Polarization Simulation

Applied mathematics: General Relativity

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Abstract

general relativity, the theory developed by Albert Einstein from the year 1907 to 1915, the very thing we call gravity was actually a curve in space-time due to massive object like neutron stars and planets which warp space around them This curvature is caused by the presence of of mass[1], the greater the mass of the object, the greater it's effects on warping the space-time around it, as the these massive objects that curve space around accelerate, the curvature around them will propagate outwards at the space of light, these propagation are known to gravitational waves[1], with some for the sources of these gravitational waves are inspiraling binary neutron star which when they orbit close to one another create gravitational waves, they are incredibly hard to detect experimentally, though there are few facilities around the world which have been able to achieve this complicated task. For this project the gravitational waves will be simulated from computation for different wave forms[2-5]. And also the discussion around Polarization of wave forms and their source, the important information they might provide in the field of astronomy and other related fields. These waves also carry energy away from their source, and is absorbed by object of masses around the source

1. INTRODUCTION

The era of gravitational-wave (GW) astronomy began with the detection of binary black hole (BBH) mergers, by the Advanced Laser Interferometer Gravitational-Wave Observatory (LIGO) detectors[5].
Detecting gravitational waves requires extremely sensitive detectors,partly due to the fact that their sources are very far away, which means over billions of light years detecting them become a very difficult problem and over the last few decades this was an impossible task task as the technology at the time was not powerful enough, on this age and era with great technological advancements,LIGO[4] and Virgo, was able to achieve this when the signal was first observed at the LIGO.
Livingston detector at 1030:43 UTC, and at the LIGO Hanford and Virgo detectors with a delay of 8 ms and 14 ms, respectively[4]. Understanding gravitational wave GWs will have a significant impact in understanding the fabric of space-time around black holes, how they form and the merger of binary system, and studying the early universe, it'

formation and parts of the universe that light is unable to reach, this will provide a break through for most mysteries of the universe this will also help verify predictions such as light and gravitational waves traveling at the same speed. Gravitational wave Gws can come from the very early universe and the electromagnetic radiation from the Big Bang is called the *Cosmic Microwave Background*, these describe how the universe was just before the Big Bang so one of the key things that gravitational waves would present is a window (i.e a snap shot just few seconds [10^{-24}]) after the universe formed. For this project, the approach is to use the expansion around flat space[2].
And simulate the polarization modes of GW, and look at how the gravitational wave (GW) affect matter, also how the modes behaviour in the absence of matter, using the properties of flat and curved space time and learn some fo the important properties of Einstein equation as the basis on which the gravitational waves (GW) in general relativity is built upon. Gravitational waves , GWs are characterized by two tensor (spin-2) polarizations only, whereas generic metric theories may allow up to six polarizations[4,6,7].

2. GRAVITATIONAL WAVE SOURCES

When the curvature varies very rapidly as a results of motion of matter, curvature ripples form, and these ripples of space-time are called gravitational waves GWs. They propagate at the speed of light from the source, gravitational waves GWs is the results of the accelerated objects(masses) and Unlike Electromagnetic waves EM which propagate through space-time, gravitational wave GWs oscillate space-time itself with the frequency range (10^{-9}Hz-10^{4}Hz)[4].
GWs are formed by the in spiral sources such as Binary black holes, Binary Newtron star, binary white-dwarfs, due to the fact that when stars orbit around each other they lose momentum in the form of gravitational wave GWs, which results in their orbital separation decreasing and merge after a while as a results, core collapse of supernovae.
figure 1 shows a binary system where two Neutron stars orbit each other and they drag space around them as they draw closer to each other in the in spiral, and are gaining speed as they move closer to each other. When they merge the resulting system has a mass that is the sum of the two masses which is shown in figure 2. The value of the frequency [12 Hz] is consistent with the orbital radius , as the orbiting star get closer to each other, the time they take to complete an orbit gets less and less as it can be seen from both figure 1 and figure 2, and the speed at which they orbit each other increases as they get very close to each other given by the following $v = 2\pi f R = 2\pi \frac{1}{T} R$ as time decreases, speed increases. As they merge the space time around them become disturbed and a gravitational wave is produced which travel outwards from the source at a speed that is equal to the speed of light. The results can be seen in figure 3

