Noida Institute of Engineering and Technology, Greater Noida

Subject: Mathematics-III

Subject Code: AAS0301A

Unit: II

Complex Variable-Integration

B Tech 3rd Sem

Mr. Raman Chauhan

Department of Mathematics

Sequence of contents

1	Name of Subject with code, Course and Subject Teacher
2	Brief Introduction of Faculty member with Photograph
3	Evaluation Scheme
4	Subject Syllabus
5	Branch wise Applications
6	Course Objective (Point wise)
7	Course Outcomes (COs)
8	Program Outcomes only heading (POs)
9	COs and POs Mapping

Sequence of contents

Sequence of contents

Evaluation Scheme

NOIDA INSTITUTE OF ENGINEERING & TECHNOLOGY, GREATER NOIDA
(An Autonomous Institute)

B. TECH (CSE)
EVALUATION SCHEME
SEMESTER-III

Sl. No.	Subject Codes	Subject Name	Periods			Evaluation Schemes				End Semester		Total	Credit
			L	T	P	CT	TA	TOTAL	PS	TE	PE		
WEEKS COMPULSORY INDUCTION PROGRAM													
1	AAS0301A	Engineering Mathematics III	3	1	0	30	20	50		100		150	4
2	ACSE0304	Discrete Structures	3	0	0	30	20	50		100		150	3
3	ACSE0306	Digital Logic & Circuit Design	3	0	0	30	20	50		100		150	3
4	ACSE0301	Data Structures	3	1	0	30	20	50		100		150	4
5	ACSE0302	Object Oriented Techniques using Java	3	0	0	30	20	50		100		150	3
6	ACSE0305	Computer Organization & Architecture	3	0	0	30	20	50		100		150	3
7	ACSE0353	Digital Logic & Circuit Design Lab	0	0	2				25		25	50	1
8	ACSE0351	Data Structures Lab	0	0	2				25		25	50	1
9	ACSE0352	Object Oriented Techniques using Java Lab	0	0	2				25		25	50	1
10	ACSE0354	Internship Assessment-I	0	0	2				50			50	1
11	ANC0301 / ANC0302	Cyber Security*/ Environmental Science*(Non Credit)	2	0	0	30	20	50		50		100	0
12		MOOCs (For B.Tech. Hons. Degree)											
		GRAND TOTAL										**1100**	**24**

Syllabus

Unit-1 (Complex Variable: Differentiation)

Limit, Continuity and differentiability, Functions of complex variable, Analytic functions, Cauchy- Riemann equations (Cartesian and Polar form), Harmonic function, Method to find Analytic functions, Conformal mapping, Mobius transformation and their properties.

Unit-2 (Complex Variable: Integration)

Complex integrals, Contour integrals, Cauchy- Goursat theorem, Cauchy integral formula, Taylor's Series, Laurent series, Liouville's Theorem, Singularities, zero of analytic function, Residues, Method of finding residues, Cauchy Residue's theorem, Evaluation of real integral of the type $\int_0^{2\pi} f(\sin\theta, \cos\theta) d\theta$ and $\int_{-\infty}^{\infty} f(x)\, dx$

Syllabus

Unit-3 (Partial Differential Equation and its Applications)

Introduction of partial differential equations, Second order linear partial differential equations with constant coefficients. Classification of second order partial differential equations, Method of separation of variables for solving partial differential equations, Solution of one and two dimensional wave and heat conduction equations.

Syllabus

Unit-4 (Numerical Techniques)

Error analysis, Zeroes of transcendental and polynomial equations using Bisection method, Regula-falsi method and Newton-Raphson method, Interpolation: Finite differences, Newton's forward and backward interpolation, Lagrange's and Newton's divided difference formula for unequal intervals. Solution of system of linear equations, Crout's method, Gauss- Seidel method. Numerical integration: Trapezoidal rule, Simpson's one third and three-eight rules, Solution of 1st order ordinary differential equations by fourth-order Runge- Kutta methods.

Unit-5 (Aptitude-III)

Time & Work, Pipe & Cistern, Time, Speed & Distance, Boat & Stream, Sitting Arrangement, Clock & Calendar.

Branch Wise Applications

- Concept of Complex variable is used in speech recognition, image processing etc.
- Function of complex variable helps in evaluation of area, which is required in many physical formulation.

Course Objective

The objective of this course is to familiarize the engineers with concept of function of complex variables, complex variables& their applications, Integral Transforms for various mathematical tasks and numerical aptitude. It aims to show case the students with standard concepts and tools from B. Tech to deal with advanced level of mathematics and applications that would be essential for their disciplines. The students will learn:

- The idea of function of complex variables and analytic functions.
- The idea of concepts of complex functions for evaluation of definite integrals
- The concepts of concept of partial differential equation to solve partial differential and its applications.

Course Objective

- The concept of finding roots by numerical method, interpolation and numerical methods for system of linear equations, definite integral and 1st order ordinary differential equations.
- The concept of problems based on Time & Work, Pipe & Cistern, Time, Speed & Distance, Boat & Stream, Sitting Arrangement, Clock & Calendar.

Course Outcomes

- **CO1:** Apply the working methods of complex functions for finding analytic functions.
- **CO2: Apply the concepts of complex functions for finding Taylor's series, Laurent's series and evaluation of definite integrals.**
- **CO3:** Apply the concept of partial differential equation to solve complex variables and problems concerned with partial differential equations

Course Outcomes

- **CO4:** Apply the concept of numerical techniques to evaluate the zeroes of the Equation, concept of interpolation and numerical methods for various mathematical operations and tasks, such as integration, the solution of linear system of equations and the solution of differential equation.
- **CO5:** Solve the problems of Time & Work, Pipe & Cistern, Time, Speed & Distance, Boat & Stream, Sitting Arrangement , Clock & Calendar.

Program Outcomes

S.No	Program Outcomes (POs)
PO 1	Engineering Knowledge
PO 2	Problem Analysis
PO 3	Design/Development of Solutions
PO 4	Conduct Investigations of Complex Problems
PO 5	Modern Tool Usage
PO 6	The Engineer & Society
PO 7	Environment and Sustainability
PO 8	Ethics
PO 9	Individual & Team Work
PO 10	Communication
PO 11	Project Management & Finance
PO 12	Lifelong Learning

CO-PO Mapping(CO1)

Sr. No	Course Outcome	PO1	PO2	PO3	PO4	PO5	PO6	PO7	PO8	PO9	PO10	PO11	PO12
1	CO 1	H	H	H	H	L	L	L	L	L	L	L	M
2	**CO 2**	**H**	**H**	**H**	**H**	**L**	**L**	**L**	**L**	**L**	**L**	**M**	**M**
3	CO 3	H	H	H	H	L	L	L	L	L	L	M	M
4	CO 4	H	H	H	H	L	L	L	L	L	L	L	M
5	CO 5	H	H	H	H	L	L	L	L	L	L	M	M

*L= Low *M= Medium *H= High

PSO

PSO	Program Specific Outcomes (PSOs)
PSO 1	To impart proper knowledge of science and mathematics related subjects to the students.
PSO 2	To enhance the skills of the students with the ability to implement the scientific concepts for betterment of the society in professional and ethical manner.
PSO 3	To prepare the students to understand physical system, mechanical components and processes to address social, technical and engineering challenges.

