Propositional Logic

UNIT-4

Discrete Structures

B.Tech (CSE)

IIIrd Sem

RAHUL KUMAR
Assistant Professor
B.Tech CSE

EVALUATION SCHEME SEMESTER-III

Sl. No.	Subject Codes	Subject Name	Periods			Evaluation Schemes				End Semester		Total	Credit
			L	T	P	CT	TA	TOTAL	PS	TE	PE		
WEEKS COMPULSORY INDUCTION PROGRAM													
1	AAS0303	Statistics and Probability	3	1	0	30	20	50		100		150	4
2	**ACSE0306**	**Discrete Structures**	**3**	**0**	**0**	**30**	**20**	**50**		**100**		**150**	**3**
3	ACSE0305	Computer Organization & Architecture	3	0	0	30	20	50		100		150	3
4	ACSE0302	Object Oriented Techniques using Java	3	0	0	30	20	50		100		150	3
5	ACSE0301	Data Structures	3	1	0	30	20	50		100		150	4
6	ACSDS0301	Foundations of Data Science	3	0	0	30	20	50		100		150	3
7	ACSE0352	Object Oriented Techniques using Java Lab	0	0	2				25		25	50	1
8	ACSE0351	Data Structures Lab	0	0	2				25		25	50	1
9	ACSDS0351	Data Analysis Lab	0	0	2				25		25	50	1
10	ACSE0359	Internship Assessment-I	0	0	2				50			50	1
11	ANC0301 / ANC0302	Cyber Security* / Environmental Science*(Non Credit)	2	0	0	30	20	50		50		1000	
12		MOOCs (For B.Tech. Hons. Degree)											
		GRAND TOTAL										**1100**	**24**

Subject Syllabus

B. TECH. SECOND YEAR (3rd Semester))-CSE/IT/CS/M.Tech. Integrated/Data Science/AI/AI-ML/IoT					
Course code		L	T	P	Credits
Course title	DISCRETE STRUCTURES	3	0	0	3

Course objective:

The subject enhances one's ability to develop logical thinking and ability to problem solving. The objective of discrete structure is to enables students to formulate problems precisely, solve the problems, apply formal proofs techniques and explain their reasoning clearly.

Pre-requisites:

1. Basic Understanding of mathematics
2. Basic knowledge algebra.
3. Basic knowledge of mathematical notations

Subject Syllabus

Course Contents / Syllabus		
Unit 1	**Set Theory, Relation, Function**	**8 Hours**
Set Theory: Introduction to Sets and Elements, Types of sets, Venn Diagrams, Set Operations, Multisets, Ordered pairs. Proofs of some general Identities on sets. **Relations:** Definition, Operations on relations, Pictorial Representatives of Relations, Properties of relations, Composite Relations, Recursive definition of relation, Order of relations. **Functions:** Definition, Classification of functions, Operations on functions, Growth of Functions. **Combinatorics:** Introduction, basic counting Techniques, Pigeonhole Principle. **Recurrence Relation & Generating function**: Recursive definition of functions, Recursive Algorithms, Method of solving Recurrences. **Proof techniques:** Mathematical Induction, Proof by Contradiction, Proof by Cases, Direct Proof.		
Unit 2	**Algebraic Structures**	**8 Hours**
Algebraic Structures: Definition, Operation, Groups, Subgroups and order, Cyclic Groups, Cosets, Lagrange's theorem, Normal Subgroups, Permutation and Symmetric Groups, Group Homomorphisms, Rings, Internal Domains, and Fields.		

Subject Syllabus

Unit 3	Lattices and Boolean Algebra	8 Hours

Ordered set, Posets, Hasse Diagram of partially ordered set, Lattices: Introduction, Isomorphic Ordered set, Well ordered set, Properties of Lattices, Bounded and Complemented Lattices, Distributive Lattices.

Boolean Algebra: Introduction, Axioms and Theorems of Boolean Algebra, Algebraic Manipulation of Boolean Expressions, Simplification of Boolean Functions.

Unit 4	Propositional Logic	8 Hours

Propositional Logic: Introduction, Propositions and Compound Statements, Basic Logical Operations, Well-formed formula, Truth Tables, Tautology, Satisfiability, Contradiction, Algebra of Proposition, Theory of Inference.

Predicate Logic: First order predicate, Well-formed formula of Predicate, Quantifiers, Inference Theory of Predicate Logic.

Subject Syllabus

Unit 5	Tree and Graph	8 Hours

Trees: Definition, Binary tree, Complete and Extended Binary Trees, Binary Tree Traversal, Binary Search Tree.

Graphs: Definition and terminology, Representation of Graphs, Various types of Graphs, Connectivity, Isomorphism and Homeomorphism of Graphs, Euler and Hamiltonian Paths, Graph Coloring

Course outcome: After completion of this course students will be able to:

Unit 1	Apply the basic principles of sets, relations & functions and mathematical induction in computer science & engineering related problems.	K3
Unit 2	Understand the algebraic structures and its properties to solve complex problems.	K2
Unit 3	Describe lattices and its types and apply Boolean algebra to simplify digital circuit.	K2, K3
Unit 4	Infer the validity of statements and construct proofs using predicate logic formulas.	K3, K5
Unit 5	Design and use the non-linear data structure like tree and graphs to solve real world problems.	K3, K6

Application in CSE

1. Discrete Structures are useful in studying and describing objects and problems in branches of computer science such as computer algorithms, programming languages.
2. Computer implementations are significant in applying ideas from discrete mathematics to real-world problems, such as in operations research.
3. It is a very good tool for improving reasoning and problem-solving capabilities.
4. Discrete mathematics is used to include theoretical computer science, which is relevant to computing.
5. Discrete structures in computer science with the help of process algebras.

Course Objective

- A course discrete structures used to represent discrete objects and relationships between these objects. These discrete structures include sets, relation, permutations, relations, graphs and trees etc.
- The subject enhances one's ability to develop logical thinking and ability to problem solving.

Course Outcome

Course Outcome (CO)	At the end of course , the student will be able to understand	Bloom's Knowledge Level (KL)
CO1	Apply the basic principles of sets, relations & functions in computer science & engineering related problems and to solve counting problem using recursive function theory.	K1,K3,K2
CO2	Define the algebraic structure of a system and use these concepts such as coding theory, cryptographic algorithms etc.	K1,K2
CO3	Apply basics of lattices in mathematical modeling	K2,K3
CO4	Infer the validity of statements and construct proofs using predicate logic formulas.	K3,K4
CO5	Design and use the non-linear data structure like tree and graph for circuit and network designing	K2,K6

CO-PO's and PSO's Mapping

	PO 1	PO 2	PO 3	PO 4	PO 5	PO 6	PO 7	PO 8	PO 9	PO10	PO11	PO12
20CS304.1	3	2	2	-	-	1	-	-	-	1	1	3
20CS304.2	3	3	2	2	1	-	-	-	-	-	2	1
20CS304.3	3	3	2	1	-	-	3	-	2	2	2	2
20CS304.4	3	3	2	1	-	-	1	-	-	3	2	2
20CS304.5	3	3	2	1	-	-	3	-	-	1	3	3

	PSO1	PSO2	PSO3	PSO4
20CS304.1	1	2	3	-
20CS304.2	1	2	3	1
20CS304.3	3	2	3	1
20CS304.4	2	3	3	-
20CS304.5	2	3	2	

End Semester Question Paper Templates (Offline Pattern/Online Pattern

Printed page: …. **Subject Code: ……………………**

Roll No: | | | | | | | | | | | | | |

<u>NOIDA INSTITUTE OF ENGINEERING AND TECHNOLOGY ,GREATER NOIDA</u>

(An Autonomous Institute Affiliated to AKTU, Lucknow)

B.Tech/B.Voc./MBA/MCA/M.Tech (Integrated)

(SEM: ….. THEORY EXAMINATION (2020-2021)

Subject ……….