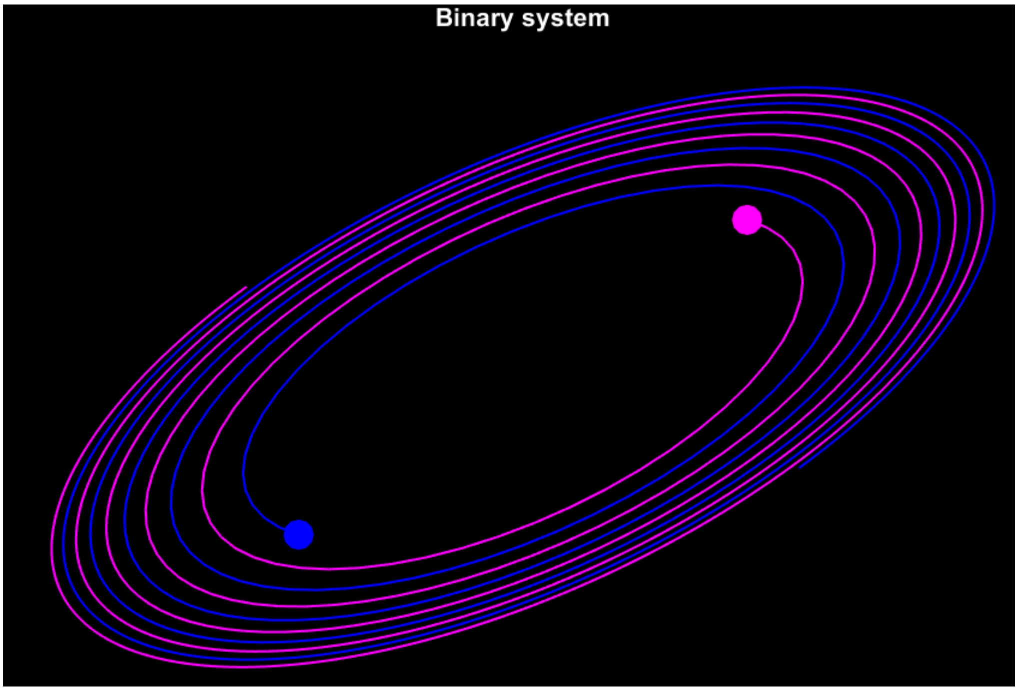

Figure 1: stars orbiting each other

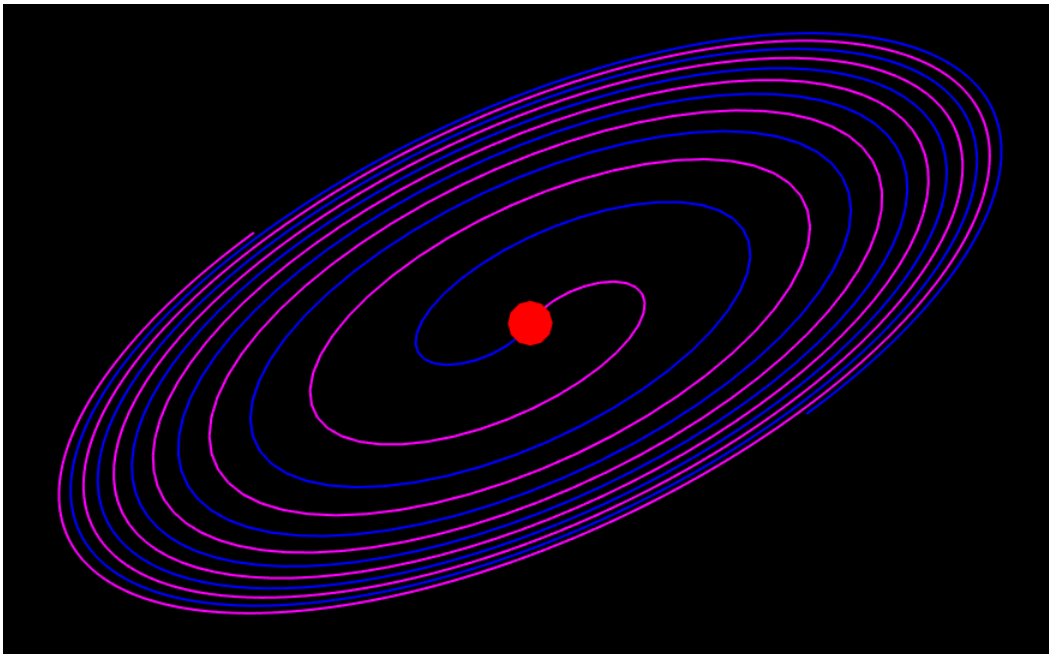

Figure 2: stars merger

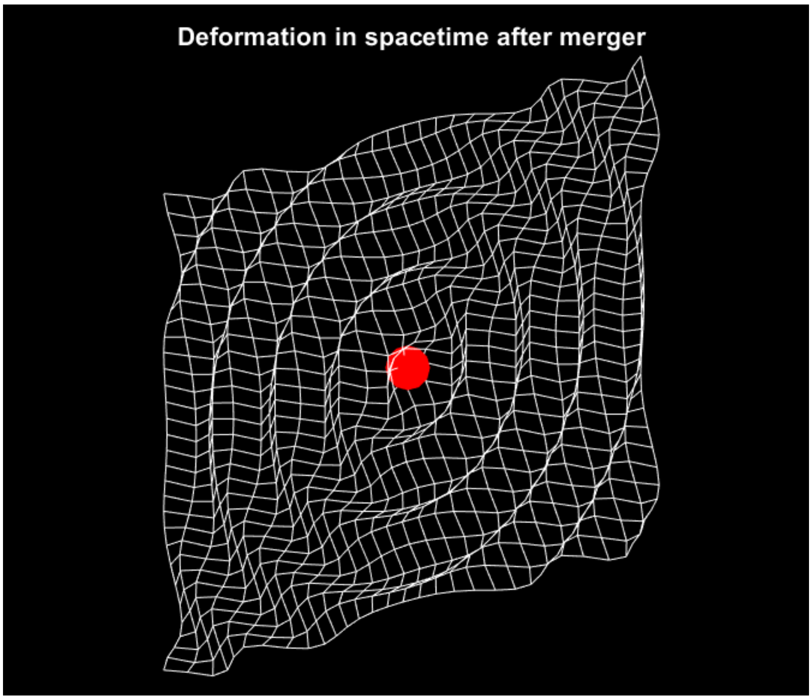

Figure 3: gravitational wave propagating outwards at light speed

so the continuous gravitational waves are said to be produced by a single massive object of mass like a rotating Neutron star. The rate a which the gravitational waves are emitted is proportional to the amount of spin the object of mass has, if it accelerates, then the amount of gravitational waves GWs emitted [produced] does not remain constant.

3. MATHEMATICAL BACKGROUND

Understanding the mathematics behind every system is essential for visualisation, this section introduces the bases on which the modeling is built, and all that will be essential on the simulation of the wave-forms.
Let first introduce the mathematical background of our project, in general relativity the source of gravitational field are matter and energy[3].