CO-PSO Mapping(CO1)

CO	PSO 1	PSO 2	PSO 3
CO.1	H	L	M
CO.2	L	M	L
CO.3	M	M	M
CO.4	H	M	M
CO.5	H	M	M

*L= Low *M= Medium *H= High

Program Educational Objectives(PEOs)

PEO-1: To have an excellent scientific and engineering breadth so as to comprehend, analyze, design and provide sustainable solutions for real-life problems using state-of-the-art technologies.

PEO-2: To have a successful career in industries, to pursue higher studies or to support entrepreneurial endeavors and to face the global challenges.

PEO-3: To have an effective communication skills, professional attitude, ethical values and a desire to learn specific knowledge in emerging trends, technologies for research, innovation and product development and contribution to society.

PEO-4: To have life-long learning for up-skilling and re-skilling for successful professional career as engineer, scientist, entrepreneur and bureaucrat for betterment of society.

End Semester Question Paper Template

- 100 Marks Question Paper Template.docx

Prerequisite and Recap(CO1)

- Knowledge of Integration
- Knowledge of Complex function

Brief Introduction about the Subject with Videos

- We will discuss properties of complex function (limits, continuity, differentiability, Analyticity and integration)
- In 3rd module we will discuss application of partial differential equations
- In 4th module we will discuss numerical methods for solving algebraic equations, system of linear equations, definite integral and 1st order ordinary differential equation.
- In 5th module we will discuss aptitude part.
- https://youtu.be/iUhwCfz18os
- https://youtu.be/ly4S0oi3Yz8
- https://youtu.be/f8XzF9_2ijs

Content

- Complex integrals,
- Contour integrals
- Cauchy- Goursat theorem
- Cauchy integral formula,
- Taylor’s series and Laurent’s series,
- Liouvilles’s theorem,
- Singularities and Classification of Singularities,
- Zeros of analytic functions,
- Residues and methods of finding residues,
- Cauchy Residue theorem,
- Evaluation of real integrals of the type $f(\sin\theta, \cos\theta)d\theta$.

Unit Objective(CO2)

- The objective of this unit is to familiarize the engineers with concept of function of complex variables. It aims to show case the students with standard concepts and tools from B.Tech to deal with advanced level of mathematics and applications that would be essential for their disciplines.

Complex Variables(CO2)

Complex Integration

Topic Objective(CO2)

We will evaluate the integration of complex function by:

- Line Integral.
- Cauchy’s Integral Theorem
- Cauchy’s Integral Formula

Line Integral in the Complex Plane(CO2)

In case of real variable, the path of integration of $\int_a^b f(x)dx$ is always along the $x - axis$ from $x = a$ to $x = b$.

But in the case of complex function $f(z)$ the path of the definite integral $\int_a^b f(z)dz$ can be along any curve from $z = a$ to $z = b$.

$$z = \mathrm{x} + iy \Rightarrow dz = dx + idy \quad \ldots\ldots\ldots\ldots(1)$$

$$dz = dx\,, \qquad \text{if } dy = 0$$

$$dz = idy\,, \qquad \text{if } dx = 0.$$

Its value depends upon the path (curve) of integration. But the value of integral from a to b remains the same along any regular curve from a to b.

Line Integral in the Complex Plane(CO2)

In case the initial point and final point coincide so that C is a closed curve, then this integral is called contour integral and is denoted by $\oint_c f(z)\,dz$. If $f(z) = u(x,y) + iv(x,y)$, then we have

$$\oint_c f(z)\,dz = \oint_c (u+iv)\,(dx+idy)$$

$$= \int_C (udx - vdy) + i\int_C (vdx + udy)$$

which shows the evaluation of the line integral of a complex function, by evaluation of two line integrals of real function .

Daily Quiz(CO2)

Q1. Evaluate the integral $\int_C \log z \, dz$ where C is the unit circle $|z| = 1$.

Q2. Evaluate the line integral $\int_C z^2 dz$ where C is the boundary of a triangle with vertices $0, 1+i, -1+i$ clockwise .

Line Integral in the Complex Plane(CO2)

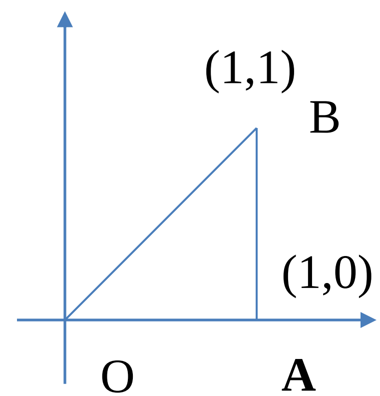
Example: Find the value of the integral $\int_0^{1+i}(x-y+ix^2)\mathrm{dz}$

(i) Along the straight line from z = 0 to z = 1+i .

(ii) Along the real axis from z = 0 to z = 1 and then along a line parallel to the imaginary axis from z=1 to z = 1+ i.

Solution: We have z = x + iy

dz = dx +i dy



(i) OB is the straight line joining z = 0 to z = 1+i.

Obviously on OB, we have y = x

dy = dx .

Now, $\int_0^1(x-y+ix^2)(1+\mathrm{i})\,dx = \int_0^1 i\,(1+\mathrm{i})\,x^2\,dx$

$$= \frac{1}{3}(-1+i)$$

Line Integral in the Complex Plane(CO2)

(ii) OA is the line from z = 0 to z =1 along the real axis and AB is the line z=1 to z = 1+i parallel to the imaginary axis.

On the line OA: y =0, z= x +iy =x and dz = dx

$$\int_0^1 (x + ix^2)dx \; = \frac{1}{2} + \frac{i}{3} \; .$$

On the line AB: x =1, z = 1+iy, dz = idy

$$\int_0^1 (1 + i - y) \; idy \; = -1 + \frac{i}{2} \; .$$

Hence $\int_0^{1+i} (x - y + ix^2)$dz along the contour OAB is

$$\int_{OA} (x - y + ix^2) \text{ dz } + \int_{AB} (x - y + ix^2) \text{ dz}$$

$$= \frac{1}{2} + \frac{i}{3} - 1 + \frac{i}{2} \; = - \frac{1}{2} + \frac{5}{6} \; i$$

Weekly Assignment(CO2)

Q.1. Evaluate $\int_0^{2+i} (\bar{z})^2 \, dz$ along the real axis from $z = 0$ to $z = 2$ and then along a line parallel $y - axis$ from $z = 2$ to $z = 2 + i$.

Q.2. Find the value of the integral $\int_0^{1+i} (x - y + ix^2) \, dz$

(a) Along the straight line from $z = 0$ to $z = 1 + i$

(b) Along real axis from $z = 0$ to $z = 1$ and then along a line parallel to theimaginary axis from $z = 1$ to $z = 1 + i$

Q.3. Evaluate $\int_0^{1+i} (x^2 - iy) \, dz$ along the path $y = x^2$

Q.4. Evaluate the line integral $\int_C z^2 dz$ where C is the boundary of a triangle with vertices $0, 1 + i, -1 + i$ clockwise .

