Determination of Bend Mode Characteristics in Dielectric Waveguides

Andrea Melloni, Federico Carniel, Raffaella Costa, and Mario Martinelli

***Abstract*—A new and effective method for the electromagnetic analysis of generic bent dielectric waveguides is presented. The method is based on the expansion of the bend mode in modes of the straight waveguide and permits determination of the shape and the phase constant of the fundamental mode of the bend waveguide with great accuracy at a very low computational cost. Simple analytical expressions of the phase constant, coupling losses, and bending-induced birefringence are derived under very general conditions. The proposed method is useful for the design and optimization of each bent section of integrated optics components.**

***Index Terms*—Integrated optics, numerical analysis, optical propagation, optical waveguides, waveguide bends.**

I. Introduction

BENT waveguides are the key building blocks of many optical integrated components. In spite of their apparent simplicity, the analysis of the propagation in a bent dielectric waveguide continues to be a challenging electromagnetic problem. Since the paper by Marcatili [1] was published more than 30 years ago, a huge amount of work, both theoretical and experimental, has been carried out on this subject.

Three factors contribute to the propagation characteristics of a bent waveguide: pure radiation losses, transition losses between the straight and the bent waveguide, and the phase constant of the propagating field. The radiation losses are the most important factor, but in practice they are ultimately determined by several practical aspects such as the roughness of the waveguide and the technological process itself. The minimum useful bending radius is somewhat determined experimentally. The waveguide is then always used at larger bending radius, and the phase constant and the coupling losses remain the two factors that influence and often determine the properties of the whole component in which they are inserted. As an example, bent waveguides are used as delay lines in an integrated Mach–Zehnder interferometers or in waveguide grating routers (WGRs). In such cases, the accurate knowledge of the phase constant of the bent sections is fundamental for the correct design of the device.

The analysis of the propagation in the bend can be studied by means of approximated techniques [1]–[6] or numerical methods [7]–[13]. The accuracy obtained by some of the available numerical techniques is excellent [10]–[13], but quite often it is difficult to take advantage from such methods for the design of the bend. On the other hand, approximated methods are not accurate enough to be used for the final design of the component. A simple and effective method that satisfies the above requirements is therefore advisable.

In this paper, we present a new method for the analysis of bent waveguides with several significant and useful characteristics. The method is based on the expansion of the bend mode in modes of the straight waveguide, including the modes under the cutoff.

The proposed method is simple: it requires only a good but classical mode solver for straight waveguides such as the beam propagation method (BPM) [14] or the Fourier decomposition method [15]. It is very efficient: once the modes of the straight waveguide are determined, the mode shape distortion and phase constant perturbation for each desired bending radius can be calculated at no extra computational cost. It is valid for every kind of dielectric waveguide, single mode as well as multimode; it is physically well based; and it gives a new explanation of the propagating mechanisms that take place in the bend. This method is useful also for the design and the optimization of each bent section of integrated optics components, even if it fails to predict the radiation losses. Moreover, simple analytical expressions of the phase constant, coupling losses, and bending-induced birefringence are derived under very general conditions.

This paper is structured as follows. Section II describes in detail the theory of the proposed method, the assumptions, the limits, and the meaning of the mode under the cutoff. In Section III, simple formulas for the determination of the phase constant perturbation and the mode shape deformation are derived and discussed. In Section IV, the results obtained with the proposed method are compared with those obtained by other well-established methods. In Section V, some numerical results relative to three typical waveguides are presented and discussed. Section VI presents our conclusions.

II. Theory

The normal modes of the bent waveguide can be described as a linear combination of solutions of the Helmoltz equation in cylindrical coordinates. The solution of such a problem is a difficult task even for a simple slab waveguide [2], and this approach is generally avoided. Instead, we propose to approximate the modes of the bent waveguide as a linear combination of the straight waveguide modes [16]. This expansion is assumed to hold in a first-order neighborhood of the cross section. In

Manuscript received March 15, 2000; revised December 18, 2000.

A. Melloni and M. Martinelli are with CoreCom, Consorzio Elaborazione e Commutazione Ottica Milano, Milan 20131, Italy, and also with the Dipartimento di Elettronica e Informazione, Politecnico di Milano, Milan 20133, Italy (e-mail: melloni@elet.polimi.it).

F. Carniel and R. Costa are with CoreCom, Consorzio Elaborazione e Commutazione Ottica Milano, Milan 20131, Italy.

Publisher Item Identifier S 0733-8724(01)02761-X.