Time: 3 Hours **Max. Marks:100**

<u>General Instructions:</u>

- All questions are compulsory. Answers should be brief and to the point.
- This Question paper consists of ………..pages & …8………questions.
- It comprises of three Sections, A, B, and C. You are to attempt all the sections.
- <u>Section A</u> -Question No- 1 is objective type questions carrying 1 mark each, Question No- 2 is very short

End Semester Question Paper Templates (Offline Pattern/Online Pattern

- **Section B** - Question No-3 is Long answer type -I questions with external choice carrying 6 marks each. You need to attempt any five out of seven questions given.
- **Section C** - Question No. 4-8 are Long answer type -II (within unit choice) questions carrying 10 marks each. You need to attempt any one part *a or b.*
- Students are instructed to cross the blank sheets before handing over the answer sheet to the invigilator.
- No sheet should be left blank. Any written material after a blank sheet will not be evaluated/checked.

End Semester Question Paper Templates (Offline Pattern/Online Pattern

		SECTION – A		CO
1.	**Attempt all parts-**		**[10×1=10]**	
	1-a.	**Question-**	**(1)**	
	1-b.	**Question-**	**(1)**	
	1-c.	**Question-**	**(1)**	
	1-d.	**Question-**	**(1)**	
	1-e.	**Question-**	**(1)**	
	1-f.	**Question-**	**(1)**	
	1-g.	**Question-**	**(1)**	
	1-h.	**Question-**	**(1)**	
	1-i.	**Question-**	**(1)**	
	1-j.	**Question-**	**(1)**	

End Semester Question Paper Templates (Offline Pattern/Online Pattern

2.	Attempt all parts-		[5×2=10]	CO
	2-a.	Question-	(2)	
	2-b.	Question-	(2)	
	2-c.	Question-	(2)	
	2-d.	Question-	(2)	
	2-e.	Question-	(2)	

End Semester Question Paper Templates (Offline Pattern/Online Pattern

SECTION – B			CO
3.	Answer any five of the following-	[5×6=30]	
	3-a. Question-	(6)	
	3-b. Question-	(6)	
	3-c. Question-	(6)	
	3-d. Question-	(6)	
	3-e. Question-	(6)	
	3-f. Question-	(6)	
	3-g. Question-	(6)	

End Semester Question Paper Templates (Offline Pattern/Online Pattern

SECTION – C				CO
4	Answer any one of the following-		[5×10=50]	
	4-a.	Question-	(10)	
	4-b.	Question-	(10)	
5.	Answer any one of the following-			
	5-a.	Question-	(10)	
	5-b.	Question-	(10)	

End Semester Question Paper Templates (Offline Pattern/Online Pattern

6.	Answer any one of the following-			
	6-a.	Question-	(10)	
	6-b.	Question-	(10)	
7.	Answer any one of the following-			
	7-a.	Question-	(10)	
	7-b.	Question-	(10)	
8.	Answer any one of the following-			
	8-a.	Question-	(10)	
	8-b.	Question-	(10)	

Topic Prerequisite & Recap (CO4)

Prerequisite

- Basic Understanding of class 10 mathematics NCERT.
- Basic Knowledge of sets and algebraic rules
- Basic Understanding of Set Theory, Relations and Functions & Algebraic Structures.
- Basic Understanding of Lattices & Boolean Algebra

Recap

Now students are able to develop there logical thinking by using Sets, Relations, Functions and Mathematical Induction, Algebraic Structures, Lattices & Boolean Algebra concepts and use in upcoming topic. i.e. Propositions.

Brief Introduction about the subject with video

Discrete mathematics is the study of mathematical structures that are fundamentally discrete rather than continuous. In contrast to real numbers that have the property of varying "smoothly", the objects studied in discrete mathematics – such as integers, graphs, and statements in logic.

- https://www.youtube.com/watch?v=Ib5njCwNMdk&list=PLBlnK6fEyqRhqJPDXcvYlLfXPh37L89g3&index=3
- https://www.youtube.com/watch?v=6kYngPvoGxU&list=PLBlnK6fEyqRhqJPDXcvYlLfXPh37L89g3&index=4
- https://www.youtube.com/watch?v=m2mf6I3g2-c&list=PLBlnK6fEyqRhqJPDXcvYlLfXPh37L89g3&index=5
- https://www.youtube.com/watch?v=tACXuzfXzSI&list=PLBlnK6fEyqRhqJPDXcvYlLfXPh37L89g3&index=6

Unit 4 Objectives

- Objective of proposition logic to write English sentences for logical expressions and vice-versa. Use standard notations of propositional logic.
- Determine if a logical argument is valid or invalid. Apply standard rules of inference including (but not limited to) Modus Ponens, Modus Tollens.
- Translate between English sentences and symbols for universally and existentially quantified statements, including statements with multiple quantifiers.

Unit Objective (CO4)

- **Propositional Logic:** Introduction, Propositions and Compound Statements, Basic Logical Operations, well formed formula, Truth tables, Tautology, Satisfiability, Contradiction, Algebra of proposition, Theory of Inference.
- **Predicate Logic:** First order predicate, well formed formula of predicate, quantifiers, Inference theory of predicate logic.

Topic Objectives: Propositions(CO4)

- The student will demonstrate the ability to use mathematical logic to solve problems
- The student will be able to:
  - represent English-language statements using symbolic logic notation.
  - use and interpret relational conjunctions (and, or, xor, not), terms of causation (if… then) and equivalence (if and only if).
  - use truth tables to analyze the truth values of compound statements based on the truth values of their components.
  - use truth tables to determine if two statements are logically equivalent.
  - use truth tables to identify tautologies and contradictions.

Propositions (CO4)

- A proposition is a statement that is either true or false (not both).

- We say that the truth value of a proposition is either true (T or 1) or false (F or 0).

Example	Beijing is the capital of China	T
	2 + 2 = 5	F
	1 + 2 = 3	T

Propositions(CO4)

The Statement/Proposition example

"Elephants are bigger than mice."

Is this a statement? Yes

Is this a proposition? Yes

What is the truth value of the proposition? True

Propositions(CO4)

The Statement/Proposition example

"520 < 111"

Is this a statement? Yes

Is this a proposition? Yes

Is it the truth value of the proposition? False

Propositions(CO4)

The Statement/Proposition example

“y > 5”

Is this a statement? Yes

Is this a proposition? No

Its truth value depends on the value of y, but this value is not Specified. We call this type of statement a propositional function or open sentence

Propositional logic (PL) [CO4]

- Propositional logic (PL) is the simplest form of logic where all the statements are made by propositions. A proposition is a declarative statement which is either true or false. It is a technique of knowledge representation in logical and mathematical form.

Example:

- a) It is Sunday.
- b) The Sun rises from West (False proposition)
- c) 3+3= 7(False proposition)
- d) 5 is a prime number.

Continue…

Following are some basic facts about propositional logic:

- Propositional logic is also called Boolean logic as it works on 0 and 1.
- In propositional logic, we use symbolic variables to represent the logic, and we can use any symbol for a representing a proposition, such A, B, C, P, Q, R, etc.
- Propositions can be either true or false, but it cannot be both.
- Propositional logic consists of an object, relations or function, and **logical connectives**.
- These connectives are also called logical operators.
- The propositions and connectives are the basic elements of the propositional logic.

Propositional logic [CO4]

- Connectives can be said as a logical operator which connects two sentences.
- A proposition formula which is always true is called **tautology**, and it is also called a valid sentence.
- A proposition formula which is always false is called **Contradiction**.
- A proposition formula which has both true and false values is called **Contingency**.
- Statements which are questions, commands, or opinions are not propositions such as "**Where is Rohini**", "**How are you**", "**What is your name**", are not propositions.

Types of Propositions

There are two types of Propositions:

Atomic Propositions

Compound propositions

Atomic Proposition: Atomic propositions are the simple propositions. It consists of a single proposition symbol. These are the sentences which must be either true or false.

Example:

a) 2+2 is 4, it is an atomic proposition as it is a **true** fact.

b) "The Sun is cold" is also a proposition as it is a **false** fact.

Compound proposition [CO4]

Compound propositions are constructed by combining simpler or atomic propositions, using parenthesis and logical connectives.

