REVIEW SUMMARY

OPTICS

Exceptional points in optics and photonics

Mohammad-Ali Miri and Andrea Alù*

BACKGROUND: Singularities are critical points for which the behavior of a mathematical model governing a physical system is of a fundamentally different nature compared to the neighboring points. Exceptional points are spectral singularities in the parameter space of a system in which two or more eigenvalues, and their corresponding eigenvectors, simultaneously coalesce. Such degeneracies are peculiar features of nonconservative systems that exchange energy with their surrounding environment. In the past two decades, there has been a growing interest in investigating such nonconservative systems, particularly in connection with the quantum mechanics notions of parity-time symmetry, after the realization that some non-Hermitian Hamiltonians exhibit entirely real spectra. Lately, non-Hermitian systems have raised considerable attention in photonics, given that optical gain and loss can be integrated as nonconservative ingredients to create artificial materials and structures with altogether new optical properties.

ADVANCES: As we introduce gain and loss in a nanophotonic system, the emergence of exceptional point singularities dramatically alters the overall response, leading to a range of exotic functionalities associated with abrupt phase transitions in the eigenvalue spectrum. Even though such a peculiar effect has been known theoretically for several years, its controllable realization has not been made possible until recently and with advances in exploiting gain and loss in guided-wave photonic systems. As shown in a range of recent theoretical and experimental works, this property creates opportunities for ultrasensitive measurements and for manipulating the modal content of multimode lasers. In addition, adiabatic parametric evolution around exceptional points provides interesting schemes for topological energy transfer and designing mode and polarization converters in photonics. Lately, non-Hermitian degeneracies have also been exploited for the design of laser systems, new nonlinear optics phenomena, and exotic scattering features in open systems.

OUTLOOK: Thus far, non-Hermitian systems have been largely disregarded owing to the dominance of the Hermitian theories in most areas of physics. Recent advances in the theory of non-Hermitian systems in connection with exceptional point singularities has revolutionized our understanding of such complex systems. In the context of optics and photonics, in particular, this topic is highly important because of the ubiquity of nonconservative elements of gain and loss. In this regard, the theoretical developments in the field of non-Hermitian physics have allowed us to revisit some of the well-established platforms with a new angle of utilizing gain and loss as new degrees of freedom, in stark contrast with the traditional approach of avoiding these elements. On the experimental front, progress in fabrication technologies has allowed for harnessing gain and loss in chip-scale photonic systems. These theoretical and experimental developments have put forward new schemes for controlling the functionality of micro- and nanophotonic devices. This is mainly based on the anomalous parameter dependence in the response of non-Hermitian systems when operating around exceptional point singularities. Such effects can have important ramifications in controlling light in new nanophotonic device designs, which are fundamentally based on engineering the interplay of coupling and dissipation and amplification mechanisms in multimode systems. Potential applications of such designs reside in coupled-cavity laser sources with better coherence properties, coupled nonlinear resonators with engineered dispersion, compact polarization and spatial mode converters, and highly efficient reconfigurable diffraction surfaces. In addition, the notion of the exceptional point provides opportunities to take advantage of the inevitable dissipation in environments such as plasmonic and semiconductor materials, which play a key role in optoelectronics. Finally, emerging platforms such as optomechanical cavities provide opportunities to investigate exceptional points and their associated phenomena in multiphysics systems. ■

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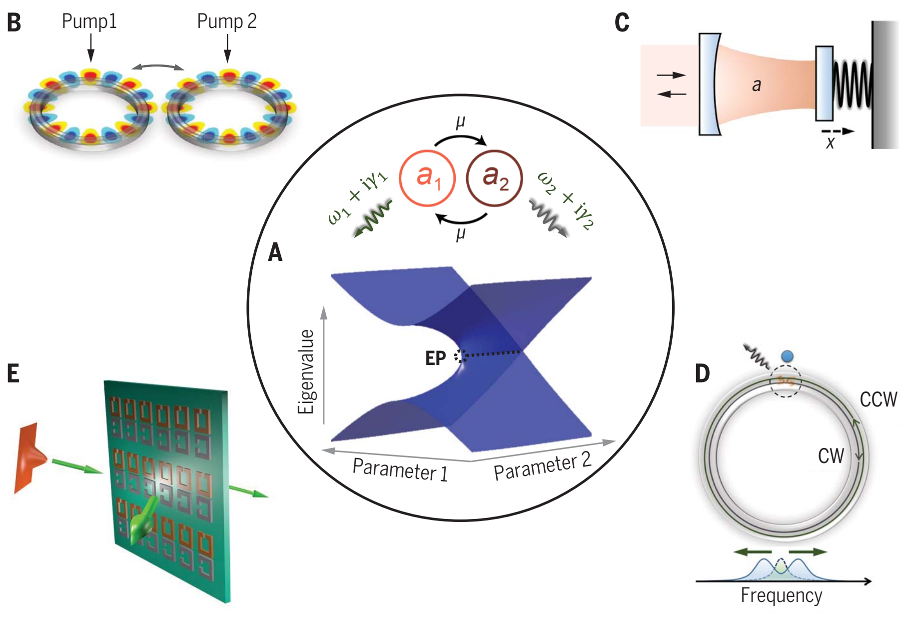

Ubiquity of non-Hermitian systems, supporting exceptional points, in photonics. (**A**) A generic non-Hermitian optical system involving two coupled modes with different detuning, $\pm\omega_{1,2}$, and gain-loss values, $\pm\gamma_{1,2}$, coupled at rate of μ. The real part of the associated eigenvalues in a two-dimensional parameter space of the system, revealing the emergence of an exceptional point (EP) singularity. a_1 and a_2 are the modal amplitudes. (**B** to **E**) A range of different photonic systems, which are all governed by the coupled-mode equations. (B) Two coupled lasers pumped at different rates. (C) Dynamical interaction between optical and mechanical degrees of freedom in an optomechanical cavity. (D) A resonator with counter-rotating whispering gallery modes. CW, clockwise; CCW, counterclockwise. (E) A thin metasurface composed of coupled nanoantennas as building blocks.

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REVIEW

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Exceptional points in optics and photonics

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Exceptional points are branch point singularities in the parameter space of a system at which two or more eigenvalues, and their corresponding eigenvectors, coalesce and become degenerate. Such peculiar degeneracies are distinct features of non-Hermitian systems, which do not obey conservation laws because they exchange energy with the surrounding environment. Non-Hermiticity has been of great interest in recent years, particularly in connection with the quantum mechanical notion of parity-time symmetry, after the realization that Hamiltonians satisfying this special symmetry can exhibit entirely real spectra. These concepts have become of particular interest in photonics because optical gain and loss can be integrated and controlled with high resolution in nanoscale structures, realizing an ideal playground for non-Hermitian physics, parity-time symmetry, and exceptional points. As we control dissipation and amplification in a nanophotonic system, the emergence of exceptional point singularities dramatically alters their overall response, leading to a range of exotic optical functionalities associated with abrupt phase transitions in the eigenvalue spectrum. These concepts enable ultrasensitive measurements, superior manipulation of the modal content of multimode lasers, and adiabatic control of topological energy transfer for mode and polarization conversion. Non-Hermitian degeneracies have also been exploited in exotic laser systems, new nonlinear optics schemes, and exotic scattering features in open systems. Here we review the opportunities offered by exceptional point physics in photonics, discuss recent developments in theoretical and experimental research based on photonic exceptional points, and examine future opportunities in this area from basic science to applied technology.

Hermiticity is a property of a wide variety of physical systems, under the assumptions of being conservative and obeying time-reversal symmetry. Hermitian operators play a key role in the theory of linear algebraic and differential operators (*1*–*4*), and they are known to exhibit real-valued eigenvalues, a property that stems from energy conservation. For a set of dynamical equations described through a Hermitian operator, the relation between initial and final states is governed by a unitary operation. Hermiticity has long been considered one of the pillars of mathematical and physical models, such as in quantum mechanics and electromagnetics. The elegance of such theories lies in powerful properties, including the completeness and orthogonality of the eigenbasis of the governing operators (*1*). However, these models are based on idealizations, like the assumption of complete isolation of a system from its surrounding environment. In principle, nonconservative elements arise ubiquitously in various forms; thus, a proper description of a realistic physical system requires a non-Hermitian Hamiltonian. Generally, nonconservative phenomena are introduced as small perturbations to otherwise Hermitian systems. Thus, the overall behavior of non-Hermitian systems has been largely extracted from their Hermitian counterparts. However, recent investigations have revealed that non-Hermitian phenomena can drastically alter the behavior of a system compared to its Hermitian counterpart. The best example of such deviation is the emergence of singularities, so-called exceptional points, at which two or more eigenvalues, and their associated eigenvectors, simultaneously coalesce and become degenerate (*5*).

