Noida Institute of Engineering and Technology, Greater Noida

Subject: Mathematics-III

Subject Code: AAS0301A

Unit: I

Complex Variable-Differentiation

Dr. Kunti Mishra

Department of Mathematics

B Tech 3rd Sem

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Brief Introduction of Faculty

Dr. Kunti Mishra
Assistant Professor
Department of Mathematics

Qualifications:
M.Sc.(Maths), M. Tech.(Gold Medalist) in Applied and Computational Mathematics, Ph.D

Ph.D. Thesis: Some Investigations in Fractal Theory
International Journal Publications: 7
International Conference Papers: 7
Area of Interests: Fixed Point Theory, Fractals
Teaching Experience: 5 years

Evaluation Scheme

NIET Greater Noida
GET FUTURE READY
Estd.2001
AN AUTONOMOUS INSTITUTE

NOIDA INSTITUTE OF ENGINEERING & TECHNOLOGY, GREATER NOIDA
(An Autonomous Institute)

B. TECH (CSE)
EVALUATION SCHEME
SEMESTER-III

Sl. No.	Subject Codes	Subject Name	Periods			Evaluation Schemes				End Semester		Total	Credit
			L	T	P	CT	TA	TOTAL	PS	TE	PE		
WEEKS COMPULSORY INDUCTION PROGRAM													
1	AAS0301A	Engineering Mathematics III	3	1	0	30	20	50		100		150	4
2	ACSE0304	Discrete Structures	3	0	0	30	20	50		100		150	3
3	ACSE0306	Digital Logic & Circuit Design	3	0	0	30	20	50		100		150	3
4	ACSE0301	Data Structures	3	1	0	30	20	50		100		150	4
5	ACSE0302	Object Oriented Techniques using Java	3	0	0	30	20	50		100		150	3
6	ACSE0305	Computer Organization & Architecture	3	0	0	30	20	50		100		150	3
7	ACSE0353	Digital Logic & Circuit Design Lab	0	0	2				25		25	50	1
8	ACSE0351	Data Structures Lab	0	0	2				25		25	50	1
9	ACSE0352	Object Oriented Techniques using Java Lab	0	0	2				25		25	50	1
10	ACSE0354	Internship Assessment-I	0	0	2				50			50	1
11	ANC0301 / ANC0302	Cyber Security*/ Environmental Science*(Non Credit)	2	0	0	30	20	50		50		100	0
12		MOOCs (For B.Tech. Hons. Degree)											
		GRAND TOTAL										**1100**	**24**

Syllabus

Unit-1 (Complex Variable: Differentiation)

Limit, Continuity and differentiability, Functions of complex variable, Analytic functions, Cauchy- Riemann equations (Cartesian and Polar form), Harmonic function, Method to find Analytic functions, Conformal mapping, Mobius transformation and their properties.

Unit-2 (Complex Variable: Integration)

Complex integrals, Contour integrals, Cauchy- Goursat theorem, Cauchy integral formula, Taylor's Series, Laurent series, Liouville's Theorem, Singularities, zero of analytic function, Residues, Method of finding residues, Cauchy Residue's theorem, Evaluation of real integral of the type $\int_0^{2\pi} f(\sin\theta, \cos\theta)d\theta$ and $\int_{-\infty}^{\infty} f(x)\, dx$

Syllabus

Unit-3 (Partial Differential Equation and its Applications)

Introduction of partial differential equations, Second order linear partial differential equations with constant coefficients. Classification of second order partial differential equations, Method of separation of variables for solving partial differential equations, Solution of one and two dimensional wave and heat conduction equations.

Syllabus

Unit-4 (Numerical Techniques)

Error analysis, Zeroes of transcendental and polynomial equations using Bisection method, Regula-falsi method and Newton-Raphson method, Interpolation: Finite differences, Newton's forward and backward interpolation, Lagrange's and Newton's divided difference formula for unequal intervals. Solution of system of linear equations, Crout's method, Gauss- Seidel method. Numerical integration: Trapezoidal rule, Simpson's one third and three-eight rules, Solution of 1st order ordinary differential equations by fourth-order Runge- Kutta methods.

Unit-5 (Aptitude-III)

Time & Work, Pipe & Cistern, Time, Speed & Distance, Boat & Stream, Sitting Arrangement, Clock & Calendar.

Branch Wise Applications

- Concept of Complex variable is used in speech recognition, image processing etc.
- Function of complex variable helps in evaluation of area, which is required in many physical formulation.

Course Objective

The objective of this course is to familiarize the engineers with concept of function of complex variables, complex variables& their applications, Integral Transforms for various mathematical tasks and numerical aptitude. It aims to show case the students with standard concepts and tools from B. Tech to deal with advanced level of mathematics and applications that would be essential for their disciplines. The students will learn:

- The idea of function of complex variables and analytic functions.
- The idea of concepts of complex functions for evaluation of definite integrals
- The concepts of concept of partial differential equation to solve partial differential and its applications.

Course Objective

- The concept of finding roots by numerical method, interpolation and numerical methods for system of linear equations, definite integral and 1^{st} order ordinary differential equations.
- The concept of problems based on Time & Work, Pipe & Cistern, Time, Speed & Distance, Boat & Stream, Sitting Arrangement, Clock & Calendar.

Course Outcome

- **CO1: Apply the working methods of complex functions for finding analytic functions.**
- **CO2:** Apply the concepts of complex functions for finding Taylor's series, Laurent's series and evaluation of definite integrals.
- **CO3:** Apply the concept of partial differential equation to solve complex variables and problems concerned with partial differential equations

Course Outcome

- **CO4:** Apply the concept of numerical techniques to evaluate the zeroes of the Equation, concept of interpolation and numerical methods for various mathematical operations and tasks, such as integration, the solution of linear system of equations and the solution of differential equation.
- **CO5:** Solve the problems of Time & Work, Pipe & Cistern, Time, Speed & Distance, Boat & Stream, Sitting Arrangement , Clock & Calendar.

Program Outcomes

S.No	Program Outcomes (POs)
PO 1	Engineering Knowledge
PO 2	Problem Analysis
PO 3	Design/Development of Solutions
PO 4	Conduct Investigations of Complex Problems
PO 5	Modern Tool Usage
PO 6	The Engineer & Society
PO 7	Environment and Sustainability
PO 8	Ethics
PO 9	Individual & Team Work
PO 10	Communication
PO 11	Project Management & Finance
PO 12	Lifelong Learning

CO-PO Mapping(CO1)

Sr. No	Course Outcome	PO1	PO2	PO3	PO4	PO5	PO6	PO7	PO8	PO9	PO10	PO11	PO12
1	**CO 1**	**H**	**H**	**H**	**H**	**L**	**L**	**L**	**L**	**L**	**L**	**L**	**M**
2	CO 2	H	H	H	H	L	L	L	L	L	L	M	M
3	CO 3	H	H	H	H	L	L	L	L	L	L	M	M
4	CO 4	H	H	H	H	L	L	L	L	L	L	L	M
5	CO 5	H	H	H	H	L	L	L	L	L	L	M	M

*L= Low *M= Medium *H= High

PSO

PSO	Program Specific Outcomes (PSOs)
PSO 1	To impart proper knowledge of science and mathematics related subjects to the students.
PSO 2	To enhance the skills of the students with the ability to implement the scientific concepts for betterment of the society in professional and ethical manner.
PSO 3	To prepare the students to understand physical system, mechanical components and processes to address social, technical and engineering challenges.

CO-PSO Mapping(CO1)

CO	PSO 1	PSO 2	PSO 3
CO.1	H	L	M
CO.2	L	M	L
CO.3	M	M	M
CO.4	H	M	M
CO.5	H	M	M

*L= Low *M= Medium *H= High

Program Educational Objectives(PEOs)

PEO-1: To have an excellent scientific and engineering breadth so as to comprehend, analyze, design and provide sustainable solutions for real-life problems using state-of-the-art technologies.

PEO-2: To have a successful career in industries, to pursue higher studies or to support entrepreneurial endeavors and to face the global challenges.