Example:

a) "It is raining today, and street is wet."

b) "Ankit is a doctor, and his clinic is in Mumbai."

Basic Logical Operations [CO4]

Basic Logical Operator:

- Basic Logical operators/Logical connectives are used to connect two simpler propositions or representing a sentence logically. We can create compound propositions with the help of logical connectives. There are mainly five connectives, which are given as follows:
- **Negation:** A sentence such as ¬ P is called negation of P. A literal can be either Positive literal or negative literal.
- **Conjunction:** A sentence which has ∧ connective such as, **P ∧ Q** is called a conjunction.

  Example: Rohan is intelligent and hardworking. It can be written as, **P= Rohan is intelligent**, **Q= Rohan is hardworking. → P∧ Q**.

Continue ...[CO4]

- **Disjunction:** A sentence which has V connective, such as **P V Q**. is called disjunction, where P and Q are the propositions.
  Example: "Ritika is a doctor or Engineer",
- Here P= Ritika is Doctor. Q= Ritika is Doctor, so we can write it as **P V Q**.
- **Implication:** A sentence such as P → Q, is called an implication. Implications are also known as if-then rules. It can be represented as
  If it is raining, then the street is wet.
  Let P= It is raining, and Q= Street is wet, so it is represented as P → Q
- **Biconditional:** A sentence such as **P⇔ Q is a Biconditional sentence, example If I am breathing, then I am alive**
  P= I am breathing, Q= I am alive, it can be represented as P ⇔ Q.

Continue… …[CO4]

- **summarized table for Propositional Logic Connectives**:

Connective symbols	Word	Technical term	Example
$\wedge$	AND	Conjunction	$A \wedge B$
$\vee$	OR	Disjunction	$A \vee B$
$\rightarrow$	Implies	Implication	$A \rightarrow B$
$\Leftrightarrow$	If and only if	Biconditional	$A \Leftrightarrow B$
$\neg$ or ~	Not	Negation	$\neg A$ or $\neg B$

Truth Table (CO4)

We will examine the truth table of following logical operators

- Negation (NOT)
- Conjunction (AND)
- Disjunction (OR)
- Exclusive or (XOR)
- Implication (if – then)
- Biconditional (if and only if)

•Truth tables can be used to show how these operators can combine propositions to compound propositions.

Negation (NOT) (CO4)

It is Unary Operator and denoted by Symbol: ¬

P	**¬P**
true (T)	false (F)
false (F)	true (T)

Conjunction (AND) (CO4)

It is binary Operator and Symbol is $\wedge$

P	Q	P$\wedge$Q
T	T	T
T	F	F
F	T	F
F	F	F

Disjunction (OR) (CO4)

It is binary Operator and Symbol is $\vee$

P	Q	P∨Q
T	T	T
T	F	T
F	T	T
F	F	F

Exclusive Or (XOR) (CO4)

It is binary Operator and Symbol is $\oplus$

P	Q	$P \oplus Q$
T	T	F
T	F	T
F	T	T
F	F	F

Implication (if - then) (CO4)

It is binary Operator and Symbol is $\rightarrow$

P	Q	P→Q
T	T	T
T	F	F
F	T	T
F	F	T

Biconditional (if and only if) (CO4)

It is binary Operator and Symbol is $\leftrightarrow$

P	Q	$P \leftrightarrow Q$
T	T	T
T	F	F
F	T	F
F	F	T

Statements and Operators(CO4)

Example 1 Statements and operators can be combined in any way to form new statements.

P	Q	$\neg P$	$\neg Q$	$(\neg P)\vee(\neg Q)$
T	T	F	F	F
T	F	F	T	T
F	T	T	F	T
F	F	T	T	T

Statements and Operators(CO4)

Example 2 Statements and operators can be combined in any way to form new statements.

P	Q	$P \wedge Q$	$\neg (P \wedge Q)$	$(\neg P) \vee (\neg Q)$
T	T	T	F	F
T	F	F	T	T
F	T	F	T	T
F	F	F	T	T

Continue… …[CO4]

Logical equivalence:

- Logical equivalence is one of the features of propositional logic. Two propositions are said to be logically equivalent if and only if the columns in the truth table are identical to each other.
- Let's take two propositions A and B, so for logical equivalence, we can write it as A⇔B. In below truth table we can see that column for ¬AV B and A→B, are identical hence A is Equivalent to B

A	B	¬A	¬AV B	A→B
T	T	F	**T**	**T**
T	F	F	**F**	**F**
F	T	T	**T**	**T**
F	F	T	**T**	**T**

Example (CO4)

The statements $\neg(P \wedge Q)$ and $(\neg P) \vee (\neg Q)$ are logically equivalent, since $\neg(P \wedge Q) \leftrightarrow (\neg P) \vee (\neg Q)$ is always true.

P	Q	$\neg(P \wedge Q)$	$(\neg P) \vee (\neg Q)$	$\neg(P \wedge Q) \leftrightarrow (\neg P) \vee (\neg Q)$
T	T	F	F	T
T	F	T	T	T
F	T	T	T	T
F	F	T	T	T

Algebra of Proposition [CO4]

Properties of Operators:

- **Commutativity:**
  - $P \wedge Q = Q \wedge P$, or
  - $P \vee Q = Q \vee P$.
- **Associativity:**
  - $(P \wedge Q) \wedge R = P \wedge (Q \wedge R)$,
  - $(P \vee Q) \vee R = P \vee (Q \vee R)$
- **Identity element:**
  - $P \wedge \text{True} = P$,
  - $P \vee \text{True} = \text{True}$.
  - **Idempotent Law:**
  - $P \vee P \equiv P$ and $P \wedge P \equiv P$

Continue…[CO4]

- **Distributive:**
  - $P \wedge (Q \vee R) = (P \wedge Q) \vee (P \wedge R)$.
  - $P \vee (Q \wedge R) = (P \vee Q) \wedge (P \vee R)$.
- **DE Morgan's Law:**
  - $\neg (P \wedge Q) = (\neg P) \vee (\neg Q)$
  - $\neg (P \vee Q) = (\neg P) \wedge (\neg Q)$.
- **Double-negation elimination: (Complement Properties)**

  $\neg (\neg P) = P$.

- **Transposition :**

- $\mathbf{P \rightarrow Q} \equiv (-Q \mathbf{\rightarrow -P})$

Tautology (CO4)

A tautology is a statement that is always true.

Examples: $\neg(P \wedge Q) \leftrightarrow (\neg P) \vee (\neg Q)$

If $S \rightarrow T$ is a tautology, we write $S \Rightarrow T$.

If $S \leftrightarrow T$ is a tautology, we write $S \Leftrightarrow T$.

P	Q	$\neg(P \wedge Q)$	$(\neg P) \vee (\neg Q)$	$\neg(P \wedge Q) \leftrightarrow (\neg P) \vee (\neg Q)$
T	T	F	F	T
T	F	T	T	T
F	T	T	T	T
F	F	T	T	T

Contradiction (CO4)

A contradiction is a statement that is always False

Examples: $R \wedge (\neg R)$ $\neg(\neg(P \wedge Q) \leftrightarrow (\neg P) \vee (\neg Q))$

The negation of any tautology is a contradiction, and the negation of any contradiction is a tautology.

Satisfiability (CO4)

- A proposition is satisfiable, if its truth table contains true at least once. Example: $p \wedge q$.
- A tautology, if it is always true. Example: $p \vee \neg p$.
- A contradiction, if it always false. Example: $p \wedge \neg p$.
- A contingency, if it is neither a tautology nor a contradiction. Example: p.

Theory of Inference [CO4]

Rules of Inference

Inference:

- In we need intelligent computers which can create new logic from old logic or by evidence, **so generating the conclusions from evidence and facts is termed as Inference**.

