COMP47750/COMP47990
Machine Learning with Python

Dimension Reduction

Part I
Feature Selection

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Based on slides by Derek Greene

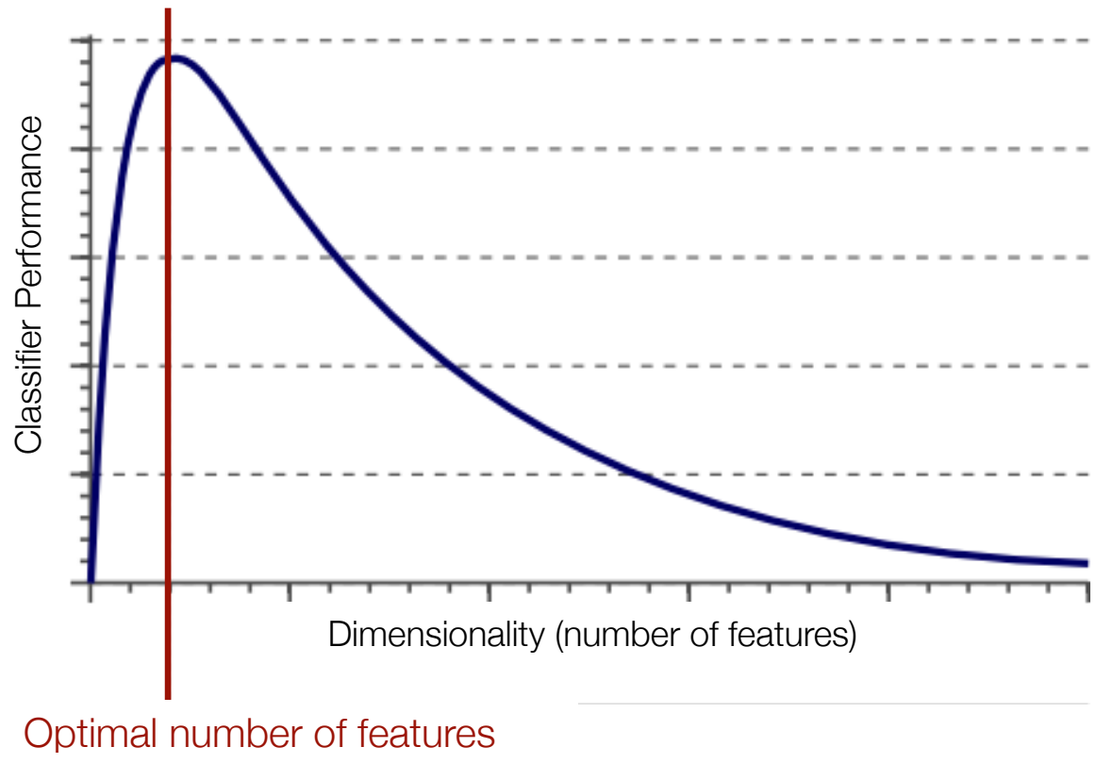
School of Computer Science

Overview

- The Curse of Dimensionality
- Dimension Reduction
- Feature Transformation v Selection
- Feature Selection in Supervised Learning
  - Filter Methods
  - Wrapper Methods
  - Permutation Importance
- Feature Transformation
  - Linear Transformations
  - Projection Methods
  - Principal Component Analysis (PCA)
  - PCA in scikit-learn

Curse of Dimensionality

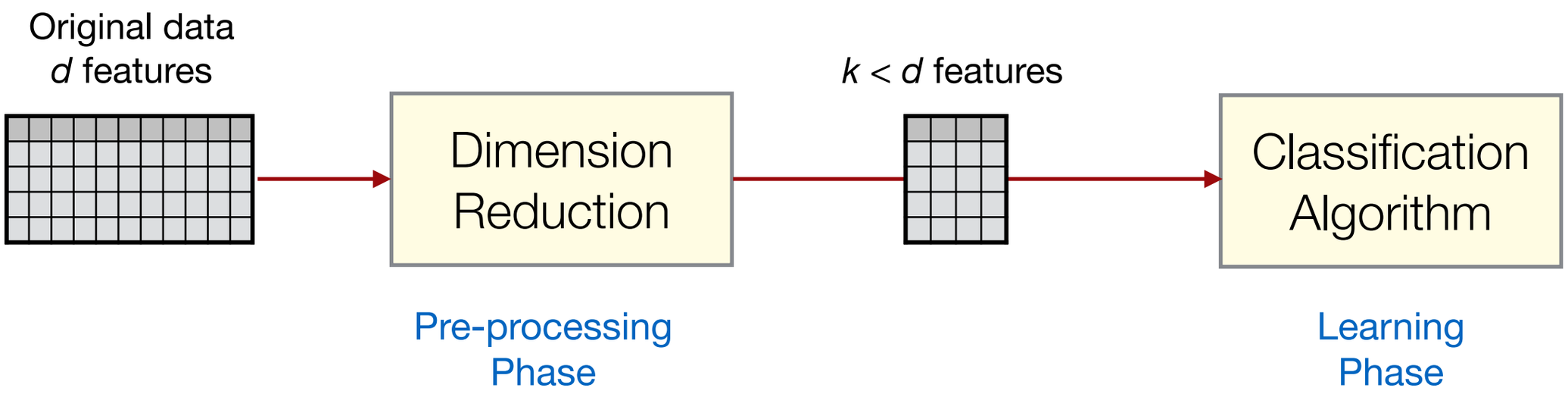
- Intuitively, adding more features (dimensions) to a dataset should provide more information about each example, making prediction easier.
- In reality, we often reach a point where adding more features no longer helps, or can even reduce predictive power.



- Curse of dimensionality: the phenomenon whereby many machine learning algorithms can perform poorly on high-dimensional data (Bellman, 1961).
- In theory, to build a good model, the number of examples required per feature increases exponentially with number of features.

Dimension Reduction

- Basic idea to try to beat the curse of dimensionality:
  1. Apply pre-processing techniques to reduce the number of features used to represent a dataset.
  2. Then build a model on the smaller feature set.
- In some (but not all) cases, the additional information that is lost by removing some features is (more than) compensated by higher classifier accuracy in the lower dimensional space.



Feature Transformation v Selection

There are two general strategies for dimension reduction:

1. Feature Transformation (Feature Extraction)

   Transforms the original features of a dataset to a completely new, smaller, more compact feature set, while retaining as much information as possible.
   e.g. Principal Components Analysis (PCA), Linear Discriminant Analysis (LDA)

2. Feature Selection

   Tries to find a minimum subset of the original features that optimises one or more criteria, rather than producing an entirely new set of dimensions for the data.

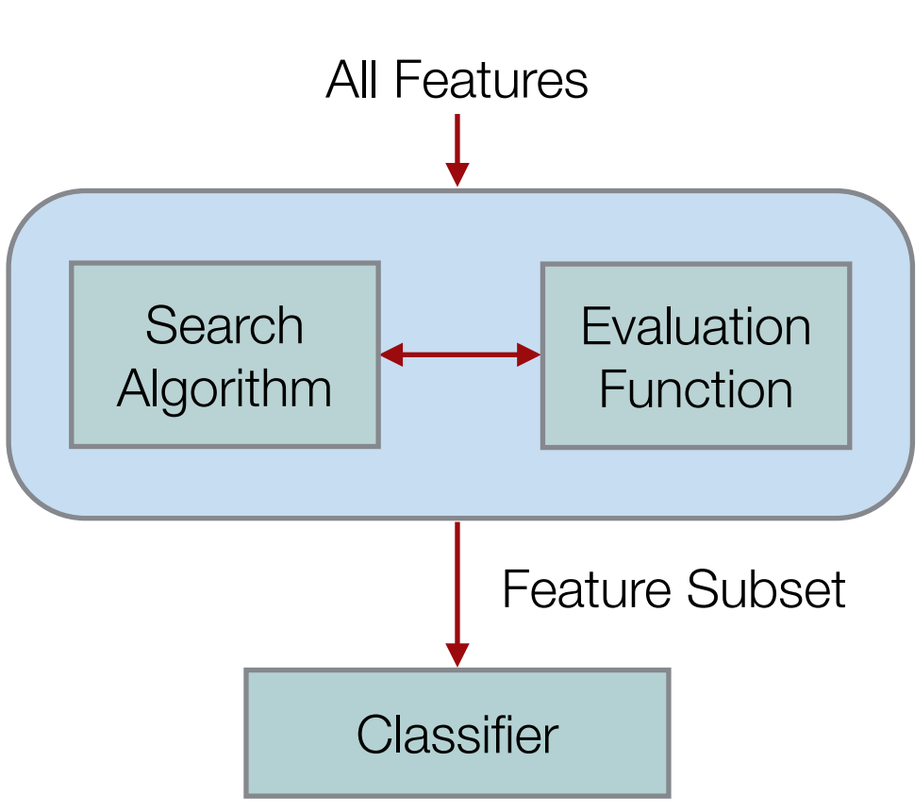
   e.g. Information Gain filter, Wrapper with sequential forward selection

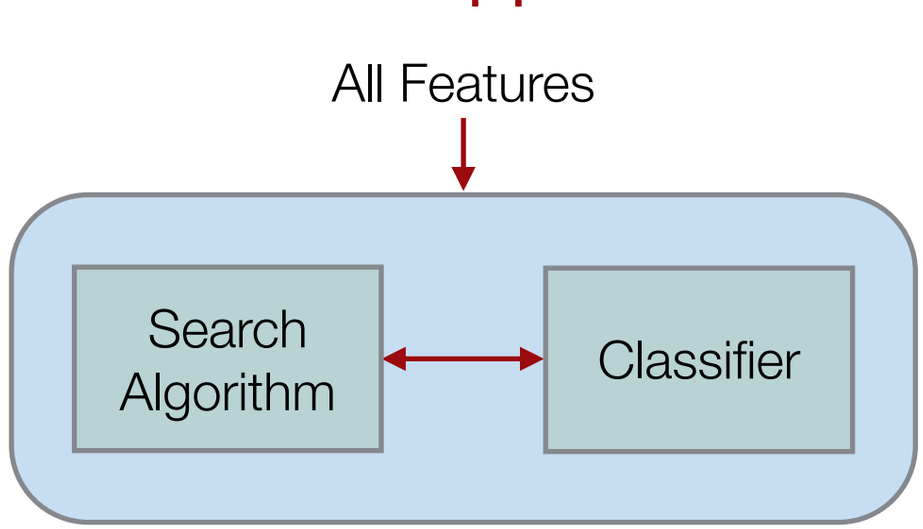
Feature Selection

- Feature Subset Selection: Find the best subset of all available features, which contains the smallest number of features that most contribute to accuracy. Discard the remaining, unimportant features.
- **Why select subset of original features?**
  1. Building a better classifier - Redundant or noisy features can damage accuracy.
  2. Knowledge discovery - Identifying useful features helps us learn more about the data.
  3. Features expensive to obtain - Test a large number of features, select a few for the final system (e.g. sensors, manufacturing).
  4. Interpretability - Selected features still have meaning. We can extract meaningful rules from the classifier.

Feature Selection Strategies

- Finding the optimal feature subset for a given dataset is difficult.
- Brute force evaluation of all feature subsets involves $\binom{d}{k}$ combinations if k is fixed, or 2^d combinations if not fixed.
- Two broad strategies for feature selection:





Filters

- Pre-processing step that ranks and “filters” features independently of the choice of classifier that will be subsequently applied.
- **Evaluation function:** How does a filter algorithm score different feature subsets to produce an overall ranking?
- Generally score the predictiveness of the features.
  - Information theoretic analysis

    e.g. Information Gain, Breiman’s Gini index
  - Statistical tests

    e.g. Chi-square statistic
  - Relief algorithm

    Filter for binary classification, based on the nearest-neighbour classification algorithm (Kira & Rendell, 1992).

Information Gain Filter

- Given a set of training examples S, where p is the proportion of positive examples, q is the proportion of negative examples.

Entropy $$H(S) = -p\log_2(p) - q\log_2(q)$$

- A feature f that is predictive of a class will give significant Information Gain (i.e. a reduction in uncertainty):

$$IG(S, f) = H(S) - \sum_{v \in values(f)} \frac{|S_v|}{S} H(S_v)$$

- **IG Filter approach**:
  1. Score all features based on their Information Gain (IG).
  2. Rank features based on their scores.
  3. Select a subset of the top ranked features to use for classification.

Information Gain Filter in scikit learn

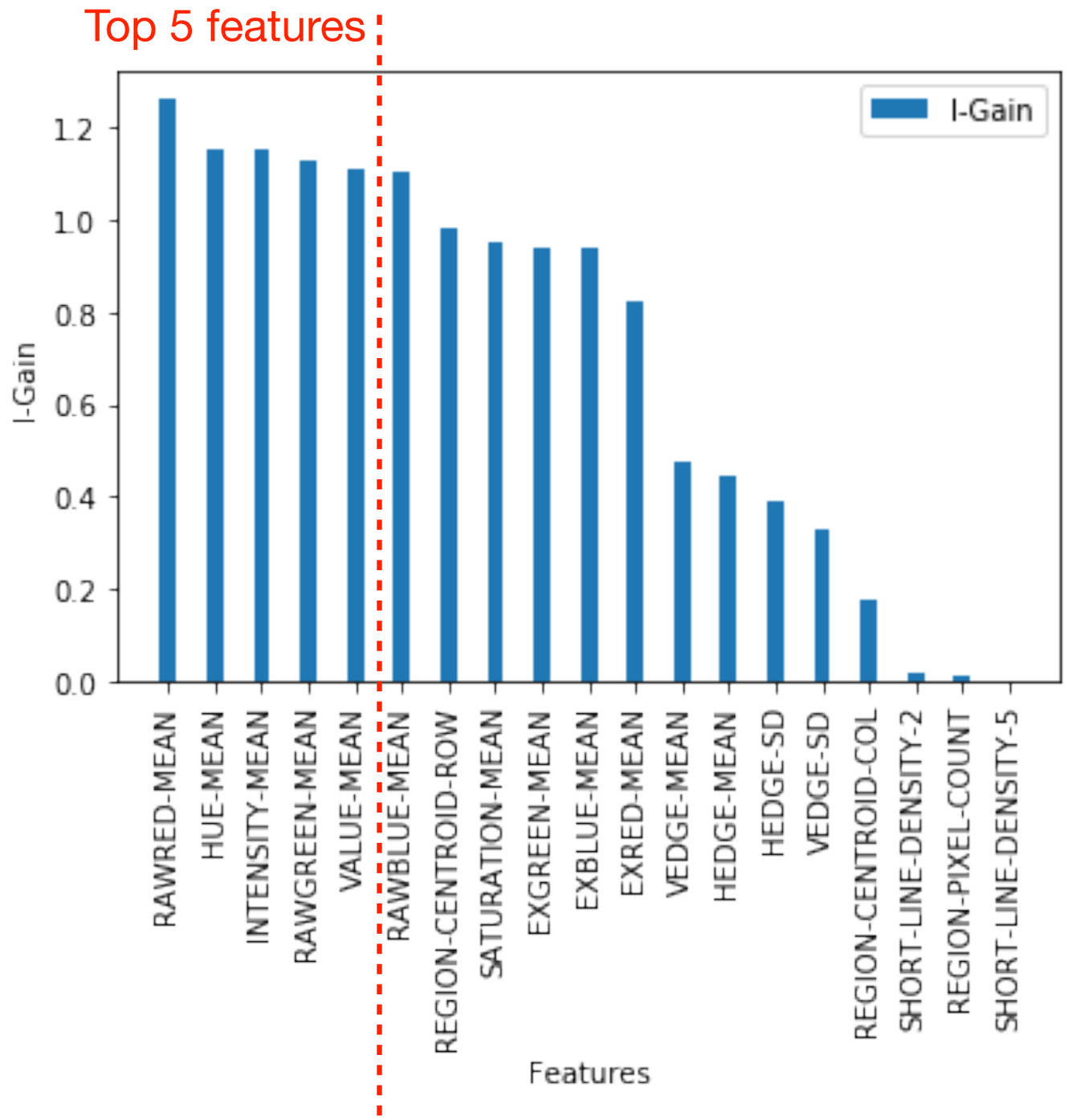
- Score each feature on information gain
- Sort data frame on this column