PEO-3: To have an effective communication skills, professional attitude, ethical values and a desire to learn specific knowledge in emerging trends, technologies for research, innovation and product development and contribution to society.

PEO-4: To have life-long learning for up-skilling and re-skilling for successful professional career as engineer, scientist, entrepreneur and bureaucrat for betterment of society.

Result Analysis

Name of Faculty	Dr. Kunti Mishra
Branch	**CS**
No. of Students	67
No. of Passed students	67
Result Percentage	**100%**
Branch	**IT**
No. of Students	68
No. of Passed students	68
Result Percentage	**100%**

End Semester Question Paper Template

- 100 Marks Question Paper Template.docx

Prerequisite and Recap(CO1)

- Knowledge of differentiation
- Knowledge of real valued function

Brief Introduction about the subject with videos

- We will discuss properties of complex function (limits, continuity, differentiability, Analyticity and integration)
- In 3rd module we will discuss application of partial differential equations
- In 4th module we will discuss numerical methods for solving algebraic equations, system of linear equations, definite integral and 1st order ordinary differential equation.
- In 5th module we will discuss aptitude part.
- https://youtu.be/iUhwCfz18os
- https://youtu.be/ly4S0oi3Yz8
- https://youtu.be/f8XzF9_2ijs

Unit Content

- Limit, Continuity and differentiability
- Functions of complex variable,
- Analytic functions
- Cauchy- Riemann equations (Cartesian and Polar form),
- Harmonic function
- Method to find Analytic functions
- Conformal mapping
- Mobius transformation and their properties.

Unit Objective(CO1)

- The objective of this course is to familiarize the engineers with concept of function of complex variables. It aims to show case the students with standard concepts and tools from B. Tech to deal with advanced level of mathematics and applications that would be essential for their disciplines.

Topic Objective(CO1)

- Gain some basic knowledge of complex numbers, complex plane, region in complex plane.

Complex variables (CO1)

Complex Variables

Complex Numbers:

A number of the form $x + iy$, where x and y are real numbers and $i^2 = -1$ is called a complex number and is denoted by z

It is also denoted by an ordered pair (x, y). Thus $z = x + iy$ or $z = (x, y)$

- The set of complex numbers is denoted by C
- If $z = x + iy$ is a complex number, then x is called the real part of z and denoted by Re(z).
- y is called the imaginary part of z and is denoted by $\mathrm{Im}(z)$.
- If $x = 0$ and $y \neq 0$ then $z = 0 + iy = iy$ is called a purely imaginary number

Complex variables(CO1)

- If $x \neq 0$ and $y = 0$ then $z = x + i0 = x$ is called a real number
- If $x = 0$ and $y = 0$ then $z = 0 + i0 = 0$ is the zero complex number

Conjugate Complex Number:

If two complex numbers differ only in the sign of the imaginary part then they are called conjugate complex number

- The conjugate of complex number $\boldsymbol{z}$ is denoted by $\bar{\boldsymbol{z}}$
- The conjugate of real number is real number itself.
- If $x + iy = 0$ then $x = 0$,$y = 0$

Complex variables(CO1)

- If $x_1 + iy_1 = x_2 + iy_2$ then $x_1 = x_2$ and $y_1 = y_2$
- If $x_1 + iy_1 = x_2 + iy_2$ then $x_1 - iy_1 = x_2 - iy_2$

Sum, difference and product of any two complex number is a complex number.

- If $z_1 = x_1 + iy_1$ and $z_2 = x_2 + iy_2$ then their

Sum: $z_1 + z_2 = (x_1 + iy_1) + (x_2 + iy_2)$
$= (x_1 + x_2) + i(y_1 + y_2)$

Difference: $z_1 - z_2 = (x_1 + iy_1) - (x_2 + iy_2)$
$= (x_1 - x_2) + i(y_1 - y_2)$

Product: $z_1 . z_2 = (x_1 + iy_1).(x_2 + iy_2)$
$= (x_1x_2 - y_1y_2) + i(x_1y_2 + y_1x_2)$

Polar form of complex numbers:

Let P(x, y) be the point which represent

$z = (x, y) = x + iy$

Let OP$= r$ and$= \angle POM = \theta$

Then from ΔOPM

$x = OM = r\cos\theta$ and $\theta y = PM = r\sin\theta$

$$\therefore z = x + iy = r(\cos\theta + \sin\theta) = re^{i\theta}$$

r is called the absolute value or the modulus of z and is denoted by $|z|$.

$\therefore r = |z| = \sqrt{x^2 + y^2} = \sqrt{z.\bar{z}}$

Complex variables(CO1)

- Geometrically, $|z|$ is the distance of point z from the origin.
- θ is called the argument of z or amplitude of z.
- It is denoted by arg z or Amp$z$

$$\therefore \theta = \arg z = \tan^{-1}\left(\frac{y}{x}\right)$$

- θ is the directed angle from the positive $\text{x} - axis$ to OP

Laws of complex numbers:

If z_1 and z_2 are two complex numbers, then

Triangle Inequality:

- $|z_1 + z_2| \leq |z_1| + |z_2|$
- $|z_1 - z_2| \leq \big||z_1| - |z_2|\big|$

Parallelogram Inequality:

- $|z_1 + z_2|^2 + |z_1 - z_2|^2 = 2(|z_1|^2 + |z_2|^2)$
- $|z_1 z_2| = |z_1||z_2|$
- $\left|\frac{z_1}{z_2}\right| = \frac{|z_1|}{|z_2|}$

Curves and regions in complex plane

Distance between two complex numbers :

The distance between two complex number $z_1$and z_2 is given by $|z_1 - z_2|$ or $|z_2 - z_1|$.

Circles : A circle with centre $z_0 = (x_0, y_0) \in C$ and$r \in R^+$is represented by $|z - z_0| = r$

Interior and exterior part of the circle $|z - z_0| = r$:

The set $\{z \in C, r \in R^+ / |z - z_0| < r\}$ indicates the interior part of the circle $|z - z_0| = r$ whereas $\{z \in C, r \in R^+ / |z - z_0| > r\}$ indicates the exterior part of the circle $|z - z_0| = r$

Complex variables(CO1)

Circular Disc: The open circular disc with centre z_0 and radius r is given by $z \in C, r \in R^+, |z - z_0| < r$.

The close circular disc with centre $z_0$and radius r is given by $z \in C, r \in R^+, |z - z_0| \leq r$

Neighbourhood:A open neighbourhood of a point z_0 is a subset of C contining an open circular disc centered at z_0

Annulus : The region between two concentric circle with Centre z_0 of radii $r_1$and $r_2(> r_1)$ can be represented by $r_1 < |z - z_0| < r_2$ such a region is called open circular ring or open annulus.

Complex variables (CO1)

Opened set: Let S be a subset of C. It is called an open set if for each points $z_0 \epsilon S$ there exist an open circular disk centered at z_0 which included in S

Closed set : A set S is called closed if its complement is open

Connected set:

A set A is said to be connected if any two points of A can be joined by finitely many line segments such that each point on the line segment is a point of A.

Complex variables(CO1)

Domain: A open connected set is called domain.

Region: It is a domain with some of its boundary points.

Closed region: It is a region together with a boundary points.

Bounded region: A region is said to be bounded if it can be enclosed in a circle of finite radius.

Recap(CO1)

- Complex variables.
- Curves and region in complex plane

Topic Objective(CO1)

- We will discuss properties of a complex function which enables us when limit exist? When function is continuous as well as differentiable and analytic?

Function of complex variable(CO1)

Complex function: If $z = x + iy$ and $w = u + iv$ are two complex variables and if to each point z of region R there corresponds at least on point w of a region R we say that w is a function of z and we write
$w = f(z)$

- If for each value of z in a region R of the z-plane there corresponds a unique value for w then w is called single valued function.

E.g . $w = z^2$ is a single valued function of z.

Function of complex variable(CO1)

Multi-valued function:

If for each value of z if more than one value of w exists then w is called multi-valued function.