The gravitational field is given by a metric tensor $g_{\mu\nu}$, in the absence of matter in the universe, then we would be living in the Minkowski space which is a flat space 4x4 metric in general relativity $\eta_{\mu\nu} = diag(-1, 1, 1, 1)$, where we have (ct, x, y, z) as coordinates of spacetime.

flat spacetime is flat Minkowski space plus a small perturbation "h"

$$g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu} \tag{3.1}$$

so the line element of Minkowski coordinates in flat spacetime, is given by this form

$$ds^2 = \eta_{\mu\nu} dx^\mu dx^\nu \tag{3.2}$$

It has been established by several people [3] that the Fierz-Pauli theory of a massless spin-2 particle in the Minkowski flat space-time is inconsistent when coupled to matter and the only consistent theory in the low frequency domain is Einstein's general relativity.

In the light of this aspect, we use Einstein's general relativity as a correct effective gravitational theory at low energies compared to the Planck scale. Since we are interested mainly in the weak field limit, we perform the weak field expansion to get the linearized solutions. After the expansion, ordinary quantum field theoretical methods are applied to the linearized gravity[2]. After the expansion any curved space geometric object is expressed as an infinite series in terms of $h_{\mu\nu}$[3], for the purpose of this work we will expand up to the terms of order $O(h^3)$, and the expansion will be first done on the contravariant metric tensor as the following

$$g^{\mu\nu} = \eta^{\mu\nu} - h^{\mu\nu} + h^{\mu\lambda}h^{\nu}_{\lambda} - h^{\mu\lambda}h_{\lambda\alpha}h^{\alpha\nu} \tag{3.3}$$

the determinant of $g_{\mu\nu}$ is then

$$g = det(g_{\mu\nu})$$

$$g = -1 - h + \frac{1}{2}(h^{\mu}_{\zeta}h^{\zeta}_{\mu} - h^2) + \frac{1}{6}(-2h^{\mu}_{\zeta}h^{\zeta}_{\gamma}h^{\gamma}_{\mu} + 3hh^{\mu}_{\zeta}h^{\zeta}_{\mu}) \tag{3.4}$$

the determinant of the tensor will be useful on the following gravitational action which is given by the following combination below

$$S = S_E + S_M$$

, where the Einstein action is given by the following

$$S_E = \frac{c^3}{16\pi G}\int d^4x\sqrt{-g}R \tag{3.5}$$

and S_M is the matter action,where $g = det(g_{\mu\nu})$ and R is the Ricci scalar, Ricci Tensor $R_{\mu\nu}$ and Riemann tensor $R_{\mu\nu\beta\rho}$[2] to expand around flat space, the gravitational action[2], and the enegy-momentum tensor of matter $T^{\mu\nu}$ is the change under the metric in Eq. (2.2) so

$$\delta S_M = \frac{1}{2c}\int d^4x\sqrt{-g}T^{\mu\nu}\delta g_{\mu\nu} \tag{3.6}$$

and some also same change in Einstein action, under the $g_{\mu\nu}$ the action principle then tells us that to maintain the physical law, we must have the total variation with respect to the inverse metric must be given by the following

$$\delta S = 0 \tag{3.7}$$

this equation must hold for any variation of the metric $\delta g_{\mu\nu}$, then we get that

$$R_{\mu\nu} - \frac{1}{2}Rg_{\mu\nu} = \frac{8\pi G}{c^4}T_{\mu\nu} \tag{3.8}$$

which is the Einstein's equation, for our purpose we will not include the cosmological constant, this is the equation relates geometry and matter and where $T_{\mu\nu}$ is the energy-momentum tensor for matter

3.1. Properties of Einstein's equation

before we get into finding the solution of the Einstein's equation and model the polarized wave forms we first discuss few properties for this equation under the assumption that there is no matter then

$$T_{\mu\nu} = 0 \tag{3.9}$$

the energy momentum tensor for matter is 0 the the right hand side of the equation is 0 so

$$R_{\mu\nu} = \frac{1}{2} R g_{\mu\nu}$$

applying the inverse metric both sides we have that

$$g^{\mu\nu} R_{\mu\nu} = \frac{1}{2} R g_{\mu\nu} g^{\mu\nu}$$

$\delta^{\mu}_{\mu} = g_{\mu\nu} g^{\mu\nu} = 4$ so then is found that $R = 0$, so as consequence of the equation above we have obtain the Racci flatness

$$R_{\mu\nu} = 0 \tag{3.10}$$

which is true in the absence of matter

3.2. Expansion in flat space-time

the theory which deals with the expansion of Einstein's field equation of motion to linear order in $h_{\mu\nu}$, and by exploiting the coordinate gauge freedom[2]

$$x^{\mu} \to x^{'\mu} = x^{\mu} + \xi^{\mu}(x) \tag{3.11}$$

so from eq. (2.1) and eq. (2.3), to linearize gravity, perturbation theory is applied to the metric tensor that describe space time geometry of eq. (2.1) , which is the metric tensor that represent the solutions of the equation of Einstein's equation.

substituting the general metric for the eq. (2.1) perturbative approximation give the following Ricci tensor[3]

$$R_{\mu\nu} = \frac{1}{2}(\partial_{\alpha}\partial_{\mu}h^{\alpha}_{\nu} + \partial_{\alpha}\partial_{\nu}h^{\alpha}_{\mu} - \partial_{\mu}\partial_{\nu}h - \Box h_{\mu\nu}) \tag{3.12}$$

where

$$\Box = \eta^{\mu\nu}\partial_{\mu}\partial_{\nu} \tag{3.13}$$

is the d'Alembart operator, and

$$h = \eta^{\mu\nu} h_{\mu\nu} \tag{3.14}$$

and is called the trace of perturbation[2], and

$$\overline{h}_{\mu\nu} = h_{\mu\nu} - \frac{1}{2}\eta_{\mu\nu} h \tag{3.15}$$

the Ricci scalar is as follows

$$R = \eta_{\mu\nu} R^{\mu\nu} = \partial_{mu}\partial_{\nu} h^{\mu\nu} - \Box h \tag{3.16}$$