The term "exceptional point" was first introduced in studying the perturbation of linear non-Hermitian operators (*6*), described by a general class of matrices $H(z)$ parameterized by the complex variable $z = x + iy$, where x is the real part, i is the imaginary unit, and y is the imaginary part. The eigenvalues $\sigma_n(z)$ and eigenvectors $|\psi_n(z)\rangle$ of H can be represented as analytic functions except at certain singularities $z = z_{EP}$ (EP, exceptional point). At such exceptional points, two eigenvalues coalesce, and the matrix H can no longer be diagonalized. The physical importance of exceptional points was pointed out in early works (*7*, *8*), in which the terminology of non-Hermitian degeneracy was used to distinguish such critical points from regular degeneracies occurring in Hermitian systems (*9*, *10*). In addition, exceptional points were referred to as branch-point singularities in investigating the quantum theory of resonances in the context of atomic, molecular, and nuclear reactions (*11*). Early experiments on microwave cavities revealed the peculiar topology of eigenvalue surfaces near exceptional points (*12*, *13*). The emergence of spectral singularities was also pointed out in the analysis of multimode laser cavities (*14*, *15*) and in time-modulated complex light potentials for matter waves (*16*).

Recently, interest in these peculiar spectral degeneracies has been sparked in a particular family of non-Hermitian Hamiltonians, the so-called parity-time (PT) symmetric systems. A Hamiltonian is PT symmetric as long as it commutes with the $\mathcal{PT}$ operator, that is, $[\mathcal{H}, \mathcal{PT}] = 0$, where the parity operator $\mathcal{P}$ represents a reflection with respect to a center of symmetry and the time operator $\mathcal{T}$ represents complex conjugation. It has been realized that PT-symmetric Hamiltonians, despite being non-Hermitian, can support entirely real eigenvalue spectra (*17*). More interestingly, it has been realized that commuting with the $\mathcal{PT}$ operator is not sufficient to ensure a real spectrum, as formally PT-symmetric Hamiltonians can undergo a phase transition to the spontaneously broken symmetry regime, in which complex eigenvalues appear. The phase transition happens as a result of a parametric variation in the Hamiltonian. Quite interestingly, the symmetry-breaking threshold point exhibits all properties of an exceptional point singularity (*17*–*23*).

Although these theoretical explorations originated in the realm of quantum mechanics, optics and photonics have proven to be the ideal platform to experimentally observe and utilize the rich physics of exceptional points (*24*–*27*). Owing to the abundance of nonconservative processes, photonics provides the necessary ingredients to realize controllable non-Hermitian Hamiltonians. Indeed, dissipation is ubiquitous in optics, because it arises from material absorption as well as radiation leakage to the outside environment. In addition, gain can be implemented in a locally controlled fashion through stimulated emission, which involves optical or electrical pumping of energy through an external source, or through parametric processes. Therefore, photonics provides a fertile ground to systematically investigate non-Hermitian Hamiltonians and exceptional points. Recent theoretical developments in the area of non-Hermitian physics have opened exciting opportunities to revisit fundamental concepts in nonconservative photonic systems with gain and loss, such as lasers, sensors, absorbers, and isolators. In these systems, exceptional points open pathways for totally new functionalities and performance. The interested reader may find detailed overviews of non-Hermitian and, in particular, PT-symmetric systems in the context of optics and photonics in recent review papers (*28*–*32*). In the present work, we discuss instead more broadly the concept of exceptional points in non-Hermitian systems. In the following, we provide an introduction to exceptional point physics and explain some of the fundamental concepts associated with such critical points. We then draw the connection with optics and photonics and show the universal occurrence of

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exceptional points in optical settings. Finally, we review recent theoretical and experimental efforts in observing exceptional points in optics and their peculiar functionalities in practical devices, presenting an outlook for the future of this exciting area of research.

Theoretical background

We begin by investigating exceptional points in a generic two-level system. Assuming that $a_{1,2}$ are the modal amplitudes of two states that evolve with the variable ξ, representing the evolution time or propagation distance, the coupled mode equations can be generally written as

$$\frac{\mathrm{d}}{\mathrm{d}\xi}\begin{pmatrix} a_1 \\ a_2 \end{pmatrix} = -\mathrm{i}\begin{pmatrix} \omega_1 - \mathrm{i}\gamma_1 & \mu \\ \mu & \omega_2 - \mathrm{i}\gamma_2 \end{pmatrix}\begin{pmatrix} a_1 \\ a_2 \end{pmatrix} \tag{1}$$

where ω is the resonance frequency of the two coupled modes, μ is the coupling coefficient, and γ is their decay rate. This particular choice of Hamiltonian system, shown in Fig. 1A, represents a large class of structures and devices of large relevance in photonics, examples of which are given in Fig. 1, such as coupled cavities (Fig. 1B) (*33*), coupled waveguides (Fig. 1C) (*34*), polarization states in the presence of small perturbations in an optical waveguide (Fig. 1D) (*35*), counter-propagating waves in Bragg gratings (Fig. 1E) (*36*), wave mixing in nonlinear crystals (Fig. 1F) (*37*), coupled optical and mechanical modes in an optomechanical cavity (Fig. 1G) (*38*), and a two-level atom in a cavity (Fig. 1H) (*39*). In the case of coupled optical resonators, for instance, $\omega_{1,2}$ in Eq. 1 represent the individual frequencies of each element, $\gamma_{1,2}$ describe their loss or gain rate, and μ represents the mutual coupling. Assuming, harmonic solutions of the form $(a_1, a_2) = (\mathrm{a}_1, \mathrm{a}_2)\mathrm{e}^{-\mathrm{i}\sigma\xi}$, the eigenvalues of the system are

$$\sigma_{\pm} = \omega_{\mathrm{ave}} - \mathrm{i}\gamma_{\mathrm{ave}} \pm \sqrt{\mu^2 + (\omega_{\mathrm{diff}} + \mathrm{i}\gamma_{\mathrm{diff}})^2} \tag{2}$$

where $\omega_{\mathrm{ave}} = (\omega_1 + \omega_2)/2$ and $\gamma_{\mathrm{ave}} = (\gamma_1 + \gamma_2)/2$, respectively, represent the mean values of resonance frequencies and loss factors, whereas $\omega_{\mathrm{diff}} = (\omega_1 - \omega_2)/2$ and $\gamma_{\mathrm{diff}} = (\gamma_1 + \gamma_2)/2$ are the differences between their resonance frequencies and loss factors.

The Hamiltonian in Eq. 1 is a function of multiple parameters. In Fig. 2, A and B, we evaluate the evolution of real and imaginary parts of the eigenvalues in the parameter space (ω_{diff}, γ_{diff}), assuming a constant coupling coefficient μ. An exceptional point occurs when the square-root term in Eq. 2 is zero, as the two eigenvalues coalesce. Assuming a real coupling constant, this happens for ($\omega_{\mathrm{diff}} = 0$; $\gamma_{\mathrm{diff}} = \pm\mu$). Figure 2, A and B, highlights the interesting topology of the branch point singularity at the exceptional point, which has important implications in the optical response of the system around this parameter point, as we discuss in the following sections.

The two-body problem investigated here is the simplest case of a non-Hermitian system. In general, exceptional points appear ubiquitously in systems with spatially discrete or continuous degrees of freedom of multiple dimensionalities. In principle, when more than two eigenvalue surfaces are involved, it is also possible that more than two surfaces simultaneously collapse at one point, creating a higher-order exceptional point (*40*, *41*). A third-order exceptional point, for example, is formed when three eigenvalues simultaneously coalesce. In this scenario, the square-root dependence of the eigenvalues around the exceptional point in Eq. 2 is replaced by a cubic root. It is worth stressing that at an exceptional point, the coalescing eigenvalues do not support independent eigenvectors, implying that, in discrete systems described by a matrix Hamiltonian, the Jordan form is no longer diagonal (*42*). This is notably different from accidental degeneracies, which occur when two eigenvalues with different eigenvectors cross. In a two-dimensional parameter space, such accidental degeneracies appear when two eigenvalue surfaces form a double cone or "diablo," forming diabolic points (*43*). In contrast with exceptional points, at the diabolic points, the eigenvectors remain linearly independent. Diabolic points emerge in various Hermitian systems, most notably in molecular reactions (*44*) and in the electronic band diagram of graphene (*45*).

