Quantum optimization in engineering: a brief review

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***Abstract*— Since its beginning, quantum computing has opened up a plethora of new perspectives and advantages., including its ability to exponentially accelerate computing processes. A large amount of research has been devoted to developing innovative methods for solving a variety of engineering problems by designing quantum versions of numerous mathematical processes. In this paper, we review the state of the art in these new techniques from the standpoint of quantum computer creation and efficiency enhancement, with an emphasis on the most widespread mathematical optimization techniques for engineering applications. It also emphasizes the obstacles and constraints associated with the use of quantum computing, as well as the major prospects for future contributions. This paper is providing a useful resource for engineering researchers who are likely to seek solutions from quantum computing.**

Keywords*— *Quantum computing, Optimization, Algorithm, Annealer,

I. Intronduction

Mathematical optimization refers to a set of approaches and methodologies used to justify judgements in a variety of domains such as physics, biology, engineering, economics, and business among others. Each situation must be represented as a mathematical model or programme task that consists of maximising or minimising an objective function while satisfying a set of contraints described in terms of decision variables. Subsequently, the problem is solved using an exact or approximate optimization method depending on the numerical complexity of the described model. The features of the computer used to execute these algorithms are critical to the speed and efficiency of these procedures. In fact, the faster and more powerful is the chosen computer, the less difficult it is to solve the problem.

The rise of quantum computers, along with their enhanced speed, is arousing the interest of researchers in the field of engineering, in particular for the conversion of state-of-the-art optimization techniques into entirely quantum versions or quantum-inspired versions that leverage the strengths and advantages of quantum computers while running on classical computers. The spectacular results to date are promising and could revolutionize the optimization field by solving NP-complete optimization problems. Nevertheless, only a few reviews have been published, and none of them has envisaged bringing together and discussing all the optimization techniques envisaged for engineering applications. Hence the aim of this paper, which is to provide researchers interested in quantum optimization with a contemporary reference that brings together the work done on these thematic tools, so that they will be aware of what has already been done and can use it to make new contributions.

To identify resources for this literature review, we used the following keywords in various combinations: quantum computing, quantum optimization, quantum-inspired optimization, quantum annealer. We found a plethora of information for this literature review, since quantum computing is currently a subject of major interest in a whole range of disciplines. Only papers complying with the following criteria were retained: a) The work must be primary research in the context of the implementation of optimization methods based on quantum computing, b) The optimization problem considered is well defined and mathematically well formulated, c) The method used and/or developed is well justified and well defined, d) The method is tested and approved. Most of the works cited in this paper were published within the last 10 years, a few published earlier have been included to introduce the concepts and/or basic works that are used to date.

The most relevant contributions are summarized in sections 3, 4 and 5 and classified into three principal categories: quantum algorithms, quantum-inspired procedures, and quantum annealers. Finally, the challenges and perspectives of the use and exploitation of quantum computing are listed and discussed in section 7.

II. Quantum computing

Though only in its early developing stages, quantum computing is increasingly winning over scientists with its promise of higher performance and faster computations than conventional computing.

A quantum computer, entirely dissimilar to regular computers, relies on quantum bits «qubits» that are built in accordance with the principle of quantum superposition where basic states |0> and |1> are superposed as depicted in Figure 1. Consequently there is an endless number of eventual states based on 0 and 1 with constantly changing weights [1, 2].

A qubit state can be operated by quantum gates, like as X, Y, and Z gates, the Hadamard and the rotation gates (among

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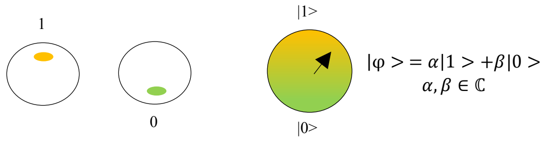
others). These gates can then be combined to build quantum circuits [3].



(a) A classical bit **(b) A quantum bit**

Fig. 1. Classical bit versus quantum bit (qubit).