then the left side of Einstein's field equation reduces to be[2]

$$R_{\mu\nu} - \frac{1}{2} R g_{\mu\nu} = \frac{1}{2}(\partial_{\alpha}\partial_{\mu} h^{\alpha}_{\mu} + \partial_{\alpha}\partial_{\nu} h^{\alpha}_{\mu} - \partial_{\mu}\partial_{\nu}\eta^{\mu\nu} h_{\mu\nu} - \Box h_{\mu} - \eta_{\mu\nu}\partial_{\beta}\partial_{\lambda} h^{\beta\lambda} + \eta_{\mu\nu}\Box h) \tag{3.17}$$

thus it is reduced to the linear, second order partial derivative in $h_{\mu\nu}$, using the Lorentz gauge, which a gauge by definition is mathematical device that doesn't change when the underlying coordinate system is shifted by infinitesimal amount,it found that

$$\partial^{\nu} \overline{h}_{\mu\nu} = 0 \tag{3.18}$$

so the Lorentz gauged field equations

$$\Box \overline{h}_{\mu\nu} = -\frac{-16\pi G}{c^4} T_{\mu\nu} \tag{3.19}$$

3.3. Polarized modes

The key predictions of general relativity is that the metric perturbation posses 2-tensor degrees of freedom which form part of six independent modes allowed , while any combinations of these polarization modes make the theory of general relativity, but a simplified investigation of these waves on matter and some and space around it illustrate how powerful they are and the potential they have for new research fields for regions of space unexplored due to the limitations of light to complex parts of space, this can done by considering polarization modes where the states are pure. The Lorentz gauged field equation in the absence of matter reduces to[2]

$$\Box \overline{h}_{\mu\nu} = 0 \tag{3.20}$$

with the plane wave solution

$$h^{TT}_{\mu\nu} = A_{\mu\nu}(K) e^{ikx} \tag{3.21}$$

where $A_{\mu\nu}(k)$ is called the polarized tensor[2], and $k^{\mu} = (\frac{w}{c}, K)$ and the waves travel at the peed of light $k^{\mu} k_{\mu} = 0$ and they are transverse, $A_{\mu\nu} k^{\mu} = 0$, where k^{μ} is the direction of propagation

The transverse-traceless (TT) gauge[2]

$$h^{0\mu} = 0, h^{\mu}_{\mu} = 0, \partial^{\mu} h_{\mu\nu} = 0 \tag{3.22}$$

reduces the field equations' 10 degrees of freedom by 8 into 2 degrees for freedom(polarization)

The perturbation in TT-gauge can be written in terms of the Amplitude ($h_{\times}$ and h_{+})

$$h^{TT}_{\mu\nu} = \begin{pmatrix} \mathrm{h}_{+} & h_{\times} & 0 \\ h_{\times} & \mathrm{h}_{+} & 0 \\ 0 & 0 & 0 \end{pmatrix}_{\mu\nu} \cos w(t - z/c) \tag{3.23}$$

where z is the direction of propagation of the gravitational wave with frequency w and c the speed of light.

3.4. Interation of Gravitational waves with test masses

To detect the propagation of gravitational wave through space, the idea is to place test particle test through space, and with this space-time curvature is detectable through the motion of the test masses that move along the geodesics of curved space-time but whose masses are so small that they produce no significant space-time curvature of their own. The idea of test masses is done to try and give the gravitational wave a physical form since they are very hard to observe in an vacuum with nothing they interact with. Since they curve space itself then even any coordinate system given is also affected by the passage of the gravitational wave. making use of Eq. (2.2), then we can find the geodesic which is the curve that parallel transports it's own tangent vector, and show that this tangenet vecotr is a constant

$$ds^2 = g_{\mu\nu} dx^\mu dx^\nu = -c^2 d\tau^2$$

$$\frac{g_{\mu\nu dx^\mu dx^\nu}}{d\tau^2} = -c^2$$

$$g_{\mu\nu}\frac{dx^\mu dx^\nu}{d\tau^2} = g_{\mu\nu}u^\mu u^\nu = -c^2$$

in a curved space the basis vector change from point to point and the curve followed by the object is given by the coordinate

$$x^\mu = x^\mu(\tau) = (ct(\tau), x(\tau), y(\tau), z(\tau))$$

so parallel transporting the tangent vector and taking the partial derivative of the basis vector, reduces to the geodesic equation

$$\frac{d^2x^\mu}{d\tau^2} + \Gamma^\mu_{\alpha\nu}\frac{dx^\alpha}{d\tau}\frac{dx^\nu}{d\tau} = 0 \qquad (3.24)$$

where the chrostoffel symbols arise because of the variation in the basis vector from point to point are given as by, this equation is the classical equation of motion of a test mass in curved background described by the metric $g_{\mu\nu}$
by applying the covarient derivative D by the proper time of the vector field along the curve $x^\mu(\tau)$ then[2]

$$\frac{D^2x^\mu}{D\tau^2} = -R^\mu_{\nu\rho\alpha}x^\rho\frac{dx^\nu}{d\tau}\frac{dx^\alpha}{d\tau} \qquad (3.25)$$

In the weak field TT gauge and in the proper detector frame, the geodesic equation x^μ is related to the polarization amplitude

$$\ddot{x}^\mu = \frac{1}{2}\ddot{h}^{TT}_{\mu\nu}x$$

(3.26)

and from the line element in Eq.(2.2) which is used to represent flate space, Eq.(2.24) that is used to model the particles that are initially at rest reduces to

$$\left.\frac{dx^\mu}{d\tau}\right|_{\tau=0} = \left.\frac{d^2x^\alpha]}{d\tau^2}\right|_{\tau=0} = 0 \qquad (3.27)$$

becomes a constant path.

