

Dynamics of soliton solutions in the chiral nonlinear Schrödinger equations

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Abstract In this study, the $(1 + 1)$ - and $(2 + 1)$ -dimensional Chiral nonlinear Schrödinger equations are investigated. These equations describe the edge states of the fractional quantum hall effect. We successfully acquired dark and bright soliton solutions to these equations by using the sine-Gordon expansion method. The constraint conditions for the existence of valid soliton are given. We present the numerical simulations to some of the obtained solutions under the choice of suitable parameters.

Keywords Chiral NLSEs · SGEM · Soliton solutions

1 Introduction

Nonlinear complex aspects in optics, plasma physics, fluid dynamics and other areas in nonlinear sciences can be expressed in the form of nonlinear Schrödinger equations (NLSEs). Nonlinear Schrödinger equations are prototypical dispersive nonlinear partial differential equations which have been derived and analyzed in various fields of nonlinear sciences [1]. These types

of equations have a wider range of applications in the fields of nonlinear sciences. Investigation of their soliton solutions is of significant importance in the studies of nonlinear science as they help in explaining the physical mechanism of a complex phenomena in nature, and this area becomes one of the most exciting and extremely active area of research [2–4]. Solitons arise on lake shore and beaches where shallow water is present, in a quantum hall effect and also in biological sciences in the context of neurosciences. Solitons are the stable localized waves that propagate in a nonlinear medium without amplitude attenuation and shape change due to the balance between the dispersion and nonlinearity [5]. Solitons play an important role in the non-perturbative developments in the quantum field theory [6–8]. Dynamics of solitons in various models have been studied by different scientists [9–34].

However, in this study, the sine-Gordon expansion method (SGEM) [35–37] is used in investigating the existence of solitons in the quantum hall effect by studying the $(1 + 1)$ - and $(2 + 1)$ -dimensional Chiral nonlinear Schrödinger equations [38, 39].

The $(1 + 1)$ -dimensional Chiral nonlinear Schrödinger equation is given by [38]

$$i\Psi_t + \Psi_{xx} - i\sigma(\Psi^*\Psi_x - \Psi\Psi_x^*)\Psi = 0, \quad (1.1)$$

where Ψ is a complex function of x and t , σ is a nonlinear coupling constant and the $*$ indicates the complex conjugate.

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The $(2 + 1)$ -dimensional Chiral nonlinear Schrödinger equation is given by [39]

$$i\Psi_t + a(\Psi_{xx} + \Psi_{yy}) + i\left(b_1(\Psi\Psi_x^* - \Psi^*\Psi_x) + b_2(\Psi\Psi_y^* - \Psi^*\Psi_y)\right)\Psi = 0, \quad (1.2)$$

where Ψ is the complex function of x and t , a is the coefficient of the dispersion terms and b_1, b_2 are nonlinear coupling constants.

Equations (1.1) and (1.2) give chiral solitons. Solitons in chiral nonlinear Schrödinger equation play a vital role in the context of quantum Hall effect, where chiral excitations are known to appear [40–42]. Several studies have been conducted on these models. Eslami et. al [39] investigated the solutions of the $(2 + 1)$ -dimensional Chiral nonlinear Schrödinger equation. Younis et al. [43] studied the chiral nonlinear Schrödinger equation, with perturbation term and a coefficient of Bohm potential. Nishino et al. [38] studied Eq. (1.1) and some soliton solutions were constructed. With aid of soliton perturbation theory, Biswas et al. [42] studied the perturbation of soliton due to the chiral nonlinear Schrödinger equation.

2 The SGEM

In this section, we present the analysis of SGEM.

Consider the sine-Gordon equation [36]

$$\Psi_{xx} - \Psi_{tt} = V^2 \sin(\Psi), \quad (2.1)$$

where $\Psi = \Psi(x, t)$ and V is a nonzero real constant.

