A NEW METHOD FOR THE COMPUTATION OF EIGENMODES IN DIELECTRIC WAVEGUIDES

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Abstract **– A new method for the computation of eigenmodes in isotropic cylindrical loss-free dielectric waveguides is proposed. Such waveguide is a cylindrical structure with the refractive index n not varied along the generatrix of cylinder. It is assumed that the waveguide is infinitely long and is in unbounded space with the constant index of refraction $n_\infty > 0$. Besides, $\max n > n_\infty$. Eigenmodes are generator-free electromagnetic waves which satisfy the homogenous Maxwell equations. We consider surface waves. Original problem formulated in unbounded domain is reduced to a linear generalized spectral problem in the circle Ω containing the domain of the cross-section of the waveguide. To approximate obtained problem Finite Element Method is used. Our method allows computing of waveguides of different cross-sections such as circle, square, rectangle, three-circle, etc.**

I. INTRODUCTION

In this work a new method for the computation of eigenmodes in isotropic cylindrical loss-free dielectric waveguides is proposed. Such waveguide is a cylindrical structure with the refractive index n being the function of transverse coordinates $n = n(x_1, x_2)$ not varied along the generatrix of cylinder. It is assumed that the waveguide is infinitely long and is in unbounded space with the constant index of refraction $n_\infty > 0$. Besides, $n_+ = \max n > n_\infty$. Eigenmodes are generator-free electromagnetic waves propagating along the waveguide. They satisfy the homogenous Maxwell equations and have the form

$$\begin{bmatrix}\mathbf{E}\\ \mathbf{H}\end{bmatrix}(x, x_3, t) = \mathrm{Re}\left(\begin{bmatrix}E\\ H\end{bmatrix}(x)\exp(i(\beta x_3 - \omega t))\right). \tag{1}$$

Here $x = (x_1, x_2)$ is the transverse coordinate vector, x_3 is the longitudinal coordinate, t is time, $\omega > 0$ is the oscillation frequency, β is the longitudinal propagation constant, E and H are the amplitudes of the vectors of electrostatic intensity $\mathbf{E}$ and magnetic intensity $\mathbf{H}$.

Dielectric waveguides eigenmodes problems are the spectral ones to find such ω and β that there exist nontrivial solutions of homogenous Maxwell equations having form (1). Surface waves are in particular interest. Their amplitudes decay exponentially at infinity and the parameters ω and β are real and positive. It is assumed that the oscillation frequency is specified on some interval. Then we need solving the spectral problem relative to the eigenvalue β and the eigenfunction (E, H) for each ω from the interval.

The graphs of functions $\beta = \beta(\omega)$ for the fundamental waves and the waves of highest type are called dispersion curves. They are of particular practical interest. These curves define the dispersion that is the deviation from the linear dependance β of ω.

Known statements of dielectric waveguides eigenmodes problems (see, for example, [2]) are inconvenient for numerical solutions. Because of the problems are formulated in all cross-section area of the waveguide, and their unbounded operators have continuous spectrum not interesting in practice. This fact is the cause of the initial problem was reduced to new problem in bounded domain. An auxiliary boundary Γ is introduced. It divides the original domain in two parts: a bounded computational domain Ω with the variable refractive index $n = n(x)$ and an unbounded domain Ω_∞ with the constant refractive index $n = n_\infty$. Boundary conditions are set on Γ. The problem in the bounded domain Ω is solved numerically. Subsequently, if needed, the solution in Ω_∞ is also determined.

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Most boundary conditions suggested on the auxiliary boundary are local and approximate. More attractive are exact nonlocal conditions. Generally, they have the form $L_\Gamma u = S_\Gamma u$, where L_Γ is the differential operator of the natural boundary condition generated by the equation of the problem and S_Γ is a nonlocal operator. The operator S_Γ is explicitly written using the separation of variables. The operator S_Γ was built in [3] in this way (the polar coordinate frame was used and Γ was a circle). Original problem was reduced to the following one

$$(A_\Omega(\beta)+S_\Gamma(p))H = p^2 BH \tag{2}$$

Here $p=(\beta^2-\varepsilon_0\mu_0\omega^2 n_\infty^2)^{1/2}$ is the transverse wave number in the environment (ε_0 is the vacuum permittivity, and μ_0 is the vacuum permeability), $H=(H_1,H_2,H_3)$, A_Ω is a bounded self-adjoint operator depending nonlinearly from β, B is a compact self-adjoint operator. The value p defines the decay rate of the amplitudes of eigenmodes at infinity. So, original problem was reduced to a parametric generalized spectral problem in the circle. The parameter p is nonlinearly included in the nonlocal boundary condition. On practice this fact complicates too much the solution of problem.

