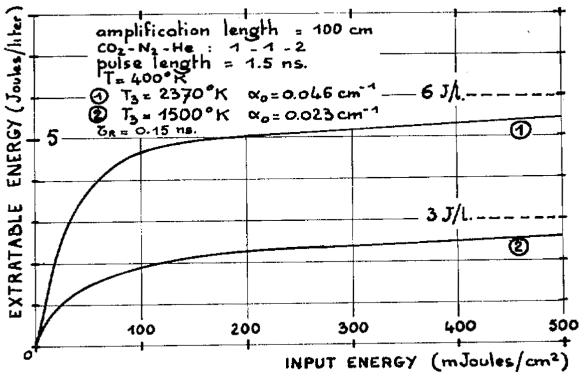

Fig. 4. Computed energy extraction versus incident pulse energy density for an amplification length $l = 1$ m, $\tau_R = 0.15$ ns, and a small signal gain $\alpha_0 = 0.046$ cm^{-1}: curve ①, $\alpha_0 = 0.023$ cm^{-1}: curve ②.

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Modes in Inhomogeneous Slab Waveguides

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***Abstract*—In this correspondence we have developed the perturbation approach for the calculation of propagation constants for some of the practical slab waveguides and have shown the applicability of the approach to systems of practical interest.**

I. INTRODUCTION

In recent years there has been a considerable amount of study on the guided propagation of electromagnetic waves through slab waveguides [1]–[6]. This has been primarily due to the rapid advancement in the preparation of slab waveguides for transmitting light beams. Numerous methods have been proposed for the preparation of slab waveguides both

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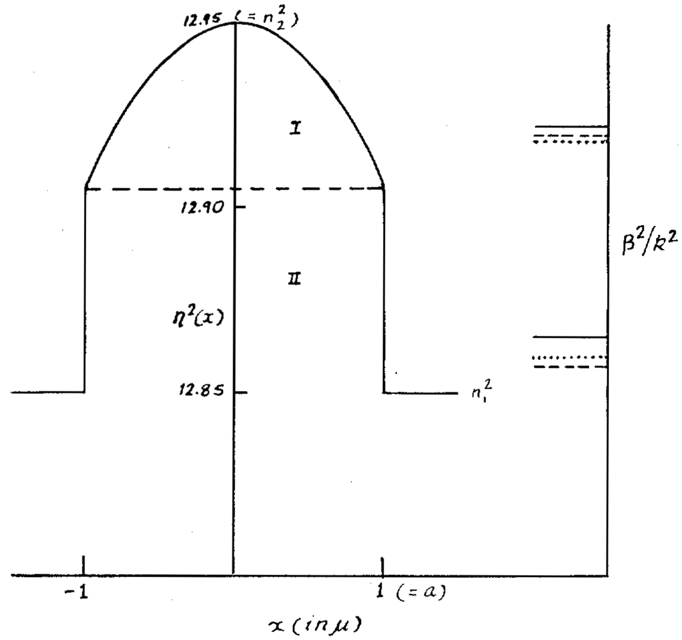

(a) (b)

Fig. 1. (a) Typical variation of $n^2(x)$ with x for a medium characterized by a refractive index variation given by (1). Regions I and II correspond to $n_2^2 > n^2 > n_2^2 - \Delta$ and $n_2^2 - \Delta > n^2 > n_1^2$, respectively. (b) The values of the propagation constants are shown. The solid lines correspond to an infinitely extended square-law medium, whereas the dashed and the dotted lines correspond to the perturbation theory developed in Section II-B and C, respectively.

in a three-layer form (thin-film deposition [7]) and in a graded-index form (by diffusion [8]–[10]).

In this correspondence we have developed a first-order perturbation approach for determining the propagation constants for a truncated square-law medium. The profile is shown in Fig. 1(a). This study enables one to find the effect of cladding on the propagation constants of an infinitely extended square-law medium. The perturbation approach has also been applied to a complimentary error-function distribution which has been assumed to be a perturbed form of an exponential distribution. These types of distributions are found in waveguides formed by diffusion techniques [9].