using this Eq.(2.26) above to study the effects of gravitational waves, first by Eq. (2.27) a ring of test masses initially at rest[2] is considered and then the effects of gravitational waves on those test masses is observed, coordinate system chosen will be the xy plane and the let the gravitational wave propagate along the z - direction, to study the motion of the test particles under the influence of GW in the xy plane[2], the +polarization is first considered

$$h_{\mu\nu}^{TT} = h_{+} \sin wt \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} \tag{3.28}$$

by integration of this system of equations , the solution

$$\delta x(t) = \frac{h_{+}}{2} x_0 \sin wt \tag{3.29}$$

$$\delta y(t) = \frac{h_{+}}{2} y_0 \sin wt \tag{3.30}$$

are obtained and again for $\times$ polarization

$$\delta x(t) = \frac{h_{\times}}{2} y_0 \sin wt \tag{3.31}$$

$$\delta y(t) = \frac{h_{\times}}{2} x_0 \sin wt \tag{3.32}$$

h_{+}*Polarization*

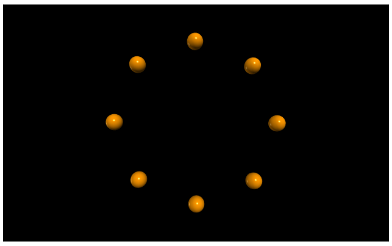

Figure 4: h_{+}

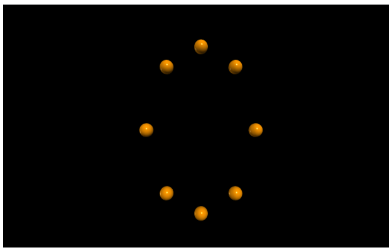

Figure 5: h_{+}

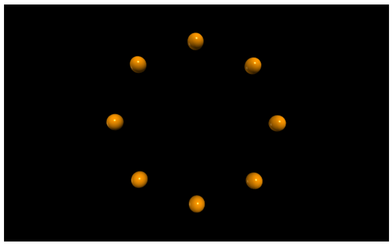

Figure 6: h_{+}

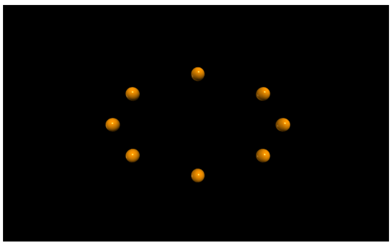

Figure 7: h_{+}

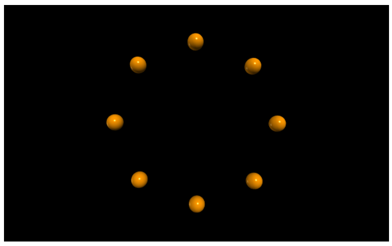

Figure 8: h_{+}

$h_\times$ *Polarization*

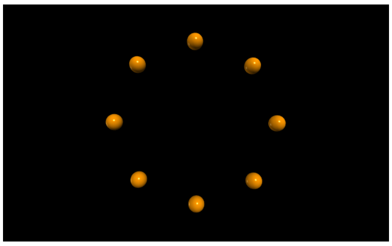

Figure 9: $h_\times$

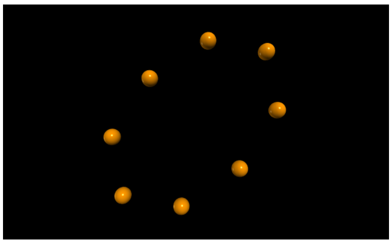

Figure 10: $h_\times$

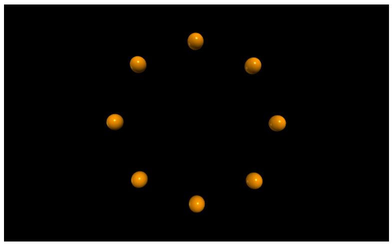

Figure 11: $h_\times$

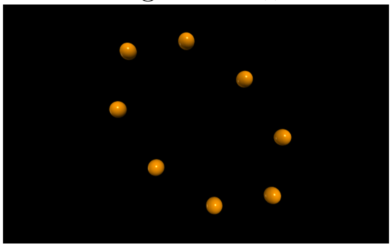

Figure 12: $h_\times$

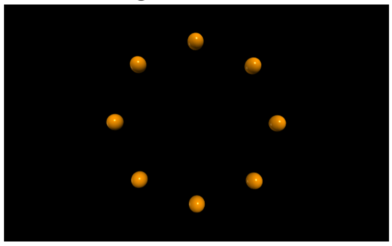

Figure 13: $h_\times$

To determine the effects of gravitational wave on matter, test particles are placed in the xy-plane with an incoming gravitational wave in the z-direction traveling perpendicular to the ring of test masses causing them to contract and expand along the x- and y- axes, the period is equal to that of the gravitational wave. The cross-polarized wave causes the test particles to expand and contract at 45^o to the motion of the plus-polarization. And how much the test particles are displaced from each other determines the amplitude of the gravitational wave, the distance between two test masses $A(x_A, y_A, z_A)$ and $B(x_B, y_B, z_B)$ which imitate weak field approximation, the TT coordinates can then be calculated from the following version of Eq. (4.2)

$$L^2 = g_{\mu\nu}dx^\mu dx^\nu$$

$$= g_{\mu\nu}(x^\mu_B - x^\mu_A)(x^\nu_B - x^\nu_A)$$

$$L^2 = (\delta_{\mu\nu} + h^{TT}_{\mu\nu})x^\mu_B x^\nu_B \tag{3.33}$$

using the spatial unit vector and the initial distance between test masses then the expression becomes

