Alex Psomas: Lecture 16.

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- Regrade requests open.
- Quiz due tomorrow.
- Quiz coming out today.
- Non-technical office hours tomorrow 1-3pm.
- Anonymous questionnaire tonight or tomorrow.

Random Variables

1. Random Variables.
2. Distributions.
3. Combining random variables.
4. Expectation

Questions about outcomes ...

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The number is a (known) function of the outcome.

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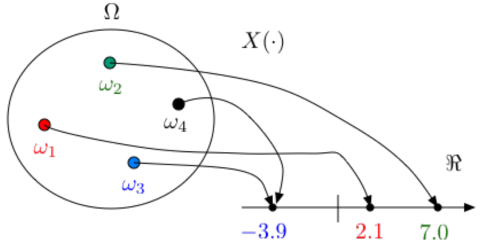

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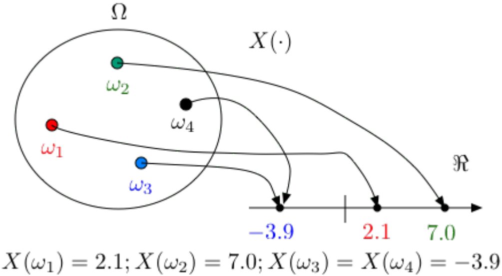

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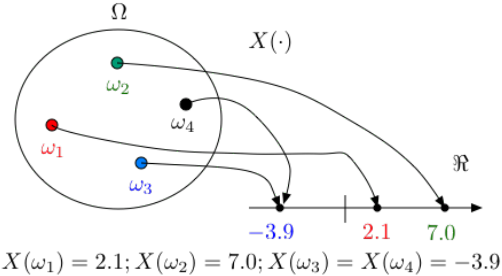

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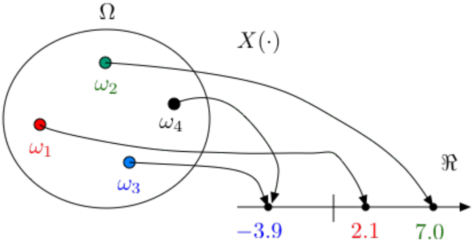

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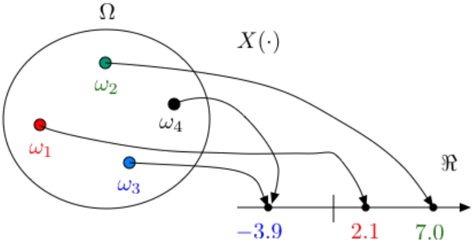

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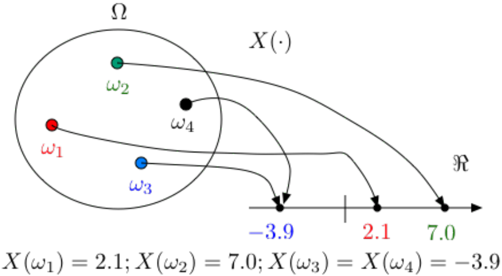

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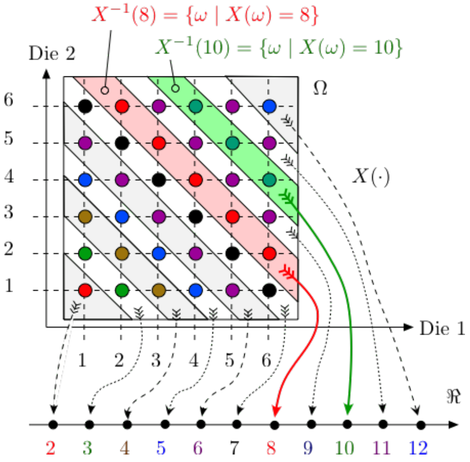