Utilizing the wave transformation $\Psi = \Psi(x, t) = U(\zeta)$, $\zeta = k(x - vt)$ in Eq. (2.1), gives the following nonlinear ordinary differential equation (NODE):

$$\Psi'' = \frac{V^2}{k^2(1 - v^2)} \sin(\Psi), \quad (2.2)$$

where $U = U(\zeta)$ and ζ stands for the width and v the velocity of the traveling wave, respectively. One can simplify Eq. (2.2) to the following forms:

$$\left[\left(\frac{U}{2}\right)'\right]^2 = \frac{V^2}{k^2(1 - v^2)} \sin^2\left(\frac{\Psi}{2}\right) + q, \quad (2.3)$$

where q is the integration constant.

Inserting $q = 0$, $w(\zeta) = \frac{U}{2}$ and $a^2 = \frac{V^2}{k^2(1 - v^2)}$ into Eq. (2.3), yields:

$$w' = a \sin(w), \quad (2.4)$$

inserting $a = 1$ into Eq. (2.4), yields

$$w' = \sin(w), \quad (2.5)$$

simplifying Eq. (2.5), we get the two significant equations as follows:

$$\sin(w) = \sin(w(\zeta)) = \frac{2qe^\zeta}{q^2 e^{2\zeta} + 1} \Big|_{q=1} = \operatorname{sech}(\zeta), \quad (2.6)$$

$$\cos(w) = \cos(w(\zeta)) = \frac{q^2 e^{2\zeta} - 1}{q^2 e^{2\zeta} + 1} \Big|_{q=1} = \tanh(\zeta), \quad (2.7)$$

where q is the integration constant.

The solution of any given nonlinear partial differential equation (NPDE) is assumed to be of the following form:

$$U(\zeta) = \sum_{i=1}^n \tanh^{i-1}(\zeta) [B_i \operatorname{sech}(\zeta) + A_i \tanh(\zeta)] + A_0, \quad (2.8)$$

according to Eqs. (2.6) and (2.7), one may rewrite Eq. (2.8) as:

$$U(w) = \sum_{i=1}^n \cos^{i-1}(w) [B_i \sin(w) + A_i \cos(w)] + A_0. \quad (2.9)$$

The value of n is determined by balancing the highest power nonlinear term and highest derivative the NODE. Inserting Eq. (2.9) and its possible derivatives into the NODE, gives an equation in different power of trigonometric functions “ $\sin^i(w)\cos^j(w)$, ($0 \leq i \leq n, 0 \leq j \leq n$)”. We collect some set of algebraic equations by summing the coefficients of trigonometric identities of the same power and equating each summation to zero. We solve this set of algebraic equations for the values of the coefficients involved. We finally insert the values of these coefficients into Eq. (2.8) to get the solutions of the given NPDE.

3 Applications

In this section, we present the applications of SGEM to the (1 + 1)- and (2 + 1)-dimensional Chiral nonlinear Schrödinger's equations [38, 39].

3.1 Application of SGEM to Eq. (1.1)

In this subsection, the SGEM is used in acquiring soliton solutions to the (1 + 1)-dimensional Chiral nonlinear Schrödinger's equation [38].

Substituting the following complex wave transformation:

$$\Psi = U(\zeta)e^{i\Omega}, \quad \zeta = c(x+vt), \quad \Omega = kx + \omega t + \varphi \quad (3.1)$$

into Eq. (1.1), one can get the relation

$$v = -2k \quad (3.2)$$

from the imaginary part and

$$c^2 U'' + 2k\sigma U^3 - (\omega + k^2)U = 0 \quad (3.3)$$

from the real part.

Balancing the terms U'' and U^3 in Eq. (3.3), yield $n = 1$.

With $n = 1$, Eqs. (2.8) and (2.9) become

$$U(\zeta) = B_1 \operatorname{sech}(\zeta) + A_1 \tanh(\zeta) + A_0 \quad (3.4)$$

and

$$U(w) = B_1 \sin(w) + A_1 \cos(w) + A_0. \quad (3.5)$$

Inserting Eq. (3.5) and its second derivative, we get an equation in trigonometric functions. After making some trigonometric identities substitutions, we collect a group of algebraic equations by equating each summation of the coefficients of the trigonometric functions with the same power to zero. To find the soliton solutions to Eq. (1.1), we insert the acquired values of the coefficients into Eq. (3.4).