III. QUANTUM OPTIMIZATION METHODS

To resolve global optimization problems, Baritompa et al. (2005) introduced BBW, an effective stochastic quantum computing method for solving the unconstrained global optimisation problems using Grover's algorithm [4]. Tang et al (2020) proposed another algorithm which uses a memetic evolution operator founded on the shuffling process and a quantum evolution operator which employs an adaptive selection mechanism to differentiate potential sinks in order to achieve a genuine equilibrium between global and local search [5]. In another hand, Alexandru et al. (2020) present new quantum strategies for accelerating many general-purpose numerical optimisation methods for minimising a function $f\colon\mathbb{R}n\rightarrow\mathbb{R}$ [6] especially backtracking line search, a non-negligible component in quasi-Newton optimisation algorithms for global optimisation under a Lipschitz constraint. In the same context, Sayed et al. (2019) created a hybrid algorithm called quantum multiverse optimization by combining the multiverse optimization algorithm with the quantum representation of the search space, and quantum operators to find the optimal trade-off between exploration and extraction of natural resources [7]. The suggested technique outperformed well-known algorithms at solving complex numerical optimisation problems in tests involving 50 unimodal and multimodal benchmark functions.

For the discrete optimization problems, Liu and Koehler (2010) explored and improved the BBW algorithm [8] in quest of solving discrete optimization problems by making it more efficient and dynamic [9]. Farhi et al. (2014) have described an approximate quantum algorithm based on a quantum circuit made up of unitary gates [10].This method was demonstrated for the MaxCut problem in bounded degree graphs. This study was continued by applying the QAOA algorithm to the restricted occurrence combinatorial problem Max E3LIN2 [11]. Wecker, Hastings and Troyer (2016) have developed a variant of QAOA for the maximum 2-satisfiability problem (MAX-2-SAT) that produces a state that is very close to the optimal state [12]. This was proved through experiments on a set of challenging examples using CFLLS-optimized annealing times.

In the context of multiobjective optimization, Da Silva and Schirru (2011) attempted to meet the challenge of optimising reactor core fuel assembly reloading by implementig an evolutionary-inspired quantum algorithm named QPBIL that combines the fundamental principles of progressive population-based learning with the linear superposition of quantum bit states [13]. In another work, Shang et al. (2014) proposed a quantum immune clonal coevolution algorithm (QICCA) based on an artificial immune system, quantum evolutionary computation, and coevolution strategy to solve dynamic multi-objective optimization problems like nominality of uniformity, local optimality, and non-convergence [14], which frequently outperform more classical algorithms, with a great ability to evolve convergent, diversified, and uniformly distributed Pareto fronts.

In addition, Zheng and Yamashiro (2010) investigated the integration of quantum operators and evolutionary algorithms, including the use of the basic quantum-inspired evolutionary algorithm to create a quantum differential evolutionary algorithm for more efficient resolution of the permutation flow shop scheduling problem expressed as aMixed integer linear program [15]. Zhisheng (2010) explored the efficiency of the quantum-behaved particle swarm optimisation (QPSO) algorithm in a power system's economic load allocation which is a non linear optimization problem [16]. Palittapongarnpim et al. (2017) used differential evolution techniques to avoid the nonconvex optimization stagnation problem [17]. These techniques has been shown to offer higher fidelity and scalability than the greedy algorithms. Quadratic In [18], Chiang (2017) addressed the problem of economic power distribution of generators using a highly effective approach that relies on particle swarm optimization with multiple updating. In addition, Khan et al (2021) have developed a quantum version of the beetle's antenna search meta-heuristic and applied it to financial portfolio optimization using realistic stock market data [19]. This quantum method outperforms particle swarm optimization and genetic algorithms.

Furthermore, in the general context of optimization, Shao (2019) designed quantum circuits to implement two variational hybrid algorithms for neural network learning [20]. These algorithms are credited with exponentially increasing the number of processable samples and polynomially increasing the sample size compared to conventional learning algorithms, while also returning information about the learning weights, allowing the results to be used to directly solve other problems. On the other hand, the approximate quantum optimization technique of Farhi et al. (2015) has been adopted and improved by Hadfield et al. (2019) to allow it to alternate more generic operator families and thus representing a larger range of states [21]. And in [22], Van Apeldoorn et al (2020) identified the steps required to speed up the solution of convex optimization problems by implementing a separation oracle that can interrogate membership oracles only by using $\tilde{O}(1)$ quantum queries. Thus, the speed-up is exponentially increased compared with conventional Omega(n) membership querying techniques. This also allowed to prove that an optimization oracle might be designed by making $\tilde{O}(n)$ quantum queries to a membership oracle. Li et al., in [23] proved that the objective Gibbs function offered better prospects compared to the energy value predicted in the variational parameter fitting. This opened up a promising new field of quantum circuit architecture for approximate optimization methods. Subsequently, the authors designed an architectural ansatz search algorithm to examine the discrete space of analog quantum circuit architectures, resulting in improved performance when applied to a full-graph Ising model. In [24], Pagano et al (2020) used an analog quantum simulator to implement an approximate low-depth quantum optimization algorithm to quantify the base-state energy of long-range interactions and the transverse field tunable Ising model, and to optimize the associated classical combinatorial problem by sampling the solutions provided using high-fidelity measurements of individual qubits, performed in a single run. Performance scales very well, and execution time is almost unrelated to the number of qubits.