II. THEORY

A. Cladded Square-Law Medium

For a cladded square-law medium, the refractive-index distribution $n(x)$ would be given by

$$\begin{aligned} n^2(x) &= n_2^2 - \Delta x^2/a^2, & |x| &\le a \\ &= n_1^2, & |x| &> a. \end{aligned} \tag{1}$$

We will carry out an analysis of the TE modes of such a waveguide. If for Ey we assume a solution of the form

$$Ey = \psi(x) \exp\left[i(\omega t - \beta z)\right]$$

then ψ can be shown to satisfy the following equation

$$\frac{d^2\psi}{dx^2} + (n^2(x)k^2 - \beta^2)\psi = 0 \tag{2}$$

where all symbols have their usual meaning.

B. Perturbation Theory for Lowest Order Modes

In order to study the effect of cladding on the lowest order modes we write the wave equation in the form

$$\frac{d^2\psi}{dx^2} + [\{n^{(0)^2}(x) + n'^2(x)\}k^2 - \beta^2]\psi = 0 \qquad (3)$$

where $n^{(0)^2}(x) = n_2^2 - \Delta x^2/a^2$ and $n'^2(x)$, which is to be considered as the perturbation, is zero for $|x| \leq a$ and $n_1^2 - n^{(0)^2}(x)$ for $|x| > a$. The modes, $\psi_n^{(0)}(x)$ for the unperturbed case [i.e., when $n'^2(x) = 0$] are the well-known Hermite–Gaussian functions [1].

In first-order perturbation theory the propagation constants would be given by

$$\beta_n^2 = \beta_n^{(0)^2} + \beta_n^{(1)^2} = \left[n_2^2k^2 - k\frac{\sqrt{\Delta}}{a}(2n+1)\right] + \left[k^2\int_{-\infty}^{\infty} n'^2(x)\,|\psi_n^{(0)}|^2\,dx\right]. \qquad (4)$$

The integral appearing in (4) can always be put in terms of error functions. For example, for $n = 0$, $\beta_0^{(1)^2}$ would be given by

$$\beta_0^{(1)^2} = k^2\left[\left(\frac{\Delta}{2\xi_0^2} - g^2\right)\operatorname{erfc}(\xi_0) + \frac{\Delta}{\sqrt{\pi}\,\xi_0}\exp(-\xi_0^2)\right] \qquad (5)$$

where $\xi_0 = a/\alpha$, $g^2 = n_2^2 - n_1^2$ and erfc represents the complimentary error function. Similarly, one can calculate the perturbation for higher order modes. For the lowest order mode, the group velocity would be given by

$$v_{g_0} = \frac{c\beta_0}{k}\left[n_2n_e - \frac{\sqrt{\Delta}}{2ak} + \left(\frac{\Delta}{4\xi_0^2} + n_1^2 - n_2n_e\right)\operatorname{erfc}(\xi_0) + \frac{\xi_0}{2\pi}\left(\frac{\Delta}{2\xi_0^2} + g^2 - \Delta\right)\exp(-\xi_0^2)\right]^{-1} \qquad (6)$$

where $n_e = n_2(1 - (\lambda/n_2)(dn_2/d\lambda))$.

C. Perturbation Theory for Modes Near Cutoff

For the analysis of modes near cutoff we assume $n^{(0)^2}(x)$ and the perturbation $n'^2(x)$ to be given by

$$\begin{aligned} n^{(0)^2}(x) &= n_2^2; \quad n'^2(x) = -\Delta x^2/a^2, \qquad [|x| \leq a] \\ &= n_1^2; \qquad\quad = 0, \qquad [|x| > a]. \end{aligned} \qquad (7)$$

The modes $\psi^{(0)}$ of the unperturbed wave equation are well known [1]. The unperturbed propagation constants $\beta^{(0)^2}$ can be obtained from the solution of the following transcendental equation [1]

$$\begin{aligned} \gamma\tan\gamma a &= \delta \quad \text{symmetric modes} \\ \gamma\cot\gamma a &= -\delta \quad \text{antisymmetric modes} \end{aligned} \qquad (8)$$

where $\gamma^2 = n_2^2k^2 - \beta^2$ and $\delta^2 = \beta^2 - n_1^2k^2$. The perturbation in the propagation constant would be given by

$$\begin{aligned} \beta^{(1)^2} &= -\int_{-a}^{a} \Delta k^2x^2/a^2\,|\psi^{(0)}|^2\,dx \\ &= -ak^2\Delta A^2\left[\frac{1}{3} \pm \left\{\frac{\cos 2\gamma a}{2\gamma^2a^2} + \left(1 - \frac{1}{2\gamma^2a^2}\right)\frac{\sin 2\gamma a}{2\gamma a}\right\}\right] \end{aligned} \qquad (9)$$

where $A = (a + 1/\delta)^{-1/2}$. The upper and lower signs correspond to symmetric and antisymmetric modes, respectively. The total number of modes would be the same as in a slab waveguide with core and cladding refractive indices n_2 and n_1, respectively.

