Assignment

Numerical Methods for PDE

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1 Question

Write a code to solve second order BVP for ODE and solve the ODE: $7y'' - 2y' - y + x = 0$ **with Boundary conditions** $y(0) = 5$ **and** $y(20) = 8$**. Play with Δx, finite difference approximations. Plot solutions for different Δx .**

1.1 Coding

```
p = @(x) (2/7);
 q = @(x) (1/7);
 r = @(x) (-x);

 aa = 0; bb = 20; alpha = 5; beta = 8; n=30;

 fprintf('    x          y    \n');
 h = (bb-aa)/n
 a = zeros(1,n+1);
 b = zeros(1,n+1);
 c = zeros(1,n+1);
 d = zeros(1,n+1);
 l = zeros(1,n+1);
 u = zeros(1,n+1);
 z = zeros(1,n+1);
 y = zeros(1,n+1);
 x = aa+h;
 a(1) = 2+(h^2)*q(x);
 b(1) = -1+(0.5)*h*p(x);
 d(1) = -(h^2)*r(x)+(1+(0.5)*h*p(x))*alpha;
m = n-1;

 for i = 2 : m
   x = aa+i*h;
   a(i) = 2+(h^2)*q(x);
   b(i) = -1+(0.5)*h*p(x);
   c(i) = -1-(0.5)*h*p(x);
   d(i) = -(h^2)*r(x);
 end

 x = bb-h;
 a(n) = 2+(h^2)*q(x);
 c(n) = -1-(0.5)*h*p(x);
 d(n) = -(h^2)*r(x)+(1-(0.5)*h*p(x))*beta;
 l(1) = a(1);
 u(1) = b(1)/a(1);
 z(1) = d(1)/l(1);

 for i = 2 : m
   l(i) = a(i)-c(i)*u(i-1);
   u(i) = b(i)/l(i);
   z(i) = (d(i)-c(i)*z(i-1))/l(i);
 end
```

```
44
45  l(n) = a(n)-c(n)*u(n-1);
46  z(n) = (d(n)-c(n)*z(n-1))/l(n);
47  y_0 = alpha;
48  y_4 = beta;
49  y(n) = z(n);
50
51  for j = 1 : m
52    i = n-j;
53    y(i) = z(i)-u(i)*y(i+1);
54  end
55  fprintf('   x            y       \n');
56  i = 0;
57  fprintf('%5.4f      %11.8f\n', aa, alpha);
58  for i = 1 : n
59    x = aa+i*h;
60    fprintf('%5.4f      %11.8f\n', x, y(i));
61  end
62  i = n+1;
63  fprintf('%5.4f      %11.8f\n', bb, beta);
64  x=0:h:20;
65  plot(x,y)
66  xlabel('x');
67  ylabel('y');
68  title('Plot for h =',h);
69  grid on;
```

1.2 Output:

1. For h=0.5

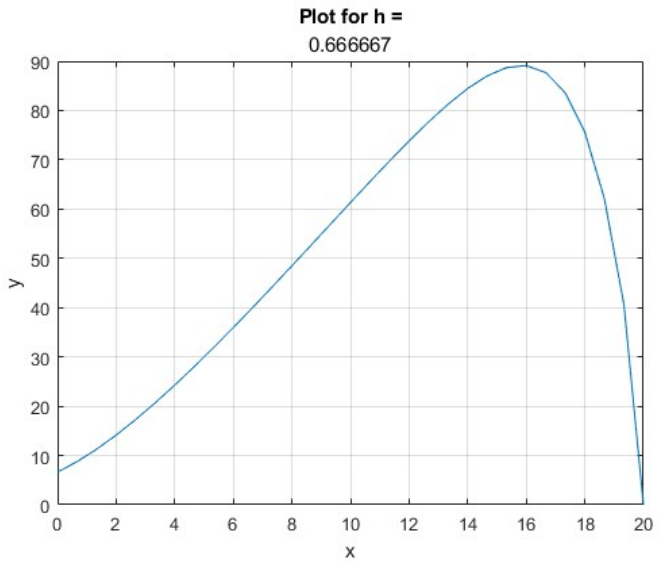

Figure 1: Plot for h=0.666

2. For h=1

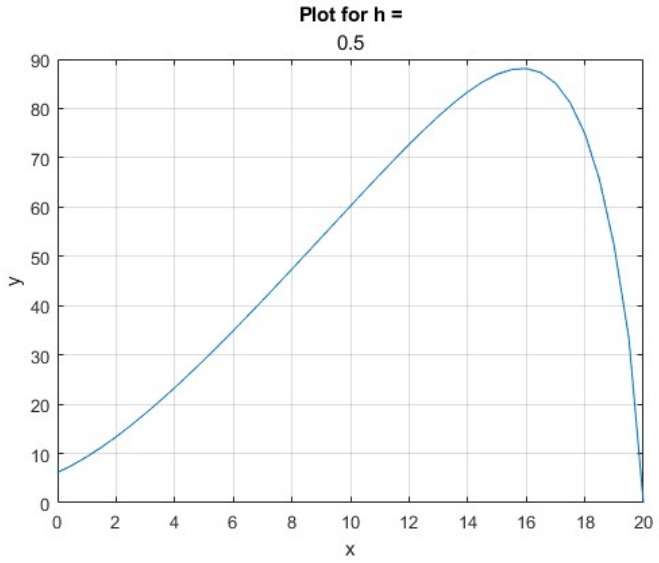

Figure 2: Plot for h=1

1.3 Analytical Solution:

Solution is: $-0.00018exp(0.54691x)+7.00018exp(-0.261203x)+x-2$

2 Question

Write a code to solve second order BVP for ODE and solve the ODE: $x''(t)=\frac{2t}{1+t^2}x'(t)-\frac{2}{1+t^2}x(t)+1$ **with boundary conditions** $x(0)=125$ **and** $x(4)=-0.95$**. Play with Δx, finite difference approximations. Plot solutions for different Δx .**