10 Feature Selection Notebook

```
from sklearn.feature_selection import SelectKBest, mutual_info_classif

mi = dict()

X_train, X_test, y_train, y_test = train_test_split(X, y,
                                                    random_state=2,
                                                    test_size=1/2)
i_scores = mutual_info_classif(X_train, y_train)

for i,j in zip(seg_data.columns,i_scores):
    mi[i]=j

df = pd.DataFrame.from_dict(mi,orient='index',columns=['I-Gain'])
df.sort_values(by=['I-Gain'],ascending=False,inplace=True)
df.head(10)
```

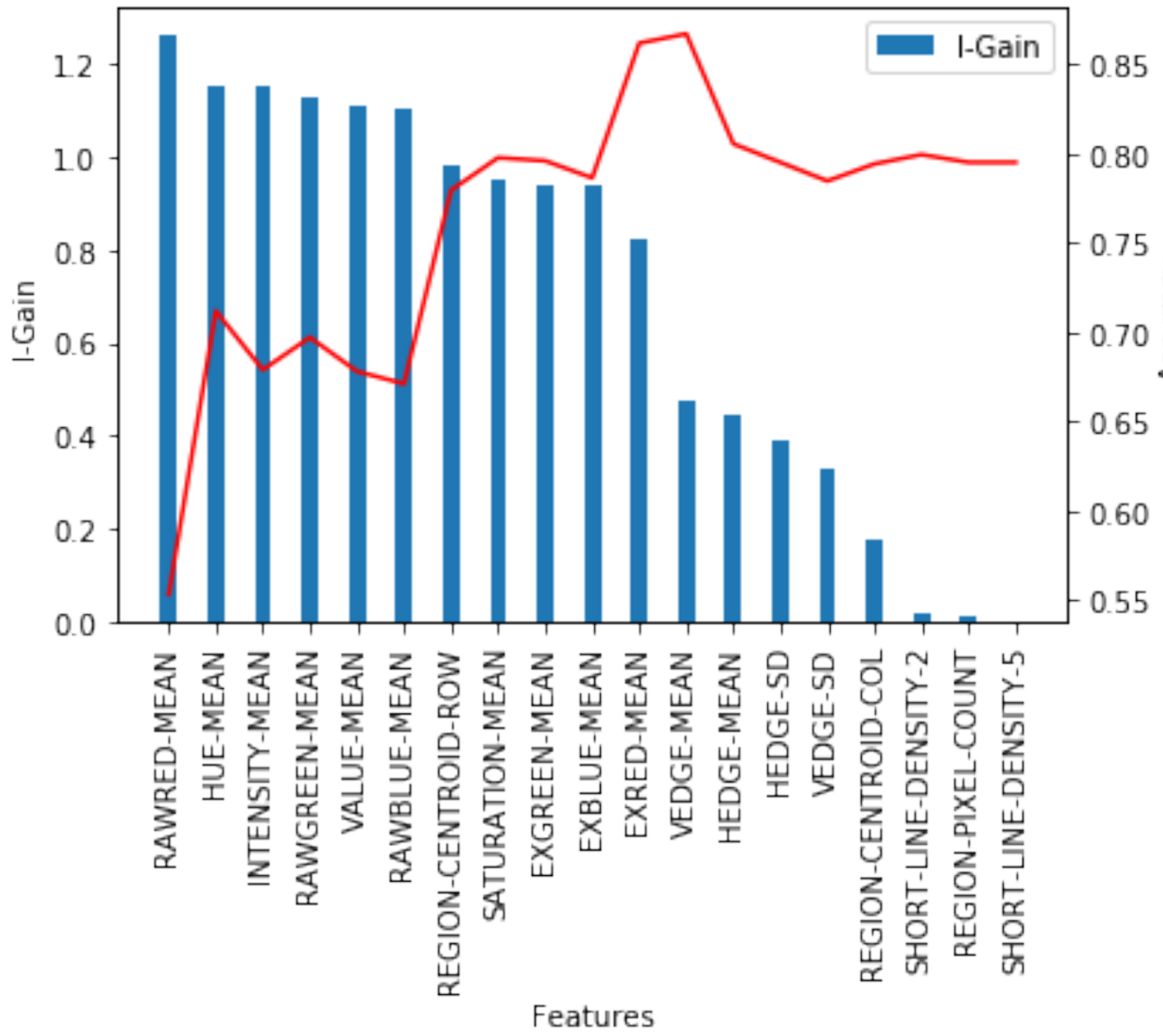
Filters - Top Features

- What to do with ranked list of features? Several basic options...
  - Select the top ranked k features.
  - Select top 50%.
  - Select features with IG > 50% of max IG score.
  - Subset of features with non-zero IG scores.



Filters - Top Features

- Better strategy for selecting k top features: evaluate classification performance using feature subsets of increasing size.
- Start with feature with highest Information Gain, then add the next feature. Measure the accuracy for each subset using cross-validation. Choose the final subset giving the highest accuracy.



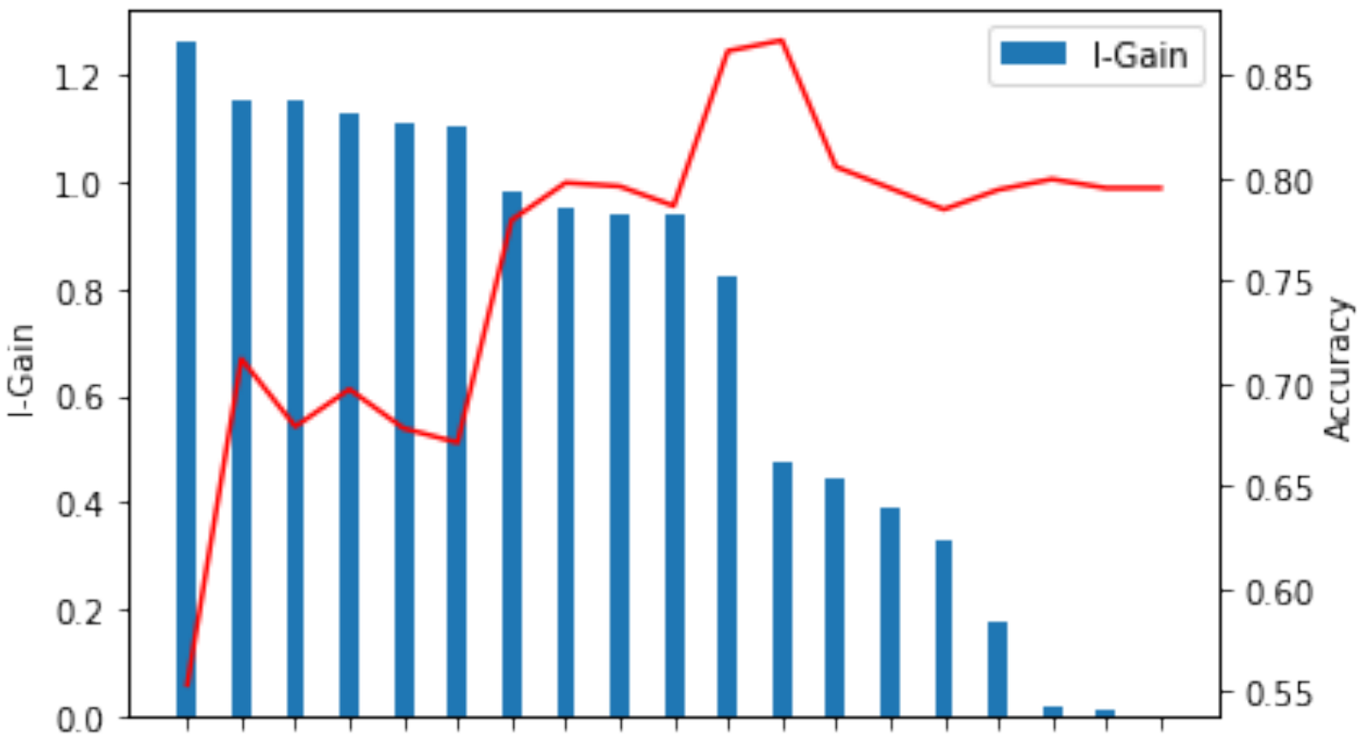
Filter Feature Selection in scikit learn

Set up data transformer

Transform train & test data

Test feature subset

```
acc_scores = []
for kk in range(1, X.shape[1]+1):
    FS_trans = SelectKBest(mutual_info_classif,
                           k=kk).fit(X_train, y_train)
    X_tR_new = FS_trans.transform(X_train)
    X_tS_new = FS_trans.transform(X_test)
    seg_NB = mnb.fit(X_tR_new, y_train)
    y_dash = seg_NB.predict(X_tS_new)
    acc = accuracy_score(y_test, y_dash)
    acc_scores.append(acc)

df['Accuracy'] = acc_scores
df.head(10)
```



	I-Gain	Accuracy
RAWRED-MEAN	1.259301	0.553247
HUE-MEAN	1.152770	0.711688
INTENSITY-MEAN	1.152625	0.678788
RAWGREEN-MEAN	1.126327	0.696970
VALUE-MEAN	1.107112	0.677922
RAWBLUE-MEAN	1.099933	0.670996
REGION-CENTROID-ROW	0.982181	0.779221
SATURATION-MEAN	0.952746	0.797403
EXGREEN-MEAN	0.938691	0.795671
EXBLUE-MEAN	0.938304	0.786147

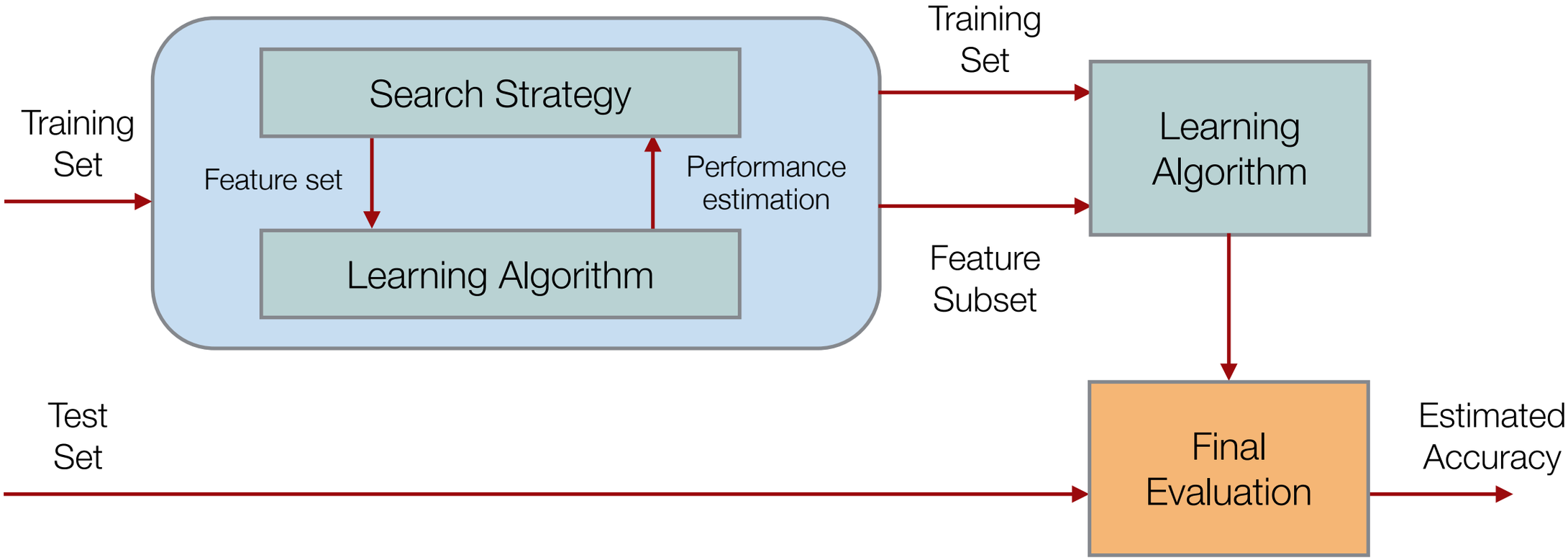
Filters - Disadvantages

Key problems with filter feature selection:

1. **No Model Bias**
    - Different features may suit different learning algorithms (Neural networks, Decision Trees, K-NN, etc.), but filters do not take this into account.
2. **No Feature Dependencies**
    - In filters, the features are considered in isolation from one another, and are not considered in context.
    - In some cases, a filter might select two predictive but correlated features, where one would be sufficient.
    - In other cases, one feature needs another feature to boost accuracy, but a filter cannot discover this.

Wrappers

- Alternative strategy: the classifier is “wrapped” in the feature selection mechanism. Feature subsets are evaluated directly based on their performance when used with that specific classifier.
- Key advantages:
  1. Takes bias of specific learning algorithm into account.
  2. Considers features in context - i.e. feature dependencies.



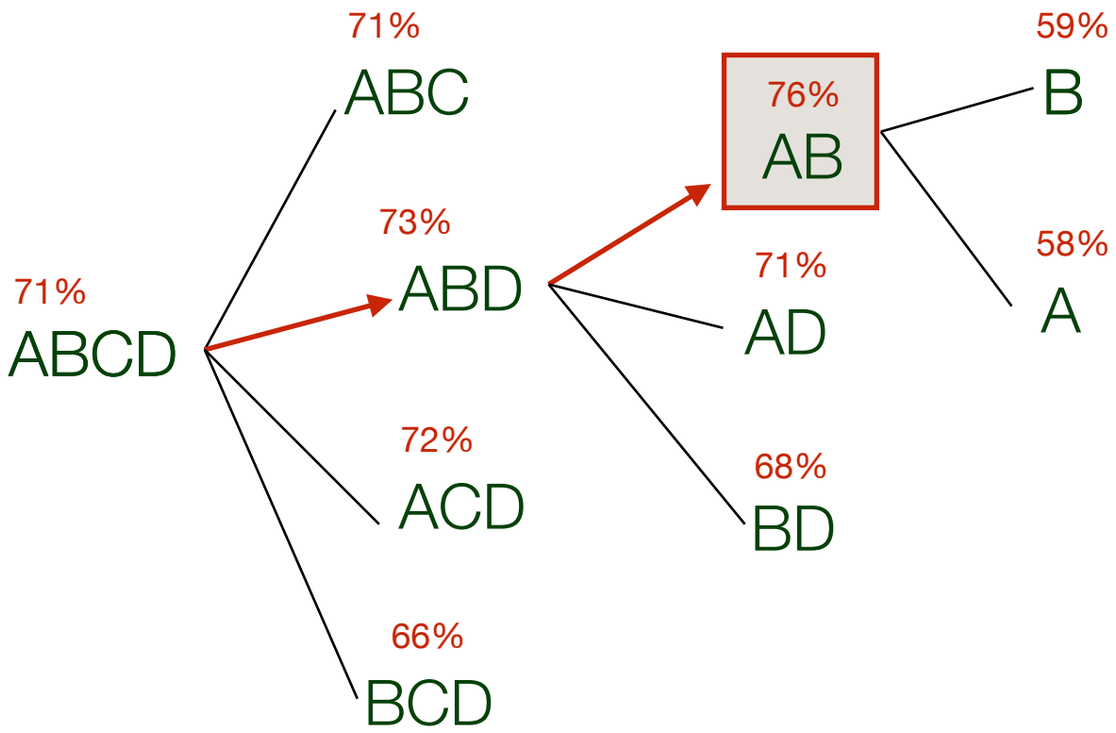
Feature Subset Search

- A key aspect of wrappers is the search strategy which generates the candidate feature subsets for evaluation.
- There are many different ways to search the space of all possible subsets. The choice of a suitable search strategy often depends on the number of features *d* in the data. Several general approaches:
  - Exhaustive search: Evaluate every possible subset of features. For *d* features there are 2^d potential subsets. For even small values of *d*, an exhaustive search over this huge space is intractable.
  - Exponential search: Returns (close to) optimal feature subset, but uses heuristics so that not every one of the 2^d possible subsets needs to be evaluated.
  - Sequential search: Fast search algorithms that choose a subset by adding or removing one feature at a time. Not guaranteed to final the optimal feature subset, but widely used.

