

## An Integrated Approach with Permutation Entropy Measure and Conventional Tests for Study on Stock Market Efficiency

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### Abstract

*Countries globally strive to create efficient stock markets to stimulate economic growth and encourage more investors. This paper deals with testing the weak form of the Efficient Market Hypothesis (EMH) using daily data across 57 global stock markets spanning from 2013 to 2023. Moreover, the study focuses on employing the combination of the Permutation Entropy measure to compute the weak form efficiency degree and several conventional tests to obtain a sufficient evaluation of the market efficiency in the sample. The traditional test results provide inconsistent conclusions on the weak form efficiency as the data series are non-normal distributed but follow a random walk pattern. However, combined with the high Permutation entropy scores estimated for the sample, it is concluded that all 57 markets are weak-form efficient. Furthermore, despite some exceptions, the developed markets tend to display the highest degrees of efficiency, followed by emerging markets, while frontier markets show the weakest efficiency.*

**Keywords:** EMH, Permutation Entropy, Weak Form Efficiency, Random Walk, Stock Market.

### Introduction

The hypothesis of market efficiency has challenged researchers globally since its appearance in the work of Fama (1970). The Efficient Market Hypothesis (EMH) assumes that the equity price incorporates all available information fully and promptly. It has stood as one of the foundational theories in finance, underpinning numerous pricing models and decision-support systems. There are three forms of market efficiency: weak form, semi-strong form, and strong form. The classification of market efficiency into these levels depends on the nature of information absorbed by the stock price. Among them, the lowest level - weak form efficiency exists when the stock price fully reflects the historical stock data, leaving no room for making abnormal profits from analyzing the relevant information in the past. At this efficiency level, the stock returns follow a random walk pattern that prevents future returns from being predicted using past data.

Achieving market efficiency, particularly in the stock market, has the potential to boost the economy and increase investor involvement. Therefore, market efficiency is not only a mere theoretical concept but also a market condition that any market aims to reach because efficiency means a high level of transparency that benefits the market's sustainability. Therefore, analyzing EMH, especially weak form efficiency, is important to investors, regulators, and researchers.

Investors who seek opportunities for excessive profit from obtaining and interpreting the historical data of price and volume can be recognized as technical analysis users. The technical analysis trading approach tries to identify the signals for stock buying and selling. If the weak form efficiency is held, there is no more opportunity for abnormal returns by solely using signals to trade. Therefore, the results of testing weak form efficiency provide investors with important suggestions and considerations for their investing strategy. For regulators, weak form efficiency requires constructing and maintaining strong regulations on

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information announcements so that all historical stock data is available and free of access to all market participants.

For researchers, the validity of the EMH, particularly its weak form, has been extensively examined (Assaf et al., 2021; Mobarek and Finorante, 2014; Terence et al., 2009; Zanin et al., 2012; Zhao et al., 2020). An overview of relevant papers reveals that the works on this topic mainly focus on testing the existence of efficiency by analyzing the pattern that stock price follows over time. In these works, the independence of current and past stock prices is evidence of price randomness, meeting the requirement of weak form efficiency.

The classical analysis methods consisting of auto-correlation, unit-root, and variance ratio tests have been applied widely (Gupta and Basu, 2007; Rosch et al., 2016; Worthington and Higgs, 2005). Although this traditional approach is still popular, it is limited because the analysis results only classify a market as efficient or inefficient without estimating the efficiency degrees. As an alternative, the entropy method has proved to be a novel approach, offering advantages in speed and effectiveness over conventional technical methods. Among various entropy measures, Permutation Entropy has become a vital tool in finance for measuring complexity, volatility, and forecasting, especially in non-linear time series making it useful in market efficiency evaluation.

Despite the abundance of studies on weak form market efficiency, there is a research gap described in some main points as follows:

Most papers assess weak form market efficiency by solely applying conventional methods or entropy measures, limiting the understanding of market efficiency in different aspects such as the level of efficiency. Moreover, there are contrasting conclusions from performing different tests using the traditional approach as it is efficient from one test's results but inefficient following results from others. While the entropy method succeeds in evaluating the weak form efficiency, a combination approach of using entropy calculation with the traditional method can obtain a stronger conclusion on the market efficiency.

Existing researchers are interested in comparing the weak form efficiencies in emerging and developed markets, they tend to ignore the investigation of efficiency in frontier markets although the frontier markets are increasingly attracting investors, not only domestic investors but also international investors.

This study aims to fill in the gap by testing the weak form efficiency of 57 markets in different levels of development from 2013 to 2023 using the Permutation Entropy measure integrated with the traditional approach. There are two research questions stated: (1) In which way the Permutation entropy can support the traditional approach to examine the weak form market efficiency? (2) Are there any differences in weak form efficiency degrees of different market groups according to the MSCI framework? The analysis results are expected to be useful for regulators to implement policies to enhance market efficiency, and for investors to design the appropriate investment strategies.

The remainder of the paper is organized as follows. Section 2 provides an overview of related research on stock market efficiency and weak form efficiency measurement. In Section 3, we detail the data sources and describe the methodology applied in this study. The efficiency analysis results are presented and discussed in Section 4. Finally, Section 5 offers the concluding remarks.

## Literature Review

### *Overview of the Efficient Market Hypothesis*

EMH is a widely discussed concept in modern finance theory. This theory was proposed by Fama (1970), who won the Nobel Prize in 2013. The center of EMH, understood as information efficiency, lies in the belief that the stock price instantly and fully reflects all related information available on the market. There is no room for price arbitrage, thus, investors cannot be expected to earn excess profits from the market.

Three forms of market efficiency, including *weak form*, *semi-strong form*, and *strong form*, differ in degrees but share the same basic theory. In weak form efficiency, the prices of stocks traded on the market follow a “Random Walk”. Therefore, predicting future stock prices using historical information is unproductive because such information is already incorporated into current prices, making it difficult to obtain excess returns (Fama 1991). The weak form efficiency suggests that investors should stop applying technical analysis for abnormal profit identification because technical analysis relies on historical prices and volumes to forecast future stock price movements. However, in a weak form efficient market, investors still have opportunities to yield higher-than-market-average profits by exploiting fundamental and inside information.