Q.5. Evaluate the integral $\int_C \log z \, dz$ where C is the unit circle $|z| = 1$

Weekly Assignment(CO2)

Q.6. Evaluate the integral $\int_C |z|\, dz$ where C is the contour

(i) The straight line from z $= -i$ to z $= i$

(ii) The left half of the unit circle $|z| = 1$ from z $= -i$ to z $= i$

Q.7. Prove that (i) $\int_C \frac{1}{z-a} dz = 2\pi i$ (ii) $\int_C (z-a)^n dz = 0$ (n is integer $\neq -1$)

where C is the circle $|z-a| = r$

Q.8. Evaluate the integral $\int_C (z - z^2) dz$ where C is the upper half of the circle $|z| = 1$. What is the value of this integral if C is the lower half of the given circle .

Q.9. Evaluate the integral $\int_C (z - z^2) dz$ where C is the upper half of the circle $|z-2| = 3$. What is the value of this integral if C is the lower half of the given circle.

Weekly Assignment(CO2)

Q.10. Evaluate the integral $\int_C \frac{2z+3}{z}\,dz$ where C is

(i) the upper half of the circle $|z| = 2$ in clockwise direction .

(ii) the lower half of the circle $|z| = 2$ in anti clockwise direction.

Cauchy’s Integral Theorem(CO2)

Statement :

If $f(z)$ is an analytic function and $f'(z)$ is continuous at all points inside and on a simple closed curve C , then

$$\int_c f(z)dz = 0$$

Daily Quiz(CO2)

Q1. Verify Cauchy's theorem for the function $f(z) = e^{iz}$ along the boundary of the triangle with vertices at the points 1+i ,-1+i and -1-i .

Cauchy's Integral Theorem(CO2)

Example 1: Evaluate $\int_C \frac{e^z}{z-2}\, dz$ where C : $|z| = 1$.

Sol: In the given integral e^z is analytic inside and on the circle

C: $|z| = 1$, z = 2 lies outside the circle, hence $\int_C \frac{e^z}{z-2}\, dz$ =0, by Cauchy's

Integral theorem.

Example 2: Evaluate $\int_C \frac{2z^2+5}{(z+2)^3(z^2+4)}\, dz$, where C is the square with vertices at 1+ i, 2+ i, 2+ 2i , 1+2i .

Sol: f(z) = $\frac{2z^2+5}{(z+2)^3(z^2+4)}$

$(z+2)^3(z^2+4)$ =0,

z = -2, (order 3) $z = \pm 2i$ (simple poles)

Cauchy's Integral Theorem(CO2)

Since the singularities does not lie inside the contour C hence by Cauchy's integral theorem $\int_C \frac{2z^2+5}{(z+2)^3(z^2+4)}\, dz = 0$

Weekly Assignment(CO2)

Q.1. Find the integral $\int_C \frac{3z^2+7z+1}{z+1} dz$, where C is the circle $|z| = \frac{1}{2}$.

Q.2. Find the integral $\int_C \frac{z+4}{z^2+2z+5} dz$,where C is the circle $|z+1| = 1$.

Q.3. Find the integral $\int_C \frac{2z^2+5}{(z+2)^3(z^2+4)} dz$, where C is the square with the vertices at $1+i$, $2+i$, $2+2i, 1+2i$.

Q.4. Find the integral $\int_C (x^2 - y^2 + 2ixy)\, dz$, where C is the contour $|z| =1$.

Q.5. Find the integral $\int_C \frac{z^2+5z+6}{z-2} dz$, where C is the circle $|z| = \frac{3}{2}$.

Weekly Assignment(CO2)

Q.6. Find the integral $\int_C \frac{e^{3iz}}{(z+\pi)} dz$, where C is the circle $|z-\pi| = 3.$

Q.7. Verify Cauchy's theorem for the function $f(z) = e^{iz}$ along the boundary of the triangle with vertices at the points 1+i ,-1+i and -1-i .

Q.8. Verify Cauchy's theorem for the function $f(z) = 3z^2 + iz - 4$ along the perimeter of sequence with vertices at $1 \pm i, -1 \pm i$.

Extension of Cauchy's integral theorem(CO2)

Statement : If $f(z)$ is analytic in the region R between two simple closed curve C_1 *and* C_2 then

$$\oint_{C_1} f(z)dz = \oint_{C_2} f(z)dz$$

Cauchy's Integral Formula:

Statement : If $f(z)$ is analytic within and on a closed curve C and 'a' is any point within C, then

$$f(a) = \frac{1}{2\pi i}\oint_C \frac{f(z)}{z-a}\mathrm{dz}\ ,$$

or $$\oint_C \frac{f(z)}{z-a}\mathrm{dz} = 2\pi i f(a)$$

Cauchy's integral formula for derivatives (CO2)

If a function $f(z)$ is analytic in a region D , then its derivative at any point $z = a$ of D is also analytic in D and is given by

$$f'(a) = \frac{1}{2\pi i}\oint_C \frac{f(z)}{(z-a)^2}\,\mathrm{dz}$$

Where C is any closed contour in D surrounding the point

$\underline{z} = a$. Then

$$f^n(a) = \frac{n!}{2\pi i}\oint_C \frac{f(z)}{(z-a)^{n+1}}\,\mathrm{dz}$$

Daily Quiz(CO2)

Use Cauchy integral formula to evaluate

$$\oint_C \frac{4-3z}{z(z-1)(z-2)}\,dz$$

where C is the circle $|z| = 3/2$.

Cauchy's Integral Theorem(CO2)

Example: Evaluate $\int_C \frac{e^{2z}}{(z^2-3z+2)}\, dz$ where C is the circle $|z| = 3$.

Sol: Since $\frac{1}{(z^2-3z+2)} = \frac{1}{(z-1)(z-2)} = \frac{1}{(z-2)} - \frac{1}{(z-1)}$ we have

$$I = \int_C \frac{e^{2z}}{(z^2-3z+2)}\, dz = \int_C \frac{e^{2z}}{(z-2)}\, dz - \int_C \frac{e^{2z}}{(z-1)}\, dz \text{ where } C: |z| = 3$$

Both the singularity z =1 and z =2 lie inside the circle $|z| = 3$.

Therefore by Cauchy's integral formula

$I = 2\pi i\ [e^{2z}]_{z=2} - 2\pi i\ [e^{2z}]_{z=1} = 2\pi i\ (e^4 - e^2)$

Recap(CO2)

We have discussed.

- Line Integral.
- Cauchy Integral Theorem
- Cauchy’s Integral Formula

Weekly Assignment(CO2)

Q.1. Evaluate $\oint_C \frac{e^{-z}}{z+1}\,dz$, where C is the circle $|z| = 2$

Q.2. Evaluate $\oint_C \frac{1}{z}\cos z\ \ dz$, where C is the ellipse $9x^2 + 4y^2 = 1$

.

Q.3. Use Cauchy integral formula to evaluate

$$\oint_C \frac{\sin \pi z^2 + \cos \pi z^2}{(z-1)(z-2)}\,dz$$

where C is the circle $|z| = 3$.

Q.4. Use Cauchy integral formula to evaluate

$$\oint_C \frac{4-3z}{z(z-1)(z-2)}\,dz$$

where C is the circle $|z| = 3/2$.

Weekly Assignment(CO2)

Q.5. Use Cauchy integral formula to evaluate

$\oint_C \frac{z}{z^2-3z+2}\, dz$, Where C is the circle $|z-2| = 1/2$.

Topic Objective(CO2)

We will discuss.

