① $h:(0,\infty)\to(0,\infty)$

h convex function

decreasing

recta que passe per $(y,h(y))$ i amb pendent $h'(y)$

$$\Rightarrow \underbrace{h(y)+h'(y)(t-y)} \underset{\text{convexity}}{\le} h(t)$$

lo haces para toda pendiente

take $t = r/2$

$y = r$

$$-\underset{\le 0}{h'(r)}\,\frac{r}{2} \le \underbrace{h(r/2)-h(r)}_{\ge 0}$$

h decreixent

$$\Rightarrow |h'(r)| \le \frac{2}{r}\,|h(r/2)-h(r)| \qquad \boxed{r>0}$$

$$\le \frac{2}{r}\operatorname*{osc}_{[r/2,\,r]} h$$

hacerlo con deriv. por +, − si no es regular.

Take $h(t) = k(\sqrt{t}) \Rightarrow h'(t) = \dfrac{k'(\sqrt{t})}{2\sqrt{t}}$

k es C' a.e.

$$|k'(\sqrt{t})| \le 2\frac{\sqrt{t}}{t}\underbrace{\operatorname*{osc}_{t/2,\,t} k(\sqrt{\cdot})}_{\le\, C t^{-\frac{n+2s}{2}}}$$

ellipticity

$$|k'(\sqrt{t})| \le C\, t^{-\frac{1+n+2s}{2}}$$

$$\boxed{\Rightarrow |k'(r)| \le \underbrace{C}_{C(n,s,\lambda,\Lambda)}\, r^{-n-2s-1} \qquad \forall r>0}$$

1 $\Rightarrow$

- $K(t)$ be Lipschitz in $(0,+\infty)$
- $[K]_{Lip(B_r^c)} \leq C_{n,s,\lambda,\Lambda}\, r^{-n-2s-1} \quad r>0$

$\boxed{\alpha \in (0,1)}$

2 $\quad L_K w = h$ in B_2

consider $\eta \qquad \eta \equiv 1$ in $B_{3/2}$
$\qquad\qquad\qquad \eta \equiv 0$ in B_2^c

$$\tilde{w} = w\eta = w(1-(1-\eta))$$

$$L_K \tilde{w} \overset{B_2}{=} L_K w - L_K(w(1-\eta)) = h - \tilde{h}$$

$x \in B_1$

$$\tilde{h}(x) = \int_{\mathbb{R}^n \setminus B_{3/2}} -w(y)(1-\eta(y))K(x-y)\,dy$$

$\boxed{\|\tilde{h}\|_{C^\alpha(B_1)}?}$

$K(z) \leq \frac{\Lambda}{|z|^{n+2s}}$ + $\quad |x-y| \geq |y|/4$ (next page)

$$|\tilde{h}(x)| \leq C\int_{\mathbb{R}^n \setminus B_{3/2}} \frac{|w(y)|}{|y|^{n+2s}} \leq C\int_{\mathbb{R}^n} \frac{|w|}{(1+|y|)^{n\dots}}$$

$$\frac{|\tilde{h}(x)-\tilde{h}(z)|}{|x-z|^\alpha} \leq C\int |w(y)| \frac{|K(x-y)-K(z-y)|}{|x-z|^\alpha}\,dy \leq C|x-z|^{1-\alpha}\int \frac{w}{(1+|y|)\dots} \leq 2\dots$$

next page

$$|x-y| \geq |y|/4$$
$$|z-y| \geq |y|/4$$

$$|x| \leq 1 \leq \tfrac{3}{2} \cdot \tfrac{3}{4} \leq \tfrac{3}{4}|y|$$
$$\Rightarrow |x-y| \geq |y| - |x| \geq |y|/4$$

$$|K(x-y) - K(z-y)| \leq |x-z| \, [K]_{Lip(B^c_{|y|/4})}$$

$$\leq C \frac{|x-z|}{|y|^{n+2s+1}} \leq C \frac{|x-z|}{|y|^{n+2s}}$$

$$\Rightarrow \quad \|\tilde{h}\|_{C^\alpha(\overline{B_1})} \leq C \int \frac{|w|}{(1+|y|)^{n+2s}} \, dy$$

$$\|\tilde{w}\|_{C^\alpha(\mathbb{R}^n)} \leq C \|w\|_{C^\alpha(\overline{B_2})}$$

"Schauder" estimates

$$L_K \tilde{w} = g \quad \text{in } B_1$$

$$\|\tilde{w}\|_{C^{\alpha+2s}(\overline{B_{1/2}})} \leq C_{n,s,\lambda,\Lambda} \left(\|g\|_{C^\alpha(\overline{B_1})} + \|\tilde{w}\|_{C^\alpha(\mathbb{R}^n)} \right)$$

$$\|\tilde{w}\|_{C^{\alpha+2s}(\overline{B_{1/2}})} = \|w\|_{C^{\alpha+2s}(\overline{B_{1/2}})}$$

$$\leq C \left(\|w\|_{C^\alpha(B_2)} + \|h\|_{C^\alpha(\overline{B_1})} + \int \frac{w}{(1+|y|)^{n+2s}} \, dy \right)$$