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REDUCTION OF RADIATION LOSS IN A CURVED DIELECTRIC SLAB WAVEGUIDE†

by

David C. Chang and Frank S. Barnes

Electromagnetics Laboratory
Department of Electrical Engineering
University of Colorado
Boulder, Colorado 80302

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TABLE OF CONTENTS

Abstract

The problem of radiation in a curved homogeneous dielectric slab-waveguide is investigated. For a symmetric guide with a large radius of curvature, phase velocities of the propagating surface-wave modes along the azimuthal direction is found to be the same as that of a straight waveguide, but additional attenuation of waves occur because of the radiation in the radial direction beyond the so-called "turning-point." A knowledge of the distance between the turning-point and the slab surface can provide adequate measure of the amount of the associated bending loss. For an asymmetric waveguide, attenuation of the wave can be either reduced or enhanced, depending upon the difference in penetration depth of the waves in regions exterior to the upper and lower slab surfaces. It is also found that substantial reduction in bending loss can be achieved by varying the refractive index of the media in the exterior regions. Explicit expressions for the case of a truncated inverse-linear profile is then determined.

1. Introduction

Generally speaking, bends in open wave guiding structures will normally introduce unwanted radiation losses as well as changes in phase velocity of propagating surface-wave modes. [Barlow and Brown, 1962; Wait, 1970; Bates and Ng, 1971]. For the special case of a curved dielectric slab waveguide, the problem of bending loss was first discussed by Elliott [1955] and later by Marcatili [1969], based upon an exact eigen-function formulation in a cylindrical coordinate, followed by subsequent approximations of the cylindrical functions in the modal characteristic equation. Such an approach, although mathematically sound, does not provide the physical insights necessary for comparison with the limiting case of a straight waveguide. More recently, Marcuse [1971] has shown that is is possible to derive bending loss from a much simpler approximate theory which assumes the field in the vicinity of a curved waveguide must be similar to that of a straight one, provided the curvature of the slab is sufficiently large. Such a theory however can not account for the change of the propagation constant from a straight to a curved waveguide in the asymmetric case. As a result, Marcuse's formula cannot adequately describe the bending loss of a slab waveguide whenever the difference in penetration depth for the regions exterior to the outer and inner surface of the slab is too great.

In this paper, an alternative presentation based upon the concept of surface-admittance is discussed. The curvature in this case is seen to effect not only the refractive-index profile inside the slab of an equivalent straight guide,but also to introduce additional surface conductances and susceptances at the slab surfaces. In the vicinity of the slab, a WKB-type modification to the field distribution enables us to draw direct analogies between the curved and straight waveguides. Physical significance of the turning point and its relationship with radiation loss are then discussed. Such a formulation alows us to discuss the possiblity of reducing radiation loss in a curved waveguide with a graded refractive-index profile. Amount of radiation reduction is then determined for a truncated, inverse linear profile.

2. Transverse-resonance condition and modal equation of a surface-wave mode.

Consider a curved, homogeneous dielectric slab waveguide of thickness D, having a radius of curvature of R and a refractive index of n (with respect to the surrounding material) as depicted in Fig. 1a. For single-mode operation at optical frequencies, parameters of the guide are chosen according to

$$(n-1) \ll 1, \quad k(n-1)\,D \sim 1 \quad \text{and} \quad R \gg D \tag{1}$$

where k is the wave number of the surrounding medium. The slab is assumed to have no variation along the z-direction so that the TE-type mode supported by the structure consists of only H_ρ, H_θ, E_z components which satisfies the following relationship:

$$H_\rho = -i(\omega\mu\rho)^{-1}\partial E_z/\partial\theta \; ; \qquad H_\theta = i(\omega\mu)^{-1}\,\partial E_z/\partial\rho, \tag{2}$$