Within the semi-strong form of EMH, stock prices incorporate rapidly and fully not only the past data of stock prices and trading volumes but also the public information. Examples of public information can be announcements on the company’s earnings or any information available forming investors’ investment decisions in this stock. Under the framework of the semi-strong form, it is believed that investors cannot consistently gain abnormal returns by analyzing historical and current data. However, there is an opportunity to earn excess returns if investors have an advantage of accessing private information.

The strong form of the EMH is the highest level of efficiency as the stock prices fully and promptly accommodate all information including historical data, public information, and private information. As a result, arbitrage opportunities are ruled out, and there is no scope for obtaining excess returns (Fama 1970).

From the introduction of the market efficiency concept, numerous studies worldwide have employed diverse methodologies to evaluate market efficiency, mostly focusing on measuring the weak form of the market. The primary and pivotal step includes testing the hypothesis of random walk price patterns, focusing on the relationship between the historical and present stock prices. Despite the prolificacy of research in exploring evidence of market efficiency, there is still an inconsistency in the findings. Various factors such as methodological approaches, evaluated markets, data frequency, and evaluation periods can influence study outcomes.

In research conducted by Worthington and Higgs (2005), a group of emerging and developed stock markets in the sample are concluded to be weak form efficient according to some test results. However, the conclusion changes for the developed markets after the variance ratio tests are applied. The Russian equity market is considered inefficient according to Terence et al. (2009) but efficient due to the analysis results in the work of McGowan (2011). Abrosimova et al. (2002) apply auto-correlation, unit-root, variance ratio tests, and model comparison approaches to stock price data in Russia and confirm the existence of weak form market efficiency in this market. Having the same study purpose, Mobarek and Finorante (2014) believe the markets of BRIC countries, including Russia, are increasing efficiency, to be fairly weak-form efficient. They also state that the results from the variance ratio and run tests are better than the ones from the serial correlation test. Urrutia (1995) also finds the resulting conflict when applying different methods for efficiency checking in Latin American countries. While the variance ratio test denied the existence of the existence of random walk, the runs test suggested its existence, thus implying markets are in weak form efficiency.

Numerous studies have highlighted factors influencing market efficiency, providing valuable insights for future research directions. Chung and Hrazdil (2010) investigate market efficiency across a sample of 193 firms selected from the largest companies trading in NYSE, and find that trading volumes tend to exhibit stronger market efficiency. Zhao (2012) and Himmelmann et al. (2012) state that market capitalization contributes to improving market efficiency. Other factors have significant impacts on market efficiency such as financial liberalization (Rejeb and Boughrara, 2013), and market integration (Aawaar, Tewari, and Liu, 2017).

### *Weak Form Efficiency Measurement*

The central idea of weak form efficiency leads to a significant number of papers trying to confirm the appearance of this efficiency in the stock markets globally. The measures used focus on examining the

ability to predict returns from historical data (Mobarek et al., 2008). They can be divided into the traditional approach that tests the random walk behavior of stock prices (Gupta and Basu, 2007) and alternative measures to recognize the predictable patterns of stock returns (Rosch et al., 2016).

The conventional methods consist of normality tests, auto-correlation tests (Laurence, 1986), and variance ratio tests (Lo, 1989). In the work of Rosch et al. (2016), the authors use the intraday returns to examine the power to predict the returns at short time horizons, like Hasbrouck and Ho (1987), and Chordia et al. (2005). They also estimate the pricing errors known as Hasbrouck errors (Hasbrouck, 1993) and Put-call parity deviations as inefficiency measures. Moreover, Rosch et al. (2016) construct a systematic index that statistically combines different measures into one because they find a co-movement in efficiency across various measures. Interestingly, the approach of examining market efficiency has been developed by incorporating physics into solving economic issues. That is entropy, a physical concept, introduced by Clausius (1865) to capture the stochastic relationship among time series. An increasing number of studies have utilized entropy in time series research in the finance world, specifically in assessing the efficiency hypothesis of stock markets.

According to Darbellay and Wuertz (2000), the advantage of using entropy for financial modeling lies in its capacity to account for nonlinear dependencies. While the assessment result from the conventional approach falls into only one of 2 categories of efficiency and inefficiency, the entropy measure provides the relative levels of efficiencies among different markets or one market in various periods. Therefore, entropy is considered the “powerful concept to characterize the diversity of patterns contained in a time series” (Alvarez-Ramirez et al., 2012). From the first form of entropy that was applied for efficiency assessment (Gulko, 1999), the estimation of entropy has been improved and classified into different types. It is also applied widely in papers conducted in different markets, such as crude oil (Messi et al., 2012) and the art market (Assaf et al., 2021).

Risso (2008) introduces the Shannon Entropy which receives 1 for market efficiency and 0 for market inefficiency. Using the data of stock indices in Japan, Malaysia, Russia, Mexico, and the US, the author measures the efficiency in those markets and reveals the positive relationship between market efficiency and stock crash probability. Shternshis et al. (2022) apply several steps to filter out some sources of efficiencies before computing the Shannon entropy to score the efficiency of the ETF market, called the filtering out method originated by Calcagnile et al. (2020).

The Approximate entropy proposed by Pincus (1991) is considered to be better at dealing with the system noise and, thus, can be an indicator of market stability (Patra and Hiremath, 2022; Pincus and Kalman, 2004). The higher the Approximate entropy estimate, the more efficiency the market achieves (Assaf et al., 2021). Similar to the Approximate entropy, the Sample entropy introduced by Richman and Moorman (2000) is created to avoid biased statistics (Delgado-Bonal and Marshak, 2019). However, both Approximate Entropy and Sample Entropy are criticized for providing a higher value than the real one (Richman and Moorman, 2000).

The application of Shannon Entropy, Sample Entropy, and Approximate Entropy in time series analysis has gained popularity in recent years (Riedl, Müller, and Wessel, 2013), because of their fast and accurate results, making them well-suited to contemporary research. Nevertheless, there are still certain limitations to these methods, as they do not account for the relations between time series values. They also receive complaints of not working well with nonlinear data systems (Zanin et al., 2012).

