401	Design and Analysis of Algorithm	3	1	0	30	20	50		100		150	4
7	ACSE0455	Microprocessor Lab	0	0	2				25		25	50	1
8	ACSE0453A	Operating Systems Lab	0	0	2				25		25	50	1
9	ACSE0451	Design and Analysis of Algorithm Lab	0	0	2				25		25	50	1
10	ACSE0459	Mini Project using Open Technology	0	0	2				50			50	1
11	ANC0402 / ANC0401	Environmental Science*/ Cyber Security*(Non Credit)	2	0	0	30	20	50		50		100	0
12		MOOCs** (For B.Tech. Hons. Degree)											
		GRAND TOTAL										**1100**	**24**

****List of MOOCs (Coursera) Based Recommended Courses for Second Year (Semester-IV) B. Tech Students**

S. No.	Subject Code	Course Name	University / Industry Partner Name	No of Hours	Credits
1	AMC0046	Algorithmic Toolbox	University of California San Diego	24	1.5
2	AMC0031	Data Structures	University of California San Diego	25	2

Syllabus

Unit-I (Statistical Techniques-I)

Introduction: Measures of central tendency: Mean, Median, Mode, Moment, Skewness, Kurtosis, Curve Fitting ,Method of least squares, Fitting of straight lines, Fitting of second degree parabola, Exponential curves ,Correlation and Rank correlation, Linear regression, nonlinear regression and multiple linear regression

Unit-II (Statistical Techniques-II)

Testing a Hypothesis, Null hypothesis, Alternative hypothesis, Level of significance, Confidence limits, p-value, Test of significance of difference of means, Z-test, t-test and Chi-square test, F-test, ANOVA: One way and Two way. Statistical Quality Control (SQC), Control Charts, Control Charts for variables (Mean and Range Charts), Control Charts for Variables (p, np and C charts).

Syllabus

Unit III (Probability and Random Variable)

Random Variable: Definition of a Random Variable, Discrete Random Variable, Continuous Random Variable, Probability mass function, Probability Density Function, Distribution functions.

Multiple Random Variables: Joint density and distribution Function, Properties of Joint Distribution function, Marginal density Functions, Conditional Distribution and Density, Statistical Independence, Central Limit Theorem (Proof not expected).

Unit IV (Expectations and Probability Distribution)

Operation on One Random Variable – Expectations: Introduction, Expected Value of a Random Variable, Mean, Variance, Moment Generating Function, Binomial, Poisson, Normal, Exponential distribution.

Syllabus

Unit V (Wavelets and applications and Aptitude-IV)

Wavelet Transform, wavelet series. Basic wavelets (Haar/Shannon/Daubechies), orthogonal wavelets, multi-resolution analysis, reconstruction of wavelets and applications.

Number System, Permutation & Combination, Probability, Function, Data Interpretation, Syllogism.

Branch Wise Application

- Data Analysis
- Artificial intelligence
- Network and Traffic modeling

Course Objectives

- The objective of this course is to familiarize the students with statistical techniques. It aims to present the students with standard concepts and tools at an intermediate to superior level that will provide them well towards undertaking a variety of problems in the discipline.

  The students will learn:

- Understand the concept of correlation, moments, skewness and kurtosis and curve fitting.
- Apply the concept of hypothesis testing and statistical quality control to create control charts.
- Remember the concept of probability to evaluate probability distributions.
- Understand the concept of Mathematical Expectations and Probability Distribution.
- Remember the concept of Wavelet Transform and Solve the problems of Number System, Permutation & Combination, Probability, Function, Data Interpretation, Syllogism.

Course Outcomes

CO1: Understand the concept of correlation, moments, skewness and kurtosis and curve fitting.

CO2: Apply the concept of hypothesis testing and statistical quality control to create control charts.

CO3: Remember the concept of probability to evaluate probability distributions

CO4: Understand the concept of Mathematical Expectations and Probability Distribution

CO5: Remember the concept of Wavelet Transform and Solve the problems of Number System, Permutation & Combination, Probability, Function, Data Interpretation, Syllogism.

Program Outcomes

S.No	Program Outcomes (POs)
PO 1	Engineering Knowledge
PO 2	Problem Analysis
PO 3	Design/Development of Solutions
PO 4	Conduct Investigations of Complex Problems
PO 5	Modern Tool Usage
PO 6	The Engineer & Society
PO 7	Environment and Sustainability
PO 8	Ethics
PO 9	Individual & Team Work
PO 10	Communication
PO 11	Project Management & Finance
PO 12	Lifelong Learning

PSOs

PSO	Program Specific Outcomes(PSOs)
PSO1	The ability to identify, analyze real world problems and design their ethical solutions using artificial intelligence, robotics, virtual/augmented reality, data analytics, block chain technology, and cloud computing
PSO2	The ability to design and develop the hardware sensor devices and related interfacing software systems for solving complex engineering problems.
PSO3	The ability to understand inter disciplinary computing techniques and to apply them in the design of advanced computing.
PSO4	The ability to conduct investigation of complex problem with the help of technical, managerial, leadership qualities, and modern engineering tools provided by industry sponsored laboratories.

CO-PO Mapping

Sr. No	Course Outcome	PO1	PO 2	PO 3	PO4	PO 5	PO 6	PO 7	PO 8	PO 9	PO10	PO11	PO12
1	CO1	H	H	H	H	L	L	L	L	L	L	L	M
2	CO2	H	H	H	H	L	L	L	L	L	L	M	M
3	CO3	H	H	H	H	L	L	L	L	L	L	M	M
4	CO4	H	H	H	H	L	L	L	L	L	L	L	M
5	CO5	H	H	H	H	L	L	L	L	L	L	M	M

*L= Low *M= Medium *H= High

CO-PSO Mapping

CO	PSO1	PSO2	PSO3	PSO4
CO.1	H	L	M	L
CO.2	L	M	L	M
CO.3	M	M	M	M
CO.4	H	M	M	M
CO.5	H	M	M	M

*L= Low *M= Medium *H= High

Program Educational Objectives(PEOs)

PEO-1: To have an excellent scientific and engineering breadth so as to comprehend, analyze, design and provide sustainable solutions for real-life problems using state-of-the-art technologies.

PEO-2: To have a successful career in industries, to pursue higher studies or to support entrepreneurial endeavors and to face the global challenges.

PEO-3: To have an effective communication skills, professional attitude, ethical values and a desire to learn specific knowledge in emerging trends, technologies for research, innovation and product development and contribution to society.

PEO-4: To have life-long learning for up-skilling and re-skilling for successful professional career as engineer, scientist, entrepreneur and bureaucrat for betterment of society.

Result Analysis

Branch	Semester	Sections	No. of enrolled Students	No. Passed Students	% Passed
CS	IV	A	67	67	100
IOT	IV	A	49	45	91.83%

End Semester Question Paper Template

Link: [100 Marks Question Paper Template.docx]

Prerequisite and Recap (CO3)

- Knowledge of Maths 1 B.Tech.
- Knowledge of Maths 2 B.Tech.
- Knowledge of Permutation and Combination.

Brief Introduction about the Subject with Videos (CO3)

- We will discuss properties of complex function (limits, continuity, differentiability, Analyticity and integration)
- In 3rd module we will discuss application of partial differential equations
- In 4th module we will discuss numerical methods for solving algebraic equations, system of linear equations, definite integral and 1st order ordinary differential equation.
- In 5th module we will discuss aptitude part.
- https://youtu.be/iUhwCfz18os
- https://youtu.be/ly4S0oi3Yz8
- https://youtu.be/f8XzF9_2ijs

Unit Objectives (CO3)

1. A basic knowledge of random variables.
2. The student is able to reflect developed mathematical methods in probability and random variable.
3. Understand the concept of random variable.
4. To explore the key properties: such as PMF, PDF etc.