Exceptional point singularities are closely related to the phenomenon of level repulsion, which has been originally explored in the context of quantum chaos, because it explains the scarcity of closely spaced levels in Wigner distributions (*46*). In photonics, level repulsion is of great interest because it marks strong coupling and hybridization between states, which is manifested as a repulsion between closely spaced eigenvalues when a parameter is adiabatically

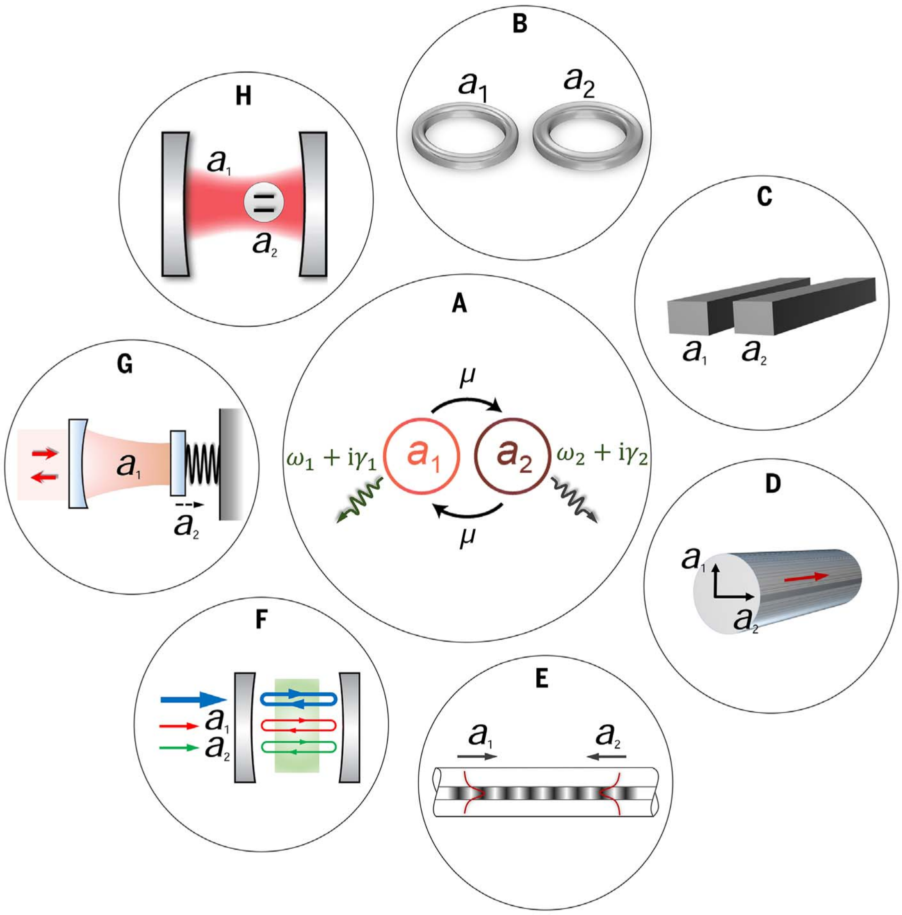

Fig. 1. A generic two-level system and its different realizations in optics and photonics. (**A**) A schematic representation of a generic two-level system composed of two coupled entities. (**B**) Two coupled optical cavities with spatially separated resonator modes. (**C**) Two evanescently coupled optical waveguides with spatially separated waveguide modes. (**D**) Coupled orthogonal polarization states in an optical waveguide. (**E**) Counter-propagating waves in a volume Bragg grating. (**F**) Signal and idler frequency components in a parametric amplifier. (**G**) Photonic and phononic degrees of freedom in an optomechanical cavity. (**H**) Coupling between a two-level atom and an optical cavity mode. The different platforms represented in (B) to (H) can be treated under a unified model depicted schematically in (A). The universality of nonconservative processes in these settings calls for a systematic understanding of non-Hermiticity in a basic two-level system as a first step toward a rigorous bottom-up approach for designing complex photonic systems in the presence of gain and loss. The arrows indicate electromagnetic waves, and different colors indicate different frequencies.

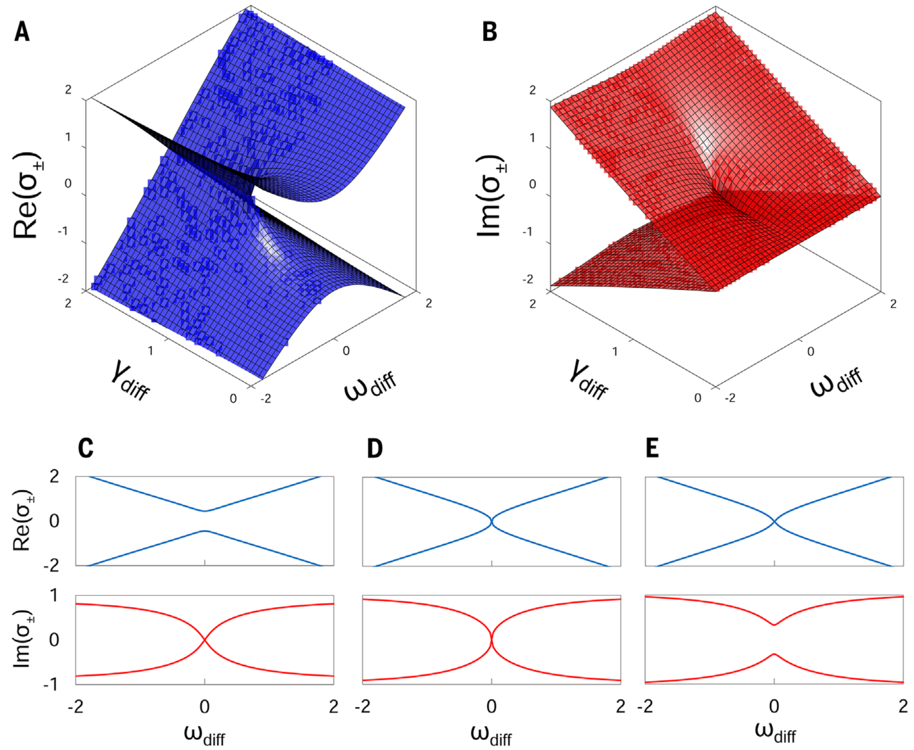

Fig. 2. Exceptional points in a non-Hermitian two-level system. (**A** and **B**) Evolution of the real (A) and imaginary (B) parts of the eigenvalues of the system described by Eq. 1 in the two-dimensional parameter space (ω_{diff}, γ_{diff}). These panels illustrate the exotic topology of the eigenvalue surfaces near an exceptional point singularity. (**C** to **E**) Eigenvalues versus ω_{diff} for different values of γ_{diff}, that is, cross sections of the surfaces depicted in (A) and (B). Owing to the presence of the exceptional point ($\gamma_{diff} = \gamma_{EP}$; $\omega_{diff} = \omega_{EP}$), depending on the value of the secondary parameter, different parameter dependence is observed for the eigenvalues. (C) For $\gamma_{diff} > \gamma_{EP}$, level repulsion occurs in the real part of the eigenvalues, whereas the imaginary parts cross. (D) For $\gamma_{diff} = \gamma_{EP}$, the real and imaginary parts coalesce at $\omega_{diff} = \omega_{EP}$. (E) For $\gamma_{diff} < \gamma_{EP}$, level crossing governs the real parts of the eigenvalues, whereas the imaginary parts repel each other.

tuned (*47*). They typically occur near an exceptional point in the real or complex parameter space. For instance, Fig. 2, C to E, shows cross sections of the eigenvalue surfaces in Fig. 2, A and B, for different values of γ_{diff}, highlighting level repulsion in either their real (Fig. 2C) or imaginary part (Fig. 2E) for values of γ_{diff} respectively larger or smaller than the critical value $\gamma_{diff} = \gamma_{EP}$, corresponding to the exceptional point condition (Fig. 2D). Level repulsion in the real (imaginary) part is accompanied by level crossing of the imaginary (real) part, as shown in Fig. 2, C to E (*48*, *49*). At the critical condition $\gamma_{diff} = \gamma_{EP}$, both real and imaginary parts of the eigenvalues coalesce, and an exceptional point is achieved. The different behavior in the three cases is determined by the topology of the involved Riemann surfaces at the given cross section. As a special case, level repulsion can arise also in Hermitian systems, such as in the case of two lossless optical resonators, in which level repulsion occurs as we detune their resonance frequency (*33*). Consistent with Fig. 2C, this phenomenon is associated with an exceptional point in the complex parameter space, as we operate at $\gamma_{diff} = 0 < \gamma_{EP}$.

