A Simple and Effective Method for Calculating the Bending Loss and Phase Enhancement of a Bent Planar Waveguide

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A simple and effective method is introduced to calculate the bending loss and phase enhancement of a bent planar waveguide. The wave field is represented in terms of Airy functions and an eigenvalue equation is derived by matching the boundary conditions and the radiation condition in the outer cladding layer. The complex propagation constant is obtained by solving the eigenvalue equation with the Newton–Raphson method, and the imaginary part of the propagation constant gives directly the bending loss of the bent waveguide. The results are compared with the previous experimental and numerical results and are shown to be highly accurate and effective. The phase enhancement due to the bending is also studied.

Introduction

Curved optical waveguides have been widely used in photonic lightwave circuits (PLCs) for optical communication in order to achieve compact sizes and other specific purposes.

The bending loss, which is one of the most important characteristics of curved wave-guides, has been extensively studied in the past. A number of methods have been developed for evaluating the radiation losses associated with bent waveguides. For example, in Heiblum and Harris [1] a conformal transform was used to convert a bent slab wave-guide to an equivalent straight waveguide with a modified refractive index. With a few proper approximations, the field can be expressed in terms of Airy functions [2]. Asymptotic expansions of Bessel functions [3, 4], WKB approximation [5], free space radiation mode (FSRM) method [6], and other analytic expressions [7] have also been used. Other numerical techniques that have been employed include the beam propagation method (BPM) [8] and an eigenmode expansion method [9].

In the present article we introduce a simple and effective method to calculate the complex propagation constant for the leaky mode of a bent planar waveguide. In Goyal et al. [2] the solution of the Helmholtz equation for a bent planar waveguide was represented in terms of Airy functions (under the assumption that the bending radius is large compared to the width of the core layer). However, it was defined in such a way that no outward radiation will occur for the bent waveguide. Hence, a leaky mode of a bent waveguide acts like a bound mode, and only a purely real propagation constant for the leaky mode can be obtained from the eigenvalue equation. In Goyal et al. and Kumar et al. [2, 10] the imaginary part (which is associated directly with the bending loss) of the propagation constant has to be found with some additional efforts by finding the width of a Lorentzian produced by plotting the ratio of the amplitude of the field in the core to the amplitude of the field in the outer cladding as a function of β. In the present article we give an expression similar to that in Rowland [11] for the resulting eigenvalue equation, from which the complex propagation constant can be determined directly. Our numerical results show that the present method is simple and effective, as well as more straightforward and accurate than the previous one. The phase enhancement (associated with the difference in the real part of the propagation constant for a bent waveguide and the corresponding straight waveguide) due to the bending is also studied in the present article.

Theory

We consider a three-layered bent waveguide whose geometry and refractive index are shown in Figure 1 in the two-dimensional space. The waveguide has a core half-width of h and is bent with a radius of R. The refractive indices of the three layers (inner cladding layer, core layer, and outer cladding layer) are denoted by n_1, n_2, and n_3, respectively. The cylindrical coordinate system (r, ϕ, z) used is also shown in Figure 1. In the present article all the fields are assumed to have harmonic time dependence $\exp(-i\omega t)$. For TE waves propagating in the ϕ direction, one can write the field component E_z in the following form