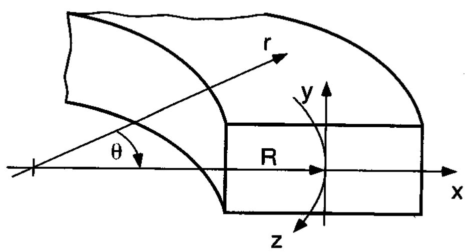

Fig. 1. The bent waveguide and its coordinate system.

other terms, we describe wave propagation in a perturbed (bent) waveguide by a combination of the unperturbed (straight) eigenmodes [17]. The assumptions and hypothesis at the base of this approach are introduced in the course of the description of the method and validated further on.

Let us define two orthogonal coordinates relative to the transverse cross-section of the waveguide x and y and a z-coordinate measured along the bend, as shown in Fig. 1. Without loss of generality, the bend is assumed in the $x-z$ plane. A first-order neighborhood of the section z is defined by $dz(x) = r(x)d\theta$, where dz is the increment along z due to an increase $d\theta$ of the bend angle and r are functions of x. Neglecting the small reflections due to the bending of the waveguide, the electric field $E_B(x, y)$ of a generic bend mode can be described in terms of forward modes $\psi_n(x, y)$ of the same straight waveguide only

$$E_B(x,y)|_{\theta=0} = \sum_{n=1}^{N} a_n \psi_n(x,y) \tag{1}$$

where a_n are the real weighting modal coefficients to be determined. A similar expansion holds for the magnetic field too. In principle, the sum in (1) extends to every mode (TE, TM, radiatives, $N = \infty, \ldots$), but under the assumption that bend modes remain TE-like or TM-like, only a few modes of the interested family can be considered.

After a propagation of dz, the mode field in the bend becomes

$$E_B(x,y)|_{\theta=d\theta} = \sum_{n=1}^{N} a_n \psi_n(x,y) e^{-\gamma_n \, dz} \tag{2}$$

where $\gamma_n = \alpha_n + j\beta_n$ is the propagation constant of the nth straight mode. Neglecting the material absorption, the attenuation constant α_n is not zero only for modes under the cutoff.

To compute the coefficients a_n, it is necessary to impose that after travelling an angular distance $d\theta$, the mode field E_B keeps undistorted, apart from a multiplying exponential term $\exp(-\nu d\theta) \cong (1-\nu d\theta)$, with ν the propagation constant of the bend modes. In other words, expanding the exponential term in (2) to the first order, the bend mode is

$$\sum_{n=1}^{N} a_n \psi_n \left(1 - \gamma_n r \; d\theta\right) = (1 - \nu d\theta) \sum_{n=1}^{N} a_n \psi_n \tag{3}$$

and after defining $r(x) = R + x$ (see Fig. 1), (3) reduces to

$$\sum_{n=1}^{N} a_n \psi_n \gamma_n (R + x) = \nu \sum_{n=1}^{N} a_n \psi_n. \tag{4}$$

In order to solve for the coefficients a_n, one can now use the orthogonality of the mode functions. By multiplying (4) by ψ_m^* and integrating over the cross section, one finds

$$a_m \gamma_m R + \sum_{n=1}^{N} a_n \gamma_n c_{mn} = \nu a_m \tag{5}$$

where

$$c_{mn} = \iint x \psi_n \psi_m^* \, dx \, dy \tag{6}$$

is the coupling coefficient between straight modes due to the bending. In deriving (5), the functions ψ_n are assumed to be orthonormal. By reducing (5) in matrix form, one gets the following matrix eigenvalue equation:

$$(R\mathbf{I} + \mathbf{C})\mathbf{G}\mathbf{a} = \boldsymbol{\nu}\mathbf{a} \tag{7}$$

where
- $\mathbf{I}$ identity matrix;
- $\mathbf{G}$ diagonal matrix whose coefficients are the propagation constants γ_n;
- $\mathbf{C}$ square matrix with coefficients c_{mn};
- $\mathbf{a}$ vector of the weighting coefficients.

Letting $\mathbf{M} = (R\mathbf{I} + \mathbf{C})\mathbf{G}$, the propagation constants of the bend modes are the eigenvalues ν of the matrix $\mathbf{M}$. The corresponding modal eigenfunctions, defined as in (1), are the eigenvectors of $\mathbf{M}$. The effective refractive indexes of the bent modes are simply given by

$$\mathbf{n_{eff}} = \frac{\lambda}{2\pi R} \, \text{Im}\,(\boldsymbol{\nu}). \tag{8}$$

From (6) and (7), it appears that the bend couples the fundamental mode to the modes with field variations in the plane of the bend only, that is, the E_{n1}^x, E_{n1}^y modes. Only these modes, therefore, have to be taken into account in the expansion (1), including the higher order modes that are below the cutoff. Properly speaking, a mode below the cutoff is not a mode but a packet of radiative modes, or plane waves, which propagate along the waveguide [15]. Each plane wave loses energy in the cladding in a different way from the other ones, and the packet broadens and changes its shape during the propagation. However, if such a mode is not far from the cutoff, its shape remains well defined, and it is still possible to define a phase velocity and a mode shape. These modes are usually called quasi-modes or leaky modes, and they have a complex propagating constant because of the radiation.