To address these limitations, Permutation Entropy was developed by Bandt and Pompe as an alternative method that considers the orderliness of information sets in the sequence by comparing the neighboring values in a series, thereby improving the accuracy of research results. Permutation Entropy has significant practical applications, and the most common use is to test the patterns and regularities of sequences in studying efficient market theory based on Fama's (1970) theory (Zanin et al., 2012). Zunino et al. (2009) utilize the estimate of permutation to score the market inefficiency in a group of 32 equity indices and returns from various countries from 1995 to 2007. The authors conclude that Permutation Entropy is positively correlated with the degree of market efficiency. The authors also analyze the forbidden patterns,

patterns that cannot be discovered because of the chaotic nature of the data series, and find a similar association between the presence of forbidden patterns and the level of market inefficiency.

Hou et al. (2017) employ Permutation Entropy to assess market efficiency and made a key finding. A decrease in the Permutation Entropy due to market impact over one year predicts a stock price decline for at least six months thereafter, offering investors a useful signal for stock price forecasting. Zhao et al. (2020) propose a more complex entropy measure, called Permutation Transition Entropy which they believe is an improvement from the original one. Working with the data in Chinese stock markets, the Permutation Transition Entropy is assessed to be successful in analyzing the complexity of the market without any assumptions on the data series.

A review of existing papers shows that Permutation Entropy has been applied in various studies related to measuring volatility, complexity, randomness, and forecasting ability of time series and serves as an effective tool for researchers and practitioners in examining market efficiency. However, the spreading of this approach is limited due to the lack of a free computational program. Therefore, Pessa and Ribeiro (2021) introduce *Ordpy*, an open-source Python package that implements various tools related to ordinal patterns for analyzing time series and images, thereby overcoming its limitations, and facilitating the broader adoption of ordinal methods, making it a preferred choice for our study.

## Research Methodology

### Description of Data Sample

The database contains daily prices of 57 stock indices representing each country's markets. The sample spans from January 15, 2013, to December 31, 2023, collected from <https://finance.yahoo.com> and <https://www.investing.com/>. The sample is categorized into three groups: developed, emerging, and frontier markets according to MSCI market classification as shown in Table 1.

**Table 1.** Markets in the Data Sample

Developed market		Emerging market		Frontier market	
Australia	Japan	Brazil	Philippines	Bulgaria	Serbia
Austria	New Zealand	Chile	Poland	Croatia	Slovenia
Belgium	Norway	Colombia	Qatar	Estonia	Sri Lanka
Canada	Portugal	Czech Republic	Saudi Arabia	Iceland	Tunisia
China (Hong Kong)	Singapore	Egypt	South Africa	Jordan	Vietnam
Denmark	Spain	Greece	Taiwan	Lithuania	
France	Sweden	Hungary	Thailand	Mauritius	
Germany	Switzerland	India	Turkey	Morocco	
Ireland	The Netherlands	Indonesia		Nigeria	
Israel	The UK	Korea		Oman	
Italy	The USA	Mexico		Pakistan	

From a sample of 57 markets, we identify an index representing each market, gathering historical price data and relevant information from January 15, 2013, to December 31, 2023. The below formula computes the index return of the stock index  $i$  in day  $t$  ( $r_{it}$ ):

$$r_{i,t} = \frac{P_{i,t}}{P_{i(t-1)}} \times 100$$

Where the  $P_{i,t}$  and  $P_{i(t-1)}$  are the index price  $i$  in day  $t$  and  $t-1$ .

Our study applied an integrated method that combines the traditional approach (tests of normal distribution, stationary, and autocorrelation) and Permutation Entropy estimate, an advanced approach receiving increasing interest recently for efficiency degree calculation.

#### *Test for Normal Distribution*

A series following a random walk is normally distributed. Therefore, the Jarque-Bera test is used to examine the normality of the historical return series. H<sub>01</sub> describes the null hypothesis for the normality test.

H<sub>01</sub>: The index return series is normally distributed.

#### *Stationary Test*

A time series follows a random walk, it is non-stationary or contains a unit root. The Augmented Dickey-Fuller (ADF) test is applied to examine the existence of a unit root in price data with H<sub>02</sub> as the null hypothesis.

H<sub>02</sub>: The index return series is non-stationary.

To strengthen the findings of the ADF test, Brooks (2014) recommends conducting a supplementary KPSS test. The KPSS test assumes that the time series is stationary by default (H<sub>03</sub>: The index return series is stationary), and it seeks to reject this hypothesis if the series is non-stationary. If the KPSS test fails to reject hypothesis H<sub>03</sub>, it indicates that the stock price series does not follow a random walk pattern. At 5% of the significance, the critical value of 0.463 is the threshold for producing the decision of null hypothesis rejection or acceptance (Kwiatkowski et al.,1992).

#### *Autocorrelation Test*

In a weak form efficient market, the stock price series absorbs the available related information rapidly and cannot be predicted from the price pattern in the past. Therefore, there should be no systematic relationship between the historical and future returns. The autocorrelation test determines whether the autocorrelations significantly deviate from zero. The null hypothesis for testing is H<sub>04</sub>.

H<sub>04</sub>: There is no autocorrelation between index return series.

#### *Permutation Entropy Estimates for Evaluating Stock Market Efficiency*

In this study, we examine the time-varying efficiency using Permutation Entropy developed by Bandt and Pompe (2002). This method combines entropy calculations based on the symbolic representation of time series while maintaining the orderliness of the sequence. Permutation Entropy offers several advantages, including simplicity, fast calculation, and robustness for analyzing various types of time series, such as chaotic, random, and real-world datasets.

Our approach to measuring informational efficiency using permutation entropy ensures consistency across different timescales and does not affect stock ranking. We calculate permutation entropy following a specific module *Ordpy*, the Python programming language was selected due to its widespread adoption in scientific computing and its robust community support.

The probability distribution resulting from this method is known as the ordinal distribution or distribution of ordinal patterns. The Bandt–Pompe (2002) approach is utilized to symbolize the data for estimating this distribution. To illustrate this process, let's consider a time series  $\{x_t\}_{t=1,\dots,N_x}$

Initially, a time series is divided into  $n_x = N_x - (d_x - 1)\tau_x$  overlapping partitions. Each partition comprises  $d_x > 1$  observation separated by  $\tau_x \geq 1$  time units. According to Bandt and Pompe's

recommendation, it is typical that we select values of  $d_x = \{4,5\}$  to ensure that  $d_x! \ll N_x$  for a reliable estimate of the ordinal probability distribution.