Inference rules:

- Inference rules are the templates for generating valid arguments. Inference rules are applied to derive proofs in artificial intelligence, and the proof is a sequence of the conclusion that leads to the desired goal.
- In inference rules, the implication among all the connectives plays an important role. Following are some terminologies related to inference rules:

Continue… [CO4]

- **Implication:** It is one of the logical connectives which can be represented as $P \rightarrow Q$. It is a Boolean expression.
- **Converse:** The converse of implication, which means the right-hand side proposition goes to the left-hand side and vice-versa. It can be written as $Q \rightarrow P$.
- **Contrapositive:** The negation of converse is termed as contrapositive, and it can be represented as $\neg Q \rightarrow \neg P$.
- **Inverse:** The negation of implication is called inverse. It can be represented as $\neg P \rightarrow \neg Q$.
- From the above term some of the compound statements are equivalent to each other, which we can prove using truth table:

Continue… [CO4]

P	Q	$P \to Q$	$Q \to P$	$\neg Q \to \neg P$	$\neg P \to \neg Q$.
T	T	T	T	T	T
T	F	F	T	F	T
F	T	T	F	T	F
F	F	T	T	T	T

- Hence from the above truth table, we can prove that $P \to Q$ is equivalent to $\neg Q \to \neg P$, and $Q \to P$ is equivalent to $\neg P \to \neg Q$.

Continue... [CO4]

Types of Inference rules:

1. Modus Ponens:

- The Modus Ponens rule is one of the most important rules of inference, and it states that if P and P → Q is true, then we can infer that Q will be true. It can be represented as:

Example:

Statement-1: "If I am sleepy then I go to bed" ==> P→ Q
Statement-2: "I am sleepy" ==> P
Conclusion: "I go to bed." ==> Q.
Hence, we can say that, if P→ Q is true and P is true then Q will be true.

- **Proof by Truth table:**

P	Q	$P \rightarrow Q$
0	0	0
0	1	1
1	0	0
1	1	1

2. Modus Tollens:

- The Modus Tollens rule state that if P→ Q is true and **¬ Q is true, then ¬ P** will also true. It can be represented as:

Notation for Modus Tollens: $$\frac{P \rightarrow Q, \ \sim Q}{\sim P}$$

- **Statement-1:** "If I am sleepy then I go to bed" ==> P→ Q

  Statement-2: "I do not go to the bed."==> ~Q

  Statement-3: Which infers that "**I am not sleepy**" => ~P

- **Proof by Truth table:**

P	Q	$\sim P$	$\sim Q$	$P \to Q$
0	0	1	1	1
0	1	1	0	1
1	0	0	1	0
1	1	0	0	1

3. Hypothetical Syllogism:

- The Hypothetical Syllogism rule state that if $P \rightarrow R$ is true whenever $P \rightarrow Q$ is true, and $Q \rightarrow R$ is true. It can be represented as the following notation:
- **Example:**

  Statement-1: If you have my home key then you can unlock my home. **$P \rightarrow Q$**
  Statement-2: If you can unlock my home then you can take my money. **$Q \rightarrow R$**
  Conclusion: If you have my home key then you can take my money. **$P \rightarrow R$**

Proof by truth table:

Continue… [CO4]

P	Q	R	$P \to Q$	$Q \to R$	$P \to R$
0	0	0	1	1	1
0	0	1	1	1	1
0	1	0	1	0	1
0	1	1	1	1	1
1	0	0	0	1	1
1	0	1	0	1	1
1	1	0	1	0	0
1	1	1	1	1	1

Continue… [CO4]

4. **Disjunctive Syllogism:**

- The Disjunctive syllogism rule state that if PVQ is true, and ¬P is true, then Q will be true. It can be represented as:

Example:

Statement-1: Today is Sunday or Monday. ==>PVQ
Statement-2: Today is not Sunday. ==> ¬P
Conclusion: Today is Monday. ==> Q

- **Proof by truth-table:**

Continue… [CO4]

5. Addition:

- The Addition rule is one the common inference rule, and it states that If P is true, then PVQ will be true.

Notation of Addition: $\frac{P}{P \vee Q}$

Example:

- **Statement:** I have a vanilla ice-cream. ==> P
  Statement-2: I have Chocolate ice-cream.
  Conclusion: I have vanilla or chocolate ice-cream. ==> (PVQ)

Proof by Truth-Table:

P	Q	$P \vee Q$
0	0	0
1	0	1
0	1	1
1	1	1

6. Simplification:

- The simplification rule state that if **P∧ Q** is true, then **Q or P** will also be true. It can be represented as:

Notation of Simplification rule: $\frac{P \land Q}{Q}$ Or $\frac{P \land Q}{P}$

- **Proof by Truth-Table:**

P	Q	$P \land Q$
0	0	0
1	0	0
0	1	0
1	1	1

7. Resolution:

- The Resolution rule state that if P∨Q and ¬ P∧R is true, then Q∨R will also be true. **It can be represented as**

Notation of Resolution $\dfrac{P \vee Q, \quad \neg P \wedge R}{Q \vee R}$

- **Proof by Truth-Table:**

P	¬ P	Q	R	$P \vee Q$	¬ P∧R	$Q \vee R$
0	1	0	0	0	0	0
0	1	0	1	0	0	1
0	1	1	0	1	1	1 ←
0	1	1	1	1	1	1 ←
1	0	0	0	1	0	0
1	0	0	1	1	0	1
1	0	1	0	1	0	1
1	0	1	1	1	0	1 ←

First-Order Predicate (CO4)

- In the topic of Propositional logic, we have seen that how to represent statements using propositional logic. But unfortunately, in propositional logic, we can only represent the facts, which are either true or false. PL is not sufficient to represent the complex sentences or natural language statements. The propositional logic has very limited expressive power. Consider the following sentence, which we cannot represent using PL logic.

"Some humans are intelligent", or

"Sachin likes cricket."

- To represent the above statements, PL logic is not sufficient, so we required some more powerful logic, such as first-order logic.

Continue…

First-Order logic:

- A predicate is an expression of one or more variables defined on some specific domain. A predicate with variables can be made a proposition by either assigning a value to the variable or by quantifying the variable.
- The following are some examples of predicates −

Let E(x, y) denote "x = y"

Let X(a, b, c) denote "a + b + c = 0"

Let M(x, y) denote "x is married to y“

- First-order logic is also known as **Predicate logic or First-order predicate logic**. First-order logic is a powerful language that develops information about the objects in a more easy way and can also express the relationship between those objects.

Continue…

-- Objects: A, B, people, numbers, colors, wars, theories, squares etc.

– **Relations: It can be unary relation such as:** red, round, is adjacent, **or n-any relation such as:** the sister of, brother of, has color, comes between

– **Function:** Father of, best friend, third inning of, end of, ...

- As a natural language, first-order logic also has two main parts:
  – **Syntax**
  – **Semantics**

Continue…

Syntax of First-Order logic:

The syntax of FOL determines which collection of symbols is a logical expression in first-order logic. The basic syntactic elements of first-order logic are symbols. We write statements in short-hand notation in FOL.

Basic Elements of First-order logic:

Following are the basic elements of FOL syntax:

Continue...

- **Constant**

  1, 2, A, John, Mumbai, cat,....
- **Variables**

  x, y, z, a, b,....
- **Predicates**

  Brother, Father, >,....
- **Function**

  sqrt, LeftLegOf, ...

- **Connectives**

  $\wedge, \vee, \neg, \Rightarrow, \Leftrightarrow$
- **Equality** ==
- **Quantifier** $\forall, \exists$

Continue…

Atomic sentences:

- Atomic sentences are the most basic sentences of first-order logic. These sentences are formed from a predicate symbol followed by a parenthesis with a sequence of terms.
- We can represent atomic sentences as **Predicate (term1, term2, ..., term n)**.

- **Example: Ravi and Ajay are brothers: => Brothers(Ravi, Ajay).**
  Chinky is a cat: => cat (Chinky).