End Semester Question Paper Template

Link:

100 Marks Question Paper Template.docx

Result Analysis

Branch	Semester	Sections	No. of enrolled Students	No. Passed Students	% Passed
CS	IV	A	67	67	100
IOT	IV	A	49	45	91.83%

Unit Content

- Expectations: Introduction
- Expected Value of a Random Variable
- Mean,
- Variance,
- Moment Generating Function,
- Binomial,
- Poisson,
- Normal,
- Exponential distribution.

Topic objective (CO4)

Mathematical expectation

- To get a general understanding of the mathematical expectation of a discrete random variable.
- To learn and be able to apply a shortcut formula for the variance of a discrete random variable.
- To be able to calculate the mean and variance of a linear function of a discrete random variable.

Mathematical expectation or expected value of a random variable:
• When variable is discrete random variable: The expected value of a discrete random variable is a weighted average of all possible values of the random variable, where the weights are the probabilities associated with the corresponding values. It is denoted by E(x)
If x denotes a discrete random variable which assumes values $x_1, x_2 .. x_n$ with corresponding probabilities $p_1, p_2 ..... p_n$respectively, then
E(x)=$x_1p_1 + x_2p_2 + \cdots + x_np_n = \sum_{i=1}^{n} p_ix_i = \sum px$
Where $\sum p = 1$

Mathematical expectation(CO4)

Mathematical expectation or expected value of a random variable:

- **When variable is discrete random variable:** The expected value of a discrete random variable is a weighted average of all possible values of the random variable, where the weights are the probabilities associated with the corresponding values. It is denoted by E(x)

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$$E(x)=x_1p_1 + x_2p_2 + \cdots + x_np_n = \sum_{i=1}^{n} p_ix_i = \sum px$$

Where $\sum p = 1$

Mathematical expectation(CO4)

rth Moment about origin in terms of expectation is written as $\mu_r = E(x-\bar{x})^r$

$\boldsymbol{Mean} = E(x) = \frac{\sum px}{\sum p} = \sum px$ because $\sum p = 1$

So E(x) represents the mean

Variance $\boldsymbol{\mu_2} = E[\{x - E(x)\}^2] = E(x^2) - [E(x)]^2$

- **When variable is continuous:** If x is a continuous random variable then expectation E(x) is written as

$$E(x) = \int_{-\infty}^{\infty} xf(x)dx = \int_{-\infty}^{\infty} xdF(x)$$

Where f(x) is probability density function.

Mathematical expectation(CO4)

Laws of Expectations :

- **Theorem 1. If C is a constant then E(C)=C.**

Proof: Function is constant so assigns value is C to each value within its domain , we have

$P(C = C) = 1$

$P(C = D) = 0, D \neq C$

Therefore $E(C) = C.1 + D.0 = C.$

Hence proved

- **Theorem 2. If a is a constant, then $\boldsymbol{E(aX) = aE(X)}$.**

Proof: Let X takes values x_1, x_2.. with corresponding probabilities p_1, p_2….. respectively, then

$E(aX) = a\, x_1p_1 + a\, x_2p_2 + \cdots = a(x_1p_1 + x_2p_2 + \cdots) = aE(X)$

Mathematical expectation(CO4)

- **Theorem 3. Expectation of the sum of two random variables**

If X and Y are two discrete random variables with finite expectations E(X) and E(Y) respectively , then the expectation of their sum exists and is the sum of their expectations i.e.

$$E(X+Y) = E(X) + E(Y)$$

Generalization: $E(X_1 + X_2 + \cdots + X_n) = E(X_1) + E(X_2) + \cdots + E(X_n)$

Or $E(\sum_{i=1}^{n} X_i) = \sum_{i=1}^{n} E(X_i)$

Note : if a and b are constant then $E(aX+b) = aE(X) + b$

- **Theorem 4.**Expectation of the product of two independent random variables i.e. E(XY) = E(X)E(Y)

Generalization:$E(X_1, X_2, \dots X_n) = E(X_1)E(X_2) \dots E(X_n)$

Mathematical expectation(CO4)

❖ **Theorem 5.** Expectation of the difference of two random variables,
i.e. E(X-Y)=E(X)-E(Y)

Q1. What is the expected value of the number of the number of points that will be obtained in a single throw with an ordinary die? Find variance also.

Solution. Here the variate is the number of points showing on a die. It assumes the values 1,2,3,4,5,6 with probability $\frac{1}{6}$ in each case.

$$E(x) = p_1x_1 + p_2x_2 + \ldots. + p_6x_6 = \frac{1}{6}\cdot 1 + \frac{1}{6}\cdot 2 + \cdots\cdots + \frac{1}{6}\cdot 6 = 3.5$$

$$\text{var}(x) = \text{E}(x^2) - [\text{E}(x^2)] = \frac{1}{6}(1^2 + 2^2 + \cdots\cdots + 6^2) - \left(\frac{7}{2}\right)^2 = \frac{35}{12}$$

Daily Quiz(CO4)

Q1. What is the mathematical expectation of the sum of points on n dice?

Recap(CO4)

- ✓ Mathematical expectation
- ✓ Mean
- ✓ Variance

Topic Objective(CO4)

In probability theory and statistics, the *moment-generating function* of a real-valued random variable is an alternative specification of its probability .

❑ MOMENT GENERATING FUNCTION:

An indirect method for computing moments is known as moment generating function. The method depends on the finding of the moments generating function.

❖ **In Case of Continuous Variable x,it is defined as**

$$M(t) = \int_a^b e^{tx} f(x) dx$$

Where integral is a function of parameter t only. The limit a, b can be $-\infty \ and \ \infty$ respectively. It is possible to associate a moment generating function with the distribution only when all the moments of the distribution are finite.

Let us see how $M(t)$ generates moments. For this let us assume

Moment Generating Function(CO4)

That $f(x)$ is a distribution function for which the integral given by(1) exsists. Then e^{tx} may be expanded in a power series and the integration may be performed term by term. It follows that

$$M(t) = \int_a^b \left(1 + tx + \frac{t^2}{2}x^2 + \cdots\right) f(x)dx$$

$$= \int_a^b f(x)dx + t\int_a^b xf(x)dx + \ldots.$$

$$= v_0 + v_1 t + v_2 . \frac{t^2}{2} + \cdots$$

Obviously, the coefficient of $\frac{t^r}{r}$ in (2) is the r^{th} moment about the origin.

Also, $\left|\frac{d^r}{dt^r} M(t)\right|_{t=0} = \left|\frac{v_r}{r} r + v_{r+1} t + \cdots\right|_{t=0} = v_r$, Thus v_r about origin $= r^{th}$ derivative of $M(t)$ with $t = 0$.

Moment Generating Function(CO4)

Although the moment generating function (m.g.f) has been defined for the variable x e.g if $z = x - m(m\ is\ mean)$,the r^{th} moment about z will r^{th} moment about z will given r^{th} moment of x about the mean m.

$$M_z(t) = M_{x-m}(t) = \int_a^b e^{tx} f(x)dx = e^{-mt} M_x(t)$$

- ❖ **In case of discrete distribution:**

The moment generating function is given by

$$M_z(t) = \sum e^{tz} P$$

By expanding e^{tz} we have

Moment Generating Function(CO4)

$$M_z(t) = \sum \left(1 + tz = \frac{t^2}{2} z^2 + \cdots\right) P$$

$$= \sum P + t \sum P\, z + \frac{t^2}{2} \sum P\, z^2 + \cdots$$

$$= v_0 + v_1 t + v_2 . \frac{t^2}{2} + \cdots$$

We get

$$v_r = \left| \frac{d^r}{dt^r} M(t) \right|_{t=0}$$

Moment generating function about any arbitrary number

$$M_{x-a}(t) = E\left(e^{t(x-a)}\right) = e^{-at} M_x(t)$$

Moment Generating Function(CO4)

Properties for moment generating function:

1. The moment generating function of the sum of two independent chance variables is the product of their respective moment generating function, i.e $M_{x+y}(t) = M_x(t) \times M_y\,(t)$
2. Effect of change of origin and scale on m.g.f.

$$M_u(t) = e^{-at/h} M_x\left(t/_h\right)$$

Q1. find the moment generating function of the discrete binomial distribution given by $P(x) = {}^nC_x p^x q^{n-x}$ where $q = 1 - p$

Also find the 1st and 2nd moment about the mean .