For a known  $d_x$  and  $\tau_x$   $w_p$  data partition can be computed by the below formula:

$$w_p = (x_p, x_{p+\tau_x}, x_{p+2\tau_x}, \dots, x_{p+(d_x-2)\tau_x}, x_{p+(d_x-1)\tau_x})$$

Where  $d_x$  is the embedding dimension and  $\tau_x$  is the embedding delay.

The permutation can be defined as  $\pi_p = (r_0, r_1, \dots, r_{d_x-1})$  of the series from 0 to  $d_x-1$  that arranges  $w_p$  in ascending order. The ordinal probability distribution  $P = \{\rho_i(\Pi_i)_{i=1,\dots,n_\pi}\}$  describes the frequency of all possible permutations. With the ordinal probability distribution P, we can compute its Shannon entropy and define the permutation entropy as:

$$H(P) = - \sum_{i=1}^{n_\pi} p_i(\Pi_i) \log p_i(\Pi_i)$$

Where the values of H are constrained within the interval [0,1] meaning that if the maximum value H=1 is attained, it signifies the state of a purely random string. The *Ordpv*'s function *permutation\_entropy* calculates the value of H directly from a time series. A high H indicates greater complexity.

### Analysis Results and Discussions

#### Market Efficiency Assessment Results from the Conventional Approach

From 57 sample markets, we select the stock index presenting one country's market with the priority criterion of having the largest number of components to cover the market. If the index fails to meet the criteria set for our research scope (e.g., launched before 15/1/2013) or lacks valuable information, an alternative representative index will be selected (one comprising the largest companies or those with the highest capitalization in the market).

Table 2 describes stock indices for 57 markets in the sample together with the corresponding standard deviations. A return standard deviation estimates the price dispersion from the mean. The higher the standard deviation, the greater the market volatility which may create opportunities for traders to earn profit. However, a higher standard deviation also reflects the market's sensitivity which can be magnified by the investor's sentiment.

**Table 2.** Description of Worldwide Stock Price Indices

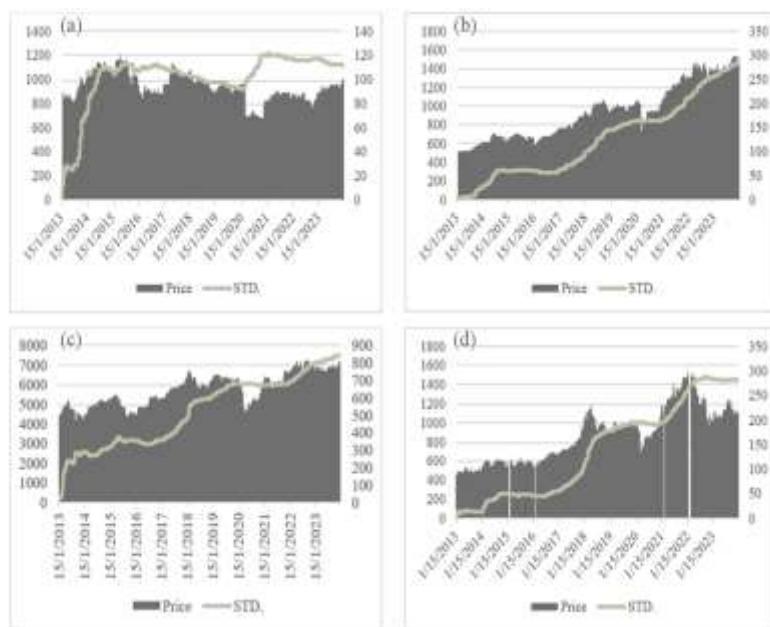
	Market	Market Index	Std.		Market	Market Index	Std.
1	Brazil (E)	BVSP	26,783.692	30	Portugal (D)	PSI 20	684.591
2	Hungary (E)	BUX Index	11,522.952	31	Denmark (D)	OMXCGI	650.142
3	Nigeria (F)	NSE	10,548.170	32	Belgium (D)	OMXCGI	650.142
4	South Africa (E)	FTSE/JSE All Share	9,916.474	33	Iceland (F)	OMXIFI	531.353
5	Pakistan (F)	KSE-100	8,207.636	34	Qatar (E)	QE Index	485.229
6	India (E)	S&P BSE 500	5,876.571	35	Austria (D)	ATX	479.318
7	Japan (D)	NIKKEI225	5,393.144	36	Estonia (F)	OMXT	384.093
8	Mexico (E)	S&P/BMV IPC	4,629.058	37	New Zealand (D)	S&P/NZX	366.353

9	Egypt (E)	EGX 30	3,904.729	38	South Korea (E)	KOSPI	355.586
10	Chile (E)	S&P/CLX IGPA	3,857.779	39	The U.K (D)	FTSE all share	311.346
11	Italy (D)	FTTILMS	3,600.528	40	Norway (D)	OSEAX	290.759
12	China (Hong Kong) (D)	SSEC	3,474.489	41	Vietnam (F)	VN Index	282.613
13	Taiwan (E)	TAIEX	3,188.764	42	Poland (E)	WIG30	281.447
14	Canada (D)	S&P/TSX Composite	2,589.358	43	Israel (D)	TA-125	268.679
15	The U.S.A (D)	NYSE Composite	2,297.708	44	Jordan (F)	ASE General	237.201
16	Germany (D)	DAX	2,272.270	45	Serbia (F)	BELEX line	223.121
17	Turkey (E)	BIST all-share	1,954.728	46	Singapore (D)	STI	203.017
18	Saudi Arabia (E)	TASI	1,826.549	47	Lithuania (F)	OMXV	199.133
19	Sri Lanka (F)	ASPI	1,640.805	48	Croatia	CROBEX	195.208
20	France (D)	CAC all share	1,414.796	49	Mauritius (F)	SEMDEX	194.007
21	Morocco (F)	MASI	1,355.461	50	Colombia (E)	COLCAP	189.789
22	Switzerland (D)	SMI	1,336.989	51	Slovenia (F)	SBI TOP	186.203
23	Ireland (D)	ISEQ All-share	1,264.577	52	Sweden (D)	OMXSPI	169.227
24	Tunisia (F)	TUNINDEX	1,261.325	53	Netherland (D)	AEX	154.907
25	Oman (F)	MSM 30	1,091.517	54	Czech Republic (E)	PX Index	154.684
26	Australia (D)	S&P ASX	875.203	55	Bulgaria (F)	BGTR 30	134.647
27	Greece (E)	ATF	708.608	56	Thailand (E)	SET	134.328
28	Indonesia (E)	JKSE	704.368	57	Spain (D)	SMSI	112.118
29	Philipines (E)	PSEi	704.368				

\*Developed, emerging, and frontier stock markets following the MSCI framework are denoted as (D), (E), and (F) respectively.