E.g . $w = \sqrt{z}$

- $w = f(z) = u(x,y) + iv(x,y)$ where $u(x,y)$ and $v(x,y)$ are known as real and imaginary parts of the function w.

E.g . $f(z) = z^2 = (x + iy)^2 = (x^2 - y^2) + i(2xy)$

$\therefore\ u(x,y) = x^2 - y^2 \ and \ v(x,y) = 2xy$

Function of complex variable(CO1)

Limit of f(z):

- A function $w = f(z)$ is said to have the limit l as z approaches a point z_0 if for given small positive number $\varepsilon > 0$ we can find positive number $\delta > 0$ such that for all $z \neq z_0$ in a disk $|z - z_0| < \delta$,we have $|f(z) - l| < \epsilon$

Symbolically, we write $\lim_{z \to z_0} f(z) = l$

Function of complex variable(CO1)

Continuity of f(z):

A function $w = f(z) = u(x, y) + iv(x, y)$ is said to be continuous at $z = z_0$ if $f(z_0)$ is defined and $\lim_{z \to z_0} f(z) = f(z_0)$

In other words if $w = f(z) = u(x, y) + iv(x, y)$ is continuous at $z = z_0$ then $u(x, y)$and $v(x, y)$ both are continuous at (x_0, y_0) and conversely if $u(x, y)$ and $v(x, y)$ both are continuous at (x_0, y_0) then $f(z)$ is continuous at $z = z_0$.

Differentiability of $f(z)$:

The derivative of a complex function $w = f(z)$ a point z_0 is written as $f'(z_0)$ and is defined by

$\frac{dw}{dz} = f'(z_0) = \lim_{\delta z \to 0} \frac{f(z_0+\delta z)-f(z_0)}{\delta z}$ provided limit exists.

Then $f(z)$ is said to be differentiable at z_0 if we write the change $\delta z = z - z_0$

since $z = z_0 + \delta z$

$$\therefore\ f'(z_0) = \lim_{\delta z \to 0} \frac{f(z) - f(z_0)}{\delta z}$$

Analytic Functions:

- A single - valued complex function $f(z)$ is said to be analytic at a point z_0 in the domain D of the z- plane, if $f(z)$ is differentiable at z_0 and at every point in some neighbourhood of z_0.
- Point where function is not analytic (i.e. it is not single valued or not) are called singular points or singularities.
- From the definition of analytic function

(1) To every point z of R, corresponds a definite value of $f(z)$.

(2) $f(z)$ is continuous function of z in the region R.

(3) At every point of z in R, $f(z)$ has a unique derivative.

Cauchy-Riemann Equation:

- $f(z)$ is analytic in domain D if and only if the first partial derivative of u and v satisfy the two equations $\frac{\partial u}{\partial x} = \frac{\partial v}{\partial y}$,

$$\frac{\partial u}{\partial y} = -\frac{\partial v}{\partial x} \text{ ....}(1)$$

The equation (1) are called C-R equations.

Note :

(i) If $f(z)$ is analytic in a domain D , then u , v satisfy C-R equations at all points in D.

(ii) C- R Condition are necessary but not sufficient.

(iii) C- R Condition are sufficient if the first partial derivative of u , v are continuous and satisfy C- R Condition.

Daily Quiz(CO1)

Q.1 Determine whether $\frac{1}{z}$ is analytic or not .

Q.2 Show that the function $e^x(\cos y + i \sin y)$ is an analytic function , find its derivative.

Q.3 Show that $f(z) = \log z$ is analytic everywhere in the complex plane except at the origin and that its derivative is $\frac{1}{z}$.

Q.4 Show that the function $f(z) = |z|^2$ is continuous everywhere but no where differentiable except at the origin .

Q.5 Show that the function defined by $f(z) = \sqrt{|xy|}$ satisfy C-R equations at the origin but is not analytic at that point .

Daily Quiz(CO1)

Q.6 Examine the nature of the function

$$f(z) = \begin{cases} \dfrac{x^2y^5(x+iy)}{x^4+y^{10}}, & z \neq 0 \\ 0 & z = 0 \end{cases}$$

in the region including the origin.

Q.7 Prove that the function $\sinh z$ is analytic and find its derivative.

Polar form of C-R equation:

To find polar form of C – R equation

Proof: Let (r, θ) be the polar coordinates of a point (x, y).

Then, $x = r\cos\theta, y = r\sin\theta$ so that

$z = x + iy = (r\cos\theta + r\sin\theta) = re^{i\theta}$

$w = u + iv = f(z) = f\left(re^{i\theta}\right)$ ....(1)

Thus u, v are now function of r and θ.

Differentiating (1) partially w.r.t r , we get

$\frac{\partial u}{\partial r} + i\frac{\partial v}{\partial r} = f'\left(re^{i\theta}\right)e^{i\theta}$............(2)

Function of complex variable (CO1)

Again differentiating (1) partially w.r.t θ , we get

$$\frac{\partial u}{\partial \theta} + i\frac{\partial v}{\partial \theta} = f'\left(re^{i\theta}\right)ire^{i\theta} \ldots\ldots\ldots\ldots(3)$$

From (2) and (3)

$$\Rightarrow \frac{\partial u}{\partial \theta} + i\frac{\partial v}{\partial \theta} = ir\left(\frac{\partial u}{\partial r} + i\frac{\partial v}{\partial r}\right)$$

$$\Rightarrow \frac{\partial u}{\partial \theta} + i\frac{\partial v}{\partial \theta} = ir\frac{\partial u}{\partial r} - r\frac{\partial v}{\partial r}$$

Equating real and imaginary parts ,we get

$$\Rightarrow \frac{\partial u}{\partial \theta} = -r\frac{\partial v}{\partial r} \quad \text{and} \quad \frac{\partial v}{\partial \theta} = r\,\frac{\partial u}{\partial r}$$

Topic objective (CO1)

- We enable to find analytic function in terms of z if we have real or imaginary part of an analytic function.

Laplace's equation:

An equation in two variables x and y given by $\frac{\partial^2 f}{\partial x^2} + \frac{\partial^2 f}{\partial y^2} = 0$ is called as Laplace's equation in two variables.

Function of complex variable(CO1)

Harmonic functions: If $f(z) = u + iv$ be an analytic function in some region R, then Cauchy-Riemann equations are satisfied. That implies

$$\frac{\partial u}{\partial x} = \frac{\partial v}{\partial y} \quad \ldots\ldots\ldots\ldots\ldots..(1)$$

$$\frac{\partial u}{\partial y} = -\frac{\partial v}{\partial x} \quad \ldots\ldots\ldots\ldots\ldots..(2)$$

Now, on differentiating (1) with respect to x and (2) with respect to y,

we get $$\frac{\partial^2 u}{\partial x^2} = \frac{\partial^2 v}{\partial x \partial y} \quad \ldots\ldots\ldots\ldots\ldots(3)$$

$$\frac{\partial^2 u}{\partial y^2} = -\frac{\partial^2 v}{\partial y \partial x} \quad \ldots\ldots\ldots\ldots(4)$$

Function of complex variable(CO1)

Assuming that $\frac{\partial^2 v}{\partial x \partial y} = \frac{\partial^2 v}{\partial y \partial x}$, adding equations (3) and (4) we get

$$\Rightarrow \frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = 0$$

Similarly, on differentiating (1) with respect to y and (2) with respect to x and subtracting them we get

$$\Rightarrow \frac{\partial^2 v}{\partial x^2} + \frac{\partial^2 v}{\partial y^2} = 0$$

Function of complex variable(CO1)

both u and v satisfy the Laplace's equation in two variables, therefore

$f(z) = u + iv$ is called as Harmonic function. This theory is known as Potential theory.

- If $f(z) = u + iv$ be an analytic function in which $u(x, y)$ is harmonic, then $v(x, y)$ is called as Harmonic conjugate of $u(x, y)$.