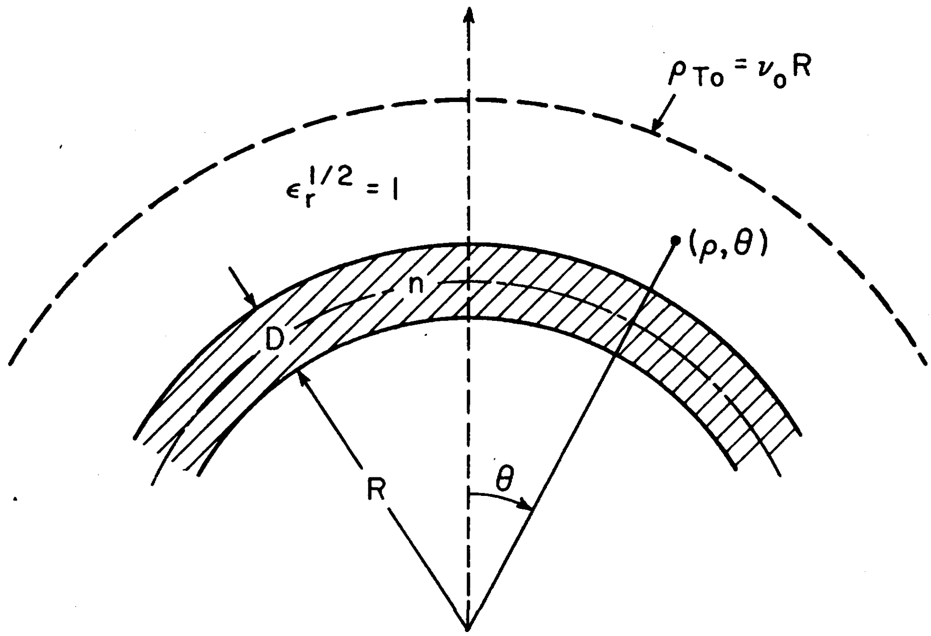
and

$$\left(\frac{1}{\rho}\frac{\partial}{\partial\rho}\,\rho\,\frac{\partial}{\partial\rho} + \frac{1}{\rho^2}\frac{\partial^2}{\partial\theta^2} + n_j^2k^2\right)E_z(\rho,\theta) = 0 \;, \tag{3}$$

where $n_j = 1$ or n depending on whether the region is outside or inside of the slab; a time-factor of $\exp(i\omega t)$ is omitted throughout. Similar expression for a TM-tape mode can be easily obtained. In (3), we can assume that the solution takes the form of

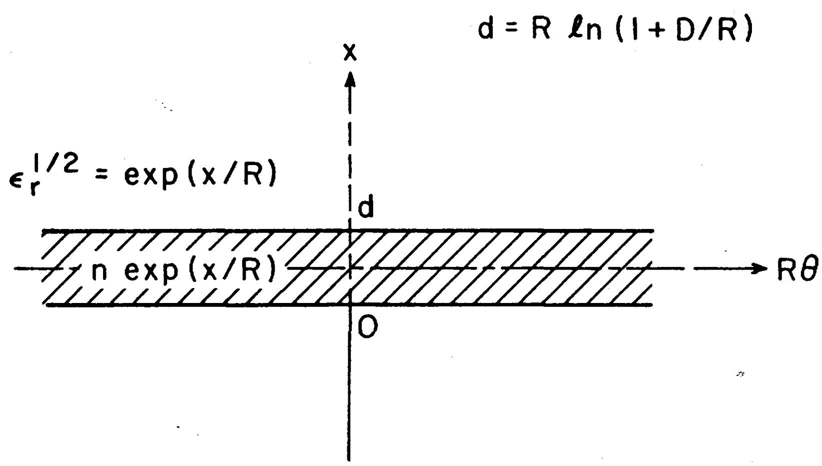
$$E_z(\rho,\theta) = \Omega_\nu(\rho)\,\exp(-ik\nu R\theta) \;, \tag{4}$$

with ν as the yet-undetermined propagation constant along the azimuthal direction, In order to compare the solution



$\rho_{To} = \nu_o R$
$\epsilon_r^{1/2} = 1$
(ρ, θ)
n
D
R
θ

(a)



$d = R\,\ell n\,(1 + D/R)$
x
$\epsilon_r^{1/2} = \exp(x/R)$
d
n exp(x/R)
$R\theta$
O

(b)

Figure 1

of (3) with that of a straight dielectric slab, we define a new coordinate system according to Figure1b:

$$x = R \ln \rho/R; \qquad y = R\theta \tag{5}$$

so that the differential equation for Ω_ν is simply given by

$$\left\{ \frac{d^2}{dx^2} + k^2[n_j^2 \exp(2x/R) - \nu^2] \right\}\Omega_\nu = 0 \, . \tag{6}$$

