Math 132A Project 1

Auto Assembly

Teo Zeng
and
Angelica Zheng

Department of Mathematics
University of California, Santa Barbara

Febuary 18th, 2023

Description of the Problem

Our company produces different types of automobiles, including trucks, small cars, and midsized luxury cars. One of the company's plants, located near Detroit, MI, puts together two kinds of midsized luxury cars. The first model, known as the Family Adventurer, is a four-door sedan that comes with standard features, vinyl seats, plastic interior, and great gas mileage. It is promoted as a wise purchase for middle-class families who are on a tight budget, and every sale of the Family Adventurer earns the company a modest profit of 3,700 dollars. The second model, the Classic Transporter, is a two-door luxury sedan that has custom features, navigational capabilities, leather seats, and a wooden interior. It is advertised as a symbol of wealth for upper-middle-class families, and each sale of the Classic Transporter generates a profit of 5,300 dollars for the company.

At present we are in the process of determining the manufacturing plan for the upcoming month. We must make a decision on the number of Family Adventurers and Classic Transporters to be produced in the plant to maximize the company's profits. We are aware that the plant has a labor-hour capacity of 48,500 hours for the month. Additionally, we know that it takes 6 hours of labor to assemble one Family Adventurer, and 10.5 hours of labor to build one Classic Transporter.

The assembly plant does not manufacture the parts required to put together the two car models. Instead, these parts are delivered from other supplier plants located in Michigan. The car parts, such as tires, windows, doors, steering wheels, and seats, are transported to the assembly plant. For the upcoming month, William is aware that only 20,000 doors will be obtainable from the door supplier. The doors are used in both the Family Adventurer and the Classic Transporter, and the supplier plant was recently impacted by a labor strike, causing the factory to shut down for a few days. As a result, the supplier plant will not be able to fulfill its monthly production quota. The 20,000 doors include 10,000 left-hand doors and 10,000 right-hand doors.

Our company has recently projected the monthly demand for various car models, and the forecast indicates that the demand for the Classic Transporter will be restricted to 3,500 cars. In contrast, there are no limits on the demand for the Family Adventurer, as long as the production capacity of the assembly plant is not exceeded.

Model

We will use linear programming to model our problem, and we will first check that our problem satisfies the four assumptions of linear programming. Let,

x_1 be the number of Family Adventurers to be produced.

x_2 be the number of Classic Transporters to be produced.

z be the profit to be maximized

Now our decision variable is x_1 and x_2 with objective function $ z = 3700x_1 + 5300x_2$

According to the lecture notes, we have

1. **Proportionality**: which means that each decision variable in every equation must appear with a constant coefficient.

   In our case, proportionality is satisfied because the contribution of our decision variables to the objective function is proportional to their values. For example, one Family Adventurer generates a profit of 3700 dollars and two Family Adventurers generate profit of 7400 dollars.

2. **Additivity**: the combined effect of the decision variables in any constraint or in the objective function is the algebraic sum of their individual weighted effects.

   In our case, additivity is satisfied because the contribution to the objective function for any of our variable is independent of the other decision variables. For example, the profit of each Family Adventurer will always be 3700 dollars regardless the profit of each Classic Transporter. And vice versa.

3. **Divisibility or continuity**: the decision variables can take on fractional (non-integer) values.

   In our case, divisibility is satisfied because when the number of cars produced are fractional, we can round up or down to obtain an integer value to satisfy the integer requirement.

4. **Certainty**: all model parameters (that is, the coefficients of the objective function, the coefficients of the constraints) are known deterministically (with certainty).

   In our case, certainty is satisfied because we were given all constraints, coefficients, objective function of the linear programming problem, which are all deterministic.

Having checked that our problem satisfies all four assumption of linear programming

We formulate our problem as the following

$$\begin{cases} \text{Maximize } \rightarrow z = 3700x_1 + 5300x_2 \\ \text{Subject to:} \\ 6x_1 + 10.5x_2 \leq 48500 \\ 4x_1 + 2x_2 \leq 20000 \\ x_2 \leq 3500 \\ x_1 \geq 0, x_2 \geq 0 \end{cases}$$

As we can see in the formulation, we want to maximize the profit z, which is subject to individual profits of each car model. The first constraint was due to labor hours. The second constraint was due to car doors and the third constraint was brought by the company forecast.