Solution: Moment generating function given about the origin is given by

Moment Generating Function(CO4)

$$M_x(t) = \sum e^{xt} P = \sum e^{tx\,n} C_x p^x q^{n-x}$$

$$= \sum {}^nC_x (e^t p)^x q^{n-x}$$

$$= (q + pe^t)^n$$

$$v_1 = \left|\frac{d}{dt} M(t)\right|_{t=0} = [n(q + pe^t)^{n-1} pe^t]_{t=0} = np$$

$$v_2 = \left|\frac{d^2}{dt^2} M(t)\right|_{t=0}$$

$$= [np\{e^t(n-1)(q + pe^t)^{n-2} pe^t + (q + pe^t)^{n-1} e^t]_{t=0}$$

Moment Generating Function(CO4)

$= [np(q + pn)]$
$= npq + n^2p^2$

Hence first and second moments about the mean are given by

$\mu_1 = 0$

Since $\bar{x} = \nu_1 = np$

$\therefore \ \mu_2 = \nu_2 - \bar{x}^2$

$\mu_2 = \nu_2 - {\nu_1}^2$
$= npq + n^2p^2 - n^2p^2$
$= npq$

Hence, mean= np, S.D. = $\sqrt{\mu_2} = \sqrt{npq}$.

Moment Generating Function(CO4)

Q2. find the moment generating function of the discrete Poisson distribution given by $P(x) = \frac{e^{-\lambda}\lambda^x}{x!}$. Also find the first and second moments about the mean.

Solution: Moment generating function about the origin is given by

$$M_x(t) = \sum e^{xt} P = \sum e^{tx} \frac{e^{-\lambda}\lambda^x}{x!}$$

$$= e^{-\lambda}\, e^{\lambda e^t} = e^{\lambda(e^t - 1)}$$

$$v_1 = \left|\frac{d}{dt} M(t)\right|_{t=0} = [e^{\lambda(e^t-1)}\lambda e^t]_{t=0} = \lambda$$

$$v_2 = \left|\frac{d^2}{dt^2} M(t)\right|_{t=0} = \lambda(\lambda + 1)$$

Moment Generating Function(CO4)

Hence , the first and second moments about mean are given by

$\mu_1 = 0$

Since $\bar{x} = \nu_1 = \lambda$

$\therefore \ \mu_2 = \nu_2 - \bar{x}^2$

$\mu_2 = \nu_2 - {\nu_1}^2$

$= \lambda(\lambda + 1) - \lambda$

$= \lambda(\lambda + 1)$

Q3. find the m.g.f. of the continuous normal distribution given by

$$f(x) = \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{1}{2}\left(\frac{x-\mu}{\sigma}\right)^2}; -\infty < x < \infty$$

Solution: Moment generating function about the origin is defineds

Moment Generating Function(CO4)

$$M_x(t) = E(e^{tx}) = \int_{-\infty}^{\infty} e^{tx} f(x)dx$$

$$= \int_{-\infty}^{\infty} e^{tx} \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{1}{2}\left(\frac{x-\mu}{\sigma}\right)^2} dx$$

$$= \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} e^{-t\sigma z} e^{-\frac{1}{2}z^2} dz; \text{ where z} = \frac{x-\mu}{\sigma}$$

$$= \frac{1}{\sqrt{2\pi}} e^{\left(\mu t+\frac{1}{2}t^2\sigma^2\right)} \int_{-\infty}^{\infty} e^{-\frac{1}{2}(z-t\sigma)^2} dz$$

$$= e^{\left(\mu t+\frac{1}{2}t^2\sigma^2\right)}.1$$

$$= e^{\left(\mu t+\frac{1}{2}t^2\sigma^2\right)}$$

Daily Quiz(CO4)

Q1. Find the moment generating function about origin of a random variable X whose probability density function is given by

$$f(x) = \frac{1}{2}e^{-|x|}.$$

Weekly Assignments (CO4)

1. Find the moment generating function about origin of random variable X whose p.m.f is given by $p(x) = pq^{x-1}, x = 1,2,3 \dots \dots$ Hence find mean and variance of the distribution.
2. Find the mean and variance of Binomial distribution.
3. Find the mean and variance of normal distribution.

Recap(CO4)

- ✓ Mathematical expectation
- ✓ Mean
- ✓ Variance
- ✓ Moment Generating Function

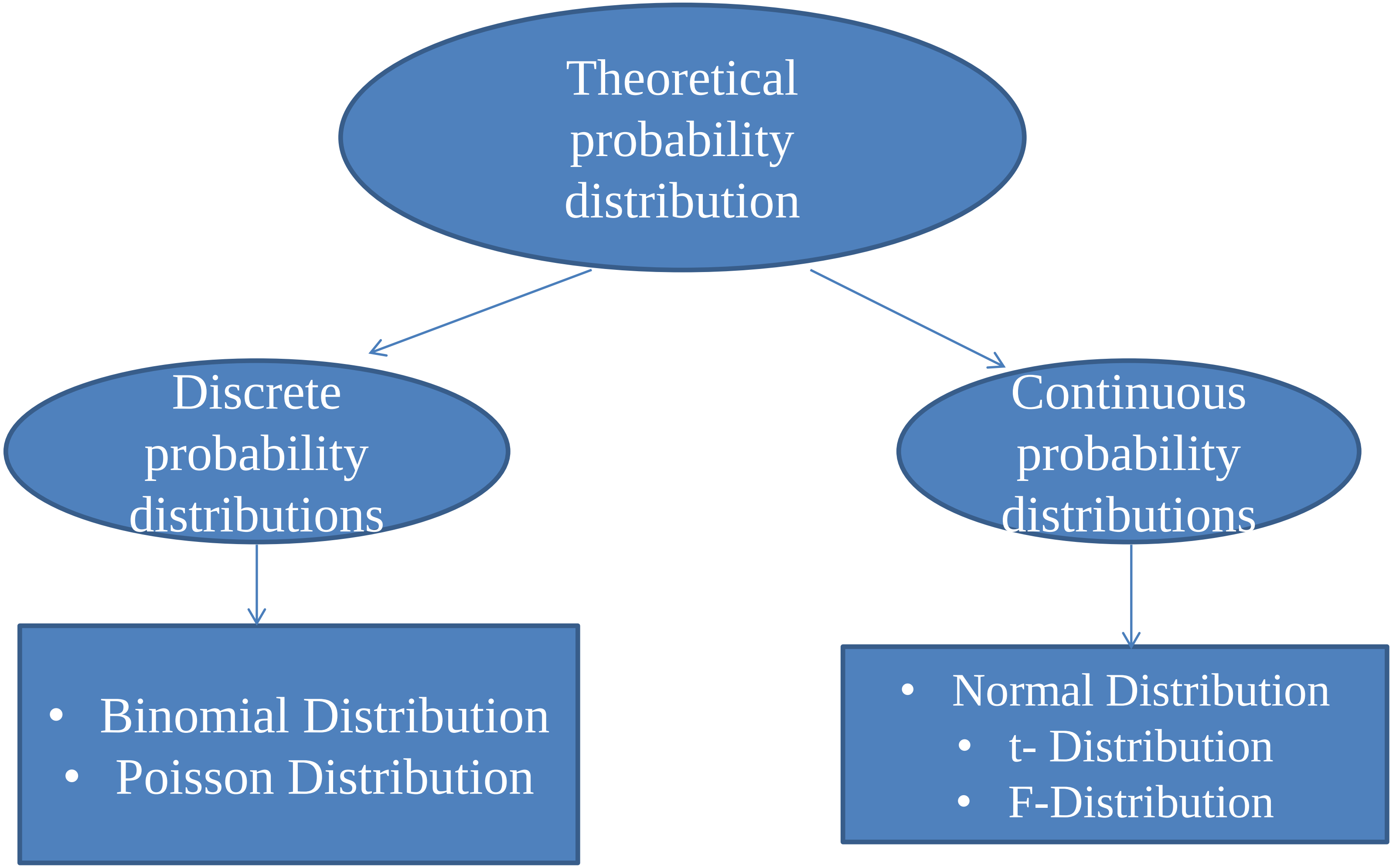
Unit Objective(CO4)