Well Formed Formula

- Well Formed Formula (wff) is a predicate holding any of the following –
- All propositional constants and propositional variables are wffs
- If x is a variable and Y is a wff, ∀xY∀xY and ∃xY∃xY are also wff
- Truth value and false values are wffs
- Each atomic formula is a wff
- All connectives connecting wffs are wffs

Continue…

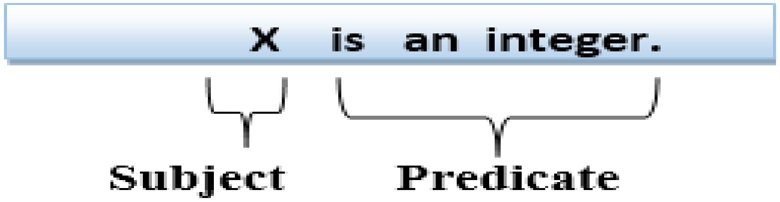
Complex Sentences:

- Complex sentences are made by combining atomic sentences using connectives.

First-order logic statements can be divided into two parts:

- **Subject:** Subject is the main part of the statement.
- **Predicate:** A predicate can be defined as a relation, which binds two atoms together in a statement.

Consider the statement: "x is an integer.", it consists of two parts, the first part x is the subject of the statement and second part "is an integer," is known as a predicate.



Quantifiers in First-order logic

- A quantifier is a language element which generates quantification, and quantification specifies the quantity of specimen in the universe of discourse.
- These are the symbols that permit to determine or identify the range and scope of the variable in the logical expression. There are two types of quantifier:
  - **Universal Quantifier, (for all, everyone, everything)**
  - **Existential quantifier, (for some, at least one).**

Continue…

Universal Quantifier:

- Universal quantifier is a symbol of logical representation, which specifies that the statement within its range is true for everything or every instance of a particular thing.
- The Universal quantifier is represented by a symbol ∀, which resembles an inverted A.
- *Note: In universal quantifier we use implication "→".*
- If x is a variable, then ∀x is read as:

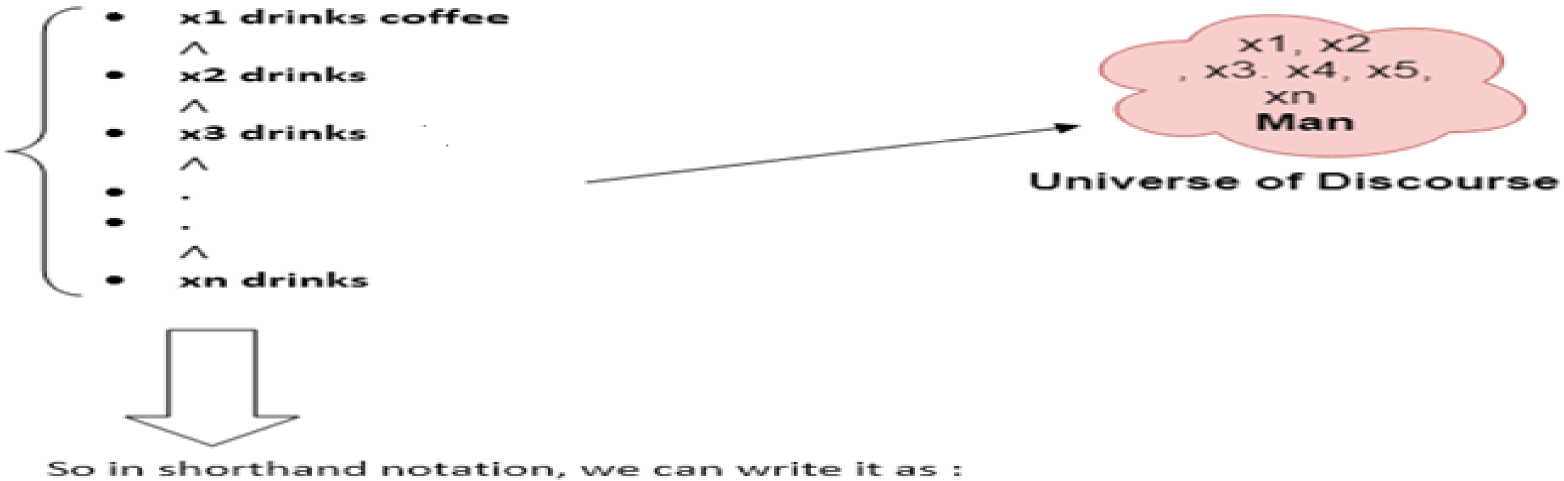
  For all x

  For each x

  For every x

Example: All man drink coffee.

- Let a variable x which refers to a cat so all x can be represented in UOD as below:



- **∀x man(x) → drink (x, coffee).**

It will be read as: There are all x where x is a man who drink coffee

Continue…

Existential Quantifier:

- Existential quantifiers are the type of quantifiers, which express that the statement within its scope is true for at least one instance of something.
- It is denoted by the logical operator ∃, which resembles as inverted E. When it is used with a predicate variable then it is called as an existential quantifier.
- *Note: In Existential quantifier we always use AND or Conjunction symbol (∧).*
- If x is a variable, then existential quantifier will be ∃x or ∃(x). And it will be read as:

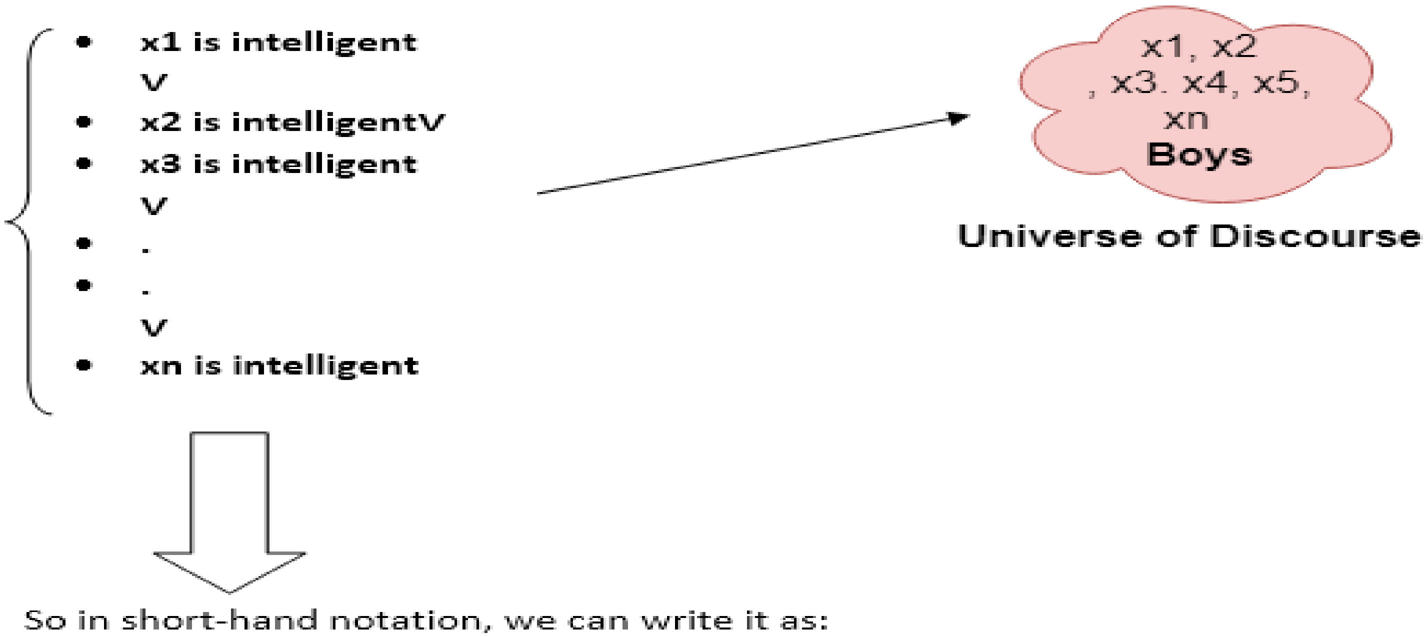
There exists a 'x.'

For some 'x.‘

For at least one 'x.'

Continue…

Example: Some boys are intelligent.



- **∃x: boys(x) ∧ intelligent(x)**

It will be read as: There are some x where x is a boy who is intelligent

Continue…

Points to remember:

- The main connective for universal quantifier ∀ is implication →.
- The main connective for existential quantifier ∃ is and ∧.