Solution of the Model

Now we will solve the problem using Excel:

	A	B	C	D	E	F
1	vars	x1	x2			
2		3766	2467			
3	profit	3700	5300	27009300		
4						
5	1	6	10.5	48499.5	<=	48500
6	2	4	2	19998	<=	20000
7	3	0	1	2467	<=	3500

We found that the optimal solution is when

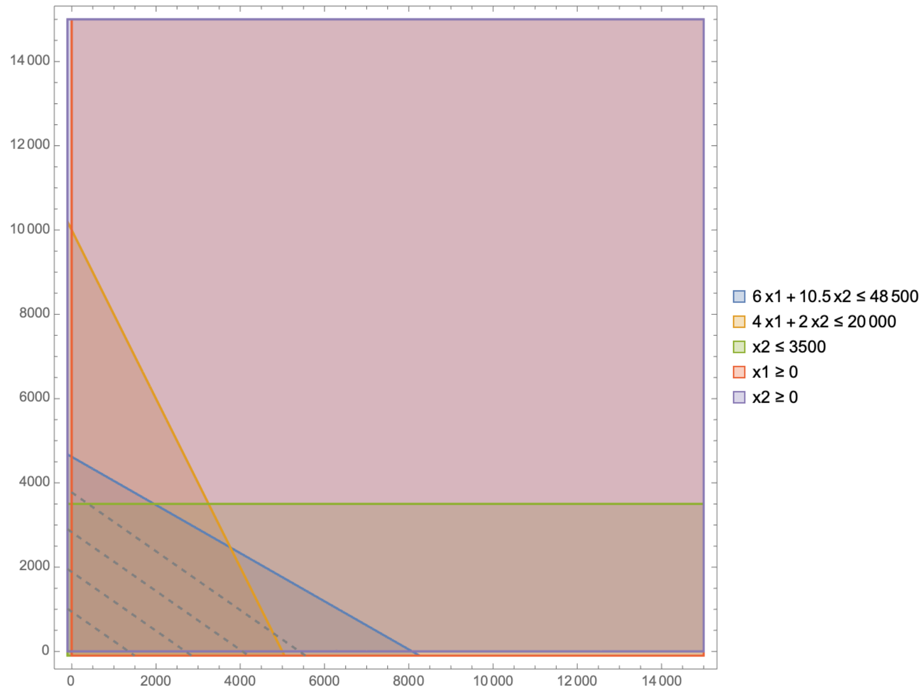
$$\begin{cases} x_1 = 3766 \\ x_2 = 2467 \end{cases}$$

Interpretation of the Solution

The result above means that the objective function is maximized, satisfying all the constraints, when the company produce 3766 Family Adventurers and 2467 Classic Transporters, generating a profit of

$$3700 \times 3766 + 5300 \times 2467 = 27009300 \text{ dollars}$$

To verify that the solution is unique, we plot out the feasible regions as shown below:



We can see from the above plot, that the conditions form a polygon boundary. As we increase the objective function z, the dashed line moved upward. Since the objective function is not parellel to any of the boundary, there exists an unique optimal to this linear programming problem. Hence there exists an unique solution.

Recommendation by Situation

The following corresponds to question (3) - (11) in the prompt

1. The marketing department knows that it can pursue a targeted $500,000 advertising campaign that will raise the demand for the Classic Transporter next month by 20 percent. Should the campaign be undertaken?

   Since the target campain cost $500000, this means that we should take out 500000 in the target function. And since this campaign will raise the demand demand for the Classic Transporter next month by 20 percent, we have the new Linear Programming Problem:

$$\begin{cases} \text{Maximize } \rightarrow z = 3700x_1 + 5300x_2 - 500000 \\ \text{Subject to:} \\ 6x_1 + 10.5x_2 \leq 48500 \\ 4x_1 + 2x_2 \leq 20000 \\ x_2 \leq 4200 \\ x_1 \geq 0, x_2 \geq 0 \end{cases}$$

We found that the optimal solution is when

$$\begin{cases} x_1 = 3766 \\ x_2 = 2467 \end{cases}$$

generating a profit of

$$3700 \times 3766 + 5300 \times 2467 - 500000 = 26509300 \text{ dollars}$$

But this is less than the original profit, which is 27009300 dollars, so William Smith **should not undertake the campaign.**

2. William knows that he can increase next months plant capacity by using overtime labor. He can increase the plants labor-hour capacity by 25 percent. With the new assembly plant capacity, how many Family Adventurers and how many Classic Transporters should be assembled?

   Since William Smith can increase the plant capacity by 25 percent, now he has $48500 \times 1.25 = 60625$ labor hours. Now we have the new linear programming problem:

$$\begin{cases} \text{Maximize } \rightarrow z = 3700x_1 + 5300x_2 \\ \text{Subject to:} \\ 6x_1 + 10.5x_2 \leq 60625 \\ 4x_1 + 2x_2 \leq 20000 \\ x_2 \leq 3500 \\ x_1 \geq 0, x_2 \geq 0 \end{cases}$$

   We found that the optimal solution is when

$$\begin{cases} x_1 = 3250 \\ x_2 = 3500 \end{cases}$$

   generating a profit of

$$3700 \times 3250 + 5300 \times 3500 = 30575000 \text{ dollars}$$

   Therefore, **William Smith should now produce** 3250 **Family Adventurers and** 3500 **Classic Transporters**.