The boundaries of the dielectric slab now occur at $x = 0$ and d where d equals to $R \ln(1+D/R)$ or approximately, D. From (6) it is clear that the effect of the curvature is to change the refractive-index profile in both the interior and exterior regions of an equivalent straight waveguide of width d. Away from the slab surface, we notice that the refractive index of the equivalent waveguide increases without bound as x approaches to infinity above the slab surface. Thus we can define a turning-point x_t according to the relationship $\exp(2x_t/R) = \nu^2$, beyond which the value of the square bracket in (6) is positive and the solution becomes oscillatory. Consequently, a dielectric slab waveguide with a finite curvature is inherently a radiating structure. It cannot continuously maintain an evanescent field outside the slab. As it will become clear later, the value of ν is slightly complex so that a finite transition region, instead of an abrupt turning-point usually exists in the exterior region. [Budden, 1961].

Inside the slab where $0 \leq x \leq d$, the refractive-index is a slow-varying function which can be well-approximated by a linear profile, i.e. $n(1+2x/R)$. Although the exact solution to the differential equation (6) is known, we nevertheless use a WKB approximation to yield [Morse and Feshbach, 1953].

$$\Omega_\nu = C_\nu Q^{-\frac{1}{2}}\{e^{-i\Phi(x)} + R_{2\nu}e^{i\Phi(x)}\}; \quad \Phi(x) = k\int_0^x Q(x)\,dx\ , \qquad (7)$$

where

$$Q(x) = [\mu^2 + 2n^2x/R]^{\frac{1}{2}}; \quad \mu = (n^2 - \nu^2)^{\frac{1}{2}}\ , \qquad (8)$$

and C_ν is an arbitrary constant. The value of $kQ(x)$ can be shown as the wave number of a local plane-wave at any specific point in the transverse plane. Thus the terms $\exp(-i\Phi)$ and $\exp(i\Phi)$ physically represent the up-and down-going waves inside the slab whereas $R_{1\nu}$ and $R_{2\nu}$ can be identified as the reflections from the lower and upper boundaries. By denoting the normalized surface admittances at $x = 0$ and d as:

$$Y_1 = -\zeta_0(H_\theta/E_z)_{x=-\varepsilon} \quad \text{and} \quad Y_2 = \zeta_0(\rho H_\theta/RE_z)_{x=d+\varepsilon}, \quad \text{as } \varepsilon \to 0 \qquad (9)$$

where $\zeta_0 = (\mu/\varepsilon)^{\frac{1}{2}}$, one can easily show from (2) and (7) that the two reflection coefficients are given by

$$R_{j\nu} = (Q_j - Y_j')/(Q_j + Y_j'); \quad Y_j' = Y_j + i(-1)^j n^2/(2kQ_j^2R),\ j=1,2 \qquad (10)$$

and Q_j is the value of Q at $x = 0$ and d, respectively. The axial propagation constant is then determined by the transverse-resonance condition inside the slab;

$$R_{1\nu}R_{2\nu}e^{-2i\Phi(d)} = 1 \; . \tag{11}$$

Here, the exponential term in (11) accounts for the phase accumlation of a wave bouncing back and forth from the two slab surfaces.

A straight guide can now be treated as a special case when R becomes infinity. The field distribution outside of the slab in this case is known to be $\exp(-k\lambda|x|)$ where $1/\lambda = (\nu^2-1)^{-\frac{1}{2}}$ is the normalized penetration depth in the exterior region. It then follows from (9) and (7) that

$$Y_1 = Y_2 = -i\lambda, \quad \Phi(d) = k\mu d \tag{12}$$

and the modal equation (11) reduces to

$$\mu_0 \tan \mu_0 d/2 = \lambda_0, \tag{13a}$$

or

$$\mu_0 \cot \mu_0 d/2 = -\lambda_0 \tag{13b}$$

with $\lambda_0^2 + \mu_0^2 = n^2 - 1$. Note that we have used the subscript "0" to distinguish this case from a curved slab. It is well known that the modal equations (13a) and (13b) allow only a finite number of real solutions which satisfy the surface-wave condition of $1 < \nu_0 < n$. Consequently, the normalized surface admittances Y_1 and Y_2 of any propagating surface-wave mode, as given by $-i(\nu^2-1)^{\frac{1}{2}}$ in (12), have to be purely reactive. [Collin, 1960].