Properties of Quantifiers:

- In universal quantifier, ∀x∀y is similar to ∀y∀x.
- In Existential quantifier, ∃x∃y is similar to ∃y∃x.
- ∃x∀y is not similar to ∀y∃x.

Continue...

- **Some Examples of FOL using quantifier**
- **1. All birds fly.**
  In this question the predicate is "**fly(bird)**."
  And since there are all birds who fly so it will be represented as follows.
  $$\forall x \; bird(x) \rightarrow fly(x).$$
- **2. Every man respects his parent.**
  In this question, the predicate is "**respect(x, y)," where x=man, and y= parent**.
  Since there is every man so will use ∀, and it will be represented as follows:
  $$\forall x \; man(x) \rightarrow respects\,(x, parent).$$

Continue…

- **3. Some boys play cricket.**
  In this question, the predicate is "**play(x, y)**," where x= boys, and y= game. Since there are some boys so we will use **∃, and it will be represented as**:
  $\exists x$ boys(x) → play(x, cricket).
- **4. Not all students like both Mathematics and Science.**
  In this question, the predicate is "**like(x, y)," where x= student, and y= subject**.
  Since there are not all students, so we will use **∀ with negation, so** following representation for this:
  $\neg\forall$ (x) [student(x) → like(x, Mathematics) ∧ like(x, Science)].

Continue…

- **5. Only one student failed in Mathematics.**

  In this question, the predicate is "**failed(x, y)," where x= student, and y= subject**.

  Since there is only one student who failed in Mathematics, so we will use following representation for this:

  ∃(x) [student(x) → failed (x, Mathematics) ∧∀ (y) [¬(x==y) ∧ student(y) → ¬failed (x, Mathematics)].

Continue…

Free and Bound Variables:

- There are two types of variables in First-order logic which are given below:
- **Free Variable:** A variable is said to be a free variable in a formula if it occurs outside the scope of the quantifier.

  Example: ∀x ∃(y)[P (x, y, z)], where z is a free variable.
- **Bound Variable:** A variable is said to be a bound variable in a formula if it occurs within the scope of the quantifier.

  Example: ∀x [A (x) B(y)], here x and y are the bound variables.

Inference in First-Order Logic

- Inference in First-Order Logic is used to deduce new facts or sentences from existing sentences. Before understanding the FOL inference rule, let's understand some basic terminologies used in FOL.

Substitution:

- Substitution is a fundamental operation performed on terms and formulas. It occurs in all inference systems in first-order logic. The substitution is complex in the presence of quantifiers in FOL. If we write **F[a/x]**, so it refers to substitute a constant "**a**" in place of variable "**x**".
- *Note: First-order logic is capable of expressing facts about some or all objects in the universe.*

Continue…

Equality:

- First-Order logic does not only use predicate and terms for making atomic sentences but also uses another way, which is equality in FOL. For this, we can use **equality symbols** which specify that the two terms refer to the same object.

Example: Brother (John) = Smith.

- As in the above example, the object referred by the **Brother (John)** is similar to the object referred by **Smith**. The equality symbol can also be used with negation to represent that two terms are not the same objects.

Example: $\neg$(x=y) which is equivalent to x $\neq$y.

Continue…

FOL inference rules for quantifier:

- As propositional logic we also have inference rules in first-order logic, so following are some basic inference rules in FOL:

Universal Generalization

Universal Instantiation

Existential Instantiation

Existential introduction

Continue…

1. Universal Generalization:

- Universal generalization is a valid inference rule which states that if premise P(c) is true for any arbitrary element c in the universe of discourse, then we can have a conclusion as ∀ x P(x).
- It can be represented as: $$\frac{P(c)}{\forall x\, P(x)}$$
- This rule can be used if we want to show that every element has a similar property.
- In this rule, x must not appear as a free variable.
- **Example:** Let's represent, P(c): "**A byte contains 8 bits**", so for **∀ x P(x)** "**All bytes contain 8 bits**.", it will also be true.

Continue…

2. Universal Instantiation:

- Universal instantiation is also called as universal elimination or UI is a valid inference rule. It can be applied multiple times to add new sentences.
- The new KB is logically equivalent to the previous KB.
- As per UI, **we can infer any sentence obtained by substituting a ground term for the variable**.
- The UI rule state that we can infer any sentence P(c) by substituting a ground term c (a constant within domain x) from **∀ x P(x) for any object in the universe of discourse**.
- It can be represented as:

$$\frac{\forall x\, P(x)}{P(c)}$$

Example:1.

- IF "Every person like ice-cream"=> ∀x P(x) so we can infer that "John likes ice-cream" => P(c)

Continue...

- **Example: 2.**

  Let's take a famous example,

  "All kings who are greedy are Evil." So let our knowledge base contains this detail as in the form of FOL:

  $\forall x$ king(x) ∧ greedy (x) → Evil (x),

- So from this information, we can infer any of the following statements using Universal Instantiation:

  King(John) ∧ Greedy (John) → Evil (John),

  King(Richard) ∧ Greedy (Richard) → Evil (Richard),

  King(Father(John)) ∧ Greedy (Father(John)) → Evil (Father(John)),

Continue…

3. Existential Instantiation:

- Existential instantiation is also called as Existential Elimination, which is a valid inference rule in first-order logic.
- It can be applied only once to replace the existential sentence.
- The new KB is not logically equivalent to old KB, but it will be satisfiable if old KB was satisfiable.
- This rule states that one can infer P(c) from the formula given in the form of ∃x P(x) for a new constant symbol c.
- The restriction with this rule is that c used in the rule must be a new term for which P(c) is true.
- It can be represented as:
- **Example:**
- From the given sentence: **∃x Crown(x) ∧ OnHead(x, John)**

Continue…

So we can infer: **Crown(K) ∧ OnHead(K, John),** as long as K does not appear in the knowledge base.

- The above used K is a constant symbol, which is called **Skolem constant**.
- The Existential instantiation is a special case of **Skolemization process**.

4. Existential introduction

- An existential introduction is also known as an existential generalization, which is a valid inference rule in first-order logic.
- This rule states that if there is some element c in the universe of discourse which has a property P, then we can infer that there exists something in the universe which has the property P.
- It can be represented as: $\frac{P(c)}{\exists x P(x)}$
- **Example: Let's say that,** "Priyanka got good marks in English." "Therefore, someone got good marks in English.

Daily Quiz(CO4)

Q1 $\neg (p \leftrightarrow q)$ is logically equivalent to:

(a) $p \leftrightarrow \neg q$ (b) $\neg p \leftrightarrow q$

(c)

¬ (p q) is logically equivalent to
A. qp
B. p¬q
C. ¬p¬q
D. ¬q¬p
Answer» B. p¬q

Q2 Which of the following option is true?

(a) If the Sun is a planet, elephants will fly

(b) 3 +2 = 8 if 5-2 = 7

(c) 1 > 3 and 3 is a positive integer

(d) -2 > 3 or 3 is a negative integer

Q3 The truth value of given statement is
'If 9 is prime then 3 is even'.

(a) False (b) True

Daily Quiz(CO4)

Q4 Let P: I am in Bangalore. , Q: I love cricket. ; then q -> p(q implies p) is:

(a) If I love cricket then I am in Bangalore

(b) If I am in Bangalore then I love cricket

(c) I am not in Bangalore

(d) I love cricket

Q5 Let P: We should be honest., Q: We should be dedicated .,R: We should be over confident.Then ‘We should be honest or dedicated but not overconfident.’ is best represented by:

(a) ~P V ~Q V R

(b) P ∧ ~Q ∧ R

(c) P V Q ∧ R

(d) P V Q ∧ ~R

Weekly Assignment(CO4)

Q1 What do you mean by logical equivalence? Explain with an example.

Q2 Construct the truth table of the following

$$\sim (P \vee (Q \wedge R)) \Leftrightarrow ((P \vee Q) \wedge (P \vee R))$$

Q3 Show that which of the following statements tautology are.

(i) $((P \vee \backsim Q) \wedge (\backsim P \vee \backsim Q)) \vee Q$

(ii) $(\sim P \wedge Q \Longrightarrow (Q \Longrightarrow P))$

Q4 Using truth table prove that: $P \leftrightarrow Q \equiv (P \rightarrow Q) \wedge (Q \rightarrow P)$.