Case 1:

$$A_0 = 0, \quad A_1 = 0, \quad B_1 = -\frac{c}{\sqrt{k\sigma}}, \quad \omega = (c-k)(c+k),$$

these coefficients give the following solution to Eq. (1.1):

$$\Psi_1(x, t) = -\frac{c}{\sqrt{k\sigma}} \operatorname{sech}[c(x-2kt)] e^{i(kx + (c-k)(c+k)t + \varphi)}, \quad (3.6)$$

with $k, \sigma \neq 0$ for valid soliton.

Case 2:

$$A_0 = 0, \quad A_1 = 0, \quad B_1 = -\frac{c(c^2 - \omega)^{1/4}}{\sqrt{\sigma(c^2 - \omega)}}, \quad k = \sqrt{c^2 - \omega},$$

these coefficients give the following solution to Eq. (1.1):

$$\Psi_2(x, t) = -\frac{c(c^2 - \omega)^{1/4}}{\sqrt{\sigma(c^2 - \omega)}} \operatorname{sech}\left[c(x - 2\sqrt{c^2 - \omega}t)\right] e^{i(\sqrt{c^2 - \omega}x + \omega t + \varphi)}, \quad (3.7)$$

with $\sigma \neq 0$ and $c^2 - \omega > 0$ for valid soliton.

Case 3:

$$A_0 = 0, \quad A_1 = 0, \quad B_1 = -\frac{\sqrt{k^2 + \omega}}{\sqrt{k\sigma}}, \quad c = \sqrt{k^2 + \omega},$$

these coefficients give the following solution to Eq. (1.1):

$$\Psi_3(x, t) = -\frac{\sqrt{k^2 + \omega}}{\sqrt{k\sigma}} \operatorname{sech}[\sqrt{k^2 + \omega}(x - 2kt)] e^{i(kx + \omega t + \varphi)}, \quad (3.8)$$

with $k, \sigma \neq 0$ and $k^2 + \omega > 0$ for valid soliton.

3.2 Application of SGEM to Eq. (1.2)

In this subsection, we apply the SGEM to the (2 + 1)-dimensional Chiral nonlinear Schrödinger's equation [39].

Substituting the following complex wave transformation:

$$\Psi = U(\zeta)e^{i\Omega}, \quad \zeta = \alpha x + \beta y - vt, \quad \Omega = px + qy + \omega t + \varphi \quad (3.9)$$

into Eq. (1.2), one can get the relation

$$v = 2a(\alpha p + \beta q) \quad (3.10)$$

from the imaginary part and

$$a(\alpha^2 + \beta^2)U'' + 2(pb_1 + qb_2)U^3 - (a(p^2 + q^2) + \omega)U = 0 \quad (3.11)$$

from the real part.

Balancing the terms U'' and U^3 in Eq. (3.3), yield $n = 1$.

Proceeding as before, we get the following solutions for Eq. (1.2):

Case 1:

$$A_0 = 0, \quad A_1 = 0, \quad B_1 = \frac{\sqrt{a(p^2 + q^2) + \omega}}{\sqrt{pb_1 + qb_2}},$$

$$\alpha = \frac{\sqrt{a(p^2 + q^2 - \beta^2) + \omega}}{\sqrt{a}},$$

we have

$$\Psi_1(x, y, t) = \frac{\sqrt{a(p^2 + q^2) + \omega}}{\sqrt{pb_1 + qb_2}} \operatorname{sech} \times \left[\beta y + \frac{\sqrt{a(p^2 + q^2 - \beta^2) + \omega}}{\sqrt{a}} x - 2a \left(\beta q + p \frac{\sqrt{a(p^2 + q^2 - \beta^2) + \omega}}{\sqrt{a}} \right) t \right] e^{i(px + qy + \omega t + \varphi)}, \quad (3.12)$$

with $a(a(p^2 + q^2 - \beta^2) + \omega) > 0$ for valid soliton.

