

math **MAMMOTH**

Grade 6-A Worktext South African Version

Revision of the basic operations

Expressions
and equations

Decimals

Ratios

Percent



L i g h t B l u e S e r i e s

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**Sample worksheet from
www.mathmammoth.com**

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Foreword

Math Mammoth Grade 6-A and Grade 6-B South African Version worktexts comprise a complete maths curriculum for sixth grade mathematics studies.

Math Mammoth South African version has been customised to South Africa in the following manners:

- The names used are South African names (instead of Jack and Jill, there are Ansie and Musa).
- The currency used in word problems is rand.
- The material is all metric. In other words, the US customary measuring units are not taught.
- Spelling is British English instead of American English.
- Paper size is A4.

Please note that the curriculum is not following the South African official syllabus for sixth grade maths. Instead, it is a copy of the US version of Math Mammoth Grade 6, aligned to the US Common Core Standards. This decision was made because of the great amount of work that would be involved in writing new lessons and reorganising old ones to match all the standards in the South African syllabus. For the most part, Math Mammoth is exceeding South African standards.

In sixth grade, students encounter the beginnings of algebra, algebraic expressions, one-variable equations and inequalities, integers and ratios. We also revise and deepen the students' understanding of rational numbers: both fractions and decimals are studied in depth, while percent is a new topic for 6th grade. In geometry, students learn to compute the area of various polygons, and also calculate volume and surface area of various solids. The last major area of study is statistics, where students learn to summarise and describe distributions using both measures of centre and variability.

The grade starts out with a revision of the four operations with whole numbers (including long division), place value and rounding. Students are also introduced to exponents and they do some problem solving.

Chapter 2 starts the study of algebra topics, delving first into expressions and equations. Students practise writing expressions in many different ways, and use properties of operations and the idea of maintaining the equality of both sides of an equation to solve simple one-step equations. We also study briefly inequalities and using two variables.

Chapter 3 has to do with decimals. This is a long chapter, as we revise all of decimal arithmetic, just using more decimal digits than in 5th grade. Students also convert measuring units in this chapter.

Ratios is a new topic (chapter 4). Students are already familiar with finding fractional parts from earlier grades, and now it is time to advance that knowledge into the study of ratios, which arise naturally from dividing a quantity into many equal parts. We study such topics as rates, unit rates, equivalent ratios and problem solving using bar models.

Percent (chapter 5) is an important topic to understand thoroughly because of its many applications in real life. The goal of this chapter is to develop a basic understanding of percent, to see percentages as decimals and to learn to calculate discounts.

In part 6-B, students study number theory, fractions, integers, geometry and statistics.

I wish you success in teaching maths!

Maria Miller, the author

Chapter 1: Revision of the Basic Operations

Introduction

The goal of the first chapter in grade 6 is to revise the four basic operations with whole numbers, place value and rounding, as well as to learn about exponents and problem solving.

A lot of this chapter is revision, and I hope this provides a gentle start for 6th grade maths. In the next chapter, we will delve into some beginning algebra topics.

The Lessons in Chapter 1

	page	span
Warm-Up: Mental Maths	9	<i>2 pages</i>
Revision of the Four Operations 1	11	<i>6 pages</i>
Revision of the Four Operations 2	17	<i>3 pages</i>
Powers and Exponents	20	<i>3 pages</i>
Place Value	23	<i>4 pages</i>
Rounding and Estimating.....	27	<i>3 pages</i>
Lessons in Problem Solving	30	<i>4 pages</i>
Chapter 1 Revision	34	<i>2 pages</i>

Helpful Resources on the Internet

Long division

Snork's Long Division Game

Interactive and guided long division practice that only accepts correct answers and truly guides the student step-by-step through long division problems. In the beginning, choose a two-digit number as the highest number you want to work with (the divisor) in order to practise with two-digit divisors.

<http://www.kidsnumbers.com/long-division.php>

Mr. Martini's Classroom: Long Division

An interactive long division tool.

<http://www.thegreatmartiniccompany.com/longarithmetic/longdivision.html>

Short Division

A page that explains short division in detail. Short division is the same algorithm as long division, but some steps are only done in one's head instead of being written down.

<http://www.themathpage.com/ARITH/divide-whole-numbers.htm>

All four operations

Math Mahjong

A Mahjong game where you need to match tiles that have the same value. It uses all four operations and has three levels.

http://www.sheppardsoftware.com/mathgames/mixed_mahjong/mahjongMath_Level_1.html

Pop the Balloons

Pop the balloons in the order of their value. You need to use all four operations.

<http://www.sheppardsoftware.com/mathgames/numberballoons/BalloonPopMixed.htm>

MathCar Racing

Keep ahead of the computer car by thinking logically, and practise the four operations at the same time.

Calculator Chaos

Most of the keys have fallen off the calculator but you have to make certain numbers using the keys that are left.

http://www.mathplayground.com/calculator_chaos.html

ArithmeTiles

Use the four operations and numbers on neighbouring tiles to make target numbers.

<http://www.primarygames.com/math/arithmetiles/>

SpeedMath Deluxe

Create an equation from the four given digits using addition, subtraction, multiplication and division. Make certain that you remember the order of operations. Includes negative numbers sometimes.

<http://education.jlab.org/smdeluxe/index.html>

Place value

Numbers

Practise place value, comparing numbers and ordering numbers with this interactive online practice.

<http://www.youtube.com/watch?v=0fKBhvDjuy0>

Keep My Place

Fill in the big numbers in this cross-number puzzle.

<http://www.counton.org/magnet/kaleidoscope2/Crossnumber/index.html>

Estimation at AAA Math

Exercises about rounding whole numbers and decimals, front-end estimation, estimating sums and differences. Each page has an explanation, interactive practice and games.

<http://www.aaamath.com/B/est.htm>

Place Value Game

Create the largest possible number from the digits the computer gives you. Unfortunately, the computer will give you each digit one at a time and you won't know what the next number will be.

<http://education.jlab.org/placevalue/index.html>

Free Exponent Worksheets

Create a variety of customisable, printable worksheets to practise exponents.

<http://www.homeschoolmath.net/worksheets/exponents.php>

Baseball Exponents

Choose the right answer from three possibilities before the pitched ball comes.

<http://www.xpmath.com/forums/arcade.php?do=play&gameid=95>

Exponents Quiz from ThatQuiz.org

Ten questions, fairly easy and not timed. You can change the parameters as you like to include negative bases, square roots and even logarithms.

<http://www.thatquiz.org/tq-2/?-j1-l4-p0>

Exponents Jeopardy

The question categories include evaluating exponents, equations with exponents and exponents with fractional bases.

<http://www.math-play.com/Exponents-Jeopardy/Exponents-Jeopardy.html>

Pyramid Math

Simple practice of either exponents, roots, LCM, or GCF. Drag the triangle with the right answer to the vase.

<http://www.mathnook.com/math/pyramidmath.html>

Exponents Battleship

A regular battleship game against the computer. Each time you “hit”, you need to answer a maths problem involving exponents (and multiplication).

<http://www.quia.com/ba/1000.html>

Exponent Battle

A card game to practise exponents. I would limit the cards to small numbers, instead of using the whole deck.

<http://www.learn-with-math-games.com/exponent-game.html>

Pirates Board Game

Steer your boat in pirate waters in this online board game, and evaluate powers.

<http://mathgames4children.com/fun-board-games/6th-grade/pirate/exponents-pirate-waters-grade-6-game.html>

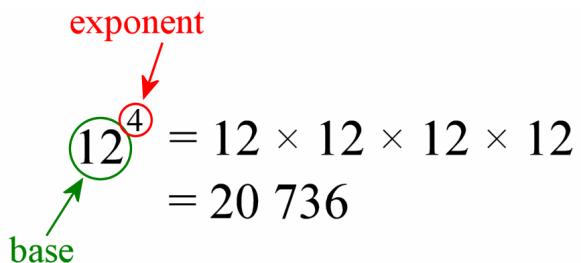
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Powers and Exponents

Exponents are a “shorthand” for writing repeated multiplications by the same number.

For example, $2 \times 2 \times 2 \times 2 \times 2$ is written 2^5 .
 $5 \times 5 \times 5 \times 5 \times 5$ is written 5^6 .

The tiny raised number is called the **exponent**. It tells us how many times the *base* number is multiplied by itself.



The expression 2^5 is read as “two to the fifth power,” “two to the fifth,” or “two raised to the fifth power.”

Similarly, 7^9 is read as “seven to the ninth power,” “seven to the ninth,” or “seven raised to the ninth power.”

The “powers of 6” are simply expressions where 6 is raised to some power: For example, 6^3 , 6^4 , 6^{45} and 6^{99} are powers of 6. What would powers of 10 be?

Expressions with the exponent 2 are usually read as something “**squared**.” For example, 11^2 is read as “**eleven squared**.” That is because it gives us *the area of a square* with the side length of 11 units.

Similarly, if the exponent is 3, the expression is usually read using the word “**cubed**.” For example, 31^3 is read as “**thirty-one cubed**” because it gives the *volume of a cube* with the edge length of 31 units.

1. Write the expressions as multiplications, and then solve them in your head.

a. $3^2 = \underline{3} \times \underline{3} = 9$

b. 1^6

c. 4^3

d. 10^4

e. 5^3

f. 10^2

g. 2^3

h. 8^2

i. 0^5

j. 10^5

k. 50^2

l. 100^3

2. Rewrite the expressions using an exponent, then solve them. You may use a calculator.



a. $2 \times 2 \times 2 \times 2 \times 2 \times 2$

b. $8 \times 8 \times 8 \times 8 \times 8$

c. 40 squared

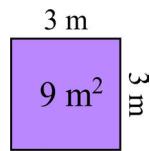
d. $10 \times 10 \times 10 \times 10$

e. nine to the eighth power

f. eleven cubed

You just learned that the expression 7^2 is read “seven *squared*” because it tells us the area of a *square* with a side length of 7 units. Let’s compare that to square metres and other units of area.

If the sides of a square are 3 m long, then its area is $3 \text{ m} \times 3 \text{ m} = 9 \text{ m}^2$ or nine square metres.



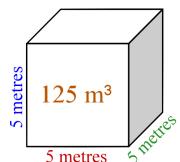
Notice that the symbol for square metres is **m²**. This means “**metre × metre**.” We are, in effect, squaring the unit *metre* (multiplying the unit of length *metre* by itself)!

Or, in the expression $9 \text{ cm} \times 9 \text{ cm}$, we multiply 9 by itself, but we also multiply the unit *cm* by itself. That is why the result is **81 cm²**, and the square centimetre (cm^2) comes from multiplying “**centimetre × centimetre**.”

We do the same thing with any other unit of length to form the corresponding unit for area, such as square kilometres or square millimetres.

In a similar way, to calculate the volume of this cube, we multiply $5 \text{ m} \times 5 \text{ m} \times 5 \text{ m} = 125 \text{ m}^3$.

We not only multiply 5 by itself three times, but also multiply the unit *metre* by itself three times (*metre × metre × metre*) to get the unit of volume “*cubic metre*” or m^3 .



3. Express the area (A) as a multiplication, and solve.

a. A square with a side of 12 kilometres:

$$A = 12 \text{ km} \times 12 \text{ km} = \underline{\hspace{2cm}}$$

b. A square with sides 6 m long:

$$A = \underline{\hspace{2cm}}$$

c. A square with a side length of 6 centimetres:

$$A = \underline{\hspace{2cm}}$$

d. A square with a side with a length of 12 cm:

$$A = \underline{\hspace{2cm}}$$

4. Express the volume (V) as a multiplication, and solve.

a. A cube with a side of 2 cm:

$$V = 2 \text{ cm} \times 2 \text{ cm} \times 2 \text{ cm} = \underline{\hspace{2cm}}$$

b. A cube with sides 10 cm long:

$$V = \underline{\hspace{2cm}}$$

c. A cube with sides 1 m in length:

$$V = \underline{\hspace{2cm}}$$

d. A cube with edges that are all 5 m long:

$$V = \underline{\hspace{2cm}}$$

5. a. The perimeter of a square is 40 cm. What is its area?

b. The volume of a cube is 64 cubic centimetres. How long is its edge?

c. The area of a square is 121 m². What is its perimeter?

d. The volume of a cube is 27 cm³. What is the length of one edge?

The powers of 10 are very special
—and very easy!

Notice that the exponent tells us *how many zeros* there are in the answer.

$$10^1 = 10$$

$$10^4 = 10\,000$$

$$10^2 = 10 \times 10 = 100$$

$$10^5 = 100\,000$$

$$10^3 = 10 \times 10 \times 10 = 1\,000$$

$$10^6 = 1\,000\,000$$

6. Fill in the patterns. In part (d), choose your own number to be the base.

Use a calculator in parts (c) and (d).



a.

$$2^1 =$$

$$2^2 =$$

$$2^3 =$$

$$2^4 =$$

$$2^5 =$$

$$2^6 =$$

b.

$$3^1 =$$

$$3^2 =$$

$$3^3 =$$

$$3^4 =$$

$$3^5 =$$

$$3^6 =$$

c.

$$5^1 =$$

$$5^2 =$$

$$5^3 =$$

$$5^4 =$$

$$5^5 =$$

$$5^6 =$$

d.

7. Look at the patterns above. Think carefully how each step comes from the previous one. Then answer.

a. If $3^7 = 2\,187$, how can you use that result to find 3^8 ?

b. Now find 3^8 without a calculator.

c. If $2^{45} = 35\,184\,372\,088\,832$, use that to find 2^{46} without a calculator.

8. Fill in.

a. 17^2 gives us the _____ of a _____ with sides _____ units long.

b. 101^3 gives us the _____ of a _____ with edges _____ units long.

c. 2×6^2 gives us the _____ of two _____ with sides _____ units long.

d. 4×10^3 gives us the _____ of _____ with edges _____ units long.

Make a pattern, called a **sequence**, with the powers of 2, starting with 2^6 and going *backwards* to 2^0 . At each step, *divide* by 2. What is the logical (though surprising) value for 2^0 from this method?

Puzzle Corner

Make another, similar, sequence for the powers of 10. Start with 10^6 and divide by 10 until you reach 10^0 . What value do you calculate for 10^0 ?

Try this same pattern for at least one other base number, n . What value do you calculate for n^0 ?

Do you think it will come out this way for every base number?

Why or why not?

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Chapter 2: Expressions and Equations

Introduction

In this chapter we concentrate on two important concepts: expressions and equations. We also touch on inequalities and graphing on a very introductory level. In order to make the learning of these concepts easier, the expressions and equations in this chapter do not involve negative numbers (as they typically do when studied in pre-algebra and algebra). The study of negative numbers is in part 6-B.

We start out by learning some basic vocabulary used to describe mathematical expressions verbally—terms such as the sum, the difference, the product, the quotient and the quantity. Next, we study the order of operations once again. A lot of this lesson is revision. The lesson *Multiplying and Dividing in Parts* is also partially revision and leads up to the lesson on distributive property that follows later. Then, we get into studying expressions in definite terms: students encounter the exact definition of an expression, a variable, and a formula, and practise writing expressions in many different ways.

The concepts of equivalent expressions and simplifying expressions are important. If you can simplify an expression in some way, the new expression you get is equivalent to the first. We study these ideas first using lengths—it is a concrete example, and hopefully easy to grasp. In the lesson *More On Writing and Simplifying Expressions* students encounter more terminology: term, coefficient and constant. In exercise 3, they write an expression for the perimeter of some shapes in two ways. This exercise is once again preparing them to understand the distributive property. Next, students write and simplify expressions for the area of rectangles and rectangular shapes. Then we study the distributive property, concentrating on the symbolic aspect and tying it in with area models.

The next topic is equations. Students learn some basics, such as, the solutions of an equation are the values of the variables that make the equation true. They use properties of operations and the idea of maintaining the equality of both sides of an equation to solve simple one-step equations. I have also included a few two-step equations as an optional topic. Lastly, in this chapter students get to solve and graph simple inequalities, and study the usage of two variables and graphing.

The Lessons in Chapter 2

	page	span
Terminology for the Four Operations	39	4 pages
Order of Operations.....	43	3 pages
Multiplying and Dividing in Parts	46	4 pages
Expressions	50	3 pages
Writing and Simplifying Expressions 1:		
Length and Perimeter	53	3 pages
More on Writing and Simplifying Expressions	56	3 pages
Writing and Simplifying Expressions 2: Area	59	5 pages
The Distributive Property	64	4 pages
Equations	68	4 pages
More Equations	72	4 pages
Inequalities	76	4 pages
Using Two Variables	80	4 pages
Mixed Revision.....	84	2 pages
Chapter 2 Revision	86	4 pages

Helpful Resources on the Internet

Calculator Chaos

Most of the keys have fallen off the calculator, but you have to make certain numbers using the keys that are left.

http://www.mathplayground.com/calculator_chaos.html

ArithmeTiles

Use the four operations and numbers on neighbouring tiles to make target numbers.

<http://www.primarygames.com/math/arithmetiles/index.htm>

Choose A Math Operation

Choose the mathematical operation(s) so that the number sentence is true. Practise the role of zero and one in basic operations or operations with negative numbers. Helps develop number sense and logical thinking.

<http://www.homeschoolmath.net/operation-game.php>

Order of Operations Quiz

A 10-question online quiz that includes two different operations and possibly brackets in each question. You can also modify the quiz parameters yourself.

<http://www.thatquiz.org/tq-1/?-j8f-la>

The Order of Operations Millionaire

Answer multiple-choice questions that have to do with the order of operations, and win a million. Can be played alone or in two teams.

<http://www.math-play.com/Order-of-Operations-Millionaire/order-of-operations-millionaire.html>

Exploring Order of Operations (Object Interactive)

The program shows an expression, and you click on the correct operation (either +, -, ×, ÷ or exponent) to be done first. The program then solves that operation, and you click on the *next* operation to be performed, *etc.*, until it is solved. Lastly, the resource includes a game where you click on the falling blocks in the sequence that the order of operations would dictate.

http://www.learnalberta.ca/content/mejhm/html/object_interactives/order_of_operations/use_it.html

Order of Operations Practice

A simple online quiz of 10 questions. Uses brackets and the four operations.

<http://www.onlinemathlearning.com/order-of-operations-practice.html>

Fill and Pour

Fill and pour liquid with two containers until you get the target amount. A logical thinking puzzle.

http://nlvm.usu.edu/en/nav/frames_asid_273_g_2_t_4.html

Balance Beam Activity

A virtual balance that poses puzzles where the student must think algebraically to find the weights of various figures. Includes three levels.

<http://mste.illinois.edu/users/pavel/java/balance/index.html>

Algebraic Expressions Millionaire

For each question you have to identify the correct mathematical expression that models a given word expression.

<http://www.math-play.com/Algebraic-Expressions-Millionaire/algebraic-expressions-millionaire.html>

Escape Planet

Choose the equation that matches the words.

http://www.harcourtschool.com/activity/escape_planet_6/

BuzzMath Practice - Algebraic Expressions

Online practice for simplifying different kinds of algebraic expressions.

<http://www.mathplayground.com/practice.php?topic=algebraic-expressions>

Expressions : Expressions and Variables Quiz

Choose an equation to match the word problem or situation.

http://www.softschools.com/quizzes/math/expressions_and_variables/quiz815.html

Equation Match

Match simple equations that have the same solution.

<http://www.bbc.co.uk/schools/mathsfile/shockwave/games/equationmatch.html>

Algebraic Reasoning

Find the value of an object based on two scales.

http://www.mathplayground.com/algebraic_reasoning.html

Algebra Puzzle

Find the value of each of the three objects presented in the puzzle. The numbers given represent the sum of the objects in each row or column.

http://www.mathplayground.com/algebra_puzzle.html

Battleship

An interesting game where the student must choose the right solution to a 1-step equation every time he hits an enemy ship. Although some of the equations involve negative solutions, the game is multiple-choice, so it's possible to guess the solution, even if the student isn't familiar with negative numbers.

<http://www.quia.com/ba/36544.html>

Algebra Meltdown

Solve simple equations using function machines to guide atoms through the reactor. But don't keep the scientists waiting too long or they blow their tops. Again, includes negative numbers.

<http://www.mangahigh.com/en/games/algebrameltdown>

Words into Equations Battleship Game

Practise expressions such as quotient, difference, product and sum.

<http://www.quia.com/ba/210997.html>

Balance when Adding and Subtracting Game

The interactive balance illustrates simple equations. Your task is to add or subtract Xs, and add or subtract 1s until you have X alone on one side.

<http://www.mathsisfun.com/algebra/add-subtract-balance.html>

Algebra Balance Scales

Similar to the one above, but you need to first put the Xs and 1s in the balance to match the given equation.

http://nlvm.usu.edu/en/nav/frames_asid_201_g_4_t_2.html — only positive numbers

http://nlvm.usu.edu/en/nav/frames_asid_324_g_4_t_2.html — includes negative numbers

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The Distributive Property

The **distributive property** states that $a(b + c) = ab + ac$

It may look like a meaningless or difficult equation to you now, but don't worry, it will become clearer!

The equation $a(b + c) = ab + ac$ means that you can *distribute* the multiplication (by a) over the sum ($b + c$) so that you multiply the numbers b and c separately by a , and add last.

You have already used the distributive property! When you separated $3 \cdot 84$ into $3 \cdot (80 + 4)$, you then multiplied 80 and 4 *separately* by 3, and added last: $3 \cdot 80 + 3 \cdot 4 = 240 + 12 = 252$. We called this using "partial products" or "multiplying in parts."

Example 1. Using the distributive property, we can write the product $2(x + 1)$ as $2x + 2 \cdot 1$, which simplifies to $2x + 2$.

Notice what happens: Each term in the sum ($x + 1$) gets multiplied by the factor 2! Graphically:

$$2(x + 1) = \underline{2x} + \underline{2 \cdot 1}$$

Example 2. To multiply $s \cdot (3 + t)$ using the distributive property, we need to multiply *both* 3 and t by s :

$$s \cdot (3 + t) = s \cdot 3 + s \cdot t, \text{ which simplifies to } 3s + st.$$

1. Multiply using the distributive property.

a. $3(90 + 5) = 3 \cdot \underline{\quad} + 3 \cdot \underline{\quad} =$	b. $7(50 + 6) = 7 \cdot \underline{\quad} + 7 \cdot \underline{\quad} =$
c. $4(a + b) = 4 \cdot \underline{\quad} + 4 \cdot \underline{\quad} =$	d. $2(x + 6) = 2 \cdot \underline{\quad} + 2 \cdot \underline{\quad} =$
e. $7(y + 3) =$	f. $10(s + 4) =$
g. $s(6 + x) =$	h. $x(y + 3) =$
i. $8(5 + b) =$	j. $9(5 + c) =$

Example 3. We can use the distributive property also when the sum has three or more terms. Simply multiply *each term* in the sum by the factor in front of the brackets:

$$5(x + y + 6) = \underline{5 \cdot x} + \underline{5 \cdot y} + \underline{5 \cdot 6}, \text{ which simplifies to } 5x + 5y + 30$$

2. Multiply using the distributive property.

a. $3(a + b + 5) =$	b. $8(5 + y + r) =$
c. $4(s + 5 + 8) =$	d. $3(10 + c + d + 2) =$

Example 4. Now one of the terms in the sum has a coefficient (the 2 in $2x$):

$$6(2x + 3) = 6 \cdot 2x + 6 \cdot 3 = 12x + 18$$

3. Multiply using the distributive property.

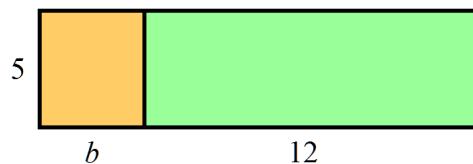
a. $2(3x + 5) =$	b. $7(7a + 6) =$
c. $5(4a + 8b) =$	d. $2(4x + 3y) =$
e. $3(9 + 10z) =$	f. $6(3x + 4 + 2y) =$
g. $11(2c + 7a) =$	h. $8(5 + 2a + 3b) =$

To understand even better why the distributive property works, let's look at an area model (this, too, you have seen before!).

The area of the whole rectangle is 5 times $(b + 12)$.

But if we think of it as *two* rectangles, the area of the first rectangle is $5b$, and of the second, $5 \cdot 12$.

