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Dear JFA Editors,

We would like to submit our paper “**Semilinear integro-differential equations, I: odd solutions with respect to the Simons cone**” to the *Journal of Functional Analysis*.

This is the first of two papers concerning saddle-shaped solutions to the semilinear equation $L_K u = f(u)$ in \mathbb{R}^{2m} , where L_K is a linear elliptic integro-differential operator and f is of Allen-Cahn type.

Saddle-shaped solutions are doubly radial, odd with respect to the Simons cone $\{(x', x'') \in \mathbb{R}^m \times \mathbb{R}^m : |x'| = |x''|\}$, and vanish only on this set. The interest on these solutions is motivated by the nonlocal version of a conjecture by De Giorgi on the Allen-Cahn equation with the aim of finding a counterexample in high dimensions. Moreover, this problem is related to the regularity theory of nonlocal minimal surfaces.

By the odd symmetry of these solutions, L_K coincides with a new operator $L_K^\mathcal{O}$ which acts on functions defined only on one side of the Simons cone, $\{|x'| > |x''|\}$, and that vanish on it. This operator $L_K^\mathcal{O}$, which corresponds to reflect a function oddly and then apply L_K , has a kernel on $\{|x'| > |x''|\}$ which is different from K . The positivity of this new kernel is of crucial importance in order to be in front of an elliptic problem.

In this first paper, we characterize the kernels K for which the new kernel is positive and therefore one can develop a theory on the saddle-shaped solution. The necessary and sufficient condition for this turns out to be that K is radially symmetric and $\tau \mapsto K(\sqrt{\tau})$ is a strictly convex function.

Assuming this, we prove an energy estimate for doubly radial odd minimizers and the existence of saddle-shaped solution. In a subsequent article, part II, further qualitative properties of saddle-shaped solutions will be established, such as their asymptotic behavior, a maximum principle for the linearized operator, and their uniqueness. To obtain these results, the setting established in this first paper is essential.

We hope that you will find the paper of interest for your journal.

Sincerely,

Juan-Carlos Felipe-Navarro and Tomás Sanz-Perela