D. Some Numerical Results

We have applied the previous analysis to calculate the total number and the propagation constants of the modes propagating through GaAs p-n junctions. We have considered a distribution in between the extreme cases considered by Zachos and Ripper [11] and Rutz [12], namely parabolic in the depletion region and constant in the p and n regions. The following values have been used in the calculations: $n_2^2 = 12.95$; $n_1^2 = 12.85$; $\Delta = 0.0448$; $a = 1$ μm; $\lambda = 8383$ Å. These correspond to typical GaAs laser medium parameters. The calculated values of β^2/k^2 before and after applying the perturbation theory are shown as horizontal lines in Fig. 1(b). For the values of parameters chosen here we see that only two modes can propagate in this waveguide.

Using the perturbation theory previously dealt with one can also study the effect of cladding on the group velocity. For example, corresponding to the values of various parameters given previously, the group velocity of the lowest order mode [see (10)] would be $0.2004c$ where c is the velocity of light in free space. The corresponding value for an infinitely extended square-law medium is $0.196c$. Thus the perturbation theory predicts a 2.24-percent increase in the group velocity of the fundamental mode due to the presence of the cladding.

E. Perturbation Theory for Planar Waveguides Formed by the Out-Diffusion Technique

Some of the waveguides fabricated by Kaminow and Carruthers [9] have the following dielectric-constant profile

$$\begin{aligned} \epsilon(x) = n^2(x) &= \epsilon_1, & x < 0 \\ &= \epsilon_0 + \Delta\epsilon\operatorname{erfc}(2x/\pi a), & x > 0. \end{aligned} \qquad (10)$$

Numerical calculations of such a profile have been reported by Smithgall and Dabby [3]. Since the above refractive-index distribution resembles the exponential distribution one can consider it to be a perturbed form of the following distribution

$$\begin{aligned} \epsilon(x) &= \epsilon_1, & x < 0 \\ &= \epsilon_0 + \Delta\epsilon\exp(-x/d), & x > 0. \end{aligned} \qquad (11)$$

The modes for such a dielectric-constant distribution are given by [5]

$$\begin{aligned} \psi &= A\,J_\nu(\xi\exp(-x/2d)), & x > 0 \\ &= B\exp(p_1x), & x < 0 \end{aligned} \qquad (12)$$

where $\nu = 2dp_0$; $\xi = 2kd\sqrt{\Delta\epsilon}$; $k = 2\pi/\lambda_0$; $p_i = (\beta^{(0)^2} - \epsilon_ik^2)^{1/2}$; $i = 0, 1$ and $\beta^{(0)}$ being the corresponding propagation constants which are determined from the solution of the following transcendental equation

$$\frac{J_{\nu-1}(\xi) - J_{\nu+1}(\xi)}{J_\nu(\xi)} = -\frac{p_i\lambda}{\pi\sqrt{\Delta\epsilon}}. \qquad (13)$$

The constants A and B in (12) are to be determined from the normalization and continuity conditions. The perturbation can be assumed to be zero for $x < 0$ and $\Delta\epsilon[\operatorname{erfc}(2x/\pi a) - \exp(-x/d)]$ for $x > 0$. Using (12) for the unperturbed eigenfunctions we have calculated the propagation constants corresponding to the system analyzed by Smithgall and Dabby [3]. The values of various parameters are $\epsilon_0 = 2.268$; $\Delta\epsilon = 0.9185$; $a = 0.56$ μm; $d = 0.4767$ μm; and $k = 12$ μm^{-1}. This set of values gave $\nu = 7.500$ and $\beta/k = 1.6425$ for the lowest order mode. Applying the first-order perturbation theory the value of β/k changes to 1.6649 which is within 0.08 percent of the exact value obtained through a numerical solution of the differential equation [3].