$$E_z(r, \phi) = E_z(r)e^{i v \phi} \tag{1}$$

where $E_z(r)$ satisfies the following Helmholtz equation,

$$\left[\frac{\partial^2}{\partial r^2} + \frac{1}{r} \frac{\partial}{\partial r} + n^2(r)k_0^2 - \frac{v^2}{r^2} \right] E_z(r) = 0 \tag{2}$$
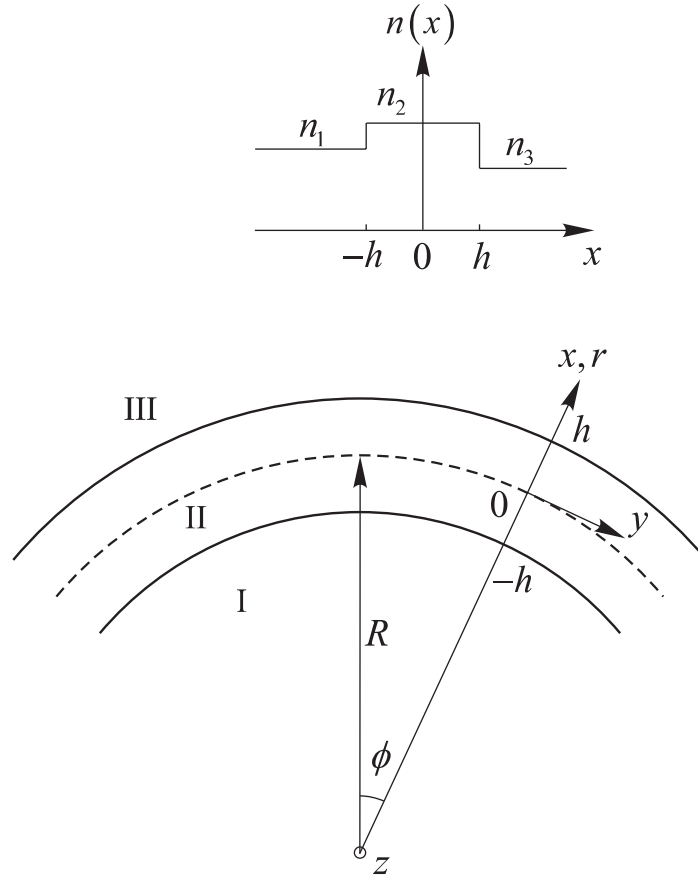
Figure 1. Schematic diagram for the geometry of the bent planar waveguide.

Replacing $E_z = \psi/\sqrt{r}$, $r = R + x$, and $\nu = \beta R$, one can rewrite Equation (2) as

$$\left[\frac{\partial^2}{\partial x^2} + k_0^2 n^2(x) - \beta^2 \frac{R^2}{(R+x)^2} + \frac{1}{4(R+x)^2} \right] \psi = 0 \qquad (3)$$

In the present article we assume that $R \gg h$. Then we can take only the first order of the Taylor expansion of

$$\frac{R^2}{(R+x)^2} \approx 1 - \frac{2x}{R} \qquad (4)$$

Equation (3) thus becomes

$$\left[\frac{\partial^2}{\partial x^2} + k_0^2 n^2(x) - \left(\beta^2 - \frac{1}{4R^2} \right) \left(1 - \frac{2x}{R} \right) \right] \psi = 0 \qquad (5)$$

Equation (5) can be further transformed into

$$\frac{d^2\psi}{dZ_i^2} - Z_i\psi = 0 \tag{6}$$

where

$$Z_i = \frac{-k_0^2 n_i^2 + \beta^2 - \frac{1}{4R^2} - ax}{a^{2/3}} \tag{7}$$

$$a = \frac{2}{R}\left(\beta^2 - \frac{1}{4R^2}\right) \tag{8}$$

Equation (6) indicates that the solution for a bent waveguide is equivalent to that for a straight waveguide that has a linear variation for the square of the refractive index in each layer [2]. The solution of Equation (6) in each layer may be represented in terms of Airy functions,

$$\psi_i = C_i \mathrm{Ai}(Z_i) + D_i \mathrm{Bi}(Z_i), \qquad i = 1, 2, 3 \tag{9}$$

Next we consider the boundary conditions. In region I, the asymptotic of Airy functions as $x \to -\infty$ (i.e., $Z_1 \to +\infty$) are given by [2, 12]

$$\mathrm{Ai}(Z_1) \to \frac{1}{2\sqrt{\pi}} Z_1^{-1/4} e^{-\xi} \tag{10}$$

$$\mathrm{Bi}(Z_1) \to \frac{1}{\sqrt{\pi}} Z_1^{-1/4} e^{\xi} \tag{11}$$

where $\xi = (2/3)Z_1^{3/2}$. Since $\mathrm{Bi}(Z_1)$ tends to infinity when $x \to -\infty$ [11], we must set