- Taylor's series
- Laurent's Expansion

Taylor's Series (CO2)

- If $f(z)$ is analytic inside a circle C at all points with its centre at the point a and radius R, then at each point z inside C
- $f(z) = f(a) + (z-a)f'(z) +$

$$\frac{(z-a)^2}{2!}f''(z)+\dots\dots\dots+\frac{(z-a)^n}{n!}f^n(z)+\cdots$$

OR

$f(z) = \sum_{n=0}^{\infty} a_n\,(z-a)^n$,where$a_n = \frac{f^n(a)}{n!}$.

Maclaurin’ s Series(CO2)

- If $f(z)$ is analytic on C_1 and C_2, and the annular region R bounded by the two concentric circle C_1 and C_2 of radius r_1 and r_2 $(r_2 < r_1)$ and with center at a then for all z in R
- $f(z) = a_0 + a_1(z-a) + a_2(z-a)^2 + \cdots + \frac{b_1}{(z-a)} + \frac{b_1}{(z-a)^2} + \cdots$

$$= \sum_{n=0}^{\infty} a_n (z-a)^n + \sum_{n=1}^{\infty} b_n (z-a)^{-n}$$

Daily Quiz(CO2)

Q1. Expand $f(z) = \frac{z}{(Z+1)(Z+2)}$ about z = -2.

l

Taylor's Series(CO2)

Example: Find the Taylor's and Laurent's series of $f(z) = \frac{-2z+3}{z^2-3z+2}$ with center at the origin.

Sol: $f(z) = \frac{-2z+3}{z^2-3z+2} = \frac{-2z+3}{(z-1)(z-2)} = \frac{-1}{(z-1)} - \frac{1}{(z-2)}$

For Taylor's expansion of f(z) we take $|z| < 1$, then

$$f(z) = (1-z)^{-1} + \frac{1}{2}\left(1 - \frac{z}{2}\right)^{-1}$$

$$= (1 + z + z^2 + \cdots \ldots \ldots . .) + \frac{1}{2}\left(1 + \frac{z}{2} + \frac{z^2}{2^2} + \cdots \ldots \ldots . .\right)$$

$$= \frac{3}{2} + \frac{5}{4}z + \frac{9}{8}z^2 + \ \ldots$$

which is the Taylor's expansion of f(z) about the origin.

Taylor's Series(CO2)

Next for Laurent's expansion of f(z) consider the annual region $1 \leq |z| \leq 2$, then

$$f(z) = \frac{-2z+3}{(z-1)(z-2)} = \frac{-1}{(z-1)} - \frac{1}{(z-2)}$$

$$= -\frac{1}{z}\left(1 + \frac{1}{z} + \frac{1}{z^2} + \cdots\right) + \frac{1}{2}\left(1 + \frac{z}{2} + \frac{z^2}{2^2} + \cdots\right)$$

$$= \left(\frac{1}{2} + \frac{z}{2^2} + \frac{z^2}{2^3} + \cdots\right) - \left(\frac{1}{z} + \frac{1}{z^2} + \frac{1}{z^3} + \cdots\right)$$

which is the Laurent's expansion about the origin.

Recap(CO2)

We have discussed:

- Taylor’s series
- Laurent’s expansion

Weekly Assignment(CO2)

Q.1. Obtain the Taylor's series expansion of $f(z) = \frac{1}{z^2-4z+3}$ about the point z = 4. Find its region of convergence.

Q.2. Expand $f(z) = \frac{z}{(Z+1)(Z+2)}$ about z=-2.

Topic Objective(CO2)

We will discuss.

- Zero and singularity
- Residue
- Cauchy’s Residue theorem

Zero of an Analytic Function(CO2)

A zero of an analytic function $f(z)$ is a value of z when $f(z) = 0$
If $f(z)$ is analytic in the neighbourhood of $z = a$,
then by Taylor's theorem,
$f(z) = a_0 + a_1(z-a) + a_2(z-a)^2 + \ldots + a_n(z-a)^n + \cdots$
Now, If $a_0 = a_1 = a_2 = \ldots\ldots\ldots\ldots = a_{n-1} = 0$ but $a_n \neq 0$,
then $f(z)$ is said to have a zero of order n at $z = a$.
The zero is said to be simple if $n = 1$.

$$\because \quad a_n = \frac{f^n(a)}{n!}$$

$\therefore$ For a zero of order m at $z = a$,
$f(a) = f'(a) = f''(a) = \ldots\ldots\ldots\ldots = f^{n-1}(a) = 0$ but $f^n(a) \neq 0$

Zero of an Analytic Function(CO2)

Thus $z = 1, 1/2, 1/3$,.......are all isolated singularities as there is no other singularity in their neighbourhood.

Thus $f(z) = a_n(z-a)^n + a_{n+1}(z-a)^{n+1}+\ldots$
$= (z-a)^n[a_n + a_{n+1}(z-a)+\ldots\ldots]$
$= (z-a)^n\phi(z)$.
Where, $\phi(z)$ is analytic and non zero at and in the neighbourhood of $z=a$.

Singularity(CO2)

Singularity:

A singularity of a function $f(z)$ is a point at which the function ceases to be analytic.

For example, if $f(z) = \dfrac{1}{z-3}$ then z=3 is a singularity of $f(z)$.

Type of Singularities:

1. Isolated Singularity: If $z = a$ is a singularity of $f(z)$ such that $f(z)$ is analytic at each point in its neighbourhood, then $z = a$ is called an isolated singularity.

Example: $f(z) = cot(\pi/z)$ is not anlytic where $tan(\pi/z) = 0$

$i.e.$ at the points $\pi/z=n\pi$ or $z=1/n$ (n=1,2,3,..........)

Singularity(CO2)

But when n is large, $z = 0$ is such a singularity that there are infinite number of other sigularities in its neighbourhood.

Thus $z = 0$ is a non−isolated singularity of $f(z)$.

The second term on RHS of Laurent′s series is called the Principal Part of $f(z)$ at the isolated singularity $z = a$.

2. Removable Singularity:

All $b_n{}'s$ are zero $\Rightarrow$ no term in Principal Part

Example: The function $f(z) = \frac{\sin(z-a)}{z-a}$ has removable singularity at $z = a$.

Singularity(CO2)

3. Essential Singularity:

Infinite number of terms in Principal Part

Example:

The function $f(z = \sin\frac{1}{z-a}$ has essential singularity at $z = a$.

Singularity(CO2)

4. Pole: Finite number of terms in Principal Part

If $z = a$ is a pole of order m of the function $f(z)$,

then $f(z)$

$$= \sum_{0}^{\infty} a_n (z-a)^n + \frac{b_1}{z-a} + \frac{b_2}{(z-a)^2} + \ldots\ldots\ldots\ldots\ldots + \frac{b_m}{(z-a)^m}$$

When m=1, pole is said to be simple, for m=2 it is a double pole.

Example: $f(z) = \frac{\sin(z-a)}{(z-a)^4}$ has a pole at $z = a$.

<u>NOTE:</u>

1. The limit point of the zeros of a function $f(z)$ is an isolated essential singularity.
2. The limit points of the poles of a function $f(z)$ is a non isolated essential singularity.

Detection of Singularity:

(a) Removable Singularity:

$\lim_{z \to a} f(z)$ exists and is finite.