Of course, these two expressions have to be equal:

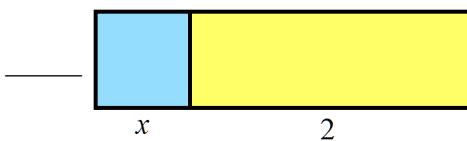


$$5 \cdot (b + 12) = 5b + 5 \cdot 12 = 5b + 60$$

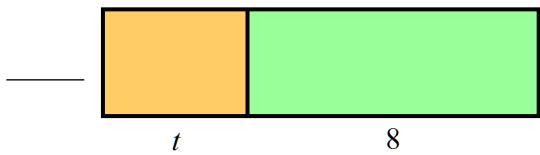
4. Write an expression for the area in two ways, thinking of one rectangle or two.

 a. $9(\underline{\hspace{1cm}} + \underline{\hspace{1cm}})$ and $9 \cdot \underline{\hspace{1cm}} + 9 \cdot \underline{\hspace{1cm}} =$	 b. $s(\underline{\hspace{1cm}} + \underline{\hspace{1cm}})$ and $s \cdot \underline{\hspace{1cm}} + s \cdot \underline{\hspace{1cm}} =$
 c. $\underline{\hspace{1cm}}(\underline{\hspace{1cm}} + \underline{\hspace{1cm}})$ and	 d.
 e.	 f.

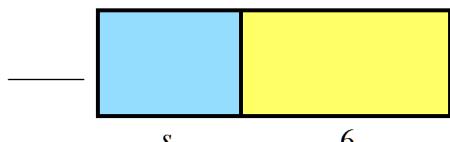
5. Find the missing number or variable in these area models.



a. $\underline{\hspace{1cm}}(x + 2) = 3x + 6$



b. $\underline{\hspace{1cm}}(t + 8) = 7t + 56$



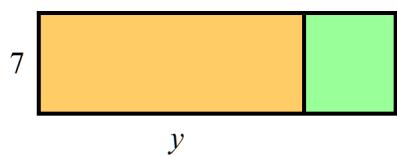
c. The total area is $9s + 54$.



d. $4(\underline{\hspace{1cm}} + 5) = 4z + 20$



e. $5(s + \underline{\hspace{1cm}}) = 5s + 30$



f. The total area is $7y + 42$.

6. Find the missing number in the equations.

a. $\underline{\hspace{1cm}}(x + 5) = 6x + 30$

b. $10(y + \underline{\hspace{1cm}}) = 10y + 30$

c. $6(\underline{\hspace{1cm}} + z) = 12 + 6z$

d. $8(r + \underline{\hspace{1cm}}) = 8r + 24$

7. Find the missing number in the equations. These are just a little bit trickier!

a. $\underline{\hspace{1cm}}(2x + 5) = 6x + 15$

b. $\underline{\hspace{1cm}}(3w + 5) = 21w + 35$

c. $\underline{\hspace{1cm}}(6y + 4) = 12y + 8$

d. $\underline{\hspace{1cm}}(10s + 3) = 50s + 15$

e. $2(\underline{\hspace{1cm}} + 9) = 4x + 18$

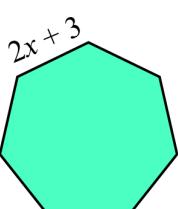
f. $4(\underline{\hspace{1cm}} + 3) = 12x + 12$

g. $5(\underline{\hspace{1cm}} + 3) = 20y + 15$

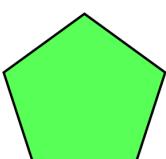
h. $8(\underline{\hspace{1cm}} + \underline{\hspace{1cm}} + 7) = 40t + 8s + 56$

8. Write an expression for the perimeter of this regular heptagon as a *product*.

Then multiply the expression using the distributive property



9. The perimeter of a regular pentagon is $15x + 5$. How long is one of its sides?



When we use the distributive property “backwards,” and write a sum as a product, it is called **factoring**.

Example 5. The sum $5x + 5$ can be written as $5(x + 1)$. We took the SUM $5x + 5$ and wrote it as a PRODUCT— something times something, in this case 5 times the quantity $(x + 1)$.

Example 6. The sum $24x + 16$ can be written as the product $8(3x + 2)$.

Notice that the numbers 24 and 16 are *both divisible by 8!* That is why we write 8 as one of the factors.

10. Think of the distributive property “backwards,” and factor these sums. Think of divisibility!

a. $6x + 6 = \underline{\quad}(x + \underline{\quad})$	b. $8y + 16 = 8(\underline{\quad} + \underline{\quad})$
c. $15x + 45 = \underline{\quad}(x + \underline{\quad})$	d. $4w + 40 = \underline{\quad}(w + \underline{\quad})$
e. $6x + 30 = \underline{\quad}(\underline{\quad} + \underline{\quad})$	f. $8x + 16y + 48 = \underline{\quad}(\underline{\quad} + \underline{\quad} + \underline{\quad})$

11. Factor these sums (writing them as products). Think of divisibility!

a. $8x + 4 = \underline{\quad}(2x + \underline{\quad})$	b. $15x + 10 = \underline{\quad}(3x + \underline{\quad})$
c. $24y + 8 = \underline{\quad}(\underline{\quad} + \underline{\quad})$	d. $6x + 3 = \underline{\quad}(\underline{\quad} + \underline{\quad})$
e. $42y + 14 = \underline{\quad}(\underline{\quad} + \underline{\quad})$	f. $32x + 24 = \underline{\quad}(\underline{\quad} + \underline{\quad})$
g. $27y + 9 = \underline{\quad}(\underline{\quad} + \underline{\quad})$	h. $55x + 22 = \underline{\quad}(\underline{\quad} + \underline{\quad})$
i. $36y + 12 = \underline{\quad}(\underline{\quad} + \underline{\quad})$	j. $36x + 9z + 27 = \underline{\quad}(\underline{\quad} + \underline{\quad} + \underline{\quad})$

12. The perimeter of a square is $48x + 16$. How long is its side?

As a shopkeeper, you need to purchase 1 000 items to get a wholesale (cheaper) price of R8 per item, so you do. You figure you might sell 600 of them. You also want to advertise a R3 discount to your customers. What should the non-discounted selling price be for you to actually earn a R500 profit from the sale of these items?

Puzzle Corner

Epilogue: It may be hard to see now where distributive property or factoring might be useful, but it IS extremely necessary later in algebra when solving equations.

To solve the problem above, you *can* figure it out without algebra, but it becomes fairly straightforward if we write an equation for it. Let p be the non-discounted price. Then $p - R3$ is the price with the discount. We get:

$$\text{What we need to take in} = \text{pay to supplier} + \text{profit}$$

$$600(p - R3) = 1\,000 \cdot R8 + R500$$

To solve this equation, one needs to use the distributive property in the very first step:

$$600p - R1\,800 = R8\,500$$

$$600p = R10\,300$$

(Can you solve this last step yourself?)

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Chapter 3: Decimals

Introduction

In this chapter we study all four operations of decimals, the metric system, and using decimals in measuring units. Most of the topics here have already been studied in 5th grade, but in 5th grade we were using numbers with a maximum of three decimal digits. This time there is no such restriction, and the decimals used can have many more decimal digits than that.

However, since the topics are the same, if the student has a good grasp of decimals already, consider assigning only 1/3 - 1/2 of the problems because the student should be able to go through this chapter quickly.

We study place value with decimals and comparing decimals up to six decimal digits. The next several lessons contain a lot of revision, just using longer decimals than in 5th grade: adding and subtracting decimals, rounding decimals, multiplying and dividing decimals, fractions and decimals, and multiplying and dividing decimals by the powers of ten.

In the lessons about dividing decimals by decimals, I have tried to explain the principle behind the common shortcut (“Move the decimal point in both the divisor and the dividend enough steps that the divisor becomes a whole number”). This shortcut actually has to do with the principle that when you multiply the divisor and the dividend by the same number (*any* number), the value of the quotient does not change. This even ties in with equivalent fractions. Many school books never explain this principle in connection with decimal division.

The last lessons in this chapter deal with measuring units and the metric system, and they round out our study of decimals.

The Lessons in Chapter 3

	page	span
Place Value with Decimals	93	<i>2 pages</i>
Comparing Decimals	95	<i>2 pages</i>
Add and Subtract Decimals	97	<i>2 pages</i>
Rounding Decimals	99	<i>3 pages</i>
Revision: Multiply and Divide Decimals Mentally	102	<i>2 pages</i>
Revision: Multiply Decimals by Decimals	104	<i>3 pages</i>
Revision: Long Division with Decimals	107	<i>2 pages</i>
Problem Solving with Decimals	109	<i>2 pages</i>
Fractions and Decimals	111	<i>3 pages</i>
Multiply and Divide by Powers of Ten	114	<i>2 pages</i>
Revision: Divide Decimals by Decimals	116	<i>3 pages</i>
Divide Decimals by Decimals 2	119	<i>2 pages</i>
Convert Metric Measuring Units	121	<i>3 pages</i>
Mixed Revision	124	<i>2 pages</i>
Chapter 3 Revision	126	<i>4 pages</i>

Helpful Resources on the Internet

Place Value Strategy

Arrange the 3 or 4 digits given by the spinner to make the largest number possible.

<http://www.decimalsquares.com/dsGames/games/placevalue.html>

Decimal Darts

Try to pop balloons with darts by estimating the balloons' height.

<http://www.decimalsquares.com/dsGames/games/darts.html>

Decimal Challenge

Try to guess a decimal number between 0 and 10. After each guess you get feedback about whether your guess was too high or too low.

<http://www.interactivestuff.org/sums4fun/decchall.html>

Beat the Clock

In this timed game, type in the decimal number for the part of a square that is shaded.

<http://www.decimalsquares.com/dsGames/games/beatclock.html>

Scales

Move the pointer to match the decimal number given to you. Refresh the page from your browser to get another problem to solve.

<http://www.interactivestuff.org/sums4fun/scales.html>

Switch

Switch around decimal numbers in a sequence to put them into ascending order. Refresh the page from your browser to get another problem to solve.

<http://www.interactivestuff.org/sums4fun/switch.html>

Smaller and Smaller Maze

Practise ordering decimal numbers to find your way through the maze.

<http://www.counton.org/magnet/kaleidoscope/smaller/index.html>

Decimal and Whole Number Jeopardy

Revise place value and comparing and rounding numbers. Also, practise number patterns.

<http://www.quia.com/cb/8142.html>

Decimals in Space

An Asteroids-style game where you first answer a question about the smallest decimal and then get to shoot asteroids, earning points based on the numbers on them.

<http://www.mathwarehouse.com/games/our-games/decimal-games/decimal-place-value-math-game>

Sock

Push the green blocks into the holes to make the target number.

<http://www.interactivestuff.org/sums4fun/sock.html>

Decimal Squares Blackjack

Play cards with decimals, trying to get as close to 2 as possible without going over.

<http://www.decimalsquares.com/dsGames/games/blackjack.html>

A Decimal Puzzle

Make every circle add up to 3.

http://nlvm.usu.edu/en/nav/frames_asid_187_g_2_t_1.html
open=instructions&from=category_g_2_t_1.html

FunBrain Decimal Power Football

Simple games for addition, subtraction, multiplication and division of decimals, including some with a missing factor or divisor. Solve a problem, and the football player moves down the field.

<http://www.funbrain.com/cgi-bin/getskill.cgi?A1=choices&A2=fb&A3=6&A4=0&A7=0>

Exploring Division of Decimals

Use a square to explore the products of two numbers with one decimal digit. The product is shown as an area.

<http://www.hbschool.com/activity/elab2004/gr6/1.html>

Decimal Speedway

Practise decimal multiplication in this fun car-racing game.

<http://www.decimalsquares.com/dsGames/games/speedway.html>

The Metric Units Tutorial—Metric Number line

A tutorial of the common metric unit prefixes and a way to convert between metric units using a “metric unit number line,” which visually shows you how many steps you need to move the decimal point.

<http://www.dmacc.edu/medmath1/METRIC%20Number%20Line%20HTML/sld001.htm>

Fractions - Decimals calculator

Convert fractions to decimals, or decimals to fractions, including repeating (recurring) decimals to any number of decimal places, which normal calculators do not do.

<http://www.maths.surrey.ac.uk/hosted-sites/R.Knott/Fractions/FractionsCalc.html>

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Fractions and Decimals

You already know how to change decimals to fractions. The number of decimal digits tells you the denominator — it is always a power of ten with as many zeros as you have decimal digits. For the numerator, just copy all the digits from the number.

Example 1. $3,0928 = \frac{30928}{10\,000}$

You can also write this as a mixed number, in which case you take the whole number part from the decimal, and the actual decimal digits from the numerator:

$$15,30599 = \frac{1\,530\,599}{100\,000} = 15 \frac{30\,599}{100\,000}$$

1. Write as fractions.

a. 0,09	b. 0,005	c. 0,045
d. 0,00371	e. 0,02381	f. 0,0000031

2. Write as fractions and also as mixed numbers.

a. 2,9302	b. 2,003814
c. 5,3925012	d. 3,0078
e. 3,294819	f. 45,00032

When changing a **fraction into a decimal**, we have several tools in our “toolbox.”

Tool 1. If the denominator of a fraction is already a power of ten, there is not much to do but to write it as a decimal. The number of zeros in the power of ten tells you the number of decimal digits you need.

$$\frac{3}{10} = 0,3 \quad \frac{451\,593}{10\,000} = 45,1593$$

3. Write as decimals.

a. $\frac{36}{100}$	b. $\frac{5\,009}{1\,000}$	c. $1 \frac{45}{1\,000}$
d. $\frac{3\,908}{10\,000}$	e. $2 \frac{593}{100\,000}$	f. $\frac{5\,903}{1\,000\,000}$
g. $\frac{45\,039\,034}{1\,000\,000}$	h. $\frac{435\,112}{10\,000}$	i. $\frac{450\,683}{100\,000}$

Tool 2. With some fractions, you can find an equivalent fraction with a denominator of 10, 100, 1 000, etc. and then write the fraction as a decimal.

$$\frac{27}{30} = \frac{9}{10} = 0,9$$

$\div 3$
 $\div 3$

$$\frac{66}{200} = \frac{33}{100} = 0,33$$

$\div 2$
 $\div 2$

$$\frac{3}{8} = \frac{375}{1000} = 0,375$$

$\times 125$
 $\times 125$

4. Write as decimals. Think of the equivalent fraction that has a denominator of 10, 100, or 1 000.

a. $\frac{1}{5}$

b. $\frac{1}{8}$

c. $1\frac{1}{20}$

d. $3\frac{9}{25}$

e. $\frac{12}{200}$

f. $8\frac{3}{4}$

g. $4\frac{3}{5}$

h. $\frac{13}{20}$

i. $\frac{7}{8}$

j. $\frac{11}{125}$

k. $\frac{24}{400}$

l. $\frac{95}{500}$

5. In these problems, you see both fractions and decimals. Either change the decimal into a fraction, or vice versa. You decide which way is easier! Then, calculate in your head.

a. $0,2 + \frac{1}{4}$

b. $0,34 + 1\frac{1}{5}$

c. $2\frac{3}{5} + 1,3$

d. $\frac{5}{8} - 0,09$

e. $0,02 + \frac{3}{4}$

f. $1,9 + 3\frac{1}{8}$

g. $\frac{14}{20} - 0,23$

h. $\frac{18}{25} + 0,07$

Tool 3. Most of the time, in order to change a fraction to a decimal, you simply treat the fraction as a division problem and divide (with a calculator or long division).

$$\frac{5}{6} = 5 \div 6 = 0,83333\dots \approx 0,83$$

6. Use long division in your notebook to write these fractions as decimals. Give your answers to three decimal digits.

a. $\frac{2}{9} =$

b. $\frac{3}{7} =$

c. $\frac{7}{16} =$

7. Use a calculator to write these fractions as decimals. Give your answers to three decimal digits.

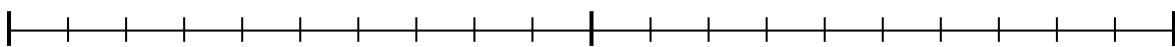
a. $\frac{1}{11} =$

b. $\frac{3}{23} =$

c. $\frac{47}{56} =$

8. Label the bold tick marks on the number line “0,” “1” and “2.” Then mark the following numbers on it where they belong.

$$0,2; \quad \frac{1}{4}; \quad 0,65; \quad 1\frac{1}{3}; \quad 0,04; \quad \frac{2}{5}; \quad 1,22; \quad 1\frac{3}{4}; \quad 1,95; \quad 1\frac{4}{5}$$



9. One bag of milk powder contains 900 g. Another contains $\frac{3}{4}$ kg.
What is the combined weight of the two?

10. A drawing measures $14\frac{5}{10}$ centimetres by $20\frac{3}{10}$ centimetres.

a. Write these mixed numbers as decimals.

b. Calculate the area of the drawing to the nearest square centimetre.

11. Rice costs R16,45 per kilogram. Mampho bought $1\frac{3}{4}$ kg of it.
Calculate the total price of Mampho’s purchase (in rand and cents).

12. Explain two different ways to calculate the price of $\frac{3}{8}$ of a litre of petrol, if one litre costs R13,30.
(You do not have to calculate the price; just explain two ways *how* to do it.)

13. Give your answer to each of the following problems as both a fraction and as a decimal.

a. $0,3 \times \frac{5}{8}$

b. $\frac{3}{4} \times 1,5$

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Convert Metric Measuring Units

The metric system has one basic unit for each thing we might measure: For length, the unit is the **metre**. For weight, it is the **gram**. And for volume, it is the **litre**.

All of the other units for measuring length, weight, or volume are *derived* from the basic units using **prefixes**. The prefixes tell us what multiple of the basic unit the *derived unit* is.

For example, centilitre is 1/100 part of a litre (*centi* means 1/100).

Prefix	Abbreviated	Meaning
kilo-	k	1 000
hecto-	h	100
deca-	da	10
-	-	(the basic unit)
deci-	d	1/10
centi-	c	1/100
milli-	m	1/1 000

Unit	Abbr	Meaning
kilometre	km	1 000 metres
hectometre	hm	100 metres
decametre	dam	10 metres
metre	m	(the basic unit)
decimetre	dm	1/10 metre
centimetre	cm	1/100 metre
millimetre	mm	1/1000 metre

Unit	Abbr	Meaning
kilogram	kg	1 000 grams
hectogram	hg	100 grams
decagram	dag	10 grams
gram	g	(the basic unit)
decigram	dg	1/10 gram
centigram	cg	1/100 gram
milligram	mg	1/1000 gram

Unit	Abbr	Meaning
kilolitre	kl	1 000 litres
hectolitre	hl	100 litres
decalitre	dal	10 litres
litre	L	(the basic unit)
decilitre	dl	1/10 litre
centilitre	cl	1/100 litre
millilitre	ml	1/1000 litre

1. Write these amounts using the basic units (metres, grams, or litres) by “translating” the prefixes. Use both fractions and decimals, like this: 3 cm = 3/100 m = 0,03 m (since “centi” means “hundredth part”).

a. $3 \text{ cm} = 3/100 \text{ m} = 0,03 \text{ m}$	b. $2 \text{ cg} = \underline{\hspace{2cm}} \text{ g} = \underline{\hspace{2cm}} \text{ g}$
$5 \text{ mm} = \underline{\hspace{2cm}} \text{ m} = \underline{\hspace{2cm}} \text{ m}$	$6 \text{ ml} = \underline{\hspace{2cm}} \text{ L} = \underline{\hspace{2cm}} \text{ L}$
$7 \text{ dl} = \underline{\hspace{2cm}} \text{ L} = \underline{\hspace{2cm}} \text{ L}$	$1 \text{ dg} = \underline{\hspace{2cm}} \text{ g} = \underline{\hspace{2cm}} \text{ g}$

2. Write the amounts in basic units (metres, grams, or litres) by “translating” the prefixes.

a. $3 \text{ kl} = \underline{\hspace{2cm}} \text{ L}$	b. $2 \text{ dam} = \underline{\hspace{2cm}} \text{ m}$	c. $70 \text{ km} = \underline{\hspace{2cm}} \text{ m}$
$8 \text{ dag} = \underline{\hspace{2cm}} \text{ g}$	$9 \text{ hl} = \underline{\hspace{2cm}} \text{ L}$	$5 \text{ hg} = \underline{\hspace{2cm}} \text{ g}$
$6 \text{ hm} = \underline{\hspace{2cm}} \text{ m}$	$7 \text{ kg} = \underline{\hspace{2cm}} \text{ g}$	$8 \text{ dal} = \underline{\hspace{2cm}} \text{ L}$

3. Write the amounts with derived units (units with prefixes) and single-digit numbers.

a. $3 \text{ 000 g} = \underline{3} \text{ } \underline{\text{kg}}$	b. $0,01 \text{ m} = \underline{\hspace{2cm}} \text{ }$	c. $0,04 \text{ L} = \underline{\hspace{2cm}} \text{ }$
$800 \text{ L} = \underline{8} \text{ } \underline{\text{ }}$	$0,2 \text{ L} = \underline{\hspace{2cm}} \text{ }$	$0,8 \text{ m} = \underline{\hspace{2cm}} \text{ }$
$60 \text{ m} = \underline{6} \text{ } \underline{\text{ }}$	$0,005 \text{ g} = \underline{\hspace{2cm}} \text{ }$	$0,007 \text{ L} = \underline{\hspace{2cm}} \text{ }$

4. Write using prefixed units.

- | | | |
|-----------------------------|--------------------------|----------------------------|
| a. 0,04 metres = 4 cm | b. 0,005 grams = 5 _____ | c. 0,037 metres = 37 _____ |
| d. 400 litres = 4 _____ | e. 0,6 metres = 6 _____ | f. 2 000 metres = 2 _____ |
| g. 0,206 litres = 206 _____ | h. 20 metres = 2 _____ | i. 0,9 grams = 9 _____ |

5. Change into the basic unit (either metre, litre, or gram). Think of the meaning of the prefix.

- | | | |
|-------------------|------------|-------------|
| a. 45 cm = 0,45 m | b. 65 mg = | c. 2 dm = |
| d. 81 km = | e. 6 ml = | f. 758 mg = |
| g. 2 kl = | h. 8 dl = | i. 9 dag = |

Example 1. Convert 2,5 cg to grams.

					2, 5	
kg	hg	dag	g	dg	cg	mg

			0,	0	2	5
kg	hg	dag	g	dg	cg	mg

Write 2,5 in the chart so that “2”, which is in the ones place, is placed in the centigrams place.

Move the decimal comma just after the grams place. Add necessary zeros. Answer: 0,025 g.

6. Write the measurements in the place value charts.

a. 12,3 m							
	km	hm	dam	m	dm	cm	mm

c. 56 cl

	kl	hl	dal	l	dl	cl	ml

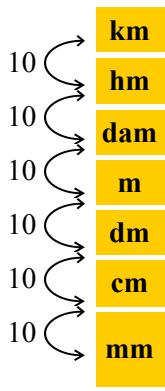
b. 78 mm							
	km	hm	dam	m	dm	cm	mm

d. 9,83 hg

	kg	hg	dag	g	dg	cg	mg

7. Convert the measurements to the given units, using the charts above.

	m	dm	cm	mm
a. 12,3 m	12,3			
b. 78 mm				78 mm
	L	dl	cl	ml
c. 56 cl				
	g	dg	cg	mg
d. 9,83 hg				



You can also convert measurements by thinking of how many steps apart the two units are in the chart and then multiplying or dividing by the corresponding power of ten.

Example 2. Convert 2,4 km into centimetres.

There are five steps from kilometres to centimetres. That means we would multiply 2,4 by 10, five times—or multiply 2,4 by 10^5 .

$$2,4 \cdot 100\,000 = 240\,000, \text{ so } 2,4 \text{ km} = 240\,000 \text{ cm.}$$

Example 3. Convert 2 900 cg into hectograms.

“Centi” and “hecto” are four steps apart, so we will divide by $10^4 = 1000$.
 $2\,900 \div 10\,000 = 0,29$, so $2\,900 \text{ cg} = 0,29 \text{ hg}$.