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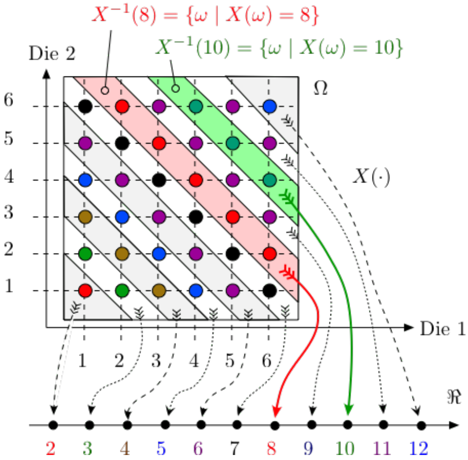

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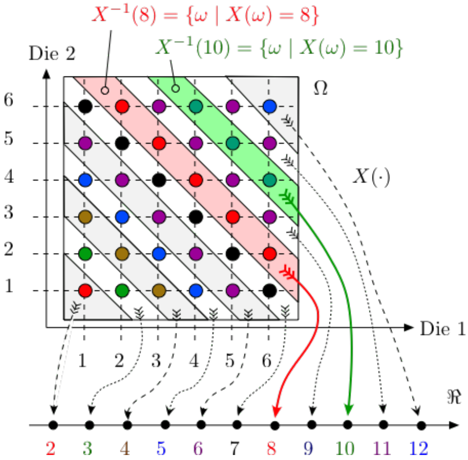

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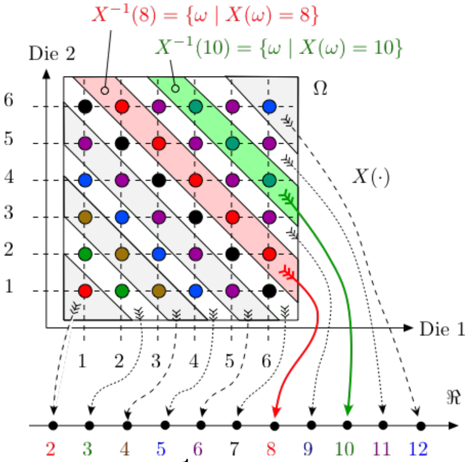

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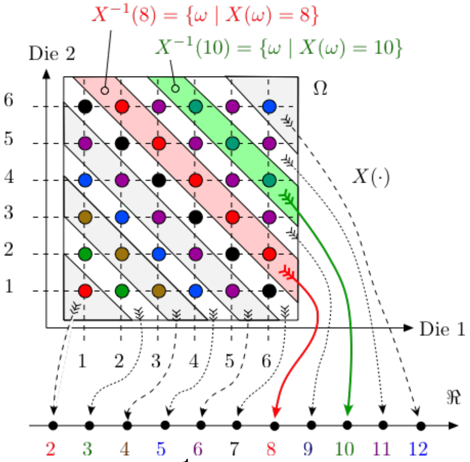

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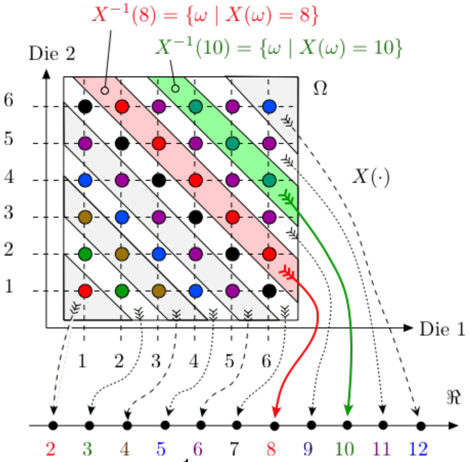