1. A basic knowledge in **probability theory**.
2. The student is able to reflect developed mathematical methods in **probability** and **statistics**.
3. Understand the concept of Probability and its usage in various business applications.
4. The **binomial distribution** model allows us to compute the **probability** of observing a specified number of "successes" when the process is repeated a specific number of times
5. **Poisson Distribution** is a tool that helps to predict the probability of certain events from happening when you know how often the event has occurred.
6. To learn the characteristics of a typical **normal curve**.
7. To explore the key properties, such as the moment-generating function, mean and variance, of a **normal** random variable.

Topic Objective(CO4)

The **probability distributions** are very much helpful for making predictions. Estimates and predictions form an **important** part of research investigation. With the help of Probability distributions, we make estimates and predictions for the further analysis.

Probability distributions(CO4)



Binomial Distribution(CO4)

Binomial Probability Distribution: Probability distribution defined as follows is known as binomial Probability distribution.

$$P(X = r) = {}^{n}C_{r} p^{r} q^{n-r}, r = 1,2 \dots n$$

Where n is no of trial which are finite ,r be the success in n trials and

$p + q = 1$, p is probability of success and q is probability of failure.

Assumptions For Binomial distribution:

- n, the number of trials is finite
- Each trial has only two possible outcomes usually called success and failure.
- All trials are independent.
- p and q is constant for all trials.

Cont…(CO4)

Recurrence or recursion formula:

$P(r) = {}^{n}C_{r} p^{r} q^{n-r} = \frac{n!}{r!(n-r)!} p^{r} q^{n-r}$….(1)

Equation (1) denote binomial distribution.

$P(r+1) = {}^{n}C_{r+1} p^{r+1} q^{n-r-1} = \frac{n!}{(r+1)!(n-r-1)!} p^{r+1} q^{n-r-1}$….(2)

By equation (1) and (2)

$$\frac{P(r+1)}{P(r)} = \frac{(n-r)!}{(n-r-1)!} \times \frac{r!}{(r+1)!} \times \frac{p}{q}$$

$P(r+1) = \frac{(n-r)}{(r+1)} \frac{p}{q} . P(r)$…..(3)

Equation (3) is known as Recurrence Formula.

Cont…(CO4)

Mean Of Binomial distribution:

$$\boldsymbol{\mu = \sum_{r=0}^{n} rP(r)}$$

For Binomial distribution $P(r) = {}^nC_r p^r q^{n-r}$

$$\mu = \sum_{r=0}^{n} r\,{}^nC_r p^r q^{n-r}$$

By expanding we have

$$\Longrightarrow 0 + 1.\,{}^nC_1 pq^{n-1} + 2.\,{}^nC_2 p^2 q^{n-2} + \cdots + n.\,{}^nC_n p^n$$

$$\Longrightarrow np[{}^{n-1}C_0 q^{n-1} + {}^{n-1}C_1 pq^{n-2} + \cdots + {}^{n-1}C_{n-1} p^{n-1}]$$

$$\Longrightarrow np(q+p)^{n-1} = np$$

Hence mean of binomial distribution is np.

Cont…(CO4)

Variance of Binomial Distribution:

$$Variance\ \sigma^2 = \sum_{r=0}^{n} r^2 P(r) - \mu^2$$

$$= \sum_{r=0}^{n} [r + r(r-1)]P(r) - \mu^2$$

$$= \sum_{r=0}^{n} rP(r) + \sum_{r=0}^{n} r(r-1)P(r) - \mu^2$$

$$= np + \sum_{r=0}^{n} r(r-1)P(r) - n^2p^2$$

$$= np + n(n-1)p^2 - n^2p^2 = np(1-p) = npq$$

Cont…(CO4)

Hence the **Variance of binomial distribution is $\boldsymbol{npq}$** and Standard deviation is $\sqrt{npq}$.

Moment generating function of binomial Distribution:

i. About origin

$$M_x(t) = E(e^{xt}) = \sum_{x=0}^{n} {}^nC_x (pe^t)^x q^{n-x} = (q + pe^t)^n$$

ii. About mean

$$M_{x-np}(t) = E\left(e^{t(x-np)}\right) = e^{-npt}E(e^{xt}) = e^{-npt}M_x(t) = e^{-npt}(q + pe^t)^n$$
$$= (qe^{-pt} + pe^{qt})^n$$

Cont…(CO4)

Applications of Binomial Distribution:

1. In problem concerning no. of defectives in sample production line.
2. In estimation of reliability of systems.
3. No. of rounds fired from a gun hitting a target.
4. In radar detection.

Q1. If 10% of bolts are produced by a machine are defective , determine the probability that out of 10 bolts chosen at random

i. 1
ii. None
iii. At most 2 bolts will be defective

Problem's based on Binomial Distribution(CO4)

Solution: let p and q are the probability of defective and non defective bolts respectively.

$p = \frac{10}{100} = \frac{1}{10}, q = 1 - \frac{1}{10} = \frac{9}{10}$ and n=10 (no of bolts chosen)

The Probability of r defective bolts out of n bolt chosen at random is given by

$P(r) = {}^{n}C_r p^r q^{n-r}$

i. Here r=1,

$$P(1) = {}^{10}C_1 \left(\frac{1}{10}\right)^1 \left(\frac{9}{10}\right)^{10-1} = 0.3874$$

ii. Here r=0

$$P(0) = {}^{10}C_0 \left(\frac{1}{10}\right)^0 \left(\frac{9}{10}\right)^{10-0} = 0.3486$$

Problem's based on Binomial distribution(CO4)

iii.Prob.that at most 2 bolts will be defective $=P(\leq 2) = P(0) + P(1) + P(2)$

$$P(2) = {}^{10}C_2 \left(\frac{1}{10}\right)^2 \left(\frac{9}{10}\right)^{10-2}$$

$=45\left(\frac{1}{100}\right)(0.43046) = 0.1937$

From(4).Required Probability$=P(0) + P(1) + P(2)$

$$= 0.3486 + 0.3874 + 0.1937 = 0.9297$$

Q2. Out of 800 families with 4 children each, how many families would be expected to have

i. 2 boys and 2 girls ii. At least one boy iii no girl

iv. Atmost two girls? Assume equal probability for boys and girls.