More recently, Harrigan et al (2021) validated the use of Google's Sycamore superconducting qubit quantum processor for combinatorial approximate optimization [25] (2021). Its performance is evaluated on the hardware's native planar connection graph using the Sherrington-Kirkpatrick model and Max-Cut, non-native problems for that require significant computation. In addition, El Gaily and Imre (2021) introduced the constrained quantum relation test (CQRT), an extended variant of the quantum existence test [26]. The CQRT can be used to convert the quantum extreme value search algorithm (QEVSA) into a limited quantum optimisation process with minimal complexity, maximum exponential speed, and high classical and quantum certainty.

IV. Quantum inspired methods

The quantum-inspired evolutionary algorithm described by Han and Kim (2002) two decades ago was one of the earliest quantum adaptations of classical optimisation methods [27]. In which one qubit is used for probabilistic representation of chromosoms while the variation operator was ensured by a quantum rotation gate to lead individuals towards better solutions and avoid premature convergence.

To tackle the global optimization problem, Jiao et al. (2008) described a quantum-inspired immune clonal algorithm (QICA) [28]. QICA antibodies are propagated, organized into subpopulations and then mapped onto multi-state gene quantum bits in their suggested efficient approach.

To handle discrete optimisation issues, Layeb (2013) used a hybrid quantum-inspired harmony search algorithm to successfully apply quantum-inspired operators such as measurement and interference on 0-1 optimization problems [29]. Zouache et al. (2016) adapted the firefly approach to develop the quantum-inspired firefly algorithm with particle swarm optimisation QIFAPSO [30]. They exploit superposition and quantum measurement to increase the control of solution diversity. QIFAPSO algorithm's efficiency was tested on various instances of the multidimensional knapsack problem and the experimental results proved its effectiveness and superiority over existing approaches. In the same context, Wang and Wang (2021) implemented a quantum-inspired differential evolution algorithm combined with gray-wolf optimizer to enhance both variety and convergence performance in high-scale scenarios for knapsack 0-1 problems [31] experimental results reveal increased global search efficiency Then, in [32], Mahmoudi et al. (2022) proposed a unique quantum-inspired metaheuristic based on the hybridization of firefly and particle swarm optimization algorithms to solve the energy-efficient clustering problem in wireless IoT networks. Simulations on networks of 10 to 100 nodes reveal that the suggested method is efficient, giving up to 15.48% energy savings and a considerable computational speedup.

In a more general context, In [33], a new approach to quantum behavior is used to represent particle motion and to solve non-linear dynamical optimization problems. This approach consists of three main mathematical phases based on the Schrödinger equation and the Monte Carlo method. More recently, Gao et al. (2020) presented a new quantum-inspired evolutionary algorithm for the robot path planning problem suited for handling large-scale optimisation challenges in both complicated static and dynamic situations, outperforming standard genetic algorithms significantly [34].

V. Quantum annealars and applications

The progress of quantum technology has led to the construction of experimental programmable quantum annealers, which have the potential to solve unconstrained binary quadratic optimization problems, and hence numerous combinatorial optimization problems of practical interest, much faster than their classical counterparts. In [35], Finnila et al (1994) presented one of the first uses of quantum annealers to generate a method for identifying the extrema of multidimensional functions. Decades later, Biswas et al. (2017) presented a quantum annealing programming overview along with an examination of three potential application areas [36]. In May 2019, Fujitsu released its Digital quantum-inspired annealing service who can be used to solve real-world combinatorial optimisation problems for clients by formulating them as unconstrained binary quadratic optimisation problems [37]. Vyskocil and Djidjev (2019) suggested a new constraint implementation scalable approach based on combinatorial design and mixed integer linear programming (MILP) for improved embedding of $\sum x_i = k$ constraints for binary variables x_i in quantum annealers [38]. Recently, Abel et al. (2021) published a thorough comparison of quantum annealing to traditional optimisation approaches such as thermal annealing, Nelder-Mead, and gradient descent [39]. According to this research, quantum annealing is clearly the best method for quickly and efficiently minimising potentials while the thermal annealing approach is somewhat more efficient at discovering the global minimum, whereas the Nelder-Mead and gradient descent methods are very prone to be locked in local minima. In addition, in [40] the diagonal thermal properties of the classic one-dimensional transverse-field Ising model were examined using the D-Wave 2000Q quantum annealing processor. The latter was found unsuitable for thermal sampling qince it could not establish the correct expectation values anticipated by the quantum Monte Carlo method.