Determination of Analytic function whose real or imaginary part is Known:

If the real or the imaginary part of any analytic function is given, other part can be determined by using the following methods :

(a) Direct Method

(b) Milne-Thomson’s Method

(c) Exact Differential equation method

(d) Shortcut Method

Function of complex variable (CO1)

(a) **<u>Direct Method:</u>** Let $f(z) = u + iv$ is an analytic function. If v is given, then we can find u in following steps

Step-I Find $\frac{\partial u}{\partial x}$ by using C-R equation $\frac{\partial u}{\partial x} = \frac{\partial v}{\partial y}$

Step-II Integrating $\frac{\partial u}{\partial x}$ with respect to x to find u with taking integrating constant $f(y)$

Step-III Differentiate u (from step-II) with respect to y. Evaluate $\frac{\partial u}{\partial y}$ containing $f'(y)$

Function of complex variable (CO1)

Step-IV Find $\frac{\partial u}{\partial y}$ by using C-R equation $\frac{\partial u}{\partial y} = -\frac{\partial v}{\partial x}$

Step-V by comparing the result of $\frac{\partial u}{\partial y}$ from step-III and step-IV, evaluate $f'(y)$

Step-VI Integrate $f'(y)$ and evaluate $f(y)$

Step-VII Substitute the value of $f(y)$ in step-II and evaluate u.

Function of complex variable(CO1)

Example: If $f(z) = u + iv$ represents the analytic complex potential for an electric field and $v = x^2 - y^2 + \frac{x}{x^2+y^2}$, determine the function u.

Solution: Given that $v = x^2 - y^2 + \frac{x}{x^2+y^2}$

Hence $\frac{\partial v}{\partial x} = 2x + \frac{(x^2+y^2)(1)-(x)(2x)}{(x^2+y^2)^2}$

$= 2x + \frac{y^2-x^2}{(x^2+y^2)^2}$ ....(1)

Again, $\frac{\partial v}{\partial y} = -2y - \frac{2xy}{(x^2+y^2)^2}$ ....(2)

So, u and v must satisfy the Cauchy's-Riemann equations:

$\frac{\partial u}{\partial x} = \frac{\partial v}{\partial y}$ ……………….(3) *and* $\frac{\partial u}{\partial y} = -\frac{\partial v}{\partial x}$ …….(4)

Function of complex variable(CO1)

Now, from equations (2) and (3)

$$\frac{\partial u}{\partial x} = -2y - \frac{2xy}{(x^2+y^2)^2}$$

On integrating with respect to x

$$u = -2y\int dx - 2y\int \frac{x}{(x^2+y^2)^2}dx$$

$$u = -2xy + \frac{y}{x^2+y^2} + f(y)$$

Differentiating with respect to y

$$\frac{\partial u}{\partial y} = -2x - \frac{y^2-x^2}{(x^2+y^2)^2} + f'(y) \quad \ldots\ldots\ldots\ldots(5)$$

Function of complex variable(CO1)

Again, from equations (1) and (4)

$$\frac{\partial u}{\partial y} = -2x - \frac{y^2-x^2}{(x^2+y^2)^2} \qquad \ldots\ldots\ldots\ldots.(6)$$

On comparing (5) and (6)

$f'(y) = 0$, So $f(y) = k$

Hence, $u = -2xy + \frac{y}{x^2+y^2} + k$

Function of complex variable(CO1)

Milne-Thomson's Method:

Let a complex variable function is given by

$f(z) = u(x, y) + iv(x, y)$ ....(1)

Where $z = x + iy$ and $x = \frac{z+\bar{z}}{2}$, $y = \frac{z-\bar{z}}{2i}$

Now , on writing $f(z) = u(x, y) + iv(x, y)$ in terms of z and $\bar{z}$

$$f(z) = u\left(\frac{z+\bar{z}}{2}, \frac{z-\bar{z}}{2i}\right) + iv\left(\frac{z+\bar{z}}{2}, \frac{z-\bar{z}}{2i}\right)$$

Considering this as a formal identity in the two variables

z and $\bar{z}$, and substituting $z = \bar{z}$

Function of complex variable(CO1)

$\Rightarrow f(z) = u(z, 0) + iv(z, 0)$ …………….(2)

Equation (2) is same as the equation (1), if we replace x by z and y by 0.

Therefore, to express any function in terms of z , replace x by z and y by 0.

This is an elegant method of finding $f(z)$, 'when its real part or imaginary

part is given' and this method is known as Milne-Thomson's Method.

Function of complex variable(CO1)

Example-1: Find the analytic function $f(z) = u(x,y) + iv(x,y)$, whose real part is . $x^3 - 3xy^2 + 3x^2 - 3y^2$.

Solution: Let $f(z) = u(x,y) + iv(x,y)$ is an analytic function.

Hence, from the question

$u = x^3 - 3xy^2 + 3x^2 - 3y^2$

Now , $\frac{\partial f}{\partial x} = \frac{\partial u}{\partial x} + i\frac{\partial v}{\partial x} = \frac{\partial u}{\partial x} - \mathrm{i}\frac{\partial u}{\partial y}$ [by using C.R. equation]

Hence , $\therefore \frac{\partial f}{\partial x} = \frac{\partial u}{\partial x} - \mathrm{i}\frac{\partial u}{\partial y} = (3x^2 - 3y^2 + 6x) + i(6xy + 6y)$

by using Milne-Thomson's method (replacing x by z and y by 0)

$\Rightarrow \frac{\partial f}{\partial z} = (3z^2 + 6z) + i(0)$

$\Rightarrow f(z) = (z^3 + 3z^2) + c$

Hence , $v = Im[z^3 + 3z^2 + ic]$

$= Im[x^3 - iy^3 + i3x^2y - 3xy^2 + 3x^2 - 3y^2 + i6xy + ic]$

$\therefore \quad v = 3x^2y - y^3 + 6xy + c$

Therefore,

$f(z) = (x^3 - 3xy^2 + 3x^2 - 3y^2) + i(3x^2y - \ y^3 + 6xy + c)$

Example-2: Find the analytic function $f(z) = u(x, y) + iv(x, y)$,

If $u - v = (x - y)(x^2 + 4xy + y^2)$.

Solution: Given that $u - v = (x - y)(x^2 + 4xy + y^2)$

$$\frac{\partial u}{\partial x} - \frac{\partial v}{\partial x} = (3x^2 + 6xy - 3y^2) \qquad \ldots\ldots\ldots(1)$$

$$\frac{\partial u}{\partial y} - \frac{\partial v}{\partial y} = (3x^2 - 6xy - 3y^2) \qquad \ldots\ldots\ldots.(2)$$

On applying Cauchy-Riemann's theorem in equation (2)

$$\Rightarrow -\frac{\partial v}{\partial x} - \frac{\partial u}{\partial x} = (3x^2 - 6xy - 3y^2) \qquad \ldots\ldots(3)$$

Now, from equations (1) and (3)

$$\frac{\partial u}{\partial x} = 6xy \quad \text{and} \quad \frac{\partial v}{\partial x} = 3y^2 - 3x^2$$

Function of complex variable(CO1)

Again , $\frac{\partial f}{\partial x} = \frac{\partial u}{\partial x} + i\frac{\partial v}{\partial x} = 6xy + i(3y^2 \ - \ 3x^2)$

By using Milne-Thomson's method

$\Rightarrow \frac{\partial f}{\partial z} = i(-3z^2)$

$\Rightarrow \ f(z) = i(-z^3) + c$

$\Rightarrow \ f(z) = i(-x^3 + iy^3 - i3x^2y + 3xy^2) + c$

$\Rightarrow \ f(z) = (3x^2y - y^3 + c) + i(3xy^2 - x^3)$

Exact Differential equation Method:

Let $f(z) = u(x, y) + iv(x, y)$ be an analytic function .