(b) Pole:

$\lim_{z \to a} f(z) = \infty$

(c) Essential Singularity:

$\lim_{z \to a} f(z)$ does not exist.

Daily Quiz(CO2)

$$(a)\ \ f(z) = \frac{z - \sin z}{z^3} \ \text{ at } z = 0.$$

$$(b)\ \ f(z) = \frac{1}{1 - e^z} \ \text{ at } z = 2\pi i.$$

$$(c)\ \ f(z) = \frac{e^{1/z}}{z^2} \ \text{ at } z = 0.$$

Weekly Assignment(CO2)

Q. 1 Discuss the nature of singularity of:

$(a)\ \ f(z) = \dfrac{z - \sin z}{z^3}$ at $z = 0$. $Ans: removable$

$(b)\ \ f(z) = \dfrac{1}{1 - e^z}$ at $z = 2\pi i$. Ans: simple pole

$(c)\ \ f(z) = \dfrac{e^{1/z}}{z^2}$ at $z = 0$. $Ans: essential$

$(d)\ \ f(z) = \frac{1}{\cos z - \sin z}$ at $z = \pi/4$. Ans: simple pole

$(e)\ \ f(z) = \dfrac{e^{2z}}{(z-1)^4}$. Ans: Pole of 4th order

Residues:

If the function $f(z)$ is analytic inside and on a simple closed contour C, then by Cauchy integral theorem, we have

$$\oint_C f(z)dz = 0.$$

When $f(z)$ has one or more isolated singular points inside C, then the Cauchy integral theorem cannot be used.

Each of these isolated singular points inside C contributes to the value of complex integral.

These contributions are called $residues$.

Residue(CO2)

Residue at Pole:

Suppose a single valued function $f(z)$ has a pole of order $m\ at\ z = a$, then

$$f(z) = \sum_{n=0}^{\infty} a_n(z-a)^n + \sum_{n=1}^{m} b_n(z-a)^{-n}\ )$$

Where $b_n = \frac{1}{2\pi i}\int_C \frac{f(z)dz}{(z-a)^{-n+1}}$ $\quad C: |z-a| = r$

Particularly, $b_1 = \dfrac{1}{2\pi i}\displaystyle\int_C f(z)dz$

The coefficient b_1 is called the residue of $f(z)$ at the pole $z = a$ and is denoted by the symbol Res(z=a).

Residue(CO2)

Method of Finding out Residues:

(1) If $f(z)$ has a simple pole at $z=a$, then

$$\text{Res}[f(z)]_{z=a} = \lim_{z\to a}(z-a)f(z).$$

(2) If $f(z)$ has a pole of order m at $z=a$, then

$$\text{Res}[f(z)]_{z=a} = \frac{1}{(m-1)!}\lim_{z\to a}\frac{d^{m-1}}{dz^{m-1}}[(z-a)^m f(z)].$$

(3) If $f(z)$ is of the form $f(z = \frac{\phi(z)}{\varphi(z)}$ has a simple pole at $z=a$,then

$$\text{Res}[f(z)]_{z=a} = \frac{\phi(a)}{\varphi'(a)}$$

Residue(CO2)

(4) Residue of $f(z)$ at $z=\infty$

$$= \lim_{z \to \infty} \{-zf(z)\}$$

or

$$= -\left[\text{coefficient of } \frac{1}{z} \text{ in the expansion of } f(z) \text{ for values of } z \text{ in the neighbourhood of } z=\infty\right]$$

Daily Quiz(CO2)

- Determine the poles of the function $f(z) = \frac{12z-7}{(z-1)^2(2z+3)}$ and the residue at each pole. Hence evaluate $\int_C f(z)dz$ where C is $|z| = 2$.

Cauchy's Residue Theorem(CO2)

Let $f(z)$ be one valued and analytic within and on a closed contour C except at a finite number of poles $z_1, z_2, z_3, \ldots, z_n$ and let $R_1, R_2, R_3, \ldots\ldots\ldots\ldots, R_n$ be respectively the residues of $f(z)$ at these poles, then

$$\int_C f(z)dz = 2\pi i(R_1 + R_2 + R_3 + \ldots\ldots\ldots\ldots + R_n)$$

$$= 2\pi \text{i(Sum of residues at the poles within } C).$$

Recap(CO2)

We have discussed.

- Zero and singularity
- Residue
- Cauchy's Residue theorem

Weekly Assignment(CO2)

Q1. Determine the poles of the function $f(z) = \dfrac{z^2}{(z-1)^2(z+2)}$ and the residue at each pole. Hence evaluate $\int_C f(z)dz$ where C is $|z| = 3$. [UPTU 2015] **Ans:**2πi

Q2. Determine the poles of the function $f(z) = \dfrac{12z-7}{(z-1)^2(2z+3)}$ and the residue at each pole. Hence evaluate $\int_C f(z)dz$ where C is $|z| = 2$. [GBTU 2011] **Ans:**0

Q3. Evaluate $\int_c \dfrac{24z-7}{(z-1)^2(2z+3)}$dz where c is the circle of radius 2 with center at the origin. [UPTU 2012] **Ans :** 0

Weekly Assignment(CO2)

Q4. Determine the poles of the function $f(z) = \dfrac{z-3}{z^2+2z+5}$ and the residue at each pole. Hence evaluate $\int_C f(z)dz$ where C is $|z+1-i| = 2$. [GBTU 2013] **Ans:**$\pi(\text{i}-2)$

Q5 Find the poles of the function $f(z) = \dfrac{1-2z}{z(z-1)(z-2)^2}$ and the residue at each pole. [AKTU 2017]

Ans: poles z = 0,1(order 1), 2(order 2). Residues: $-\dfrac{1}{4}, -1, \dfrac{5}{4}$

Topic Objective(CO2)

We are going to discuss:

- Evaluation of real integrals using residue theorem.

Evaluation of Real Integral(CO2)

Integration Round the unit circle of the type $\int_0^{2\pi} f(\cos\theta, \sin\theta)\, d\theta$:

Where, $f(\cos\theta, \sin\theta)$ is a rational function of $\cos\theta$ and $\sin\theta$.

Consider a circle of unit radius with centre at origin, as contour.

$\sin\theta = \frac{1}{2i}\left(z - \frac{1}{z}\right)$, $\cos\theta = \frac{1}{2}\left(z + \frac{1}{z}\right)$

Where z $= re^{i\theta} = 1.\, e^{i\theta} = e^{i\theta}$.

As we know $z = e^{i\theta}, dz = ie^{i\theta} d\theta = izd\theta$

or, $d\theta = \frac{dz}{iz}$.

The integration is converted into a function of z .Then apply Cauchy's residue theorem to evaluate the integral.

Weekly Assignment(CO2)

Q.1. Evaluate the integral $\int_0^{2\pi} \frac{d\theta}{5-3\cos\theta}$.

Q.2. Evaluate the integral $\int_0^{2\pi} \frac{d\theta}{a+b\sin\theta}$ if $a > |b|$.

Q.3. Using contour integration, Evaluate $\int_0^{2\pi} \frac{d\theta}{a+b\cos\theta}$

Where $a > |b|$ hence or otherwise Evaluate $\int_0^{\pi} \frac{d\theta}{a+b\cos\theta}$

Evaluation of Real Integrals (CO2)

Such integral can be reduced the contour integral if

(1) $f_2(x)$ has no real roots .