VI. Addressing existing quantum optimisation algorithms problems

Ajagekar and You (2019) have examined the possibility of using open-source software tools as a first preliminary step in applying quantum computing to optimization problems concerning energy systems, as well as the restrictions of high-tech quantum computers in this field [41]. On the other hand, Greplova (2019) established a data-driven optimization strategy and game tactics to facilitate the identification of states of matter with the required attributes to alleviate the metallurgy problem in quantum computing [42]. To address the problems associated with the DEA differential evolution algorithm, such as insufficient diversity in the subsequent search phase, slow and early convergence and low solution quality, Deng et al (2021) proposed a more powerful variant of DEA by coordinating the computational features of the quantum evolution algorithm with the split and conquer principle of the cooperative coevolution algorithm, while using quantum chromosome coding to increase population diversity and quantum rotation to increase convergence speed[43]. In addition, Moussa, Calandra and Dunjko (2020) used machine learning techniques to compare the Quantum Approximate Optimization Algorithm (QAOA) with the approximation algorithm of Goemans and Williamson on the Max-Cut problem [44], with an accuracy well over 96% after cross-validation. In addition, Egger et al. (2021) showed how to start quantum optimisation from a state that corresponds to the solution of the relaxed combinatorial optimisation problem. They also examined the necessary features of the algorithm utilised for this purpose [45].

VII. Challenges and Prospects

The need for quantum development tools has grown with the effervescence of scientific research in this field. Actually, only a few cloud-based quantum systems offering access to limited qubit-based services for testing theoretical developments have become quantum development tools [49] in the hope and perspective that quantum computers available for research and industrial use [46]. The development and customization of hardware to meet the needs of quantum research and development appears to be a major challenge for the ambitious wishing to break into this sector. These systems, now in use, also present considerable challenges, such as: accuracy and fault tolerance, which compromise the reliability of qubits [47], the high sensitivity of physical quantum states to noise on current hardware platforms, inevitably leading to errors in algorithm execution [48], In addition, due to the highly advanced nature of quantum information processing, quantum computers can pose significant security problems for current cryptographic algorithms. Much remains to be done to find methods for ensuring a secure transition between traditional computers and their quantum counterparts. Progress in this sector is quick, and it is difficult to forecast when users will conclude that the benefits of quantum computers outweigh the burden of having to accept and adapt to ever-changing ways of utilising them.

In the meanwhile, research continues to progress. Improving on what has already been achieved will offer many opportunities, such as optimizing quantum algorithms by reducing computational resources given the limited number of qubits available for validation [2, 3, 49, 50]. All the shortcomings listed above represent opportunities for further study and work to make the quantum computing field more effective and more affordable. On the other hand, calibration of quantum systems will also become necessary in the near future. Standards will be needed to analyze and evaluate their performance and market impact.

VIII. Conclusion

Quantum computing is becoming a matter of course as more mathematical tools used in various sectors, such as engineering, discover quantum equivalents for optimization procedures. For the first time, this literature review evaluates more than two decades of application of these mathematical optimization tools, with an emphasis on work devoted to their implementation for the building of quantum computers, performance improvement, and optimization approaches. The contributions studied in this paper underscored and highlighted the significance and power of quantum computing in solving very complex real-life problems with high efficiency and speed. It also made it possible to identify the difficulties and restrictions connected with quantum computers, such as their availability for research and industrial applications, precision, fault tolerance, and qubit reliability. In addition, it offers opportunities for further development and enormous potential in this sector. This synthesis will provide a good starting point for engineering researchers wishing to use quantum computing to work out their problems.

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