Cont…(CO4)

Solution: Probability for boys and girls are equal

$p = probability\ of\ having\ a\ boy = \frac{1}{2},$

$q = probability\ of\ having\ a\ girl = \frac{1}{2}$ n=4 N=800

i. The expected number of families having 2 boys and 2 girls

$$= 800\,{}^4C_2\left(\frac{1}{2}\right)^2\left(\frac{1}{2}\right)^2 = 300$$

ii. The expected number of families having at least one boy

$$= 800\left[{}^4C_1\left(\frac{1}{2}\right)^3\left(\frac{1}{2}\right)^1 + {}^4C_2\left(\frac{1}{2}\right)^2\left(\frac{1}{2}\right)^2 + {}^4C_3\left(\frac{1}{2}\right)^1\left(\frac{1}{2}\right)^3 + {}^4C_4\left(\frac{1}{2}\right)^4\right]$$

$$= 750$$

Cont…(CO4)

iii. The expected number of families having no girl i.e. having 4 boys =

$$800\left[{}^4C_4\left(\frac{1}{2}\right)^4\right] = 50$$

iv. The expected number of families having almost two girls i.e. having at least 2 boys

$$= 800\left[{}^4C_2\left(\frac{1}{2}\right)^2\left(\frac{1}{2}\right)^2 + {}^4C_3\left(\frac{1}{2}\right)^1\left(\frac{1}{2}\right)^3 + {}^4C_4\left(\frac{1}{2}\right)^4\right] = 550$$

Daily Quiz(CO4)

1. Four persons in a group of 20 are graduates. If 4 persons are selected at random from 20, find the probability that all 4 are graduates.

   Ans: 0.0016

2. The Prob. that a bulb produced by a factory will fuse after use of 150 days is 0.05. Find the probability that out of 5 such bulbs at least one bulb will fuse after use of 150 days of use.

   Ans: $1 - \left(\frac{19}{20}\right)^5$

Recap (CO4)

- ✓ Mathematical expectation
- ✓ Mean
- ✓ Variance
- ✓ Moment Generating Function
- ✓ Binomial distribution

Poisson Distribution(CO4)

Poisson distribution: Probability distribution defined as follows is known as Poisson Probability distribution.

$$P(X = r) = \frac{e^{-\lambda}\lambda^r}{r!}, (r = 0,1,2,3 \dots .)$$

Where λ $finite\ number = np$.

Recurrence formula for Poisson Distribution:

Poisson distribution $P(r) = \frac{e^{-\lambda}\lambda^r}{r!} \dots \dots \dots (1)$

$P(r+1) = \frac{e^{-\lambda}\lambda^{r+1}}{(r+1)!}$........(2)

$$\frac{P(r+1)}{P(r)} = \frac{\lambda r!}{(r+1)!} = \frac{\lambda}{r+1}$$

Cont…(CO4)

$$P(r+1) = \frac{\lambda}{r+1} P(r), r = 01,2,3 \ldots \ldots$$

This is called the recurrence or recursion formula for Poisson distribution.

Mean of the Poisson distribution:

Mean $\mu = \sum_{r=0}^{\infty} rP(r)$

$$= \sum_{r=0}^{\infty} r.\frac{e^{-\lambda}\lambda^r}{r!}$$

$$= e^{-\lambda} \sum_{r=1}^{\infty} \frac{\lambda^r}{(r-1)!}$$

Cont…(CO4)

$$= e^{-\lambda}\left(\lambda + \frac{\lambda^2}{1!} + \frac{\lambda^3}{2!} + \cdots.\right)$$

$$= \lambda e^{-\lambda}\left(1 + \frac{\lambda}{1!} + \frac{\lambda^2}{2!} + \cdots.\right)$$

$$= \lambda e^{-\lambda}\left(1 + \frac{\lambda}{1!} + \frac{\lambda^2}{2!} + \cdots.\right)$$

$$= \lambda e^{-\lambda} e^{\lambda} = \lambda$$

Mean $= \lambda$ for Poisson distribution.

Variance of Poisson Distribution:

$$Variance\ \sigma^2 = \sum_{r=0}^{\infty} r^2 P(r) - \mu^2$$

Cont…(CO4)

$$= \sum_{r=0}^{\infty} r^2 \frac{e^{-\lambda}\lambda^r}{r!} - \mu^2$$

$$= e^{-\lambda} \sum_{r=0}^{\infty} \frac{r^2 \lambda^r}{r!} - \lambda^2$$

$$= e^{-\lambda} \left[\frac{1^2 \lambda^1}{1!} + \frac{2^2 \lambda^2}{2!} + \frac{3^2 \lambda^3}{3!} + \cdots \right] - \lambda^2$$

$$= \lambda e^{-\lambda} \left[1 + \frac{2\lambda}{1!} + \frac{3\lambda^2}{2!} + \cdots \right] - \lambda^2$$

$$= \lambda e^{-\lambda} \left[1 + \frac{(1+1)\lambda}{1!} + \frac{(1+2)\lambda^2}{2!} + \cdots \right] - \lambda^2$$

$$= \lambda e^{-\lambda} \left[\left(1 + \frac{\lambda}{1!} + \frac{\lambda^2}{2!} + \cdots \right) + \left(\frac{\lambda}{1!} + \frac{2\lambda^2}{2!} + \cdots \right) \right] - \lambda^2$$

Cont…(CO4)

$$= \lambda e^{-\lambda}\left[e^{\lambda} + \lambda\left(1 + \frac{\lambda}{1!} + \frac{\lambda^2}{2!} + \cdots\right)\right] - \lambda^2$$
$$= \lambda e^{-\lambda}\left[e^{\lambda} + \lambda e^{\lambda}\right] - \lambda^2$$
$$= e^{\lambda} e^{-\lambda} \lambda[1 + \lambda] - \lambda^2$$
$$= \lambda[1 + \lambda] - \lambda^2$$
$$= \lambda$$

Hence , the Variance of the Poisson distribution is also λ.

Applications of Poisson Distribution:

i. Arrival pattern of the defective vehicles in a workshop.

ii. Patients in hospitals.

iii. Telephone calls.

iv. Emission of radioactive (α) particles.

Problem based on Poisson Distributions(CO4)

Q1. If the Variance of the Poisson distribution is 2 , find the probability for r=1,2,3,4 from the recurrence relation of the Poisson Distribution. Also find $P(r \geq 4)$.

Solution: Given that Variance $= 2 = \lambda$

Recurrence relation for Poisson distribution

$$P(r+1) = \frac{\lambda}{r+1} P(r) = \frac{2}{r+1} P(r) \ldots \ldots (1)$$

Poisson Distribution $P(r) = \frac{e^{-\lambda}\lambda^r}{r!}$

So $P(r) = \frac{e^{-2}2^r}{r!} \Longrightarrow P(0) = \frac{e^{-2}2^0}{0!} \Longrightarrow e^{-2} = 0.1353$

Now putting r=0,1,2,3 in equation (1)

Cont…(CO4)

$$P(1) = 2P(0) = 2 \times 0.1353 = 0.2706$$
$$P(2) = \frac{2}{2}P(1) = \frac{2}{2} \times 0.2706 = 0.2706$$
$$P(3) = \frac{2}{3}P(2) = \frac{2}{3} \times 0.2706 = 0.1804$$
$$P(4) = \frac{2}{4}P(3) = \frac{1}{2} \times 0.1804 = 0.0902$$

Now to calculate $P(r \geq 4), we\ have$

$$P(r \geq 4) = 1 - [P(0) + P(1) + P(2) + P(3)]$$
$$= 1 - [0.1353 + 0.2706 + 0.2706 + 0.1804]$$
$$= 0.1431$$

Q2. Fit a Poisson distribution to the following data and calculate theoretical frequencies.