Q5 Using logical equivalent formulas, show that

$$\sim (P \vee (\sim P \wedge Q)) \equiv \sim P \wedge \sim Q.$$

Q6 Express $(\sim p \rightarrow r) \wedge (q \longleftrightarrow q)$ in its principle conjunctive normal form

Q7 Define Minterms and Maxterms and find the principal disjunctive normal form of the following:- $(p \wedge q) \vee (\sim p \wedge r) \vee (q \wedge r)$

Faculty Video Links, Youtube & NPTEL Video Links and Online Courses Details(CO4)

Youtube/other Video Links

- https://www.youtube.com/watch?v=Ib5njCwNMdk&list=PLBlnK6fEyqRhqJPDXcvYlLfXPh37L89g3&index=3
- https://www.youtube.com/watch?v=6kYngPvoGxU&list=PLBlnK6fEyqRhqJPDXcvYlLfXPh37L89g3&index=4
- https://www.youtube.com/watch?v=m2mf6I3g2-c&list=PLBlnK6fEyqRhqJPDXcvYlLfXPh37L89g3&index=5
- https://www.youtube.com/watch?v=tACXuzfXzSI&list=PLBlnK6fEyqRhqJPDXcvYlLfXPh37L89g3&index=6
- https://www.youtube.com/watch?v=ccz-w2JMsTM&list=PLBlnK6fEyqRhqJPDXcvYlLfXPh37L89g3&index=7

MCQs(CO4)

Q1 Which of the following statement is a proposition?

(a) Get me a glass of milkshake

(b) God bless you!

(c) What is the time now?

(d) The only odd prime number is 2

Q2 Let P: I am in Delhi. , Q: Delhi is clean. ; then q ^ p(q and p) is:

(a) Delhi is clean and I am in Delhi

(b) Delhi is not clean or I am in Delhi

(c) I am in Delhi and Delhi is not clean

(d) Delhi is clean but I am in Mumbai

Q3 The compound propositions p and q are called logically equivalent if ________ is a tautology.

(a) $p \leftrightarrow q$ (b) $p \rightarrow q$ (c) $\neg (p \vee q)$ (d) $\neg p \vee \neg q$

MCQs (CO4)

Q4 $p \rightarrow q$ is logically equivalent to:

(a) $\neg p \vee \neg q$ (b) $p \vee \neg q$ (c) $\neg p \vee q$ (d) $\neg p \wedge q$

Q5 $p \vee q$ is logically equivalent to:

(a) $\neg q \rightarrow \neg p$ (b) $q \rightarrow p$ (c) $\neg p \rightarrow \neg q$ (d) $\neg p \rightarrow q$

Q6 Which of the following statement is correct?

(a) $p \vee q \equiv q \vee p$

(b) $\neg(p \wedge q) \equiv \neg p \vee \neg q$

(c) $(p \vee q) \vee r \equiv p \vee (q \vee r)$

(d) All of mentioned

Q7 $(p \rightarrow q) \wedge (p \rightarrow r)$ is logically equivalent to:

(a) $p \rightarrow (q \wedge r)$ (b) $p \rightarrow (q \vee r)$

(c) $p \wedge (q \vee r)$ (d) $p \vee (q \wedge r)$

MCQs (CO4)

Q4 $p \rightarrow q$ is logically equivalent to:

(a) $\neg p \vee \neg q$ (b) $p \vee \neg q$ (c) $\neg p \vee q$ (d) $\neg p \wedge q$

Q5 $p \vee q$ is logically equivalent to:

(a) $\neg q \rightarrow \neg p$ (b) $q \rightarrow p$ (c) $\neg p \rightarrow \neg q$ (d) $\neg p \rightarrow q$

Q6 Which of the following statement is correct?

(a) $p \vee q \equiv q \vee p$

(b) $\neg(p \wedge q) \equiv \neg p \vee \neg q$

(c) $(p \vee q) \vee r \equiv p \vee (q \vee r)$

(d) All of mentioned

Q7 $(p \rightarrow q) \wedge (p \rightarrow r)$ is logically equivalent to:

(a) $p \rightarrow (q \wedge r)$ (b) $p \rightarrow (q \vee r)$

(c) $p \wedge (q \vee r)$ (d) $p \vee (q \wedge r)$

MCQs (CO4)

Q8 Which of the following statement is a proposition?

a) Get me a glass of milkshake

b) God bless you!

c) What is the time now?

d) The only odd prime number is 2

Q9 The truth value of given statement is

‘4+3=7 or 5 is not prime’.

a) False

b) True

Q10. Which of the following option is true?

a) If the Sun is a planet, elephants will fly

b) 3 +2 = 8 if 5-2 = 7

c) 1 > 3 and 3 is a positive integer

d) -2 > 3 or 3 is a negative intege

MCQs (CO4)

Q11 What is the value of x after this statement, assuming initial value of x is 5?‘If x equals to one then x=x+2 else x=0’.

a) 1 b) 3

c) 0 d) 2

Q12 Let P: I am in Bangalore. , Q: I love cricket. ; then q -> p(q implies p) is:

a) If I love cricket then I am in Bangalore

b) If I am in Bangalore then I love cricket

c) I am not in Bangalore

d) I love cricket

Q13. Let P:If Sahil bowls, Saurabh hits a century. ,Q: If Raju bowls , Sahil gets out on first ball. Now if P is true and Q is false then which of the following can be true?

a) Raju bowled and Sahil got out on first ball

b) Raju did not bowled

c) Sahil bowled and Saurabh hits a century

d) Sahil bowled and Saurabh got out

MCQs (CO4)

Q14 The truth value of given statement is
'If 9 is prime then 3 is even'.
a) False
b) True

Q15 Let P: I am in Delhi. , Q: Delhi is clean. ; then q ^ p(q and p) is:
a) Delhi is clean and I am in Delhi
b) Delhi is not clean or I am in Delhi
c) I am in Delhi and Delhi is not clean
d) Delhi is clean but I am in Mumbai

Q16 Let P: This is a great website, Q: You should not come back here.
Then 'This is a great website and you should come back here.' is best represented by:
a) ~P V ~Q
b) P ∧ ~Q
c) P V Q
d) P ∧ Q

MCQs (CO4)

Q17. Let P: We should be honest., Q: We should be dedicated .,R: We should be overconfident.
Then 'We should be honest or dedicated but not overconfident.' is best represented by:

a) ~P V ~Q V R

b) P ∧ ~Q ∧ R

c) P V Q ∧ R

d) P V Q ∧ ~R

Q18. A compound proposition that is always ___________ is called a tautology.

a) True

b) False

Q19 Let P (x) denote the statement "x >7." Which of these have truth value true?

a) P (0)

b) P (4)

c) P (6)

d) P (9)

MCQs (CO4)

Q20. The statement," Every comedian is funny" where C(x) is "x is a comedian" and F (x) is "x is funny" and the domain consists of all people.

a) $\exists x(C(x) \land F(x))$

b) $\forall x(C(x) \land F(x))$

c) $\exists x(C(x) \rightarrow F(x))$

d) $\forall x(C(x) \rightarrow F(x))$

GLOSSARY QUESTION (CO4)

1. Propositional logic uses symbols to stand for statements and VALUES
2. The truth value of ‘4+3=7 or 5 is not prime
3. Single inference rule is another name
4. clause can be viewed as a single lateral of disjunction sentences
5. sentences can be created by using single propositional symbo

1. FALSE
2. TRUTH
3. unit clause
4. Atomic
5. Resolution

6. to compute the truth of any sentence
7. modus ponens are derived From which rule
8. modus ponens are derived From which rule
9. total number of proposition symbols in artificial intelligence

6 Inference rule
7. Atomic sentences
8. Semantics of propositional
9. 2

Old Question Papers(CO4)

1. Rewrite the following statements without using conditional:

(a) If it is cold, he wears a hat.

(b) If productivity increase, then wages rise.

