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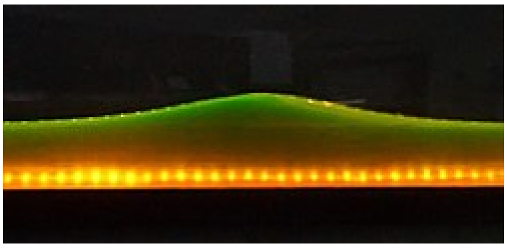
# Soliton

In mathematics and physics, a **soliton** or **solitary wave** is a self-reinforcing wave packet that maintains its shape while it propagates at a constant velocity. Solitons are caused by a cancellation of nonlinear and dispersive effects in the medium. (Dispersive effects are a property of certain systems where the speed of a wave depends on its frequency.) Solitons are the solutions of a widespread class of weakly nonlinear dispersive partial differential equations describing physical systems.



Solitary wave in a laboratory wave channel

The soliton phenomenon was first described in 1834 by John Scott Russell (1808–1882) who observed a solitary wave in the Union Canal in Scotland. He reproduced the phenomenon in a wave tank and named it the "Wave of Translation".

## Contents



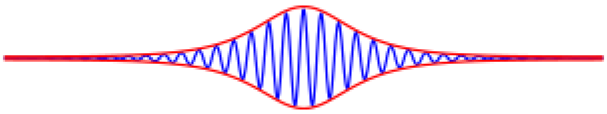
## Definition

A single, consensus definition of a soliton is difficult to find. Drazin & Johnson (1989, p. 15) ascribe three properties to solitons:

1. They are of permanent form;
2. They are localized within a region;
3. They can interact with other solitons, and emerge from the collision unchanged, except for a phase shift.

More formal definitions exist, but they require substantial mathematics. Moreover, some scientists use the term *soliton* for phenomena that do not quite have these three properties (for instance, the 'light bullets' of nonlinear optics are often called solitons despite losing energy during interaction).[1]

## Explanation



A hyperbolic secant (sech) envelope soliton for water waves: The blue line is the carrier signal, while the red line is the envelope soliton.

Dispersion and nonlinearity can interact to produce permanent and localized wave forms. Consider a pulse of light traveling in glass. This pulse can be thought of as consisting of light of several different frequencies. Since glass shows dispersion, these different frequencies travel at different speeds and the shape of the pulse therefore changes over time. However, also the nonlinear Kerr effect occurs; the refractive index of a material at a given frequency depends on the light's amplitude or strength. If the pulse has just the right shape, the Kerr effect exactly cancels the dispersion effect, and the pulse's shape does not change over time, thus is a soliton. See soliton (optics) for a more detailed description.

Many exactly solvable models have soliton solutions, including the Korteweg–de Vries equation, the nonlinear Schrödinger equation, the coupled nonlinear Schrödinger equation, and the sine-Gordon equation. The soliton solutions are typically obtained by means of the inverse scattering transform, and owe their stability to the integrability of the field equations. The mathematical theory of these equations is a broad and very active field of mathematical research.

Some types of tidal bore, a wave phenomenon of a few rivers including the River Severn, are 'undular': a wavefront followed by a train of solitons. Other solitons occur as the undersea internal waves, initiated by seabed topography, that propagate on the oceanic pycnocline. Atmospheric solitons also exist, such as the morning glory cloud of the Gulf of Carpentaria, where pressure solitons traveling in a temperature inversion layer produce vast linear roll clouds. The recent and not widely accepted soliton model in neuroscience proposes to explain the signal conduction within neurons as pressure solitons.

A topological soliton, also called a topological defect, is any solution of a set of partial differential equations that is stable against decay to the "trivial solution". Soliton stability is due to topological constraints, rather than integrability of the field equations. The constraints arise almost always because the differential equations must obey a set of boundary conditions, and the boundary has a nontrivial homotopy group, preserved by the differential equations. Thus, the differential equation solutions can be classified into homotopy classes.

No continuous transformation maps a solution in one homotopy class to another. The solutions are truly distinct, and maintain their integrity, even in the face of extremely powerful forces. Examples of topological solitons include the screw dislocation in a crystalline lattice, the Dirac string and the magnetic monopole in electromagnetism, the Skyrmion and the Wess–Zumino–Witten model in quantum field theory, the magnetic skyrmion in condensed matter physics, and cosmic strings and domain walls in cosmology.