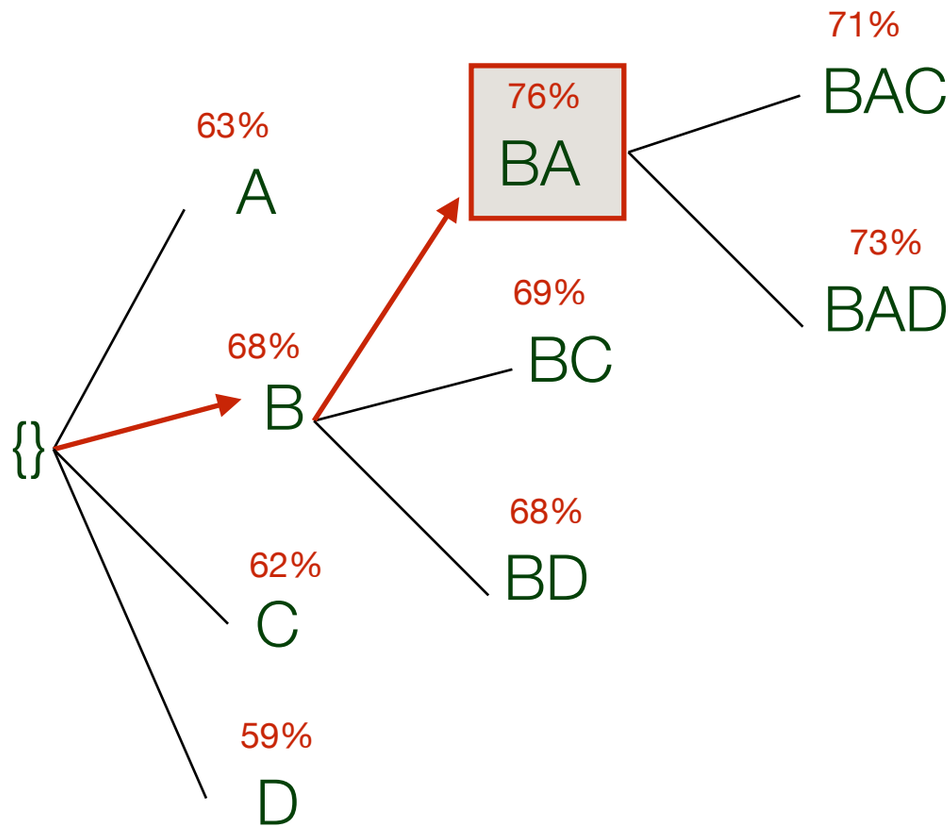
Sequential Search

- In sequential search, performance estimation guides the subset search process (e.g. overall classification accuracy).
- The most basic sequential search methods use a heuristic stepwise approach, either:
  - Forward Sequential Selection
    - Start with an empty subset.
    - Find the most informative feature and add it to the subset.
    - Repeat until there is no improvement by adding features.
  - Backward Elimination
    - Start with the complete set of features.
    - Remove the least informative feature.
    - Repeat until there is no improvement by dropping features.

Sequential Search



Example: Backward elimination from 4 features (ABCD) to 2 features (AB).



Example: Forward selection from no features to 2 features (BA).

- Backward elimination tends to find better models, can find subsets with interacting features. But tends to be slower.
- Forward selection starts with small subsets, so requires less running time if stopped early.

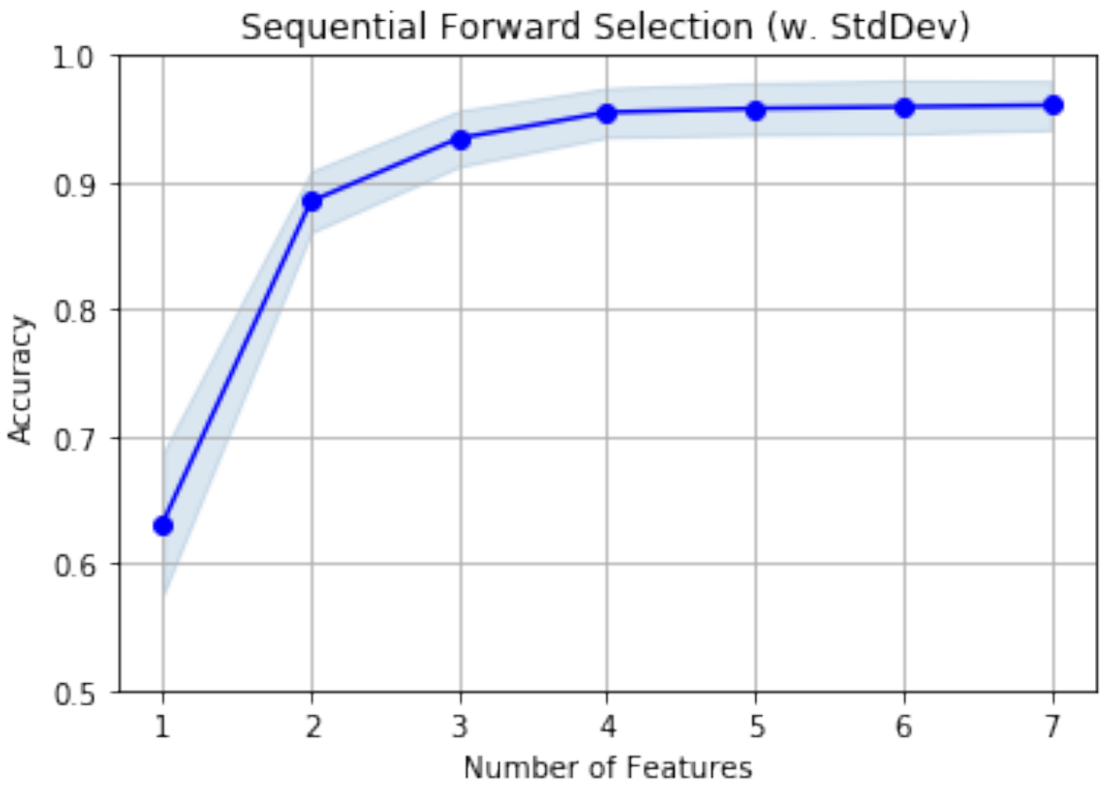
Wrapper-base Feature Selection in Python

- Support Wrapper FSS in scikit-learn is limited
- mlxtend has a slightly better implementation:
  - http://rasbt.github.io/mlxtend/USER_GUIDE_INDEX/
  - http://rasbt.github.io/mlxtend/user_guide/feature_selection/SequentialFeatureSelector/

Wrapper Feature Selection with MLxtend

- Forward Sequential Search

```
from sklearn.neighbors import KNeighborsClassifier
from mlxtend.feature_selection import SequentialFeatureSelector as SFS
sfs_forward = SFS(knn,
                  k_features=7,
                  forward=True, floating=False,
                  verbose=1, scoring='accuracy',
                  cv=10, n_jobs = -1)

sfs_forward = sfs_forward.fit(X, y)
```




```
'REGION-CENTROID-ROW',
‘VEDGE-MEAN',
'HEDGE-MEAN',
'RAWRED-MEAN',
'EXRED-MEAN',
'EXGREEN-MEAN',
'SATURATION-MEAN'
```

Wrappers - Disadvantages

There are two main disadvantages with wrapper methods:

1. Computational Cost

- Significant running time, particularly when the number of features is large. Far slower than filters.
- Computational bottleneck is the requirement to retrain the model many different times on different feature subsets.

2. Overfitting

- Risk of overfitting, particularly when the number of training examples is insufficient.
- We can end up finding the best feature subset for the training data, which does not work well for new unseen data.

Permutation Importance

Idea! If you want to find out how important something is in a process, break (or disable) it to see what happens.

- Look at the impact of randomly permuting values of that variable in test cases and reclassifying these permuted cases.
- If the error increases significantly when a variable is *noised* in this way
  - ⇒ that variable is important.
- If the error does not increase
  - ⇒ that variable is not useful for the classification.

Permutation Importance

No	Age	BMI	BP	Res.
1	60	20	140	Ok
2	60	21	145	Ok
3	85	23	130	Ok
4	81	22	160	No
5	70	24	170	No
•	•	•	•	•
•	•	•	•	•
•	•	•	•	•
6	72	26	135	No
7	81	26	145	No
8	66	23	155	No

- To score importance of BMI
  - Fit a classifier on the data
  - Calculate a baseline accuracy
  - Shuffle feature values in the test set
  - Calculate accuracy again
  - Examine increase in error
    - against error without shuffling

```
knn = KNeighborsClassifier(n_neighbors=3)
knn.fit(X_train, y_train)
knn_perm = permutation_importance(knn, X_train, y_train,
                                  n_repeats=10, random_state=0)
```

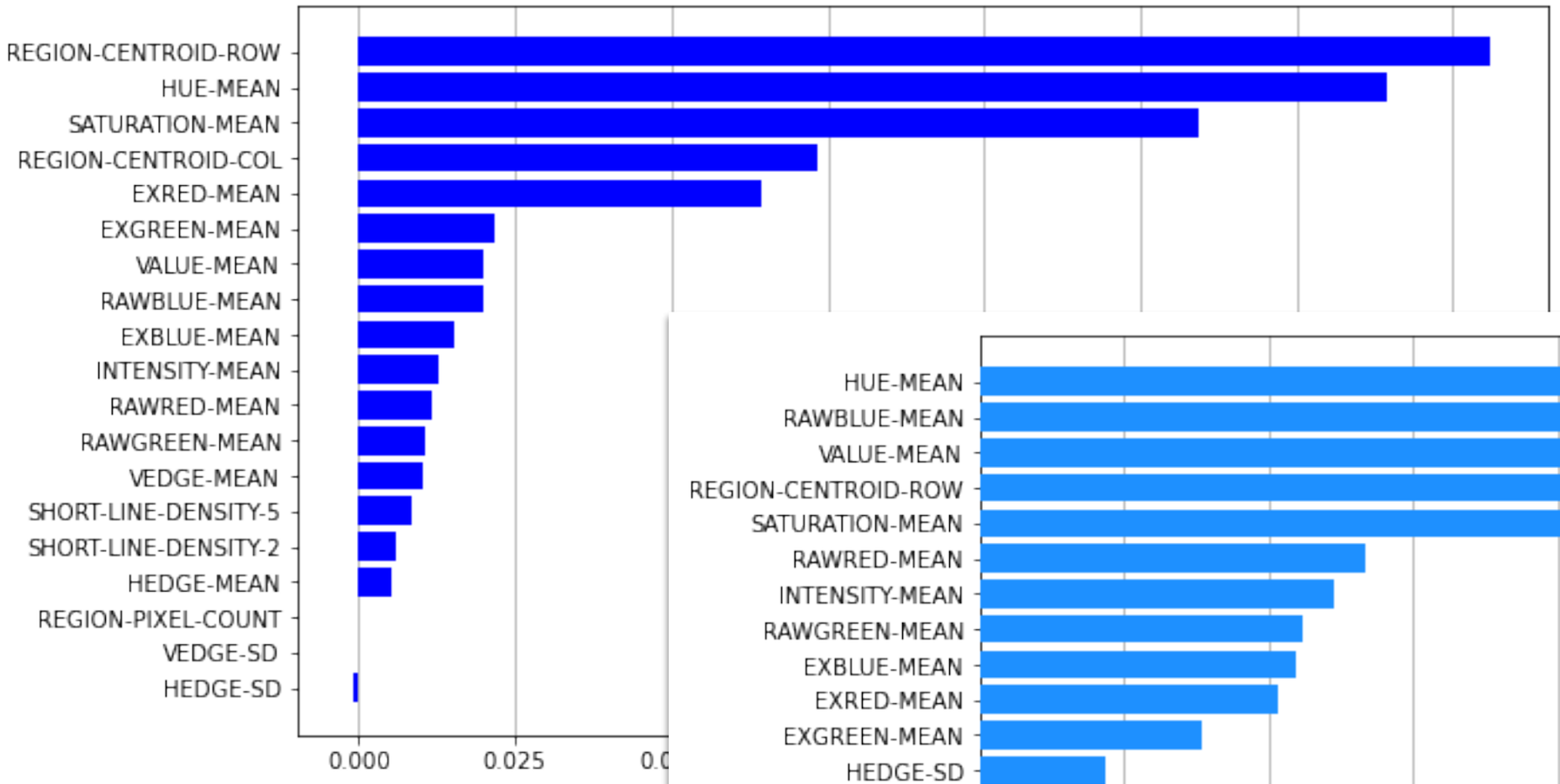
`knn_perm.importances_mean` Returns an array of importance scores