8. Convert the measurements. You can write the numbers in the place value charts or count the steps.

- | | |
|------------------------|-------------------------|
| a. 560 cl = _____ L | b. 0,493 kg = _____ dag |
| c. 24,5 hm = _____ cm | d. 491 cm = _____ m |
| e. 35 200 mg = _____ g | f. 32 dal = _____ cl |
| g. 0,483 km = _____ dm | h. 0,0056 km = _____ cm |
| i. 1,98 hl = _____ dl | j. 9,5 dl = _____ L |

kl	hl	dal	l	dl	cl	ml
kg	hg	dag	g	dg	cg	mg

km	hm	dam	m	dm	cm	mm

9. Each measurement has a mistake, either in the unit or in the decimal comma. Correct them.

- | | |
|---|-----------------------------------|
| a. The length of a pencil: 13 m | b. The length of an eraser: 45 cm |
| c. Measurement of Father's waist: 9,2 m | d. The height of a room: 0,24 m |
| e. Jack's height: 1,70 mm | f. Jenny's height: 1,34 cm |

10. Find the total ...

- | | |
|--|--|
| a. ... weight of books that weigh individually:
1,2 kg, 1,04 kg, 520 g and 128 g. | b. ... volume of containers whose individual volumes are:
1,4 L, 2,25 L, 550 ml, 240 ml and 4 dl. |
|--|--|

11. A dropper measures 4 ml. How many full droppers can you get from a 2-dl bottle?

12. Once a day, a nurse has to give a patient 3 mg of medicine for each kilogram of body weight. The patient weighs 70 kg. How many days will it take for the patient to have received 2 g of medicine?

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Chapter 4: Ratios

Introduction

In this chapter we concentrate on the concept of ratio and various applications involving ratios and rates.

The chapter starts out with the basic concepts of ratio, rate and unit rate. The lesson *Equivalent Rates* allows students to solve a variety of word problems involving ratios and rates. We also connect the concept of rates (specifically, tables of equivalent rates) with ordered pairs, use equations (such as $y = 3x$) to describe these tables, and plot the ordered pairs in the coordinate plane.

Next, we study various kinds of word problems involving ratios and use a bar model to solve these problems in two separate lessons. These lessons tie ratios in with the student's previous knowledge of bar models as a tool for problem solving.

Lastly, students encounter the concept of aspect ratio, which is simply the ratio of a rectangle's width to its height, and solve a variety of problems involving aspect ratio.

The Lessons in Chapter 4

	page	span
Ratios and Rates	132	<i>4 pages</i>
Unit Rates	136	<i>2 pages</i>
Using Equivalent Rates	138	<i>4 pages</i>
Ratio Problems and Bar Models 1	142	<i>3 pages</i>
Ratio Problems and Bar Models 2	145	<i>3 pages</i>
Aspect Ratio	148	<i>2 pages</i>
Mixed Revision	150	<i>2 pages</i>
Chapter 4 Revision	152	<i>2 pages</i>

Helpful Resources on the Internet

Practice with Ratios

An online quiz from Regents Exam Prep Center that includes both simple and challenging questions and word problems concerning ratios.

<http://www.regentsprep.org/Regents/math/ALGEBRA/AO3/pracRatio.htm>

Ratio Pairs Matching Game

Match cards representing equivalent ratios.

Easy: <http://nrich.maths.org/4824> Challenge: <http://nrich.maths.org/4821>

Equivalent Ratios Workout

10 online practice problems.

<http://www.math.com/school/subject1/practice/S1U2L1/S1U2L1Pract.html>

Ratio Stadium

A multi-player online racing game for matching equivalent ratios. The student with the fastest rate of correct answers will win the race.

<http://www.arcademicskillbuilders.com/games/ratio-stadium/>

Dirt Bike Proportions

A racing game where you need to find the unknown in a simple proportion. This game would actually work equally well for practising equivalent fractions because the proportions are quite simple.

<http://www.arcademicskillbuilders.com/games/dirt-bike-proportions/dirt-bike-proportions.html>

All About Ratios - Quizzes

Online quizzes about same and different ratios.

<http://math.rice.edu/~lanius/proportions/index.html>

Free Ride

An interactive activity about bicycle gear ratios. Choose the front and back gears, which determines the gear ratio. Then choose a route, pedal forward, and make sure you land exactly on the five flags.

<http://illuminations.nctm.org/ActivityDetail.aspx?ID=178>

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Using Equivalent Rates

Example 1. If Sipho can ride his bike to a town that is 33 kilometres away in 45 minutes, how far can he ride in 1 hour?

Let's form some equivalent rates, starting with 33 kilometres per 45 minutes and hoping to arrive at so many kilometres per 60 minutes.

However, it is not easy to go directly from 45 minutes to 60 minutes (1 hour). So, let's first figure the rate for 15 minutes, which is easy.

Why? Because to get from 45 minutes to 15 minutes you simply divide both terms of the rate by 3.

Then from 15 minutes, we can easily get to 60 minutes: Just multiply both terms by 4. We find that he can ride 44 kilometres in one hour.

$$\frac{33 \text{ kilometres}}{45 \text{ minutes}} = \frac{11 \text{ kilometres}}{15 \text{ minutes}} = \frac{44 \text{ kilometres}}{60 \text{ minutes}}$$

$\div 3$ $\times 4$
 $\div 3$ $\times 4$

1. Write the equivalent rates.

a. $\frac{15 \text{ km}}{3 \text{ hr}} = \frac{\boxed{}}{1 \text{ hr}} = \frac{15 \text{ min}}{\boxed{}} = \frac{45 \text{ min}}{\boxed{}}$

b. $\frac{R6}{45 \text{ min}} = \frac{\boxed{}}{15 \text{ min}} = \frac{\boxed{}}{1 \text{ hr}} = \frac{\boxed{}}{1 \text{ hr } 45 \text{ min}}$

c. $\frac{3 \text{ cm}}{8 \text{ m}} = \frac{\boxed{}}{2 \text{ m}} = \frac{\boxed{}}{12 \text{ m}} = \frac{\boxed{}}{20 \text{ m}}$

d. $\frac{115 \text{ words}}{2 \text{ min}} = \frac{\boxed{}}{1 \text{ min}} = \frac{\boxed{}}{3 \text{ min}}$

2. a. Joshua can ride 10 kilometres in 16 minutes. How long will it take him to ride 55 kilometres?
Use the equivalent rates.

$$\frac{10 \text{ kilometres}}{16 \text{ minutes}} = \frac{5 \text{ kilometres}}{\boxed{\text{minutes}}} = \frac{55 \text{ kilometres}}{\boxed{\text{minutes}}}$$

b. How many kilometres can Joshua ride in 40 minutes?

3. An automobile can go 80 kilometres on 8 litres of petrol.

a. How many litres of petrol would the automobile need for a trip of 95 kilometres?
Use the equivalent rates below.

$$\frac{80 \text{ kilometres}}{8 \text{ litres}} = \frac{10 \text{ kilometres}}{\boxed{\text{litres}}} = \frac{95 \text{ kilometres}}{\boxed{\text{litres}}}$$

b. How far can the automobile travel on 15 litres of petrol?

Example 2. You get 20 erasers for R59,00. How much would 22 erasers cost?

You can solve this problem in several ways. Let's use a table of rates this time.

Cost (C)			R29,50	R59,00	
Erasers (E)	1	2	10	20	22

First, find the cost for 10 erasers, and then the cost for 2. After that, you can get the cost for 22 by adding.

Ten erasers will cost half of R59,00. Two erasers will cost one-fifth of that (divide by 5 to find it!).

Lastly, add the cost of 20 erasers to the cost of 2 erasers to get the cost for 22 erasers.

Note 1: In the table, each pair of numbers is a rate. For example, R59,00 for 20 erasers (or R59,00/20 erasers) is a rate, and so is R29,50 for 10 erasers.

Note 2: We can write an equation relating the Cost (C) and the number of Erasers (E). You will find that easily from the unit rate (the price for one): $C = 2,95E$. In other words, the cost is 2,95 times the number of erasers.

4. Finish solving the problem in the example above.

5. How many erasers would you get with R11,80?

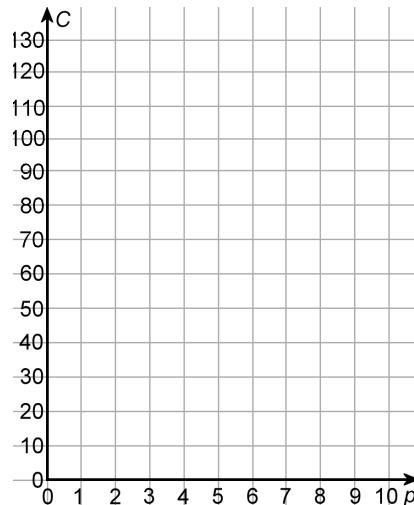
6. On average, Thipe makes a basket nine times out of twelve shots when he is practising. How many baskets can he expect to make when he tries 200 shots? A table of rates can help you solve this.

baskets							
shots							

7. a. Three pairs of socks cost R39. Fill in the table of rates.

The variable C stands for cost, and p for pairs of socks.

C			39						
p	1	2	3	4	5	6	7	8	9



b. Each number pair in the table *is* a rate, but we can also view them as points with two coordinates.
Plot the number pairs in the coordinate grid.

c. Write an equation relating the cost (C) and the number of pairs of socks (p).

8. a. You get 30 pencils for R61,50. How much would 52 pencils cost?

Cost						
Pencils						

b. Write an equation relating the cost (C) and the number of pencils (P).

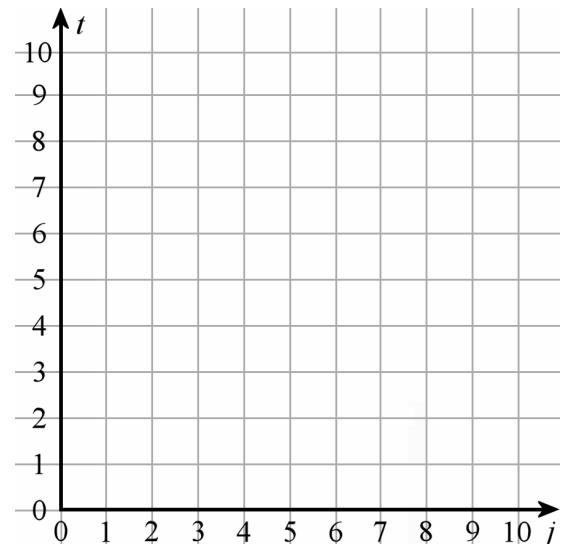
9. When Jemima makes 4 litres of tea (a pot full), she needs five jars for the tea. From this, we get the rate of 4 litres / 5 jars.

- a. Fill in the table. The variable t stands for the amount of tea, and j for the number of jars.

t					4				
j	1	2	3	4	5	6	7	8	9

- b. Plot the number pairs from the table in this coordinate grid.
c. How many jars will Jemima need for 20 litres of tea?

- d. If Jemima has 16 jars full of tea, how many litres of tea is in them?

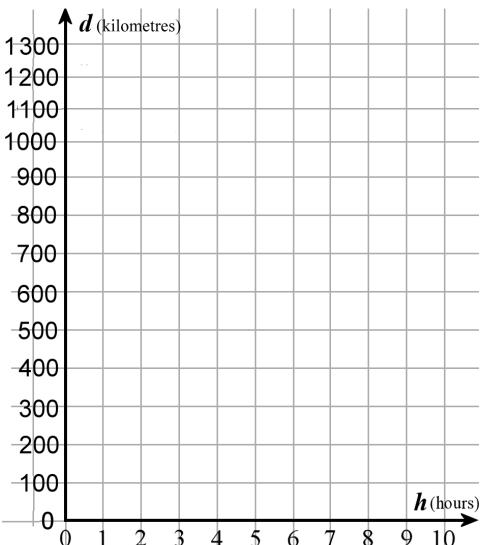


10. a. A train travels at a constant speed of 120 kilometres per hour. Fill in the table of rates.

d									
h	1	2	3	4	5	6	7	8	9

- b. Write an equation relating the distance (d) and the number of hours (h).

- c. Plot the points in the grid on the right. The variable h stands for hours, and d for distance.



11. Another train travels at the constant speed of 90 kilometres per hour. Fill in the table of rates. Then, plot the points in the same coordinate grid as for the train in #10.

d					
h	1	2	3	4	5

d					
h	6	7	8	9	10

12. How can you see from the graph which train travels faster?

13. The plot shows the walking speeds for two people (t is in minutes, d is in kilometres). Your task is to fill in the two ratio tables below. To make that easier, first find the dots that are at places where the lines cross, so that you can easily read the coordinates. (Hint: For some of the points, you will need to use fractions and mixed numbers.)

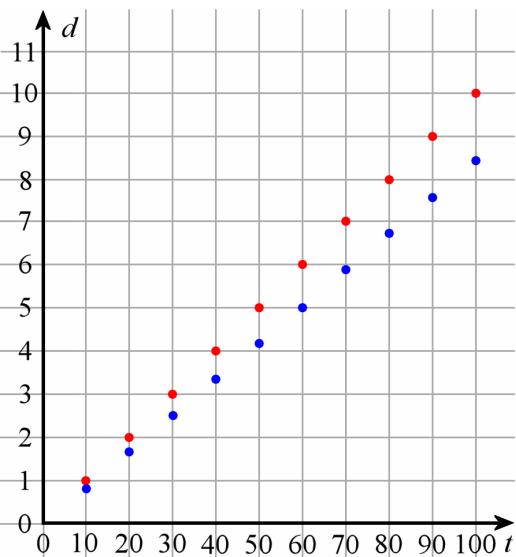
Person 1 (red dot)

d (km)									
t (min)									

Person 2 (blue dot)

d (km)									
t (min)									

- a. What is the speed of the first person in kilometres per hour?



- b. What is the speed of the second person in kilometres per hour?

14. Train 1 travels at a constant speed of 384 kilometres in three hours. Train 2 travels 784 kilometres in seven hours. Which train is faster?

15. Find which is a better deal by comparing the unit rates: R106 for eight bottles of shampoo, or R93 for six bottles of shampoo?

16. In a poll of 1 000 people, 640 said they liked blue.

- a. Simplify this ratio to lowest terms:

640 people *out of* 1 000 people = _____ people *out of* _____ people

- b. Assuming the same ratio holds true in another group of 100 people, how many of those people can we expect to like blue?

- c. Assuming the same ratio holds true in another group of 225 people, how many of those people can we expect to like blue?

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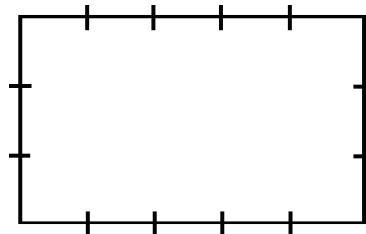
Aspect Ratio

You might have heard about the **aspect ratio** of the screens of televisions, computer monitors and other monitors. The aspect ratio is simply **the ratio of a rectangle's width to its height**.

Example 1. A rectangle's width and height are in a ratio of 5:3.

This means the aspect ratio is 5:3. If the rectangle's perimeter is 64 cm, what are its width and its height?

Let's draw the rectangle. Working from the 5:3 aspect ratio, let's divide the sides into "parts," or the same-sized segments, 5 for the width, and 3 for the height. We can see in the picture that perimeter is made up of 16 of these "parts." Since $64 \div 16 = 4$, each part is 4 cm long.



Therefore, the rectangle's width is $5 \times 4 \text{ cm} = 20 \text{ cm}$, and its length is $3 \times 4 \text{ cm} = 12 \text{ cm}$.

1. The width and height of a rectangle are in a ratio of 9:2.
 - a. Draw the rectangle, and divide its width and length into parts according to its aspect ratio.
 - b. If the rectangle's perimeter is 220 cm, find its width and its height.
2. A rectangle's width is three times its height, and its perimeter is 120 mm. Find the rectangle's width and its height.
3. Find the aspect ratio of each rectangle:
 - a. a rectangle whose height is $2/5$ of its width
 - b. a rectangle whose height is five times its width
 - c. a square
4. The door of a refrigerator is $4/9$ as wide as it is tall.
 - a. What is the ratio of the door's width to its height?
 - b. If the door is 54 cm wide, how tall is it?

5. Little Mary drew a picture on a rectangular piece of paper that was 15 centimetres wide and 25 centimetres high.
- Write the aspect ratio, and simplify it to the lowest terms.
 - If this picture were enlarged to be 45 cm wide, how high would it be? Use equivalent ratios.
6. Mr. Miller is ordering custom-made windows for his new house. He is considering windows of these sizes: 70 cm \times 90 cm, 80 cm \times 100 cm, 90 cm \times 110 cm and 100 cm \times 120 cm.
- Write the aspect ratios of all the windows and simplify them to lowest terms.
 - Do any of the windows share exactly the same aspect ratio when simplified? If so, then which ones? (That would mean they would have exactly the same shape.)
7. A sandbox is two times as wide as it is long.
- What is its aspect ratio?
 - The perimeter of the sandbox is 6 m. Find its length and width.
 - Find its area.
8. Two television sets have the same perimeter, 150 cm. The aspect ratio of one is 16 : 9, and the aspect ratio of the other is 4 : 3.
- Find the length and width of each television.
 - Which television has the larger area?
9. The area of a square is 49 sq. cm. If two of these squares are put side by side, we get a rectangle.
- Find the aspect ratio of that rectangle.
 - Find the perimeter of the rectangle.

Don't confuse area with perimeter.

The aspect ratio pertains to the *length* and *width*, not to the area. However, once you know the length and the width, you can calculate the area.

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Chapter 5: Percent

Introduction

The concept of percent builds on the student's understanding of fractions and decimals. Specifically, students should be very familiar with the idea of finding a fractional part of a whole (such as finding $\frac{3}{4}$ of R240). Students who have used Math Mammoth have been practising that concept since 4th grade. One reason why I have emphasised finding a fractional part of a whole so much in the earlier grades is specifically to lay a groundwork for the concept of percent. Assuming the student has mastered how to find a fractional part a whole, and can easily convert fractions to decimals, then studying the concept of percent should not be difficult.

The first lesson, *Percent*, practises the concept of percent as a hundredth part and how to write fractions and decimals as percentages. Next, we study how to find a percentage when the part and the whole are given (for example, if 15 out of 25 club members are girls, what percentage of them are girls?).

The following two lessons have to do with finding a certain percentage of a given number or quantity. First, we study how to do that using mental maths techniques. For example, students find 10% of R400 by dividing R400 by 10. Next, students find a percentage of a quantity using decimal multiplication, both manually and with a calculator. For example, students find 17% of 45 km by multiplying $0,17 \times 45$ km.

I prefer teaching students to calculate percentages of quantities using decimals, instead of using percentage proportion or some other method (such as changing 17% into the fraction $17/100$ for calculations). That is because using decimals is simpler: we simply change the percentage into a decimal and multiply, instead of having to build a proportion or use fractions. Also, decimals will be so much easier to use later on when solving word problems that require the usage of equations.

Next is a lesson about discounts, which is an important application from everyday life. Then we go on to the lesson *Practice with Percent*, which contrasts the two types of problems students have already studied: questions that ask to calculate a given percentage of a number and questions that ask to find the percentage. For example, the first type of question could be “*What is 70% of R380?*” and the second type could be “*What percentage is R70 of R380?*”

The last lesson lets students find the total when the percentage and the partial amount are known. For example: “Three-hundred and twenty students, which is 40% of all students, take PE. How many students are there in total?” We solve these with the help of bar models.

I have made several videos to match these lessons. You can watch them here:
<http://www.mathmammoth.com/videos/percent.php>

The Lessons in Chapter 5

	page	span
Percent	157	<i>4 pages</i>
What Percentage...?	161	<i>2 pages</i>
Percentage of a Number (Mental Maths)	163	<i>3 pages</i>
Percentage of a Number: Using Decimals	166	<i>3 pages</i>
Discounts	169	<i>2 pages</i>
Practice with Percent	171	<i>3 pages</i>
Finding the Total When the Percentage Is Known	174	<i>2 pages</i>
Mixed Revision	176	<i>2 pages</i>
Revision: Percent	178	<i>2 pages</i>

Helpful Resources on the Internet

Percent videos by Maria

Videos on percent-related topics that match the lessons in this chapter!

<http://www.mathmammoth.com/videos/percent.php>

Games & Tools

Virtual Manipulative: Percentages

An interactive tool where you fill in any two of the three boxes (whole, part and percent), and it will calculate the missing part and show the result visually in two ways.

http://nlvm.usu.edu/en/nav/frames_asid_160_g_2_t_1.html

Mission: Magnetite

Hacker tries to drop magnetite on Motherboard. To stop him, match up percentages, fractions and images showing fractional parts.

<http://pbskids.org/cyberchase/media/games/percent/index.html>

Fractions and Percent Matching Game

A simple matching game: match fractions and percentages.

http://www.mathplayground.com/matching_fraction_percent.html

Fraction/Decimal/Percent Jeopardy

Answer the questions correctly, changing between fractions, decimals and percentages.

<http://www.quia.com/cb/34887.html>

Flower Power

Grow flowers and harvest them to make money in this addictive order-'em-up game. Practise ordering decimals, fractions and percentages. The game starts with ordering decimals (daisies), and proceeds into fractions (tulips or roses).

<http://www.mangahigh.com/en/games/flowerpower>

Percent Shopping

Choose toys to purchase. In level 1, you find the sale price when the original price and percentage discount are known. In level 2, you find the percentage discount when the original price and the sale price are known.

http://www.mathplayground.com/percent_shopping.html

Penguin Waiter

Simple game where you calculate the correct tip to leave for the penguin waiter.

<http://www.funbrain.com/penguin/>

Worksheets

Percent worksheets

Create an unlimited number of free customisable percent worksheets to print.

<http://www.homeschoolmath.net/worksheets/percent-decimal.php>

<http://www.homeschoolmath.net/worksheets/percent-of-number.php>

<http://www.homeschoolmath.net/worksheets/percentages-words.php>

Worksheets & quizzes for percentages, ratios, and proportions

Several online quizzes and a few PDF worksheets for these topics.

<http://www.math4children.com/Topics/Percentages>

Tutorials

Percentages of Something

See simple percentages illustrated in different ways.

<http://www.bbc.co.uk/skillswise/game/ma16perc-game-percentages-of-something>

A Conceptual Model for Solving Percent Problems

Explanation of how to use a 10 x 10 grid to explain the basic concept of percentage, AND solve various types of percentage problems.

<http://illuminations.nctm.org/LessonDetail.aspx?id=L249>

Meaning of Percent -- Writing Fractions as Percents

Free percent lessons from Math Goodies.

http://www.mathgoodies.com/lessons/vol4/meaning_percent.html

http://www.mathgoodies.com/lessons/vol4/fractions_to_percents.html

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Percentage of a Number (Mental Maths)

100% of something means *all* of it. 1% of something means 1/100 of it.

Since one percent means “a hundredth part,” calculating a percentage of a quantity is the same thing as finding a fractional part of it. So **percentages are really fractions!**

How much is 1% of 200 kg? This means how much is 1/100 of 200 kg? It is simply 2 kg.

To find 1% of something (1/100 of something), divide by 100.

Do you remember how to divide by 100 in your head? Just move the decimal comma two places to the left. For example, 1% of 540 is 5,4, and 1% of 8,30 is 0,083.

To find 2% of some quantity, first find 1% of it, and double that.

For example, let's find 2% of R6. Since 1% of R6 is R0,06, then 2% of R6 is R0,12.

To find 10% of some quantity, divide by 10.

Why does that work? It is because 10% is 10/100, which equals 1/10. So 10% is 1/10 of the quantity!

For example, 10% of R780 is R78. And 10% of R6,50 is R0,65.

(To divide by 10 in your head, just move the decimal comma one place to the left.)

Can you think of a way to find 20% of a number? (Hint: Start with finding 10% of the number.)

1. Find 10% of these numbers.

a. 700 _____

b. 321 _____

c. 60 _____

d. 7 _____

2. Find 1% of these numbers.

a. 700 _____

b. 321 _____

c. 60 _____

d. 7 _____

3. One percent of Mother's pay cheque is R22. How much is her total pay cheque?

4. Fill in the table. Use mental maths.

percentage ↓ number →	1 200	80	29	9	5,7
1% of the number					
2% of the number					
10% of the number					
20% of the number					

5. Fill in this guide for using mental maths with percentages:

Mental Math and Percentage of a Number	
50% is $\frac{1}{2}$. To find 50% of a number, divide by _____.	50% of 244 is _____.
10% is $\frac{1}{10}$. To find 10% of a number, divide by _____.	10% of 47 is _____.
1% is $\frac{1}{100}$. To find 1% of a number, divide by _____.	1% of 530 is _____.
To find 20%, 30%, 40%, 60%, 70%, 80%, or 90% of a number, <ul style="list-style-type: none"> • First find _____ % of the number, and • then multiply by 2, 3, 4, 6, 7, 8, or 9. 	10% of 120 is _____. 30 % of 120 is _____. 60 % of 120 is _____.