Figure 1 illustrates the deviation from the average price index of selected indices which are representatives of developed markets, emerging markets, and frontier markets. As observed in the figure, the developed markets, panels (a) and (c), have smaller price fluctuations than the emerging and frontier markets (panels b&d).

**Figure 1.** Price And Standard Deviation of Several Stock Markets



Note: Panel (a), (b), (c), and (d) represent Spain, Indonesia, Norway, and Vietnam respectively.

#### Normal Distribution Test Results

Table 2 summarizes the results of the Jarque-Bera test applied to various stock market indices across different countries. Only Spain (SMSI) and Jordan (ASE General) have result values other than 0 but still smaller than 5% indicating low normality. Every other listed stock market index has a p-value of 0, indicating strong evidence against the null hypothesis of normality. This suggests that the return distributions of these indices are not normally distributed, often exhibiting skewness or fat tails. Therefore, all the selected stock markets do not follow the normality and hence, do not follow the random walk hypothesis.

**Table 2.** Jarque – Bera Test Results

Market	Market index	Jarque – Bera test (p-value)	Market	Market index	Jarque – Bera test (p-value)
Spain	SMSI	0.0023	Iceland	OMXIP1	0.000
Indonesia	JKSE	0.000	Czech Republic	PX index	0.000
Norway	OSEAX	0.000	Canada	S&P/TSX Composite	0.000
Mexico	S&P/BMV IPC	0.000	Netherland	AEX index	0.000
Poland	WIG30	0.000	Thailand	SET index	0.000
Brazil	BVSP	0.000	Greece	ATF	0.000
Philippines	PSEi	0.000	Bulgaria	BGTR 30	0.000
Belgium	OMXCGI	0.000	Singapore	STI	0.000
Ireland	ISEQ All-share	0.000	Slovenia	SBITOP	0.000
Hungary	BUX index	0.000	Estonia	OMXT	0.000
South Africa	FTSE/JSE All-share	0.000	Colombia	COLCAP	0.000

Japan	N225	0.000	India	S&P BSE 500	0.000
Germany	DAX index	0.000	Croatia	CROBEX index	0.000
The U.K	FTSE All-share	0.000	Egypt	EGX 30	0.000
Australia	S&P ASX	0.000	Jordan	ASE General	0.0421
Sweden	OMXSPI	0.000	Lithuania	OMXV	0.000
Israel	TA-125	0.000	Turkey	BIST All-share	0.000
The U.S.A	NYSE Composite	0.000	Morrocco	MASI	0.000
Italy	FTITLMS	0.000	Vietnam	VNI	0.000
South Korea	KOSPI	0.000	Pakistan	KSE-100	0.000
Serbia	BELEX line	0.000	Qatar	QE Index	0.000
Chile	S&P/CLX IGPA	0.000	Saudi Arabia	TASI	0.000
China (Hong Kong)	SSEC	0.000	Nigeria	NSE	0.000
Taiwan	TAIEX	0.000	Tunisia	TUN Index	0.000
New Zealand	NZCI	0.000	Oman	MSM 30	0.000
Portugal	PSI 20	0.000	Sri Lanka	ASPI	0.000
Denmark	OMXCGI	0.000	Mauritius	SEMDEX	0.000
Switzerland	SMI	0.000			
Austria	ATX	0.000			
France	CAC All-share	0.000			

### Stationary Test Results

Table 3 presents the stationary analysis results of 57 series of stock return indices from conducting ADF and KPSS. According to ADF analysis, at 95% confidence, the null hypothesis (H02: The index series are non-stationary) is accepted in almost all markets except for the Philippines, Belgium, Singapore, and Colombia. Using the critical value threshold of 0.463, the KPSS test results reject the null hypothesis on the existence of the stationary feature in all markets in the sample excluding Poland, Singapore, and Mauritius.

**Table 3.** ADF Test and KPSS Test Results of Stock Markets

Market	ADF test (p-value)	KPSS test	Market	ADF test (p-value)	KPSS test
Spain	0.1053	2.1027	Iceland	0.7526	5.4830
Indonesia	0.5515	4.5958	Czech Republic	0.7475	3.6874
Norway	0.9246	6.1060	Canada	0.6034	5.4442
Mexico	0.381	1.9836	Netherland	0.7419	5.9339
Poland	0.1154	0.2952	Thailand	0.0668	0.7664
Brazil	0.8665	6.0343	Greece	0.2723	1.6797
Philippines	0.0322	1.0480	Bulgaria	0.9671	5.2662
Belgium	0.0236	2.8109	Singapore	0.0032	0.1892
Ireland	0.3055	4.4978	Slovenia	0.8454	5.3564

Hungary	0.9318	5.8195	Estonia	0.8276	5.8421
South Africa	0.6739	5.2385	Colombia	0.0441	1.9443
Japan	0.6938	5.8020	India	0.9965	5.8055
Germany	0.4931	5.4636	Croatia	0.7472	2.1384
The U.K.	0.0732	2.8820	Egypt	0.9843	3.0338
Australia	0.4611	5.7919	Jordan	0.7418	0.9261
Sweden	0.771	5.7345	Lithuania	0.8153	6.3232
Israel	0.5174	5.0801	Turkey	0.9991	3.3641
The U.S.A	0.6194	5.6342	Morocco	0.2219	3.2292
Italy	0.3726	3.6097	Vietnam	0.5816	5.4450
South Korea	0.4457	3.3704	Pakistan	0.6726	3.6054
Serbia	0.6768	5.3899	Qatar	0.2088	3.0108
Chile	0.7367	2.9698	Saudi Arabia	0.7084	2.9077
China (Hong Kong)	0.1074	1.5525	Nigeria	0.9991	2.3806
Taiwan	0.9198	5.4410	Tunisia	0.9142	5.6581
New Zealand	0.6392	6.6058	Oman	0.7019	5.0898
Portugal	0.0854	1.0199	Sri Lanka	0.2708	2.5305
Denmark	0.9809	6.0616	Mauritius	0.2296	0.2666
Switzerland	0.4984	5.4781			
Austria	0.1972	2.7829			
France	0.734	5.7136			