Cont…(CO4)

Deaths:	0	1	2	3	4
Frequencies	122	60	15	2	1

Mean of Poisson distribution $\lambda = \frac{\sum fx}{\sum f}$

$$= \frac{0 \times 122 + 1 \times 60 + 2 \times 15 + 3 \times 2 + 4 \times 1}{122 + 60 + 15 + 2 + 1} \sum f = \text{N} = 200$$

$$= 0.5$$

Required Poisson distribution

$$= N.\frac{e^{-\lambda}\lambda^r}{r!} = 200\,\frac{e^{-0.5}(0.5)^r}{r!} = (121.306)\,\frac{(0.5)^r}{r!}$$

Cont…(CO4)

r	N.P(r)	Theoretical frequencies
0	$(121.306)\dfrac{(0.5)^0}{0!} = 121.306$	121
1	$(121.306)\dfrac{(0.5)^1}{1!} = 60.653$	61
2	$(121.306)\dfrac{(0.5)^2}{2!} = 15.163$	15
3	$(121.306)\dfrac{(0.5)^3}{3!} = 2.527$	3
4	$(121.306)\dfrac{(0.5)^4}{4!} = 0.3159$	0
		Total=200

Recap (CO4)

- ✓ Mathematical expectation
- ✓ Mean
- ✓ Variance
- ✓ Moment Generating Function
- ✓ Binomial distribution
- ✓ Poisson distribution

Topic objective of Normal Distribution(CO4)

- To define the probability density function of a **normal** random variable.
- To learn the characteristics of a typical **normal curve**.
- To explore the key properties, such as the moment-generating function, mean and variance, of a **normal** random variable.

Normal Distribution(CO4)

Normal Distribution: The general equation of the normal distribution is given by

$$f(x) = \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{1}{2}\left(\frac{x-\mu}{\sigma}\right)^2}$$

Basic properties for Normal distributions:

i. $f(x) \geq 0$

ii. $\int_{-\infty}^{\infty} f(x)dx = 1$

i.e. the total area under the normal curve above x-axis is 1.

iii. Normal curve is symmetrical about its mean.

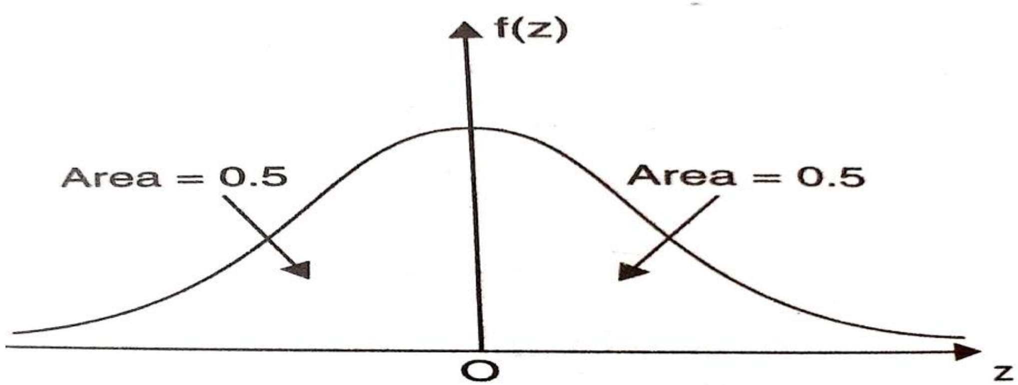
iv. The mean, mode and median coincide for this distribution.

Cont…(CO4)

Standard form of the normal distribution: The probability density function for the normal distribution in standard form is given by

$$f(z) = \frac{1}{\sqrt{2\pi}} e^{-\frac{1}{2}z^2}$$

By taking $z = \frac{x-\mu}{\sigma}$, standard normal curve is formed . The total area under the curve is 1 and it is divided into two parts by z=0.



Mean and Variance of Normal distribution(CO4)

Mean of Normal Distribution: A.M of continuous distribution f(x) is given by

$$A.M(\bar{x}) = \frac{\int_{-\infty}^{\infty} xf(x)dx}{\int_{-\infty}^{\infty} f(x)dx}$$

$\bar{x} = \int_{-\infty}^{\infty} xf(x)dx$ because $\int_{-\infty}^{\infty} f(x)dx = 1$

So $\bar{x} = \int_{-\infty}^{\infty} x \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{1}{2}\left(\frac{x-\mu}{\sigma}\right)^2} dx$

Let $\frac{x-\mu}{\sigma} = z$ so that $x = \mu + \sigma z$ therefore dx $= \sigma dz$

$$\bar{x} = \int_{-\infty}^{\infty} (\mu + \sigma z) \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{1}{2}z^2} \sigma dz$$

$$= \mu \int_{-\infty}^{\infty} \frac{1}{\sqrt{2\pi}} e^{-\frac{1}{2}z^2} dz + \frac{\sigma}{\sqrt{2\pi}} \int_{-\infty}^{\infty} z e^{-\frac{1}{2}z^2} dz$$

Because $\int_{-\infty}^{\infty} \frac{1}{\sqrt{2\pi}} e^{-\frac{1}{2}z^2} dz = 1$ so in above equation

$$= \mu + \frac{\sigma}{\sqrt{2\pi}} \int_{-\infty}^{\infty} z e^{-\frac{1}{2}z^2} dz$$

$$= \mu + \frac{\sigma}{\sqrt{2\pi}} \int_{-\infty}^{\infty} e^{-\frac{1}{2}z^2} d\left(\frac{z^2}{2}\right)$$

$$= \mu + \frac{\sigma}{\sqrt{2\pi}} \left(e^{-\frac{1}{2}z^2}\right)_{-\infty}^{\infty}$$

Cont…(CO4)

$\bar{x} = \mu$

Variance of Normal distribution :

$$= \int_{-\infty}^{\infty} (x - \bar{x})^2 f(x) dx$$

$$= \int_{-\infty}^{\infty} x^2 f(x) dx \ -\bar{x}^2 \ldots \ldots \ldots \ldots (1)$$

Let I $= \int_{-\infty}^{\infty} x^2 f(x) dx \ = \int_{-\infty}^{\infty} x^2 \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{1}{2}\left(\frac{x-\bar{x}}{\sigma}\right)^2} dx$

$\frac{x-\bar{x}}{\sigma} =$ z so that $x = \bar{x} + \sigma z$ therefore d$x = \sigma dz$

$$I = \int_{-\infty}^{\infty} (\bar{x} + \sigma z)^2 \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{1}{2}z^2} \sigma dx$$

$$= -\frac{\sigma^2}{\sqrt{2\pi}} \left(z e^{-\frac{1}{2}z^2}\right)_{-\infty}^{\infty} + \frac{\sigma^2}{\sqrt{2\pi}} \int_{-\infty}^{\infty} e^{-\frac{1}{2}z^2} dx + \bar{x}^2$$

$$= 0 + \sigma^2 + \bar{x}^2$$

$$= \sigma^2 + \bar{x}^2$$

In equation(1)

Variance $= \sigma^2 + \bar{x}^2 - \bar{x}^2 = \sigma^2$

s.d. of normal distribution is σ.

Problem based on Normal distribution(CO4)

Q1. A sample of 100 dry battery cells tested to find the length of the life produced the following results :

$$\bar{x} = 12\ hours, \sigma = 3\ hours$$

Assuming the data to be normally distributed, what percentage of battery cells are expected to have life

i. More than 15 hours

ii. Less than 6 hours

iii. between 10 and 14 hours

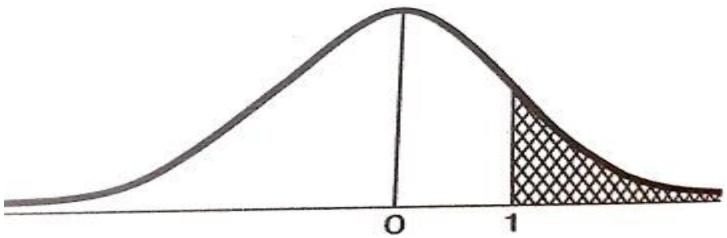
Solution: x denotes the life of dry battery cells .