2. Whenever Ram and Shyam are present in the party then there is some trouble in the party. Today there is no trouble in the party. Hence Ram and Shyam are not present in the party. Write it in symbolic notation

3. Let p be "Kailash reads Newsweek" , let "Kailash reads the New Yorker" and let r be "Kailash reads Times". Write each of following in symbolic form:

(a) Kailash reads Newsweeks or The New Yorker, but not Times.

(b) Kailash reads Newsweeks and The New Yorker or he does not read Newsweeks and Times.

(c) It is not true that Kailash reads Newsweek but not Times.

Old Question Papers(CO4)

4. Write the contra positive of the implication: “if it is Sunday then it is a holiday

5. Show that the propositions $p \rightarrow q and\ \neg p \vee q$ are logically equivalent.

7. Explain various Rules of Inference for Propositional Logic.

8. Prove the validity of the following argument “if the races are fixed so the casinos are crooked, then the tourist trade will decline. If the tourist trade decreases, then the police will be happy. The police force is never happy. Therefore, the races are not fixed.

9. What is a tautology, contradiction and contingency? Show that $(p \vee q) \wedge (\neg p \vee r) \rightarrow (q \vee r)$ is a tautology, contradiction or contingency.

Old Question Papers(CO4)

10.Show that the premises “It is not sunny this afternoon and it is colder than yesterday,” “We will go swimming only if it is sunny,” “If we do not go swimming, then we will take a canoe trip,” and “If we take a canoe trip, then we will be home by sunset” lead to the conclusion “We will be home by sunset.”

Old Question Papers (CO4)

Printed Page 1 of 2 Sub Code:ECS303

Paper Id: 910048 Roll No:

B. Tech.
(SEM-III) THEORY EXAMINATION 2019-20
DISCRETE MATHEMATICAL STRUCTURES

Time: 3 Hours ***Total Marks: 100***

Note: 1. Attempt all Sections. If require any missing data; then choose suitably.
2. Any special paper specific instruction.

SECTION A

1. Attempt *all* questions in brief. **2 x 10 = 20**

a. What are the ordered pairs in the less than or equal to relation, which contains (a, b) if $a \leq b$, on the set $\{0, 1, 2, 3\}$?

b. Build a digital circuit that produces the output $(p \vee \neg r) \wedge (\neg p \vee (q \vee \neg r))$ when given input bits p, q, and r.

c. Explain Ring and Field.

d. Show that De Morgan's laws hold in a Boolean algebra?

e. How many functions are there from a set with m elements to a set with n elements? How many of them are one to one?

f. Define planar and bipartite graphs.

g. Is the "divides" relation on the set of positive integers symmetric? Is it antisymmetric?

h. Show that $\neg (p \vee q)$ and $\neg p \wedge \neg q$ are logically equivalent.

i. Define Monoid, Group and abelian group. Give example.

j. Define Graph coloring. What is its application?

SECTION B

2. Attempt any *three* of the following: **10 x 3 = 30**

a. A total of 1240 student have taken a course in Spanish, 887 have taken a course in French, and 122 have taken a course in Russian. Further, 111 have taken courses in both Spanish and French, 31 have taken courses in both Spanish and Russian, and 22 have taken courses in both French and Russian. If 2100 students have taken at least one of Spanish, French, and Russian, how many students have taken a course in all three languages?

b. If H and K are any two subgroups of a group G then show that H U K can be a subgroup if and only if $H \subseteq K$ or $K \subseteq H$

c. Define a totally ordered set. Which elements of the poset ({2, 4, 5, 10, 12, 20, 25}, |) are maximal, and which are minimal, where | is divide operation?

d. Determine whether each of the compound propositions $(p \vee \neg q) \wedge (q \vee \neg r) \wedge (r \vee \neg p)$, $(p \vee q \vee r) \wedge (\neg p \vee \neg q \vee \neg r)$, and $(p \vee \neg q) \wedge (q \vee \neg r) \wedge (r \vee \neg p) \wedge (p \vee q \vee r) \wedge (\neg p \vee \neg q \vee \neg r)$ is satisfiable.

e. Use generating functions to solve the recurrence relation $a_k = 3a_{k-1} + 4^{k-1}$ with the initial condition $a_0 = 1$.

Old Question Papers (CO4)

Printed Page 2 of 2 Sub Code:ECS303

Paper Id: 910048 Roll No:

SECTION C

3. Attempt any *one* part of the following: 10 x 1 = 10

(a) Is the "divides" relation on the set of positive integers transitive? What is the reflexive and symmetric closure of the relation $R = \{(a, b) \mid a > b\}$ on the set of positive integers?

(b) Prove that 21 divides $4^{n+1} + 5^{2n-1}$ whenever n is a positive integer.

4. Attempt any *one* part of the following: 10 x 1 = 10

(a) Show that following four matrices forms a group under matrix multiplication.

$$A = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}, B = \begin{pmatrix} -1 & 0 \\ 0 & 1 \end{pmatrix}, C = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}, D = \begin{pmatrix} -1 & 0 \\ 0 & -1 \end{pmatrix}$$

(b) What do you mean by cosets of a subgroup? Consider the group Z of integers under addition and the subgroup H = {...., -12, -6, 0, 6 12, ...} considering of multiple of 6

(i) Find the cosets of H in Z

(ii) What is the index of H in Z.

5. Attempt any *one* part of the following: 10 x 1 = 10

(a) Construct a circuit for a full subtractor using AND gates, OR gates, and inverters. A full subtractor has two bits and a borrow as input and produces as output a difference bit and a borrow.

(b) Define a Lattice. Determine whether $(P(S), \subseteq)$ is a lattice where S is a set {1, 2, 3, 4}. Also Show that every totally ordered set is a lattice.

6. Attempt any *one* part of the following: 10 x 1 = 10

(a) Explain various Rules of Inference for Propositional Logic.

(b) Show that the premises "It is not sunny this afternoon and it is colder than yesterday," "We will go swimming only if it is sunny," "If we do not go swimming, then we will take a canoe trip," and "If we take a canoe trip, then we will be home by sunset" lead to the conclusion "We will be home by sunset."

7. Attempt any *one* part of the following: 10 x 1 = 10

(a) Construct the binary tree using following in-order and post-order traversal.

In-order : DBMINEAFCJGK

Post-order : ABDEIMNCFGJK

Also Find the Pre-order of the constructed Binary Tree.

(b) How many positive integers between 1000 and 9999 both inclusive

i) are divisible by 9?

ii) are not divisible by either 5 or 7?

iii) have distinct digits?

iv) are divisible by 5 but not by 7?

v) are divisible by 5 or 7?

Old Question Papers (CO4)

For some more Old Question Papers go to the link below.

https://drive.google.com/drive/folders/1LBqJvyWPNRCdAcr9Sag4TzECfnLgRIQn?usp=sharing

Expected Questions for Exam(CO4)

Q1. Construct the truth table for the compound proposition $(p \rightarrow q) \leftrightarrow (\neg p \rightarrow \neg q)$.

Q2. Write the symbolic representation and give its contra positive statement of "If it rains today, then I buy an umbrella"

Q3. Show that $(p \rightarrow r) \wedge (q \rightarrow r) and\ (p \vee q) \rightarrow r$ are logically equivalent.

Q4. Without using truth table show that $p \rightarrow (q \rightarrow p) \Leftrightarrow \neg p \rightarrow (p \rightarrow q)$.

Q5. Is $\neg p \wedge (p \vee q)) \rightarrow q$ a tautology?

Q6. What are the contra positive, the converse and the inverse of the conditional statement " If you work hard then you will be rewarded".

RECAP OF UNIT (CO4)

- Now you were able to understand the concepts discrete structures include sets, relation , functions, Algebraic Structure, poset, lattice, Boolean algebra, predicate logic and prepositional laogic etc.

- After understanding propositional logic students will be able to design of computing machines, artificial intelligence, definition of data structures for programming languages etc.

- Propositional Logic is concerned with statements to which the truth values, “true” and “false”, can be assigned.

- The subject enhances one’s ability to develop logical thinking and ability to problem solving.

Thank You