3. Approximate propagation constant of a curved, homogeneous slab

Provided that the radius of curvature is large compared with the thickness of the dielectric slab, the value of ν associated with a curved slab waveguide usually can be obtained as a small perturbation of a straight guide. Thus, if we designate the change in the propagation constant as $\Delta\nu = \nu - \nu_0$ and allow that both $\Delta\nu$ and $1/R$ to be small, we have from (7) and (8) the following approximate expressions:

$$d \sim D(1-1/R),$$

$$\lambda \sim \lambda_0(1+\nu_0\Delta\nu/\lambda_0^2); \qquad \mu \sim \mu_0(1-\nu_0\Delta\nu/\mu_0^2) \;,$$

$$Q \sim \mu_0[1-\nu_0\Delta\nu/\mu_0^2 + n^2x/(\mu_0^2R)] \;,$$

$$\exp(-i2\Phi) \sim \exp(-i2k\mu_0 d)[1+ik\nu_0 D(2\Delta\nu - \nu_0 D/R)/\mu_0].$$

Likewise, the change in the surface admittance can be expressed in the form of

$$Y_j = Y_j(R \to \infty) + \Delta Y_j(\nu_0)/kR \; ; \qquad j = 1,2, \tag{15}$$

where $Y_j(R \to \infty) = -i\lambda$ as given in (12) and the value of $\Delta Y_j(\nu_0)$ is to be determined later from an explicit knowledge of the field distribution outside the slab. After some algebraic manipulations, it is not difficult to show that the use of (14), (15) and (11) yields

$$\Delta\nu = (\nu_0/L_e R)\{\frac{D}{2}\ [D + k\frac{2n^2\lambda_0}{(n^2-1)\nu_0^2}] - i\left(\frac{\mu_0}{k\nu_0}\right)^2 (\Delta Y_1 + \Delta Y_2)/(n^2-1)\}\ ;$$

$$L_e = D + 2/(\lambda_0 k) \tag{16}$$

Thus, a part of $\Delta\nu$ is now attributed explicitly to the change in surface admittances at the slab boundaries. It is important to note that attenuation of the surface-wave, i.e. the imaginary part of $\Delta\nu$, is a direct result of additional surface conductances introduced at the boundaries by the curvature of the original waveguide.

In order to determine the value of ΔY_1 and ΔY_2, a more detailed knowledge of the external field distribution is required. For the region away from the lower surface of the slab, i.e. $x < 0$, it is seen from (6) that the transverse wave number given by $k[\exp(2x/R) - \nu_0^2]^{\frac{1}{2}}$, is always purely imaginary. This implies the approximate field of a curved guide should remain evanescent as it moves away from the inner side of the slab. Thus, the approximate solution in this region takes the form of

$$\Omega_\nu = B_\nu W^{-\frac{1}{2}} e^{-\psi_i(x)} \quad ; \quad \psi_i(x) = k\int_x^0 W(x)\,dx\ , \tag{17}$$

where B_ν is an arbitrary constant; $W(x) = (\lambda^2 - 2x/R)^{\frac{1}{2}}$ and $\lambda = (\nu^2-1)^{\frac{1}{2}}$. The substitution of (17) and (2) into (9) yields immediately the following result:

$$Y_1 = -i\lambda - i/(2\lambda^2 kR) \quad \text{or} \quad \Delta Y_1 = -\ i/(2\lambda_0^2)\ , \tag{18}$$

which according to (16), indicates that the change in surface admittance at the inner boundary will not produce any attenuation of the surface-wave.

The situation at the upper surface of the curved slab is quite different; as mentioned earlier, there exists a transition region near the turning point in the exterior region beyond which the electromagnetic field has to propagate rather than decay exponentially. The turning-point x_t is defined as the "distance" where the effective refractive index of the medium becomes zero. Thus, it follows from (6) that

$$x_t = R \ln \nu \text{ or } \rho_t = \nu R \tag{19}$$

To facilitate an analytical continuation of the solutions below and above the turning point, another evanescent field which represents the reflected field toward the slab boundary is found to be necessary:

$$\Omega_\nu = A W^{-\frac{1}{2}} [e^{-\psi_0(x)} + \Gamma e^{+\psi_0(x)}] ; \quad \psi_0(x) = k\int_d^x W(x)\,dx \tag{20}$$