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Nonlinear Optical Susceptibility of Thiogallate $CdGa_2S_4$

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***Abstract*—The nonlinear optical coefficient d_{36} of $CdGa_2S_4$ was measured to be 5 times larger than d_{31} ($LiNbO_3$). This large nonlinearity is in good agreement with theory.**

Introduction

There is a large and continuing interest in tertiary chalcopyrites [1]–[3] (e.g., $AgGaS_2$) due to their large nonlinear optical susceptibilities d_{36} and their favorable phase-matching properties. Although (as discussed in detail previously [4]) $AgGaS_2$ has a rather large nonlinear coefficient [3.5 times larger than $d_{31}(LiNbO_3)$] the magnitude of d_{36} is significantly reduced from the optimum possible value. The reason for this [4] is that the Ag–S bond has a *negative* nonlinearity which subtracts from the larger *positive* Ga–S nonlinearity, thereby reducing the net crystal nonlinear coefficient $d_{36}(AgGaS_2)$. Thus, in addition to being theoretically interesting, there could be significant device potential in investigating a similar chalcopyrite with the negative Ag–S bond replaced by a positive bond such as Cd–S. The tertiary thiogallate (defect chalcopyrite) $CdGa_2S_4$ should therefore have an even larger nonlinear coefficient than $AgGaS_2$ (as suggested previously [4]). Because of this and our success in growing good-quality single crystals of $CdGa_2S_4$, we have measured $d_{36}(CdGa_2S_4)$.

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Experimental

$CdGa_2S_4$ was first reported by Hahn *et al.* [5] to crystallize with the tetragonal thiogallate structure (class $\bar{4}$). Crystals have been grown by both vapor transport [6], [7] and by the Stockbarger method [8], and the optical [9], [10] and luminescence [11] properties of both pure and Cu-doped $CdGa_2S_4$ have been investigated.

Our crystals were prepared using stoichiometric amounts of 99.999-percent pure CdS and Ga_2S_3 and 0.1-mole percent of 99.9999-percent pure Ga, which were enclosed in evacuated and sealed silica ampoules. These ampoules were heated to 1060°C in a horizontal furnace and then slowly cooled at a rate of 1°C/h. Inspection afterwards showed that a melt had formed in the process. The furnace had a natural temperature gradient inside, being cooler at the ends, and therefore nucleation began at the ends of the ampoule forming crystals by directional freezing. Crystals with dimensions up to 1 cm were grown in this manner.

Because our $CdGa_2S_4$ crystals had excellent-quality [1, 1, 2] growth faces, we found it convenient to use a large [1, 1, 2] plate ($\sim$1 cm^2 in area) which was polished to a thickness of $\sim$1 mm for our measurements. The second-harmonic coefficient was measured with the Maker-fringe technique [12], using a single TEM_{00}-mode YAG laser. The sample rotation was about a vertical axis which contained the projection of the c axis, and both the fundamental and second-harmonic fields were polarized vertically (V). The effective nonlinear coefficient (at normal incidence) for such a geometry is easily seen to be

$$(d_{eff})_V = 3 \sin^2 \theta \cos \theta d_{36} \tag{1}$$

where θ is the angle between the c axis and the crystal plate. For a [1, 1, 2] plate of $CdGa_2S_4$ (for which $c/a = 1.808$) this angle θ is 32.5°. From the Maker-fringe spacing, the coherence length at 1.064 μm was found to be

$$l_c = 2.10 \ \mu\text{m}. \tag{2}$$

Using Hobden's [10] value for the index of refraction at $2\omega = 0.532$ μm and our experimental result for l_c we find

$$n_\omega = 2.327 \quad n_{2\omega} = 2.453. \tag{3}$$

The other linear property required is the absorption, which we found to be negligible at the fundamental and to have the value

$$\alpha_{2\omega} = 0.85 \text{ cm}^{-1} \tag{4}$$

at the harmonic frequency. The transparency range of 0.45–13 μm is shown in Fig. 1.

By comparing the second harmonic from the $CdGa_2S_4$ plate with that produced by a quartz reference [12], and making use of (1)–(4) we found

$$d_{36}(CdGa_2S_4)/d_{11}(SiO_2) = 80 \pm 15 \text{ percent.} \tag{5}$$

Therefore, using [13] $d_{11}(SiO_2) = 0.80 \times 10^{-9}$ ESU at 1.064 μm we have

$$d_{36}(CdGa_2S_4) = 64 \times 10^{-9} \text{ ESU} \tag{6}$$