In this work using ideas in [3] we reduce original problem in equivalent way to a linear bounded generalized spectral problem having the form

$$C(p)\mathrm{H} = \beta^2 D(p)\mathrm{H} \tag{3}$$

where $\mathrm{H}=(H_1,H_2)\subset[W_2^1(\Omega)]^2$, $C(p)$ is a bounded operator, $D(p)$ is a compact operator. Both operators are self-adjoint and they contain nonlocal boundary operators. We use the transverse wave number p in the environment around the waveguide as a fixed parameter. Solving problem (3) for every fixed $p>0$ we find β^2 and eigenvectors H. Then if it is necessary we easily get the dependance β on $\omega=((\beta^2-p^2)/(n_\infty^2\varepsilon_0\mu_0))^{1/2}$ from the dependance β on p.

II. Problem-Solving Procedure

A. Statement of Problem

The spectral waveguide theory is based on Maxwell's homogenous equations

$$\operatorname{rot}\mathbf{E} = -\mu_0\frac{\partial \mathbf{H}}{\partial t}, \qquad \operatorname{rot}\mathbf{H} = \varepsilon_0 n^2\frac{\partial \mathbf{E}}{\partial t}.$$

Studying eigenmodes of the form (1) according to [3] we obtain problem (2). Parameter β as a rule is considered as a fixed one in problem (2), and parameter p needs to find. The range of definition of these parameters is characterized by the set $K=\{(\beta,p):\beta>0,\ 0<p<(1-(n_\infty/n_+)^2)^{1/2}\beta\}$.

B. Method of Solution

Method of solution is based on new statement of the problem. Let us fix the parameter p in problem (2). Then making not complicated transformations we obtain the following equivalence problem: find all $(\beta,p)\in K$ and $\mathrm{H}\in[W_2^1(\Omega)]^2\setminus\{0\}$, satisfying an equation (3). To approximate problem (3) Finite Element Method is used.

III. Numerical Experiments

To check the proposed method waveguides of different cross-sections have been computed. Dispersion curves for a circular cylindrical waveguide with radius of cross-section equal to 1 are shown on Fig. 1. Parameters of the waveguide are as follows: $\varepsilon_0=\mu_0=1$, $n=2^{1/2}$, $n_\infty=1$. This example is interesting as exact solutions are known in this case (see, for example, [1]). Firm lines are approximate solutions, dot lines are exact solutions. Dispersion curves for a rectangle dielectric waveguide with sides 1.5×1 are shown on Fig. 2. Parameters of the waveguide are as follows: $\varepsilon_0=\mu_0=1$, $n=2.08^{1/2}$, $n_\infty=1$. Experimental data are known in this case [4]. They are denoted as little circles on the plot. Dispersion curves for a three-circle dielectric waveguide are shown on Fig. 3. Parameters of the waveguide are exactly the same as in the case of a circle waveguide. The cross-section of this waveguide is three mutually tangent circles of radius 0.4.

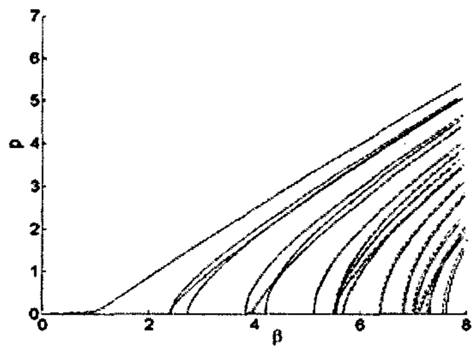

Fig. 1. Dispersion curves for a circle dielectric waveguide.

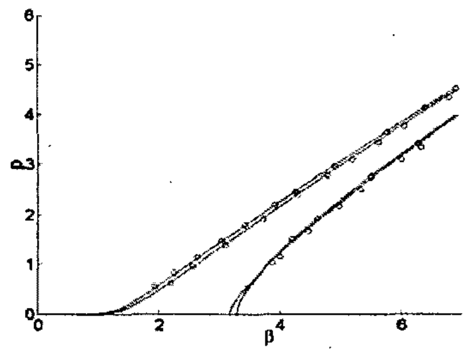

Fig. 2. Dispersion curves for a rectangle dielectric waveguide.

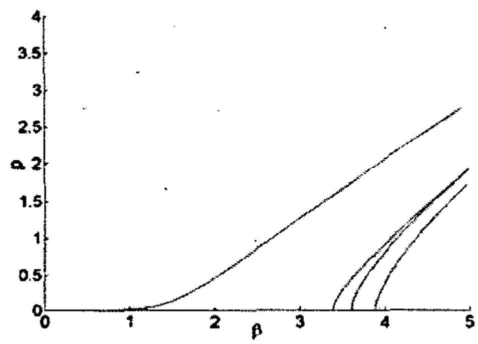

Fig. 3. Dispersion curves for a three-circle dielectric waveguide.

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