6. Find the percentages. Use mental maths.

a. 10% of 60 kg _____ 20% of 60 kg _____	b. 10% of R14 _____ 30% of R14 _____	c. 10% of 5 m _____ 40% of 5 m _____
d. 1% of R60 _____ 4% of R60 _____	e. 10% of 110 cm _____ 70% of 110 cm _____	f. 1% of R1 330 _____ 3% of R1 330 _____

7. Daniel pays a 20% income tax on his R2 100 salary.

- a. How many rand is the tax?
- b. How much money does he have left after paying the tax?
- c. What percentage of his salary does he have left?

8. Nomvula pays 30% of her R3 100 salary in taxes. How much money does she have left after paying the tax?

9. Identify the errors that these children made. Then find the correct answers.

a. Find 90% of R55. Peter's solution: 10% of R55 is R5,50 So, I subtract $100\% - R5,50 = R94,50$	b. Find 6% of R1 400. Karen's solution: 1% of R1 400 is R1,40. So, 6% is six times that, or R8,40.
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Some more mental maths “tricks”	
90% of a quantity First find 10% of the quantity and then subtract that from 100% of it.	25% of a quantity 25% is the same as $1/4$. So, to find 25% of a quantity, divide it by 4.
12% of a quantity First find 10% of it. Then find 1% of it, and use that 1% to find 2% of it. Then add the 10% and the 2%.	75% of a quantity 75% is $3/4$. First find $1/4$ of the quantity and multiply that by 3.

10. Find percentages of the quantities.

a. 50% of 26 cm _____	b. 25% of 40 mm _____	c. 80% of 45 m _____
d. 75% of R4,40 _____	e. 90% of 1,2 m _____	f. 25% of 120 kg _____

11. Fill in the mental maths method for finding 12% of R65.

10% of R65 is R_____. 1% of R65 is R_____. 2% of R65 is R_____.

Now, add to get 12% of R65: R_____ + R_____ = R_____

12. Fill in the mental maths shortcut for finding 24% of 44 kg.

25% of 44 kg is _____ kg. 1% of 44 kg is _____ kg.

Subtract _____ kg – _____ kg = _____ kg

13. From her cell phone bill, Hannah sees that of the 340 text messages she sent last month, 15% were sent during the night at a cheaper rate. How many messages did Hannah send at night? During the day?

14. A herd of 40 horses had some bay, some chestnut and some white horses. Thirty percent of them are bay, and 45% are chestnut. How many horses are white?

15. A college has 1 500 students, and 12% of them ride the bus. Another 25% walk to the college. How many students do not do either?

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Discounts

Other than figuring Value Added Tax, the area of life in which you will probably most often need to use percentages is in calculating discounts.

A guitar that costs R600 is 20% off. What is the sale price?

Method 1. We calculate 20% of R600. That is the discounted amount in *rand*. Then we subtract that from the original price, R600.

20% of R600 is R120. And $R600 - R120 = R480$. So the sale price is R480.

Method 2. Since 20% of the price has been removed, 80% of the price is *left*. By calculating 80% of the original price, you will get the new discounted price: $0,8 \times R600 = R480$

Two methods for calculating the discounted price:

1. Calculate the discount amount as a percentage of the original price. Then subtract.
2. Find what percentage of the price is left. Then calculate that percentage of the normal price.

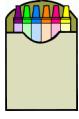
1. All of these items are on sale. Calculate the discount in rand and the resulting sale price.

a.  Price: R200 20% off Discount amount: R _____ Sale price: R _____	b.  Price: R10 40% off Discount amount: R _____ Sale price: R _____	c.  Price: R95 30% off Discount amount: R _____ Sale price: R _____
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2. A frisbee that originally cost R25 was on sale for 20% off.

Mashudu tried to calculate the discounted price this way: $R25 - R20 = R5$. What did she do wrong? Find the correct discounted price.

3. All these items are on sale. Find the discounted price.

a. Price: R6,20 25% off  Discount amount: R _____ Discounted price: R _____	b. Price: R130 25% off  Discount amount: R _____ Discounted price: R _____	c. Price: R950 30% off  Discount amount: R _____ Discounted price: R _____
d. Price: R80 40% off  Discount amount: R _____ Discounted price: R _____	e. Price: R8,00 10% off  Discount amount: R _____ Discounted price: R _____	f. Price: R7,30 50% off  Discount amount: R _____ Discounted price: R _____

You can often use **estimation** when calculating the discounted price.

Example 1. A bicycle that costs R598,95 is discounted by 25%. What is the discounted price?

To estimate, round the original price of the bicycle to R600. Then, 25% of R600 is R150 (it is 1/4 of it). So the discounted price is about R450.

Example 2. A dress that costs R425,90 is discounted by 28%. What is the discounted price?

Round the discount percentage to 30%, and the price of the dress to R430. 10% of R430 is R43. 30% of R430 is three times that much, or R129. Subtract using rounded numbers: $R430 - R130 = R300$.

4. Estimate the discounted price.

- a. 30% off of a book that costs R39,90
- b. 17% off of a sandwich that costs R12,50
- c. 75% off of a swimming cap that costs R75,50

5. Which is a better deal? Estimate using rounded numbers and mental maths.

- a. 75% off of a brand-name pair of sunglasses that costs R199
OR an equivalent off-brand pair of sunglasses for R44,99.

- b. 40% off of a new pair of sandals that costs R89
OR a used pair, like new, of the same sandals for R39,90.

6. A company sells a computer program for R540,00. They estimate they would sell 50 copies of it in a week, with that price. If they discount the price by 25%, they think they could sell 100 copies.

Estimate which way they would earn the most money.

Example 3. A lunchbox costing R50 is discounted and now costs only R35. What is the discount percentage?

Think about what *fraction* of the price “disappeared.” Then, write that fraction as a percentage.

We see that R15 of the price “went away.” The fraction of the price that was taken off is thus $15/50$. Now we simply rewrite $15/50$ as $30/100$, which is, as a percentage, 30%. So it was discounted by 30%.

7. Find the discount percentage.

- a. A scarf: original price, R50; discounted price, R45.
- b. A netball: original price, R40; discounted price, R30.
- c. Gloves: original price, R25; discounted price, R20.

8. Which of these methods work for calculating a discounted price of 25% off of R46?

$$0,25 \times R46$$

$$0,75 \times R46$$

$$R46 - \frac{R46}{25}$$

$$R46 - \frac{R46}{4}$$

$$\frac{R46}{4}$$

$$\frac{R46}{4} \times 3$$

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math **MAMMOTH**

Grade 6-B Worktext South African Version

Factoring

Fractions

Integers

Geometry

Statistics



L i g h t B l u e S e r i e s

Sample worksheet from
By Maria Miller
www.mathmammoth.com

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EDITION 10/2015

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**Sample worksheet from
www.mathmammoth.com**

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Foreword

Math Mammoth Grade 6-A and Grade 6-B South African Version worktexts comprise a complete maths curriculum for sixth grade mathematics studies.

Math Mammoth South African version has been customised to South Africa in the following manners:

- We use South African names (instead of Jack and Jill, we use Ansie and Musa).
- The currency used in word problems is rand.
- The material is all metric. In other words, the US customary measuring units are not used.
- Spelling is British English instead of American English.
- Paper size is A4.

Please note that the curriculum is not following the South African official syllabus for sixth grade maths. Instead, it is a copy of the US version of Math Mammoth Grade 6, aligned to the US Common Core Standards. This decision was made because of the great amount of work that would be involved in writing new lessons and reorganising old ones to match all the standards in the South African syllabus. For the most part, Math Mammoth is exceeding South African standards.

The Grade 6-A text covered the first half of the topics for 6th grade: a revision of the four basic operations, an introduction to algebra, and decimals, ratios and percent.

This part B covers the remainder of the topics for 6th grade: using factoring to help to calculate the Greatest Common Factor (GCF) and Least Common Multiple (LCM); addition, subtraction, multiplication and division of fractions; integers (negative numbers) and graphing in the coordinate plane; finding the area of a polygon; and statistical distributions.

Chapter 6 first revises prime factorisation and then applies those principles to using the greatest common factor to simplify fractions and the least common multiple to find common denominators.

After a thorough revision of the fraction operations from fifth grade (addition, subtraction and multiplication), Chapter 7, Fractions, then focuses on the new topic for sixth grade: the division of fractions. The chapter also includes ample practice in solving problems with fractions.

Chapter 8 introduces students to integers (signed numbers). They plot points in all four quadrants of the coordinate plane, reflect and translate simple figures, and learn to add and subtract with negative numbers. (The multiplication and division of integers will be studied in 7th grade.)

The next chapter, Geometry, focuses on calculating the area of polygons. After a brief revision of terminology for triangles and quadrilaterals and drawing fundamentals, this new topic is presented simply in a logical progression: first, the area of a right triangle; next, the area of a parallelogram; then, the area of any triangle; and finally, the area of a polygon. The chapter then expands the already learned concepts into developing nets for simple solids to calculate their surface area and to figuring the volume of rectangular prisms with the length of the edges in fractions.

The final chapter is about statistics. Beginning with the concept of a statistical distribution, students learn about measures of centre and measures of variability. They also learn how to make dot plots, histograms, boxplots, and stem-and-leaf plots as ways to summarise and analyse distributions.

I wish you success in teaching maths!

Maria Miller, the author

Chapter 6: Prime Factorisation, GCF and LCM

Introduction

The topics of this chapter belong to a branch of mathematics known as *number theory*. Number theory has to do with the study of whole numbers and their special properties. In this chapter, we revise prime factorisation and study the greatest common factor (GCF) and the least common multiple (LCM).

The main application of factoring and the greatest common factor in arithmetic is in simplifying fractions, so that is why I have included a lesson on that topic. However, it is not absolutely necessary to use the GCF when simplifying fractions, and the lesson emphasises that fact.

The concepts of factoring and the GCF are important to understand because they will be carried over to algebra, where students will factor polynomials. In this chapter, we lay the groundwork for that by using the GCF to factor simple sums, such as $27 + 45$. For example, a sum such as $27 + 45$ factors into $9(3 + 5)$.

Similarly, the main use for the least common multiple in arithmetic is to find the smallest common denominator for adding fractions, and we study that topic in this chapter in connection with the LCM.

Primes are fascinating “creatures,” and you can let students read more about them by accessing the Internet resources listed below. The really important, but far more advanced, application of prime numbers is in cryptography. Some students might be interested in reading additional material on that subject—please see the list below for Internet resources.

The Lessons in Chapter 6

	page	span
The Sieve of Eratosthenes and Prime Factorisation	10	<i>3 pages</i>
Using Factoring When Simplifying Fractions	13	<i>3 pages</i>
The Greatest Common Factor (GCF)	16	<i>3 pages</i>
Factoring Sums	19	<i>3 pages</i>
The Least Common Multiple (LCM)	22	<i>4 pages</i>
Mixed Revision	26	<i>2 pages</i>
Chapter 6 Revision	28	<i>2 pages</i>

Helpful Resources on the Internet

Primes

An Interactive Sieve of Eratosthenes

To find all prime numbers below a given number, use a sieve of Eratosthenes to “weed out” all the composite numbers.

<http://www.cut-the-knot.org/Curriculum/Arithmetic/Eratosthenes.shtml>

Another Interactive Sieve of Eratosthenes

Click on a number to remove its multiples from the grid.

http://nlvm.usu.edu/en/nav/frames_asid_158_g_3_t_1.html?open=instructions

Primes, Factors and Divisibility – Explorer at CountOn.org

Lessons explaining divisibility tests, primes and factors.

<http://www.counton.org/explorer/primes/>

Prime Number Calculator

This calculator tests if a number is a prime, and tells you its smallest divisor if it is not a prime.

<http://www.basic-mathematics.com/prime-number-calculator.html>

Prime Numbers as Building Blocks – Euclid’s Greatest Discovery

A short video about the fundamental theorem of arithmetic: that each composite number has a unique prime factorisation.

<http://www.youtube.com/watch?v=5kl28hmhin0>

The Prime Pages

Learn more about primes on this site: the largest known primes, finding primes, how many are there and more.

<http://primes.utm.edu/>

Primality of 1 from Wikipedia

Discussing whether 1 should or should not be counted as a prime number.

http://en.wikipedia.org/wiki/Prime_number#Primality_of_one

Arguments for and Against the Primality of 1

<http://primefan.tripod.com/Prime1ProCon.html>

Prime factorisation

Factorization Forest

For each number you factorise, you will get to grow a tree in your forest! You can choose between 6 different trees.

<http://mrnussbaum.com/forest/>

Factor Trees at Math Playground

Factor numbers to their prime factors using an interactive factor tree, or find the GCF and LCM of numbers.

<http://www.mathplayground.com/factortrees.html>

MathGoodies Interactive Factor Tree Game

Type in a missing number from the factor tree, and the program will find the other factor and continue drawing the tree as needed.

http://www.mathgoodies.com/factors/prime_factors.html

Free Worksheets for Prime Factorization

Generate free, printable worksheets for prime factorisation or for finding all the factors of a given number.

Customise the worksheets in various ways (difficulty level, spacing, font size, number of problems.)

<http://www.homeschoolmath.net/worksheets/factoring.php>

The Cryptoclub. Using Mathematics to Make and Break Secret Codes (book)

Cryptoclub kids strive to break the codes of secret messages, and at the same time learn more and more about encrypting and decrypting. The book contains problems to solve at the end of each chapter, little tips and historical information about how cryptography has been used over the centuries. By solving the problems you can actually learn to do all of it yourself.

<http://www.amazon.com/gp/product/156881223X?tag=homeschoolmat-20>

Greatest common factor and least common multiple

Fruit Shoot — Greatest Common Factor

Shoot the fruit that has the greatest common factor of two given numbers. There are three levels and two different speeds.

<http://www.sheppardsoftware.com/mathgames/fractions/GreatestCommonFactor.htm>

Fruit Shoot — Least Common Multiple

Shoot the fruit that has the least common multiple of two given numbers. There are three levels and two different speeds.

<http://www.sheppardsoftware.com/mathgames/fractions/LeastCommonMultiple.htm>

Factors and Multiples Jeopardy Game

A jeopardy game where the questions have to do with factors, multiples, prime factorisation, GCF and LCM.

<http://www.math-play.com/Factors-and-Multiples-Jeopardy/Factors-and-Multiples-Jeopardy.html>

Factors, LCM, and GCF: Activity from Math Playground

Choose “Find the prime factorization of 2 numbers, GCF, and LCM.” First, you find the prime factorisation of two different numbers, using the factor tree. Once that is done, the activity shows you a Venn diagram. Drag the factors of the two numbers into the correct areas, then figure out their GFC and LCM.

<http://www.mathplayground.com/factortrees.html>

Free Worksheets for Greatest Common Factor and Least Common Multiple

Generate free, printable worksheets for GCF and LCM. Customise the worksheets in various ways (choose number range, font size, etc.)

http://www.homeschoolmath.net/worksheets/GCF_LCM.php

Least Common Multiple Tutorial

An animated tutorial and exercises for the least common multiple from e-learning for Kids.

<http://e-learningforkids.org/Courses/EN/M1105>

Factors Millionaire Game

A millionaire game where the questions have to do with factors, prime numbers and the greatest common factor.

<http://www.math-play.com/Factors-Millionaire/Factors-Millionaire.html>

Greatest Common Factor at ThatQuiz.org

10-question quiz, not timed, difficulty level 5 (medium). You can also change the parameters to your liking.

<http://www.thatquiz.org/tq-r/?-j2-l5-p0>

Least Common Multiple at ThatQuiz.org

10-question quiz, not timed, difficulty level 5 (medium). You can also change the parameters to your liking.

<http://www.thatquiz.org/tq-r/?-j4-l5-p0>

GCF and LCM Quiz

10-question quiz, not timed, difficulty level 5 (medium). You can also change the parameters to your liking.

<http://www.thatquiz.org/tq-r/?-j4-l5-p0>

Math Problems with LCM & GCF

A quiz of 10 word problems involving the usage of the greatest common factor and the least common multiple.

<http://www.funtrivia.com/playquiz/quiz2715661f17598.html>

Greatest Common Factor Activity from Glencoe

First, the activity asks you to click on all the factors of two numbers (which represent how many apples and oranges there are to bag). Then, you find which one of them is the GCF. Next, it gives you practice problems for finding all the factors of a number, finding common factors of two numbers and finding the GCF of two numbers.

http://www.glencoe.com/sites/common_assets/mathematics/mc1/cim/chapter_04/M1_06/M1_06_dev_100.html

Snowball Fight!

Multiple-choice questions on the LCM of two numbers. When you click a right answer, the game throws a snowball for you.

<http://www.fun4thebrain.com/beyondfacts/lcmsnowball.html>

Pyramid Math

This includes games for GCF, LCM, exponents and square roots. The question to solve appears on the right, under "example." Choose the triangular tile with the correct answer, and drag it to the solution vase. Includes easy and hard levels, timed and non-timed versions.

<http://www.mathnook.com/math/pyramidmath.html>

Factors (for revision)

Product game

For two players: Each selects a factor, and the computer colours the product. The first one to get four in a row wins.

<http://illuminations.nctm.org/ActivityDetail.aspx?ID=29>

Factor Feeder

In this Flash game use the arrow keys to move your Pacman-style character to eat the given number's factors while avoiding numbers that are not factors.

<http://hoodamath.com/games/factorfeeder.php>

Sliding Tile Factorization Game

Slide a number over another to capture it, if it is a factor of the other. Number 1 is only supposed to be used to capture a prime number.

http://www.visualmathlearning.com/Games/sliding_factors.html

Snake

Eat factors, multiples and prime numbers in this remake of the classic game.

<http://www.spacetime.us/arcade/play.php?game=2>

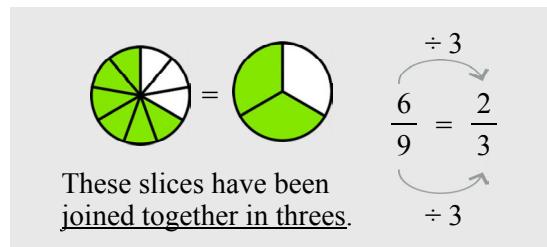
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Using Factoring When Simplifying Fractions

On this page, we will revise simplifying fractions. Let your teacher decide if you can skip this page.

You are used to seeing the process of **simplifying fractions** like this: →

In simplifying fractions, we divide both the numerator and the denominator by the same number. The fraction becomes *simpler*, which means that the numerator and the denominator are now *smaller* numbers than they were before.

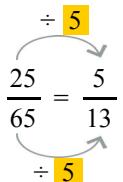


However, this does NOT change the actual value of the fraction. It is the “same amount of pie” as it was before. It is just cut differently.

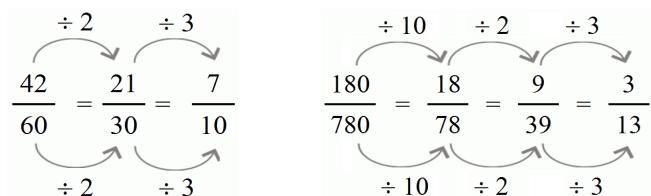
We can simplify a fraction only if its numerator and denominator are divisible by the same number:

We can simplify $\frac{25}{65}$ because both 25 and 65 are divisible by 5: →

We *cannot* simplify $\frac{11}{20}$ because 11 and 20 do not have any common divisors except 1.



You can simplify in multiple steps. Just start somewhere, using the divisibility tests. The goal is to simplify the fraction to **lowest terms**, where the numerator and the denominator have no common factors.



1. Simplify the fractions to the lowest terms, if possible.

a. $\frac{12}{36}$	b. $\frac{45}{55}$	c. $\frac{15}{23}$	d. $\frac{13}{6}$
e. $\frac{15}{21}$	f. $\frac{19}{15}$	g. $\frac{17}{24}$	h. $\frac{24}{30}$

2. Simplify the fractions. Use your knowledge of divisibility.

a. $\frac{95}{100}$	b. $\frac{66}{82}$	c. $\frac{69}{99}$
d. $\frac{120}{600}$	e. $\frac{38}{52}$	f. $\frac{72}{84}$

A new notation

Let's start using a new way to keep track of simplifying fractions. When we simplify a fraction, we will just cross out the old numerator and denominator and write the new numerator above the fraction and the new denominator below it, as illustrated at the right.

$$\frac{\cancel{12}}{\cancel{20}} = \frac{3}{5}$$

The number you divide by (the 4) does *not* get indicated in any way! You only think it: "I divide 12 by 4, and get 3. I divide 20 by 4, and get 5."

$$\frac{\cancel{35}}{\cancel{55}} = \frac{7}{11}$$

You may not see any advantage over the "old" method yet, but this shortcut will come in handy soon.

3. Simplify the fractions. Write the simplified numerator above and the simplified denominator below the old ones.

a. $\frac{14}{16}$	b. $\frac{33}{27}$	c. $\frac{12}{26}$	d. $\frac{9}{33}$	e. $\frac{42}{28}$
--------------------	--------------------	--------------------	-------------------	--------------------

Using factoring when simplifying

Carefully study the example on the right where we factor 96/144.

$$\frac{96}{144} = \frac{\cancel{8}^2 \times \cancel{12}^1}{\cancel{12}^3 \times \cancel{12}^1} = \frac{2}{3}$$

- First we factor (write) 96 as 8×12 , and 144 as 12×12 .
- Then we simplify in two steps:
 1. 8 and 12 are both divisible by 4, so they simplify into 2 and 3.
 2. 12 and 12 are divisible by 12, so they simplify into 1 and 1. Essentially, they cancel each other out.

For comparison, the "old" way looks like this:

$$\frac{96}{144} = \frac{8}{12} = \frac{2}{3}$$

$\div 12 \quad \div 4$
 $\cancel{96}^{\div 12} = \cancel{8}^{\div 4} = \cancel{12}^{\div 4} = \frac{2}{3}$

Let's study some more examples.
(Remember that they don't show the number that you divide by.)

$$\frac{42}{105} = \frac{\cancel{7}^1 \times \cancel{6}^2}{\cancel{3}^5 \times \cancel{3}^1} = \frac{2}{5}$$

$$\frac{45}{150} = \frac{\cancel{9}^3 \times \cancel{5}^1}{\cancel{3}^{10} \times \cancel{5}^1} = \frac{3}{10}$$

4. The numerator and the denominator have already been factored in some problems. Your task is to simplify.

a. $\frac{56}{84} = \frac{7 \times 8}{21 \times 4} =$	b. $\frac{54}{144} = \frac{6 \times 9}{12 \times 12} =$	c. $\frac{120}{72} = \frac{10 \times \boxed{12}}{\boxed{12} \times 9} =$
d. $\frac{80}{48} = \frac{\boxed{16} \times 8}{\boxed{16} \times 8} =$	e. $\frac{36}{90} = \frac{6 \times 6}{\cancel{9}^3 \times \cancel{10}^2} =$	f. $\frac{28}{140} = \frac{7 \times 4}{\cancel{7}^1 \times \cancel{20}^4} =$

Simplify “criss-cross”

These examples are from the previous page. This time the 45 in the numerator has been written as 5×9 instead of 9×5 . We can cancel out the 5 from the numerator with the 5 from the denominator (we simplify criss-cross).

Also, we can simplify the 9 in the numerator and the 30 in the denominator criss-cross. The other example (simplifying 42/105) is similar.

This same concept can be applied to make multiplying fractions easier.

$$\frac{45}{150} = \frac{\cancel{5}^1 \times \cancel{9}^3}{\cancel{30}^6 \times \cancel{5}^1} = \frac{3}{10}$$

$$\frac{42}{105} = \frac{\cancel{7}^1 \times \cancel{6}^2}{\cancel{3}^1 \times \cancel{35}^5} = \frac{2}{5}$$

5. Simplify.

a. $\frac{14}{84} = \frac{2 \times 7}{21 \times 4} =$	b. $\frac{54}{150} = \frac{9 \times \boxed{6}}{10 \times \boxed{6}} =$	c. $\frac{138}{36} = \frac{2 \times \boxed{6}}{\boxed{6} \times 4} =$
d. $\frac{27}{20} \times \frac{10}{21} =$	e. $\frac{75}{90} = \frac{\cancel{5}^1 \times 15}{\cancel{9}^3 \times 10} =$	f. $\frac{48}{45} \times \frac{55}{64} =$

In this example, the simplification is done in two steps. In the first step, 12 and 2 are divided by 2, leaving 6 and 1. In the second step, 6 and 69 are divided by 3, leaving 2 and 23.

These two steps can also be done without rewriting the expression. The 6 and 69 are divided by 3 as before. This time we simply did not rewrite the expression in between but just continued on with the numbers 6 and 69 that were already written there.