## History

In 1834, John Scott Russell describes his *wave of translation*.[nb 1] The discovery is described here in Scott Russell's own words:[nb 2]

> I was observing the motion of a boat which was rapidly drawn along a narrow channel by a pair of horses, when the boat suddenly stopped – not so the mass of water in the channel which it had put in motion; it accumulated round the prow of the vessel in a state of violent agitation, then

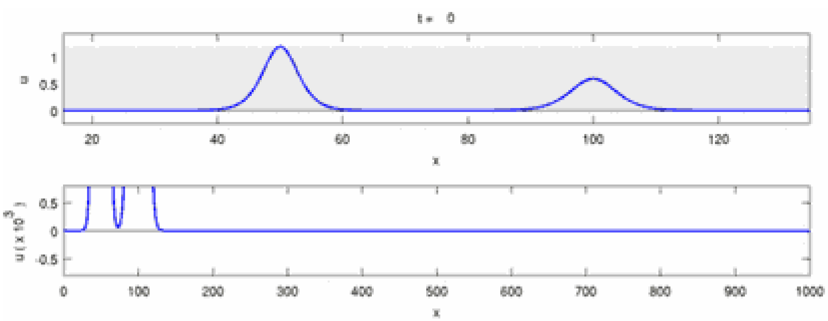
> suddenly leaving it behind, rolled forward with great velocity, assuming the form of a large solitary elevation, a rounded, smooth and well-defined heap of water, which continued its course along the channel apparently without change of form or diminution of speed. I followed it on horseback, and overtook it still rolling on at a rate of some eight or nine miles an hour, preserving its original figure some thirty feet long and a foot to a foot and a half in height. Its height gradually diminished, and after a chase of one or two miles I lost it in the windings of the channel. Such, in the month of August 1834, was my first chance interview with that singular and beautiful phenomenon which I have called the Wave of Translation.[2]

Scott Russell spent some time making practical and theoretical investigations of these waves. He built wave tanks at his home and noticed some key properties:

- The waves are stable, and can travel over very large distances (normal waves would tend to either flatten out, or steepen and topple over)
- The speed depends on the size of the wave, and its width on the depth of water.
- Unlike normal waves they will never merge – so a small wave is overtaken by a large one, rather than the two combining.
- If a wave is too big for the depth of water, it splits into two, one big and one small.

Scott Russell's experimental work seemed at odds with Isaac Newton's and Daniel Bernoulli's theories of hydrodynamics. George Biddell Airy and George Gabriel Stokes had difficulty accepting Scott Russell's experimental observations because they could not be explained by the existing water wave theories. Their contemporaries spent some time attempting to extend the theory but it would take until the 1870s before Joseph Boussinesq[3] and Lord Rayleigh published a theoretical treatment and solutions.[nb 3] In 1895 Diederik Korteweg and Gustav de Vries provided what is now known as the Korteweg–de Vries equation, including solitary wave and periodic cnoidal wave solutions.[4][nb 4]

In 1965 Norman Zabusky of Bell Labs and Martin Kruskal of Princeton University first demonstrated soliton behavior in media subject to the Korteweg–de Vries equation (KdV equation) in a computational investigation using a finite difference approach. They also showed how this behavior explained the puzzling earlier work of Fermi, Pasta, Ulam, and Tsingou.[6]



An animation of the overtaking of two solitary waves according to the Benjamin–Bona–Mahony equation – or BBM equation, a model equation for (among others) long surface gravity waves. The wave heights of the solitary waves are 1.2 and 0.6, respectively, and their velocities are 1.4 and 1.2.

The *upper graph* is for a frame of reference moving with the average velocity of the solitary waves.

The *lower graph* (with a different vertical scale and in a stationary frame of reference) shows the oscillatory tail produced by the interaction.[5] Thus, the solitary wave solutions of the BBM equation are not solitons.

In 1967, Gardner, Greene, Kruskal and Miura discovered an inverse scattering transform enabling analytical solution of the KdV equation.[7] The work of Peter Lax on Lax pairs and the Lax equation has since extended this to solution of many related soliton-generating systems.