Such a phenomenon is well-known in the asymptotic theory of differential equations; the "reflection coefficient" Γ in (20) is commonly referred to as the Stoke's constant and can be shown as equal to

$$\Gamma = -(i/2)\ e^{-2\tau};\ \tau = k\int_d W(x)\,dx \tag{21}$$

[Budden, 1961]. It is of interest to note that the reflected evanescent field, while it is significant in providing an analytical continuation of the solution in the transition region near the turning-point, nevertheless is exponentially small at the slab boundary. Equation (20) then suggests the field in the vicinity of the slab indeed resembles that of a straight waveguide. Substitution of (20) and (2) into (9) yields

$$Y_2 = -iw_2(1-\Gamma)/(1+\Gamma) + i/(2w_2^2 kR) \;;\quad w_2 = (\lambda^2 - 2d/R)^{\frac{1}{2}} \tag{21a}$$

or alternatively,

$$\Delta Y_2 = k\lambda_0 Re^{-2\tau} + i(1+2kd\lambda_0)/(2\lambda_0^2) \tag{21b}$$

Thus, the evanescent field reflected from the turning-point yields the only surface conductance term at the slab boundary. Accordingly, the axial propagation constant of the surface wave can now be determined from (16), (18) and (21). After some manipulation, it is not difficult to show that

$$\exp(ik\nu R\theta) = \exp[ik\nu_0(R+D/2)\theta - \alpha R\theta] , \tag{22}$$

where

$$\alpha = \frac{\lambda_0 \mu_0^2}{(n^2-1)\,\nu_0 L_e} e^{-2\tau} , \tag{23}$$

Here, the expression for L_e, as given by (16) has the significance as representing the width of the slab plus the portion of the exterior region where the field is most concentrated. Equation (22) indicates that the phase velocity of the wave along the median line remains essentially unchanged from the straight waveguide. However, attenuation of the wave may occur continuously along the guide due to radiation beyong the turning-point. Interestingly, the factor τ can be expressed explicitly in terms of the "distance" between the approximate location of the turning-point and the upper boundary of the slab as follows.

$$\tau = \frac{k\lambda^3}{3} R - \lambda_0 kd = \tfrac{2}{3}k\lambda_0(\rho_{T0}-R-D); \quad \rho_{T0} = \nu_0 R ; \tag{24}$$

In the derivation of (24), use has been made of the approximations that $|\lambda^2| > d/R$ and $\lambda^2 = 2(\nu-1)$ from the assumption made in (1); $\nu = \nu_0(1+D/2R)$ from (22). It then follows from (23) and (24) that a smaller attenuation can be achieved simply by an increase of this "distance" $\rho_{T0}-R-D$, or a decrease in the penetration depth λ_0^{-1}.

Following a similar approach, we can now write down the result for an asymmetric waveguide:

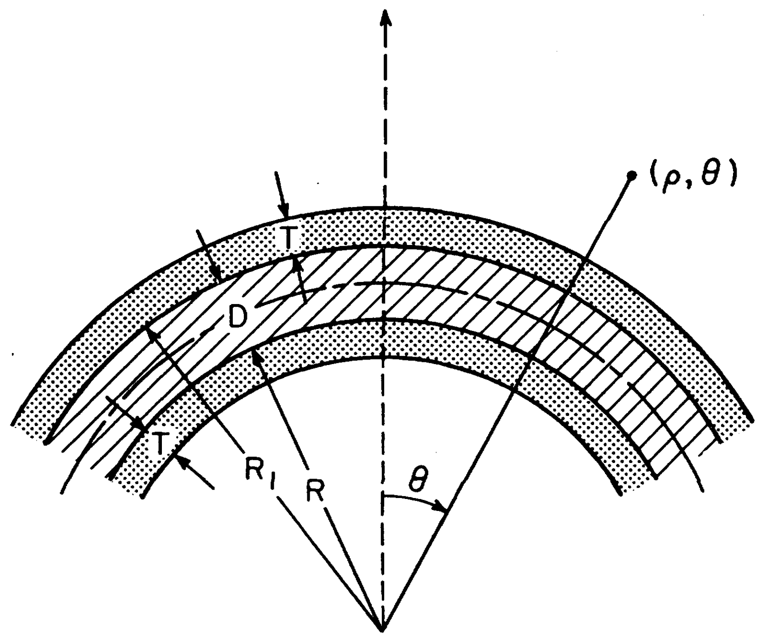
$$\exp(ik\nu R\theta) = \exp\{ik\nu_0[R + \tfrac{1}{2}(D+\delta)]\theta - \alpha_1 R\theta\} , \tag{25}$$