And $z = \frac{x-\bar{x}}{\sigma} = \frac{x-12}{3}$

i. When x = 15 then z = 1

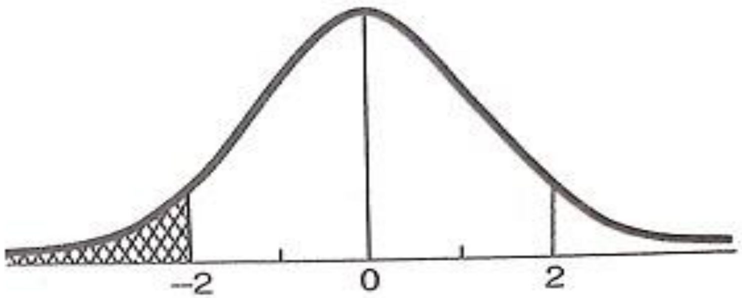
Cont…(CO4)

Therefore $P(x > 15) = P(z > 1)$
$= P(0 < z < \infty) - P(0 < z < 1)$
$= 0.5 - 0.3413 = 0.1587 = 15.87\%$



ii. When x = 6 then z = −2

Therefore
$P(x < 6) = P(z < -2) = P(z > 2)$
$= P(0 < z < \infty) - P(0 < z < 2)$
$= 0.5 - 0.4772 = 0.0228 = 2.28\%$



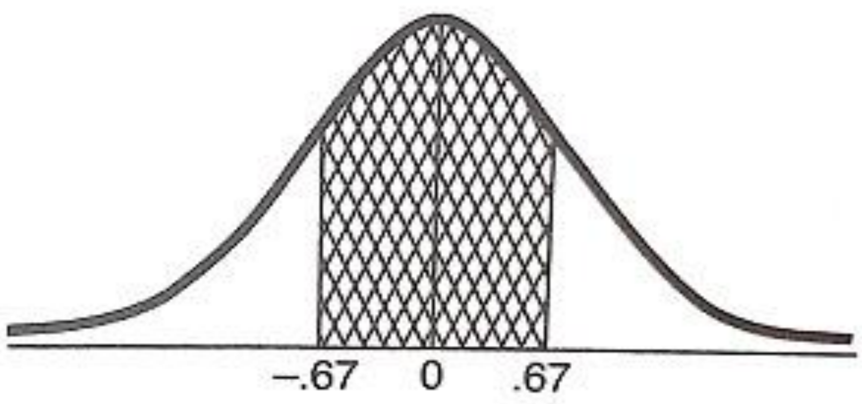
Cont…(CO4)

iii. When $x = 10$ then $z = -0.67$

when $x = 14$ then $z = 0.67$

$P(10 < x < 14) = P(-0.67 < z <).67)$

$= 2P(0 < z < 0.67) = 2 \times 0.2485 = 49.70\%$



Daily Quiz(CO4)

1. In a distribution exactly Normal, 31% of the items are under 45 and 8% are over 64. What are the mean and Standard deviation of this Distribution? It is given that if

$$f(t) = \frac{1}{\sqrt{2\pi}} \int_0^t e^{\frac{-x^2}{2}} dx \text{ then } f(0.5) = 0.19, f(1.4) = 0.42.$$

Recap(CO4)

- ✓ Mathematical expectation
- ✓ Mean
- ✓ Variance
- ✓ Moment Generating Function
- ✓ Binomial distribution
- ✓ Poisson distribution
- ✓ Normal Distributions

Exponential Distribution(CO4)

Exponential distribution:

A continuous random variable $X = x$ which has the following Pdf

$$f(x) = \begin{cases} \lambda e^{-\lambda x} & for\ x \geq 0 \\ 0 & for\ x < 0 \end{cases}$$

Where λ is a parameter.

is called exponential distribution.

Mean of Exponential distribution: We know that mean is given by

$$\text{Mean} = E(X = x) = \int_{-\infty}^{\infty} xf(x)dx$$

$$= \int_{0}^{\infty} x\lambda e^{-\lambda x}\, dx$$

$$= \lambda \int_{0}^{\infty} xe^{-\lambda x}\, dx$$

Exponential Distribution(CO4)

$= \lambda \int_0^\infty x^{2-1}\ e^{-\lambda x}\, dx,$ which is a Gamma function

$$= \lambda \frac{\Gamma 2}{\lambda^2} = \frac{1}{\lambda}$$

$$\left[Mean = \frac{1}{\lambda}\right]$$

Variance of Exponential distribution:

We know that the variance is given by

$$\text{Variance}(X = x) = E(x^2) - (E(x))^2 \ \ldots\ldots\ldots.(1)$$

Now $\quad E(x^2) = \int_{-\infty}^{\infty} x^2 f(x) dx$

$$= \int_0^\infty x^2 \lambda e^{-\lambda x}\, dx$$

$= \lambda \int_0^\infty x^{3-1} e^{-\lambda x}\, dx$, which is a Gamma function

$$= \lambda \frac{\Gamma 3}{\lambda^3} = \frac{2}{\lambda^2}$$

Exponential Distribution(CO4)

From (1), Variance$(x) = E(x^2) - [E(x)]^2$

$$V(x) = \frac{2}{\lambda^2} - \left[\frac{1}{\lambda}\right]^2 = \frac{1}{\lambda^2}$$

Hence **Variance**$(\boldsymbol{x}) = \frac{\mathbf{1}}{\boldsymbol{\lambda^2}}$

Q1. The length of Telephone conversation is an exponential variate with mean 3minutes. Find the Probability that call

(a) End in less than 3 minutes.

(b) tabes between 3 to 5 minutes.

Solution. Given, Mean$= 3 = \frac{1}{\lambda}$

$$\Rightarrow \lambda = \frac{1}{3}$$

$$P(x < 3) = \int_0^3 f(x)dx = \int_0^3 \lambda e^{-\lambda x}\, dx = \int_0^3 \frac{1}{3} e^{-x/3} dx$$

Exponential Distribution(CO4)

$$= \frac{1}{3}\left[\frac{e^{-x/3}}{-1/3}\right]_0^3 = -(e^{-1} - e^{-0}) = 1 - \frac{1}{e}$$

$$P(3 < x < 5) = \int_3^5 f(x)dx = \int_3^5 \lambda e^{-\lambda x} dx$$

$$= \frac{1}{3}\int_3^5 e^{-x/3} dx = \frac{1}{3}\left[\frac{-x/3}{-1/3}\right]_3^5 = -\left[e^{-5/3} - e^{-1}\right]$$

$$= \frac{1}{e} - \frac{1}{e^{5/3}}$$

Daily Quiz(CO4)

Q1. The length of Telephone conversation is an exponential variate with mean 7 minutes. Find the Probability that call

(a) End in less than 7 minutes.

(b) tabels between 7 to 10 minutes.

Weekly Assignment(CO4)

Q1. The length of Telephone conversation is an exponential variate with mean 5 minutes. Find the Probability that call

(a) End in less than 5 minutes.

(b) tabels between 5 to 10 minutes.

Q2. The experience shows that 4 accidents occur in a plant on an average per month. Calculate the probabilities of less than 3 accidents in a certain month. Use Poisson distribution. (Given e^{-4}=0.01832).