If this looks too confusing, you do not have to write it in such a compact manner. You can rewrite the expression before simplifying it some more.

$$\frac{48}{138} = \frac{\cancel{12}^6 \times 4}{\cancel{2}^1 \times 69} = \frac{6 \times 4}{1 \times \cancel{69}^{23}} = \frac{8}{23}$$

$$\frac{48}{138} = \frac{\cancel{12}^2 \times 4}{\cancel{2}^1 \times \cancel{69}^{23}} = \frac{8}{23}$$

6. Simplify the fractions to lowest terms, or simplify before you multiply the fractions.

a. $\frac{88}{100}$	b. $\frac{84}{102}$	c. $\frac{85}{105}$
d. $\frac{8}{5} \times \frac{8}{20} =$	e. $\frac{72}{120}$	f. $\frac{104}{240}$
g. $\frac{35}{98}$	h. $\frac{5}{7} \times \frac{17}{15} =$	i. $\frac{72}{112}$

The Greatest Common Factor (GCF)

Let's take two whole numbers. We can then list all the factors of each number, and then find the factors that are common in both lists. Lastly, we can choose the greatest or largest among those "common factors." That is the **greatest common factor** of the two numbers. The term itself really tells you what it means!

Example 1. Find the greatest common factor of 18 and 30.

The factors of 18: 1, 2, 3, 6, 9 and 18.

The factors of 30: 1, 2, 3, 5, 6, 10, 15 and 30.

Their common factors are 1, 2, 3 and 6. The greatest common factor is 6.

Here is a **method to find all the factors of a given number.**

Example 2. Find the factors (divisors) of 36.

We check if 36 is divisible by 1, 2, 3, 4, 5 and so on. Each time we find a divisor, we write down *two* factors.

- 36 is divisible by 1. We write $36 = 1 \cdot 36$, and that equation gives us two factors of 36: both the smallest (**1**) and the largest (**36**).
- Next, 36 is also divisible by 2. We write $36 = 2 \cdot 18$, and that equation gives us two more factors of 36: the second smallest (**2**) and the second largest (**18**).
- Next, 36 is divisible by 3. We write $36 = 3 \cdot 12$, and now we have found the third smallest factor (**3**) and the third largest factor (**12**).
- Next, 36 is divisible by 4. We write $36 = 4 \cdot 9$, and we have found the fourth smallest factor (**4**) and the fourth largest factor (**9**).
- Finally, 36 is divisible by 6. We write $36 = 6 \cdot 6$, and we have found the fifth smallest factor (**6**) which is also the fifth largest factor.

We know that we are done because the list of factors from the "small" end (1, 2, 3, 4, 6) has met the list of factors from the "large" end (36, 18, 12, 9, 6).

Therefore, all of the factors of 36 are: 1, 2, 3, 4, 6, 9, 12, 18 and 36.

1. List all of the factors of the given numbers.

a. 48	b. 60
c. 42	d. 99

2. Find the greatest common factor of the given numbers. Your work above will help!

a. 48 and 60	b. 42 and 48	c. 42 and 60	d. 99 and 60
--------------	--------------	--------------	--------------

3. List all of the factors of the given numbers.

a. 44	b. 66
c. 28	d. 56
e. 100	f. 45

4. Find the greatest common factor of the given numbers. Your work above will help!

a. 44 and 66	b. 100 and 28	c. 45 and 100	d. 45 and 66
e. 28 and 44	f. 56 and 28	g. 56 and 100	h. 45 and 28

Example 3. What is the greatest common factor useful for?

It can be used to simplify fractions. For example, let's say you know that the GCF of 66 and 84 is 6. Then, to simplify the fraction $\frac{66}{84}$ to lowest terms, you divide both the numerator and the denominator by 6. →

$$\frac{66}{84} = \frac{11}{14}$$

However, it is *not* necessary to use the GCF when simplifying fractions. You can always simplify in several steps. See the example at the right. →

Or, you can *simplify by factoring*, like we did in the previous lesson:

$$\frac{66}{84} = \frac{6 \cdot 11}{7 \cdot 6 \cdot 2} = \frac{11}{14}$$

In fact, these other methods might be quicker than using the GCF.

5. Simplify these fractions, if possible. Your work in the previous exercises can help!

a. $\frac{48}{66}$

b. $\frac{42}{44}$

c. $\frac{42}{48}$

d. $\frac{99}{60}$

e. $\frac{48}{100}$

f. $\frac{100}{99}$

g. $\frac{56}{28}$

h. $\frac{44}{99}$

Using prime factorisation to find the greatest common factor (optional)

Another, more efficient way to find the GCF of two or more numbers is to use the prime factorisations of the numbers to find *all* of the common prime factors. The product of those common prime factors forms the GCF.

Example 4. Find the GCF of 48 and 84.

The prime factorisations are: $48 = 2 \cdot 2 \cdot 2 \cdot 2 \cdot 3$ and $84 = 2 \cdot 2 \cdot 3 \cdot 7$.

We see that the common prime factors are 2 and 2 and 3. Therefore, the GCF is $2 \cdot 2 \cdot 3 = 12$.

Example 5. Find the GCF of 75, 105 and 125.

The prime factorisations are: $75 = 3 \cdot 5 \cdot 5$, $105 = 3 \cdot 5 \cdot 7$ and $150 = 2 \cdot 3 \cdot 5 \cdot 5$.

The common prime factors for all of them are 3 and 5. Therefore, the GCF of these three numbers is $3 \cdot 5 = 15$.

6. Find the greatest common factor of the numbers.

a. 120 and 66

b. 36 and 136

c. 98 and 76

d. 132 and 72

e. 45 and 76

f. 64 and 120

7. Find the greatest common factor of the given numbers.

a. 75, 25 and 90

b. 54, 36 and 40

c. 18, 24 and 36

d. 72, 60 and 48

Find the greatest common factor of 187 and 264.

Puzzle Corner

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Chapter 7: Fractions

Introduction

This chapter begins with a revision of fraction arithmetic from fifth grade — specifically, addition, subtraction, simplification and multiplication of fractions. Then it focuses on the new topic: division of fractions.

The introductory lesson on the division of fractions presents the concept of reciprocal numbers and ties the reciprocity relationship to the idea that division is the appropriate operation to solve questions of the form, “How many times does this number fit into that number?” For example, we can write a division from the question, “How many times does $1/3$ fit into 1?” The answer is, obviously, 3 times. So we can write the division $1 \div (1/3) = 3$ and the multiplication $3 \times (1/3) = 1$. These two numbers, $3/1$ and $1/3$, are reciprocal numbers because their product is 1.

Students learn to solve questions like that through using visual models and writing division sentences that match them. The eventual goal is to arrive at the shortcut for fraction division—that each division can be changed into a multiplication by taking the reciprocal of the divisor, which is often called the “invert (flip)-and-multiply” rule.

However, that “rule” is just a shortcut. It is necessary to memorise it, but memorising a shortcut doesn’t help students make sense conceptually out of the division of fractions—they also need to study the concept of division and use visual models to better understand the process involved.

In two lessons that follow, students apply what they have learned to solve problems involving fractions or fractional parts. A lot of the problems in these lessons are revision in the sense that they involve previously learned concepts and are similar to problems students have solved earlier, but many involve the division of fractions, thus incorporating the new concept presented in this chapter.

The Lessons in Chapter 7

	page	span
Revision: Add and Subtract Fractions and Mixed Numbers	34	4 pages
Add and Subtract Fractions: More Practice	38	3 pages
Revision: Multiplying Fractions 1	41	3 pages
Revision: Multiplying Fractions 2	44	3 pages
Dividing Fractions: Reciprocal Numbers	47	5 pages
Divide Fractions	52	4 pages
Problem Solving with Fractions 1	56	3 pages
Problem Solving with Fractions 2	59	3 pages
Mixed Revision	62	2 pages
Fractions Revision	64	3 pages

Helpful Resources on the Internet

General

Fraction Models

Explore improper fractions, mixed numbers, decimals and percentages. The activity includes several models: bar graphs, area models, pie charts and set diagrams. Adjust the numerators and denominators to see how it changes the representations of the fractions in the models.

<http://illuminations.nctm.org/ActivityDetail.aspx?ID=11>

Visual Fractions

Great site for studying all aspects of fractions: identifying, renaming, comparing, adding, subtracting, multiplying and dividing. Each topic is illustrated by either a number line or a circle with a Java applet. There are also a couple of games, for example: Make Cookies for Grampy.

<http://www.visualfractions.com>

Conceptua Math Fractions Tools

Free and interactive tools for fractions: identify them, add or subtract them, estimate with them, compare them, find equivalent fractions, multiply or divide them, find common denominators and more. Each activity uses several fraction models such as fraction circles, horizontal and vertical bars, number lines, etc. that allow students to develop a conceptual understanding of fractions. A free registration is required.

<http://www.conceptuamath.com/app/tool-library>

Fraction Games at Sheppard Software

Games for practicing adding and subtracting fractions, simplifying fractions, and finding equivalent fractions and the fraction of a set.

<http://www.sheppardsoftware.com/mathgames/menus/fractions.htm>

Fraction Lessons at MathExpression.com

Tutorials, examples and videos to explain all of the basic topics in fractions.

<http://www.mathexpression.com/learning-fractions.html>

Visual Math Learning

Free tutorials with some interactivity about all the fraction operations. Emphasises visual models and lets students interact with those.

http://www.visualmathlearning.com/pre_algebra/chapter_9/chap_9.html

Online Fraction Calculator

Add, subtract, multiply, or divide fractions and mixed numbers.

http://www.homeschoolmath.net/worksheets/fraction_calculator.php

Fraction Worksheets: Addition, Subtraction, Multiplication, and Division

Create custom-made worksheets for the four operations with fractions and mixed numbers.

<http://www.homeschoolmath.net/worksheets/fraction.php>

Fraction Worksheets: Equivalent Fractions, Simplifying, Convert to Mixed Numbers

Create custom-made worksheets for some other fraction operations.

<http://www.homeschoolmath.net/worksheets/fraction-b.php>

Addition and Subtraction

Fraction Videos 1: Addition and Subtraction

My own videos that cover equivalent fractions and addition and subtraction of like and unlike fractions and of mixed numbers.

http://www.mathmammoth.com/videos/fractions_1.php

MathSplat

Click on the right answer to addition problems involving like fractions, or the bug splats on your windshield!

<http://fen.com/studentactivities/MathSplat/mathsplat.htm>

Adding Fractions

Illustrates how to find the common denominator when adding two unlike fractions using interactive pie models.

http://nlvm.usu.edu/en/nav/frames_asid_106_g_3_t_1.html

Adding and Subtracting Fractions with Uncommon Denominators Tool at Conceptua Fractions

A tool that links a visual model to the procedure for adding two unlike fractions. A free registration is required.

<https://www.conceptuamath.com/app/tool/adding-fractions-with-uncommon-denominators>

<https://www.conceptuamath.com/app/tool/subtracting-fractions-with-uncommon-denominators>

Old Egyptian Fractions

Puzzles to solve: Add fractions like a true Old Egyptian Maths Cat!

<http://www.mathcats.com/explore/oldegyptianfractions.html>

Fraction Bars Blackjack

The computer gives you two fraction cards. You have the option of getting more or “holding.” The object is to get as close as possible to 2, without going over, by adding the fractions on your cards.

http://fractionbars.com/Fraction_Bars_Black_Jack/

Multiplication and Division

Fraction Videos 2: Multiplication and Division

My own videos that cover multiplying and dividing fractions.

http://www.mathmammoth.com/videos/fractions_2.php

Multiply Fractions Jeopardy

A jeopardy-style game. Choose a question by clicking on the tile that shows the number of points you will win.

<http://www.quia.com/cb/95583.html>

Fractions Mystery Picture Game

Solve problems where you find a fractional part of a quantity to uncover a picture.

<http://www.dositey.com/2008/math/mystery2.html>

Math Basketball - Dividing Fractions Game

First make a basket, and then you get to solve a fraction division problem with multiple-choice answers.

<http://www.math-play.com/math-basketball-dividing-fractions-game/math-basketball-dividing-fractions-game.html>

Soccer Math - Dividing Fractions Game

In order to kick the ball and score points, you first have to answer maths problems correctly.

<http://www.math-play.com/soccer-math-dividing-fractions-game/soccer-math-dividing-fractions-game.html>

Number Line Bars

Fraction bars that illustrate visually how many times a fraction “fits into” another fraction.

http://nlvm.usu.edu/en/NAV/frames_asid_265_g_2_t_1.html?open=activities&from=category_g_2_t_1.html

Fraction Worksheets: Addition, Subtraction, Multiplication, and Division

Create custom-made worksheets for fraction addition, subtraction, multiplication and division.

<http://www.homeschoolmath.net/worksheets/fraction.php>

Comparing Fractions

Comparison Shoot Out

Choose level 2 or 3 to compare fractions and shoot the soccer ball into the goal.

<http://www.fuelthebrain.com/Game/play.php?ID=47>

Comparing Fractions—XP Math

Simple timed practice with comparing two fractions.

<http://xpmath.com/forums/arcade.php?do=play&gameid=8>

Comparing Fractions Tool at Conceptua Fractions

An interactive tool where students place numbers, visual models and decimals on a number line.

<http://www.conceptuamath.com/app/tool/comparing-fractions>

Fractional Hi Lo

The computer has selected a fraction. You make guesses and it tells you if your guess was too high or too low.

<http://www.theproblemsite.com/games/hilo.asp>

Comparing/Ordering Fractions Worksheets

Create free worksheets for comparing two fractions or ordering 3-8 fractions. Compare fractions with the same numerator or with the same denominator, or compare a fraction to 1/2 or other fractions. You can also include images (fraction pies).

http://www.homeschoolmath.net/worksheets/comparing_fractions.php

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Dividing Fractions: Reciprocal Numbers

First, let's revise a little:

How many times does one number go into another?

From this situation, you can always write a division, even if the numbers are fractions!

How many times does go into ?

Three times. We write the division: $2 \div \frac{2}{3} = 3$.

Then check the division: $3 \times \frac{2}{3} = \frac{6}{3} = 2$.

1. Solve. Write a division. Then write a multiplication that checks your division.

a. How many times does go into ?

$$3 \div \frac{1}{3} = \underline{\quad}$$

Check: $\underline{\quad} \times \frac{1}{3} =$

b. How many times does go into ?

$$\div \frac{\underline{\quad}}{\underline{\quad}} = \underline{\quad}$$

Check: $\underline{\quad} \times \frac{\underline{\quad}}{\underline{\quad}} =$

c. How many times does go into ?

$$\div \frac{\underline{\quad}}{\underline{\quad}} = \underline{\quad}$$

Check:

d. How many times does go into ?

$$\div \frac{\underline{\quad}}{\underline{\quad}} = \underline{\quad}$$

Check:

2. Solve. Think how many times the fraction goes into the whole number. Can you find a *pattern* or a *shortcut*?

a. $3 \div \frac{1}{6} =$

b. $4 \div \frac{1}{5} =$

c. $3 \div \frac{1}{10} =$

d. $5 \div \frac{1}{10} =$

e. $7 \div \frac{1}{4} =$

f. $4 \div \frac{1}{8} =$

g. $4 \div \frac{1}{10} =$

h. $9 \div \frac{1}{8} =$

The shortcut is this:

$$5 \div \frac{1}{4} \\ \downarrow \quad \downarrow \\ 5 \times 4 = 20$$

$$3 \div \frac{1}{8} \\ \downarrow \quad \downarrow \\ 3 \times 8 = 24$$

$$9 \div \frac{1}{7} \\ \downarrow \quad \downarrow \\ 9 \times 7 = 63$$

Notice that $1/4$ inverted (upside down) is $4/1$ or simply 4. We call $1/4$ and 4 reciprocal numbers, or just reciprocals. So the shortcut is: multiply by the reciprocal of the divisor.

Does the shortcut make sense to you? For example, consider the problem $5 \div (1/4)$. Since $1/4$ goes into 1 exactly four times, it must go into 5 exactly $5 \times 4 = 20$ times.

Two numbers are reciprocal numbers (or reciprocals) of each other if, when multiplied, they make 1.

$\frac{3}{4}$ is a reciprocal of $\frac{4}{3}$, because $\frac{3}{4} \times \frac{4}{3} = \frac{12}{12} = 1$.	$\frac{1}{7}$ is a reciprocal of 7, because $\frac{1}{7} \times 7 = \frac{7}{7} = 1$.
---	--

You can find the reciprocal of a fraction $\frac{m}{n}$ by flipping the numerator and denominator: $\frac{n}{m}$.

This works, because $\frac{m}{n} \times \frac{n}{m} = \frac{n \times m}{m \times n} = 1$.

To find the reciprocal of a mixed number, first write it as a fraction, then “flip” it.

Since $2\frac{3}{4} = \frac{11}{4}$, its reciprocal number is $\frac{4}{11}$.

3. Find the reciprocal numbers. Then write a multiplication with the given number and its reciprocal.

a. $\frac{5}{8}$

$$\frac{5}{8} \times \boxed{\frac{\textcolor{blue}{\square}}{\textcolor{blue}{\square}}} = 1$$

b. $\frac{1}{9}$

$$\boxed{\frac{\textcolor{blue}{\square}}{\textcolor{blue}{\square}}} \times \boxed{\frac{\textcolor{blue}{\square}}{\textcolor{blue}{\square}}} = 1$$

c. $1\frac{7}{8}$

$$\boxed{\frac{\textcolor{blue}{\square}}{\textcolor{blue}{\square}}} \times \boxed{\frac{\textcolor{blue}{\square}}{\textcolor{blue}{\square}}} = 1$$

d. 32

$$32 \times \boxed{\frac{\textcolor{blue}{\square}}{\textcolor{blue}{\square}}} = 1$$

e. $2\frac{1}{8}$

$$\boxed{\frac{\textcolor{blue}{\square}}{\textcolor{blue}{\square}}} \times \boxed{\frac{\textcolor{blue}{\square}}{\textcolor{blue}{\square}}} = 1$$

4. Write a division sentence to match each multiplication above.

a. $1 \div \boxed{\frac{\textcolor{blue}{\square}}{\textcolor{blue}{\square}}} = \boxed{\frac{\textcolor{blue}{\square}}{\textcolor{blue}{\square}}}$

b. $1 \div \boxed{\frac{\textcolor{blue}{\square}}{\textcolor{blue}{\square}}} = \boxed{\frac{\textcolor{blue}{\square}}{\textcolor{blue}{\square}}}$

c. $1 \div \boxed{\frac{\textcolor{blue}{\square}}{\textcolor{blue}{\square}}} = \boxed{\frac{\textcolor{blue}{\square}}{\textcolor{blue}{\square}}}$

d. $\underline{\quad} \div \boxed{\frac{\textcolor{blue}{\square}}{\textcolor{blue}{\square}}} = \boxed{\frac{\textcolor{blue}{\square}}{\textcolor{blue}{\square}}}$

e. $\underline{\quad} \div \boxed{\frac{\textcolor{blue}{\square}}{\textcolor{blue}{\square}}} = \boxed{\frac{\textcolor{blue}{\square}}{\textcolor{blue}{\square}}}$

Read the following explanation and really try to understand it. It is important!

Now let's try to **make some sense visually** out of how reciprocal numbers fit into the division of fractions.

We can think of the division $1 \div (2/5)$ as asking, “**How many times does $2/5$ fit into 1?**”

Using pictures: How many times does  go into  ?

From the picture we can see that  goes into  two times, and then we have $1/5$ left over.

But how many times does $\frac{2}{5}$ fit into the leftover piece, $\frac{1}{5}$? How many times does  go into  ?

That is like trying to fit a TWO-part piece into a hole that holds just ONE part.

Only $1/2$ of the two-part piece fits! So, $2/5$ fits into $1/5$ exactly half a time.

So we found that, in total, $2/5$ fits into 1 exactly **$2\frac{1}{2}$ times**. We can write the division $1 \div \frac{2}{5} = 2\frac{1}{2}$ or $\frac{5}{2}$.

Notice, we got $1 \div \frac{2}{5} = \frac{5}{2}$. Checking that with multiplication, we get $\frac{5}{2} \times \frac{2}{5} = 1$. Reciprocals again!

One more example. We can think of the division $1 \div (5/7)$ as asking, “**How many times does $5/7$ fit into 1?**”

Using pictures: How many times does  go into  ?

From the picture we can see that  goes into  just once, and then we have $2/7$ left over.

But how many times does $\frac{5}{7}$ fit into the leftover piece, $\frac{2}{7}$? How many times does  go into  ?

The five-part piece fits into a hole that is only big enough for two parts just $2/5$ of the way.

So $5/7$ fits into one exactly $1\frac{2}{5}$ times. The division is $1 \div \frac{5}{7} = 1\frac{2}{5}$ or $1 \div \frac{5}{7} = \frac{7}{5}$. Reciprocals again!

5. Solve. Think how many times the given fraction fits into one whole. Write a division.

a. How many times does  go into  ?

$$1 \div \frac{\text{---}}{\text{---}} =$$

b. How many times does  go into  ?

$$1 \div \frac{\text{---}}{\text{---}} =$$

c. How many times does  go into  ?

$$1 \div \frac{\text{---}}{\text{---}} =$$

d. How many times does  go into  ?

$$1 \div \frac{\text{---}}{\text{---}} =$$

e. How many times does  go into  ?

$$1 \div \frac{\text{---}}{\text{---}} =$$

f. How many times does  go into  ?

$$1 \div \frac{\text{---}}{\text{---}} =$$

6. Solve. Think how many times the given fraction fits into the other number. Write a division.

a. How many times does  go into  ?

$$2 \div \frac{\text{---}}{\text{---}} =$$

b. How many times does  go into  ?

$$\frac{\text{---}}{\text{---}} \div \frac{\text{---}}{\text{---}} =$$

c. How many times does  go into  ?

$$3 \div \frac{\text{---}}{\text{---}} =$$

d. How many times does  go into  ?

$$\frac{\text{---}}{\text{---}} \div \frac{\text{---}}{\text{---}} =$$

SHORTCUT: instead of dividing, multiply by the reciprocal of the divisor.

Study the examples to see how this works.

How many times does go into ?

$$\frac{3}{4} \div \frac{1}{3}$$

$$\downarrow \quad \downarrow$$

$$\frac{3}{4} \times 3 = \frac{9}{4} = 2\frac{1}{4}$$

Answer: 2 1/4 times.

Does it make sense?

Yes, fits into a little more than two times.

How many times does go into ?

$$\frac{7}{4} \div \frac{2}{5}$$

$$\downarrow \quad \downarrow$$

$$\frac{7}{4} \times \frac{5}{2} = \frac{35}{8} = 4\frac{3}{8}$$

Answer: 4 3/8 times.

Does it make sense?

Yes, goes into over four times.

How many times does go into ?

$$\frac{2}{9} \div \frac{2}{7}$$

$$\downarrow \quad \downarrow$$

$$\cancel{\frac{2}{9}} \times \frac{7}{\cancel{2}} = \frac{7}{9}$$

Answer: 7/9 of a time.

Does it make sense?

Yes, because does not go into even one full time!

Remember: There are *two changes* in each calculation:

1. Change the division into multiplication.
2. Use the reciprocal of the divisor.

7. Solve these division problems using the shortcut. Remember to check to make sure your answer makes sense.

a. $\frac{3}{4} \div 5$

$\downarrow \quad \downarrow$

$$\frac{3}{4} \times \frac{1}{5} =$$

b. $\frac{2}{3} \div \frac{6}{7}$

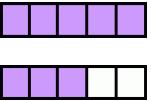
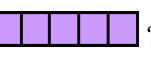
c. $\frac{4}{7} \div \frac{3}{7}$

d. $\frac{2}{3} \div \frac{3}{5}$

e. $4 \div \frac{2}{5}$

f. $\frac{13}{3} \div \frac{1}{5}$

8. a. Write a division to match the situation on the right.

How many times does  fit into ?

We have $\frac{8}{5}$, which is eight pieces, trying to fit into five pieces... so they fit $\frac{8}{5}$ of the way.

- b. Check your division by multiplication.

9. Fill in.

$$2 \div \frac{3}{4} = ?$$

Or, how many times does  go into ?

First, let's solve how many times  goes into .

Since $1 \div \frac{3}{4} = \frac{4}{3}$, it goes into one  =  times.

If $\frac{3}{4}$ fits into  times, then it fits into  double that many times, or _____ times.

We get the same answer by using the shortcut:

$$\begin{array}{r} 2 \div \frac{3}{4} \\ \downarrow \quad \downarrow \\ 2 \times \frac{4}{3} = \end{array}$$

10. Fill in.

$$\frac{2}{7} \div \frac{5}{6} = ?$$

Or, how many times does  go into ?