$$\alpha_1 = \left(\frac{\alpha L_e}{L_e-\delta}\right) e^{-2k\nu_0\lambda_0\delta/3}; \quad k\delta = \left(\frac{1}{\lambda_0} - \frac{1}{\lambda_1}\right) , \tag{26}$$

where $\lambda_1^{-1} = (\nu_0^2 - n_1^2)^{-\frac{1}{2}}$ and n_1 is the normalized penetration depth and the refractive index of the medium external to the lower boundary of the slab, respectively. Equation (25) agrees in form with the result obtained by Marcuse [1971]. However the attenuation constant α_1 is modified by the additional exponential factor in the present analysis. As a result, the attenuation of wave can be either enhanced or reduced, depending upon the extent of asymmetry as given by the value of δ in (26).

4. Reduction of radiation loss

One of the significant conclusions we can draw from the foregoing analysis is that the amount of radiation loss, i.e. the value of α, is explicitly related to the "distance" measured from the outer slab surface to the approximate location of the turning-point. Thus, a reduction in radiation loss can be in principle achieved by a relocation of the turning-point. As it is evident from (6), a complete removal of this point is indeed possible whenever the surrounding medium has a refractive index profile of $\exp(-x/R)$ or in the cylindrical coordinate, an equivalent profile of R/ρ. This means an inverse-linear profile will ideally produce no radiation. Unfortunately, such a graded profile is not physically plausible since it requires the refractive index to vanish at infinity. One alternative is truncate the inverse-linear profile at some finite distance away from the slab in order to achieve a substantial reduction, if not complete removal, of radiation. To examine such a possibility, we assume the refractive-index profile in the immediate vicinity of the slab is to be in the form of R_1/ρ for $R_1 \leq \rho \leq R_2$ where $R_1 = R+D$ is the location of the upper slab surface and $T = R_2-R_1$ is the width of the graded region (Fig. 2). It then follows directly from (6) that the solution in this region is given by



$$\epsilon_r^{1/2} = \begin{cases} R/(R-T), & \text{for } (R-T) < \rho; \\ R/\rho, & (R-T) < \rho < R; \\ n, & R < \rho < R_1; \\ R_1/\rho, & R_1 < \rho < (R_1+T); \\ R_1/(R_1+T), & \rho > (R_1+T) \end{cases}$$

Figure 2

$$\Omega_\nu = F_\nu [e^{-k\lambda_2(x-d)} + \Gamma_2 e^{k\lambda_2(x-d)}] \; ; \quad d \le x \le d+t \, , \tag{27}$$

where F_ν is an arbitrary constant; $(d+t) = R\ell n R_2/R$ and $\lambda_2^{-1} = (\nu^2 - R_1^2/R^2)^{-\frac{1}{2}}$ is the normalized penetration depth of wave in this region. According to (9), the normalized surface-admittance Y_2 appeared at the slab boundary $x = d$ is then related to the surface-admittance Y_3 at $x = d+t$ by the following expression

$$Y_2 = -i\lambda_2 [1-2(\frac{i\lambda_2 + Y_3}{i\lambda_2 - Y_3}) e^{-2k\lambda_2 t}] \, , \tag{28}$$

provided that the attenuation of wave is sufficiently large in the x-direction, i.e. $k\lambda_2 t > 1$. If now we assume a constant refractive index of R_1/R_2 for the extended region where $x \ge t+d$, expression for the surface admittance Y_3 at the transition $x = t+d$ is readily obtainable from (21a,b) when the penetration depth λ^{-1} is replaced by $\lambda_3^{-1} = (\nu^2 - R_1^2/R^2)^{-\frac{1}{2}} \sim (\nu^2 - 2d/R)^{-\frac{1}{2}}$. Thus by retaining only the leading terms, we obtain from (28)

$$Y_2 = i\lambda + \Delta Y_2/(kR) ; \quad \Delta Y_2 = ikd/\lambda_0 + [k\lambda_0 R e^{-2\tau'} + i/(2\lambda_0^2)] e^{-2k\lambda_0 T} \, , \tag{29}$$

where $\tau' = k \int_{d+t} [\lambda^2 - 2(x-t)/R]^{\frac{1}{2}} dx = \lambda_3 R/3 - \lambda_0 d$, which is the same as in (24). A comparison of (29) with (21b) clearly indicates that the change in Y_2 is substantially reduced by the factor $\exp(-2k\lambda_0 T)$.