$$L = \sqrt{L_0^2(\delta_{\mu\nu} + h^{TT}_{\mu\nu}n^\mu n^\nu)}$$

$$= L_0\sqrt{1 + h^{TT}_{\mu\nu}n^\mu n^\nu}$$

which is approximately equals to

$$L \approx L_0(1 + \frac{1}{2}h^{TT}_{\mu\nu}n^\mu n^\nu) \tag{3.34}$$

so the change of the gravitational wave

$$\frac{h}{2} = \frac{\delta L}{L_0}$$

where the amplitude of the gravitational wave is given by h. The distance between the test masses thus changes with the time according to the time variation of the wave if the wave has the definite frequency k, and amplitude[8] so then the above expression become

$$\frac{\delta L}{L_0} = \frac{1}{2} a \sin(kt + \delta) \tag{3.35}$$

the fractional change in distance along the x=axis oscillates periodically with half the amplitude of the gravitational wave[8]. To calculate the distance between the test masses it is the following new coordinates(X, Y) are used with z = 0[8]

$$X = (1 - \frac{1}{2} a \sin kt)x, Y = (1 - \frac{1}{2} a \sin kt)y \tag{3.36}$$

the another one of the polarization $[h_\times]$ is the rotation of the above from the angle in the x-axis by the 45^o, as it observed. then the new polarization can be set to the following[8]

$$x = \frac{1}{\sqrt{2}}(x' + y') \tag{3.37}$$

and

$$y = \frac{1}{\sqrt{2}}(x' - y') \tag{3.38}$$

4. ORBITING SYSTEM AND CIRCULAR POLARIZATION

When Objects with mass orbit each other they drag space around them which creates a gravitational wave, so the for this part, two object of masses m_1 and m_2 are the source of gravitational wave GW, they orbit each other in the xy-plane with the origin as the centre of mass of the system, like from the previous section above, this system can represent system like binary systems, binary neutron stars that orbit each other[10]. The effects of the gravitational wave from the orbit would be the same as if the gravitational wave is from a single mass orbiting at radius R so to find the centre of mass of the two objects of mass the following expression is used

$$\nu = \frac{m_1 m_2}{m_1 + m_2} \tag{4.1}$$

and the radius of the orbiting body is given by the expression

$$R = \vec{r_1} - \vec{r_2} \tag{4.2}$$

The results from this case will have a gravitational wave that has both $h_\times$ Polarization and h_+ Polarization at the same time and the gravitational waves GWs angular frequency will be 2 times that of the orbiting system. The circular Polarization is then represented by the figure below

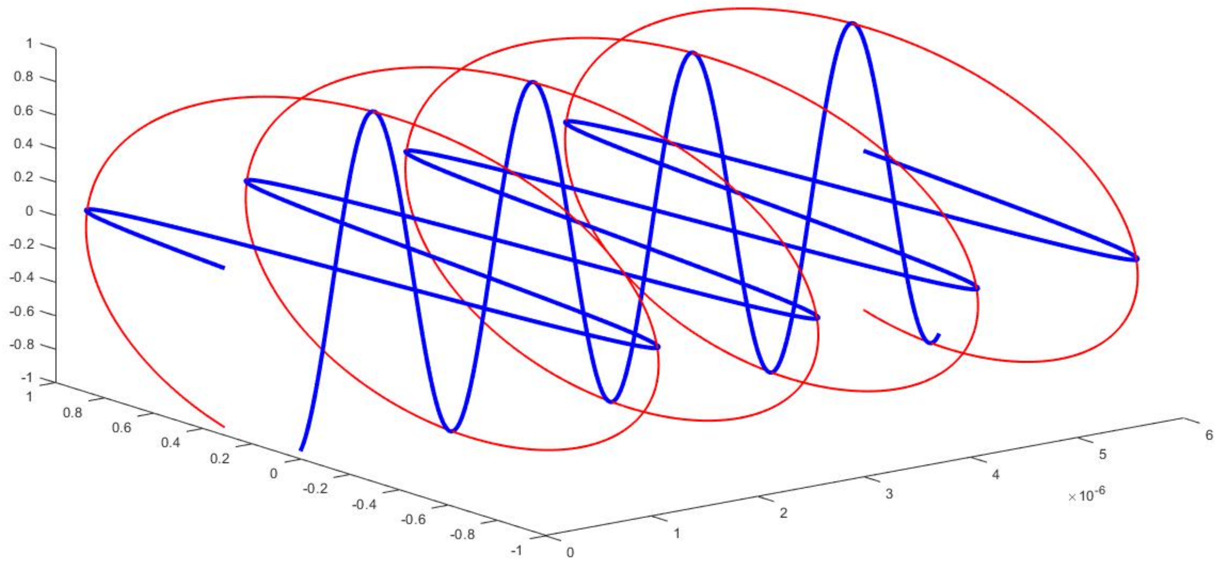

Figure 14: Circular Polarization

5. GRAVITATIONAL WAVE

Since gravitational waves, have physical effects on objects with masses, then it must mean that the gravitational waves carry energy[9]. Binary stars also have Gravitational waves carry energy away from the sources, as the binary stars orbit and get closer to each other they lose energy and angular momentum. Gravitational GWs on the above test ring of particles stretches and squeezes them in x and y direction given by Eq.(3.29) and Eq.(3.30). Since the gravitational wave carry energy away from the source then the energy absorbed by the test particles is

$$E_k = \frac{m}{2}(\delta\dot{x}^2 + \delta\dot{x}^2) \tag{5.1}$$

though it is true that gravitational waves carry energy away from the source however it cannot be localized and much like the electromagnetic wave, gravitational waves GWs energy is conserved is conserved and their energy curve space-time and they also be absorbed by detectors.