In the context of exceptional points, an especially relevant class of non-Hermitian two-level systems are those satisfying PT symmetry. In the context of quantum mechanics, a Hamiltonian $\mathcal{H}$ is PT symmetric when $[\mathcal{H}, \mathcal{PT}] = 0$, where $\mathcal{P}$ and $\mathcal{T}$ respectively represent parity and time operators. In photonics, this corresponds to the case in which loss in one region is balanced by gain in another symmetric region (*50*). For the two-level system of Eq. 1, considering that the parity and time operators respectively act as $\mathcal{P}(a,b) = (b,a)$ and $\mathcal{T}(a,b) = (a^*, b^*)$, where a and b are two variables, the conditions of PT symmetry are satisfied for $\omega_1 = \omega_2 \equiv \omega$ and $\gamma_1 = -\gamma_2 \equiv \gamma$. The response of this system is governed by the interplay of two major processes: the gain and loss contrast γ and the mutual coupling μ. An exceptional point arises at the critical condition $\mu = \gamma$. Here, the exceptional point marks the onset of a transition from purely real eigenvalues, associated with oscillatory solutions $\exp(\pm i|\sigma_\pm|\xi)$, where ξ is the evolution variable, to purely imaginary eigenvalues associated with growing or decaying solutions $\exp(\pm|\sigma_\pm|\xi)$. This transition is often referred to as spontaneous symmetry breaking, because the eigenvalues change their behavior despite the fact that the governing evolution operator preserves its symmetry. The behavior of the eigenvalues of a PT-symmetric system is shown in Fig. 3A, highlighting the bifurcation associated with the spontaneous symmetry breakdown at the exceptional point.

In Eq. 1, we assumed that the coupling μ is a real parameter, whereas in principle, it can become complex, involving dissipation. For instance, in several scenarios, coupling between two states is mediated through a continuum of radiation modes, for which the energy partially leaks to the outside environment (*51*). Examples include radiative coupling between subwavelength nanoparticles (*52*) as well as channel-mediated coupling of microring lasers (*53*). Independent of the coupling mechanism, exceptional points also arise in this case. According to Eq. 2, assuming a purely imaginary coupling $\mu = i\mu_i$, exceptional points emerge for ($\omega_{diff} = \pm\mu_i$; $\gamma_{diff} = 0$). In this case, the exceptional point arises for a frequency detuning equal to the mutual coupling between cavities.

The discussion on exceptional points presented so far has been built on Hamiltonian systems, or, in broader terms, on dynamical systems, that evolve in time and space through a linear operator. A large body of photonic systems, however, are open, coupled to a continuum of radiation modes, as in the case of optical waveguides coupled to cavities or finite-sized scatterers illuminated by impinging optical fields. Such systems are better described through a scattering matrix, which directly relates outgoing waves and incoming waves. The scattering matrix can be compared with the time-evolution operator, that is, $\mathcal{U} = \exp(-i\mathcal{H}\xi)$ in Hamiltonian systems. Indeed, in a scattering medium without material gain or loss, the scattering matrix is unitary, with all its eigenvalues located on the unit circle (*54*). In the presence of loss and/or gain, however, the norms are not preserved, and the eigenvalues can, in general, be located inside or outside the unit circle. Quite interestingly, similar to Hamiltonian systems, exceptional points can also emerge in the scattering matrix formalism when two or more eigenvalues and their associated eigenvectors coalesce (*55*). A basic example is a PT-symmetric Fabry-Perot resonator involving two materials with balanced gain and loss (Fig. 3B). At a given frequency, for an increasing gain and loss contrast, the scattering-matrix eigenvalues bifurcate from the unit circle at an exceptional point singularity, as shown in Fig. 3B. Here, the exceptional point marks the onset of the broken symmetry regime, in which amplification of the wave excitation becomes the dominant response of the PT-symmetric scatterer.

Exceptional points in photonics

Exceptional points arise in several optical and photonic systems. In the previous section, we introduced a general class of two-level systems described through coupled-mode equations, pointing out the conditions to achieve a second-order exceptional point. Integrated photonic waveguides and cavities, in particular, provide a controllable platform to observe exceptional points. In

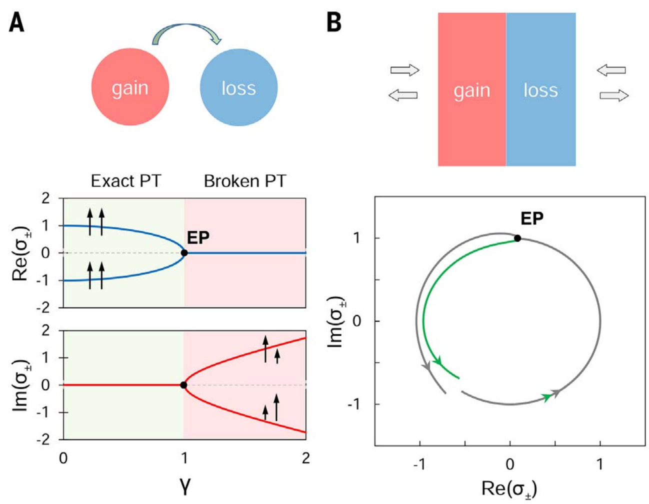

Fig. 3. PT symmetry in closed and open systems. PT-symmetric systems form an interesting class of non-Hermitian settings, which share certain similarities with Hermitian systems. In the case of a two-level system (Fig. 1), PT symmetry is realized for $\omega_1 = \omega_2 \equiv \omega$ and $\gamma_1 = -\gamma_2 \equiv \gamma$, that is, when the individual levels share the same real part but exhibit opposite values of the imaginary parts (gain and loss). (**A**) A PT-symmetric system of two coupled waveguides (top) with gain (red) and loss (blue), and the corresponding eigenvalues (bottom) versus the gain-loss contrast γ. This figure reveals a transition in the eigenvalues from purely real (exact PT symmetry) to purely imaginary (broken PT symmetry). Interestingly, the PT symmetry–breaking threshold point reveals all the properties of an exceptional point singularity. In this figure, the arrows represent the intensity of the eigenmodes in both the exact and broken PT regimes. (**B**) A PT-symmetric Fabry-Perot resonator (top) and the eigenvalues of its scattering matrix (bottom) evolving as a function of the frequency of excitation. In this case, an exceptional point marks a transition in the eigenvalue evolution, breaking away from the unit circle. The geometries of (A) and (B) represent examples of Hamiltonian and scattering settings.

integrated photonic platforms, exceptional points and phase transitions have been observed in coupled passive optical waveguides, where controllable loss in one of the channels was utilized (56) (Fig. 4, A and B). In the context of PT symmetry, spontaneous symmetry breaking at the exceptional point was demonstrated in a coupled arrangement of optical waveguides with balanced gain and loss (50). In other works, coupled optical cavities with gain and loss were utilized to observe a PT-symmetric phase transition (57, 58) (Fig. 4, D and E). The first demonstration of exceptional points in periodic structures was achieved in time-domain lattices (59) (Fig. 4C), induced through the propagation of short laser pulses in two coupled fiber loops of a slightly different lengths with alternating gain and loss. This propagation creates a quantum walk of pulses governed by PT-symmetric evolution equations, described through a peculiar band structure as in spatially periodic structures. In addition, exceptional points have been demonstrated in photonic crystal slabs (60), in which out-of-plane radiation losses due to the finite thickness of the dielectric slab result in the merging of two eigenfrequency bands, inducing a ring of exceptional points in the wave number space. Among other realizations, exceptional points have also been experimentally demonstrated in chaotic optical cavities (61). In all these photonic systems, operation around the exceptional points enables a singular optical response.