(2) The degree of $f_2(x)$ is greater than that of $f_1(x)$ by at least two.

Procedure:

Let $f(x) = \frac{f_1(x)}{f_2(x)}$

Consider $f_2(x) = \int_C f(z)dz$.

Evaluation of Real Integrals (CO2)

Where C is a curve , consisting of the upper half C_R of the circle $|z| = R$ and part of the real axis from –R to R.

If there are no poles of $f(z)$ on the real axis , the circle $|z| = R$ which is arbitrary can be taken such that there is no singularity on its circumference C_R in the upper half of the plane, but possibly some poles inside the contour C specified

above.

Using Cauchy's Residue theorem, we have

$\int_C f(z)dz = 2\pi i$ (sum of residue of $f(z)$ at the poles within C)

i.e. $\int_{-R}^{R} f(x)dx + \int_{C_R} f(z)dz = 2\pi i$(sum of residue of $f(z)$ at the poles within C)

Evaluation of Real Integrals (CO2)

or, $\int_{-R}^{R} f(x)dx = -\int_{C_R} f(z)dz + 2\pi i$ (sum of residue of $f(z)$ at the poles within C)

$\therefore \lim\limits_{R\to\infty} \int_{-R}^{R} f(x)dx = -\lim\limits_{R\to\infty} \int_{C_R} f(z)dz + 2\pi i$

(sum of residue of $f(z)$ at the poles within C) …………(1)

Now, $\lim\limits_{R\to\infty} \int_{C_R} f(z)dz = \int_0^{\pi} f(Re^{i\theta})\, \mathrm{R}ie^{i\theta} \mathrm{d}\theta = 0$

$\therefore \int_{-\infty}^{\infty} f(x)dx = 2\pi i$ (sum of residue within C)

Recap(CO2)

We have discussed.

- Evaluation of real integrals using residue theorem.

Weekly Assignment(CO2)

Q.1. Apply calculus of residues to prove that

$$\int_0^\infty \frac{dx}{(x^2+a^2)^2} = \frac{\pi}{4a^3}; \mathrm{a} > 0$$

Q.2. Evaluate $\int_0^\infty \frac{\cos mx}{(x^2+1)} dx$

Q.3. Evaluate $\int_0^\infty \frac{\sin x}{x} dx$

Q.4. Evaluate $\int_{-\infty}^\infty \frac{x \sin \pi x}{(x^2+2x+5)} dx$

Q.5. Evaluate $\int_{-\infty}^\infty \frac{x^2-x+2}{(x^4+10x^2+9)} dx$

Video Links (CO2)

Video Links:

- https://www.youtube.com/playlist?list=PLzJaFd3A7DZuyLLbmVpb9e9V3Q9cYBL
- https://www.youtube.com/playlist?list=PLbMVogVj5nJS_i8vfVWJG16mPcoEKMuWT
- https://youtu.be/b5VUnapu-qs
- https://youtu.be/b5VUnapu-qs
- https://youtu.be/yV_v6zxADgY
- https://youtu.be/2ZBcbFhrfOg
- https://youtu.be/dlK0E0OG39k
- https://youtu.be/qjpLIlVo_6E

MCQ(CO2)

Q1. $\int_0^{2+i}(x^2+iy)dz$ along the path $y = x$ is equal to

(a) $\left(\frac{2}{3}+\frac{14}{3}i\right)$ (b) $\left(\frac{3}{2}+\frac{3}{14}i\right)$ (c) $\left(\frac{2}{3}-\frac{14}{3}i\right)$ (d) $\left(\frac{3}{2}-\frac{4}{14}i\right)$

Ans. (a)

Q2. If there is no pole inside and on the contour, then the value of integral is

(a) ∞ (b) 0 (c) –1 (d) 1

Ans. (b)

Glossary Question(CO2)

Q1. Pick the correct option from the glossary:

(i) $\cos z$

(ii) $\sin z$

(iii) $\log(1+z)$

(iv) $\tan z$

A. $z - z^2/2 + z^3/3 - \ldots$.

B. $1 - z^2/2! + z^4/4! - \ldots$.

C. $z - z^3/3! + z^5/5! - \ldots$.

D. $z + z^3/3 + 2z^5/15 + \ldots$.

First Sessional Paper

Printed page:2 **Subject Code: AAS0301A**

Roll No:

<u>NOIDA INSTITUTE OF ENGINEERING AND TECHNOLOGY, GREATER NOIDA</u>

(An Autonomous Institute)

Affiliated to Dr. A.P. J. Abdul Kalam Technical University, Uttar Pradesh, Lucknow

Course: B.Tech **Branch: CSE/IT/CS**

Semester: III **Sessional Examination: I** **Year: (2020-2021)**

Subject Name: Eng. Maths III

Time: 1.15Hours **[SET-1]** **Max. Marks:30**

<u>General Instructions:</u>

- This Question paper consists of 2 pages & 5 questions. It comprises of three Sections, A, B, and C.
- **<u>Section A</u>** -Question No- 1 is objective type questions carrying 1 mark each, Question No- 2 is very short answer type carrying 2 mark each. You are expected to answer them as directed.
- **<u>Section B</u>** - Question No-3 is short answer type questions carrying 5 marks each. You need to attempt any two out of three questions given.
- **<u>Section C</u>** -Question No. 4 & 5 Long answer type (within unit choice) questions carrying 6 marks each. You need to attempt any one-part *a* or *b*.

First Sessional Paper

- No sheet should be left blank. Any written material after a blank sheet will not be evaluated/checked.
- **Blooms Level:** *K1: Remember, K2: Understand, K3: Apply, K4: Analyze, K5: Evaluate, K6: Create*

		SECTION – A	**[8]**	**CO**	**Blooms level**
1.	**Attempt all parts**		**(4×1=4)**	**CO**	
	a.	$\lim_{z\to 0} \frac{z}{\bar{z}}$ (i) Limit exists (ii) Limit does not exist (iii) Limit exists and equal to 1 (iv) None of these	**(1)**	**1**	**K5**
	b.	If $f(z) = \frac{z}{z^2+9}$ then (i) $f(z)$ is continuous (ii) $f(z)$ is discontinuous at $z = \pm 3i$ (iii) $\lim_{z\to i} \frac{z}{z^2+9} = -\frac{i}{8}$ (iv) Both B & C	**(1)**	**1**	**K2**
	c.	Function $f(z) = z\|z\|$ is (i) Analytic anywhere (ii) Not analytic anywhere (ii) Harmonic (iv) None of these	**(1)**	**1**	**K3**

First Sessional Paper

- No sheet should be left blank. Any written material after a blank sheet will not be evaluated/checked.
- **Blooms Level:** *K1: Remember, K2: Understand, K3: Apply, K4: Analyze, K5: Evaluate, K6: Create*

		SECTION – A	**[8]**	**CO**	**Blooms level**
1.	**Attempt all parts**		**(4×1=4)**	**CO**	
	a.	$\lim_{z\to 0} \frac{z}{\bar{z}}$ (i) Limit exists (ii) Limit does not exist (iii) Limit exists and equal to 1 (iv) None of these	**(1)**	**1**	**K5**
	b.	If $f(z) = \frac{z}{z^2+9}$ then (i) $f(z)$ is continuous (ii) $f(z)$ is discontinuous at $z = \pm 3i$ (iii) $\lim_{z\to i} \frac{z}{z^2+9} = -\frac{i}{8}$ (iv) Both B & C	**(1)**	**1**	**K2**
	c.	Function $f(z) = z\lvert z\rvert$ is (i) Analytic anywhere (ii) Not analytic anywhere (ii) Harmonic (iv) None of these	**(1)**	**1**	**K3**