Case-I If $\boldsymbol{u(x, y)}$ is given

We know that $dv = \frac{\partial v}{\partial x}dx + \frac{\partial v}{\partial y}dy$

$= -\frac{\partial u}{\partial y}dx + \frac{\partial u}{\partial x}dy$ [By C-R equations]

Let $dv = Mdx + Ndy$, therefore $M = -\frac{\partial u}{\partial y}$ and $N = \frac{\partial u}{\partial x}$

Again , $\frac{\partial M}{\partial y} = -\frac{\partial^2 u}{\partial y^2}$ and $\frac{\partial N}{\partial x} = \frac{\partial^2 u}{\partial x^2}$

Function of complex variable(CO1)

Therefore $\frac{\partial M}{\partial y} - \frac{\partial N}{\partial x} = -\left(\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2}\right) = 0$ [$\because u$ is a harmonic function]

$\therefore \frac{\partial M}{\partial y} = \frac{\partial N}{\partial x}$ [Satisfies the condition for exact differential equation]

Hence , $v = \int \left(-\frac{\partial u}{\partial y}\right) dx + \int \left(\frac{\partial u}{\partial x}\right) dy + c$

Function of complex variable (CO1)

Case-II : If $v(x, y)$ is given

We know that $du = \frac{\partial u}{\partial x}dx + \frac{\partial u}{\partial y}dy$

$= \frac{\partial v}{\partial y}dx - \frac{\partial v}{\partial x}dy$[By C-R equations]

Let $du = Mdx + Ndy$, therefore $M = \frac{\partial v}{\partial y}$ and $N = -\frac{\partial v}{\partial x}$

Again , $\frac{\partial M}{\partial y} = \frac{\partial^2 v}{\partial y^2}$ and $\frac{\partial N}{\partial x} = -\frac{\partial^2 v}{\partial x^2}$

Therefore $\frac{\partial M}{\partial y} - \frac{\partial N}{\partial x} = \left(\frac{\partial^2 v}{\partial x^2} + \frac{\partial^2 v}{\partial y^2}\right) = 0$ [$\because$ v is a harmonic function]

$\therefore$ $\frac{\partial M}{\partial y} = \frac{\partial N}{\partial x}$[Satisfies the condition for exact differential equation]

Hence , $u = \int \left(\frac{\partial v}{\partial y}\right) dx + \int \left(-\frac{\partial v}{\partial x}\right) dy + c$

Example-1: If $u = x^3 - 3xy^2 + 3x + 1$, determine v for which $f(z)$ is an analytic function .

Solution: Given that $u = x^3 - 3xy^2 + 3x + 1$

By using exact differential equation method, v can be written as

$$v = \int \left(-\frac{\partial u}{\partial y}\right) dx + \int \left(\frac{\partial u}{\partial x}\right) dy + c$$

$$\Rightarrow v = \int (6xy) dx + \int (-3y^2 + 3)\, dy + c$$

$$\Rightarrow v = 3x^2y - y^3 + 3y + c$$

Function of complex variable (CO1)

Example-2 : If $u = y^3 - 3x^2y$, determine v for which $f(z)$ is an analytic function.

Solution : Given that $u = y^3 - 3x^2y$

By using exact differential equation method, v can be written as

$$v = \int \left(-\frac{\partial u}{\partial y}\right) dx + \int \left(\frac{\partial u}{\partial x}\right) dy + c$$

$$\Rightarrow v = \int (3y^2 - 3x^2) dx + \int (0)\, dy + c$$

$$\Rightarrow v = -3xy^2 + x^3 + +c$$

Daily Quiz(CO1)

Q.1. Show that the following functions are harmonic and find their harmonic conjugate functions. Also find the analytic function $f(z)$

(i) $u = \frac{1}{2} log(x^2 + y^2)$ (ii) $v = sinh\, x\, cos\, y$

Q.2. Show that the following functions are harmonic , and also find the analytic function $f(z)$

(i) $u(x, y) = x^4 - 6x^2y^2 + y^4$ (ii) $u(x, y) = x^3 - 3xy^2$

(iii) $u(x, y) = e^x\, cos\, y$

Daily Quiz(CO1)

Q.3. If $u(x, y) = e^x(x\cos y - y\sin y)$ is harmonic function, find an analytic function $f(z) = u(x, y) + iv(x, y)$ such that $f(1) = e$.

Q.4. Find an analytic function $f(z)$ whose imaginary part is $e^{-x}(x\cos y + y\sin y)$.

Q.5. If $u - v = (x - y)(x^2 + 4xy + y^2)$ and $f(z)$ is an analytic function . Find $f(z)$ in terms of z.

Recap(CO1)

- Complex variables.
- Curves and region in complex plane
- Limit
- Continuity
- Differentiability
- Analyticity
- Harmonic function
- Milne’s Thomson Method

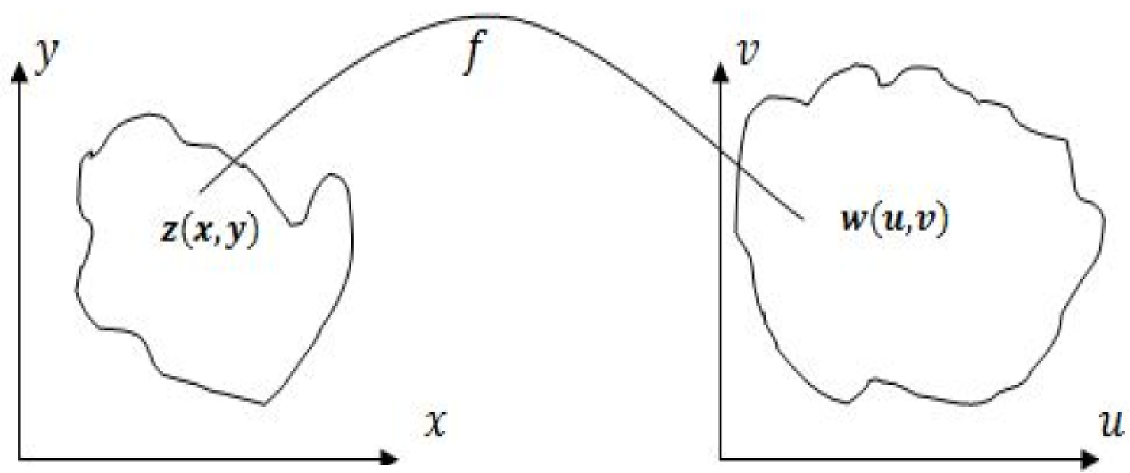
Topic objective (CO1)

- Conformal mapping can be used in scattering and diffraction problems. For scattering and diffraction problem of plane electromagnetic waves, the mathematical problem involves finding a solution to scaler wave function which satisfies both boundary condition and radiation condition at infinity.

Conformal Mapping (CO1)

Conformal Mapping: Mapping is a mathematical technique used to convert one mathematical problem and its solution into another. It involves the study of complex variables.

Let a complex variable function $z = x + iy$ define in z-plane have to convert in another complex variable function $f(z) = w = u + iv$ define in w −plane. This process is called conformal mapping.



Mapping

Conformal Mapping (CO1)

- Conformal mapping is a mathematical technique used to convert one mathematical problem and its solution into another preserving both angles and shape of infinites small figures but not necessarily their size.

<u>The necessary Condition for Conformal mapping:</u>

- If $w = f(z)$ represents a Conformal mapping of the domain D in the z-plane into a domain D of the w −plane. Then $f(z)$ is an analytic function in the domain D.

Conformal Mapping(CO1)

Example: Determine and sketch the image of $|z| = 1$ under the transformation $w = z + i$.