First, let's solve how many times  goes into .

Since $1 \div \frac{5}{6} = \frac{6}{5}$, it goes into one  =  times.

If $\frac{2}{7}$ fits into  times, then it fits into  exactly $\frac{5}{6}$ as many times as it fits into 1,

which is $\frac{5}{6} \times \frac{6}{5} = \frac{30}{30} =$

We get the same answer by using the shortcut:

$$\begin{array}{r} \frac{2}{7} \div \frac{5}{6} \\ \downarrow \quad \downarrow \\ \frac{2}{7} \times \frac{6}{5} = \end{array}$$

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Chapter 8: Integers

Introduction

In chapter 8, students are introduced to integers, the coordinate plane in all four quadrants, and integer addition and subtraction. The multiplication and division of integers will be studied in seventh grade.

Integers are introduced using the number line to relate them to the concepts of temperature, elevation and money. We also study briefly the ideas of absolute value (an integer's distance from zero) and the opposite of a number.

Next, students learn to locate points in all four quadrants and how the coordinates of a figure change when it is reflected across the x or y -axis. Students also move points according to given instructions and find distances between points with the same first coordinate or the same second coordinate.

Adding and subtracting integers is presented through two main models: (1) movements along the number line and (2) positive and negative counters. With the help of these models, students should not only learn the shortcuts, or "rules," for adding and subtracting integers, but also understand *why* these shortcuts work.

A lesson about subtracting integers explains the shortcut for subtracting a negative integer from three different viewpoints (as a manipulation of counters, as movements on a number line and as a distance or difference). There is also a roundup lesson for addition and subtraction of integers.

The last topic in this chapter is graphing. Students will plot points on the coordinate grid according to a given equation in two variables (such as $y = x + 2$), this time using also negative numbers. They will notice the patterns in the coordinates of the points and the pattern in the points drawn in the grid and also work through some real-life problems.

The Lessons in Chapter 8

	page	span
Integers	71	<i>3 pages</i>
Coordinate Grid	74	<i>4 pages</i>
Coordinate Grid Practice	78	<i>3 pages</i>
Addition and Subtraction as Movements	81	<i>3 pages</i>
Adding Integers: Counters	84	<i>3 pages</i>
Subtracting a Negative Integer	87	<i>2 pages</i>
Add and Subtract Roundup	89	<i>2 pages</i>
Graphing	91	<i>4 pages</i>
Mixed Revision	95	<i>2 pages</i>
Integers Revision	97	<i>3 pages</i>

Helpful Resources on the Internet

Free worksheets for the Coordinate Grid

Generate printable and customisable worksheets for plotting points and shapes and for moving and reflecting shapes in the coordinate grid. Options include limiting to the first quadrant or all quadrants, scaling, image size, workspace and border.

http://www.homeschoolmath.net/worksheets/coordinate_grid.php

Ordering integers

Number Balls Game

Click on the rotating number balls in ascending order.

<http://www.sheppardsoftware.com/mathgames/numberballs/numberballsAS2.htm>

Negative Numbers Hat Game

Put the numbered hats on people's heads in the right order.

<http://www.primaryresources.co.uk/online/negnumorder.swf>

Order Negative Numbers

Drag and drop the numbers in the right order onto the ladder (scroll down the page a bit to see the activity).

http://www.bbc.co.uk/bitesize/ks3/mathsmaths/negative_numbers/revision/2/

Addition and subtraction

The section for "all operations" below has more games for adding and subtracting.

Colour Chips Addition

The user drags positive/negative chips to the working area, then combines them in pairs to see the sum.

http://nlvm.usu.edu/en/nav/frames_asid_161_g_2_t_1.html

Colour Chips Subtraction

Drag positive and negative chips and zero pairs into the working area as instructed, then subtract.

http://nlvm.usu.edu/en/nav/frames_asid_162_g_3_t_1.html

Line Jumper

You see a number line and an addition or subtraction problem. Click the right answer on the number line.

<http://www.funbrain.com/linejump>

Space Coupe to the Rescue

By choosing a positive or negative number, the player controls the vertical position of a spaceship. If the spaceship reaches the same vertical position as a virus pod, the pod is destroyed.

<http://pbskids.org/cyberchase/games/negativenumbers>

Red and Black TripleMatch Game for Adding Integers

This is a fun card game for 2-5 people to practise adding integers.

<http://mathmamawrites.blogspot.com/2010/07/black-and-red-triplematch-card-game-for.html>

Adding and Subtracting Integers Gizmos from Explorelearning.com

Interactive simulations that illustrate adding and subtracting integers on a number line or with chips. Includes an exploration guide and assessment questions. You can get a 5-minute access for free, or a free 30-day trial account.

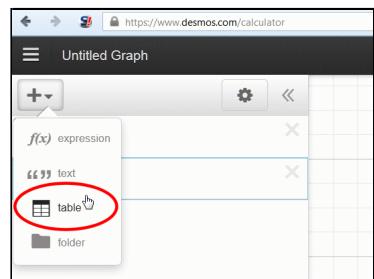
<http://www.explorelearning.com/index.cfm?method=cResource.dspResourcesForCourse&CourseID=211>

Graphing

Desmos Graphing Calculator

A versatile, easy-to-use, and free graphing calculator. To practise plotting points and lines as learned in this chapter, add an item from the “+” button and choose “table.” Fill in x and y values, and Desmos will plot the points. You can then type the equation of the line in the form $y = (\text{something})$, such as $y = 2x$, and check if the line goes through your points.

<https://www.desmos.com/calculator>



Meta-Calculator 2.0

Choose Graphing Calculator. You can enter an equation to be graphed, or choose “plot points” from the top menu to enter individual points.

<http://www.meta-calculator.com/online/>

Graph Mole

A fun game about plotting points in a coordinate plane. Plot the points before the mole eats the vegetables.

<http://funbasedlearning.com/algebra/graphing/default.htm>

Catch the Fly

Wait for the fly to land on the coordinate grid, then type its coordinates, and a frog will eat it.

http://hotmath.com/hotmath_help/games/ctf/ctf_hotmath.swf

Looking for the Top Quark Game

Each player receives six quarks that they hide on a grid. The players use coordinates to find their opponent’s hidden quarks.

<http://education.jlab.org/topquarkgame>

Coordinate Grid Quiz from ThatQuiz.org

This quiz has 10 questions and asks to either plot a point or give the coordinates of a given point. You can also modify the quiz parameters to your liking.

<http://www.thatquiz.org/tq-7/?-j8-l5-m2kc0-na-p0>

Co-ordinate Game

You will see a red circle on the grid. Enter its co-ordinates and click check.

http://www.bgfl.org/bgfl/custom/resources_ftp/client_ftp/ks3/mathssupport/games/game1.htm

All operations / General

Arithmetic Four (Connect the Four game)

Practise any or all of the four operations with integers. First you answer a maths problem, then you can do your move in a connect-the-four game. Choose addition and subtraction to practise the operations learned in this chapter.

<http://www.shodor.org/interactivate/activities/ArithmeticFour>

Flashcards with Negative Numbers

AplusMath.com has interactive flashcards for integer addition, subtraction, multiplication and division. Choose addition and subtraction to practise the topics in this chapter.

<http://www.aplusmath.com/Flashcards/sub-nflash.html>

Create Integers Worksheets

Use the basic operations worksheet generator to make worksheets for integers within a certain range of negative to positive numbers.

<http://www.homeschoolmath.net/worksheets/basic-operations-worksheets.php>

**Sample worksheet from
www.mathmammoth.com**

How to Teach Integers

A helpful article for teachers about techniques for teaching integer operations.

<http://www.homeschoolmath.net/teaching/integers.php>

Free Downloadable Integer Fact Sheets

http://www.homeschoolmath.net/download/Add_Subtract_Integers_Fact_Sheet.pdf

http://www.homeschoolmath.net/download/Multiply_Divide_Integers_Fact_Sheet.pdf

The History of Negative Numbers

Although they seem normal to us now, in the past negative numbers have spurred controversy and been called “fictitious” or worse.

http://nrich.maths.org/public/viewer.php?obj_id=5961

http://www.classzone.com/books/algebra_1/page_build.cfm?content=links_app3_ch2&ch=2

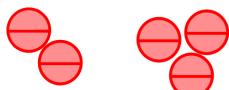
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Adding Integers: Counters

Addition of integers can be modelled using **counters**. We will use green counters with a “+” sign for positives and red counters with a “–” sign for negatives.



Here we have the sum $2 + 3$. There is a group of 2 positives and another of 3 positives.



This picture shows the sum $(-2) + (-3)$. We *add* negatives and positives. In total, there are five negatives, so the sum is -5 .

$$\text{Green counter} + \text{Red counter} = 0$$

$$1 + (-1) = 0$$

One positive counter and one negative counter *cancel* each other. In other words, their sum is zero!



$$2 + (-2) = 0$$

Two negatives and two positives also cancel each other. Their sum is zero.



$$3 + (-1) = 2$$

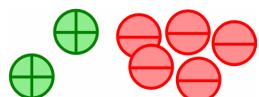
Here, one “positive-negative” pair is cancelled (you can cross it out!). We are left with 2 positives.



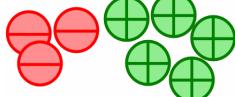
$$(-4) + 3 = -1$$

Now the negatives outweigh the positives. Pair up three negatives with three positives. Those cancel out. There is still one negative left.

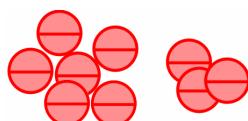
1. Refer to the pictures and add. Remember each “positive-negative” pair is cancelled.



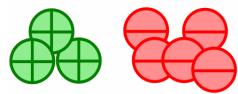
a. $2 + (-5) = \underline{\hspace{2cm}}$



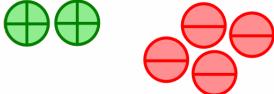
b. $(-3) + 5 = \underline{\hspace{2cm}}$



c. $(-6) + (-3) = \underline{\hspace{2cm}}$



d. $3 + (-5) = \underline{\hspace{2cm}}$

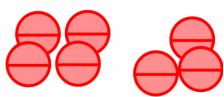


e. $2 + (-4) = \underline{\hspace{2cm}}$

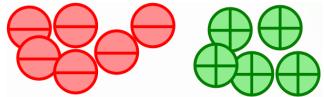


f. $(-8) + 5 = \underline{\hspace{2cm}}$

2. Write addition sentences (equations) to match the pictures.



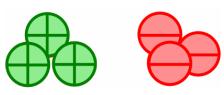
a.



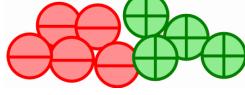
b.



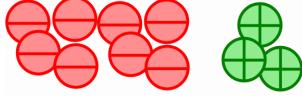
c.



d.



e.



f.

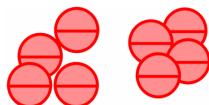
A note on notation

We can write an elevated minus sign to indicate a negative number: $\bar{4}$.

Or we can write it with a minus sign and brackets: (-4) .

We can even write it without the brackets if the meaning is clear: -4 .

So $\bar{4} + \bar{4} = \bar{8}$ is the same as $(-4) + (-4) = (-8)$, which is the same as $-4 + (-4) = -8$.



You *should* write the brackets if you have $+$ and $-$, or two $-$ signs, next to each other.

So, do *not* write “ $8 + -4$ ”; write “ $8 + (-4)$.” And do not write “ $3 - -3$ ”; write “ $3 - (-3)$.”

3. Think of the counters. Add.

a. $7 + (-8) =$ $(-7) + 8 =$	b. $(-7) + (-8) =$ $7 + 8 =$	c. $5 + (-7) =$ $7 + (-5) =$	d. $50 + (-20) =$ $10 + (-40) =$
e. $\bar{2} + \bar{4} =$ $\bar{6} + 6 =$	f. $10 + \bar{1} =$ $\bar{10} + \bar{1} =$	g. $\bar{8} + 2 =$ $\bar{8} + \bar{2} =$	h. $\bar{9} + \bar{1} =$ $9 + \bar{1} =$

4. Rewrite these sentences using symbols, and solve the resulting sums.

a. The sum of seven positives and five negatives.

b. Add -3 and -11 .

c. Positive 100 and negative 15 added together.

5. Write a sum for each situation and solve it.

a. Your checking account is overdrawn by R50. (This means your account is negative). Then you earn R60. What is the balance in your account now?

b. Hannah owed her mother R20. Then, she borrowed R15 more from her mother. What is Hannah's “balance” now?

6. Consider the four expressions $2 + 6$, $(-2) + (-6)$, $(-2) + 6$ and $2 + (-6)$. Write these expressions in order from the one with **least** value to the one with **greatest** value.

7. Find the number that is missing from the equations.

a. $-3 + \underline{\hspace{2cm}} = -7$	b. $-3 + \underline{\hspace{2cm}} = 3$	c. $3 + \underline{\hspace{2cm}} = (-7)$
d. $\underline{\hspace{2cm}} + (-15) = -22$	e. $2 + \underline{\hspace{2cm}} = -5$	f. $\underline{\hspace{2cm}} + (-5) = 0$

Comparing number line jumps and counters

We can think of $-5 + (-3)$ as five negatives and three negatives, totalling 8 negatives or -8 . We also know that $-5 - 3$ is like starting at -5 and jumping three steps towards the left on the number line, ending at -8 .

Since both have the same answer, the two expressions $-5 + (-3)$ and $-5 - 3$ are equal:

$$-5 + (-3) = -5 - 3$$

It is as if the “ $+ -$ ” in the middle is changed into a single $-$ sign. This, indeed, is a *shortcut!*

Similarly, $2 + (-7)$ is the same as $2 - 7$. Either (1) think of having 2 positive and 7 negative counters, totalling 5 negatives, (2) or think of being at 2 and taking 7 steps to the left, ending at -5 .

When solving integer problems, you can think of number line jumps or of counters, whichever is easier.

8. Compare how $-7 + 4$ is modelled on the number line and with counters.

- On the number line, $-7 + 4$ is like starting at _____, and moving _____ steps to the _____, ending at _____.
- With counters, $-7 + 4$ is like _____ negatives and _____ positives added together. We can form _____ negative-positive pairs that cancel, and what is left is _____ negatives.

9. Add.

a. $4 + (-10) =$ $-6 + 8 =$	b. $-8 + (-8) =$ $7 + (-8) =$	c. $-5 + (-7) =$ $12 + (-5) =$	d. $11 + (-2) =$ $-10 + 20 =$
--------------------------------	----------------------------------	-----------------------------------	----------------------------------

10. a. Find the value of the expression $x + (-4)$ for four different values of x .
You can choose the values.

b. For which value of x does the expression $x + (-4)$ have the value 0?

11. Solve the problems, and observe the patterns.

a. $3 - 2 =$ $3 - 3 =$ $3 - 4 =$ $3 - 5 =$ $3 - 6 =$	b. $-7 - 0 =$ $-7 - 1 =$ $-7 - 2 =$ $-7 - 3 =$ $-7 - 4 =$	c. $-5 + 0 =$ $-5 + 1 =$ $-5 + 2 =$ $-5 + 3 =$ $-5 + 4 =$	d. $-6 + 6 =$ $-6 + 7 =$ $-6 + 8 =$ $-6 + 9 =$ $-6 + 10 =$
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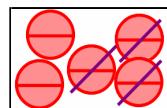
Subtracting a Negative Integer

We have already looked at such subtractions as $3 - 5$ or $-2 - 8$, which you can think of as number line jumps. But what about **subtracting a negative integer**? What is $5 - (-4)$? Or $(-5) - (-3)$?

Let's look at this kind of expression with a "double negative" in several different ways.

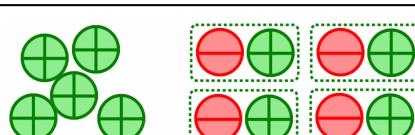
1. Subtraction as "taking away":

We can model subtracting a negative number using counters.
 $(-5) - (-3)$ means we start with 5 negative counters, and then we *take away* 3 negative counters. That leaves 2 negatives, or -2 .



$5 - (-4)$ cannot easily be modelled that way, because it is hard to take away 4 negative counters when we do not have any negative counters to start with. But you *could* do it this way:

Start out with 5 positives. Then *add* four positive-negative pairs, which is just adding zero! Now you can take away four negatives. You are left with nine positives.



Start out with 5. Add four positive-negative pairs, which amount to zero. Lastly, cross out four negatives. You are left with nine positives.

2. Subtracting a negative number as a number line jump:

$5 - (-4)$ is like standing at 5 on the number line, and getting ready to subtract, or go to the left. But, since there is a minus sign in front of the 4, it "turns you around" to face the positive direction (to the right), and you take 4 steps to the right instead. So, $5 - (-4) = 5 + 4 = 9$.

$(-5) - (-3)$ is like standing at -5 , ready to go to the left, but the minus sign in front of 3 turns you "about face," and you take 3 steps to the right instead. You end up at -2 .

3. Subtraction as a difference/distance:

To find the difference between 76 and 329, you subtract $329 - 76 = 253$ (the smaller-valued number from the bigger-valued one). If you subtract the numbers the other way, $76 - 329$, the answer is -253 .

By the same analogy, we can think of $5 - (-4)$ as meaning the difference (distance) between 5 and -4 . From the number line we can see the distance is 9.

$(-5) - (-3)$ could be the distance between -5 and -3 , except it has the larger number, -3 , subtracted from the smaller number, -5 .

If we turn them around, $(-3) - (-5)$ would give us the distance (difference) between those two numbers, which is 2. Then, $(-5) - (-3)$ would be the opposite of that, or -2 .

Two negatives make a positive!

You have probably already noticed that, any way you look at it, we can, in effect, replace those two minuses in the middle with a + sign.

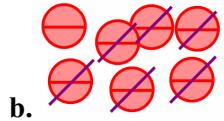
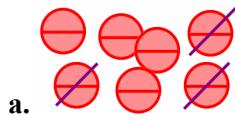
In other words, $5 - (-4)$ has the same answer as $5 + 4$.

And $(-5) - (-3)$ has the same answer as $-5 + 3$.

It may look a bit strange, but it works out really well.

$$\begin{array}{r} 5 - (-4) \\ \underline{+} \quad \underline{-} \\ 5 + 4 = 9 \end{array}$$
$$\begin{array}{r} (-5) - (-3) \\ \underline{+} \quad \underline{-} \\ (-5) + 3 = -2 \end{array}$$

1. Write a subtraction sentence to match the pictures.



2. Write an addition or subtraction sentence to match the number line movements.

- a. You are at -2 . You jump 6 steps to the left.
- b. You are at -2 . You get ready to jump 6 steps to the left,
but turn around at the last minute and jump 6 steps to the right instead.

3. Find the distance between the two numbers. Then, write a matching subtraction sentence. To get a positive distance, remember to *subtract the smaller number from the bigger number*.

a. The distance between 3 and -7 is _____.

Subtraction: _____ $-$ _____ = _____

b. The distance between -3 and -9 is _____.

Subtraction: _____ $-$ _____ = _____

c. The distance between -2 and 10 is _____.

Subtraction: _____ $-$ _____ = _____

d. The distance between -11 and -20 is _____.

Subtraction: _____ $-$ _____ = _____

4. Solve. Remember the shortcut: you can change each double minus “ $--$ ” into a plus sign.

a. $-8 - (-4) =$

$8 - (-4) =$

$-8 + (-4) =$

$8 + (-4) =$

b. $-1 - (-5) =$

$1 - (-5) =$

$-1 - 5 =$

$1 - 5 =$

c. $12 - (-15) =$

$-12 + 15 =$

$-12 - 15 =$

$12 + (-15) =$

5. Connect with a line the expressions that are equal (have the same value).

a. $10 - (-3)$

$10 + (-3)$

a.

$10 - 3$

$10 + 3$

b.

$-9 + 2$

$-9 - 2$

$-9 + (-2)$

$-9 - (-2)$

6. Write an integer addition or subtraction to describe the situations.

- a. A roller coaster begins at 27 m above ground level.
Then it descends 32 metres.

- b. Mike has R 25 . He wants to buy a toy aeroplane from his friend that costs R 40 .
How much will he owe his friend?

Solve $-1 + (-2) - (-3) - 4$.

Puzzle Corner

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Chapter 9: Geometry

Introduction

The main topics in this chapter include

- the area of triangles
- the area of polygons
- nets and the surface area of prisms and pyramids
- conversions between units of area
- the volume of prisms with sides of fractional length

We start out by revising quadrilaterals and the basic drawing of shapes. To do the drawing problems students will need to know how to use a ruler to measure lengths and a protractor to measure angles.

The focus of the chapter is on learning to find the area of polygons. To reach this goal, we follow a step-by-step development. First, we study how to find the area of a right triangle, which is very easy, as a right triangle is always half of a rectangle. Next, we build on the concept that the area of a parallelogram is the same as the area of the related rectangle to develop the usual formula for the area of a parallelogram as the product of its base times its height. This formula for the area of a parallelogram gives us a way to generalise finding the area of any triangle as half of the area of the corresponding parallelogram.

Finally, the area of a polygon can be determined by dividing it into triangles, finding the areas of those triangles, and summing them. Students also practise their new skills in the context of a coordinate grid. They draw polygons in the coordinate plane and find the lengths of their sides, perimeters and areas.

Nets and surface area is the next major topic. Students draw nets and determine the surface area of prisms and pyramids using nets. They learn how to convert between different area units, not using conversion factors or formulas, but using logical reasoning where they learn to determine those conversion factors themselves.

Lastly, we study the volume of rectangular prisms, this time with edges of fractional length. (Students have already studied this topic in fifth grade for prisms with the length of the edges being a whole number.) The basic idea is to prove that the volume of a rectangular prism *can* be calculated by multiplying its edge lengths even when the edges have fractional lengths. To that end, students need to think how many little cubes with edges $1/2$ or $1/3$ unit go into a larger prism. Once we have established the formula for volume, students solve some problems concerning the volume of rectangular prisms.

The Lessons in Chapter 9

	page	span
Quadrilaterals Revision	104	<i>2 pages</i>
Drawing Problems	106	<i>2 pages</i>
Area of Right Triangles	108	<i>2 pages</i>
Area of Parallelograms	110	<i>3 pages</i>
Area of Triangles	113	<i>3 pages</i>
Area of Polygons	116	<i>3 pages</i>
Polygons in the Coordinate Grid	119	<i>3 pages</i>
Area and Perimeter Problems	122	<i>2 pages</i>
Nets and Surface Area 1	124	<i>3 pages</i>
Nets and Surface Area 2	127	<i>4 pages</i>
Converting Between Area Units	131	<i>2 pages</i>
Volume of a Rectangular Prism with Sides of Fractional Length	133	<i>3 pages</i>
Volume Problems	136	<i>2 pages</i>
Mixed Revision	138	<i>3 pages</i>
Geometry Revision	141	<i>3 pages</i>

Helpful Resources on the Internet

Use these online resources as you see fit to supplement the main text.

Area

Free worksheets for the area of triangles, quadrilaterals and polygons

Generate printable and customisable worksheets to practise finding the area of triangles, parallelograms, trapeziums, or polygons in the coordinate grid. Options include choosing either the first or all quadrants, scaling, image size, workspace and border.

http://www.homeschoolmath.net/worksheets/area_triangles_polygons.php

BBC Bitesize - Area

Brief revision “bites,” including a few interactive questions, about the area of triangles, parallelograms and compound shapes.

<http://www.bbc.co.uk/bitesize/ks3/mathematics/measures/area/revision/4/>

Geometry Area/Perimeter Quiz from ThatQuiz.org

An online quiz that asks either the area or the perimeter of rectangles, triangles, parallelograms and trapeziums. You can also modify the quiz parameters to your liking, for example to omit a shape, or instead of solving for the area, solve for an unknown side when perimeter/area is given.

<http://www.thatquiz.org/tq-4/?-j1i00f-lc-p0>

Area Tool

Use this tool to determine how the length of the base and the height of a figure can be used to determine its area. Can you find the similarities and differences between the area formulas for trapeziums, parallelograms and triangles?

<http://illuminations.nctm.org/Activity.aspx?id=3567>

Triangle Explorer

Practise calculating the area of a triangle using this interactive tool.

<http://www.shodor.org/interactivate/activities/TriangleExplorer/>

Area of a Triangle

In this activity, you try to make a parallelogram using a copy of the given triangle.

<http://illuminations.nctm.org/Activity.aspx?id=4160>

Area of a Parallelogram

In this activity, you rearrange the pieces of a parallelogram to make a rectangle.