3. William knows that overtime labor does not come without an extra cost. What is the maximum amount he should be willing to pay for all overtime labor beyond the cost of this labor at regular time rates?

   Without the overtime, the company earns a profit of 27009300 dollars. With overtime, the company now earns a profit of 30575000 dollars. **So the maximum amount of money William Smith would like to pay is** 3565700 **dollars.**

$$30575000 - 27009300 = 3565700 \text{ dollars}$$

4. William explores the option of using both the targeted advertising campaign and the overtime labor-hours. The advertising campaign raises the demand for the Classic Transporter by 20 percent, and the overtime labor increases the plants labor-hour capacity by 25 percent.

Implementing both advertising and overtime labor will give the new linear programming problem:

$$\begin{cases} \text{Maximize } \rightarrow z = 3700x_1 + 5300x_2 - 500000 \\ \text{Subject to:} \\ 6x_1 + 10.5x_2 \leq 60625 \\ 4x_1 + 2x_2 \leq 20000 \\ x_2 \leq 4200 \\ x_1 \geq 0, x_2 \geq 0 \end{cases}$$

We found the solution of this linear programming problem to be

$$\begin{cases} x_1 = 2957 \\ x_2 = 4084 \end{cases}$$

which generates a profit of

$$3700 \times 2957 + 5300 \times 4084 - 500000 = 32086100 \text{ dollars}$$

Therefore, William Smith should now produce 2957 **Family Adventurers and** 4084 **Classic Transporters**.

5. Knowing that the advertising campaign costs $\$500,000$ and the maximum usage of overtime labor-hours cost $\$1,600,000$, is the solution found in part (6) a wise decision compared to the solution found in the beginning?

Since the overtime labor costs 1600000 dollars, the new profit for implementing both advertising and overtime labor yields a profit of

$$32086100 - 1600000 = 30486100 \text{ dollars}$$

And we have $30486100 > 27009300$ dollars we implementing both **is a wise decision**.

6. The company has determined that dealerships are actually heavily discounting the price of the Family Adventurers to move them off the lot. Because of a profit-sharing agreement with its dealers, the company is therefore not making a profit of \$3700 on the Family Adventurer but is instead making a profit of $\$2,800$.

   There will be four scenarios. Notice that **we assume that the overtime pay does not cost extra money.**

   1. using both advertising and overtime labor

      Now the linear programming problem becomes:

      $$\begin{cases} \text{Maximize } \rightarrow z = 2800x_1 + 5300x_2 - 500000 \\ \text{Subject to:} \\ 6x_1 + 10.5x_2 \leq 60625 \\ 4x_1 + 2x_2 \leq 20000 \\ x_2 \leq 4200 \\ x_1 \geq 0, x_2 \geq 0 \end{cases}$$

      We found that the solution to this linear programming problem is

      $$\begin{cases} x_1 = 2754 \\ x_2 = 4200 \end{cases}$$

      generating a profit of

      $$2800 \times 2754 + 5300 \times 4200 - 500000 = 29471200 \text{ dollars}$$

   2. using overtime labor only

      Now the linear programming problem becomes:

      $$\begin{cases} \text{Maximize } \rightarrow z = 2800x_1 + 5300x_2 \\ \text{Subject to:} \\ 6x_1 + 10.5x_2 \leq 60625 \\ 4x_1 + 2x_2 \leq 20000 \\ x_2 \leq 3500 \\ x_1 \geq 0, x_2 \geq 0 \end{cases}$$

      We found that the solution to this linear programming problem is

      $$\begin{cases} x_1 = 3250 \\ x_2 = 3500 \end{cases}$$

      generating a profit of

      $$2800 \times 3200 + 5300 \times 3500 = 27650000 \text{ dollars}$$

3. using advertising only

   Now the linear programming problem becomes:

$$\begin{cases} \text{Maximize } \to z = 2800x_1 + 5300x_2 - 500000 \\ \text{Subject to:} \\ 6x_1 + 10.5x_2 \leq 48500 \\ 4x_1 + 2x_2 \leq 20000 \\ x_2 \leq 4200 \\ x_1 \geq 0, x_2 \geq 0 \end{cases}$$