The peculiar properties of exceptional points have also been investigated in open scattering systems involving gain and loss. In particular, it has been shown that a PT-symmetric Fabry-Perot cavity, similar to the one discussed in Fig. 3B, can simultaneously act as a laser and a coherent perfect absorber at the exceptional point (55, 62). This interesting behavior, occurring as a result of the coalescence of a pair of poles and zeroes of the scattering matrix eigenvalue, has been recently demonstrated in an integrated semiconductor resonator with active and passive regions (63). Non-Hermitian optical gratings with alternating layers of materials with different levels of loss or gain reveal another interesting aspect of exceptional points (64, 65). In such systems, whereas reciprocity enforces equal transmission in both directions, the reflection coefficients can be completely different. In a Hermitian system, equal transmission coefficients also require equal magnitude of the reflection coefficients, but in non-Hermitian systems, this is not the case. The contrast in reflection amplitudes is maximized at the exceptional point, where the reflection from one direction becomes zero and the reflection from the other direction can be very large, thus inducing unidirectional invisibility (65). In a similar fashion, it has been shown that a two-layer structure with gain and loss can exhibit one-way reflectionless behavior at a particular frequency, thus inducing an anisotropic transmission resonance (66). At the exceptional point, the photonic bandgap closes, whereas the coupling between counter-propagating waves becomes unidirectional (67). Unidirectional invisibility has been observed in different settings, including in integrated semiconductor waveguide gratings (68), organic composite films (69), time-domain lattices (59), and coupled acoustic resonators (70). Similar ideas have been utilized in microring resonators to create integrated laser devices supporting modes with definite angular momentum when the system is biased at an exceptional point (71). In addition, it has been shown that properly engineered defects in microring resonators can create an exceptional point that instead induces chirality between counter-rotating modes (72–74). It has also been shown that non-Hermitian scattering systems operating around the exceptional points can induce other interesting phenomena, such as negative refraction (75) and unidirectional cloaking (76, 77).

Coherently prepared, multilevel warm atomic vapors provide another controllable platform to realize complex optical potentials. In such systems, strong pump laser beams can create waveguiding effects for weak probe beams where, under proper detuning, both gain and loss can be achieved in Raman-active systems (78). In this regard, the realization of complex potentials supporting exceptional points have been theoretically proposed in three- and four-level atoms (79, 80) and experimentally demonstrated in coupled atomic vapor cells (81), as well as in PT-symmetric optical lattices (82).

Even though the discussion here is primarily focused on linear operators, it is important to also stress the relevance of exceptional points in nonlinear systems. The connection of non-Hermiticity to nonlinear systems is multifold: First, most nonlinear configurations in optics and photonics are accompanied by losses, and second, active devices are, by nature, nonlinear. Therefore, lasers, amplifiers, and saturable absorbers are all examples of devices in which nonlinearity and non-Hermiticity coexist. In addition, nonlinear optical effects can create interactions between different wave components. A high-intensity pump, for example, initiates energy exchange between lower-intensity wave components that are governed by a linearized operator. Such an operator is, by essence, non-Hermitian, given the energy exchange between pump and probe through the nonlinearity.

The interplay of nonconservative and nonlinear effects is of special interest, given that optical materials with strong nonlinearities necessarily suffer from large absorption (83). Therefore, concepts from non-Hermitian physics are sought to provide strategies to take advantage of losses in such nonlinear materials. In this regard, the conjunctive use of nonlinear processes with gain and loss have been suggested as a viable route to achieve optical nonreciprocity (84, 85). In addition, it has been shown that laser systems exhibit exotic behavior such as anomalous pump dependence near the exceptional point singularity (86, 87), as well as reduced lasing threshold with increased losses

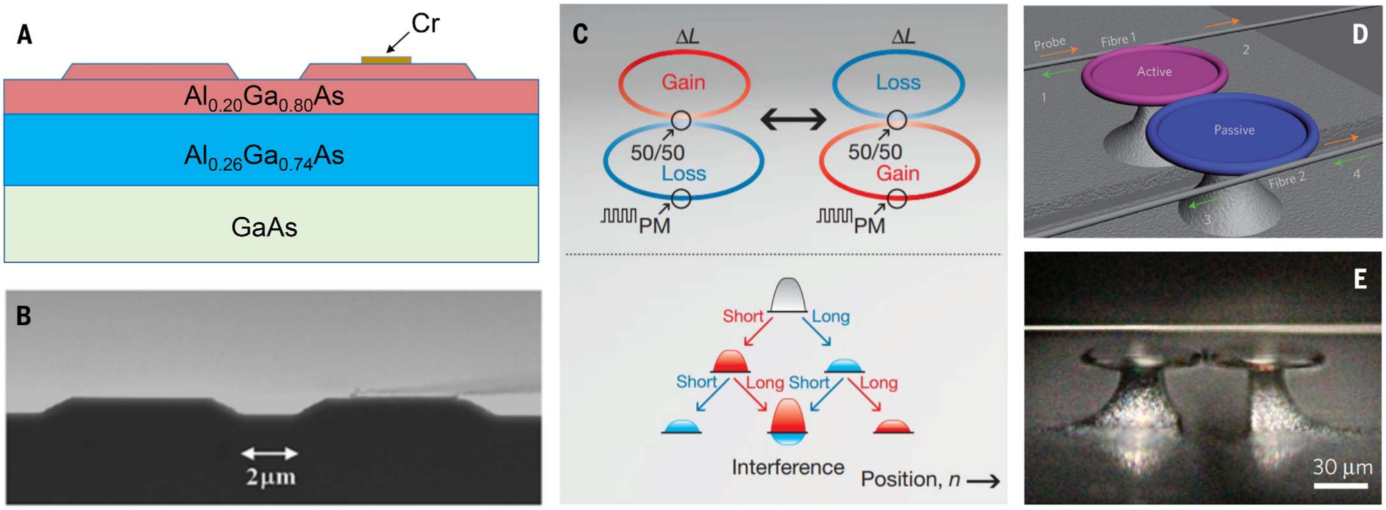

Fig. 4. Experimental demonstration of exceptional points in various optical settings. (**A** and **B**) Coupled integrated photonic waveguides (A) fabricated through a multilayer $Al_xGa_{1-x}As$ heterostructure (B), for which thin layers of chromium of different widths were utilized to impart different amount of losses in one of the waveguides (*56*). In this setting, couplers with different losses on one arm were used to observe mode symmetry breaking beyond the critical loss contrast associated with the exceptional point. (**C**) The propagation of laser pulses in coupled fiber loops of slightly different lengths (ΔL) with alternating gain and loss creates a quantum walk of pulses which is governed by a PT-symmetric operator (*59*). In this temporal lattice, the onset of complex eigenvalues associated with the band merging effect at the exceptional point was experimentally demonstrated. PM represents a phase modulator that creates an effective potential for the light pulses. (**D** and **E**) Coupled microring resonators with gain and loss have been used to probe the exceptional point through the mode splitting of the resonance eigenmodes (*57*, *58*). In (D), the numbers indicate the four ports that are used to probe the system, and orange and green arrows represent waves propagating in forward and backward directions, respectively. [Credits: (A) and (B) reprinted with permission from (*56*), copyright 2009 by the American Physical Society; (C), (D), and (E) reprinted from (*59*), (*57*), and (*59*), respectively, with permission from Springer Nature]

(*88*). The impact of non-Hermiticity on nonlinear waves in bulk and periodic systems has been also explored, after the realization that PT-symmetric potentials support optical solitons (*89*). Indeed, although dissipative nonlinear systems have been largely investigated (*90*), recent developments in the area of PT symmetry have sparked interest in the exploration of new integrated nonlinear systems combining gain and loss (*29*, *91*, *92*). In addition, solitary waves in PT-symmetric potentials have been experimentally demonstrated in time-domain lattices (*93*). Nonlinear wave-mixing processes, such as sum and difference frequency generation and optical parametric amplification, are other examples of non-Hermitian systems in which external coupling through a pump beam mediates the interactions (*94*).

At this point, it is worth stressing that exceptional points are not necessarily difficult to find in optical setups because they occur ubiquitously in the wave number space, even in conservative systems in which no gain or loss is involved. In these scenarios, a part of a Hermitian system can be considered non-Hermitian, because it exchanges energy with the rest of the system. Possibly the best-known example of these trivial exceptional points is the total internal reflection at the interface of two materials. In this case, light transmitted at the interface of two media critically depends on the incidence angle of the impinging light. In particular, at a critical angle, a phase transition occurs in the propagation wave number of the second medium, which goes from being real to complex valued. Other well-known examples of exceptional points in the wave number space are the cut-off frequency of a closed waveguide or the edge of a photonic bandgap in periodic structures. In addition, a volume Bragg grating, in which alternating layers of two different materials with refractive indices n_1 and n_2 create a photonic bandgap for a range of incoming frequencies, supports an exceptional point. In this structure, the wave number of the counter-propagating waves follows a square-root dispersion in terms of the incoming wave frequency. Whereas in the propagation band the wavenumber is real, inside the bandgap it becomes complex, and an exceptional point marks this transition. Similar to the exceptional points emerging in complex potentials, the photonic bandgap in gratings exhibits interesting properties, such as a vanishing group velocity (*95*).