In practice, only leaky modes with an attenuation constant α_n much smaller then the phase constant β_n play an important role. This condition is often verified only for the first leaky mode. The number of modes to include in the expansion (1) is therefore very small: the fundamental mode, all the guided modes with field variations in the plane of the bend only, and the first horizontal leaky mode, each one belonging to the same family, TE (E^x) or TM (E^y).

Waveguide modes can be easily calculated with classical mode solvers such as the BPM [14] or the Fourier decomposition method [15]. For modes under the cutoff in weakly guiding structures, it is advisable to use large computational windows or the method proposed in [18].

The main advantage of the proposed method is that once the modes of the straight waveguide are known, the computation of the bend mode and its phase velocity is greatly simplified with respect to other numerical methods; it is very fast and accurate. Moreover, as shown in the next section, the dependence with respect to the bending radius of bend mode characteristics such as phase velocity, birefringence, mode shape, coupling losses with a straight waveguide, and so on can be derived analytically, simplifying the investigation and the optimization of a large number of optical components.

III. The Bend Mode

The shape and the phase constant perturbations to the mode of a generic straight waveguide, induced by the bending, can be calculated with the effective numerical method proposed in the previous section. In the important and common case of a monomode waveguide, only two modes, the fundamental mode and the first horizontal leaky mode (E_{21}^x, E_{21}^y), are sufficient to predict the bent waveguide behavior with an excellent accuracy.

The proposed method can be summarized as following. First, the fundamental mode and the first mode below the cutoff are numerically calculated with a good mode solver; then the overlap integrals c_{mn} (6) are computed. Finally, the eigenvalues and eigenvectors are obtained from the eigenvalue equation [(7)]. In the following, simple expressions of the phase constant and the mode shape of a bent monomode waveguide are derived and discussed. First, however, a comment on the role of the attenuation constants of the various modes is necessary.

A. Attenuation Constants

The imaginary part of the eigenvalues ν is related to the phase constant of the bend mode, through (8). The real part of ν, however, fails to give any information on the attenuation of the bend mode. The radiation losses, in fact, depend on the behavior of the field far away from the core [2], where it is radiative. According to this simple model, the bend mode can be seen as the fundamental straight mode distorted by the first leaky mode, and therefore the radiative field is not accurately described. The fact that the phase constant is correctly determined but not the attenuation is not surprising; it is a typical property of perturbation techniques.

Leaky modes enter into the description of a bend mode only if their attenuation constant α_n is very small compared to the phase constant β_n. If highly radiative leaky modes are required in the mode expansion, this means that the propagation in a bent waveguide is probably compromised. In any case, the attenuation constant α_n can be neglected for two reasons. First, it contributes to the useless real part of ν, that it is without physical meaning; secondly, it gives a negligible correction to the phase constant of the fundamental bend mode. This is numerically verified in Section IV.

By neglecting α_n, the mode functions ψ_n can be assumed real. The eigenvalue problem (7) and the coupling coefficients c_{mn} allow a more simple determination of the shape and the phase constant of the bend mode, as explained in the next two sections.

B. Phase Constant

If only two modes are taken into account, once the coupling coefficients c_{mn} are calculated, the eigenvalue problem can be solved analytically. The expression of the effective index of the bend mode (8) is simple to calculate but it is rather long. Instead, it is more interesting to expand the expression in power series for large bending radius and keep only the firsts terms. By solving (7) neglecting α_2, the phase constant β_B of the fundamental bend mode results as

$$\beta_B = \beta_1 + \beta_1 \frac{c_{11}}{R} + \frac{\beta_1 \beta_2 c_{12}^2}{(\beta_1 - \beta_2) R^2} + \cdots. \tag{9}$$

Clearly, for very large bending radius, β_B tends to the phase constant β_1 of the straight waveguide fundamental mode, while for finite radius the correcting terms in (9) must be considered. The first correcting term vanishes if the refractive index profile of the waveguide is symmetric in the plane of the bend because the mode is even with respect to the center of the waveguide [see (6)], and thus $c_{11} = 0$. The same holds for the other higher order odd terms. Hence, the perturbation to the phase constant in bent symmetric waveguides is $\Delta\beta = \beta_B - \beta_1 = B/R^2$, as predicted in [3] and discussed in [12]. From (9), parameter B is simply

$$B = \frac{\beta_1 \beta_2 c_{12}^2}{(\beta_1 - \beta_2)}. \tag{10}$$

Equation (10) states that at a given radius, the phase constant perturbation is higher for weakly guiding structures because the difference between the phase constants of the two first modes is smaller with respect to strongly confined waveguides. This simple formula shows that B is positive, with β_1 greater than β_2, a result that is consistent with the slowing down of the propagating field in the bend. For sharp bends, it could be necessary to solve numerically the eigenvalue equation (7) instead of using (9) or (10), but in such a case the strong radiation losses can compromise the propagation in the bend.