Solution: let $w = u + iv$

$u + iv = x + iy + i = x + i(y + 1)$

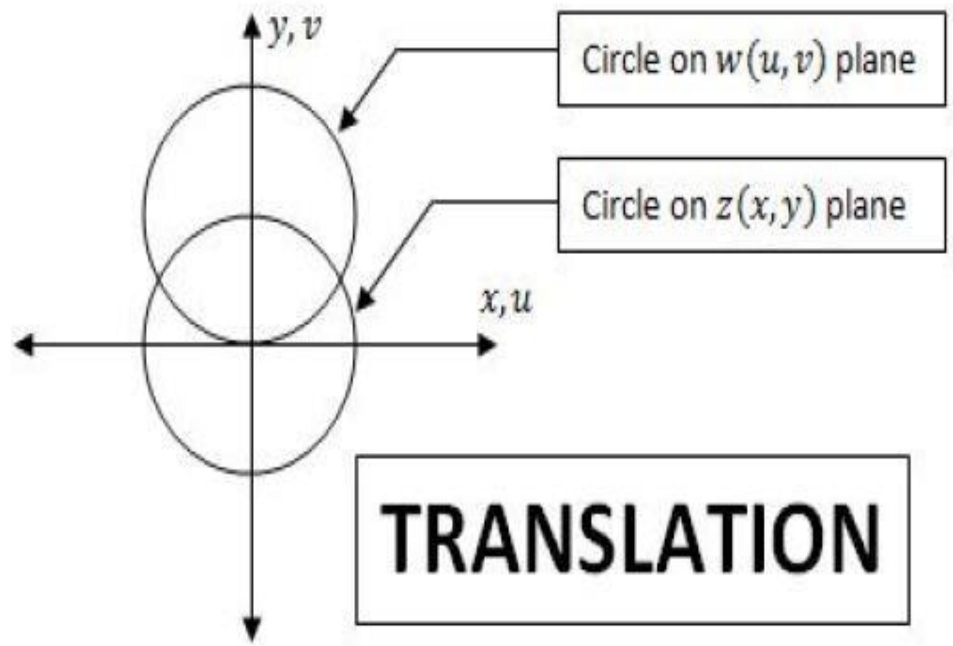
Hence $x = u$ & $y = v - 1$

Again from the question $|z| = 1$

$x^2 + y^2 = 1$

$u^2 + (v - 1)^2 = 1$, which represent a circle in (u, v) plane.

Conformal Mapping(CO1)



Daily Quiz(CO1)

1. Find the image of $x = 2$ under the transformation $w = \frac{1}{z}$.
2. Find the image of the circle $|z| = 2$ under the transformation $w = 3z$.
3. For the conformal transformation $w = z^2$, show that the coefficient of magnification at $z = 2 + i$ is $2\sqrt{5}$.
4. For the conformal transformation $w = z^2$, show that the angle of rotation at $z = 2 + i$ is $tan^{-1}(0.5)$.

Recap(CO1)

- Complex variables.
- Curves and region in complex plane
- Limit
- Continuity
- Differentiability
- Analyticity
- Harmonic function
- Milne’s Thomson Method
- Conformal Map

Topic objective(CO1)

- Bilinear Transformation is used for transforming an analog filter to a digital filter.

Bilinear/Mobius Transformation(CO1)

Bilinear Transformation: A transformation of the form $w = \frac{az+b}{cz+d}$, where a, b, c, d are complex constants and $ad - bc \neq 0$ is called a bilinear transformation.

Every bilinear transformation $w = \frac{az+b}{cz+d}$, $ad - bc \neq 0$ is the combination of the basic transformations

I. Translation: $w = z + c$

II. Rotation: $w = e^{i\alpha} z$

III. Magnification: $w = cz$

IV. Inversion: $w = \frac{1}{z}$

Mobius Transformation(CO1)

Cross-ratio: let a set of point $z = \{z_1, z_2, z_3\}$ and its mappings are $\mathrm{w} = \{w_1, w_2, w_3\}$, then the ratio

$$\frac{(w - w_1)(w_2 - w_3)}{(w - w_3)(w_2 - w_1)} = \frac{(z - z_1)(z_2 - z_3)}{(z - z_3)(z_2 - z_1)}$$

Is called cross- ratio.

Example. Determine the bilinear transformation which maps the points $z = 0, -1, i$ onto $w = i, 0, \infty$. Also, find the image of the unit circle $|z| = 1$.

Solution: The bilinear transformation mapping $z = 0, -1, i$ onto $w = i, 0, \infty$ respectively given by

$$\frac{(w - w_1)(w_2 - w_3)}{(w - w_3)(w_2 - w_1)} = \frac{(z - z_1)(z_2 - z_3)}{(z - z_3)(z_2 - z_1)}$$

Mobius Transformation(CO1)

$$\frac{(w-w_1)\left(\frac{w_2}{w_3}-1\right)}{\left(\frac{w}{w_3}-1\right)(w_2-w_1)}=\frac{(z-z_1)(z_2-z_3)}{(z-z_3)(z_2-z_1)}$$

$$\Rightarrow \frac{(w-i)(-1)}{(-1)(0-i)}=\frac{(z-0)(-1-i)}{(z-i)(-1-0)}$$

$$\Rightarrow \frac{w-i}{-i}=\frac{z(1+i)}{z-i}$$

$$\Rightarrow w-i=\frac{z(-i+1)}{z-i}$$

$$\Rightarrow w=\frac{z(-i+1)}{z-i}+i$$

$$\Rightarrow w=\frac{z+1}{z-i} \qquad \text{-(1)}$$

Which is the required bilinear transformation.

Mobius Transformation(CO1)

Eqn.(1), can be written as $z = \frac{iw+1}{w-1}$.

Now, $|z| = 1$

$\Rightarrow \left|\frac{iw+1}{w-1}\right| = 1$

$\Rightarrow |iw + 1| = |w - 1|$

$\Rightarrow |i(u + iv) + 1| = |u + iv - 1|$

$\Rightarrow |(1 - v) + iu| = |(u - 1) + iv|$

$\Rightarrow (1 - v)^2 + u^2 = (u - 1)^2 + v^2$

$\Rightarrow 1 + v^2 - 2v + u^2 = u^2 + 1 - 2u + v^2$

$\Rightarrow u - v = 0 \; or \; v = u.$

Hence the image of unit circle $|z| = 1$ in z-plane is a straight line making an angle $\pi/_4$ to u-axis and passing through origin in w-plane.

Daily Quiz(CO1)

Q1. Find the points of invariant of the transformation $w = \frac{2z+3}{z+2}$.

Q2. Find the fixed points under the transformation $w = \frac{2z-5}{z+4}$.

Q3. Find the bilinear transformations whose fixed points are 1 and 2.

Weekly Assignment(CO1)

Q1. Determine whether $\frac{1}{z}$ is analytic or not .

Q2. Show that the function $e^x(\cos y + i \sin y)$ is an analytic function , find its derivative.

Q3. Show that $f(z) = \log z$ is analytic everywhere in the complex plane except at the origin and that its derivative is $\frac{1}{z}$.

Q4. Show that the function $f(z) = |z|^2$ is continuous everywhere but no where differentiable except at the origin .

Q5. Show that the function defined by $f(z) = \sqrt{|xy|}$ satisfy C-R equations at the origin but is not analytic at that point.

Weekly Assignment(CO1)

Q6. Find an analytic function $f(z)$ whose imaginary part is $e^{-x}(x\cos y + y\sin y)$

Q7. If $u - v = (x-y)(x^2+4xy+y^2)$ and $f(z)$ is an analytic function . Find $f(z)$ in terms of z

Weekly Assignment(CO1)

Q8. Find the image of $x = 2$ under the transformation $w = \frac{1}{z}$.

Q9. Find the image of the circle $|z| = 2$ under the transformation $w = 3z$.

Q10. For the conformal transformation $w = z^2$, show that the coefficient of magnification at $z = 2 + i$ is $2\sqrt{5}$.

Q11. For the conformal transformation $w = z^2$, show that the angle of rotation at $z = 2 + i$ is $tan^{-1}(0.5)$.

Q12. Find the points of invariant of the transformation $w = \frac{2z+3}{z+2}$.

Q13. Find the fixed points under the transformation $w = \frac{2z-5}{z+4}$.

Q14. Find the bilinear transformations whose fixed points are 1 and 2.