First Sessional Paper

	d.	There exists no analytic function $f(z)$ if (i) $real\ f(z) = y - 2x$ (ii) $real\ f(z) = y^2 - 2x$ (ii) $real\ f(z) = y^2 - x^2$ (iv) $real\ f(z) = y - x$	**(1)**	**1**	**K2**
2.	**Attempt all parts**		**(2×2=4)**	**CO**	
	a.	Show that if $f(z)$ is analytic and $Imf(z) = constant$ then $f(z)$ is constant.	**(2)**	**1**	**K3**
	b.	Find the bilinear transformation which maps the points $z = 0,1,\infty$ into the points $w = i, -1, -i$ respectively.	**(2)**	**1**	**K5**
		SECTION – B			
3.	**Answer any two of the following-**		**[2×5=10]**	**CO**	
	a.	Examine the nature of the function $f(z) = \frac{x^3y(y-ix)}{x^6+y^2}$, $z \neq 0, f(0) = 0$, prove that $\frac{f(z)-f(0)}{z} \to 0$ as $z \to 0$ along any radius vector but not as $z \to 0$ in any manner and also that $f(z)$ is not analytic at $z = 0$.	**(5)**	**1**	**K4**
	b.	Find the image of $\|z - 1\| = 1$ under the transformation $w = \frac{1}{z}$.	**(5)**	**1**	**K5**
	c.	Show that $f(z) = \cos z$ is analytic in entire complex plane.	**(5)**	**1**	**K3**

First Sessional Paper

		SECTION – C			
4	**Answer any one of the following-**		**[2×6=12]**	**CO**	
	a.	Determine an analytic function $f(z)$ in terms of z whose real part is $\frac{\sin 2x}{\cosh 2y - \cos 2x}$.	**(6)**	**1**	**K5**
	b.	If $w = \varphi + i\psi$ represent the complex potential for an electric field and $\psi = x^2 - y^2 + \frac{x}{x^2+y^2}$. Determine the function φ.	**(6)**	**1**	**K5**
5.	**Answer any one of the following-**				
	a.	Determine an analytic function $f(z)$ in terms of z if $3u + v = 3\sin x \cos hy + \cos x . \sin hy$.	**(6)**	**1**	**K5**
	b.	Find an analytic function $f(z)$ in terms of z if $Re[f'(z)] = 3x^2 - 4y - 3y^2$ and $f(1+i) = 0$ & $f'(0) = 0$.	**(6)**	**1**	**K5**

Second Sessional Paper

Printed page:2 **Subject Code: AAS0301A**

Roll No:

NOIDA INSTITUTE OF ENGINEERING AND TECHNOLOGY, GREATER NOIDA

(An Autonomous Institute)

Affiliated to Dr. A.P. J. Abdul Kalam Technical University, Uttar Pradesh, Lucknow

Course: B.Tech **Branch: CSE/IT/CS**

Semester: III **Sessional Examination: II** **Year: (2020-2021)**

Subject Name: Eng. Maths III

Time: 1.15Hours **[SET-1]** **Max. Marks:30**

General Instructions:

- This Question paper consists of 2 pages & 5 questions. It comprises of three Sections, A, B, and C.
- **Section A** -Question No- 1 is objective type questions carrying 1 mark each, Question No- 2 is very short answer type carrying 2 mark each. You are expected to answer them as directed.
- **Section B** - Question No-3 is short answer type questions carrying 5 marks each. You need to attempt any two out of three questions given.
- **Section C** -Question No. 4 & 5 Long answer type (within unit choice) questions carrying 6 marks each. You need to attempt any one-part *a* or *b*.

Second Sessional Paper

1	Attempt all parts		(4×1=4)	
	a.	$\int_0^{2+i}(x^2+iy)dz$ along the path $y=x$ is equal to $(i)\ \left(\frac{2}{3}+\frac{14}{3}i\right)$ $(ii)\ \left(\frac{3}{2}+\frac{3}{14}i\right)$ $(iii)\ \left(\frac{2}{3}-\frac{14}{3}i\right)$ (iv) None of these	(1)	CO2
	b.	Residue of $z\cos(1/z)$ at $z = 0$ is (i) 0 (ii) 1 (iii) $-1/2$ (iv) $1/2$	(1)	CO2
	c.	The region of validity for Taylor's series about $z=0$ of the function e^z is (i) $\|z\|=0$ (ii) $\|z\|<1$ (iii) $\|z\|>1$ (iv) $\|z\|<\infty$	(1)	CO2
	d.	If $f(z)=\frac{\sin z}{z^4}$, then $z=0$ is (i) Removable singularity (ii) Pole of order 4 (iii) Pole of order 3 (iv) None of these	(1)	CO2

Second Sessional Paper

2.	**Attempt all parts**		**(2×2=4)**	
	a.	State Cauchy Integral formula.	**(2)**	**CO2**
	b.	Evaluate the integral $\int_C \|z\|\, dz$ where C is the left half of the unit circle $\|z\| = 1$ from $z = -i$ to $z = i$.	**(2)**	**CO2**
SECTION – B			**[10 Marks]**	
3.	**Answer any two of the following-**		**(2×5=10)**	
	a.	Verify Cauchy integral theorem for $f(z) = z^2$ taken over the boundary of square with vertices $1 \pm i, -1 \pm i$.	**(5)**	**CO2**
	b.	Using Cauchy integral formula, evaluate $\int_C \frac{z^2+1}{z^2-1} dz$ where C is circle (i) $\|z\| = 3/2$ (ii) $\|z-1\| = 1$	**(5)**	**CO2**
	c.	Evaluate $\int_C \frac{1}{z^2(z^2-4)e^z} dz$ where C is $\|z\| = 1$.	**(5)**	**CO2**

Second Sessional Paper

SECTION – C			**[12 Marks]**	
4	**Answer any one of the following-**		**(1×6=6)**	
	a.	Expand $f(z) = \frac{1}{(z+1)(z+3)}$ $(i)\quad \lvert z\rvert < 1 \qquad\qquad (ii)\quad 1 < \lvert z\rvert < 3$	**(6)**	**CO2**
	b.	State & Prove Cauchy Residue Theorem.	**(6)**	**CO2**
5.	**Answer any one of the following-**		**(1×6=6)**	
	a.	Evaluate $\int_0^{2\pi} \frac{1}{5+4\cos\theta}\, d\theta$ using contour integration.	**(6)**	**CO2**
	b.	Prove that $\int_0^{\infty} \frac{dx}{(x^2+1)^2} = \frac{\pi}{4}$ using contour integration.	**(6)**	**CO2**

Third Sessional Paper

Printed page:2 **Subject Code: AAS0301A**

Roll No:

NOIDA INSTITUTE OF ENGINEERING AND TECHNOLOGY, GREATER NOIDA

(An Autonomous Institute)

Affiliated to Dr. A.P. J. Abdul Kalam Technical University, Uttar Pradesh, Lucknow

Course: B.Tech **Branch: CSE/IT/CS**

Semester: III **Sessional Examination: III** **Year: (2021-2022)**

Subject Name: Eng. Maths III

Time: 1.15 Hours **[SET-2]** **Max. Marks:30**

General Instructions:

- This Question paper consists of 2 pages & 5 questions. It comprises of three Sections, A, B, and C.
- **Section A** -Question No- 1 is objective type questions carrying 1 mark each, Question No- 2 is very short answer type carrying 2 mark each. You are expected to answer them as directed.
- **Section B** - Question No-3 is short answer type questions carrying 5 marks each. You need to attempt any two out of three questions given.
- **Section C** -Question No. 4 & 5 Long answer type (within unit choice) questions carrying 6 marks each. You need to attempt any one-part *a* or *b*.