The results given by (9) and (10) are general and valid for every kind of waveguide and state that parameter B by itself characterizes the phase constant perturbation of a symmetric bent waveguide.

C. Mode Shape

The mode shape of the fundamental bend mode is defined by the eigenvectors. This mode can be seen as the combination of the fundamental mode of the straight waveguide of amplitude a_1 with the second mode of amplitude a_2. The coefficient a_2 characterizes the shape of the bend mode. It is a direct measure of its distortion respect to the straight waveguide mode. The sign of a_2 indicates the direction of the bend, right or left.

As for the eigenvalues, also for the coefficients a_i an analytical expression can be found. Instead of considering the full

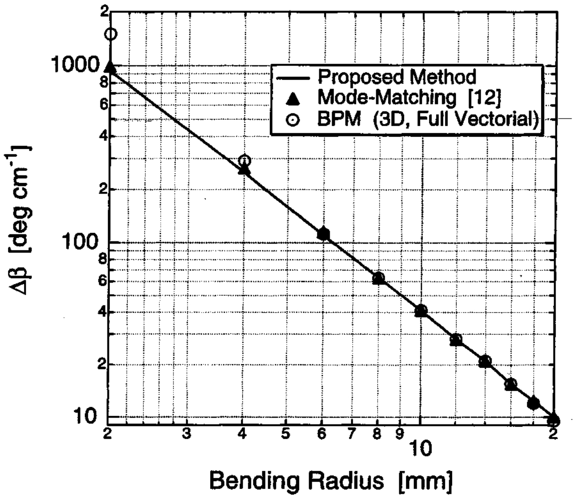

Fig. 2. Comparison between bend-induced phase-constant perturbation $\Delta\beta$ versus the bending radius calculated with the proposed method, the BPM [14], and the mode-matching [12].

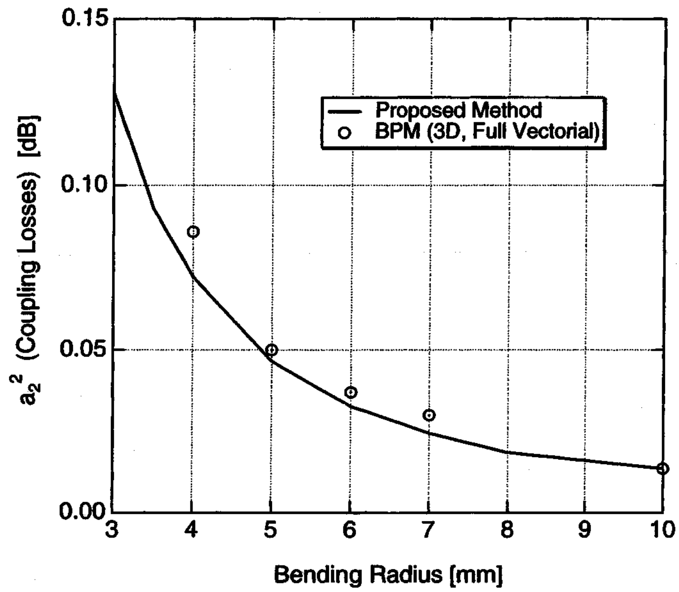

Fig. 3. Comparison between coupling losses versus the bending radius calculated with the proposed method and the BPM [14].

expression, however, a simple form of the coefficient a_2 is obtained by keeping only the first term of its power expansion for large R, which for symmetric waveguides is

$$a_2 \simeq \frac{c_{12}\beta_2}{R(\beta_1 - \beta_2)} = \frac{A}{R} \tag{11}$$

where the radius R must be entered with a sign that indicates a right or left curve, so that the shape of the bend mode remains shifted outward from the bend. The excitation of the second order mode is higher for weakly guiding structures, where the difference between the phase constant of the two modes is smaller.

Coefficient a_2, that is, parameter A, is an important and useful parameter that characterizes, by itself, the shape of the mode. For example, in the transition between the straight and the bent waveguide, the power "lost" on the second mode is proportional to $|a_2|^2$, corresponding to the projection of φ_2 over E_B. The transition losses go as R^{-2}, as can be predicted by the overlap integral of two Gaussian functions with an offset equal to the shift of the maximum of the bent mode respect to that of the straight guided mode [7]. However, (11) predicts that the behavior R^{-2} of this loss contribution is a more general property and is valid for every kind of waveguide.