Applications in nanophotonics

The exotic properties of exceptional points open interesting possibilities for advanced light manipulation. In this section, we present an overview of some of the recent theoretical and experimental developments in the exploration of exceptional points for applications in photonics. As in other areas of physics, in photonics, perturbation theory is an important mathematical tool to tackle a range of problems without having to deal with complex full-wave equations. Owing to the singularity at exceptional points, as well as the dimensionality collapse in the eigenvector space, standard perturbation theory, however, does not apply at such points. The perturbation problem can be introduced as $\mathcal{H} = \mathcal{H}_0 + \varepsilon\mathcal{H}_1$ where we want to find the behavior of the eigenvalues $\sigma_n(\varepsilon)$ and eigenvectors $|\psi_n(\varepsilon)\rangle$ of $\mathcal{H}$ for $\varepsilon \ll 1$, where ε is the perturbation parameter. In general, such a perturbation problem can be divided into regular and singular problems (*96*). In the regular case, a power-series solution with integral powers of ε exists, that is, $\sigma(\varepsilon) = \sigma_0 + \sum_{n=1}^{\infty} c_n\varepsilon^n$, where c_n are the series coefficients, with a finite radius of convergence. However, in the case of an exceptional point singularity, such a solution does not converge. At a singularity, the exact solution at $\varepsilon = 0$ is of a fundamentally different nature compared with its neighboring points $\varepsilon \to 0$ (*96*). At a second-order exceptional point, the series solution

$$\sigma_\pm(\varepsilon) = \sigma_0 + \sum_{n=1}^{\infty} (\pm 1)^n c_n \varepsilon^{n/2} \tag{3}$$

exists, where σ_0 is the eigenvalue at the exceptional point. The radius of convergence of this series in the complex ε plane is determined by the nearest exceptional point. In a similar manner, for a kth-order exceptional point the nth term in the perturbation series is $\varepsilon^{n/k}$, with a dominant first-order term of $\varepsilon^{1/k}$. For small perturbations, this term is considerably larger than the linear term ε, which occurs at regular points, enabling extra sensitivity to the parameter ε of a system when biased at the exceptional point singularity. This property has been proposed to achieve enhanced mode splitting between counter-propagating whispering gallery modes of a microring resonator in the presence of nanoparticles (97). The prospect of utilizing exceptional points for enhanced mode splitting has been experimentally demonstrated in microtoroid cavities (98, 99) (Fig. 5A). In addition, integrated microring resonators with externally controllable perturbations have been utilized to induce second- and third-order exceptional points, where ½ and ⅓ power-law exponents in mode splitting have been demonstrated (100) (Fig. 5B). Although it has been pointed out that enhanced sensitivity at the exceptional point does not necessarily correspond to enhanced precision in sensing instruments (101) and that quantum noise should be considered to assess the ultimate performance of these exceptional point sensors (102), sensors appear to be an interesting application area for these concepts. In this area, it has also been shown that a scaled form of PT symmetry can be used for enhanced sensor telemetry (103).

Another interesting application of exceptional points is mode discrimination in multimode laser cavities (104). A common issue in laser systems is that often several transverse or longitudinal modes may simultaneously lase. In this regard, it has been suggested to complement the active multimode laser cavity with a passive cavity that ideally exhibits an equal amount of loss. In this scenario, the overall level of loss is increased in the entire system, given that each mode overlaps with the loss region, and thus the gain threshold is expected to increase. However, a large discrimination between lasing thresholds of different modes is obtained at the exceptional point supported by this PT-symmetric system. In this case, the modes are split into two classes that are equally distributed between the active and passive regions, as well as modes that are localized either in the gain or loss cavity. The first class of modes remains neutral, whereas the modes located in the gain enter the gain regime. As a result, the passive cavity prevents some of the modes from lasing. More interestingly, this structure creates a large discrimination between the lasing thresholds of the fundamental mode with its closest competing counterpart. Assuming g_0 and g_1 to be the gain coefficients for fundamental and competing modes, respectively, in the coupled-cavity system, the discrimination is governed by $\sqrt{g_0^2 - g_1^2}$, which can be considerably larger than $g_0 - g_1$ in a single laser cavity. This approach has been utilized to enforce single longitudinal-mode operation in coupled microring lasers (105) (Fig. 6A) and in single rings with embedded active-passive gratings (106) (Fig. 6B). Similar strategies have been utilized to filter out transverse modes in ring resonators with large cross sections (107), in optically and electrically pumped stripe lasers (108, 109) (Fig. 6C), and in microdisc

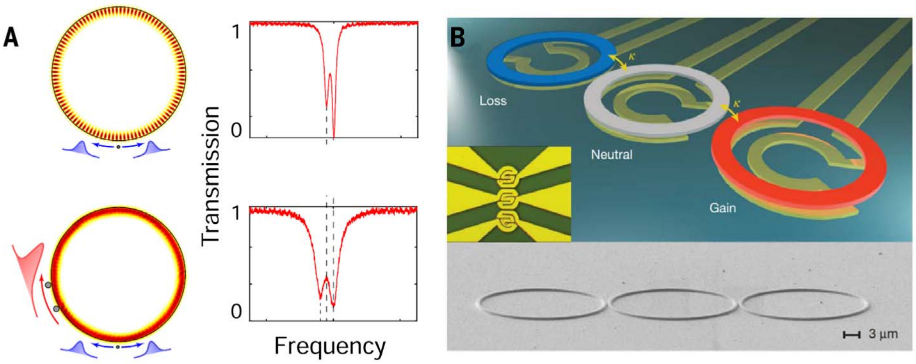

Fig. 5. Demonstration of enhanced perturbation near an exceptional point singularity. (**A**) Sensing a nanoparticle with a microtoroid resonator biased at an exceptional point (99). Blue arrows and curve indicate light pulses propagating in counter-rotating whispering gallery modes, and the red arrow and curve indicate a backscattering pulse due to the presence of additional scatterers (shown with two gray circles), which help to bias the system at an exceptional point. (**B**) Three coupled microring resonators creating a third-order exceptional point (100). κ represents the strength of coupling between adjacent microrings. [Credits: (A) and (B) reprinted from (99) and (100), respectively, with permission from Springer Nature]

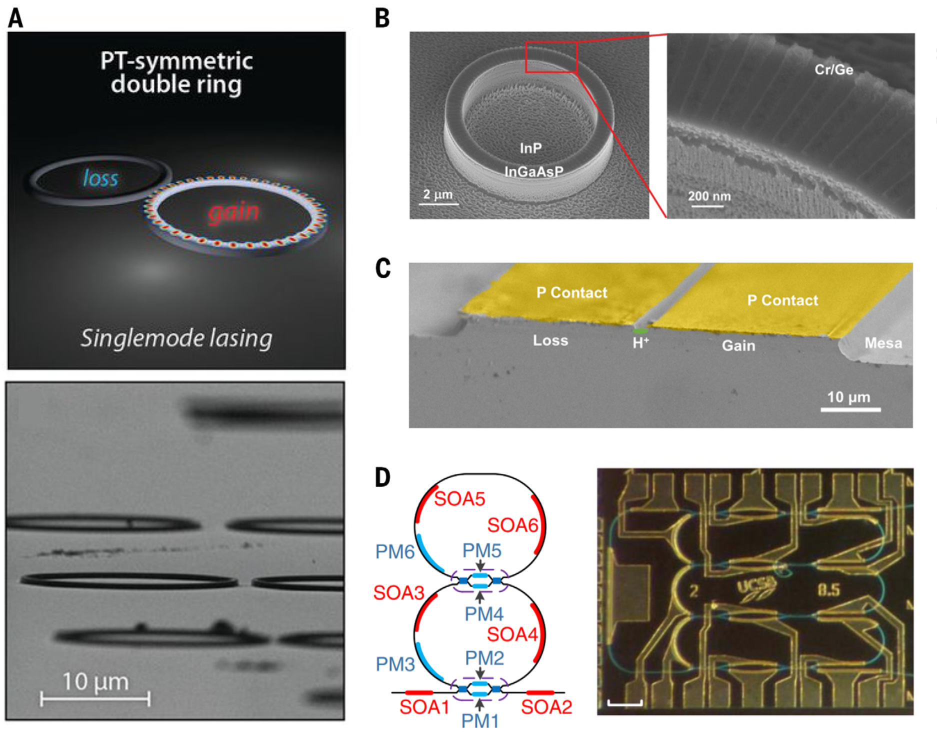

Fig. 6. PT-symmetric laser arrangement and its different realizations. (**A**) Coupled active-passive microring resonators (105), with a scanning electron microscope (SEM) image shown at the bottom. (**B**) SEM image of a microring resonator with an embedded gain-loss grating (106). (**C**) SEM image of coupled stripe lasers (109). (**D**) A schematic of integrated coupled microring lasers (left) and a photograph of the fabricated system (right), where the scale bar represents 200 μm (111). PM, phase modulator; SOA, semiconductor optical amplifier. [Credits: (A) and (B) reprinted from (105) and (106), respectively, with permission; (C) reprinted from (109) with permission from John Wiley and Sons; (D) reprinted from (111) with permission from Springer Nature]

lasers (110). In addition, integrated coupled microring lasers have been demonstrated with single-mode operation at telecommunication wavelengths (111) (Fig. 6D).