The results indicate the agreement between ADF and KPSS tests in the confirmation of non-stationary existence in 52 data series associated with 52 out of 57 stock markets. Those markets are, therefore, weak-form efficient according to ADF and KPSS test results excluding Singapore market. The conflict in the analysis results from ADF and KPSS tests exists in five rg markets, consisting of the Philippines, Belgium, Colombia, Poland, and Mauritius (highlighted in Table 3). Therefore, we need to use results from other tests to decide whether these markets are efficient or not.

#### Autocorrelation Test Results

**Table 4.** Ljung-Box Test Results of Stock Markets

Market	Market index	Ljung-Box test (p-value)	Market	Market index	Ljung – Box test (p-value)
Spain	SMSI	0.0000	Iceland	OMXIP1	0.0000
Indonesia	JKSE	0.0000	Czech Republic	PX index	0.0000
Norway	OSEAX	0.0000	Canada	S&P/TSX Composite	0.0000
Mexico	S&P/BMV IPC	0.0000	Netherland	AEX index	0.0000
Poland	WIG30	0.0000	Thailand	SET index	0.0000
Brazil	BVSP	0.0000	Greece	ATF	0.0000
Philippines	PSEi	0.0000	Bulgaria	BGTR 30	0.0000
Belgium	OMXCGI	0.0000	Singapore	STI	0.0000
Ireland	ISEQ All-share	0.0000	Slovenia	SBITOP	0.0000

Hungary	BUX index	0.0000	Estonia	OMXT	0.0000
South Africa	FTSE/JSE All-share	0.0000	Colombia	COLCAP	0.0000
Japan	N225	0.0000	India	S&P BSE 500	0.0000
Germany	DAX index	0.0000	Croatia	CROBEX index	0.0000
The U.K	FTSE All-share	0.0000	Egypt	EGX 30	0.0000
Australia	S&P ASX	0.0000	Jordan	ASE General	0.0000
Sweden	OMXSPI	0.0000	Lithuania	OMXV	0.0000
Israel	TA-125	0.0000	Turkey	BIST All-share	0.0000
The U.S.A	NYSE Composite	0.0000	Morocco	MASI	0.0000
Italy	FTTILMS	0.0000	Vietnam	VNI	0.0000
South Korea	KOSPI	0.0000	Pakistan	KSE-100	0.0000
Serbia	BELEX line	0.0000	Qatar	QE Index	0.0000
Chile	S&P/CLX IGPA	0.0000	Saudi Arabia	TASI	0.0000
China (Hong Kong)	SSEC	0.0000	Nigeria	NSE	0.0000
Taiwan	TAIEX	0.0000	Tunisia	TUN Index	0.0000
New Zealand	NZCI	0.0000	Oman	MSM 30	0.0000
Portugal	PSI 20	0.0000	Sri Lanka	ASPI	0.0000
Denmark	OMXCGI	0.0000	Mauritius	SEMDEX	0.0000
Switzerland	SMI	0.0000			
Austria	ATX	0.0000			
France	CAC All-share	0.0000			

Table 4 shows the autocorrelation test of stock markets through the Ljung-Box test. All p-values are significant at 5%, indicating the dependence in the price series. In other words, there are significant correlations between the past and present values of stock market returns and thus, violate the weak form of market efficiency.

Summarizing the test results of data normality, stationary, and auto-correlation reveals the different opinions on the data characteristics (Table 5). A market is marked “True” if the test outcome supports the weak form efficiency hypothesis or “False” if the test outcome rejects the hypothesis. There are some blank boxes, mostly related to stationary tests’ outcomes, indicating conflicts in results.

The majority of stock markets exhibit non-normal return distributions, implying potential risks associated with extremely unexpected events due to the analysis results of normal distribution and auto-correlation. However, most stock markets display non-stationary, suggesting that their statistical properties evolve, which follows the random walk, a main feature of market efficiency. Therefore, there is no strong evidence of accepting or rejecting the market efficiency in the weak form of the research sample. Therefore, we further quantify the degree of randomness in the daily stock price series, which is the Permutation Entropy using the *Ordpy* module and a Python application.