IV. Assessment of the Method

The assessment and the reliability of the proposed method is verified here by comparing our results with those obtained by others methods available in literature. The comparisons on the parameters $\Delta\beta$ and a_2 are carried out using a monomode buried waveguide 5.2 μm wide and 4.8 μm high [12], with an index difference $\Delta n = 0.64\%$. The choice of this waveguide is a good test for the proposed method because the first leaky mode is highly radiative and is more difficult to compute with respect to those of rib or ridge waveguides. The comparison refers to the TE-like mode at a wavelength of 1550 nm, but similar results are obtained also for TM-like modes and other wavelengths.

At first, the two straight modes are calculated using a standard BPM [14]. The attenuation constant of the leaky mode is $\alpha_2 = 23$ dB, more than four orders of magnitude smaller than β_2. The waveguide is symmetric in the plane of the bend, and so only the coupling coefficient c_{12} is calculated through the integral (6). Neglecting α_2, the coefficients A and B are calculated by means of (10) and (11), and a_2 and $\Delta\beta$ are determined for any desired bending radius without any further time-consuming numerical simulations.

Fig. 2 reports the comparison between the perturbations $\Delta\beta$ of the phase constants versus the bending radius, calculated with the proposed method, with a BPM and with the mode-matching method presented in [12]. The agreement between the three methods is very good, and we found $B = 401$ deg mm, as in [12]. The BPM instead begins to be inaccurate for a bending radius of about 5 mm, while for smaller radius, the dimensions of the computational window become critical and the calculation is time consuming.

For this waveguide, the phase constant β_B, calculated taking into account the leaky mode attenuation, differs from the value obtained neglecting α_2 by less than 10^{-6}, for bending radius lower that 4 mm. The effect of the leaky mode attenuation can be always neglected, even in the case of higher attenuation because the coupling with the first mode is weaker. For this reason, all the results presented in the next section are obtained considering only one leaky mode and neglecting its attenuation.

The comparison between the amplitude a_2 of the second mode calculated with the proposed method and with the BPM is shown in Fig. 3. According to many authors [5], the transition losses in the junction between the straight waveguide and the bent waveguide is given by the overlap integral between the bend mode and the straight mode. In the present model, the transition loss corresponds to the square of the coefficient a_2, even if we prefer to refer to it as "second mode excitation." In Fig. 3, the agreement between the two methods is satisfactory,

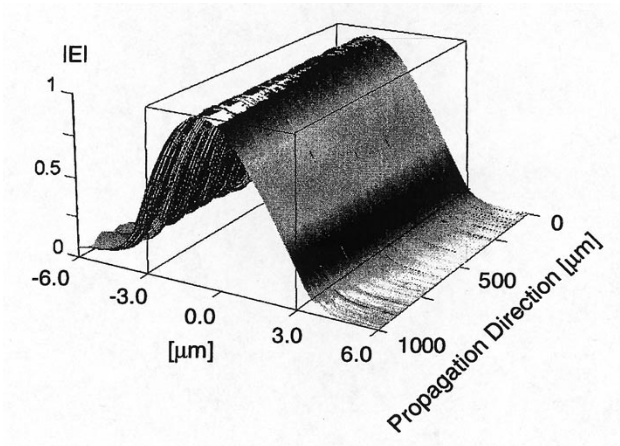

Fig. 4. Propagation of the fundamental bend mode in a bent waveguide.

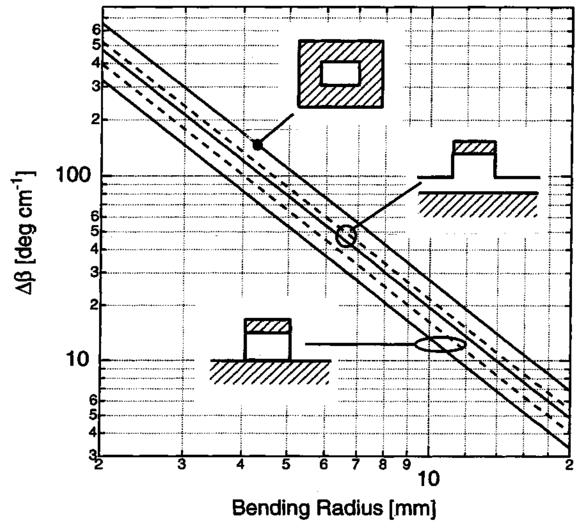

Fig. 5. Bend-induced phase constant perturbation $\Delta\beta$ versus the bending radius for both TE (——) and TM (- - -) polarizations.