2.Solution Coding

```
p = @(x) (2*x/(1+x^2));
 q = @(x) (-2/(1+x^2));
 r = @(x) (1);

 aa = 0; bb = 4; alpha = 125; beta = -0.95; n=100;

 fprintf('    x              y    \n');
 h = (bb-aa)/n
 a = zeros(1,n+1);
 b = zeros(1,n+1);
 c = zeros(1,n+1);
 d = zeros(1,n+1);
 l = zeros(1,n+1);
 u = zeros(1,n+1);
 z = zeros(1,n+1);
 y = zeros(1,n+1);
 x = aa+h;
 a(1) = 2+(h^2)*q(x);
 b(1) = -1+(0.5)*h*p(x);
 d(1) = -(h^2)*r(x)+(1+(0.5)*h*p(x))*alpha;
m = n-1;
```

```
22
23  for i = 2 : m
24    x = aa+i*h;
25    a(i) = 2+(h^2)*q(x);
26    b(i) = -1+(0.5)*h*p(x);
27    c(i) = -1-(0.5)*h*p(x);
28    d(i) = -(h^2)*r(x);
29  end
30
31  x = bb-h;
32  a(n) = 2+(h^2)*q(x);
33  c(n) = -1-(0.5)*h*p(x);
34  d(n) = -(h^2)*r(x)+(1-(0.5)*h*p(x))*beta;
35  l(1) = a(1);
36  u(1) = b(1)/a(1);
37  z(1) = d(1)/l(1);
38
39  for i = 2 : m
40    l(i) = a(i)-c(i)*u(i-1);
41    u(i) = b(i)/l(i);
42    z(i) = (d(i)-c(i)*z(i-1))/l(i);
43  end
44
45  l(n) = a(n)-c(n)*u(n-1);
46  z(n) = (d(n)-c(n)*z(n-1))/l(n);
47  y_0 = alpha;
48  y_4 = beta;
49  y(n) = z(n);
50
51  for j = 1 : m
52    i = n-j;
53    y(i) = z(i)-u(i)*y(i+1);
54  end
55  fprintf('  x            y      \n');
56  i = 0;
57  fprintf('%5.4f     %11.8f\n', aa, alpha);
58  for i = 1 : n
59    x = aa+i*h;
60    fprintf('%5.4f     %11.8f\n', x, y(i));
61  end
62  i = n+1;
63  fprintf('%5.4f     %11.8f\n', bb, beta);
64
65
66  x=0:h:4;
67  plot(x,y)
68  xlabel('x');
69  ylabel('y');
70  title('Plot for h=',h);
71  grid on;
```

2.1 Output:

1. For h=0.2

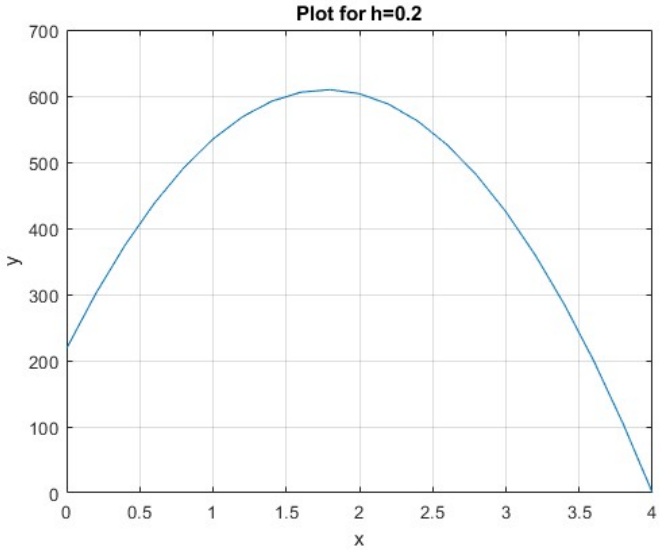

Figure 3: Plot for h=0.2

2. For h= 0.04

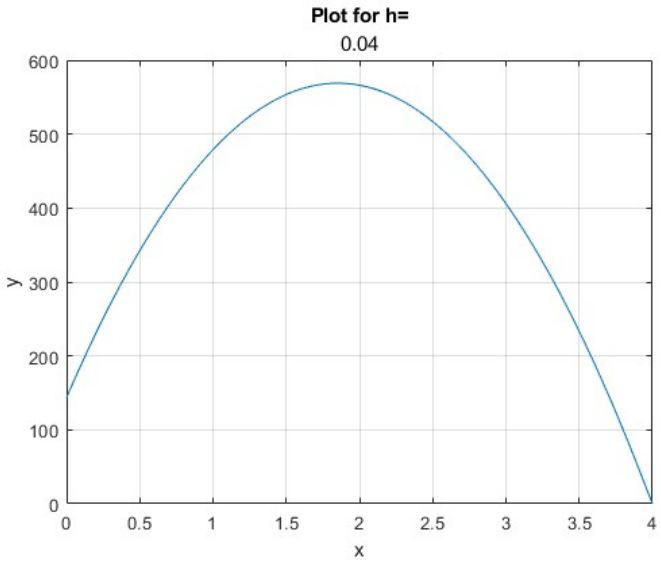

Figure 4: Plot for h=0.02

2.2 Analytical Solution:

Solution is: $-125(x^2-1)-((2arctan(4))+(15log(17)/8-(37801/80)))x-(x^2)+2xarctan(x)-(1)log(1+x^2)+(1)(x^2)log(1+x^2)$