<http://illuminations.nctm.org/Activity.aspx?id=4158>

Math Playground: Party Designer

You need to design areas for the party, such as a crafts table, a food table, a seesaw and so on, so that they have the given perimeters and areas.

<http://www.mathplayground.com/PartyDesigner/PartyDesigner.html>

Coordinate Grid

Free worksheets for the Coordinate Grid

Generate printable and customisable worksheets for plotting points and shapes or for moving and reflecting shapes in the coordinate grid. Options include choosing either the first or all quadrants, scaling, image size, workspace and border.

http://www.homeschoolmath.net/worksheets/coordinate_grid.php

Volume & Surface Area

Worksheets for the Volume and Surface Area of Rectangular Prisms

Customisable worksheets for volume/surface area of cubes and rectangular prisms. Includes the option of using fractional edge lengths.

www.homeschoolmath.net/worksheets/volume_surface_area.php

2-D and 3-D Shapes

Learn about different solids: rotate them and see their nets.

http://www.bgfl.org/bgfl/custom/resources_ftp/client_ftp/ks2/mathsf/3d

Geometric Solids

Manipulate (rotate) various geometric solids by dragging with the mouse and see their nets. Count the number of faces, edges and vertices.

<http://illuminations.nctm.org/Activity.aspx?id=3521>

Cuboid Exploder and Isometric Shape Exploder

These interactive demonstrations let you see either various cuboids (also known as boxes or rectangular prisms) or various shapes made of unit cubes and then “explode” them into their unit cubes, thus illustrating volume.

<http://www.teacherled.com/resources/cuboidexplode/cuboidexplodeload.html> and

<http://www.teacherled.com/resources/isoexplode/isoexplodeload.html>

Sample worksheet from

www.mathmammoth.com

Volume Shoot Game

Shoot (select) the shapes with a given volume in cubic units.

<http://www.sheppardsoftware.com/mathgames/geometry/shapeshoot/VolumeShapesShoot.htm>

Interactivate: Surface Area and Volume

Explore or calculate the surface area and volume of rectangular prisms and triangular prisms. You can change the base, height and depth interactively.

<http://www.shodor.org/interactivate/activities/SurfaceAreaAndVolume>

Geometry Volume/Surface Area Quiz

An online quiz that asks for either the volume or surface area of cubes and prisms. You can modify the quiz parameters to your liking, for example to solve only for volume, only for surface area, or even for the length of an unknown side with the volume or surface area given.

<http://www.thatquiz.org/tq-4/?-j3vu0-1c-m2kc0-na-p0>

Making Cuboids

An interactive activity to explore the surface area and volume of a cuboid, calculate them, or find the volume when the areas of the faces are known.

<http://www.mrbartonmaths.com/resources/keystage3/shape/Volume%20and%20Surface%20Area%20of%20Cuboids.swf>

Volume of right rectangular prisms with fractional edges

Word problems from OpusMath: Choose the ones you want and then build a Word document from them. Answer keys available. Free registration required.

<http://www.opusmath.com/common-core-standards/6.g.2-find-the-volume-of-a-right-rectangular-prism-with-fractional-edge-lengths>

Just for Fun

Online Kaleidoscope

Create your own kaleidoscope pattern with this interactive tool.

http://www.zefrank.com/dtoy_vs_byokal/

Interactive Tangram Puzzle

Place the tangram pieces so they form the given shape.

http://nlvm.usu.edu/en/nav/frames_asid_112_g_2_t_1.html

Interactivate! Tessellate

An online, interactive tool for creating your own tessellations. Choose a shape, then edit its corners or edges. The program automatically changes the shape so that it will tessellate (tile) the plane. Then push the tessellate button to see your creation!

<http://www.shodor.org/interactivate/activities/Tessellate>

National Library of Virtual Manipulatives for Interactive Mathematics: Geometry

A collection of interactive geometry activities: Congruent triangles, fractals, geoboard, golden rectangle, ladybug leaf, ladybug mazes, platonic solids, tangrams, tessellations, transformations and more.

http://nlvm.usu.edu/en/nav/category_g_3_t_3.html

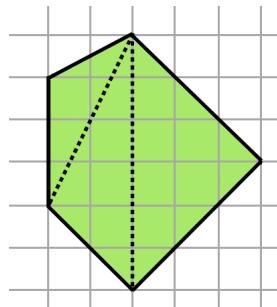
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Area of Polygons

To calculate the **area of polygons**, all you have to do is divide them into easy shapes, such as rectangles and triangles. Calculate the area of each easy shape separately, and add them to find the total area.

1. This figure is called a _____.

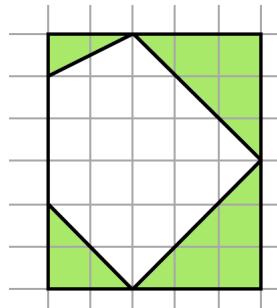
Calculate its area using the three triangles.
For each triangle, use the *vertical* side as the base.



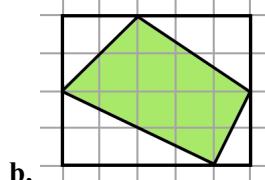
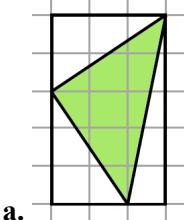
2. Here is another way of calculating the area of the same figure.

1. Calculate the area of the rectangle that encloses the figure.
2. Calculate the areas of the four shaded triangles.
3. Subtract.

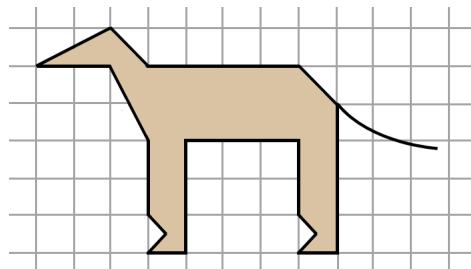
Use this method and verify that you get the same result as above.



3. Find the areas of the shaded figures.

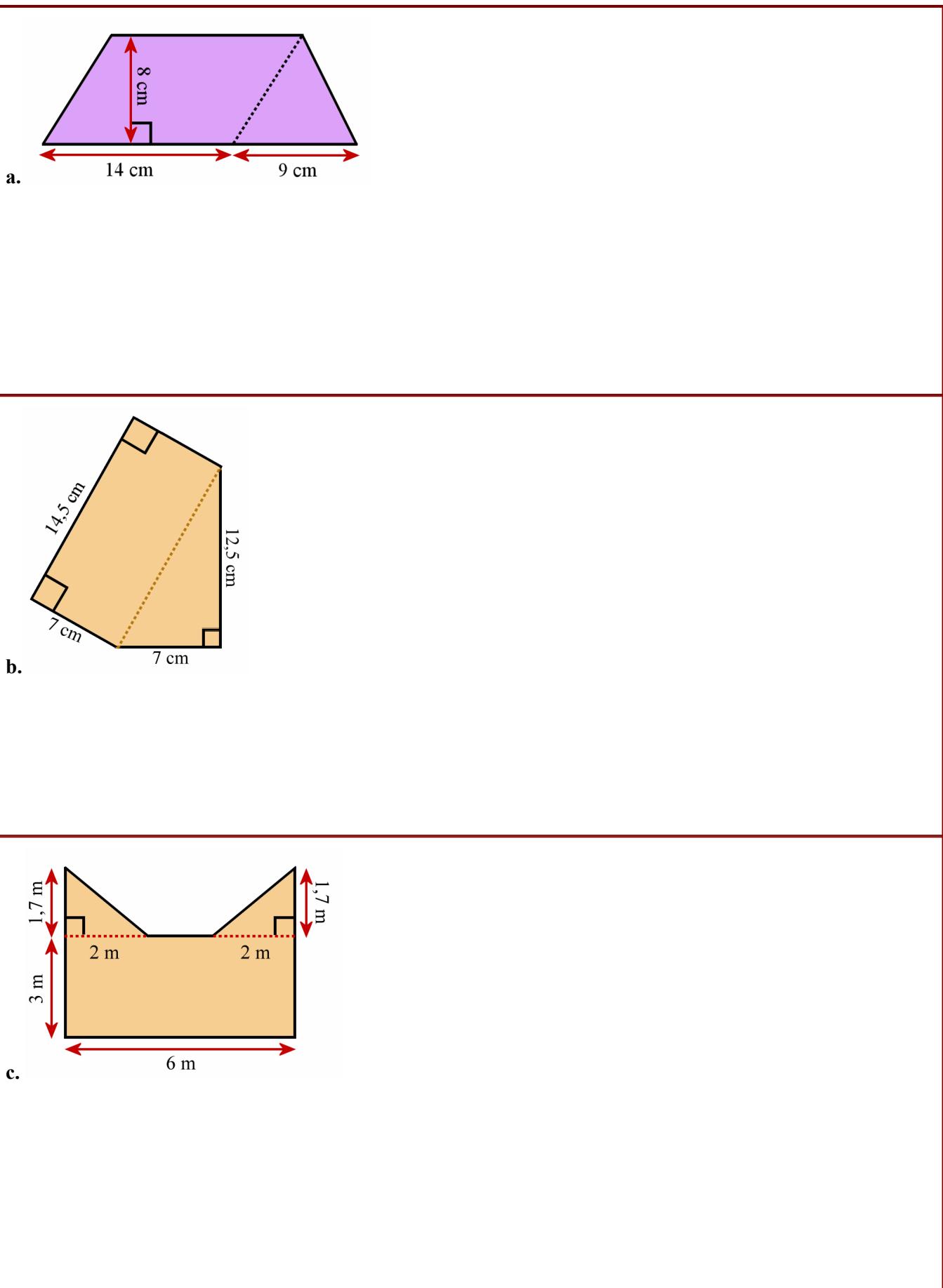


4. a. The side of each little square in the drawing on the right is 1 centimetre.
Find the area of the polygon.



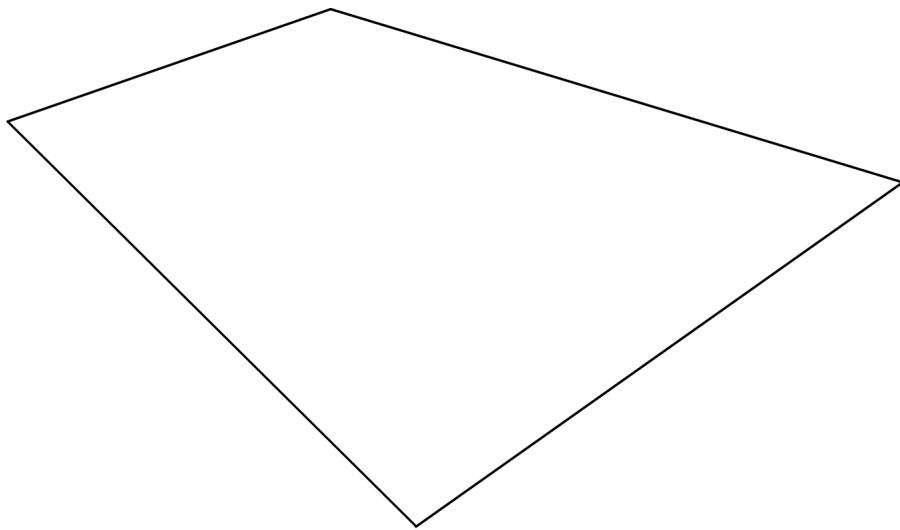
- b. Imagine that the side of each little square is 2 centimetres instead.
What is the area now?

5. Calculate the total area of the figures.



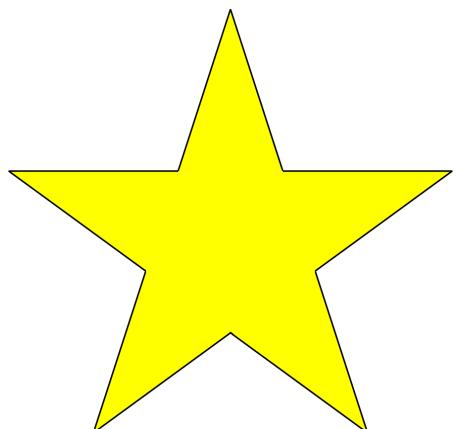
6. Divide this quadrilateral into two triangles, and then find its area in square centimetres.

You may use a calculator.



Puzzle Corner

Measure what you need to from this star to find: (a) its perimeter in centimetres and (b) its area in square centimetres.



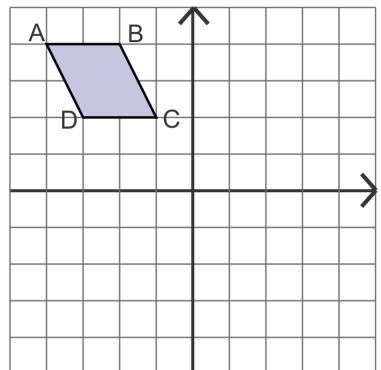
Polygons in the Coordinate Grid

1. Name the polygons, transform them, and find their area.

a. What is this polygon called?

b. Reflect it in the x -axis.

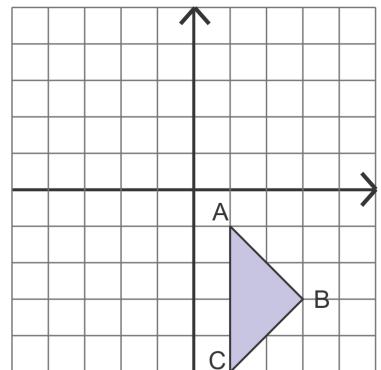
c. Find its area.



d. Classify this triangle according to its sides and angles.

e. Reflect it in the y -axis.

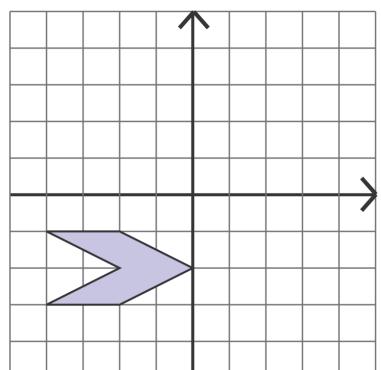
f. Find its area.



g. What is this polygon called?

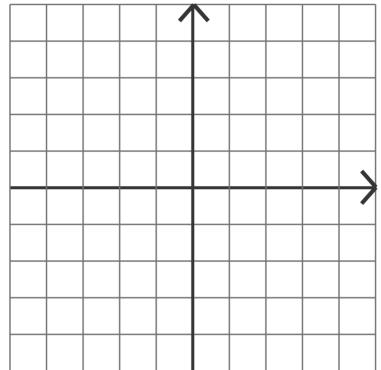
h. Move it 4 units up, and 2 units to the right.

i. Find its area.



j. Draw any pentagon using grid points as vertices.

k. Find its area.



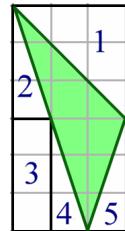
Here is a neat way to **find the area of any polygon whose vertices are points in the grid**.

(1) Draw a rectangle around the polygon. (2) Divide the area between the polygon and the rectangle into triangles and rectangles. (3) Calculate those areas. (4) Subtract the calculated areas from the total area of the large rectangle to find the area of the polygon.

Example 1. To find the area of the coloured triangle, we draw a rectangle around it that is 3 units by 6 units. Then we find the areas marked with 1, 2, 3, 4 and 5:

- 1: a triangle; $3 \times 3 \div 2 = 4,5$ square units
- 2: a triangle; $1 \times 3 \div 2 = 1,5$ square units
- 3: a rectangle; $1 \times 3 = 3$ square units
- 4: a triangle; $1 \times 3 \div 2 = 1,5$ square units
- 5: a triangle; $1 \times 3 \div 2 = 1,5$ square units

The total for the shapes 1, 2, 3, 4 and 5 is 12 square units.



Therefore, the area of the coloured triangle is 18 square units – 12 square units = 6 square units.

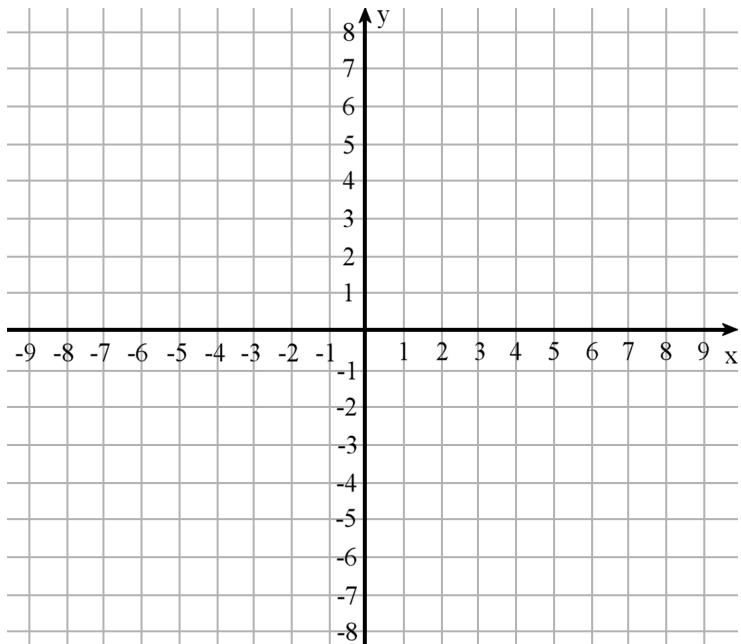
2. Find the area of a triangle with given vertices.

a. $(-8, 7)$, $(-5, 3)$ and $(4, 0)$.

b. $(-7, -2)$, $(-2, -1)$ and $(-4, -7)$.

3. Draw a quadrilateral in the grid with vertices $(8, 5)$, $(3, 4)$, $(4, -5)$, $(7, -6)$

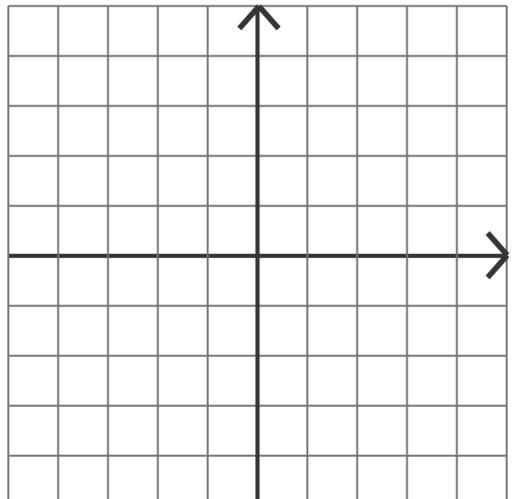
Use the same technique to find its area.



4. The points $(1, 2)$, $(2, 4)$, $(-1, 2)$, $(-2, 4)$ are four vertices of a water fountain in the shape of a regular octagon. The other four points are found by reflecting these four in the y -axis.

Find the length of one side of the fountain.

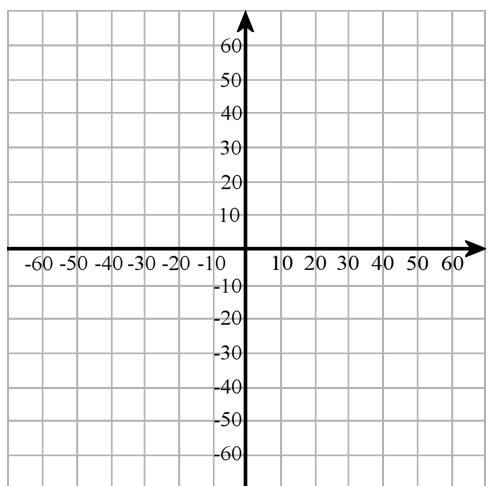
Find its perimeter.



5. What polygon is formed when you join the following points in order with line segments?

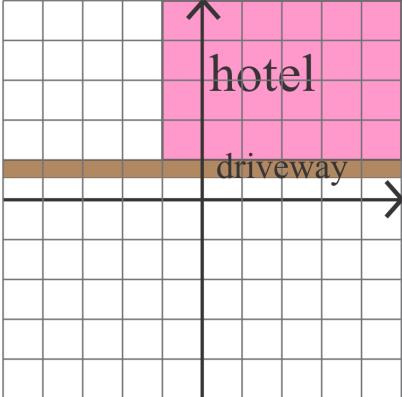
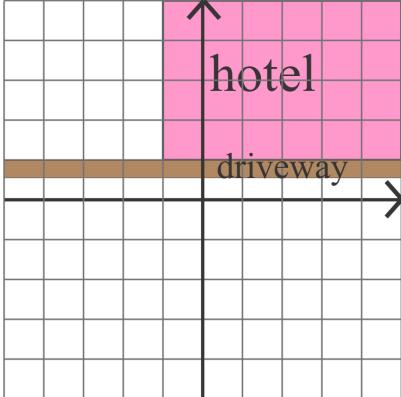
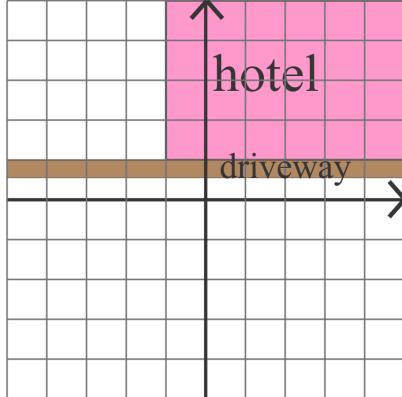
$(-35, -40), (-35, 40), (-20, 40), (20, -15), (20, 40), (35, 40), (35, -40), (20, -40), (-20, 15), (-20, -40)$ and $(-35, -40)$

Challenge: Find its area.



6. A hotel wants to build a swimming pool with a total area of 100 to 140 square metres. One of its sides has to be 12,5 metres.

- Suggest three different rectangular shapes and draw them in the grids below. Each unit in the grid is 5 m.
- For each pool, give the coordinates of the four corners of the pool.
- For each pool, calculate the pool's distance to the driveway.

Pool Design 1:	Pool Design 2:	Pool Design 3:
		
Coordinates of the corners:	Coordinates of the corners:	Coordinates of the corners:
Area:	Area:	Area:
Distance to the driveway:	Distance to the driveway:	Distance to the driveway:

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Chapter 10: Statistics

Introduction

The fundamental theme in our study of statistics is the concept of *distribution*. In the first lesson, students learn what a distribution is—basically, it is *how* the data is distributed. The distribution can be described by its centre, spread and overall shape. The shape is read from a graph, such as a dot plot or a bar graph.

Two major concepts when summarising and analysing distributions are its centre and its variability. First we study the centre, in the lessons about mean, median and mode. Students not only learn to calculate these values, but also relate the choice of measures of centre to the shape of the data distribution and the type of data.

In the lesson *Measures of Variation* we study range, interquartile range and mean absolute deviation. The last one takes many calculations, and the lesson gives instructions on how to calculate it using a spreadsheet program, such as Excel.

Then in the next lessons, students learn to make several different kinds of graphs: histograms, boxplots and stem-and-leaf plots. In those lessons, students continue summarising distributions by giving their shape, a measure of centre and a measure of variability.

The Lessons in Chapter 10

	page	span
Understanding Distributions	148	5 pages
Mean, Median and Mode	153	2 pages
Using Mean, Median and Mode	155	3 pages
Measures of Variation	158	5 pages
Making Histograms	163	3 pages
Boxplots	166	4 pages
Stem-and-Leaf-Plots	170	3 pages
Mixed Revision	173	3 pages
Statistics Revision	176	3 pages

Helpful Resources on the Internet

Mean, Median, Mode, Range, etc.

Using and Handling Data

Simple explanations of how to find the mean, the median and the mode.

<http://www.mathsisfun.com/data/central-measures.html>

Math Goodies Interactive Statistics Lessons

Clear lessons with examples, interactive quiz questions, practice exercises and challenge exercises over topics that include range, arithmetic mean, non-routine mean, median and mode.

http://www.mathgoodies.com/lessons/toc_vol8.htm

Measures of Center and Quartiles Quiz from ThatQuiz.org

An online quiz about the measures of centre and quartiles in boxplots, stem-and-leaf plots and dot plots.

<http://www.thatquiz.org/tq-5/?-jr0t0-l1-p0>

Mean, Median, and Mode

Lesson on how to calculate the mean, median and mode for a set of data given in different ways. It also has interactive exercises.

http://www.cimt.plymouth.ac.uk/projects/mepres/book8/bk8i5/bk8_5i2.htm

Measures Activity

Enter your own data and the program will calculate mean, median, mode, range and some other statistical measures.

<http://www.shodor.org/interactivate/activities/Measures>

Landmark Shark Game

You are dealt five number cards, and using them as your data set, you need to choose which of the range, median, or mode is the largest number.