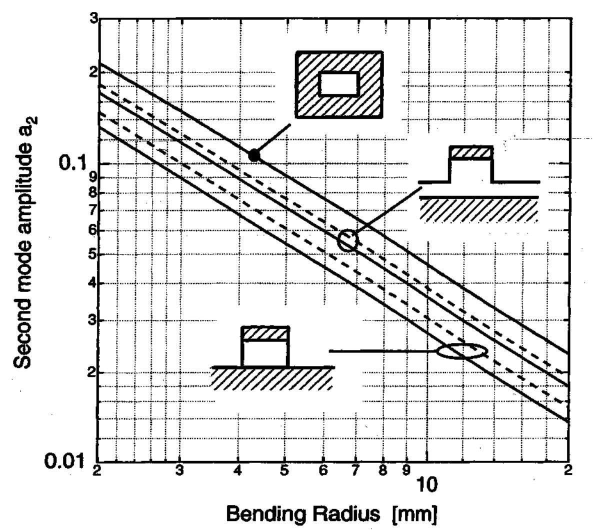

Fig. 6. Second mode amplitude a_2 versus the bending radius for both TE (——) and TM (- - -) polarizations.

also taking into account that the results of the BPM are affected by an uncertainty due to the oscillatory nature of the insertion loss [5]. The BPM, however, confirms the behavior R^{-2} of the coefficient a_2 [19].

As a last comparison, it is shown that the bend mode calculated as the linear combination of just the first two modes is an excellent approximation for every useful bending radius. After the bend mode was determined, it is launched in the bent waveguide and propagated by means of a BPM. Fig. 4 shows that such a mode couples without losses to the bent waveguide and propagates undistorted except for a small amplitude decrease. In the figure, the waveguide appears straight because it was transformed through a conformal mapping for practical reasons. The bend is toward the right, and the small radiation losses are visible on the left side of the figure. For this test, a ridge waveguide with a bending radius of 1 mm has been considered in order to have a significant field deformation and small radiation losses. If the fundamental mode of the straight waveguide was launched in this bend, the field would undergo strong oscillations due to the coupling between the fundamental bent mode and higher order radiative modes.

This test demonstrates that the bend mode can be approximated, at least for any practical application, with the linear combination of the first two modes of the same straight waveguide. Once again, note that the bend mode can be calculated for every desired bending radius without any further time-consuming simulations.

V. Numerical Results

In this section, a comparison between the bending properties of different waveguides is presented. Three type of waveguides are considered and discussed: a 5.2 by 5.2 μm^2 buried waveguide, a rib waveguide with a 2-μm-thick basement, and a ridge waveguide 8.5 μm wide, 6 μm thick, and with an upper cladding 4 μm thick. The index difference is $\Delta n = 0.69\%$ and the analysis is carried out at a wavelength of 1550 nm. The three waveguides present an attenuation of 0.1 dB/rad at bending radius of about 5, 3, and 2 mm, respectively. A sketch of the three structures is reported in Figs. 5 and 6.

It is interesting to note that in the rib waveguide, the second vertical mode E_{12}^x, E_{12}^y is guided. This mode, however, is neither excited by the bend nor coupled to the fundamental or to the first horizontal leaky mode, and has to be excluded from the mode expansion. Hence, also the analysis of the rib waveguide requires only two modes, even if it is bimodal.

In Fig. 5, the phase perturbation $\Delta\beta$ of the three considered waveguides versus the bending radius is reported, calculated by solving the eigenvalue problem (7). Both TE-like and TM-like mode perturbations are shown. The phase constant of the buried waveguide is more sensible to the bending with respect to the other two waveguides, and the ridge waveguide is the less perturbed one. The reason is that in the ridge waveguide, the second mode phase constant β_2 is influenced by the air around the waveguide, and the difference $\beta_1 - \beta_2$ appearing in (10) is higher than in buried waveguide.

The parameters B relative to the three considered waveguides and both TE and TM polarizations are reported in

Table I. Parameter B is a good approximation for the phase perturbation: at the bending radius for which the radiation losses are 0.1 dB/rad, the values of $\Delta\beta$ calculated by using the parameter B and by solving the eigenvalue equation [(7)] are within the 2%. As widely known, $\Delta\beta$ is an important parameter to be controlled in the design of every structure based on delay lines realized with bent waveguides, such as Mach–Zehnder interferometer based filters, WGRs, or ring resonators. In fact, both in Mach–Zehnder interferometers and in ring resonators, the dependence on the curvature of the propagation constant of the bend mode can cause a frequency shift of the filter channels, which has to be controlled for an accurate design. In WGR structures, an improper control of this parameter has more evident consequences in the broadening of the channel bandwidth and in an increase of the crosstalk.