Permutation Importance

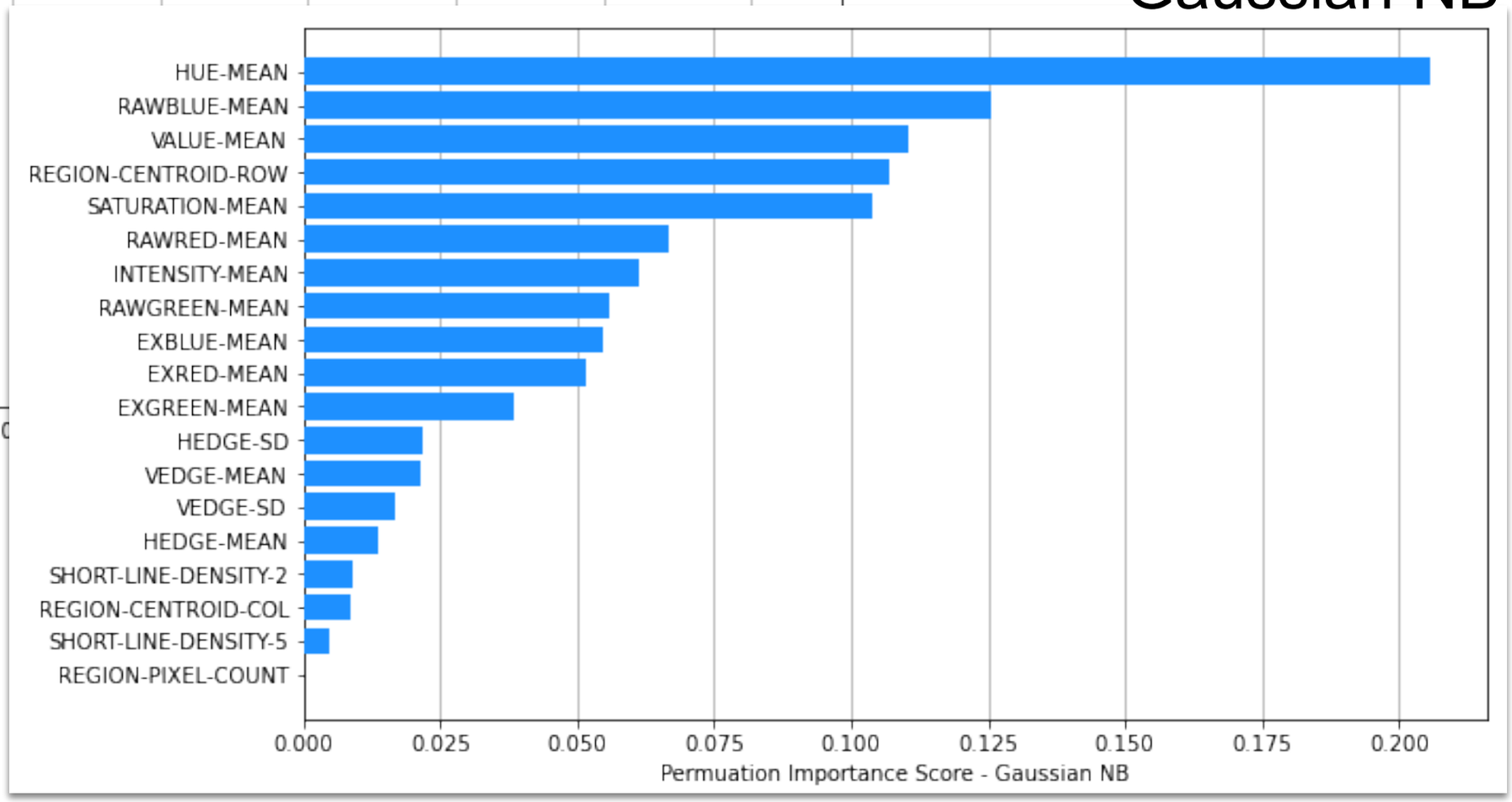
10 Feature Selection Notebook

Original idea from Random Forests
But works for any classifier

k-NN



Gaussian NB



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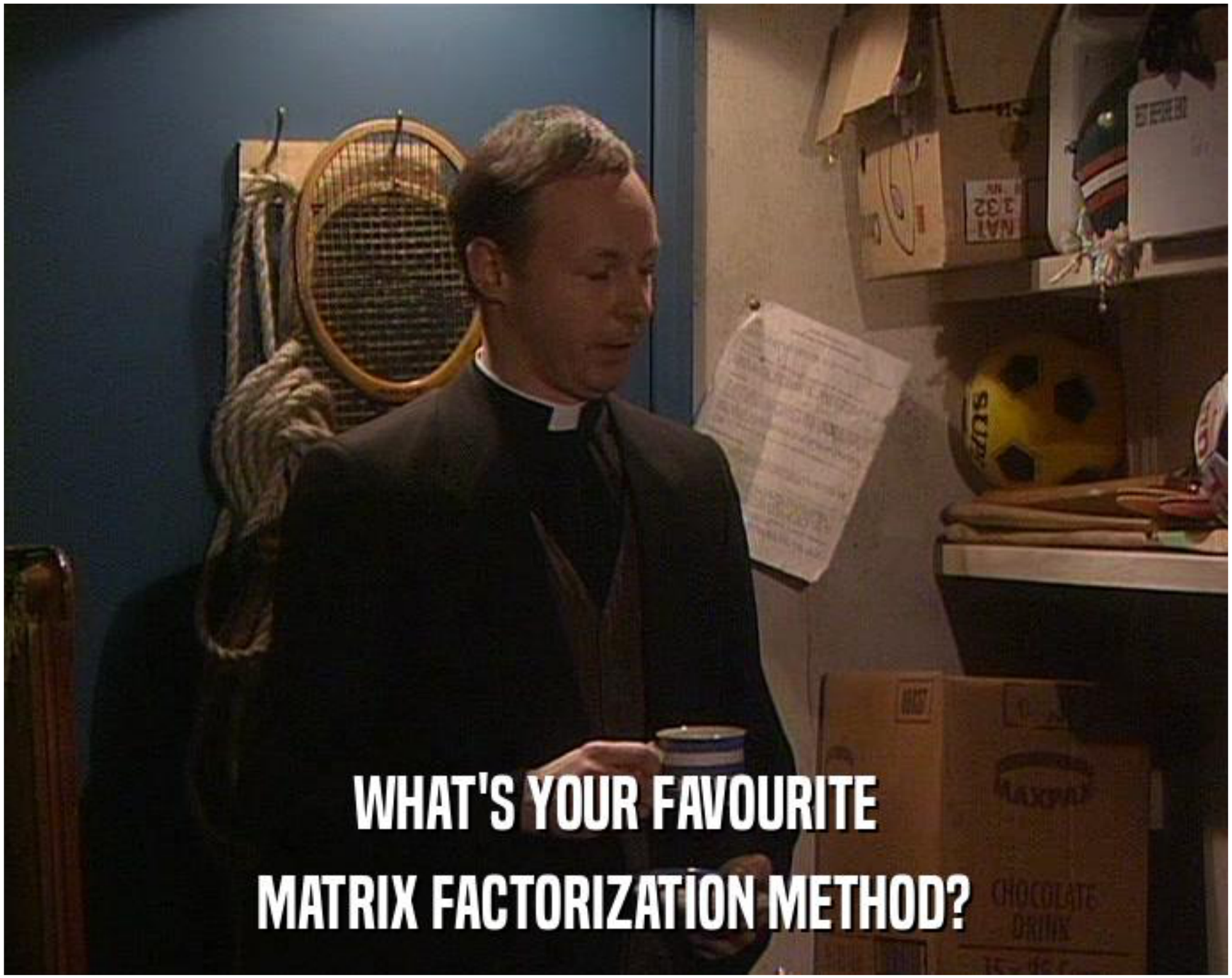


With credit to Kareem Carr

Feature Selection v Transformation

Feature Selection

- Tries to find a minimum subset of the original features that optimises one or more criteria, rather than producing an entirely new set of dimensions for the data.

Feature Transformation (Feature Extraction)

- A popular alternative strategy in data pre-processing is to transform the data into an entirely different format.
- Examples represented by one set of features are transformed to another new set of features.
- Resulting features can be more compact and less noisy, resulting in more accurate predictions.
- Typically involve a linear transformation of the original data.

Feature Selection v Transformation

	region-centroid-col	region-centroid-row	region-pixel-count	vedge-mean	vedge-sd	hedge-mean	hedge-sd	intensity-mean
x1	218.0	178.0	9.0	0.8	0.5	1.1	0.5	59.6
x2	113.0	130.0	9.0	0.3	0.3	0.3	0.4	0.9
x3	202.0	41.0	9.0	0.9	0.8	1.1	1.0	123.0
x4	32.0	173.0	9.0	1.7	1.8	9.0	6.7	43.6
x5	61.0	197.0	9.0	1.4	1.5	2.6	1.9	49.6
x6	149.0	185.0	9.0	1.6	1.1	3.1	1.9	49.3
x7	197.0	229.0	9.0	1.4	1.6	1.2	0.6	17.7
x8	29.0	111.0	9.0	0.4	0.2	0.6	0.2	5.4
...	...	...	...	...	...	...	...	...

Feature Selection:
Select subset of the original features, use them to represent the data.

	region-centroid-row	hedge-mean	hedge-sd	intensity-mean
x1	178.0	1.1	0.5	59.6
x2	130.0	0.3	0.4	0.9
x3	41.0	1.1	1.0	123.0
x4	173.0	9.0	6.7	43.6
x5	197.0	2.6	1.9	49.6
x6	185.0	3.1	1.9	49.3
x7	229.0	1.2	0.6	17.7
x8	111.0	0.6	0.2	5.4
...	...	...	...	...

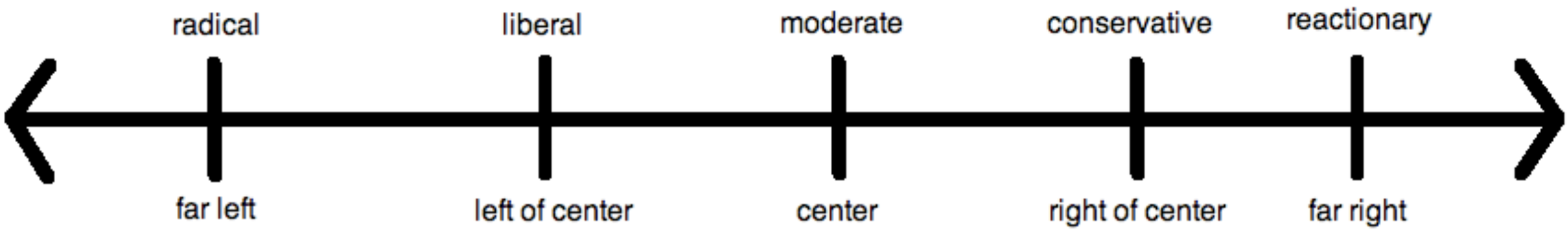
Feature Transformation:
Create a new set of dimensions, use them to represent the data.

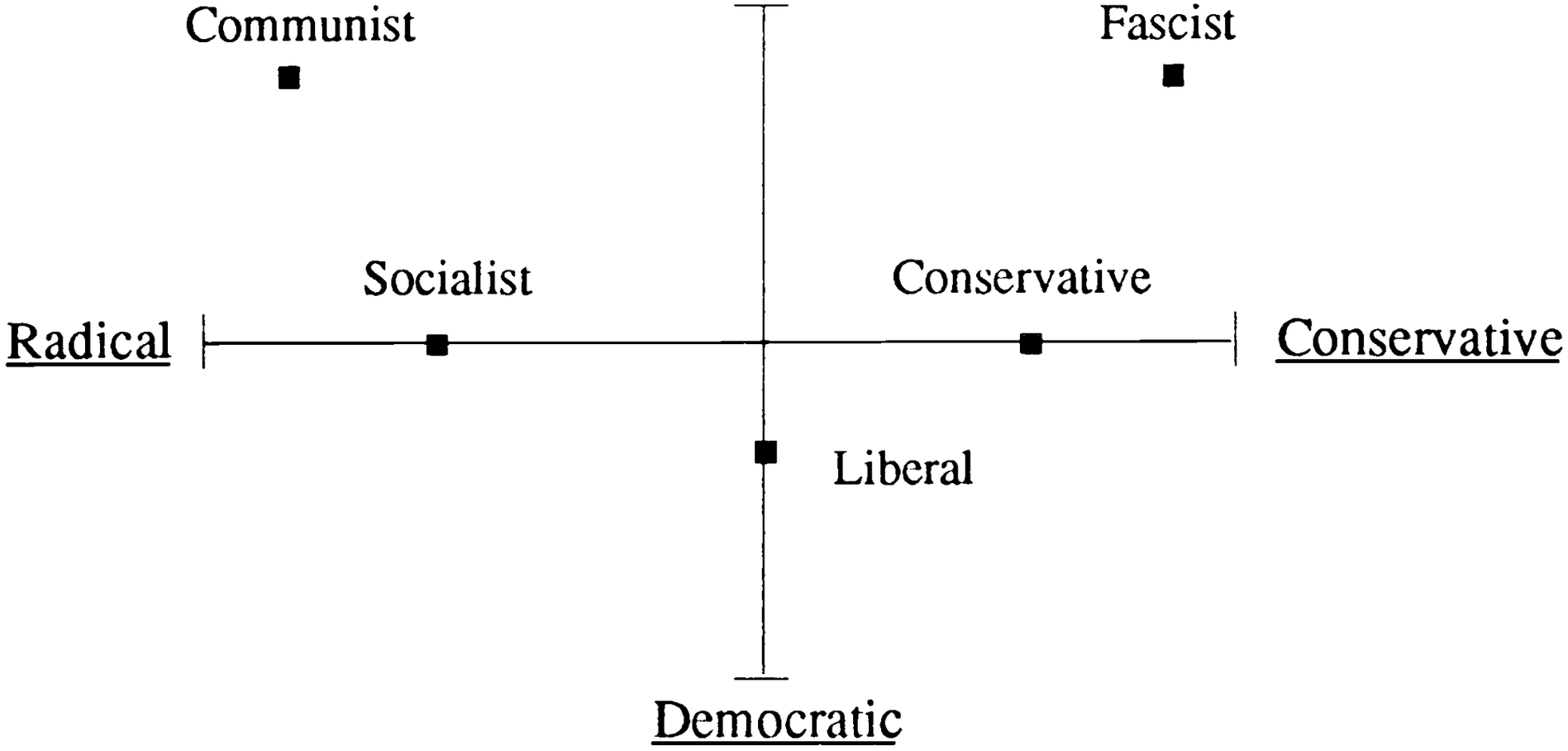
	PC1	PC2	PC3	PC4
x1	95.04	19.51	36.68	37.97
x2	-13.34	11.55	19.53	-29.57
x3	76.50	-2.37	-112.33	41.45
x4	-90.95	5.04	44.07	29.96
x5	-61.17	13.49	61.95	43.10
x6	26.18	16.23	49.07	34.47
x7	74.49	26.46	100.29	21.20
x8	-97.65	5.20	2.54	-29.51
...	...	...	...	...

- 2D projection of a 3D object

But

- *Intrinsic* dimension of surface of the Earth is 2D.
- Earth surface occupies a 2D *manifold* in a 3D space.





Linear Transformations

- In a linear transformation, we map data to new variables, which are linear functions of the original variables.
- **Example:** Bill owes Mary £5 and $15 after a holiday. Current exchange rates:

  €:£ → 1.15:1, €:$ → 0.93:1
- Based on rates, Bill owes €(1.15 x 5 + 0.93 x 15) = €19.70
- We can view this as a linear transformation…

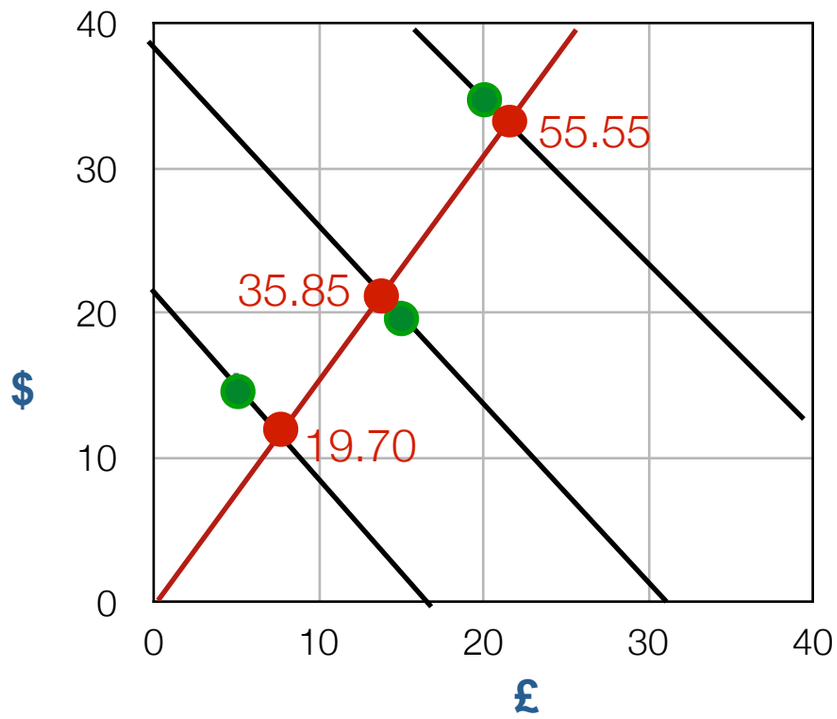
£	$
5	15
15	20
20	35

x

1.15
0.93

=

€
19.70
35.85
55.55



Matrix Multiplication Perspective

£	$
5	15
15	20
20	35

3 x *2*

x

1.15
0.93

2 x *1*

=

€
19.70
35.85
55.55

3 x *1*

2 x 1 matrix maps 2D data to 1D

More generally

d x k matrix maps d D data to k D

$$\underset{n \times d}{\mathbf{Y}} \; \underset{d \times k}{\mathbf{P}} = \underset{n \times k}{\mathbf{X}'}$$