Note that solitons are, by definition, unaltered in shape and speed by a collision with other solitons.[8] So solitary waves on a water surface are *near*-solitons, but not exactly – after the interaction of two (colliding or overtaking) solitary waves, they have changed a bit in amplitude and an oscillatory residual is left behind.[9]

Solitons are also studied in quantum mechanics, thanks to the fact that they could provide a new foundation of it through de Broglie's unfinished program, known as "Double solution theory" or "Nonlinear wave mechanics". This theory, developed by de Broglie in 1927 and revived in the 1950s, is the natural continuation of his ideas developed between 1923 and 1926, which extended the wave-particle duality introduced by Albert Einstein for the light quanta, to all the particles of matter. In 2019, researchers from Tel-Aviv university measured an accelerating surface gravity water wave soliton by using an external hydrodynamic linear potential. They also managed to excite ballistic solitons and measure their corresponding phases.[10]

# In fiber optics

Much experimentation has been done using solitons in fiber optics applications. Solitons in a fiber optic system are described by the Manakov equations. Solitons' inherent stability make long-distance transmission possible without the use of repeaters, and could potentially double transmission capacity as well.[11]

| Year | Discovery |
|---|---|
| 1973 | Akira Hasegawa of AT&T Bell Labs was the first to suggest that solitons could exist in optical fibers, due to a balance between self-phase modulation and anomalous dispersion.[12] Also in 1973 Robin Bullough made the first mathematical report of the existence of optical solitons. He also proposed the idea of a soliton-based transmission system to increase performance of optical telecommunications. |
| 1987 | Emplit et al. (1987) – from the Universities of Brussels and Limoges – made the first experimental observation of the propagation of a dark soliton, in an optical fiber. |
| 1988 | Linn Mollenauer and his team transmitted soliton pulses over 4,000 kilometers using a phenomenon called the Raman effect, named after Sir C. V. Raman who first described it in the 1920s, to provide optical gain in the fiber. |
| 1991 | A Bell Labs research team transmitted solitons error-free at 2.5 gigabits per second over more than 14,000 kilometers, using erbium optical fiber amplifiers (spliced-in segments of optical fiber containing the rare earth element erbium). Pump lasers, coupled to the optical amplifiers, activate the erbium, which energizes the light pulses. |
| 1998 | Thierry Georges and his team at France Telecom R&D Center, combining optical solitons of different wavelengths (wavelength-division multiplexing), demonstrated a *composite* data transmission of 1 terabit per second (1,000,000,000,000 units of information per second), not to be confused with Terabit-Ethernet.<br><br>The above impressive experiments have not translated to actual commercial soliton system deployments however, in either terrestrial or submarine systems, chiefly due to the Gordon–Haus (GH) jitter. The GH jitter requires sophisticated, expensive compensatory solutions that ultimately makes dense wavelength-division multiplexing (DWDM) soliton transmission in the field unattractive, compared to the conventional non-return-to-zero/return-to-zero paradigm. Further, the likely future adoption of the more spectrally efficient phase-shift-keyed/QAM formats makes soliton transmission even less viable, due to the Gordon–Mollenauer effect. Consequently, the long-haul fiberoptic transmission soliton has remained a laboratory curiosity. |
| 2000 | Cundiff predicted the existence of a vector soliton in a birefringence fiber cavity passively mode locking through a semiconductor saturable absorber mirror (SESAM). The polarization state of such a vector soliton could either be rotating or locked depending on the cavity parameters.[13] |
| 2008 | D. Y. Tang *et al.* observed a novel form of higher-order vector soliton from the perspectives of experiments and numerical simulations. Different types of vector solitons and the polarization state of vector solitons have been investigated by his group.[14] |

# In biology

Solitons may occur in proteins[15] and DNA.[16] Solitons are related to the low-frequency collective motion in proteins and DNA.[17]

A recently developed model in neuroscience proposes that signals, in the form of density waves, are conducted within neurons in the form of solitons.[18][19][20] Solitons can be described as almost lossless energy transfer in biomolecular chains or lattices as wave-like propagations of coupled conformational and electronic disturbances.[21]

# In magnets

In magnets, there also exist different types of solitons and other nonlinear waves.[22] These magnetic solitons are an exact solution of classical nonlinear differential equations — magnetic equations, e.g. the Landau–Lifshitz equation, continuum Heisenberg model, Ishimori equation, nonlinear Schrödinger equation and others.