<http://media.emgames.com/emgames/demosite/playdemo.html?activity=M5A006&activitytype=dcr&level=3>

Train Race Game

Calculate the median and range of travel times for four different trains, then choose a good train to take based on your results.

<http://www.bbc.co.uk/schools/mathsfайл/shockwave/games/train.html>

Range and interquartile range quiz

An online multiple-choice quiz that is created randomly. Refresh the page (or press F5) to get another quiz.

<http://www.glencoe.com/sec/math/studytools/cgi-bin/msgQuiz.php4?isbn=0-07-825200-8&chapter=12&lesson=2&&headerFile=4>

Mean Deviation

A simple explanation about what the mean absolute deviation is, how to find it, and what it means.

<http://www.mathsisfun.com/data/mean-deviation.html>

Mean Absolute Deviation

Several videos explaining how to calculate the mean absolute deviation of a data set.

<http://www.onlinemathlearning.com/measures-variability-7sp3.html>

Working with the Mean Absolute Deviation (MAD)

A tutorial and questions where you are asked to create line plots with a specified mean absolute deviation.

http://www.learner.org/courses/learningmath/data/session5/part_e/working.html

GCSE Bitesize Mean, Mode and Median Lessons

Explanations with simple examples.

<http://www.bbc.co.uk/schools/gcsebitesize/mathsf/statistics/measuresofaveragerev1.shtml>

Graphing and Graphs

Bar Chart Virtual Manipulative

Build your bar chart online using this interactive tool.

http://nlvm.usu.edu/en/nav/frames_asid_190_g_1_t_1.html?from=category_g_1_t_1.html

An Interactive Bar Grapher

Graph data sets in bar graphs. The colour, thickness and scale of the graph are adjustable. You can input your own data, or you can use or alter pre-made data sets. Uses Java.

<http://illuminations.nctm.org/ActivityDetail.aspx?ID=63>

Create a Graph

Children can create bar graphs, line graphs, pie graphs, area graphs and xyz graphs to view, print and save.

<http://nces.ed.gov/nceskids/createagraph/default.aspx>

Statistics Interactive Activities

(scroll down to Statistics and Probability concepts)

A set of interactive tools for exploring histograms, pie charts, boxplots, stem-leafs, and mean, median, variance and standard deviation of data. You can enter your own data or explore the examples.

<http://www.shodor.org/interactivate/activities/tools.html>

PlotLy

A comprehensive, collaborative data analysis and graphing tool. Bring data in from anywhere, do the maths, graph it with interactive plots (scatter, line, area, bar, histogram, heatmap, box and more) and export it.

<http://plot.ly>

Graphs Quiz from ThatQuiz.org

This quiz asks questions about different kinds of graphs (bar, line, circle graph, multi-bar, stem-and-leaf, boxplot, scattergraph). You can modify the quiz parameters to your liking, such as to plot the graph, answer different kinds of questions about the graph, or find mean, median, or mode based on the graph.

<http://www.thatquiz.org/tq-5/math/graphs>

Math Goodies Interactive Data & Graphs Lessons

Clear lessons with examples and interactive quiz questions, covering the concept and construction of line graphs, bar graphs, circle graphs, comparing graphs and exercises.

http://www.mathgoodies.com/lessons/toc_vol11.html

Statistics Gizmos from Explorelearning.com

Interactive exploration activities online, with lesson plans. Topics include box-and-whisker plots, mean, median, and mode, histograms, stem-and-leaf plots and more. This is an excellent resource. The gizmos work for 5 minutes for free. You can also sign up for a free trial account.

<http://www.explorelearning.com/index.cfm?method=cResource.dspResourcesForCourse&CourseID=222>

Create a Histogram

Explore already-made histograms using given sets of data, or use your own data to make your own. Try changing the interval size (the bin size) to see how it affects the graph.

<http://www.shodor.org/interactivate/activities/Histogram/>

Make Your Own Boxplot

Enter values from your own data, and this web page creates your box-and-whisker plot for you.

<http://www.mrnussbaum.com/graph/bw.htm>

Create a Boxplot

You can explore boxplots using the given sets of data, or make your own. Try adding more data to the existing data sets and see how the plot changes.

<http://www.shodor.org/interactivate/activities/BoxPlot/>

Make Your Own Stem-and-Leaf Plot

Enter values from your own data, and this web page creates your stem-and-leaf plot for you.

<http://www.mrnussbaum.com/graph/sl.htm>

Stem-and-Leaf Plots Quiz

An online multiple-choice quiz that is created randomly. Refresh the page (or press F5) to get another quiz.

<http://www.glencoe.com/sec/math/studytools/cgi-bin/msgQuiz.php4?isbn=0-07-825200-8&chapter=12&lesson=1&&headerFile=4>

Boxplots Quiz

An online multiple-choice quiz that is created randomly. Refresh the page (or press F5) to get another quiz.

<http://www.glencoe.com/sec/math/studytools/cgi-bin/msgQuiz.php4?isbn=0-07-825200-8&chapter=12&lesson=3&&headerFile=4>

Circle Grapher

A tool to graph data sets in a circle graph. You can input your own data or alter a pre-made data set.

<http://illuminations.nctm.org/activitydetail.aspx?id=60>

Statistics - Facts & Figures

GapMinder

Visualising human development trends (such as poverty, health, gaps, income on a global scale) via stunning, interactive statistical graphs. This is an interactive, dynamic tool and not just static graphs. Download the software or the reports for free.

<http://www.gapminder.org/data/>

WorldOdometers

World statistics updated in real time. Useful for general educational purposes - for some stunning facts.

<http://www.worldometers.info>

UN Data

The United Nations offers the ability to search across its statistical databases, including education, human development, population, trade and gender.

<http://data.un.org>

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Using Mean, Median and Mode

Whether you use mean, median, or mode depends both

- on the **type of data and**
- on the **shape of distribution.**

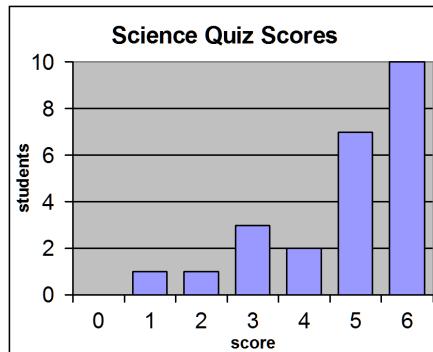
Example 1. This distribution of science quiz scores is heavily skewed to the left, and its “peak” is at 6. Which of the three measures of centre would best describe this distribution?

Let's calculate the mean, median and mode.

Mode: We can see from the graph that the mode is 6.

Median: There are 24 students. The students' actual scores are 1, 2, 3, 3, 3, 3, 4, 4, 4, 5, 5, 5, 5, 5, 5, 5, 6, 6, 6, 6, 6, 6, 6, 6, 6, 6.

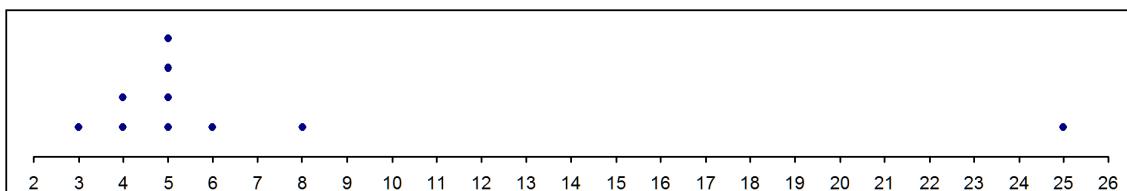
The median is the average of the 12th and 13th scores, which is 5.



The mean is $\frac{1 + 2 + 3 + 3 + 3 + 4 + 4 + 4 + 5 + 5 + 5 + 5 + 5 + 6 + 6 + 6 + 6 + 6 + 6 + 6 + 6 + 6}{24} = 4,79167 \approx 4,79$.

Notice that the mean is less than 5, but the two highest bars on the graph are at 5 and 6. In this case, the mean does *not* describe the peak of the distribution very well because it actually falls outside the peak! Both the median and the mode do describe it well.

1. a. Find the mean, median and mode of this data set: 3, 4, 4, 5, 5, 5, 5, 6, 8, 25.



mean _____ median _____ mode _____

b. Which of the three, mean, median, or mode, best describes the centre of this data?

Clearly, either the _____ or the _____, but *not* the _____!

The _____ is off from the central peak of the distribution.

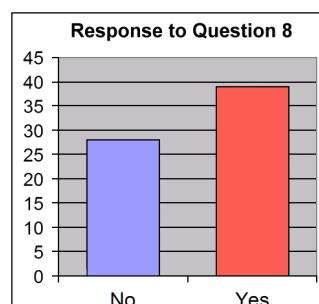
The reason for this is that the data item “25” throws it off. This 25 is very different from the other data items in the set, and could even be a typing error! Such an item is called an **outlier**.

2. The graph shows the response to a certain question in a survey.

It was measured as a *yes/no* question. Which of the below are possible to determine? (Mark with an “x”).

_____ mean _____ median _____ mode

Hint: Imagine what the original data that was used to create the graph looks like.



Guidelines for using the mean, median and mode

- The *mode* can be used with any type of data.
- The *median* can only be used if the data can be put in order.
- The *mean* can only be used if the data is numerical.

Sometimes, the median and the mean do not fall where the peak of the distribution is.

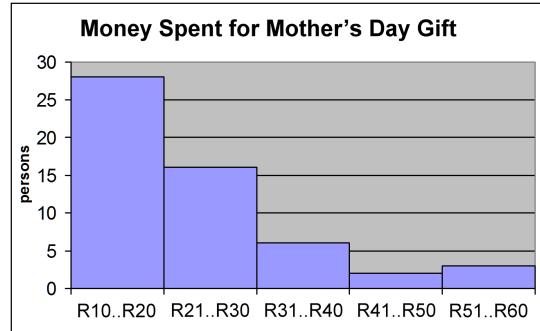
- The mean works best if the distribution is fairly close to a bell shape and does not have outliers.
- If the distribution is very skewed or has outliers, it is better to use median than mean.

3. Jordan asked 55 teenagers about how much money they spent to purchase Mother's Day gifts.

a. Which of the numbers R21 and R19 is the mean?

Which is the median?

b. Would mean or median better describe this data? Why?



c. Approximately what percentage of these teenagers spent R20 or less on a Mother's Day gift?

- Name what is being studied (usually the *title* of the graph tells you this).
- Describe how the data was measured and in what units. For example, the respondents have given numerical answers in rand. Or perhaps they chose either "yes" or "no."
- Indicate whether the mean, median, or mode can be calculated. You do not have to find the mean, even when it is possible.

Hint: Think what kind of data was used to create the graph (the original data).

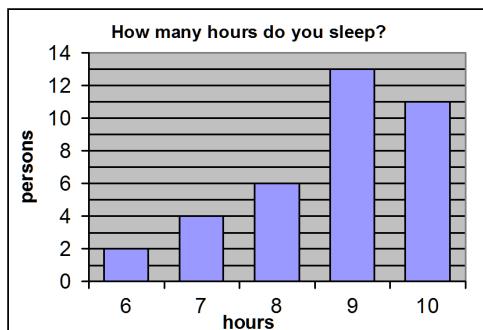
a. What is being measured or studied? _____

How is it measured?

Which are possible? (Mark with an "x").

mean median mode

The mode is: _____ The median is: _____



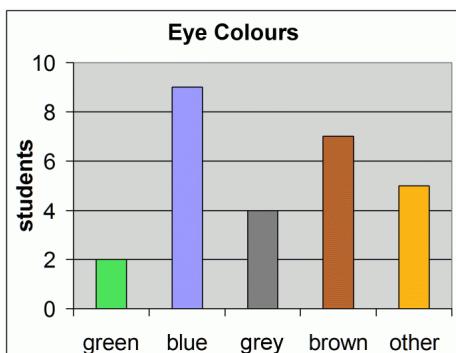
b. What is being measured or studied? _____

How is it measured?

Which are possible? (Mark with an "x").

mean median mode

The mode is: _____ The median is: _____



For the following data sets:

- Create a dot plot or a bar graph.
- Name your graph.
- Describe the shape of the distribution.
- Indicate how many observations there are.
- Choose measure(s) of centre that describe the peak of the distribution, and calculate them.

5. a. The length of words on three pages in a certain children's story book:

7 5 6 8 3 6 6 2 4 2 2 3 3 4 4 3 5 5 4
5 4 3 2 5 2 1 4 4 7 5 4 8 3 3 3 3 3 5
5 3 4 2 3 1 6 2 5 4 4 3 4 3 2 8

Here is the same data sorted:

1 1 2 2 2 2 2 2 3 3 3 3 3 3 3 3 3 3 4 4 4
4 4 4 4 4 4 4 5 5 5 5 5 5 5 6 6 6 6 7 7 8 8 8

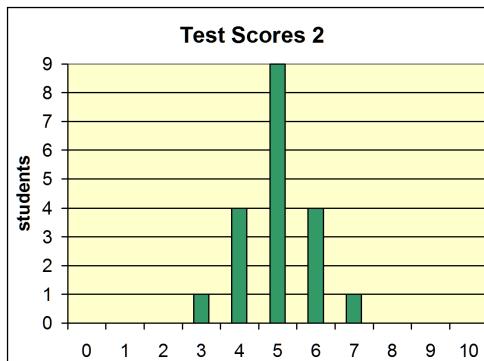
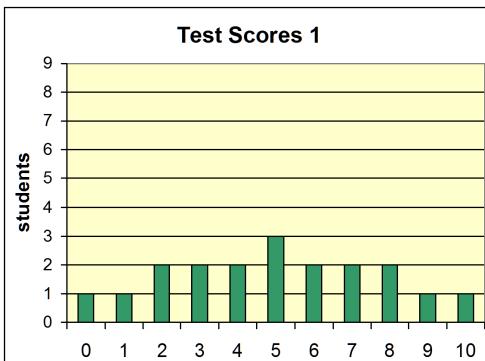
- b. A restaurant asked its customers some questions about their food and service. One question was, "How would you rate the meal you ate today?" There were five possible answers: "excellent," "good," "normal," "not so good," and "poor." The customers' responses are listed below:

normal poor excellent good good excellent good
normal not so good excellent good good good
normal normal good excellent good good good
not so good not so good excellent good

Puzzle Corner

Can you find a quick, *mental maths* method for calculating the mean for this data set? 102, 94, 99, 105, 96, 107, 101, 104 (the weights of a litter of kittens at birth, in grams)

Measures of Variation



Look at the two graphs. The first gives the scores for test 1 and the second for test 2. *Both* sets of data have a mean of 5,0 and a median of 5. Yet the distributions are very different. If you were told just the mean and median, you would not know that!

How are they different? In test 1, the students got a wide range of different scores; the data is very scattered and **varies a lot**. In test 2, nearly all of the students got a score from 4 to 6. The data is concentrated, or *clustered*, around 5.

We have several ways of measuring the **variation** in a distribution.

One way is to use range. Simply put, **range is the difference between the largest and smallest data items**.

Example 1. For test 1, the smallest score is 0 and the largest is 10. The range is 10.

For test 2, the smallest score is 3 and the largest is 7. The range is 4. Clearly, the range is much smaller for test 2, indicating the data is clustered. The larger range of test 1 scores means the data is much more scattered.

Another measure of variation is the **interquartile range**.

To determine this measure, we first identify the **quartiles**, which are the **numbers that divide the data into quarters**. The **interquartile range is the difference between the first and third quartiles**. Since the quartiles divide the data into quarters, exactly half of it lies between the first and third quartiles. The smaller this measure is, the more concentrated the data is.

Example 2 shows how to determine the interquartile range.

Example 2. The scores for test 1 are: 0, 1, 2, 2, 3, 3, 4, 4, 5, 5, 5, 6, 6, 7, 7, 8, 8, 9, 10.

Find the interquartile range.

We need to divide the data into quarters. Finding the median naturally divides the data into two halves:

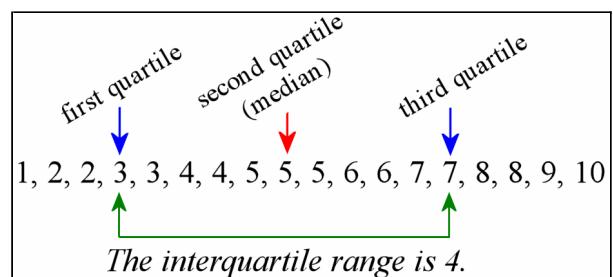
0, 1, 2, 2, 3, 3, 4, 4, 5, **5**, 5, 6, 6, 7, 7, 8, 8, 9, 10

Now we take *the lower half of the data*, excluding the median, and find *its median*. 0, 1, 2, 2, **3**, 3, 4, 4, 5. That is the **first quartile**.

Similarly, the *median of the upper half of the data* is the **third quartile**: 5, 6, 6, 7, **7**, 8, 8, 9, 10

The median itself is the **second quartile**.

Together, the three quartiles divide the data into quarters. The interquartile range is the difference between the third and first quartile, or in this case $7 - 3 = \underline{4}$.



1. Find the median and interquartile range of the data sets.

- First, find the median.
- Next, find the median of the lower half of the data (excluding the median itself).
- Then, find the median of the upper half of the data (excluding the median itself).

a. 5, 5, 6, 6, 7, 7, 7, 7, 7, 8, 8, 8, 9, 10, 10

first quartile _____ median _____ third quartile _____ interquartile range _____

b. 2, 2, 3, 4, 5, 5, 5, 5, 6, 6, 6, 7, 7, 7, 9, 9

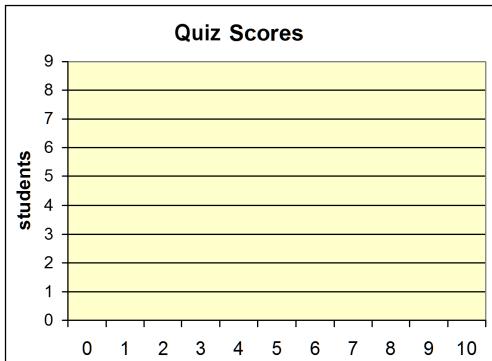
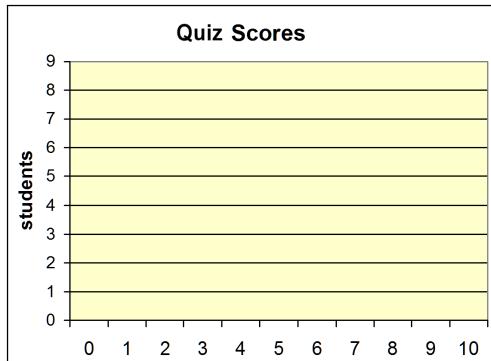
first quartile _____ median _____ third quartile _____ interquartile range _____

c. Let's say the data sets in (1a) and (1b) are the quiz scores of two groups of students.

Which group did better in general?

In which group did the quiz scores vary more?

Also, make bar graphs for the quiz scores of the two groups.



2. Find the range and the interquartile range of the data sets.

a. The number of paid vacation days in a year of the employees in a small firm:

6 8 10 10 11 11 12 12 12 13 13 14 14 14 15 17 18 20 24

range _____

1st quartile _____ median _____ 3rd quartile _____

interquartile range _____

b. The height of some children in centimetres:

136 138 139 139 140 140 140 140 140 141 141 141 142 144 144 145 147

range _____

1st quartile _____ median _____ 3rd quartile _____

interquartile range _____

Mean absolute deviation

Yet another measure of variation is the **mean absolute deviation**. Use it only if you use the mean as your measure of centre.

To be brief, mean absolute deviation measures *how much, on average, the various data items deviate (differ) from the mean*. In other words, for each data item, we calculate how much it differs from the mean, and then we calculate the average of those differences.

It is called “absolute” deviation because we use the absolute values of the differences from the mean. In other words, those differences are always taken to be positive, never negative.

It is easier to calculate mean absolute deviation using a computer because it involves so many calculations. Some calculators may also have it. Example 3 will make the process clear.

Example 3. Calculate the mean and the mean absolute deviation for the ages of people in a gymnastics group:

62 60 65 63 70 64 78 71 68 66 70

The mean is $\frac{62 + 60 + 65 + 63 + 70 + 64 + 78 + 71 + 68 + 66 + 70}{11} = 67$.

So the average age of the members is 67 years. But on average, how much do their ages *differ* from this mean of 67 years? That is what mean absolute deviation tells us.

The table below shows how the calculations can be arranged in a table. We put the data items in one column, and then we calculate the difference of each data item from the mean in another column. You can skip the column titled “difference from the mean”, and go direct to the absolute difference, if you like.

The mean absolute deviation is abbreviated as *m.a.d.* in the bottom row. It is calculated as the mean of the numbers in the last column (the absolute differences) and the answer is 4.

	age	difference from the mean	absolute difference
	62	-5	5
	60	-7	7
	65	-2	2
	63	-4	4
	70	3	3
	64	-3	3
	78	11	11
	71	4	4
	68	1	1
	66	-1	1
	70	3	3
mean	67	<i>m.a.d.</i>	4

What does it mean to say that the mean absolute deviation is 4? It means that, on average, the members’ ages differ from the mean of 67 years by 4 years.

Calculating mean absolute deviation using a spreadsheet program (Excel, LibreOffice Calc, etc.)

1.

	A	B	C	D	E
1	ages		differences	absolute differences	
2		62	-5	5	
3		60	-7	7	
4		65	-2	2	
5		63	-4	4	
6		70	3	3	
7		64	-3	3	
8		78	11	11	
9		71	4	4	
10		68	1	1	
11		66	-1	1	
12		70	3	3	
13					
14	mean	67		4	

To calculate the mean of a set of data, in the cell where you want the calculation to appear, type:

=AVERAGE(B2:B12)

When you type the formula in the cell, it appears in the formula bar at the top, as in the image. A formula always starts with an equals sign.

Press "ENTER" to see the answer, 67.

2.

	A	B	C	D	E
1	ages		differences	absolute differences	
2		62	-5	5	
3		60	-7	7	
4		65	-2	2	
5		63	-4	4	
6		70	3	3	
7		64	-3	3	
8		78	11	11	
9		71	4	4	
10		68	1	1	
11		66	-1	1	
12		70	3	3	
13					
14	mean	67		4	

Next we calculate the difference between each item of data and the mean.

Type "=B2 - \$B\$14" to subtract the values in cells B2 and B14.

The dollar signs in \$B\$14 make it an **absolute reference**, so it doesn't change when you copy and paste the formula into another cell. Pasting the formula into the cells below is a quick way to get the spreadsheet to calculate those values, too.

3.

	A	B	C	D	E
1	ages		differences	absolute differences	
2		62	-5	5	
3		60	-7	7	
4		65	-2	2	
5		63	-4	4	
6		70	3	3	
7		64	-3	3	
8		78	11	11	
9		71	4	4	
10		68	1	1	
11		66	-1	1	
12		70	3	3	
13					
14	mean	67		4	

Now we calculate the absolute value of each difference.

In cell D2 type "=ABS(C2)" to calculate the absolute value of the number in cell C2. Then copy cell D2 and paste it into the cells below it to copy the formula and adjust the reference in it automatically.

4.

	A	B	C	D	E
1	ages		differences	absolute differences	
2		62	-5	5	
3		60	-7	7	
4		65	-2	2	
5		63	-4	4	
6		70	3	3	
7		64	-3	3	
8		78	11	11	
9		71	4	4	
10		68	1	1	
11		66	-1	1	
12		70	3	3	
13					
14	mean	67		4	

Lastly, we are ready to calculate the mean absolute deviation by taking the average of the values in cells D2 to D12. In the cell where you want the value to appear, type "=AVERAGE(D2:D12)".

The answer "4" will then appear in the cell after you press "ENTER."

3. Calculate the mean and the mean absolute deviation for these sets of data. You can use a spreadsheet program on a computer if you have access to one.

a.

Art Club - members' ages			
	age	difference from mean	absolute difference
	7		
	9		
	9		
	10		
	12		
	13		
	13		
	13		
	14		
	14		
	15		
	15		
mean		m.a.d.	

b.

Prices of MP3 players			
	price	difference from mean	absolute difference
	629		
	630		
	633		
	634		
	634		
	635		
	635		
	635		
	636		
	636		
	637		
	639		
	642		
mean		m.a.d.	

4. Draw a dot plot for each data set in question 3.

a.

b.

5. For each data set above, answer the questions.

- What is the shape of the distribution?
- Based on the shape, which measure of centre, mean or median, would better describe the data?
- Based on the best measure of centre, which measure of variation should be chosen, interquartile range or mean absolute deviation?