In a bent waveguide, the TE-like and TM-like modes are differently perturbed, and this difference depends on the asymmetry of the waveguide. In the ridge waveguide, for example, $B_{\rm TE}$ and $B_{\rm TM}$ differ by more than the 20%. The birefringence Δn_B induced by the bending is given by

$$\Delta n_B = \frac{\lambda}{2\pi R^2}(B_{\rm TE} - B_{\rm TM}) \tag{12}$$

and depends on the bending radius as R^{-2}. The product $R^2\Delta n_B$ is a measure of the bending-induced birefringence and is also reported in Table I for the three considered waveguides. The knowledge of the dependence of the bend-induced birefringence on the curvature can be helpful for the design of low birefringence bent waveguides: a good choice of the curvature radius can minimize the total birefringence of the bend, in particular when the straight and bend-induced waveguide birefringence are opposite in sign.

The second comparison is carried out on the coefficient a_2. The behavior of a_2 versus R is shown in Fig. 6, and the corresponding parameter A is reported in the Table I. Again, the buried waveguide appears as the most sensible to the bending, and its mode presents the same distortion of the other two waveguides but at a larger bending radius. In the buried waveguide, the TE- and TM-like modes present the same distortion, while for the rib and ridge waveguides, there is a difference between parameter A for the two polarizations. As was outlined in Section IV, the knowledge of parameter A, and consequently coefficient a_2, is a direct way to estimate the coupling losses between a straight and a curved waveguide. Starting from these results, the phenomenon of mode coupling along the bend in order to optimize the bend to straight waveguides transition and the whole bend design is under study. More details will be described in a forthcoming paper.

The wavelength dependence of the two important parameters A and B depends on the behavior of the coupling coefficient c_{12}. The dependence is in any case rather weak, as already observed in [12] for the coefficient B.

In conclusion, parameters A and B define the behavior of the bent waveguide with great accuracy. As a result, they are two important characteristic parameters on which the design and the optimization of each bent dielectric waveguide can be based.

TABLE I
PARAMETERS A AND B AND BEND-INDUCED BIREFRINGENCE $R^2\Delta n_B$

		Buried	Rib	Ridge
A_{TE}	[mm]	0.461	0.358	0.272
A_{TM}	[mm]	0.460	0.385	0.305
B_{TE}	[deg mm]	274.2	194.6	133.2
B_{TM}	[deg mm]	273.0	212.9	162.5
$R^2\Delta n_B$	[mm^2]	5.16×10^{-6}	-7.88×10^{-5}	-1.26×10^{-4}

VI. CONCLUSION

An efficient and useful technique for the analysis of bent dielectric waveguides with reasonably low radiation losses has been proposed and discussed. The technique is based on the expansion of the bent modes in modes of the straight waveguide and requires only a standard mode solver. The proposed method is simple to implement. The mode shape distortion and the phase constant for each desired bending radius can be calculated at a very low computational cost.

In this paper, it has been demonstrated on a theoretical basis that the shape and the phase constant of the bending mode are univocally and accurately defined by two important characteristic parameters named A and B.

The comparisons between the results obtained with the proposed method and those obtained with other methods available in the literature show the great accuracy of the proposed technique. As an example, the bending characteristics of a buried, rib, and ridge waveguide have been analyzed in detail and compared on the basis of the two parameters A and B. Birefringence and polarization properties are discussed as well.

In conclusion, the proposed method is of general validity, is physically well based, and is useful for the design and optimization of the bent sections of every integrated optics component.

ACKNOWLEDGMENT

The authors thank Dr. G. G. Gentili for many valuable and fruitful discussions.

REFERENCES

[1] E. A. J. Marcatili, “Bends in optical dielectric guides,” *Bell Syst. Tech. J.*, pp. 2103–2132, Sept. 1969.
[2] D. Marcuse, “Bending loss of the asymmetric slab waveguide,” *Bell Syst. Tech. J.*, vol. 50, no. 8, pp. 2551–2563, Oct. 1971.
[3] D. Marcuse, “Field deformation and loss caused by curvature of optical fibers,” *J. Opt. Soc. Amer.*, vol. 66, no. 4, pp. 311–320, Apr. 1976.
[4] D. C. Chang and E. F. Kuester, “Radiation and propagation of a surface-wave mode on a curved open waveguide of arbitrary cross section,” *Radio Sci.*, vol. 11, no. 5, pp. 449–457, May 1976.
[5] R. Baets and P. E. Lagasse, “Loss calculation and design of arbitrarily curved integrated-optic waveguides,” *J. Opt. Soc. Amer.*, vol. 73, no. 2, pp. 177–182, Feb. 1983.
[6] M. Chunsheng and L. Shiyong, “Optical characteristics of bent dielectric rectangular waveguides,” *Opt. Quantum Electron.*, no. 19, pp. 83–92, 1987.
[7] J. Saijonmaa and D. Yevick, “Beam-propagation analysis of loss in bent optical waveguides and fibers,” *J. Opt. Soc. Amer.*, vol. 73, no. 12, pp. 1785–1791, Dec. 1983.