As illustrated in Fig. 7A, an interesting aspect of exceptional points consists of their exotic topological features in the parameter space. This discussion falls into the broad context of topological photonics, an area of optics research that has produced considerable excitement in recent years. Inspired by the unusual physics of topological insulators in condensed-matter physics, topological phenomena in photonics have been shown to arise in sophisticated periodic structures, ranging from gyromagnetic photonic crystals (112), arrays of helical waveguides (113), arrays of microring resonators (114), bianisotropic or magnetized metacrystals (115), dielectric chiral metasurfaces (116), and time-modulated lattices (117). In these systems, highly unusual photon transport, characterized by one-way propagation along the edges of the sample, arises within bandgaps delimited by bands with distinct topological properties. That their optical properties are related to a topological feature makes the response inherently robust to disorder and imperfections. Analogously, exceptional points represent an interesting example of topological features arising in simple coupled dynamical systems as a result of the interplay between interaction and dissipation. According to Fig. 7A, a loop of eigenvalues that encircle a base point identifies a topological object, given that it cannot be continuously deformed to a single point without crossing the base point.

The rigorous analysis of these features can be carried out using results from condensed-matter physics, in which the topological band theory of non-Hermitian Hamiltonians has been rigorously investigated in (118). Specifically, it was shown that non-Hermitian band structures exhibit a topological invariant associated with the gradient of the band in momentum space (119). Inspired by the periodic table of topological insulators, a systematic classification of topological phases of non-Hermitian systems has also been presented (118). An interesting problem in this context is to adiabatically change the parameters of a non-Hermitian system such that the exceptional point is dynamically encircled, as depicted in Fig. 7B. In a Hermitian system, when adiabatically changing the parameters along a closed path, the two eigenvectors are bound to return to their original form, apart from acquiring a possible geometric phase (120). In the case of non-Hermitian systems, instead, parametric cycling an exceptional point interchanges the instantaneous eigenvectors, whereas only one picks up the geometric phase (13, 121–123). In principle, this behavior does not occur, even for arbitrarily slow dynamic cycling of the exceptional point, given that the adiabatic theorem breaks down in case of non-Hermitian systems. Indeed, under such conditions, depending on the direction of rotation, one of the two eigenstates dominates at the end of the parametric cycle. This interesting topological response provides a scheme for topologically robust energy conversion between different states.

On the basis of this principle, topological energy transfer has been recently demonstrated in a multimode optomechanical cavity in which two mechanical modes of a membrane are coupled and coherently controlled through a laser beam (124) (Fig. 7C). In addition, dynamical cycling of exceptional points is explored in a microwave waveguide in which a robust asymmetric transmission between even and odd modes is demonstrated (125) (Fig. 7D). In addition, it has been shown that this concept can provide opportunities for polarization manipulation (126, 127). In particular, one can create an omnipolarizer in which the output light is polarized along a specific direction irrespective of the polarization of the input state (Fig. 7E). For propagation along the opposite direction, on the other hand, the output is populated in the orthogonal polarization.

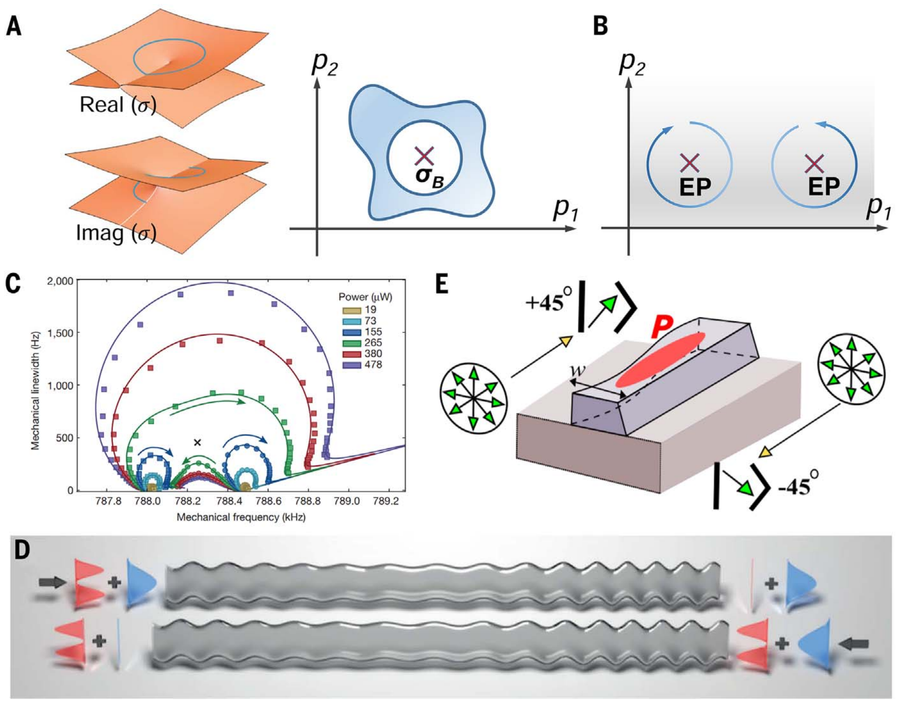

Fig. 7. Chiral mode conversion through dynamically cycling an exceptional point. (**A**) The eigenvalue surfaces near an exceptional point (left). Although a loop of eigenvalues containing a base point can be continuously deformed into a circle, it cannot be shrined into a point without crossing the base point (right) (118, 119). p_1 and p_2 represent two parameters. (**B**) Two different possibilities of encircling an exceptional point (EP) cycling along opposite directions. (**C**) The experimental probing of the complex eigenvalues of two mechanical oscillators driven adiabatically through optical fields (124). The cross indicates the location of the exceptional point. (**D**) Asymmetric conversion between the even and odd modes of a waveguide, when the loss and detuning are adiabatically controlled in order to encircle an exceptional point (125). Blue and red curves indicate two modes of the waveguide, and the arrow indicates the direction of propagation. (**E**) An adiabatic conversion between orthogonal polarization states (126). Green arrows show the propagation direction, yellow arrows indicate the polarization state, P is the pumping, and w is the channel width. [Credits: (A) reprinted with permission from (118), copyright 2018 by the American Physical Society, and (119); (C) and (D) reprinted from (124) and (125) with permission from Springer Nature; (E) reprinted with permission from (126), copyright 2017 by the American Physical Society]

Conclusions and outlook

The peculiar features of exceptional points, associated with their unusual parameter dependence in the eigenvalue spectrum of non-Hermitian systems, enable exciting opportunities for a wide range of applications. These applications arise in scenarios in which interaction among different modes in the presence of dissipation and/or amplification is involved. In such circumstances, coupling and gain-loss mechanisms can be engineered and utilized to induce and control exceptional points, to take advantage of the strong and anomalous parameter dependence of the system around them.

We envision future opportunities to exploit these singular responses in photonics for advanced dispersion engineering. As a relevant recent example, level repulsion in the group velocity dispersion between coupled cavities has been used to control the modal dispersion of an individual cavity. This has been utilized to create anomalous dispersion, which is of great importance in four-wave mixing and parametric frequency comb generation (128–130). However, the full potential of coupled waveguide or cavity arrangements for dispersion manipulation is still largely unexplored, and multiple coupled cavities or metamaterials may be envisioned to take full advantage of exceptional points in the context of dispersion engineering.