$$\underset{\text{samples}}{n} \begin{bmatrix} y_{1,1} & y_{1,2} & \cdots & y_{1,d} \\ y_{2,1} & y_{2,2} & \cdots & y_{2,d} \\ \vdots & \vdots & & \vdots \\ y_{n,1} & y_{n,2} & \cdots & y_{n,d} \end{bmatrix} \times \begin{bmatrix} p_{1,1} & \cdots & p_{1,k} \\ p_{2,1} & \cdots & p_{2,k} \\ \vdots & & \vdots \\ p_{d,1} & \cdots & p_{d,k} \end{bmatrix} = \begin{bmatrix} x_{1,1} & \cdots & x_{1,k} \\ x_{2,1} & \cdots & x_{2,k} \\ \vdots & & \vdots \\ x_{n,1} & \cdots & x_{n,k} \end{bmatrix} \underset{\text{samples}}{n}$$

d features — k features

Matrix Multiplication in Python

£	$
5	15
15	20
20	35

3 x *2*

x

1.15
0.93

2 x *1*

=

€
19.70
35.85
55.55

3 x *1*

Use numpy arrays
.dot method: matrix multiplication

Set up data array

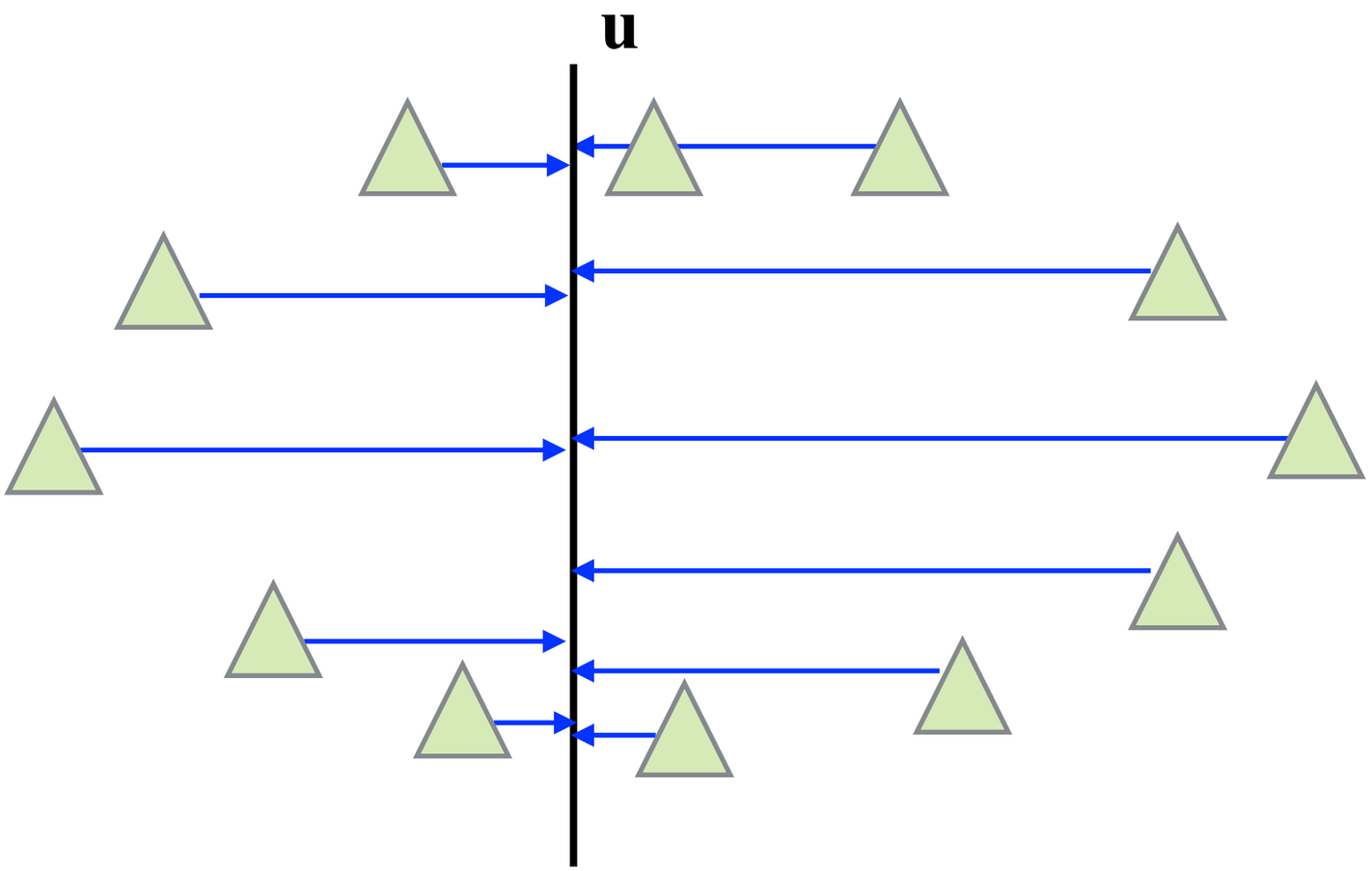
Exchange rate array

Matrix multiplication

```
import numpy as np

cur = np.array([[5,15],
                [15,20],
                [20,35]])

er = np.array([1.15,0.93])

cur.dot(er)

array([19.7 , 35.85, 55.55])
```

Projection Methods

- Projection methods map the original *d*-dimensional space to a new *(k < d)*-dimensional space, with the minimum loss of information.
- "Good" spaces for projections are characterised by preserving most of the useful information in the data - the variation in the data.



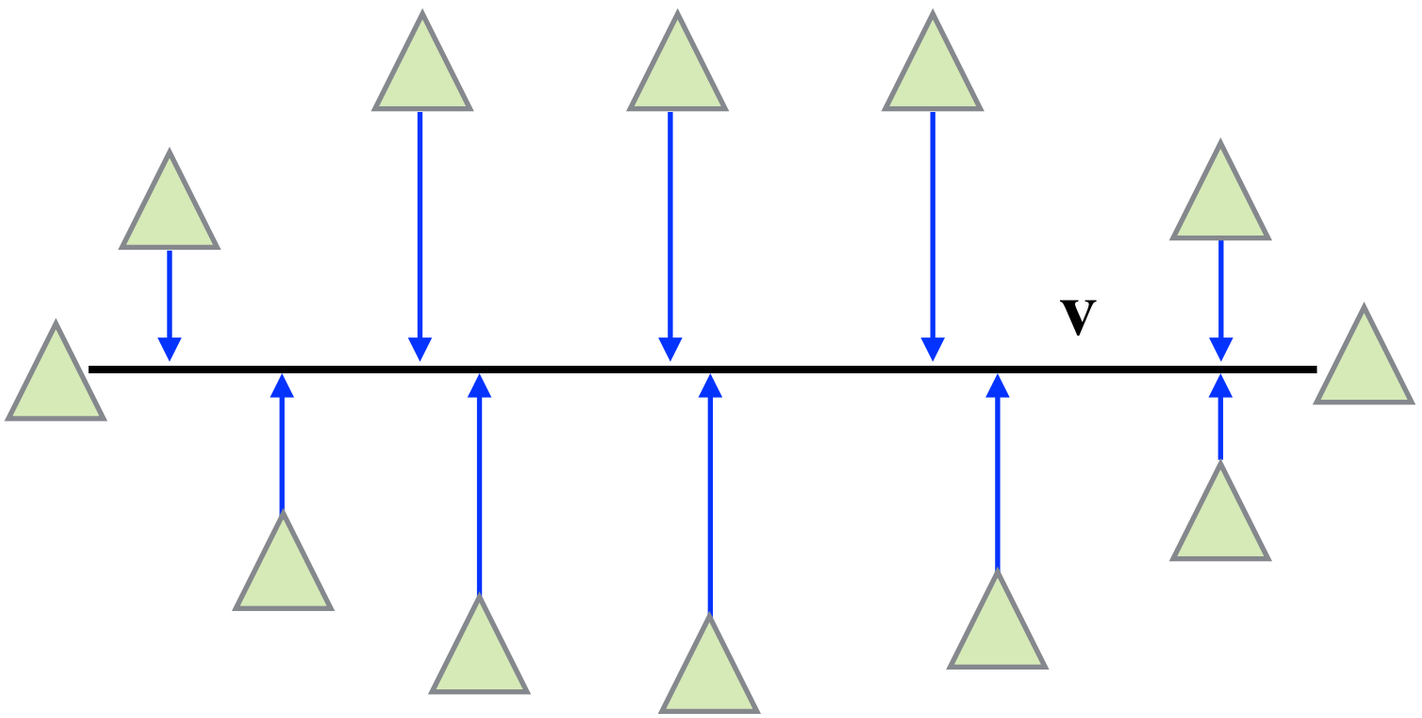
georgemdallas.wordpress.com

If we project the original data to this new dimension $\boldsymbol{u}$, the data is not very spread out.

The dimension does not have high variance.

Projection Methods

- Projection methods map the original *d*-dimensional space to a new *(k < d)*-dimensional space, with the minimum loss of information.
- "Good" spaces for projections are characterised by preserving most of the useful information in the data - the variation in the data.

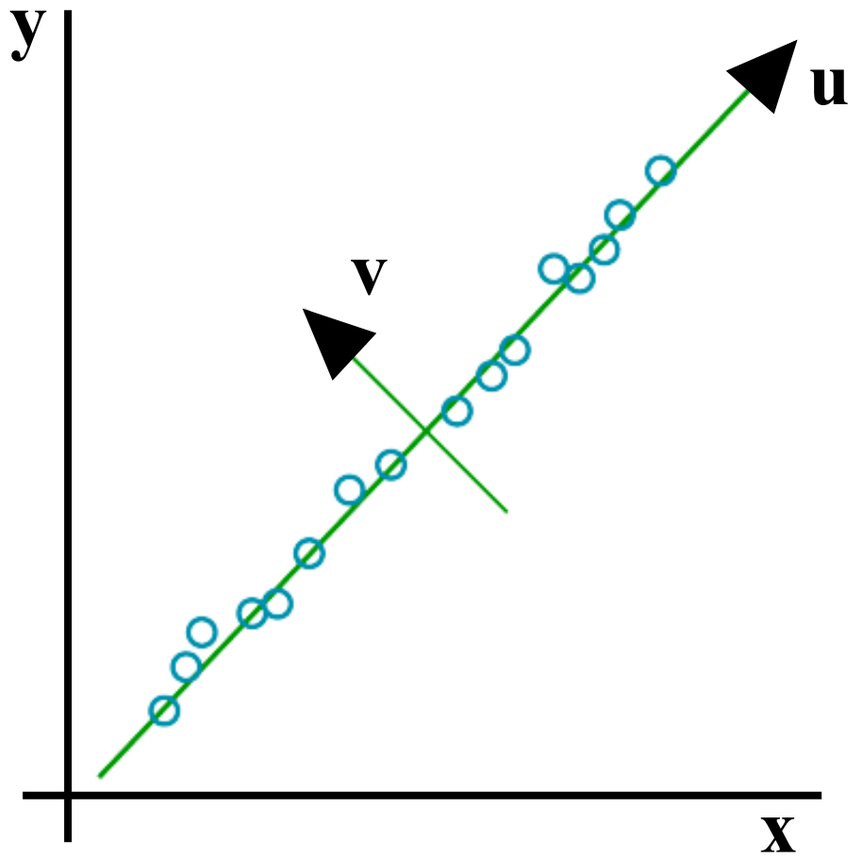


If we project to the alternative new dimension ***v***, the data is far more spread out.

This new dimension has more variation - i.e. more information has been preserved.

Principal Component Analysis (PCA)

- To minimise the loss of information, we need to find new dimension(s) which maximise the variation.



Given data in terms of dimensions *(x,y)*, the principal direction in which the data varies is along the ***u*** axis.

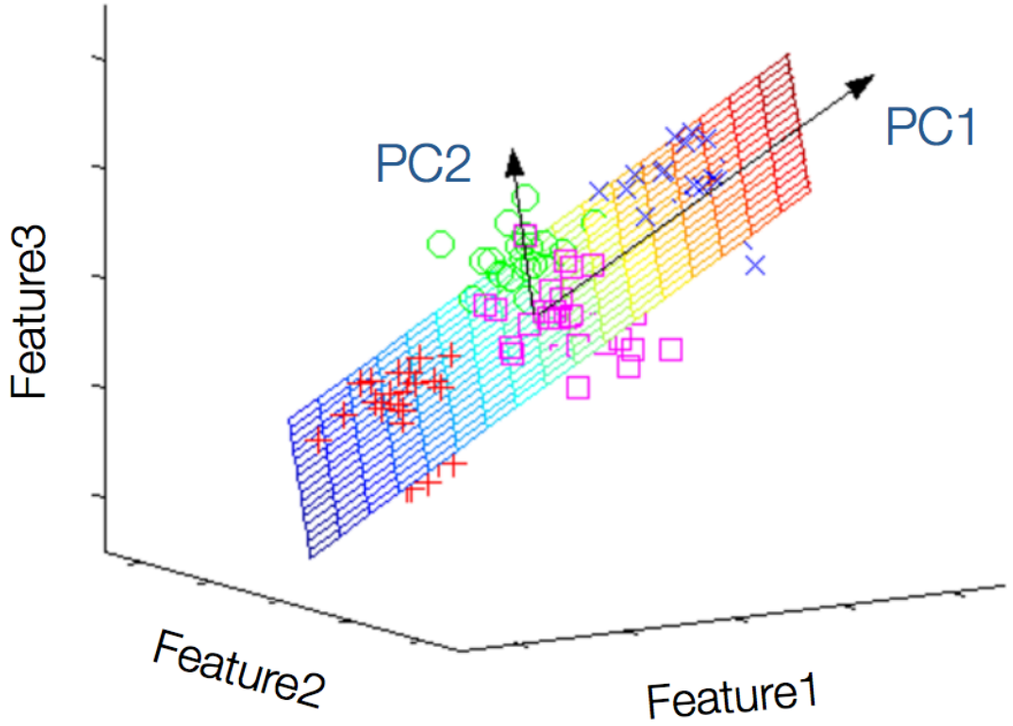
Very little variation in the data in the direction of ***v***.

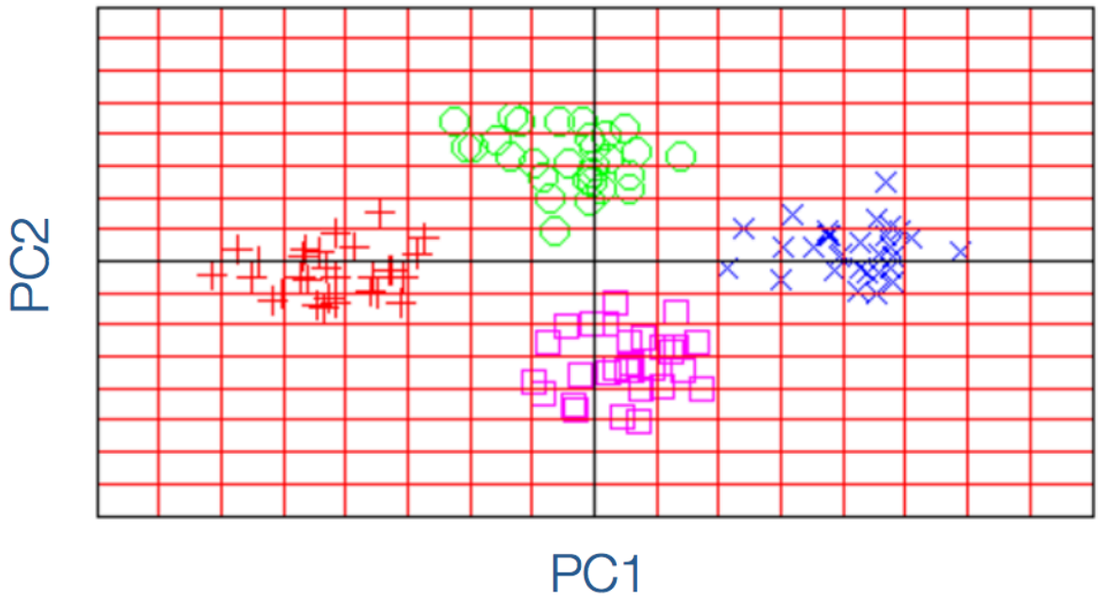
➡ Select the ***u*** dimension to represent the data.