Case 2:

$$A_0 = 0, \quad A_1 = 0, \quad B_1 = \frac{\sqrt{a(p^2 + q^2) + \omega}}{\sqrt{pb_1 + qb_2}},$$

$$\beta = \frac{\sqrt{a(p^2 + q^2 - \alpha^2) + \omega}}{\sqrt{a}},$$

we have

$$\Psi_2(x, y, t) = \frac{\sqrt{a(p^2 + q^2) + \omega}}{\sqrt{pb_1 + qb_2}} \operatorname{sech} \times \left[\alpha x + \frac{\sqrt{a(p^2 + q^2 - \alpha^2) + \omega}}{\sqrt{a}} y - 2a \left(\alpha p + q \frac{\sqrt{a(p^2 + q^2 - \alpha^2) + \omega}}{\sqrt{a}} \right) t \right] e^{i(px + qy + \omega t + \varphi)}, \quad (3.13)$$

with $a(a(p^2 + q^2 - \alpha^2) + \omega) > 0$ for valid soliton.

Case 3:

$$A_0 = 0, \quad A_1 = \frac{\sqrt{\omega(\alpha^2 + \beta^2)}}{\sqrt{(pb_1 + qb_2)(p^2 + q^2 + 2(\alpha^2 + \beta^2))}},$$

$$B_1 = 0,$$

$$a = -\frac{\omega}{p^2 + q^2 + 2(\alpha^2 + \beta^2)},$$

we have

$$\Psi_3(x, y, t) = \frac{\sqrt{\omega(\alpha^2 + \beta^2)}}{\sqrt{(pb_1 + qb_2)(p^2 + q^2 + 2(\alpha^2 + \beta^2))}} \tanh \times \left[\alpha x + \beta y + \frac{2\omega(\alpha p + \beta q)}{p^2 + q^2 + 2(\alpha^2 + \beta^2)} t \right] e^{i(px + qy + \omega t + \varphi)}, \quad (3.14)$$

with $\omega(\alpha^2 + \beta^2)(pb_1 + qb_2)(p^2 + q^2 + 2(\alpha^2 + \beta^2)) > 0$ for valid soliton.

Case 4:

$$A_0 = 0, \quad A_1 = \frac{\sqrt{-a(\alpha^2 + \beta^2)}}{\sqrt{pb_1 + qb_2}}, \quad B_1 = 0,$$

$$\omega = -a(p^2 + q^2 + 2(\alpha^2 + \beta^2)),$$

we have

$$\Psi_4(x, y, t) = \frac{\sqrt{-a(\alpha^2 + \beta^2)}}{\sqrt{pb_1 + qb_2}} \tanh[\alpha x + \beta y - 2a(\alpha p + \beta q)t] e^{i(px + qy - a(p^2 + q^2 + 2(\alpha^2 + \beta^2))t + \varphi)}, \quad (3.15)$$

with $-a(\alpha^2 + \beta^2)(pb_1 + qb_2) < 0$ for valid soliton.

4 Results and discussion

This section discusses the reported results in this study. For Eq. (1.1), bright soliton solutions which are Eqs. (3.6), (3.7) and (3.8) are reported. For (1.2), bright and dark soliton solutions which are Eqs. (3.12), (3.13) and (3.14), (3.15), respectively, are reported. Bright and dark solitons propagate in a nonlinear dispersive media. Dark soliton describes the solitary waves with lower intensity than the background while bright soliton describes the solitary waves whose peak intensity is larger than the background [44].