$$D_1 = 0 \tag{12}$$

In region III as $x \to +\infty$ (i.e., $Z_3 \to -\infty$), we have

$$\mathrm{Ai}(Z_3) \to \frac{1}{\sqrt{\pi}}(-Z_3)^{-1/4} \sin\left(\xi + \frac{\pi}{4}\right) \tag{13}$$

$$\mathrm{Bi}(Z_3) \to \frac{1}{\sqrt{\pi}}(-Z_3)^{-1/4} \cos\left(\xi + \frac{\pi}{4}\right) \tag{14}$$

where $\xi = (2/3)(-Z_3)^{3/2}$ (note that Z_3 is negative when x is sufficiently large). In order to let the field in region III have only an outward propagating leaky wave when $x \to +\infty$, we should have

$$C_3 = i D_3 \tag{15}$$

Under such a condition $C_3\mathrm{Ai}(Z_3)+D_3\mathrm{Bi}(Z_3)$ is proportional to $\exp[i(\xi+\pi/4)]$. Then the product $\exp(-i\omega t)[C_3\mathrm{Ai}(Z_3)+D_3\mathrm{Bi}(Z_3)]$ has a factor $\exp[i(-\omega t+\xi)]$, which represents

an outward-going wave as the phase increases when r increases or t decreases. Note that unlike the asymptotic of Hankel function [4, 13], the outward phase is not correct when x tends to $+\infty$ since Equation (9) is valid only if $x \ll R$; however, the solution should be accurate enough for large values for the radius R and relatively small values of x near the waveguide.

For TE waves the continuity of ψ and its first derivative at $x = \pm h$ will give the following system of equations:

$$A \begin{pmatrix} C_1 & C_2 & D_2 & D_3 \end{pmatrix}^T = 0 \tag{16}$$

where

$$A = \begin{Bmatrix} -\mathrm{Ai}[Z_1(-h)] & \mathrm{Ai}[Z_2(-h)] & \mathrm{Bi}[Z_2(-h)] & 0 \\ -\mathrm{Ai}'[Z_1(-h)] & \mathrm{Ai}'[Z_2(-h)] & \mathrm{Bi}'[Z_2(-h)] & 0 \\ 0 & \mathrm{Ai}[Z_2(h)] & \mathrm{Bi}[Z_2(h)] & -i\mathrm{Ai}[Z_3(h)] - \mathrm{Bi}[Z_3(h)] \\ 0 & \mathrm{Ai}'[Z_2(h)] & \mathrm{Bi}'[Z_2(h)] & -i\mathrm{Ai}'[Z_3(h)] - \mathrm{Bi}'[Z_3(h)] \end{Bmatrix} \tag{17}$$

In order to have non-zero solution for Equation (16), the determinant of the coefficient matrix A should be zero, i.e.,

$$f(\beta) \equiv \det A = 0 \tag{18}$$

The root of Equation (18) gives the complex propagation constant β of the leaky mode. The magnitude of the radiative loss can be obtained from the imaginary part β_{im} of the propagation constant.

It is worthwhile to note that the leaky mode defined by Equations (9), (12), and (15) differs from that defined in Goyal et al. [2], where C_3 is assumed to be 0 for the eigenvalue β of the quasi-mode. The reason for their assumption, as discussed in Goyal et al. [2], is to have a decaying field distribution, which is similar to a real (not leaky) mode distribution, as x increases from h to point x_1 where $Z_3(x_1) = 0$ (see the vertical dotted line around $x = 11$ μm in Figure 2). Since $\mathrm{Ai}[Z_3(x)]$ is a monotonically increasing function of x in this region, in Goyal et al. [2] it is intentionally excluded from the field distribution. This, however, is not appropriate for a leaky mode for two reasons. First, it leads to a lossless propagation of the leaky wave (acts just like a real mode). The eigenvalue equation under such a condition gives a purely real value for the propagation constant, which is not physically correct since the propagation constant must have an imaginary part (associated with the leaky nature of the bent waveguide). Second, the real solution of the quasi-mode field makes it more complicated to obtain the power that leaks from the bent waveguide. In Goyal et al. [2] the leakage loss (which should be directly related to the imaginary part of β) has to be found in an implicit way [14]. Here with our definition both the real and imaginary parts of the propagation constant can be directly determined from Equation (16). Note that our outward-going factor $\exp[i(\xi + \pi/4)]$ is somewhat different from the asymptotic behavior of Hankel function of the first type (corresponding to a conventional outward-going wave). This is due to the replacement of Equation (4).