Following a similar approach we can readily show that the surface admittance at the lower slab boundary is given by

$$Y_1 = -i\lambda + \Delta Y_1/(kR); \quad \Delta Y_1 = (-i/2\lambda_0^2)e^{-2k\lambda_0 T}, \tag{30}$$

when the region below the slab surface is replaced by a truncated inverse-linear profile of same width, having a refractive index of R/ρ for $(R-T) < \rho < R$ and $R/(R-T)$ for $\rho < (R-T)$. Substitution of (29) and (30) into (16) after some algebraic manipulation, yields

$$\exp(ik\nu R\theta) = \exp\{ik\nu_0(R+D/2)\theta - \alpha' R\theta\} \tag{31a}$$

where

$$\alpha' = \alpha e^{-2k\lambda_0 T} \quad . \tag{31b}$$

Thus, the attenuation of wave is reduced by the factor $\exp(-2k\lambda_0 T)$. It should be noted that the refractive index of the slab is assumed to remain constant in the above derivation. Because of the decrease in refractive index in the exterior region, reduction in radiation loss is realized essentially by increasing the distance of the turning-point by an amount corresponding to the width of the graded region. However, different conclusion may be obtained when the refractive index of the material inside the slab is replaced by a similarly graded profile of $n(R/\rho)$ for $R < \rho < R_1$. In such case, the phase accumulation inside the slab vanishes and the expressions in (14) have to be modified according to

$$Q \sim \mu_0(1-\nu_0\Delta\nu/\mu_0^2); \quad \exp(-i2\Phi) \sim \exp(-i2k\mu_0 d)(1+i2k\nu_0 D\Delta\nu/\mu_0) \quad . \tag{32}$$

Substitution of (32) into the modal equation (11) subsequently yield the solution as

$$\exp(ik\nu R\theta) = \exp(ik\nu_0 R\theta - {}''R\theta) \; ; \tag{33a}$$

$$\alpha'' = \alpha e^{-2k\lambda_0 (T-D/3)} \tag{33b}$$

Thus, a reduction of radiation loss is possible only when $T > D/3$. On the other hand, the phase velocity of the wave is now identical to that of a straight waveguide.

4. Concluding Remark

In this paper, the surface-wave propagation of an open wave-guiding structure consisted of a curved, dielectric slab is studied. For a waveguide with a large radius of curvature ($d/R << 1$), characteristics of surface-wave modes are shown to be equivalent to that of a straight guide of same dimension having a linear profile inside the slab and a perturbed surface admittance at the slab boundaries. For a uniform and symmetrical slab guide, the phase accumulation inside the slab contributes directly to the change in azimuthal propagation constant, while the additional surface conductance at the upper boundary is related to the bending loss. To the first-order approximation, phase velocity of a surface-wave mode propagates along the median of the slab remains unchanged, but the attenuation constant in the axial direction is given by

$$\alpha = \frac{\lambda_0 \mu_0^2}{(n^2-1)\nu_0 L_e} e^{-2\tau} \; ; \quad \tau = \frac{2}{3} k\lambda_0 (\rho_{T0} - R - D)$$

which indicates that the distance from the slab surface to the virtual location of the turning-point gives an adequate measure of the amount of bending loss. For a slab guide of fixed curvature, reduction in radiation loss can be accomplished by increasing the refractive index of the slab, which in effect confines the surface-wave field more to the inside of the slab and removes the turning-point further from the slab.

An alternative way to reduce the radiation loss in radiation is to grade the refractive index of the slab guide with a truncated inverse-linear profile. Provided that the radial attenuation across the truncated region is large enough ($k\lambda_0 T > 1$), the axial attenuation is reduced from a non-graded, uniform guide by the factor of $\exp(-2k\lambda_0 T)$ or $\exp(-2k\lambda_0 [T-D/3])$, depending upon whether the refractive-index inside the slab surface is similarly graded or not. To illustrate the amount of reduction achievable, we can choose a typical single-mode waveguide having a refractive-index of $n^2 = 1.01$ inside the slab, and a slab-width given by $V = k(n^2-1)^{\frac{1}{2}} d = 2.2$. The value of $k\lambda_0 d$ is then determined from (13) to be approximately 2, for the principle TE-type surface-wave mode. [see Fig. 5, Marcuse, 1971]. Thus, the amount of reduction in radiation is approximately 1/400 and 1/110 for a thickness of $T = 1.5D$ in the two cases.

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