# In nuclear physics

Atomic nuclei may exhibit solitonic behavior.[23] Here the whole nuclear wave function is predicted to exist as a soliton under certain conditions of temperature and energy. Such conditions are suggested to exist in the cores of some stars in which the nuclei would not react but pass through each other unchanged, retaining their soliton waves through a collision between nuclei.

The Skyrme Model is a model of nuclei in which each nucleus is considered to be a topologically stable soliton solution of a field theory with conserved baryon number.

# Bions

The bound state of two solitons is known as a *bion*,[24][25][26] or in systems where the bound state periodically oscillates, a *breather*.

In field theory *bion* usually refers to the solution of the Born–Infeld model. The name appears to have been coined by G. W. Gibbons in order to distinguish this solution from the conventional soliton, understood as a *regular*, finite-energy (and usually stable) solution of a differential equation describing some physical system.[27] The word *regular* means a smooth solution carrying no sources at all. However, the solution of the Born–Infeld model still carries a source in the form of a Dirac-delta function at the origin. As a consequence it displays a singularity in this point (although the electric field is everywhere regular). In some physical contexts (for instance string theory) this feature can be important, which motivated the introduction of a special name for this class of solitons.

On the other hand, when gravity is added (i.e. when considering the coupling of the Born–Infeld model to general relativity) the corresponding solution is called *EBIon*, where "E" stands for Einstein.

# See also

- Compacton, a soliton with compact support
- Freak waves may be a Peregrine soliton related phenomenon involving breather waves which exhibit concentrated localized energy with non-linear properties.[28]
- Nematicons

- Non-topological soliton, in quantum field theory
- Nonlinear Schrödinger equation
- Oscillons
- Pattern formation
- Peakon, a soliton with a non-differentiable peak
- Q-ball a non-topological soliton
- Sine-Gordon equation
- Soliton (topological)
- Soliton distribution
- Soliton hypothesis for ball lightning, by David Finkelstein
- Soliton model of nerve impulse propagation
- Topological quantum number
- Vector soliton

# Notes

1. "Translation" here means that there is real mass transport, although it is not the same water which is transported from one end of the canal to the other end by this "Wave of Translation". Rather, a fluid parcel acquires momentum during the passage of the solitary wave, and comes to rest again after the passage of the wave. But the fluid parcel has been displaced substantially forward during the process – by Stokes drift in the wave propagation direction. And a net mass transport is the result. Usually there is little mass transport from one side to another side for ordinary waves.
2. This passage has been repeated in many papers and books on soliton theory.
3. Lord Rayleigh published a paper in *Philosophical Magazine* in 1876 to support John Scott Russell's experimental observation with his mathematical theory. In his 1876 paper, Lord Rayleigh mentioned Scott Russell's name and also admitted that the first theoretical treatment was by Joseph Valentin Boussinesq in 1871. Joseph Boussinesq mentioned Russell's name in his 1871 paper. Thus Scott Russell's observations on solitons were accepted as true by some prominent scientists within his own lifetime of 1808–1882.
4. Korteweg and de Vries did not mention John Scott Russell's name at all in their 1895 paper but they did quote Boussinesq's paper of 1871 and Lord Rayleigh's paper of 1876. The paper by Korteweg and de Vries in 1895 was not the first theoretical treatment of this subject but it was a very important milestone in the history of the development of soliton theory.

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# External links

### Related to John Scott Russell

- John Scott Russell and the solitary wave (http://www.ma.hw.ac.uk/~chris/scott_russell.html)
- John Scott Russell biography (http://www.ma.hw.ac.uk/solitons/LocalHeroes/sr.html)
- Photograph of soliton on the Scott Russell Aqueduct (http://www.ma.hw.ac.uk/solitons/soliton1b.html)

### Other

- Heriot–Watt University soliton page (http://www.ma.hw.ac.uk/solitons/)
- Helmholtz solitons, Salford University (https://web.archive.org/web/20110716073821/http://www.cse.salford.ac.uk/profiles/gsmcdonald/solitons.php)
- Short didactic review on optical solitons (http://www.opfocus.org/index.php?topic=story&v=5&s=5)
- Filmed collision between two Solitons (https://www.youtube.com/watch?v=G-xHbcMm_Gs) on YouTube