Since the imaginary part of β is relatively small compared to the real part, we can avoid the cumbersome search for the root of Equation (18) in the whole complex plane by using the following procedure. First, a real approximation of β is obtained by solving
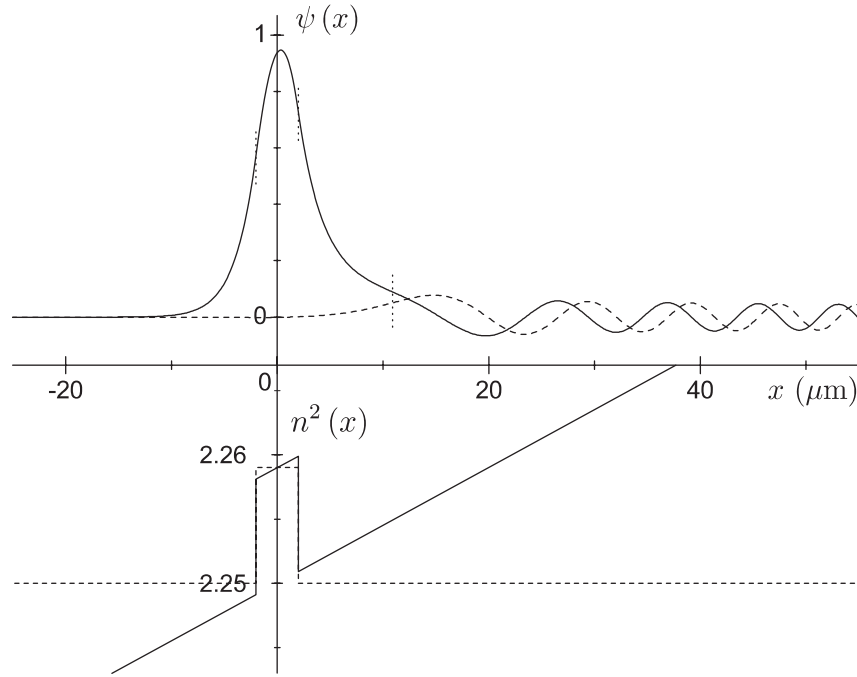
Figure 2. Plot of the real part (solid line) and the imaginary part (dashed line) of the electric field for a bent waveguide with $h = 2.0 \ \mu$m, $n_2 = 1.503$, $n_1 = n_3 = 1.50$, $R = 1$ cm, and $\lambda = 1.0$ μm. The three vertical dotted lines indicate the positions $x = \pm h$ and $x = x_1$, where $z(x_1) = 0$. Lower: The refractive index profile for the bent waveguide (dashed line) and its straight waveguide equivalent (solid line).

the real part of Equation (18), which is

$$
\begin{vmatrix}
-\text{Ai}[Z_1(-h)] & \text{Ai}[Z_2(-h)] & \text{Bi}[Z_2(-h)] & 0 \\
-\text{Ai}'[Z_1(-h)] & \text{Ai}'[Z_2(-h)] & \text{Bi}'[Z_2(-h)] & 0 \\
0 & \text{Ai}[Z_2(h)] & \text{Bi}[Z_2(h)] & -\text{Bi}[Z_3(h)] \\
0 & \text{Ai}'[Z_2(h)] & \text{Bi}'[Z_2(h)] & -\text{Bi}'[Z_3(h)]
\end{vmatrix} = 0
\tag{19}
$$

The real root β_0 found should be in accordance with that in Goyal et al. [2] since the associate code is the same when setting $C_3 = 0$. Then using the Newton–Raphson method, the complex solution of Equation (18) can be found iteratively by

$$
\beta_{k+1} = \beta_k - \frac{f(\beta_k)}{f'(\beta_k)}
\tag{20}
$$

until $|\beta_{k+1} - \beta_k| < \varepsilon$ (a small parameter). For convenience, the first derivative $f'(\beta_k)$ can be replaced by the center difference of $f(\beta_k)$. In Equation (20), the Airy functions for complex argument need to be evaluated.