6. DISCUSSION AND CONCLUSION

This simulation from these codes was to model the existence of gravitational waves GW, two programming languages are for these codes (python and Matlab) due to the fact that thy provide powerful and simple functionalities depending on the type of simulation. The effects of gravitational wave of the ring of test particle is done on python, see **Appendix A** for the code, and the simulation for Neutron star merger is done on MATLAB, see **Appendix B** for this code[10]. On the table below are the parameters and their values respectively used on the ring of particle simulations

Wave forms	Parameters	Input values
$h_\times$	k, x0, y0, m, dt	2.0 Hz, 1.0 m, 1.0 m, 1.0 Kg, 0.01
h_+	k, x0, y0, m, dt	2.0 Hz, 1.0 m, 1.0 m, 1.0 Kg, 0.01

This code simulates two wave forms h_+ and $h_\times$, so when running the code, the type of wave form you want simulated has to be selected by inputting + or ×. For the MATLAB code on Appendix B which simulates the merger of two Neutron stars from the in spiral, merger and ring-down.the disturbance in space-time has a Amplitude of 12 Hz,The simulation first start with two Neutron stars orbiting each other and as they get closer and closer to each other thier relative speed increases until the two star merge and create giant star which is the size of the two star added together. Then the simulation jumps to another frame where the view of disturbed space-time is shown.

7. ACKNOWLEDGMENTS

I am grateful to Professor Amanda Weltman who mentioned this project to me on one of the meetings, and i am also grateful to Irvin Martinez for his suppose and contribution on ideas for Mathematical background.

A. APPENDIX: GRAVITATIONALWAVE.PY

```
from vpython import*
import numpy as np
from math import sin, cos
#from matlabplo

#_____________________________________________________
# constants
y0 = 1.0
x0 = 1.0
Polarization = "+"
k  = 2.0
M = 1.0 # mass of the test particles in the xy plane
Cords = [1 , -1 , 0,
       1/np.sqrt(2), -1/np.sqrt(2)]

#_______________________________________________________
```

```python
# h_{+} Polarization
def Model_Plus():
    #_____________________________________________________
    # test mass [particles] that
    #will be affected by gravitational wave
    testMass1 = sphere(radius = 0.1, color = color.orange, pos =
        vector(Cords[0],Cords[2],Cords[2]))
    testMass2 = sphere(radius = 0.1, color = color.orange, pos =
        vector(Cords[1],Cords[2],Cords[2]))
    testMass3 = sphere(radius = 0.1, color = color.orange, pos =
        vector(Cords[2],Cords[0],Cords[2]))
    testMass4 = sphere(radius = 0.1, color = color.orange, pos =
        vector(Cords[2],Cords[1],Cords[2]))
    testMass5 = sphere(radius = 0.1, color = color.orange, pos =
        vector(Cords[3],Cords[3],Cords[2]))
    testMass6 = sphere(radius = 0.1, color = color.orange, pos =
        vector(Cords[3],Cords[4],Cords[2]))
    testMass7 = sphere(radius = 0.1, color = color.orange, pos =
        vector(Cords[4],Cords[4],Cords[2]))
    testMass8 = sphere(radius = 0.1, color = color.orange, pos =
        vector(Cords[4],Cords[3],Cords[2]))
    #_____________________________________________________
    # motion of the test mass as the wave pass by
    xyLine = [testMass1,testMass2,testMass3,testMass4]
    xyQuad = [testMass5,testMass6,testMass7,testMass8]
    dt = 0.01
    t = 0
    while (t < 100):
        rate(100)
        dx = 1-(1/2)*x0*sin(k*t)
        dy = 1+(1/2)*y0*sin(k*t)
        dx1 = 1/np.sqrt(2)-(1/2)*x0*sin(k*t)
        dy1 = 1/np.sqrt(2)+(1/2)*y0*sin(k*t)
        xyLine[0].pos.x = dx
        xyQuad[0].pos.x = dx1
        xyQuad[0].pos.y =dy1
        xyQuad[1].pos.x = -dx1
        xyQuad[1].pos.y = dy1
        xyLine[1].pos.x =-dx
        xyQuad[2].pos.x =-dx1
        xyQuad[2].pos.y =-dy1
        xyQuad[3].pos.x = dx1
        xyQuad[3].pos.y = -dy1
        xyLine[2].pos.y = dy
        xyLine[3].pos.y =-dy
        t+=dt
```

```python
#________________________________________________________
        # h_{x} Polarization
def Model_Cross():
    #____________________________________________________
    # test mass [particles] that
    #will be affected by gravitational wave
    testMass1 = sphere(radius = 0.1, color = color.orange, pos =
        vector(Cords[0],Cords[2],Cords[2]))
    testMass2 = sphere(radius = 0.1, color = color.orange, pos =
        vector(Cords[1],Cords[2],Cords[2]))
    testMass3 = sphere(radius = 0.1, color = color.orange, pos =
        vector(Cords[2],Cords[0],Cords[2]))
    testMass4 = sphere(radius = 0.1, color = color.orange, pos =
        vector(Cords[2],Cords[1],Cords[2]))
    testMass5 = sphere(radius = 0.1, color = color.orange, pos =
        vector(Cords[3],Cords[3],Cords[2]))
    testMass6 = sphere(radius = 0.1, color = color.orange, pos =
        vector(Cords[3],Cords[4],Cords[2]))
    testMass7 = sphere(radius = 0.1, color = color.orange, pos =
        vector(Cords[4],Cords[4],Cords[2]))
    testMass8 = sphere(radius = 0.1, color = color.orange, pos =
        vector(Cords[4],Cords[3],Cords[2]))
    #____________________________________________________
    # motion of the test mass as the wave pass by
    xyLine = [testMass1,testMass2,testMass3,testMass4]
    xyQuad = [testMass5,testMass6,testMass7,testMass8]
    dt = 0.01
    t = 0
    while (t < 100):
        rate(100)
        dx =(1/2)*x0*sin((1/4)*np.pi*k*t);
        dy =(1/2)*y0*sin((1/4)*np.pi*k*t)                           # the wave
            is passing to xy-plane and the z-direction
        dx1 = 1/np.sqrt(2)-(1/2)*x0*sin((1/4)*np.pi*k*t);
        dy1 = 1/np.sqrt(2)-(1/2)*y0*sin((1/4)*np.pi*k*t)
        dx2 = 1/np.sqrt(2)+(1/2)*x0*sin((1/4)*np.pi*k*t);
        dy2 = 1/np.sqrt(2)+(1/2)*y0*sin((1/4)*np.pi*k*t)
        xyLine[0].pos.x = dx; xyLine[0].pos.y = 1
        xyQuad[0].pos.x = -dx1;
        xyQuad[0].pos.y =dy1
        xyQuad[1].pos.x = dx2;
        xyQuad[1].pos.y = dy2
        xyLine[1].pos.x = -dx;
        xyLine[1].pos.y = -1
        xyQuad[2].pos.x =-dx2;
        xyQuad[2].pos.y =-dy2
        xyQuad[3].pos.x = dx1;
```

```
        xyQuad[3].pos.y = -dy1
        xyLine[2].pos.y = -dy;
        xyLine[2].pos.x = -1
        xyLine[3].pos.y = dy; xyLine[3].pos.x =1
        t+=dt
#____________________________________________________________
        # function calls for the Polarizations
choice = input("Enter your Polarization choice [h_{+} or h_{x}] as + or x:
    \n")
if (choice==Polarization):
    print(Model_Plus())
else:
    print(Model_Cross())
```