Q3. Net profit of 400 companies is normally distributed with a mean profit of Rs. 150 lakhs and a standard deviation of Rs. 20 lakhs. Find the number of companies whose profits(Rs. Lakhs) are between 100 and 138.Also find the minimum profit of top 15% companies. (Area for Z=2.5 ,0.35 and 0.6 are 0.4938,1.04 and 0.2251)

Weekly Assignment(CO4)

Q4. In a sample of 1000 cases, the mean of a certain test is 14 and standard deviation is 2.5. Assuming distribution to be normal find

i. How many students score between 12 and 15?

ii. How many score above 18?

iii. How many score below 8?

iv. How many score 16?

Given that $\phi(z) = \frac{1}{\sqrt{2\pi}}\int_0^z e^{-z^2/2}dz$, $\phi(0.4) = 0.155, \phi(0.8) = 0.2881, \phi(1.6) = 0.445, \phi(0.6) = 0.0225, \phi(1) = 0.341$).

Q5. A large number of measurement is normally distributed with a mean 65.5 inches and S.D. of 6.2 inches. Find the percentage of measurement that fall between 54.8 inches and 68.8 inches. Given that area (at z = 1.73) = 0.4582 and area (at z = .53) = 0.2019 .

Weekly Assignment(CO4)

Q6. In a distribution exactly Normal, 31% of the items are under 45 and 8% are over 64. What are the mean and Standard deviation of this Distribution? It is given that if $f(t) = \frac{1}{\sqrt{2\pi}}\int_0^t e^{\frac{-x^2}{2}}dx$then $f(0.5) = 0.19, f(1.4) = 0.42$

Q7. The life of army shoes is normally distributed with mean 8 months and standard deviation 2 months.If 5000 airs are insured, how many pairs would you expected to need replacement after 12 months?

Q8. Find the mean and variance of the Binomial, Poisson and Normal distribution.

Weekly Assignment(CO4)

Q9.The probability that a pen manufactured by a company will be defective is $1/10$. If 12 such pens are manufactured, find the probability that

i. Exactly two will be defective
ii. At least two will be defective
iii. None will be defective.

Q10.It is given that 2% of the electric bulbs manufactured by a company are defective. Using Poisson distribution find the probability that a sample of 200 bulbs will contain

i. No defective bulb
ii. Two defective bulbs
iii. Atmost three defective bulbs.

Weekly Assignment(CO4)

Q11.In a test on 2000 electric bulbs,it was found that the life of a particular make, was normally distributed with an average life of 2040 hours and S.D. of 60 hours, estimate the number of bulbs likely to burn for

i. More than 2150 hours

ii. Less than 1950 hours

iii. More than 1920 hours but less than 2160 hours

Q12.In a distribution exactly Normal, 7% of the items are under 35 and 89% are under 63. What are the mean and Standard deviation of this Distribution?

Faculty Video Links, Youtube & NPTEL Video Links and Online Courses Details

- Self made Video Link:
- **https://youtu.be/6pZXCcoeYiU**
- **https://youtu.be/izT2QpldbnU**
- https://youtu.be/UaLNsZQK8fo
- Suggested Video inks:

https://youtu.be/L3wQw0wva3g

https://youtu.be/n9qpktdFfLU

https://youtu.be/_Qlxt0HmuOo

https://youtu.be/YSwmpAmLV2s

https://youtu.be/KLnGOL_AUgA

https://youtu.be/cQp_bJdxjWw

https://youtu.be/geB0A7CPGaQ

https://youtu.be/zmyh7nCjmsg

https://youtu.be/ohquDY3fZqk

https://youtu.be/izGZLnB-mEo

https://youtu.be/q48uKU_KWas

MCQ s(CO4)

Q1. Suppose that a random variable x has normal distribution with mean 9 and variance 9. Then the value of c such that $P(x > c) = 0.16$ is-(Given that $\phi(1) = 0.34$)

i. 12

ii. 1

iii. 10

iv. None of these

Q2 .The life of army shoes is normally distributed with mean 8 months and standard deviation 2 months. If 5000 pairs are issued, how many pairs would be expected to need replacement after 12 months are-(Given that $P(z \geq 2) = 0.228$)

i. 114

ii. 4886

iii. 115

iv. 4890

MCQ s(CO4)

a. $\frac{100}{3}$

b. $\frac{200}{3}$

c. $\frac{50}{3}$

d. None of these

Q3. Suppose that a book of 600 pages contains 40 printing mistakes. Assume that these errors are randomly distributed throughout the book and x, the number of errors per page has a Poisson distribution then what is probability that 10 pages selected at random will be free of errors

i. 0.54

ii. 0.15

iii. 0.51

iv. None of these

MCQ s(CO4)

Q4. The probability that a bomb dropped from a plane will strike the target is $\frac{1}{5}$. If six bombs are dropped then the probability that exactly two will strike the target is

i. 0.345

ii. 0.145

iii. 0.245

iv. None of these

Q5. For the standard normal variate z mean and variance are-

i. 0,1

ii. μ, σ^2

iii. 1,0

iv. σ^2, μ

Glossary(CO4)

Q1. The distribution of the number of road accidents per day in a city is poisson with mean 4. find the number of days out of 100 days when there will be

i. No accident

ii. At least two accident

iii. At most three accident

iv. Between two and five accident

Pick the correct option from glossary

a. 91

b. 43

c. 39

d. 2

Glossary(CO4)

Q2.In 800 families with 4 children each ,how many families would be expected to have

i. 2 boys and 2 girls
ii. No girl
iii. At least one boy
iv. At most two girls (Assume equal probabilities for boys and girls)

Pick the correct option from glossary

a. 750
b. 550
c. 50
d. 300

Old Question Papers

First Sessional Set-1 (CSE,IT,CS,ECE,IOT).docx

Second Sessional Set-2 (CSE,IT,CS,ECE,IOT).docx

Maths IV PUT.docx

Maths IV final paper_2022.pdf

University Expected question(CO4)

Q1. Find the mean and variance of the binomial distribution.

Q2. If the probability of hitting a target is 10% and 10 shots are fired independently. What is the probability that the target will be hit at least once?

Q3. If there are 3 misprints in a book of 1000 pages find the probability that a given page will contain

i. No misprint

ii. More than 2 misprints

Q4. If the heights of 300 students are normally distributed with mean 64.5 inches and standard deviation 3.3 inches. Find the height below which 99% of the student lie.

(Given that $\phi(z) = \frac{1}{\sqrt{2\pi}} \int_0^z e^{-z^2/2} dz$, $\phi(2.327) = 0.49$).

University Expected question(CO4)

Q7. The experience shows that 4 accidents occur in a plant on an average per month. Calculate the probabilities of less than 3 accidents in a certain month. Use Poisson distribution. (Given $e^{-4}=0.01832$).

Q8. As a result of tests on 20,000 electric bulbs manufactured by a company it was found that the life time of a bulb was normally distributed with an average life of 2040 hours and standard deviation of 60 hours. On the basis of the information, estimate the number of bulbs that is expected to burn for

(i) more than 2150 hours, and (ii) less than 1960 hours.

Q9. Write short note on

i. Binomial distribution

ii. Poisson distribution

Q10. Show that mean of binomial distribution is np.

Unit Recap(CO4)

Students have taught the importance of the following topics….

With the concept of probability to evaluate probability distributions:

- ✓ Mathematical expectation
- ✓ Mean
- ✓ Variance
- ✓ Moment Generating Function
- ✓ Binomial distribution
- ✓ Poisson distribution
- ✓ Exponential Distribution

References (CO4)

1. **Introduction to Statistics** - P.K. Giri & J. Banerjee.
2. **Statistical Models: Theory and Practice** by David Freedman.
3. Richard I. Levin, David S.Rubin **"Statistics for Management",** Pearson Education
4. Anderson, Sweeney and Williams **"Statistics for Business and Economics",** Cengage Learning.

Thank You