Figure 2 shows an example of the field distribution in a bent waveguide together with the refractive index profile for the bent waveguide and the equivalent straight waveguide structure showing the effect of the bend. Note that the only approximation in this method is Equation (4), which results in the square of the effective refractive index linear with x in

Table 1

Real (effective index) and imaginary (bending loss) parts of the propagation constant

R/h	β_0/k_0	β_{re}/k_0	$2h\beta_{im}$	$2hk_0\Gamma^a$
0.1×10^4	1.501 811 920 930	1.502 106 427 872	1.1918×10^{-2}	$[1.9464 \times 10^{-2}]$
				$[1.3995 \times 10^{-2}]$
0.3×10^4	1.501 706 844 215	1.501 710 769 924	8.9454×10^{-4}	$[9.1012 \times 10^{-4}]$
				$[9.0063 \times 10^{-4}]$
0.5×10^4	1.501 637 704 694	1.501 637 774 280	9.0990×10^{-5}	$[9.1031 \times 10^{-5}]$
				$[9.1000 \times 10^{-5}]$
0.7×10^4	1.501 615 376 460	1.501 615 377 357	8.6214×10^{-6}	$[8.6216 \times 10^{-6}]$
				$[8.6215 \times 10^{-6}]$
0.9×10^4	1.501 606 676 523	1.501 606 676 533	7.7555×10^{-7}	$[7.7561 \times 10^{-7}]$
				$[7.7561 \times 10^{-7}]$
1.0×10^4	1.501 604 253 474	1.501 604 253 474	2.3039×10^{-7}	$[2.3042 \times 10^{-7}]$
				$[2.3042 \times 10^{-7}]$
1.1×10^4	1.501 602 492 868	1.501 602 492 868	6.8194×10^{-8}	$[6.8205 \times 10^{-8}]$
				$[6.8205 \times 10^{-8}]$
1.3×10^4	1.501 600 151 710	1.501 600 151 710	5.9340×10^{-9}	$[5.9354 \times 10^{-9}]$
				$[5.9354 \times 10^{-9}]$
1.5×10^4	1.501 598 703 766	1.501 598 703 766	5.1348×10^{-10}	$[5.1365 \times 10^{-10}]$
				$[5.1365 \times 10^{-10}]$

^{a}From Goyal et al. [2].

each layer. This should be a very good approximation for the fields near the waveguide when $R \gg h$. Moreover, unlike to the earlier WKB analysis of bent waveguides [5], which gives approximate solution to the exact conformally transformed wave equation for the bent waveguide, in the present method the approximation is made directly to the waveguide structure, which is more straightforward and makes it easier to evaluate the validity of the approximation. No additional approximation is required to solve the wave equation.

In order to compare the method with the previous methods, we have carried out the calculations with the same waveguide parameters as used in Thyagarajan et al. [15] and Goyal et al. [2]. These parameters are $h = 2.0$ μm, $n_1 = n_3 = 1.50$, $n_2 = 1.503$, and $\lambda = 1.0$ μm. The results are tabulated in Table 1, where β_0 is the zeroth-order real approximation obtained from Equation (19), and β_{re} and β_{im} are the real and imaginary parts of β obtained from Equation (20). The power inside the core layer decreases as $P(\phi) = P(0) \exp[-(2h\beta_{im})(R/h)\phi]$. The bending losses calculated with the method of Goyal et al. [2] are also shown in Table 1 for comparison. In Goyal et al. [2] the bending losses are calculated through an eigenvalue equation for a real β (as mentioned before). Specifically, the imaginary part of the propagation constant is obtained by evaluating $k_0\Gamma$, where Γ is the half-width-half-maximum (HWHM) of a Lorentzian determined by the ratio $(C_2^2 + D_2^2)/(C_3^2 + D_3^2)$ as a function of β in the neighborhood of the real eigenvalue of β. The results obtained at both $\beta = \beta_0$ and $\beta = \beta_{re}$ are tabulated in Table 1 in the round and square brackets, respectively. Since β_0 is essentially the same as the real propagation constant obtained in Goyal et al. [2], the values in the round brackets represent the results from this method. From Table 1 one sees that the results for the