In order to have clear and good understanding of the physical properties of the constructed dark and bright soliton solutions, under the choice of the suitable values of parameters, the 3D, 2D and the contour graphs are plotted. The perspective view of the bright and dark solitons, Eqs. (3.7), (3.12) and (3.15) can be seen in the 3D graphs which appear in the (a) parts of Figs. 1, 2 and 3, respectively. The propagation pattern of the wave along the x -axis for Eqs. (3.7), (3.12) and (3.15) is illustrated in the 2D graphs which are located at the

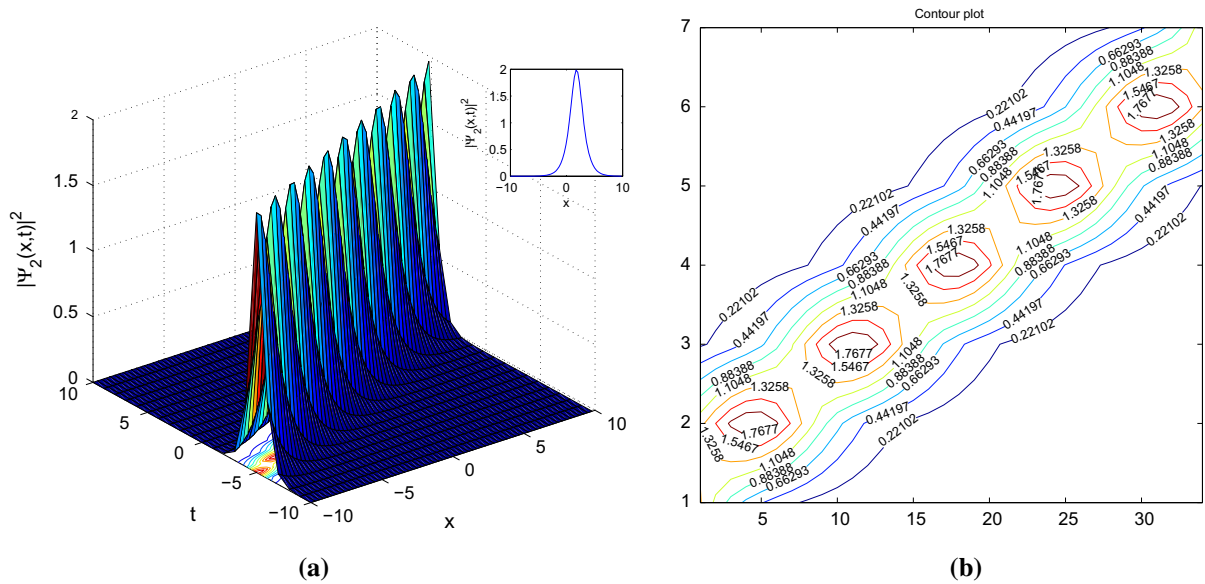
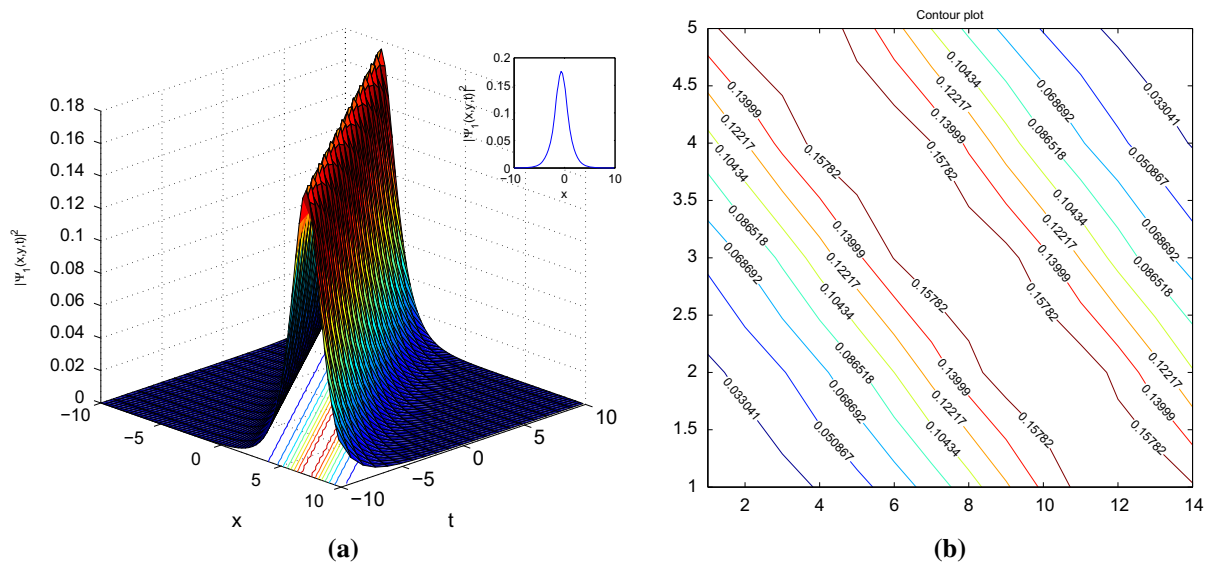