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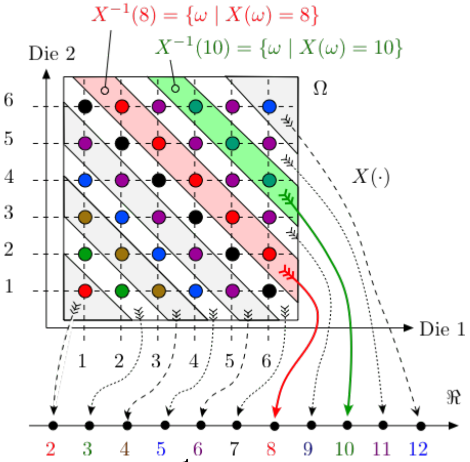

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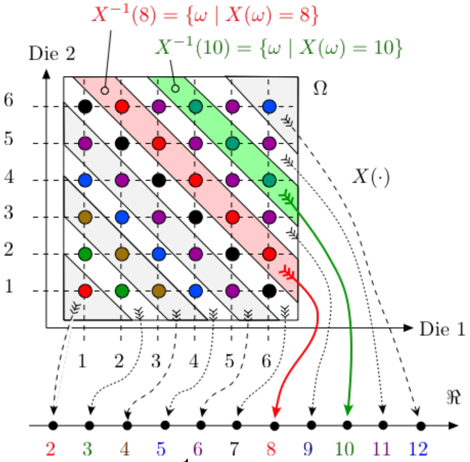

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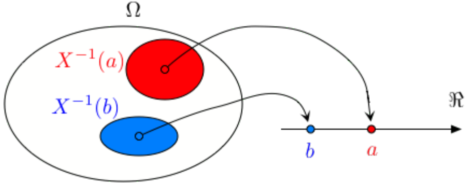

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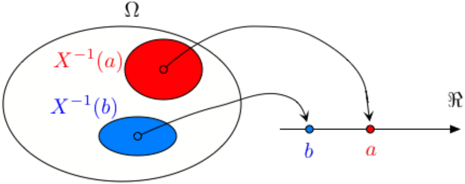

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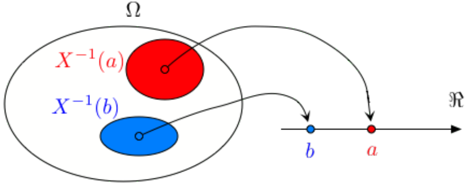

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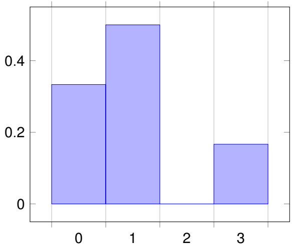

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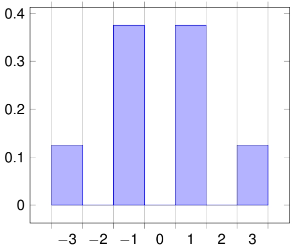

Number of dots.

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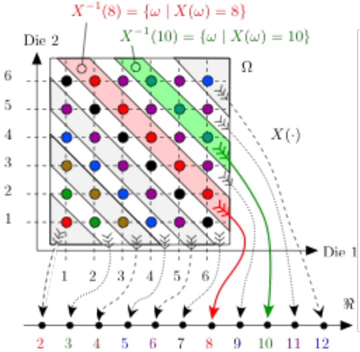

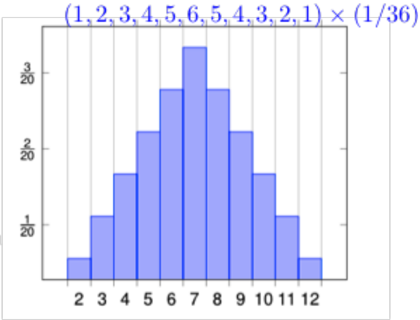

The Bernoulli distribution

Flip a coin, with heads probability p.