B. APPENDIX: GRAVITATIONAL_EFFECT_ANIMATION.M

```
function [] = Gravitational_effect_animation()
% Display parameters
lapse = 0.001;
%sample_step = 12;
Psi = zeros(31,31);
dt = 1;
h = figure;
set(h,'Position',get(0,'screensize'));
set(gcf,'Color',[0 0 0]);
phase_dif = 2*pi:- pi/12: pi/12;
Amplitude = 12;
%___________________________________________________________________________
Theta = logspace(0,log10(12*pi),865);
Radius = 12*pi+1-Theta;
U = [log(Radius).*cos(Theta);
    log(Radius).*sin(Theta)];
% Rotation matrix x axis
Rm = @(Theta)[cos(Theta),-sin(Theta);
             sin(Theta),cos(Theta)];
V = Rm(pi)*U;
[Y,Z] = meshgrid(1:31,1:31);
% Display settings
for s = 1:12:865
    hold on;
    plot(U(1,s),U(2,s),'o','Color',[1 0 1],'linewidth',13);
    plot(V(1,s),V(2,s),'o','Color',[0 0 1],'linewidth',13);
    line(U(1,1:s),U(2,1:s),'Color',[1 0 1],'linewidth',1);
    line(V(1,1:s),V(2,1:s),'Color',[0 0 1],'linewidth',1);
    ax = gca; ax.Clipping = 'off';axis off;
    title('Binary system','fontsize',16,'Color',[1 1 1]);
```

```
    view(3);camroll(70);zoom(1.8);
    dt =dt+ 1;
    drawnow;
    frame = getframe(h);
    im = frame2im(frame);
    [D,cm] = rgb2ind(im,256);
    % Write to the .gif file
    if s == 1
        imwrite(D,cm,'Binary_system','gif', 'loopcount',Inf,'delaytime',lapse);
    else
        imwrite(D,cm,'Binary_system','gif','writemode','append','delaytime',lapse);
    end
    clf;
end
%merged system
hold on;
ax = gca;
ax.Clipping = 'off';
line(U(1,1:length(Theta)),U(2,1:length(Theta)),'Color',[1 0 1],'linewidth',1);
line(V(1,1:length(Theta)),V(2,1:length(Theta)),'Color',[0 0 1],'linewidth',1);
plot(U(1,s),U(2,s),'o','Color',[1 0 0],'linewidth',26);axis off;
title('Binary system','fontsize',16,'Color',[1 1 1]);
view(3);camroll(50);
[D,cm] = rgb2ind(frame2im(getframe(h)),256);
imwrite(D,cm,'Binary_system','gif','writemode','append','delaytime',2*lapse);
for k = 1:length(phase_dif)
    for n = -floor(0.5*31):floor(0.5*31)
        u = n + ceil(0.5*31);
        for p = -floor(0.5*31):floor(0.5*31)
            v = p + ceil(0.5*31);
            if strcmp('+','x')
                Theta = angle(p+n*1i) +0.25*pi;
            else
                Theta = angle(p+n*1i);
            end
            Psi(u,v) =sin(norm([norm([n;p])*cos(Theta),
            norm([n;p])*sin(Theta)])+phase_dif(1,length(phase_dif)-k+1));
        end
    end
    mesh(Psi,Y-16,Z-16,Psi), hold on;colormap([1 1 1]);alpha 0;
    %Binary system
    for s = 1:12:865
        plot3(-0.4,0,0,'o','Color',[1 0 0],'linewidth',22), hold on;
    end
    % Features & other settings
    ax = gca;
    ax.Clipping = 'off';
```

```
    set(ax,'Color',[0 0 0]);
    axis equal;
    title('Deformation in spacetime after merger','FontSize',16,'Color',[1 1
        1]), hold on;
    view([60,30]); camdolly(0, 0.05, 0); zoom(1.0);
    drawnow;
    [D,cm] = rgb2ind(frame2im( getframe(h)),256);
    % Write to the .gif file
    if k == 1
        imwrite(D,cm,'Gravitational_effect_animation','gif',
            'loopcount',Inf,'delaytime',lapse);
    else
        imwrite(D,cm,'Gravitational_effect_animation','gif',
        'writemode','append','delaytime',lapse);
    end
    clf;
end
end
```

References

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[10] Cf., for instance, Hobson et al. (2006), Ch. 17, or Misner et al. (1973), Ch. 35 for introductions