Blooms Level: K1: Remember, K2: Understand, K3: Apply, K4: Analyze, K5: Evaluate, K6: Create

Third Sessional Paper

		SECTION – A	[8 Marks]	CO	Blooms level
1	**Attempt all parts**		**(4×1=4)**		
	a.	The solution of PDE $(D+4D'+5)^2 z = 0$ is (i) $z = e^{-5x} f_1(y-4x) + xe^{-5x} f_2(y-4x)$ (ii) $z = e^{-5x} f_1(y+4x) + xe^{-5x} f_2(y+4x)$ (iii) $z = e^{5x} f_1(y+4x) + xe^{5x} f_2(y+4x)$ (iv) None of these	**(1)**	**CO3**	**K5**
	b.	PDE: $Bu_{xx} + Au_{xy} + Cu_{yy} + f(x,y,u,u_x,u_y) = 0$ is elliptic if __________	**(1)**	**CO3**	**K4**
	c.	While solving a PDE using a Variable Separable method, we equate the ratio to a Constant which? (i) Can be Positive or Negative Integer or Zero (ii) Can be Positive or Negative rational number or Zero (iii) Must be a Positive Integer (iv) Must be a Negative Integer	**(1)**	**CO3**	**K1**
	d.	$\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = 0$ is two-dimensional heat equation in ________state.	**(1)**	**CO3**	**K1**

Third Sessional Paper

2.	**Attempt all parts**		**(2×2=4)**		
	a.	Find the P.I. of $(D^2 - 2DD')z = \sin x.\cos 2y$	**(2)**	**CO3**	**K5**
	b.	Classify the PDE: $yu_{xx} + (x+y)u_{xy} + xu_{yy} = 0$ about the line $y = x$.	**(2)**	**CO3**	**K4**
SECTION – B			**[10 Marks]**		
3.	**Answer any two of the following-**		**[2×5=10]**		
	a.	Solve the PDE $4\frac{\partial u}{\partial x} + \frac{\partial u}{\partial y} = 3u$ subject to the condition $u(0,y) = 4e^{-y} - e^{-5y}$ by method of separation of variables.	**(5)**	**CO3**	**K5**
	b.	Solve the PDE: $(D^2 + DD' - 6D'^2)z = y \sin x$	**(5)**	**CO3**	**K5**
	c.	Solve the PDE: $\left(D^2 - D'^2 - 3D + 3D'\right)z = xy + e^{x+2y}$	**(5)**	**CO3**	**K5**

Third Sessional Paper

SECTION – C			**[12 Marks]**		
4	**Answer any one of the following-**		**[2×6=12]**		
	a.	A tightly stretched string with fixed end points $x = 0$ and $x = l$ is initially in a position is given by $y = y_0\, sin^3 \frac{\pi x}{l}$. If it released from rest from this position, find the displacement $y(x, t)$.	**(6)**	**CO3**	**K5**
	b.	Solve the PDE $\frac{\partial^2 u}{\partial y^2} + \frac{\partial u}{\partial x} = 0$ subject to the condition: $u(x, 0) = 0, u(x, \pi) = 0, u(0, y) = 4 \sin 3y$ by method of separation of variables.	**(6)**	**CO3**	**K5**
5.	**Answer any one of the following-**				
	a.	Find the temperature of the bar of length 2 whose ends are kept at zero and internal surface insulated by if the initial temperature is $sin\frac{\pi x}{2} + 3 \sin\frac{5\pi x}{2}$.	**(6)**	**CO3**	**K5**
	b.	Find the solution of Laplace equation subject to the condition: $u(0, y) = u(1, y) = u(x, 0) = 0, u(x, 1) = 100 \sin \pi\, x$	**(6)**	**CO3**	**K5**

Expected Questions (CO2)

Q1. If $f(z)$is a regular function of z, prove that

$$\left(\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial y^2}\right)|f(z)|^2=4|f'(z)|^2.$$

Q2. If $u-v=(x-y)(x^2+4xy+y^2)$ and $f(z)$ is an analytic function . Find $f(z)$ in terms of z.

Q3. Find the bilinear transformation which maps the points $z=0,1,\infty$ into the points $w=\ i,-1,-i$ respectively.

Q4. Consider the transformation $z=\sqrt{2}e^{\frac{i\pi}{4}}z$ and determine the region R' of w –plane corresponding to rectangular region R bounded by the lines $x=0,y=0\ ,x=2$ and $y=1$ in z-plane.

Expected Questions(CO2)

Q5. Expand $\frac{1}{(z+1)(z+3)}$ in the region $|z| < 1$.

Q6. Use Cauchy integral formula to evaluate $\oint_c \frac{\sin\pi z^2+\cos\pi z^2}{(z-1)(z-2)}\,dz$ where C is the circle $|z| = 3$.

Q7. Evaluate $\int_C \frac{e^{2z}}{(z^2-3z+2)}\,dz$ where C is the circle $|z| = 3$.

Q8. Evaluate $\int_0^\infty \frac{\cos mx}{(x^2+1)}\,dx$.

Q9. Evaluate $\int_0^\infty \frac{\sin x}{x}\,dx$.

Recapof Unit(CO2)

We discussed following points in this unit.

- Limit, Continuity and differentiability
- Functions of complex variable,
- Analytic functions
- Cauchy- Riemann equations (Cartesian and Polar form),
- Harmonic function
- Method to find Analytic functions
- Conformal mapping
- Mobius transformation and their properties.

References

- Erwin Kreyszig, Advanced Engineering Mathematics, 9th Edition, John Wiley & Sons, 2006.
- P. G. Hoel, S. C. Port and C. J. Stone, Introduction to Probability Theory, Universal Book Stall, 2003(Reprint).
- S. Ross: A First Course in Probability, 6th Ed., Pearson Education India, 2002.
- W. Feller, An Introduction to Probability Theory and its Applications, Vol. 1, 3rd Ed., Wiley, 1968.

References

- R.K. Jain and S.R.K. Iyenger: Advance Engineering Mathematics; Narosa Publishing House, New Delhi.
- B.S. Grewal, Higher Engineering Mathematics, Khanna Publishers, 35th Edition, 2000. 2.T.Veerarajan : Engineering Mathematics (for semester III), Tata McGraw-Hill, New Delhi.
- J.N. Kapur: Mathematical Statistics; S. Chand & Sons Company Limited, New Delhi.
- D.N.Elhance,V. Elhance & B.M. Aggarwal: Fundamentals of Statistics; Kitab Mahal Distributers, New Delhi.

Thank You