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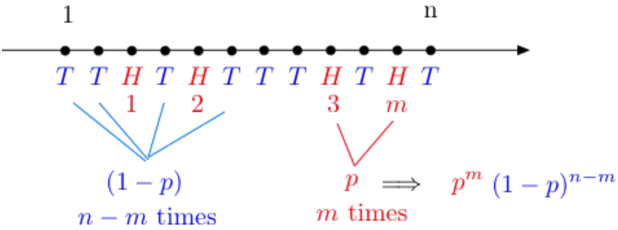
$$Pr[X = i] = \binom{n}{i} p^i (1-p)^{n-i}, i = 0, 1, \ldots, n : B(n,p) \text{ distribution}$$

The binomial distribution.



$\binom{n}{m}$ outcomes with m Hs and $n-m$ Ts

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Expectation.

How did people do on the midterm?

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Distribution.

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Summary of distribution?

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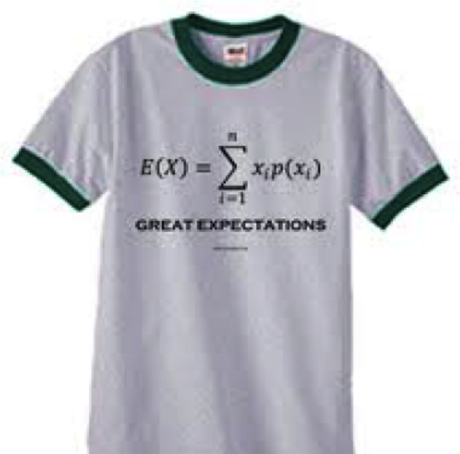

Expectation - Intuition

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We use this frequentist interpretation as a definition.

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This (nontrivial) result is called the Law of Large Numbers.

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Our intuition matches the math.

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Apparently: expected value is not a common value, by any means.

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The random variable X is sometimes written as

$$1\{\omega \in A\} \text{ or } 1_A(\omega).$$

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Using Linearity - 1: Dots on dice

Roll a die n times.

X_m = number of dots on roll m.

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Hence,

$$E[X] = \frac{7n}{2}.$$

Using Linearity - 2: Fixed point.

Hand out assignments at random to n students.

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Note that linearity holds even though the X_m are not independent (whatever that means).

Using Linearity - 3: Binomial Distribution.

Flip n coins with heads probability p.

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Flip n coins with heads probability p. X - number of heads

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Today's gig: St. Petersburg paradox

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We start with a pot of 2 dollars.

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We start with a pot of 2 dollars.

Flip a fair coin.

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Flip a fair coin. If it’s tails, you take the pot.

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Flip a fair coin. If it's tails, you take the pot. If it's heads, I double the pot.

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We start with a pot of 2 dollars.

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So, if the sequence is HHT, you make 8 dollars.

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How much would you we willing to pay?

Today's gig: St. Petersburg paradox

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Today's gig: St. Petersburg paradox

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Is there a trick here?

Today’s gig: St. Petersburg paradox

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What if I didn't have infinite money?

Today’s gig: St. Petersburg paradox

What if I didn’t have infinite money?

Banker	Bankroll	Expected value of lottery
Friendly game	\$100	\$7.56
Millionaire	\$1,000,000	\$20.91
Billionaire	\$1,000,000,000	\$30.86
Bill Gates (2015)	\$79,200,000,000[5]	\$37.15
U.S. GDP (2007)	\$13.8 trillion[6]	\$44.57
World GDP (2007)	\$54.3 trillion[6]	\$46.54
Googolaire	$\$10^{100}$	\$333.14

Summary

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- $E[X] := \sum_a a Pr[X = a]$.

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- $Pr[X = a] := Pr[X^{-1}(a)] = Pr[\{\omega \mid X(\omega) = a\}]$.
- $Pr[X \in A] := Pr[X^{-1}(A)]$.
- The distribution of X is the list of possible values and their probability: $\{(a, Pr[X = a]), a \in \mathscr{A}\}$.
- $g(X, Y, Z)$ assigns the value .... .
- $E[X] := \sum_a a Pr[X = a]$.
- Expectation is Linear.

Summary

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- $B(n, p)$.