[8] K. Thyagarajan, M. R. Shenoy, and A. K. Ghatak, "Accurate numerical method for the calculation of bending loss in optical waveguides using a matrix approach," *Opt. Lett.*, vol. 12, no. 4, pp. 296–298, Apr. 1987.

[9] H. Deng, G. H. Jin, J. Harari, J. P. Vilcot, and D. Decoster, "Investigation of 3D semivectorial finite-difference beam propagation method for bent waveguides," *J. Lightwave Technol.*, vol. 16, pp. 915–922, May 1998.

[10] H. J. M. Bastiaansen, J. M. van der Keur, and H. Blok, "Rigorous, full-vectorial source-type integral equation analysis of circularly curved channel waveguides," *IEEE Trans. Microwave Theory Tech.*, vol. 43, pp. 401–409, Feb. 1995.

[11] S. Kim and A. Gopinath, "Vector analysis of optical dielectric waveguide bends using finite-difference method," *J. Lightwave Technol.*, vol. 14, pp. 2085–2092, Sept. 1996.

[12] W. Wang, R. Scotti, and D. J. Muehlner, "Phase compensation of bent silica-glass optical channel waveguide devices by vector-wave mode-matching method," *J. Lightwave Technol.*, vol. 15, pp. 538–545, Mar. 1997.

[13] W. W. Lui, C. L. Xu, T. Hirono, K. Yokoyama, and W. P. Huang, "Full-vectorial propagation in semiconductor optical bending waveguides and equivalent straight waveguide approximations," *J. Lightwave Technol.*, vol. 16, pp. 910–914, May 1998.

[14] C. L. Xu, W. P. Huang, and S. K. Chaudhuri, "Efficient and accurate vector mode calculations by beam propagation method," *J. Lightwave Technol.*, vol. 11, pp. 1209–1215, July 1993.

[15] D. Marcuse, *Theory of Dielectric Optical Waveguides*, 2nd ed. New York: Academic, 1991.

[16] A. Melloni, R. Costa, F. Carniel, and M. Martinelli, "An effective method for the analysis of bent dielectric waveguides," in *Proc. LEOS'99*, Nov. 1999, pp. 641–642.

[17] A. Yariv and P. Yeh, *Optical Wave in Crystals*. New York: Wiley, 1984.

[18] S. J. Hewlett and F. Ladouceur, "Fourier decomposition method applied to mapped infinite domains: Scalar analysis of dielectric waveguides down to modal cutoff," *J. Lightwave Technol.*, vol. 13, pp. 375–383, Mar. 1995.

[19] T. Kitoh, N. Takato, M. Yasu, and M. Kawachi, "Bending loss reduction in silica-based waveguides by using lateral offsets," *J. Lightwave Technol.*, vol. 13, pp. 555–562, Apr. 1995.

Andrea Melloni graduated and received the Ph.D. degree in electronic engineering from the Politecnico di Milano, Milan, Italy, in 1989 and 1992, respectively.

Since 1992, he has been an Assistant Professor at Politecnico di Milano. From 1989 to 1995, his field of research included the modeling and computer-aided design of microwave devices and numerical methods for electromagnetic (scattering) problems. In 1995, he joined the Photonic group at Politecnico di Milano, and since then, his main activities are on high bit-rates communication systems, fiber nonlinearities, analysis, modeling and design of passive integrated optics components, and optical fiber sensors. He is responsible for the Integrated Optics Lab of CoreCom.

Federico Carniel was born in Milan, Italy, in 1974. He graduated in electronic engineering from Politecnico di Milano, Milan, Italy, in 1999.

Since 1999, he has been with Cisco Photonics, Italy, working in the system design and research group.

Raffaella Costa received the Laurea Degree cum laude in Physics from Universita' degli Studi di Milano, Milan, Italy, in 1996.

From 1995 to 1998, she was with the Photonic Unit of Italtel working on the design of passive integrated optics devices. Then, she joined CoreCom, where she is with the Integrated Optics lab working on modeling and characterization of integrated optic devices for high-density WDM systems.

Mario Martinelli received the Laurea Degree cum laude in nuclear-electronic engineering from Politecnico di Milano, Milan, Italy, in 1976.

In 1978, he joined the Quantum Electronics Department of CISE Advanced Technologies, Milan, Italy, where, in 1986, he was appointed Head of the Coherent Optics Section. In 1992, he was appointed Associated Professor of Optics Communications by Politecnico di Milano, where he activated the first Italian course on the subjec. In 1993, he founded the Photonic Lab at the Electronics and Information Department. Since 1995, he has been Director of CoreCom, a research consortium between Politecnico di Milano and Pirelli Cavi e Sistemi, whose main area of activity is in the field of Optical Processing and Photonic Switching. In 2000, he was appointed Full Professor in Optics Communications at Politecnico di Milano.