Fig. 1 a The 2D and 3D, b contour plots of Eq. (3.7)



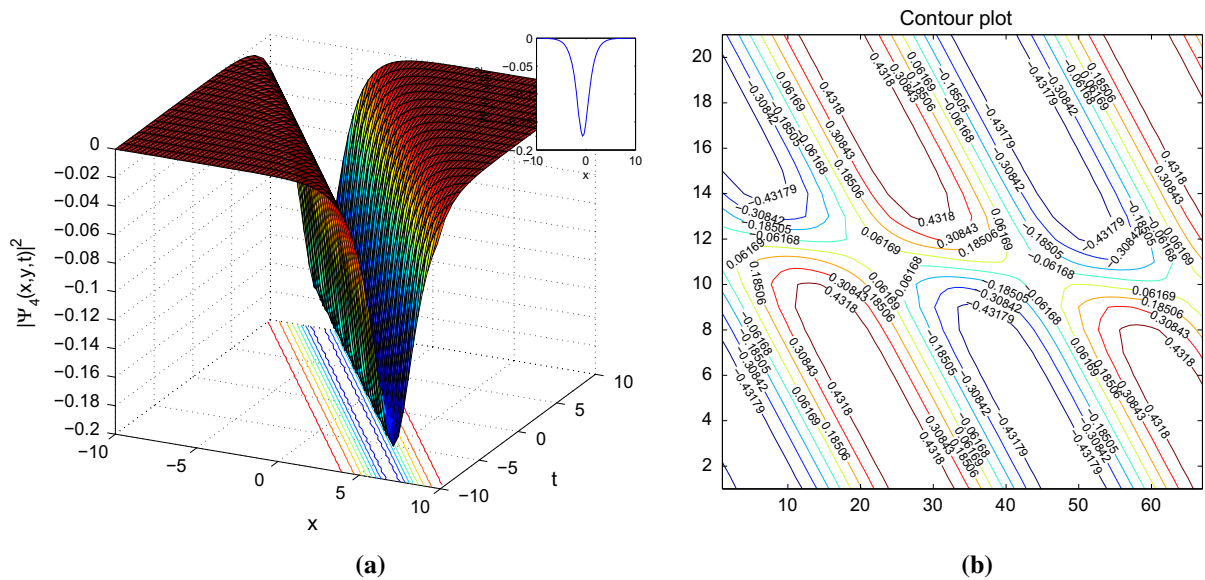


Fig. 3 **a** The 2D and 3D, **b** contour plots of Eq. (3.15)

the obtained solutions satisfy their corresponding equation. We plot the 2D, 3D and contour plots of some of the obtained solutions. The results presented in this study may be important in explaining the physical meaning of some nonlinear models arising in nonlinear sciences. We observed that sine-Gordon expansion method is a simple and powerful mathematical tool that gives good results when applied to complex nonlinear models.

Compliance with ethical standards

Conflict of interest The authors declare that they have no conflict of interest.

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