**Table 5.** Results Checklist from the Conventional Approach

Stock market	Normal distribution test	Stationarity test	Auto-	Stock market	Normal distribution test	Stationarity test	Auto-
			correlation test				correlation test
Spain	FALSE	TRUE	FALSE	Czech Republic	FALSE	TRUE	FALSE
Indonesia	FALSE	TRUE	FALSE	Canada	FALSE	TRUE	FALSE
Norway	FALSE	TRUE	FALSE	Netherlands	FALSE	TRUE	FALSE
Mexico	FALSE	TRUE	FALSE	Thailand	FALSE	FALSE	FALSE
Poland	FALSE		FALSE	Greece	FALSE	TRUE	FALSE
Brazil	FALSE	TRUE	FALSE	Bulgaria	FALSE	TRUE	FALSE
Philippines	FALSE		FALSE	Singapore	FALSE	FALSE	FALSE
Belgium	FALSE		FALSE	Slovenia	FALSE	TRUE	FALSE
Ireland	FALSE	TRUE	FALSE	Estonia	FALSE	TRUE	FALSE
Hungary	FALSE	TRUE	FALSE	Colombia	FALSE		FALSE
South Africa	FALSE	TRUE	FALSE	India	FALSE	TRUE	FALSE
Japan	FALSE	TRUE	FALSE	Croatia	FALSE	TRUE	FALSE
Germany	FALSE	TRUE	FALSE	Egypt	FALSE	TRUE	FALSE
The U.K	FALSE	TRUE	FALSE	Jordan	FALSE	TRUE	FALSE
Australia	FALSE	TRUE	FALSE	Lithuania	FALSE	TRUE	FALSE
Sweden	FALSE	TRUE	FALSE	Turkey	FALSE	TRUE	FALSE
Israel	FALSE	TRUE	FALSE	Morocco	FALSE	TRUE	FALSE
The U.S.A	FALSE	TRUE	FALSE	Vietnam	FALSE	TRUE	FALSE
Italy	FALSE	TRUE	FALSE	Pakistan	FALSE	TRUE	FALSE
South Korea	FALSE	TRUE	FALSE	Qatar	FALSE	TRUE	FALSE
Serbia	FALSE	TRUE	FALSE	Saudi Arabia	FALSE	TRUE	FALSE
Chile	FALSE	TRUE	FALSE	Nigeria	FALSE	TRUE	FALSE
China (Hong Kong)	FALSE	TRUE	FALSE	Tunisia	FALSE	TRUE	FALSE
Taiwan	FALSE	TRUE	FALSE	Oman	FALSE	TRUE	FALSE
New Zealand	FALSE	TRUE	FALSE	Sri Lanka	FALSE	TRUE	FALSE
Portugal	FALSE	TRUE	FALSE	Mauritius	FALSE		FALSE
Denmark	FALSE	TRUE	FALSE				
Switzerland	FALSE	TRUE	FALSE				
Austria	FALSE	TRUE	FALSE				
France	FALSE	TRUE	FALSE				
Iceland	FALSE	TRUE	FALSE				

### *Permutation Entropy Results Analysis*

We rank the stock markets based on the degree of absolute market efficiency displayed in Table 6. Markets with entropy values closer to 1 are considered highly weak-form efficient, showing no correlations between the current price of a stock and its historical price. If the entropy value is around zero, it suggests that the stock return can be predicted from the past data which violates the so that the market is inefficient at a low level.

According to Table 6, Spain ranks first place; thus, it is considered the most efficient market in the sample. The high position of Spain comes from the success of the application of MiFIR (The Markets in Financial Instruments Directive) regulation that increases the financial markets' transparency in Europe. The developed and emerging markets dominate the high entropy value group while most frontier markets achieve low entropy scores. Five markets consisting of Poland, the Philipines, Belgium, Colombia, and Mauritius receiving different opinions in auto-correlation tests are also ranked differently. Although Mauritius obtains the smallest entropy value in the sample, all six markets are weak-form efficient according to Entropy computation. Singapore is weak-form inefficient from conventional tests' results but still efficient according to the Permutation Entropy estimated.

There are 70% of the top 20 countries are developed markets. The remaining countries belong to emerging markets. In addition, 4 emerging markets (Mexico, Brazil, Hungary, and South Africa) are ranked closely to the developed market group. The results show that developed markets exhibit the highest level of information efficiency, characterized by random and unpredictable prices, followed by emerging markets, and frontier markets have the weakest efficiency. This aligns with previous research findings that developed stock markets tend to be more efficient (Zhang, 1999, Zunino et al., 2010).

**Table 6.** Efficiency Ranking from Permutation Entropy Estimates

Stock market	Stock market index	Obs.	Calculated Entropy	Ranking	Market cap. (billion USD)
Spain (D)	SMSI	2830	0.925363	1	1,329.71
Indonesia (E)	JKSE	2678	0.921197	2	755.25
Norway (D)	OSEAX	2753	0.920577	3	405.05
Mexico (E)	S&P/BMV IPC	2758	0.919173	4	569.46
Poland (E)	WIG30	2738	0.918112	5	366.02
Brazil (E)	BVSP	2715	0.917844	6	993.4
Philipines (E)	PSEi	2680	0.917824	7	301.26
Belgium (D)	OMXCGI	2806	0.916448	8	701.38
Ireland (D)	ISEQ All-share	2794	0.914774	9	130.62
Hungary (E)	BUX index	2731	0.911635	10	43.14
South Africa (E)	FTSE/JSE All-share	2738	0.910226	11	1,163.16
Japan (D)	N225	2712	0.910159	12	5,863.30
Germany (D)	DAX index	2780	0.909852	13	2,182.88
The U.K (D)	FTSE All-share	2510	0.908933	14	2,271.20
Australia (D)	S&P ASX	2856	0.908093	15	1,792.93
Sweden (D)	OMXSPI	2754	0.906376	16	1,027.91
Israel (D)	TA-125	2688	0.904417	17	262.6