In a similar fashion, coupled-cavity arrangements offer exciting prospects to design new semiconductor lasers with highly desired functionalities. Although modern semiconductor laser sources exist in the entire optical spectrum, their coherence properties are not sufficient for many applications. In particular, key requirements for laser sources, such as stable and narrowband frequency operation, as well as frequency tunability, can be achieved through coupled-cavity geometries (*131*–*134*) (Fig. 8B). Even though this scheme has been previously applied to semiconductor lasers at specific frequencies, it remains to be explored in other, arguably more practical, sources and at different frequencies. In this regard, coupled-cavity techniques in conjunction with non-Hermitian designs provide an exciting strategy to systematically address the current challenges in integrated laser sources by taking advantage of the strong parameter dependence of such structures near exceptional points.

Mode conversion in a compact integrated photonic device is another important functionality that can largely benefit from exceptional points, in terms of reduced footprint and inherent robustness to disorder. Even though rigorous optimization techniques allow for inverse design of such structures, often resulting in complex structures that require advanced fabrication technologies, alternative designs with reduced complexity are highly desirable. In this vein, adiabatic perturbation of a structural parameter inducing an exceptional point–induced controllable level repulsion can provide a simple approach for hybridization and adiabatic exchange of modes. Recently, it has been shown that in optical ridge waveguides with different cladding and buffer materials, varying the waveguide width induces a strong coupling between transverse electric and magnetic polarizations of different spatial orders (*135*). As a result, adiabatic tapering of the waveguide width along the propagation direction can efficiently convert polarization states as well as spatial-mode orders (*136*, *137*). As shown schematically in Fig. 8C, the inclusion of selective gain and loss in such geometries provides an alternative degree of freedom to control the mode-conversion efficiency. In addition, hybridization between multiple modes through higher-order exceptional points can initiate the simultaneous conversion among a large number of modes. The full ramifications of these concepts become very powerful new tools in photonic engineering.

The quest for integration of optical setups on a chip requires integrated implementation of fundamental elements such as laser sources with critical power and coherence demands, isolators and circulators, mode convertors, and so on. In this regard, multimode structures have proven to provide a great opportunity to achieve desired functionalities and realize compact devices. This trend naturally calls for a bottom-up approach in designing photonic devices in an abstract modal picture in which three ingredients are relevant: (i) modal detuning, (ii) mode coupling, and (iii) modal gain and/or loss. The role of the first two processes has been largely explored in the past in the context of coupled-mode theory. The third mechanism, on the other hand, has been largely unexplored. As we discussed in this survey, the interplay of these phenomena can result into totally new opportunities for photonics, associated with the emergence of exceptional points that notably alter the eigenvalue surfaces. Therefore, notions from exceptional point physics can provide new designs for realizing multimode integrated photonic devices. This creates opportunities for theoretical and experimental research focused on exploring the fundamental bounds of accessible performance, such as bandwidth and sensitivity, of photonic devices operating at exceptional points. It is worth stressing that inducing exceptional points through gain and loss imposes difficulties in experimental photonics. This is because optical gain is limited to certain materials and is not generally compatible with all platforms, and loss is generally undesired for various purposes. At the same time, suitable settings for investigating and fruitfully exploiting exceptional points arise in systems that inherently involve optical gain or loss, such as semiconductor lasers, saturable absorbers, and plasmonic structures, among others.

Along different lines, remaining to be investigated are the interesting physics arising from the propagation of classical light at exceptional point singularities. Recent theoretical investigations, for example, suggest dynamical slowing and stopping of light in coupled waveguides at exceptional points (*138*), as well as photonic catastrophe in optical lattices (*139*). In addition, a point of interest would be to explore these phenomena in new platforms. An emerging playground to explore the rich physics of exceptional points is provided by hybrid photonic platforms

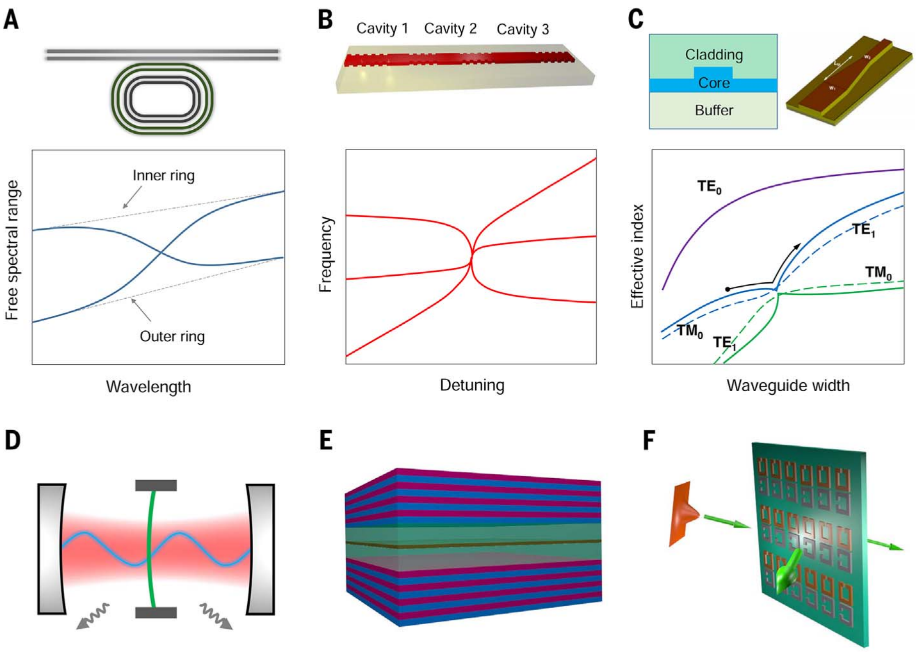

Fig. 8. Application of exceptional points in multimode photonic integrated circuits and new platforms to investigate exceptional points. (**A** to **C**) Applications. (A) Hybridization of eigenfrequencies in coupled microring resonators (top) creates two branches with strong dispersion (bottom) (*130*). The anomalous dispersion can be utilized for frequency comb generation. (B) Wavelength manipulation in three coupled-cavity lasers through a strong dispersion at a third-order exceptional point (*133*). (C) Level repulsion of modes with different polarization provides an opportunity for compact polarization mode conversion (*135*, *136*). A parametric evaluation of the eigenmodes of a rib waveguide (top left) versus the waveguide width reveals a level repulsion between transverse electric (TE) and transverse magnetic (TM) polarizations (bottom). Therefore, tapering of the waveguide width over a finite distance (top right) can result in an adiabatic polarization conversion. (**D** to **F**) New platforms. (D) Multimode optomechanical cavities provide a flexible platform for investigating exceptional points. (E) Exciton-polaritons in semiconductor cavities offer an alternative multiphysics structure for realizing exceptional points. (F) Coupled nanoantennas can be designed as non-Hermitian building blocks of optical metasurfaces. [Credits: (A) reprinted from (*130*) with permission from Springer Nature; (B) reprinted from (*133*) with permission from AIP Publishing; (C) reprinted with permission from (*135*) and (*136*), copyright 2011 and 2012, respectively, Optical Society of America]

that integrate other degrees of freedom beyond optics, exploiting the interaction between different phenomena. In particular, cavity optomechanics, relying on the strong coupling between optics and mechanical motion, offers a reconfigurable, inherently non-Hermitian platform that can be externally controlled through pump lasers with proper intensity and phase (*140*) (Fig. 8D). Operating in the red and blue sideband detuning of the pump beam can effectively control loss or gain for the optical modes involved, opening exciting opportunities for PT symmetry and exceptional points in a low-noise nanophotonic integrated environment. Similarly, cavity polaritons, because of their inherent non-Hermitian properties, can provide another platform for investigating and utilizing exceptional points (*141*) (Fig. 8E).

Finally, it is worth mentioning the potential of utilizing exceptional point singularities in optical scattering problems, where the coupling between discrete localized metastable states and a continuum of radiation states is concerned. Interest in photonic bound states embedded in the continuum is increasing, owing to their interesting properties (*142*–*144*). Such settings can, in general, be treated as non-Hermitian problems, for which a point of interest would be to explore the connection between radiation leakage and exceptional points emerging in the continuum, as observed in recent experiments (*145*). In addition, similar concepts can be utilized in designing coupled optical nanoantennas as non-Hermitian building blocks of metasurfaces in order to create scattering surfaces with desired phase, frequency, and polarization response (Fig. 8F). In addition to the radiative losses of dielectric inclusions, the inherent loss in metallic inclusions at optical frequencies can be turned into an opportunity to realize and exploit exceptional points in properly designed geometries (*146*). We envision exciting opportunities in translating the concepts of exceptional point physics to quantum nanophotonic and low-photon hybrid systems.

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