   We found that the solution to this linear programming problem is

$$\begin{cases} x_1 = 735 \\ x_2 = 4199 \end{cases}$$

   generating a profit of

$$2800 \times 735 + 5300 \times 4199 - 500000 = 23812700 \text{ dollars}$$

4. using neither advertising nor overtime labor

   Now the linear programming problem becomes:

$$\begin{cases} \text{Maximize } \to z = 2800x_1 + 5300x_2 \\ \text{Subject to:} \\ 6x_1 + 10.5x_2 \leq 48500 \\ 4x_1 + 2x_2 \leq 20000 \\ x_2 \leq 3500 \\ x_1 \geq 0, x_2 \geq 0 \end{cases}$$

   We found that the solution to this linear programming problem is

$$\begin{cases} x_1 = 1960 \\ x_2 = 3499 \end{cases}$$

   generating a profit of

$$2800 \times 1960 + 5300 \times 3499 = 24032700 \text{ dollars}$$

Comparing all four scenarios, option 1 generates a profit of 29471200 dolloars, which is the most. So William Smith should **consider implenting both advertising and overtime labor.**

7. The company has discovered quality problems with the Family Adventurer by randomly testing Adventurers at the end of the assembly line. Inspectors have discovered that in over 60 percent of the cases, two of the four doors on an Adventurer do not seal properly. Because the percentage of defective Adventurers determined by the random testing is so high, the floor supervisor has decided to perform quality control tests on every Adventurer at the end of the line. Because of the added tests, the time it takes to assemble one Family Adventurer has increased from 6 to 7.5 hours.

   With the new time to assemble the Adventurer, assume that William is not implementing advertisesment or overtime labor, and the profit of adventurer did not drop, we have the linear programming problem as follows:

$$\begin{cases} \text{Maximize } \rightarrow z = 3700x_1 + 5300x_2 \\ \text{Subject to:} \\ 7.5x_1 + 10.5x_2 \leq 48500 \\ 4x_1 + 2x_2 \leq 20000 \\ x_2 \leq 3500 \\ x_1 \geq 0, x_2 \geq 0 \end{cases}$$

   We found that the optimal solution is when

$$\begin{cases} x_1 = 1568 \\ x_2 = 3499 \end{cases}$$

   generating a profit of

$$3700 \times 1568 + 5300 \times 3499 = 24346300 \text{ dollars}$$

   Therefore, William Smith should now produce 1568 **Family Adventurers and** 3499 **Classic Transporters**.

8. The board of directors of the automobile company wishes to capture a larger share of the luxury sedan market and therefore would like to meet the full demand for Classic Trasnporters. They ask William to determine by how much the profit of his assembly plant would decrease as compared to the profit found in part (1). They then ask him to meet the full demand for Classic Transporters if the decrease in profit is not more than $\$2,000,000$.

   If the full demand of Classic Transporters are met, then the Linear Programming Problem becomes:

$$\begin{cases} \text{Maximize } \rightarrow z = 3700x_1 + 5300x_2 \\ \text{Subject to:} \\ 6x_1 + 10.5x_2 \leq 48500 \\ 4x_1 + 2x_2 \leq 20000 \\ x_2 = 3500 \\ x_1 \geq 0, x_2 \geq 0 \end{cases}$$

We found that the optimal solution is when

$$\begin{cases} x_1 = 1958 \\ x_2 = 3500 \end{cases}$$

generating a profit of

$$3700 \times 1958 + 5300 \times 3500 = 25403000 \text{ dollars}$$

And comparing the profit in (1),

$$27009300 - 25403000 = 1606300 \text{ dollars} < 2000000 \text{ dollars}$$

So **William can meet the full demand.**

9. William now makes his final decision by combining all the new considerations described in parts (6), (7), and (8). What are his final decisions on whether to undertake the advertising campaign?

   Combining all the situations in (6), (7), (8), we formulate the linear programming problem as follows:

$$\begin{cases} \text{Maximize } \rightarrow z = 2800x_1 + 5300x_2 - 500000 - 1600000 \\ \text{Subject to:} \\ 6x_1 + 10.5x_2 \leq 60625 \\ 4x_1 + 2x_2 \leq 20000 \\ x_2 \leq 4200 \\ x_1 \geq 0, x_2 \geq 0 \end{cases}$$

   We found that the solution to this linear programming problem is

$$\begin{cases} x_1 = 2754 \\ x_2 = 4200 \end{cases}$$

   generating a profit of

$$2800 \times 2754 + 5300 \times 4200 - 500000 - 1600000 = 27871200 \text{ dollars}$$

$$27871200 \text{ dollars} > 27009300 \text{ dollars}$$

Since the profit is higher than the original profit based on the decisions made in part(6),(7),(8), **William Smith should now produce** 2754 **Family Adventurers and** 4200 **Classic Transporters**.

Conclusion

Given all William Smith's options and possible situations, he should **undertake advertisement and use overtime labor** in any circumstances in order to maximize profit.