- Use Principal Component Analysis (PCA) - an unsupervised projection method which performs dimensionality reduction, while trying to keep as much of the variance in the data as possible.

Principal Component Analysis (PCA)

- **Example:** Transformation of original data (3 dimensions) to a lower dimensional space (2 dimensions) via PCA.
- Using PCA, we can identify the two-dimensional plane that optimally describes the highest variance of the data. Each resulting dimension is a linear combination of the original 3 dimensions.





Eigenvectors and Eigenvalues

- Given an input matrix $\mathbf{X}$, an eigenvector of the matrix is a non-zero vector $\mathbf{v}$ that satisfies the equation:

$$\mathbf{X}v = \lambda v$$

  where the corresponding number λ is called an eigenvalue.

- Eigendecomposition is the factorisation of a matrix into its eigenvalues and eigenvectors.

$$\mathbf{X} = \begin{pmatrix} 1.0000 & 0.5000 & 0.3330 & 0.2500 \\ 0.5000 & 1.0000 & 0.6667 & 0.5000 \\ 0.3333 & 0.6667 & 1.0000 & 0.7500 \\ 0.2500 & 0.5000 & 0.7500 & 1.0000 \end{pmatrix}$$

4 x 4 symmetric matrix

$$\mathbf{\Lambda} = \begin{pmatrix} 2.5361 & 0 & 0 & 0 \\ 0 & 0.8483 & 0 & 0 \\ 0 & 0 & 0.4078 & 0 \\ 0 & 0 & 0 & 0.2077 \end{pmatrix}$$

4 eigenvalues

Eigenvectors and values exist in pairs: every eigenvector has a corresponding eigenvalue.

$$\mathbf{V} = \begin{pmatrix} -0.37775 & -0.81052 & -0.44217 & -0.06988 \\ -0.53223 & -0.18762 & 0.74199 & 0.36221 \\ -0.56139 & 0.30099 & 0.04872 & -0.76926 \\ -0.50881 & 0.46611 & -0.50155 & 0.52169 \end{pmatrix}$$

4 eigenvectors

Eigendecomposition in *sklearn*

```
X = np.array([[1, 0.5, 0.333, 0.25],
              [0.5, 1, 0.667, 0.5],
              [0.333, 0.667,1, 0.75],
              [0.25, 0.5, 0.75, 1]
             ])
In [24]:
eVal, eVec = np.linalg.eig(X)
In [25]:
vec1 = eVec[:,0]
val1 = eVal[0]
```

$$\mathbf{X}v = \lambda v$$

```
X.dot(vec1)
Out[37]:
array([-0.95799966, -1.34996974, -1.42377554, -1.29038555])

val1 * vec1
Out[38]:
array([-0.95799966, -1.34996974, -1.42377554, -1.29038555])
```

Notebook ’10 Eigen’

Eigenvectors in PCA

- Each eigenvector has a direction - i.e. it is a dimension.
- Eigenvectors of symmetric matrices are orthogonal to each other - i.e. they point in completely different directions.
- Each eigenvalue is a number indicating how much variance there is in the data in that direction.
- The eigenvector with the largest eigenvalue has the most variance, making it a useful dimension for projection.

- Principal Components (PCs): New dimensions constructed as linear combinations of the original features, which are uncorrelated with one another. Constructed from eigenvectors.
- The first PC accounts for the most variability in the data. The next PC has the highest variance possible under the constraint that it is uncorrelated with the first PC, and so on…

Covariance Matrix

- To assess variability in the data, we can measure the covariance between the original features - i.e. the tendency for two features x and y to vary in the same direction:
  - Do features x and y tend to increase together?
  - Or does feature y decrease as feature x increases?
- We estimate the covariances based on all *n* examples.
- By measuring the covariance between all pairs of features, we can create a symmetric covariance matrix.

Original n x d matrix

$$\mathbf{C} = \mathbf{Y}^{\mathsf{T}}\mathbf{Y}/(n-1) = \begin{bmatrix} c_{11} & c_{12} & .. & c_{1d} \\ c_{21} & c_{22} & .. & c_{2d} \\ .. & .. & .. & .. \\ c_{d1} & c_{d2} & .. & c_{dd} \end{bmatrix}$$

Applying PCA

- **Input:** A dataset matrix $\mathbf{X}$ with n examples (i.e. rows). The features of this matrix (i.e columns) might potentially be correlated.
- **PCA Process:**
  1. Calculate the mean and std deviation of the columns of $\mathbf{X}$.
  2. Subtract the column means from each row of $\mathbf{X}$ and divide by the std deviation to create the normalised centred matrix $\mathbf{Y}$.
  3. Calculate the covariance matrix $\mathbf{C} = \mathbf{Y}^\mathsf{T}\mathbf{Y}/(n-1)$
  4. Calculate the eigenvectors of the covariance matrix $\mathbf{C}$.
  5. The Principal Components (PCs) are given by the eigenvectors of $\mathbf{C}$. The *i*-th PC is given by the eigenvector corresponding to the *i*-th largest eigenvalue of $\mathbf{C}$.
  6. Select an appropriate number of PCs k and use them to produce a new reduced n x k representation of the dataset.

Top k eigenvectors make up the transformation matrix $\mathbf{P}$

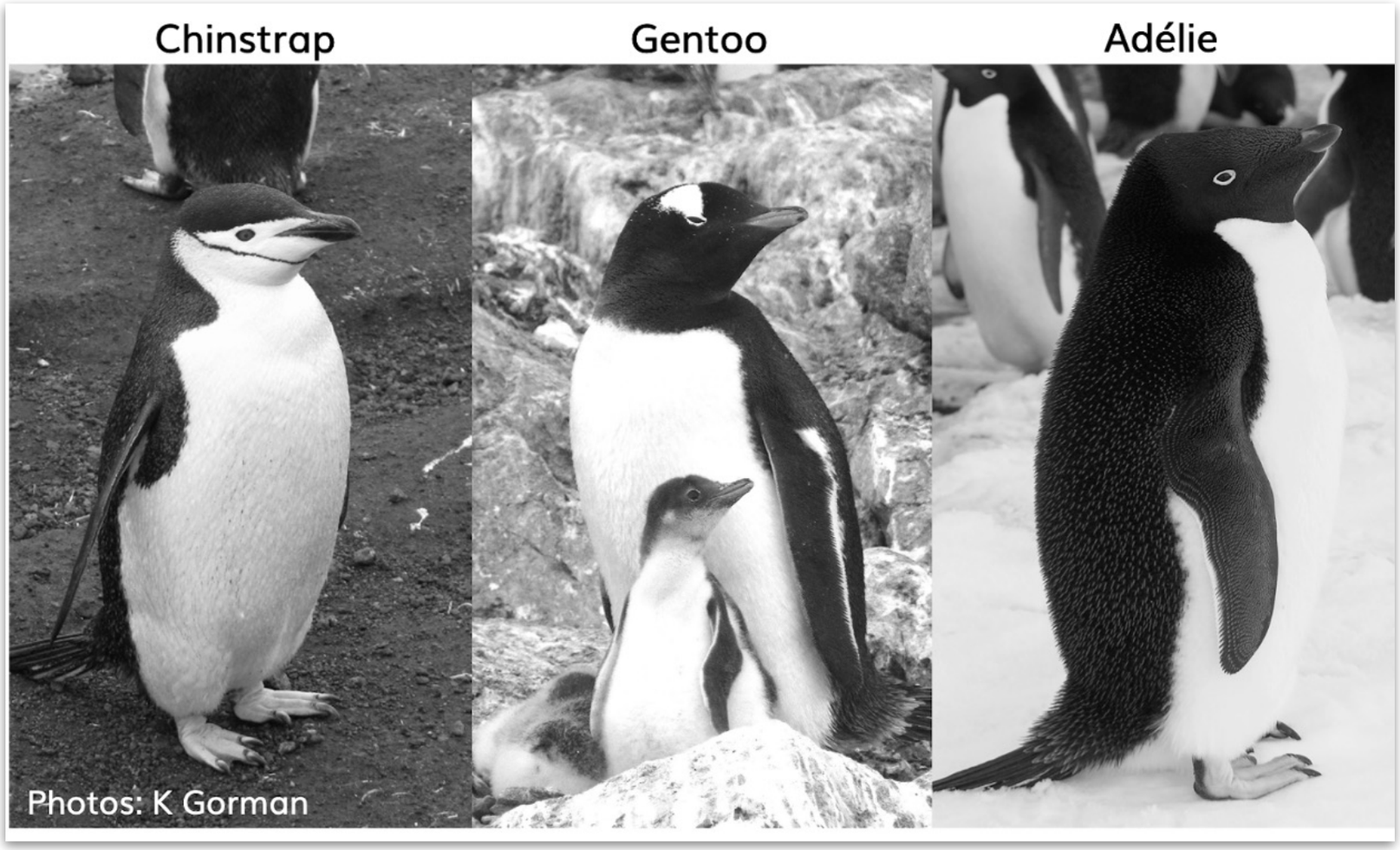
New data representation $\underset{n \text{ x } k}{\mathbf{X}'} = \underset{n \text{ x } d}{\mathbf{Y}}\ \underset{d \text{ x } k}{\mathbf{P}}$

Penguin dataset

Kristen Gorman

Allison Horst

https://github.com/allisonhorst



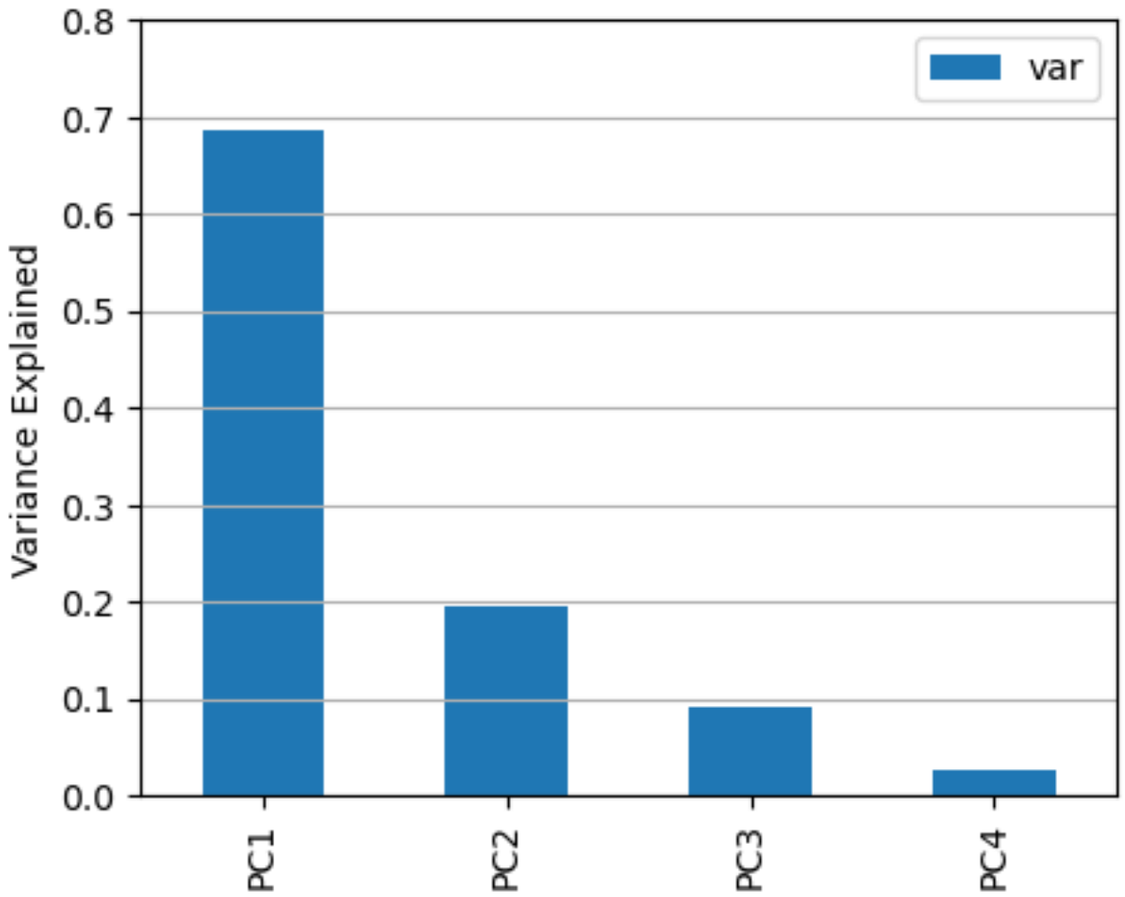
3 classes, four features

- bill length
- bill depth
- flipper length
- body mass

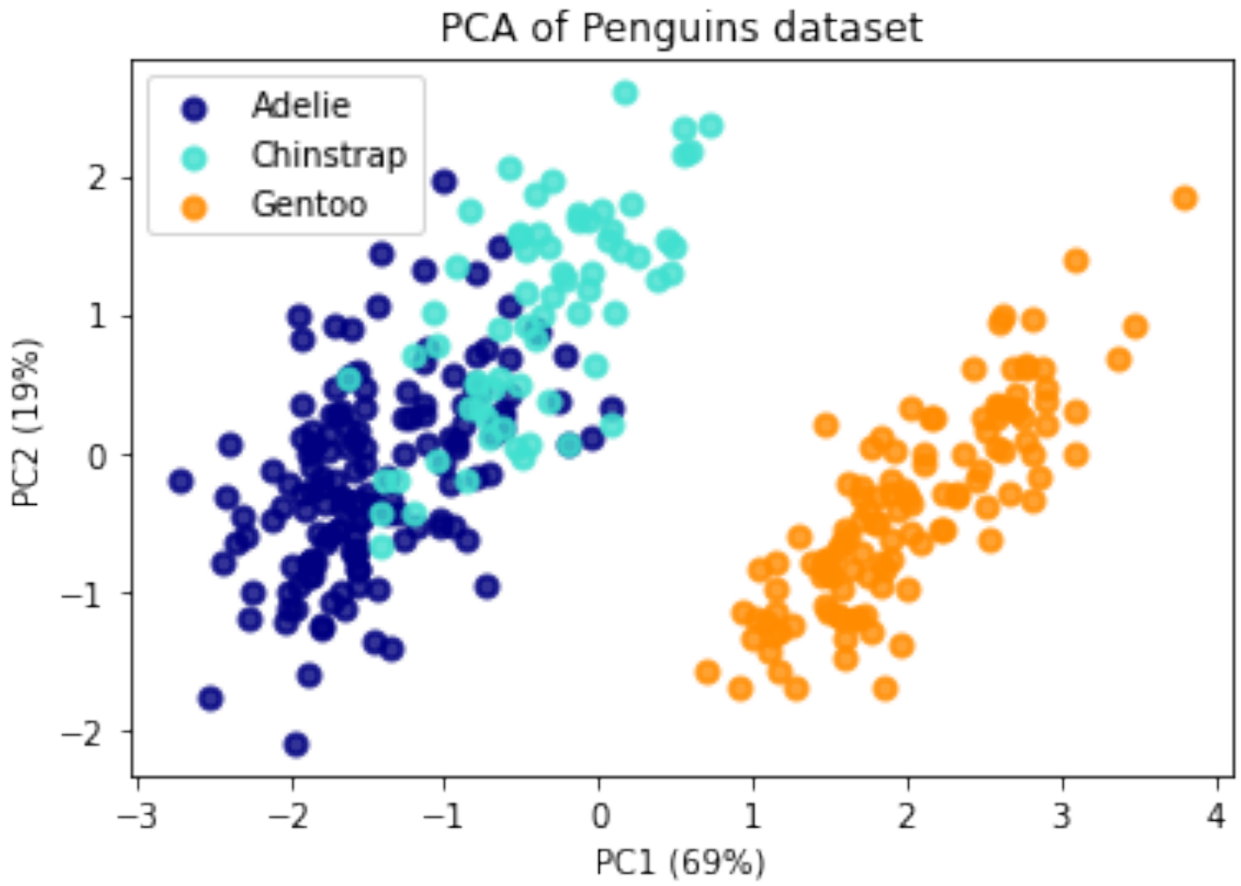
Applying PCA

We generally select the k PCs with the highest variance.