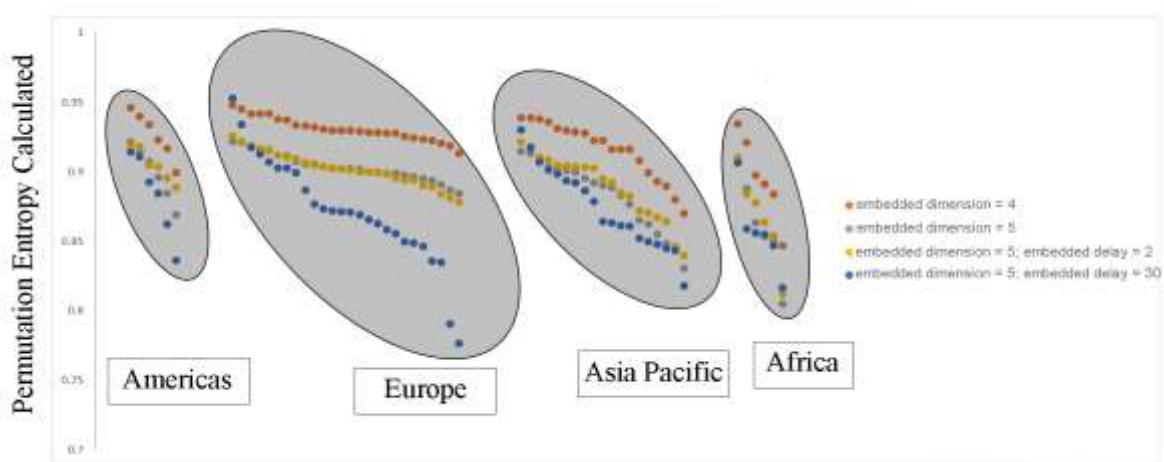
The U.S.A (D)	NYSE Composite	2710	0.904127	18	25,000.00
Italy (D)	FTTITLMS	2801	0.904116	19	738.55
South Korea (E)	KOSPI	2698	0.9038	20	1,981.60
Serbia (F)	BELEX line	2757	0.903372	21	3.97
Chile (E)	S&P/CLX IGPA	2728	0.903356	22	178.8
China (Hong Kong) (D)	SSEC	2666	0.903204	23	3,960.18
Taiwan (E)	TAIEX	2690	0.902983	24	1,856.37
New Zealand (D)	NZCI	3030	0.902432	25	93.75
Portugal (D)	PSI 20	2806	0.901826	26	106.69
Denmark (D)	OMXCGI	2738	0.901745	27	727.68
Switzerland (D)	SMI	2757	0.900892	28	1,771.17
Austria (D)	ATX	2745	0.899676	29	138.9
France (D)	CAC All-share	2806	0.899629	30	3,372.43
Iceland (F)	OMXIFI	2728	0.899363	31	19.43
Czech Republic (E)	PX index	2743	0.897956	32	3,970.38
Canada (D)	S&P/TSX Composite	2750	0.895315	33	3,387.60
Netherland (D)	AEX index	2806	0.89517	34	1,579.52
Thailand (E)	SET index	2669	0.894532	35	521.52
Greece (E)	ATF	2699	0.893755	36	96.68
Bulgaria	BGTR 30	2709	0.893521	37	6.83
Singapore (D)	STI	2631	0.891918	38	608.17
Slovenia (F)	SBITOP	2739	0.889796	39	10.13
Estonia (F)	OMXT	2750	0.888558	40	5.59
Colombia (E)	COLCAP	2672	0.888387	41	79.96
India (E)	S&P BSE 500	2711	0.884328	42	4,378.99
Croatia (F)	CROBEX index	2727	0.883774	43	25.36
Egypt (E)	EGX 30	2671	0.883696	44	55.67
Jordan (F)	ASE General	2681	0.881931	45	23.9
Lithuania (F)	OMXV	2729	0.880655	46	5.05
Turkey (E)	BIST All-share	2751	0.877954	47	341.16
Morrocco (F)	MASI	2726	0.877383	48	60.14
Vietnam (F)	VNI	2736	0.871388	49	186
Pakistan (F)	KSE-100	2714	0.870216	50	3.28
Qatar (E)	QE Index	2728	0.866629	51	171.6
Saudi Arabia (E)	TASI	2735	0.86369	52	3,002.49
Nigeria (F)	NSE	2712	0.86341	53	45.5
Tunisia (F)	TUN Index	2741	0.853542	54	7.98
Oman (F)	MSM 30	2698	0.846003	55	61.83
Sri Lanka (F)	ASPI	2609	0.839424	56	13.12

Mauritius (F)	SEMDEX	2737	0.811197	57	5.8
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	Developed market	Emerging market	Frontier market
<b>Mean</b>	0.905684	0.898794	0.872104
<b>Median</b>	0.904122	0.902983	0.879019
<b>Maximum</b>	0.925363	0.921197	0.903372
<b>Minimum</b>	0.891918	0.863690	0.811197
<b>Std. Dev.</b>	0.008219	0.017741	0.024473
<b>Skewness</b>	0.660233	-0.498357	-0.994045
<b>Kurtosis</b>	3.101780	2.224607	3.452392
<b>Jarque-Bera</b>	1.607825	1.262451	2.771438
<b>Probability</b>	0.447574	0.531940	0.250144
<b>Observations</b>	22	19	16

**Table 7.** Permutation Entropy Results Across Market Groups

The descriptive statistics in Table 7 demonstrate that the developed markets have the highest mean, median, maximum, and minimum among the three groups. However, they have the smallest standard deviation in Entropy scores which is evidence of greater stability of market efficiency. Comparing results between the median and mean, the developed market group exhibits a right-skewed distribution, while the emerging and frontier market groups are left-skewed distributed.

**Figure 2.** Entropy Estimates with Different Embedded Dimensions and Delays.

Using different embedded dimensions and delays, Figure 2 illustrates the most meaningful result (illustrated in the yellow series) is the one with the selected dimension of 5 and delay of 2. Among different continents, Africa exhibits the most scattered entropy, while Europe displays the most uniform one. This is because the stock market sample is primarily concentrated in Europe and the Asia-Pacific region, while dispersed in the Americas and Africa. Particularly, the European stock market experiences the highest level of volatility, fluctuating within a wide range compared to the other groups.

## Conclusion and Future Research

Examining the market efficiency is not new but still interesting and the efficiency results are still controversial due to the fruitfulness of the methods in finding evidence of weak form efficiency. Therefore, we conduct this study to explore whether a combination of Permutation Entropy calculation with traditional efficiency testing methods can successfully explore the weak form efficiency of 57 markets (developed, emerging, and frontier) worldwide.

The analysis results of the traditional method show that most of the market index returns in the sample are non-normal distributed but still follow the random walk according to stationary test results. Markets of Poland, the Philippines, Belgium, Colombia, and Mauritius even receive different conclusions of random walk patterns due to the contrast results from the ADF test and KPSS test in checking the stationary. Therefore, the inconsistency in the analysis findings provides unclear evidence about the weak form market efficiency.

To obtain a more convincing conclusion, the data sample is continued to be investigated by applying the Permutation Entropy estimate and the findings depict a clear description of market efficiency by ranking the efficiency degree from the highest to lowest. All entropy scores computed are bigger than 0 so that all markets in the sample are weak-form efficient. Our findings are consistent with existing literature, indicating that developed markets with strict requirements of information disclosure exhibit high levels of information efficiency.

Despite the confirmation of weak form efficiency, stock markets tend to have return distributions that deviate from normality, indicating potential risks from extreme events (fat tails). Additionally, the significant autocorrelation suggests that there are opportunities to earn profits from researching historical market data for excess returns.

For future research, authors may also improve the validity of the study by increasing the number of observations in the sample. Moreover, determining the threshold of entropy indicator of a highly efficient market or low efficient market will serve as a direction for research in this field.

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