Faculty Video Links, Youtube & NPTEL Video Links and Online Courses Details(CO1)

Video Links:

- https://www.youtube.com/playlist?list=PLzJaFd3A7DZuyLLbmVpb9e9V3Q9cYBL
- https://www.youtube.com/playlist?list=PLbMVogVj5nJS_i8vfVWJG16mPcoEKMuWT
- https://youtu.be/b5VUnapu-qs
- https://youtu.be/b5VUnapu-qs
- https://youtu.be/yV_v6zxADgY
- https://youtu.be/2ZBcbFhrfOg
- https://youtu.be/dlK0E0OG39k
- https://youtu.be/qjpLIlVo_6E

MCQ's(CO1)

1. $\lim\limits_{z \to 0} \frac{z}{\bar{z}}$

A. Limit exists

B. Limit does not exist

C. Limit exists and equal to 1

D. None of these

2. If $f(z) = \frac{z}{z^2+9}$ then

A. f(z) is continuous

B. f(z) is discontinuous at $z = \pm 3i$

C. $\lim\limits_{z \to i} \frac{z}{z^2+9} = -\frac{i}{8}$

D. Both B & C

MCQ's(CO1)

3. Analytic function is also known as

A. Regular function

B. Holomorphic function

C. Both A & B

D. None of these

4. The harmonic conjugate of the function $u = x^2 - y^2 - y$ is

A. $-4xy + y$

B. $2xy + y$

C. $2xy + x$

D. None of these

Glossary Questions(CO1)

1. Pick the correct option from glossary:

(i)$f(z) = \frac{1}{z}$

(ii) $f(z) = \cos z$

(iii) $f(z) = \log z$

(iv) $f(z) = e^{iz}$

A. Is analytic in entire complex plane except origin

B. Is analytic in entire complex plane

C. $f'(z) = \frac{1}{z}$

D. Has imaginary part $e^{-y} \sin x$

Glossary Questions(CO1)

2. Pick the correct option from glossary:

(i) $u(x, y) = x^2 + y^2$

(ii) $u(x, y) = 2x(1 - y)$

(iii) $v(x, y) = e^x$

(iv) $v(x, y) = 2xy$

A. $u(x, y)$ is not harmonic function

B. $u(x, y)$ is harmonic function

C. $v(x, y)$ is not harmonic function

D. $v(x, y)$ is harmonic function

First Sessional Paper

Printed page:2 **Subject Code: AAS0301A**

Roll No: | | | | | | | | | | | | | | |

NOIDA INSTITUTE OF ENGINEERING AND TECHNOLOGY, GREATER NOIDA

(An Autonomous Institute)

Affiliated to Dr. A.P. J. Abdul Kalam Technical University, Uttar Pradesh, Lucknow

Course: B.Tech **Branch: CSE/IT/CS**

Semester: III **Sessional Examination: I** **Year: (2020-2021)**

Subject Name: Eng. Maths III

Time: 1.15Hours **[SET-1]** **Max. Marks:30**

General Instructions:

- This Question paper consists of 2 pages & 5 questions. It comprises of three Sections, A, B, and C.
- **Section A** -Question No- 1 is objective type questions carrying 1 mark each, Question No- 2 is very short answer type carrying 2 mark each. You are expected to answer them as directed.
- **Section B** - Question No-3 is short answer type questions carrying 5 marks each. You need to attempt any two out of three questions given.
- **Section C** -Question No. 4 & 5 Long answer type (within unit choice) questions carrying 6 marks each. You need to attempt any one-part *a* or *b*.

First Sessional Paper

- No sheet should be left blank. Any written material after a blank sheet will not be evaluated/checked.
- **Blooms Level:** *K1: Remember, K2: Understand, K3: Apply, K4: Analyze, K5: Evaluate, K6: Create*

		SECTION – A	**[8]**	**CO**	**Blooms level**
1.	**Attempt all parts**		**(4×1=4)**	**CO**	
	a.	$\lim_{z \to 0} \frac{z}{\bar{z}}$ (i) Limit exists (ii) Limit does not exist (iii) Limit exists and equal to 1 (iv) None of these	**(1)**	**1**	**K5**
	b.	If $f(z) = \frac{z}{z^2+9}$ then (i) $f(z)$ is continuous (ii) $f(z)$ is discontinuous at $z = \pm 3i$ (iii) $\lim_{z \to i} \frac{z}{z^2+9} = -\frac{i}{8}$ (iv) Both B & C	**(1)**	**1**	**K2**
	c.	Function $f(z) = z\|z\|$ is (i) Analytic anywhere (ii) Not analytic anywhere (ii) Harmonic (iv) None of these	**(1)**	**1**	**K3**

First Sessional Paper

	d.	There exists no analytic function $f(z)$ if (i) $real\ f(z) = y - 2x$ (ii) $real\ f(z) = y^2 - 2x$ (ii) $real\ f(z) = y^2 - x^2$ (iv) $real\ f(z) = y - x$	(1)	1	K2
2.	Attempt all parts		(2×2=4)	CO	
	a.	Show that if $f(z)$ is analytic and $Imf(z) = constant$ then $f(z)$ is constant.	(2)	1	K3
	b.	Find the bilinear transformation which maps the points $z = 0,1,\infty$ into the points $w = i,-1,-i$ respectively.	(2)	1	K5
SECTION – B					
3.	Answer any **two** of the following-		[2×5=10]	CO	
	a.	Examine the nature of the function $f(z) = \frac{x^3y(y-ix)}{x^6+y^2}$, $z \neq 0, f(0) = 0$, prove that $\frac{f(z)-f(0)}{z} \to 0$ as $z \to 0$ along any radius vector but not as $z \to 0$ in any manner and also that $f(z)$ is not analytic at $z = 0$.	(5)	1	K4
	b.	Find the image of $\|z - 1\| = 1$ under the transformation $w = \frac{1}{z}$.	(5)	1	K5
	c.	Show that $f(z) = \cos z$ is analytic in entire complex plane.	(5)	1	K3

First Sessional Paper

		SECTION – C			
4	**Answer any one of the following-**		**[2×6=12]**	**CO**	
	a.	Determine an analytic function $f(z)$ in terms of z whose real part is $\frac{\sin 2x}{\cosh 2y - \cos 2x}$.	**(6)**	**1**	**K5**
	b.	If $w = \varphi + i\psi$ represent the complex potential for an electric field and $\psi = x^2 - y^2 + \frac{x}{x^2+y^2}$. Determine the function φ.	**(6)**	**1**	**K5**
5.	**Answer any one of the following-**				
	a.	Determine an analytic function $f(z)$ in terms of z if $3u + v = 3\sin x \cos hy + \cos x.\sin hy$.	**(6)**	**1**	**K5**
	b.	Find an analytic function $f(z)$ in terms of z if $Re[f'(z)] = 3x^2 - 4y - 3y^2$ and $f(1+i) = 0$ & $f'(0) = 0$.	**(6)**	**1**	**K5**

Second Sessional Paper

Printed page:2 **Subject Code: AAS0301A**

Roll No:

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Second Sessional Paper

		SECTION – A	**[8 Marks]**	
1	**Attempt all parts**		**(4×1=4)**	
	a.	$\int_0^{2+i}(x^2+iy)dz$ along the path $y=x$ is equal to $(i)\ \left(\frac{2}{3}+\frac{14}{3}i\right)$ $(ii)\ \left(\frac{3}{2}+\frac{3}{14}i\right)$ $(iii)\ \left(\frac{2}{3}-\frac{14}{3}i\right)$ (iv) None of these	**(1)**	**CO2**
	b.	Residue of $z\,cos\,(1/z)$ at $z\ =\ 0$ is $(i)\quad 0\qquad (ii)\ 1\qquad (iii)\quad -1/2\qquad (iv)\quad 1/2$	**(1)**	**CO2**
	c.	The region of validity for Taylor's series about $z=0$ of the function e^z is $(i)\ \ \lvert z\rvert=0\quad (ii)\ \ \lvert z\rvert<1\qquad (iii)\ \lvert z\rvert>1\qquad (iv)\ \ \lvert z\rvert<\infty$	**(1)**	**CO2**
	d.	If $f(z)=\frac{sinz}{z^4}$, then $z=0$ is (i) Removable singularity (ii) Pole of order 4 (iii) Pole of order 3 (iv) None of these	**(1)**	**CO2**