Example: Apply PCA to *Penguins* data set, and examine amount of variance in each PC.

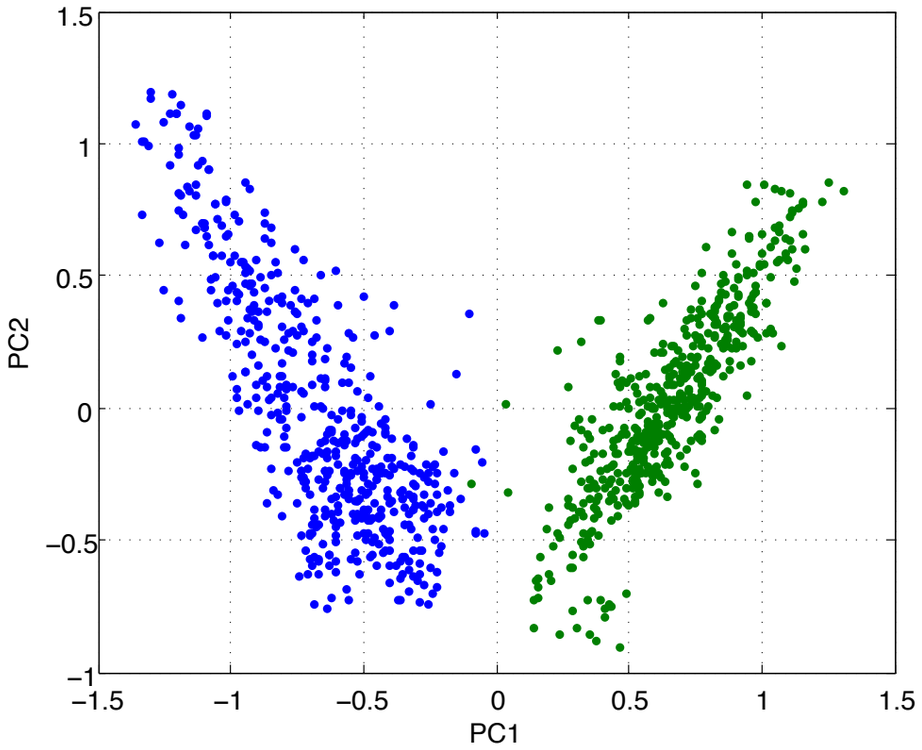


The first two PCs account for ~88% of the variance in the data.

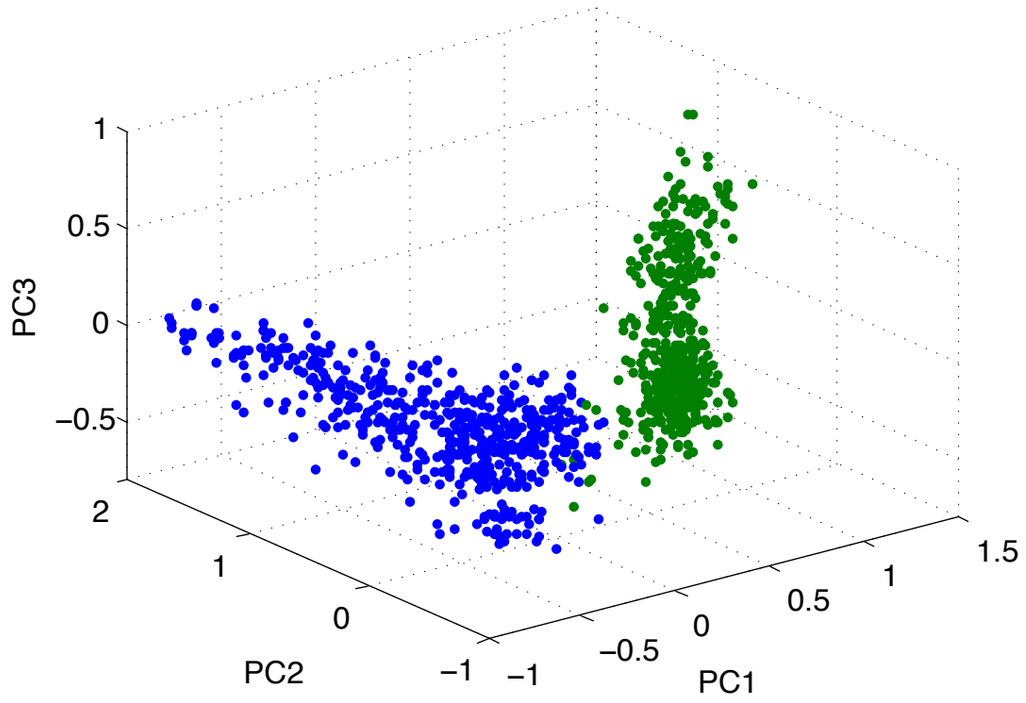


Example: PCA

- Collection of 1,021 BBC news articles on business + sport, represented by high-dimensional space with 5,570 features (words).
- Applying PCA allows us to visualise the data in a low dimensional space using a small number of PCs.



2 leading principal components (PCs)



3 leading principal components (PCs)

Python code notebook 10 PCA

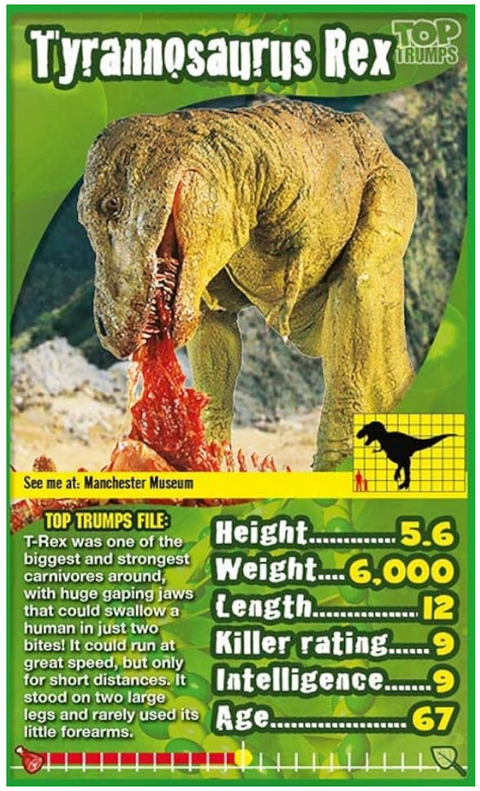
Scale the features to N(0,1)

Fit the PCA and transform

pca object contains info on explained variance

```
from sklearn.decomposition import PCA
from sklearn.preprocessing import StandardScaler

X = penguins_all[['bill_length_mm', ‘bill_depth_mm',
                  'flipper_length_mm', 'body_mass_g']]
y = penguins_all['species']

X_scal = StandardScaler().fit_transform(X)

pca = PCA(n_components=4)
X_r = pca.fit(X_scal).transform(X_scal)

# Proportion of variance explained for each components
pca.explained_variance_ratio_
Out[28]:
array([0.68633893, 0.19452929, 0.09216063, 0.02697115])
```

Dinosaurs Top Trumps data

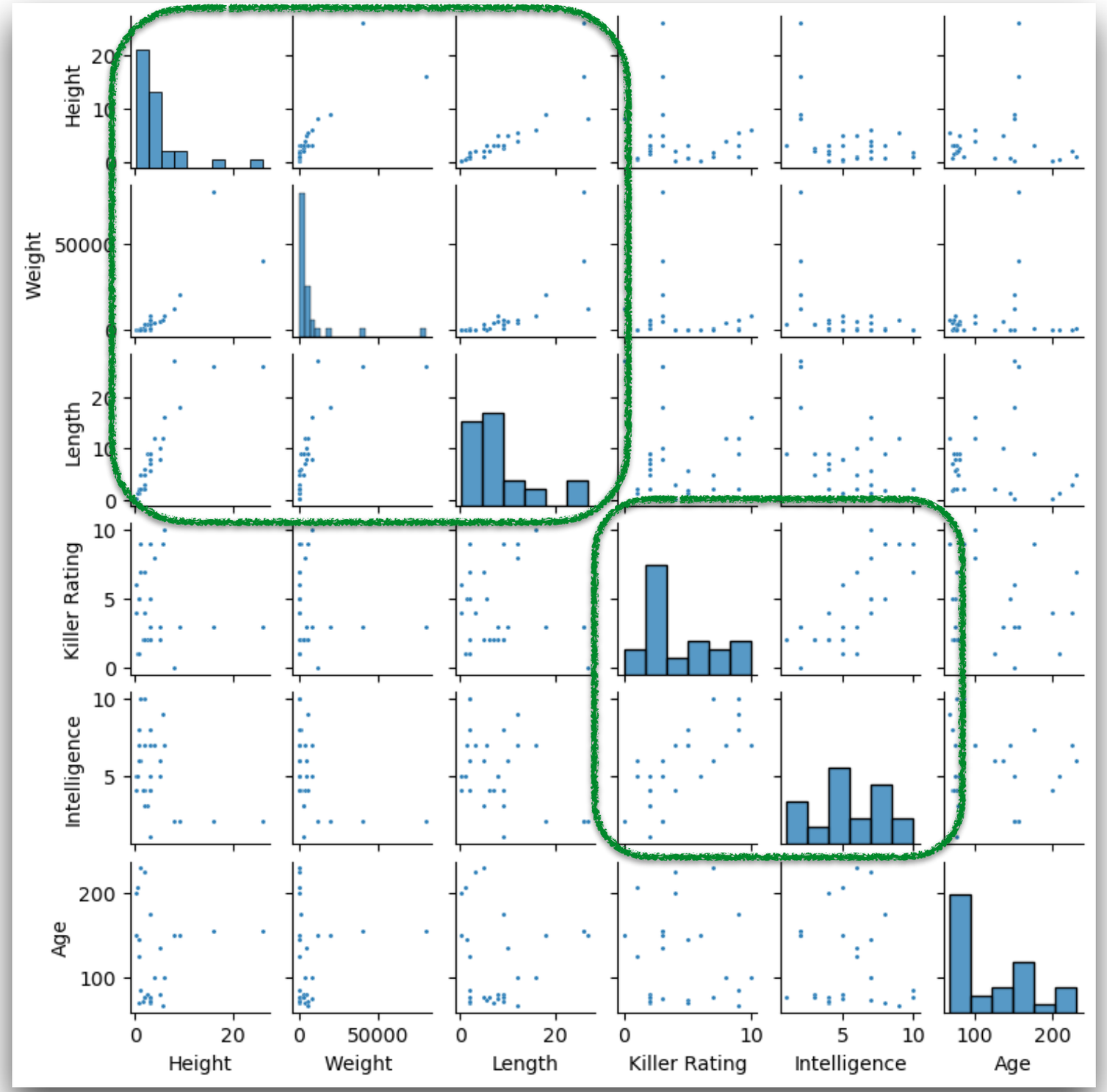


Name	Height	Weight	Length	Killer Rating	Intelligence	Age
Suchomimus	4.0	4000	12.0	8	7	100
Velociraptor	1.0	15	2.0	9	10	85
Triceratops	3.0	5500	9.0	2	4	72
Tyrannosaurus rex	5.6	6000	12.0	9	9	67
Diplodocus	8.0	12000	27.0	0	2	150
...	...	...	...	...	...	...

Correlations

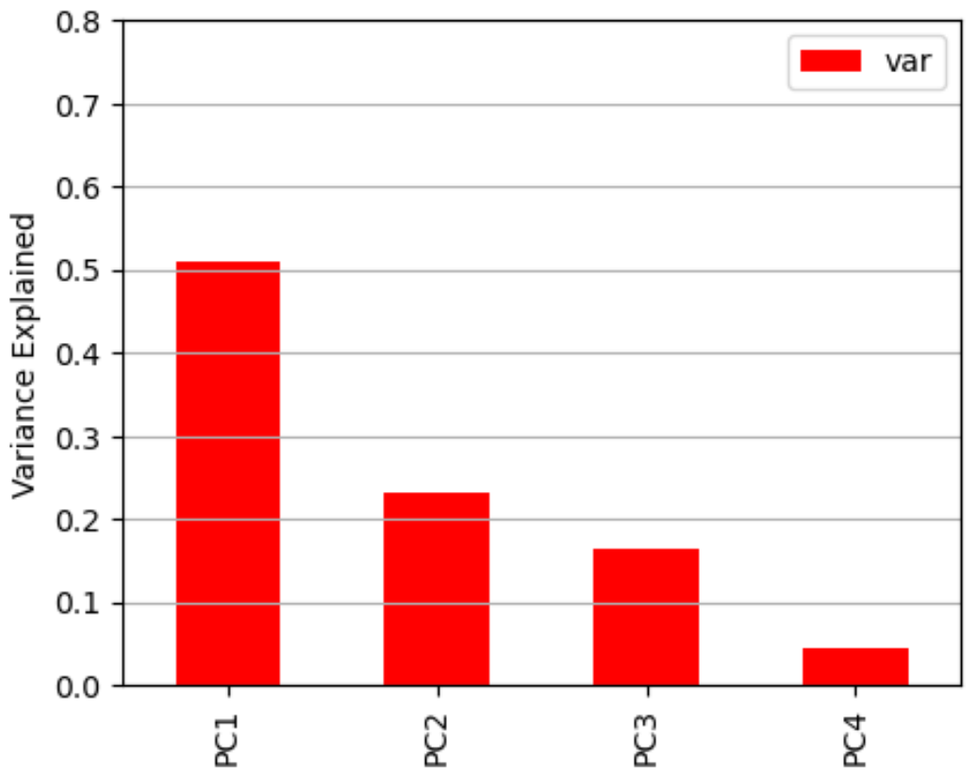
Notebook ’10 PCA’

- Correlations exist
  - Height, Weight, Length
  - Killer Rating, Intelligence



PCA on the Dinosaur data

```
from sklearn.decomposition import PCA
from sklearn.preprocessing import StandardScaler

TTDino_df = pd.read_csv('DinosaursTT.csv')

y = TTDino_df.pop('Name').values
X = TTDino_df

X_scal = StandardScaler().fit_transform(X)

pcaDino = PCA(n_components=4)
X_r = pcaDino.fit(X_scal).transform(X_scal)
pcaDino.explained_variance_ratio_
```