Second Sessional Paper

2.	**Attempt all parts**		**(2×2=4)**	
	a.	State Cauchy Integral formula.	**(2)**	**CO2**
	b.	Evaluate the integral $\int_C |z|\,dz$ where C is the left half of the unit circle $|z| = 1$ from $z = -i$ to $z = i$.	**(2)**	**CO2**
SECTION – B			**[10 Marks]**	
3.	**Answer any two of the following-**		**(2×5=10)**	
	a.	Verify Cauchy integral theorem for $f(z) = z^2$ taken over the boundary of square with vertices $1 \pm i, -1 \pm i$.	**(5)**	**CO2**
	b.	Using Cauchy integral formula, evaluate $\int_C \frac{z^2+1}{z^2-1}\,dz$ where C is circle $(i)\quad |z| = 3/2 \qquad (ii)\quad |z-1| = 1$	**(5)**	**CO2**
	c.	Evaluate $\int_C \frac{1}{z^2(z^2-4)e^z}\,dz$ where C is $|z| = 1$.	**(5)**	**CO2**

Second Sessional Paper

SECTION – C			**[12 Marks]**	
4	**Answer any one of the following-**		**(1×6=6)**	
	a.	Expand $f(z) = \frac{1}{(z+1)(z+3)}$ $(i)\ \ |z| < 1 \qquad\qquad (ii)\ \ 1 < |z| < 3$	**(6)**	**CO2**
	b.	State & Prove Cauchy Residue Theorem.	**(6)**	**CO2**
5.	**Answer any one of the following-**		**(1×6=6)**	
	a.	Evaluate $\int_0^{2\pi} \frac{1}{5+4\cos\theta} d\theta$ using contour integration.	**(6)**	**CO2**
	b.	Prove that $\int_0^{\infty} \frac{dx}{(x^2+1)^2} = \frac{\pi}{4}$ using contour integration.	**(6)**	**CO2**

Third Sessional Paper

Printed page:2 **Subject Code: AAS0301A**

Roll No: | | | | | | | | | | | | | | |

<u>NOIDA INSTITUTE OF ENGINEERING AND TECHNOLOGY, GREATER NOIDA</u>

(An Autonomous Institute)

Affiliated to Dr. A.P. J. Abdul Kalam Technical University, Uttar Pradesh, Lucknow

Course: B.Tech **Branch: CSE/IT/CS**

Semester: III **Sessional Examination: III** **Year: (2021-2022)**

Subject Name: Eng. Maths III

Time: 1.15 Hours **[SET-2]** **Max. Marks:30**

<u>General Instructions:</u>

- This Question paper consists of 2 pages & 5 questions. It comprises of three Sections, A, B, and C.
- **<u>Section A</u>** -Question No- 1 is objective type questions carrying 1 mark each, Question No- 2 is very short answer type carrying 2 mark each. You are expected to answer them as directed.
- **<u>Section B</u>** - Question No-3 is short answer type questions carrying 5 marks each. You need to attempt any two out of three questions given.
- **<u>Section C</u>** -Question No. 4 & 5 Long answer type (within unit choice) questions carrying 6 marks each. You need to attempt any one-part *a* or *b*.

Blooms Level: K1: Remember, K2: Understand, K3: Apply, K4: Analyze, K5: Evaluate, K6: Create

Third Sessional Paper

		SECTION – A	[8 Marks]	CO
1	**Attempt all parts**		**(4×1=4)**	
	a.	The solution of PDE $(D+4D'+5)^2 z = 0$ is (i) $z = e^{-5x} f_1(y-4x) + xe^{-5x} f_2(y-4x)$ (ii) $z = e^{-5x} f_1(y+4x) + xe^{-5x} f_2(y+4x)$ (iii) $z = e^{5x} f_1(y+4x) + xe^{5x} f_2(y+4x)$ (iv) None of these	**(1)**	**CO3**
	b.	PDE: $Bu_{xx} + Au_{xy} + Cu_{yy} + f(x, y, u, u_x, u_y) = 0$ is elliptic if ________	**(1)**	**CO3**
	c.	While solving a PDE using a Variable Separable method, we equate the ratio to a Constant which? (i) Can be Positive or Negative Integer or Zero (ii) Can be Positive or Negative rational number or Zero (iii) Must be a Positive Integer (iv) Must be a Negative Integer	**(1)**	**CO3**
	d.	$\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = 0$ is two-dimensional heat equation in ________state.	**(1)**	**CO3**

Third Sessional Paper

2.	**Attempt all parts**		**(2×2=4)**	
	a.	Find the P.I. of $(D^2 - 2DD')z = \sin x . \cos 2y$	**(2)**	**CO3**
	b.	Classify the PDE: $yu_{xx} + (x+y)u_{xy} + xu_{yy} = 0$ about the line $y = x$.	**(2)**	**CO3**
<u>SECTION – B</u>			**[10 Marks]**	
3.	**Answer any <u>two</u> of the following-**		**[2×5=10]**	
	a.	Solve the PDE $4\frac{\partial u}{\partial x} + \frac{\partial u}{\partial y} = 3u$ subject to the condition $u(0,y) = 4e^{-y} - e^{-5y}$ by method of separation of variables.	**(5)**	**CO3**
	b.	Solve the PDE: $(D^2 + DD' - 6D'^2)z = y \sin x$	**(5)**	**CO3**
	c.	Solve the PDE: $\left(D^2 - D'^2 - 3D + 3D'\right)z = xy + e^{x+2y}$	**(5)**	**CO3**

Third Sessional Paper

4	**Answer any one of the following-**		**[2×6=12]**	
	a.	A tightly stretched string with fixed end points $x = 0$ and $x = l$ is initially in a position is given by $y = y_0\, sin^3 \frac{\pi x}{l}$. If it released from rest from this position, find the displacement $y(x, t)$.	**(6)**	**CO3**
	b.	Solve the PDE $\frac{\partial^2 u}{\partial y^2} + \frac{\partial u}{\partial x} = 0$ subject to the condition: $u(x, 0) = 0, u(x, \pi) = 0, u(0, y) = 4 \sin 3y$ by method of separation of variables.	**(6)**	**CO3**
5.	**Answer any one of the following-**			
	a.	Find the temperature of the bar of length 2 whose ends are kept at zero and internal surface insulated by if the initial temperature is $sin \frac{\pi x}{2} + 3 \sin \frac{5\pi x}{2}$.	**(6)**	**CO3**
	b.	Find the solution of Laplace equation subject to the condition: $u(0, y) = u(1, y) = u(x, 0) = 0, u(x, 1) = 100 \sin \pi\, x$	**(6)**	**CO3**

Expected Questions (CO1)

Q1. If $f(z)$is a regular function of z, prove that $\left(\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial y^2}\right)|f(z)|^2=4|f'(z)|^2$.

Q2. If $u-v=(x-y)(x^2+4xy+y^2)$ and $f(z)$ is an analytic function . Find $f(z)$ in terms of z.

Q3. Find the bilinear transformation which maps the points $z=0,1,\infty$ into the points $w=i,-1,-i$ respectively.

Q4. Consider the transformation $z=\sqrt{2}e^{\frac{i\pi}{4}}z$ and determine the region R' of w –plane corresponding to rectangular region R bounded by the lines $x=0, y=0$, $x=2$ and $y=1$ in z-plane.

Recap of Unit

- Functions of complex variable
- Limit, Continuity and differentiability
- Analytic functions
- Cauchy- Riemann equations (Cartesian and Polar form),
- Harmonic function
- Method to find Analytic functions
- Conformal mapping
- Mobius transformation and their properties.

References

Text Books

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- B.S. Grewal, Higher Engineering Mathematics, Khanna Publishers, 35th Edition, 2000. 2.T.Veerarajan : Engineering Mathematics (for semester III), Tata McGraw-Hill, New Delhi.
- J.N. Kapur: Mathematical Statistics; S. Chand & Sons Company Limited, New Delhi.
- D.N.Elhance,V. Elhance & B.M. Aggarwal: Fundamentals of Statistics; Kitab Mahal Distributers, New Delhi.

Thank You