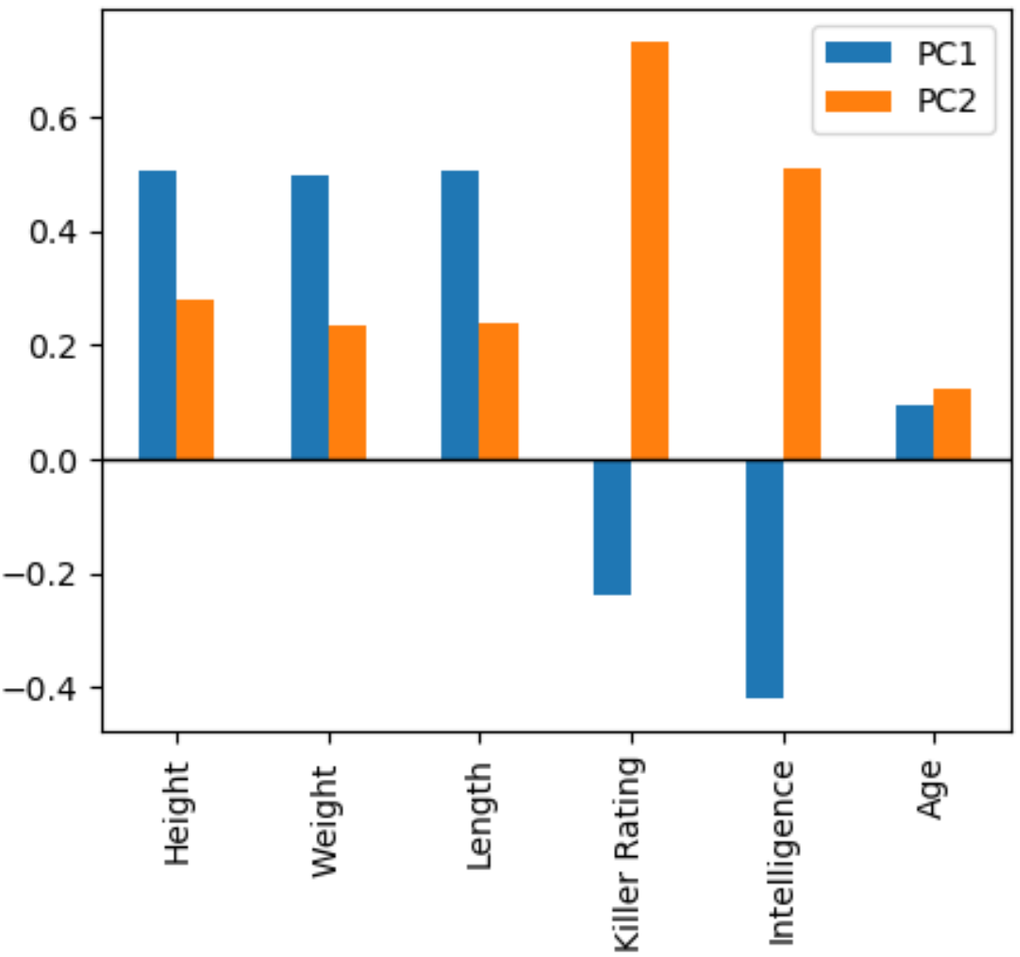


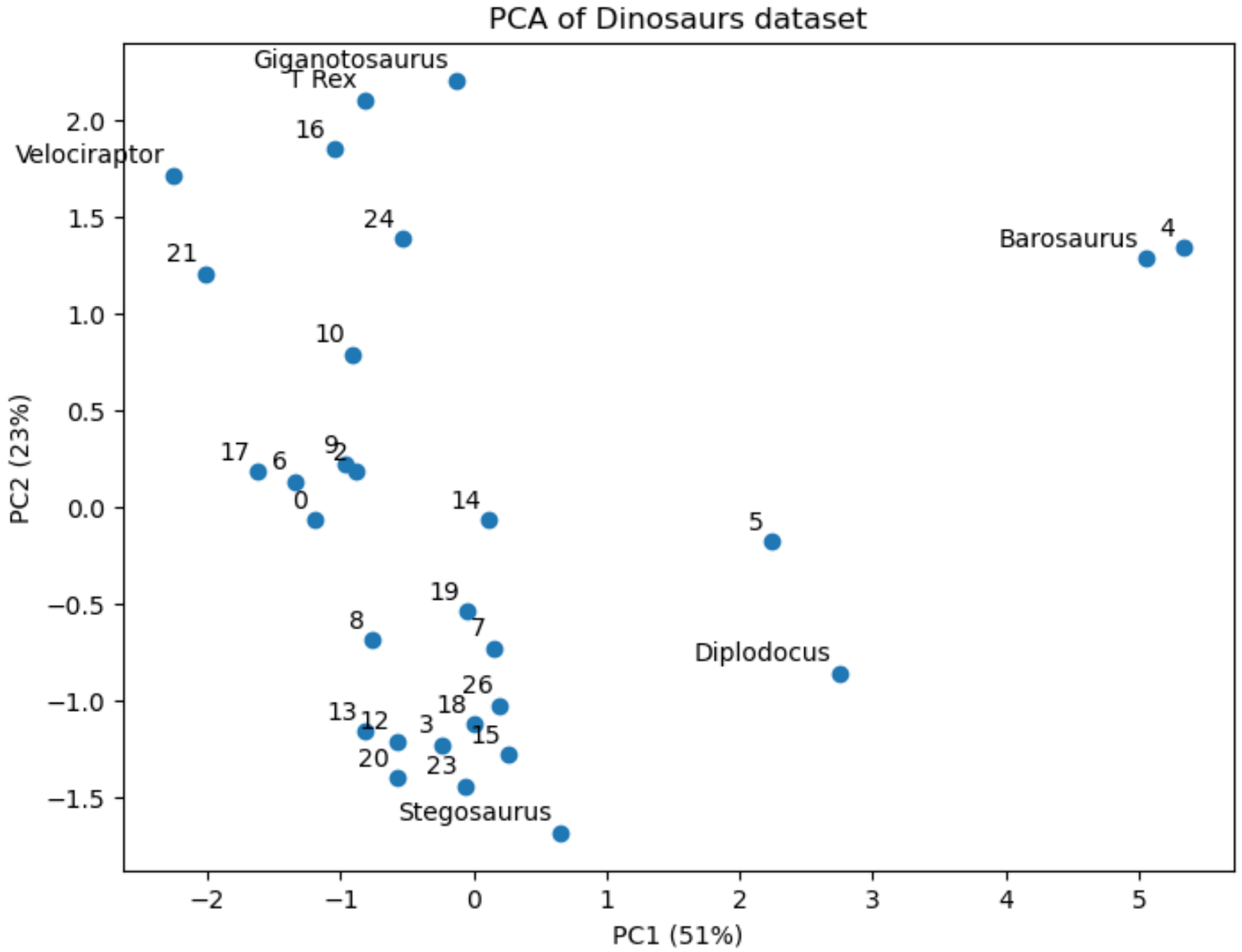
Name	Height	Weight	Length	Killer Rating	Intelligence	Age
Suchomimus	4.0	4000	12.0	8	7	100
Velociraptor	1.0	15	2.0	9	10	85
Triceratops	3.0	5500	9.0	2	4	72
Tyrannosaurus rex	5.6	6000	12.0	9	9	67
Diplodocus	8.0	12000	27.0	0	2	150
...	...	...	...	...	...	...

Dinosaur data - 1st two PCs

```
pcaDino.components_[:2].round(2)
Out[28]:
array([[ 0.51,  0.5 ,  0.5 , -0.24, -0.42,  0.09],
       [ 0.28,  0.23,  0.24,  0.73,  0.51,  0.12]])
```

```
X_r = pcaDino.fit().transform(X_scal)
plt.scatter(X_r[:, 0], X_r[:, 1])
```



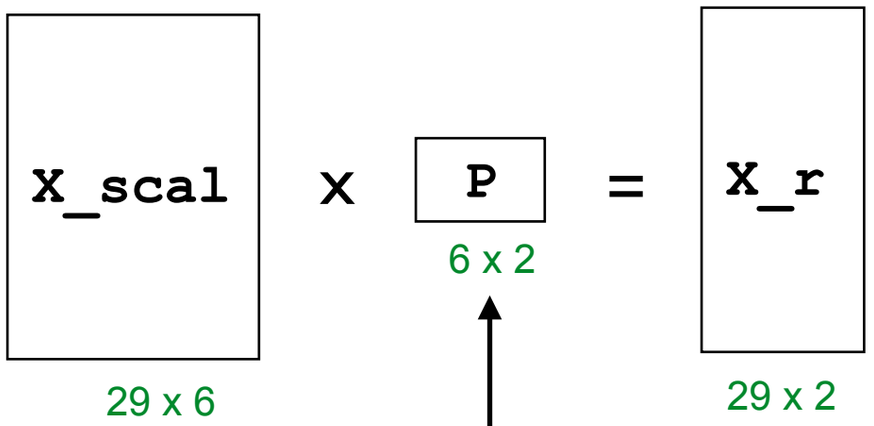


Remember:

- Feature transformation as matrix multiplication

d x k matrix maps d D data to k D

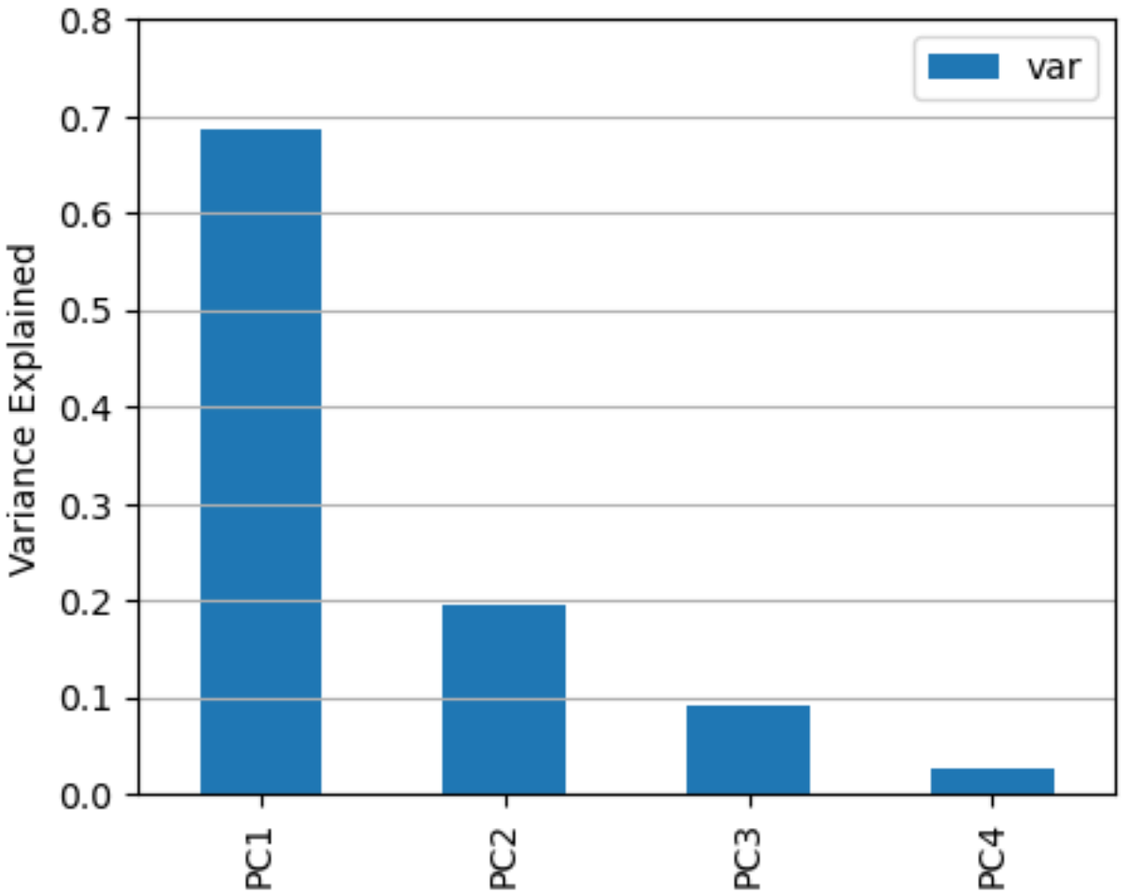
$$\underset{n \times d}{\mathbf{Y}} \; \underset{d \times k}{\mathbf{P}} = \underset{n \times k}{\mathbf{X}'}$$



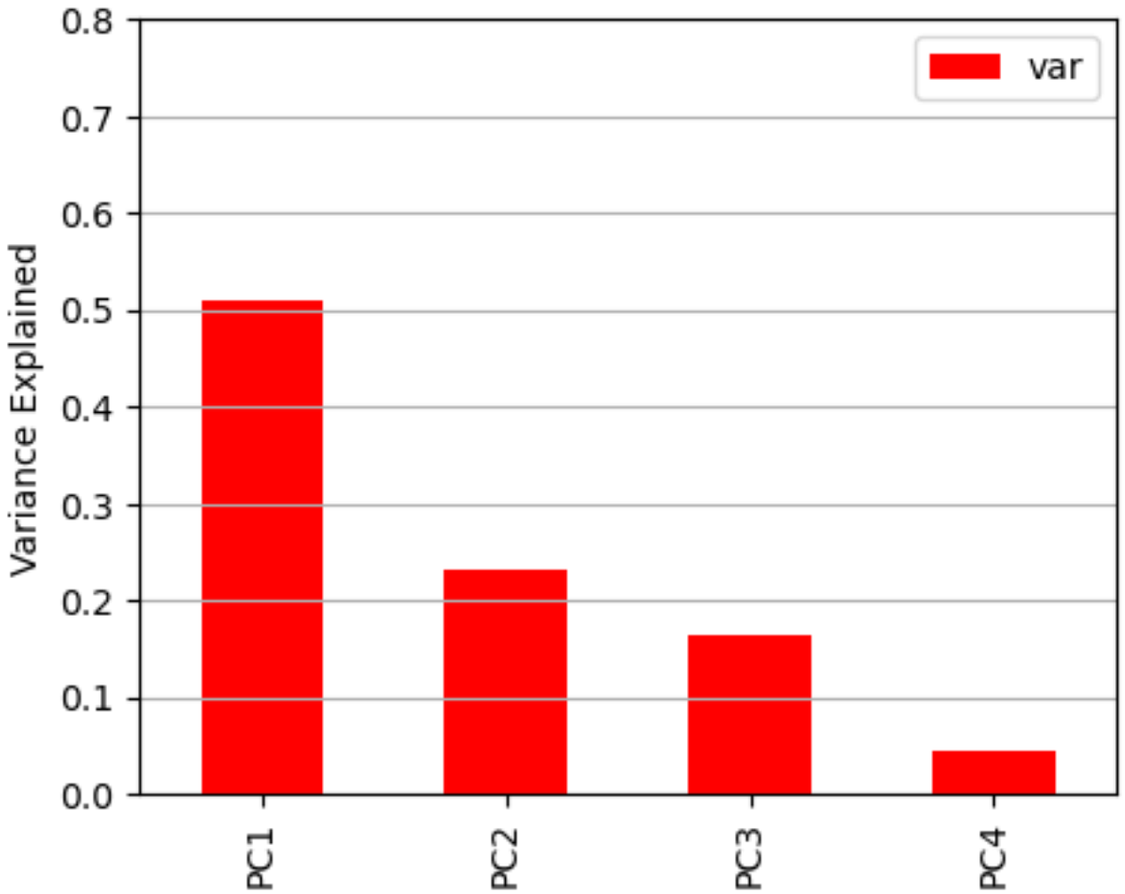
Variance Explained

- Compare inherent variance in both datasets

Penguin



Dinosaurs



Overview

- The Curse of Dimensionality
- Dimension Reduction
- Feature Transformation v Selection
- Feature Selection in Supervised Learning
  - Filter Methods
  - Wrapper Methods
  - Permutation Importance
- Feature Transformation
  - Linear Transformations
  - Projection Methods
  - Principal Component Analysis (PCA)
  - PCA in scikit-learn

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