bending loss agree quite well for large values of R; however, there are some discrepancies when R is not sufficiently large. It should be noted that the bending loss obtained from our method is more accurate than the one obtained from the previous method since there is no other approximation that has been made to obtain the complex propagation constant. In the previous method, the real propagation constant of a quasi-bound mode (which is very close to the real part of the propagation constant for the radiative mode if R is large enough, but still not exactly the same, as seen in Table 1) was used to produce the bending loss. This arouses the major error. As seen in Table 1, the use of $\beta = \beta_{re}$ in the previous method gives a result (in square brackets) closer to ours. Moreover, Γ is obtained through some additional approximations as it is determined approximately by $\Gamma \approx D_3^2/(dC_3/d\beta)^2$. It is safe to say that Γ is somewhat overestimated since no matter how large R is, the bending losses obtained from the previous method are larger than those obtained from our method.

Hence we see that the bending losses are obtained in a much more straightforward and accurate approach in this work. It is also worthwhile to note the efficiency of the present approach. Once we have the real β_0, it needs only several iterations (typically no more than 5) of Equation (20) to obtain the complex propagation constant with a relative precision higher than 10^{-14}.

The real part of the propagation constant is important for calculating the correct phase of the light wave in a bent waveguide. As expected, the values for β_{re} are generally different from those for β_0 (larger than the latter, and the difference increases as the bending increases (i.e., R decreases, see Table 1).

For TM modes, the above formulae need only a small revision. The wave equation is exactly the same except that the non-vanishing field components are H_z, E_x, and E_y (instead of E_z, H_x, and H_y for TE modes). The boundary condition is the continuity of H_z and $E_y = (-i/\omega\varepsilon)\partial H_z/\partial x$. This will lead to an equation almost the same as Equation (16), except that the $\text{Ai}'[Z_i(x_b)]$ and $\text{Bi}'[Z_i(x_b)]$ (where $x_b = \pm h$) in Equation (16) should be replaced with $(1/n_i^2)\text{Ai}'[Z_i(x_b)]$ and $(1/n_i^2)\text{Bi}'[Z_i(x_b)]$, respectively.

Numerical Examples and Discussions

First we consider a buried SiO$_2$ waveguide (fabricated by the well-known silica-on-silicon technology), which has the typical effective refractive indices (averaged along the z-direction with the effective index method [16, 17]) $n_1 = n_3 = 1.46$ and $n_2 = 1.47$. Here we choose $h = 2.5\ \mu$m and $\lambda = 1.55\ \mu$m. The results are shown in Figure 3 for both the TE and TM modes. The bending loss is given in db/cm by $(0.2/\ln 10)\pi R\beta_{im}$, where β_{im} is in m^{-1}. As seen from Figure 3, both β_{re} and β_{im} decrease as R increases. As $R \to \infty$, the effective refractive index (averaged along the radial direction) β_{re}/k_0 tends to 1.466 852 363 (for TE mode) and 1.466 828 153 (for TM mode), while $\beta_{im} \to 0$. As in the case of straight waveguides, the effective refractive index for the TM mode is lower than that for the TE mode for the case of bent waveguides. Note that the effective refractive index for the bent waveguide is higher than that for the corresponding straight waveguide. This will cause some phase error in, for example, the arrayed waveguides (which are bent) in the design of an AWG demultiplexer (e.g., [18] for AWGs) if one does not correct the phase enhancement due to the bending.

The bending loss difference for the TE and TM modes, which is too small (compared with the large variation of the bending loss for different R) to be seen in Figure 3, can be seen in Table 2 for 90° bent waveguides. Explicitly, the values are determined by $(10/\ln 10)\pi R\beta_{im}$. As one can see in Table 2, the bending loss for the TM mode is higher
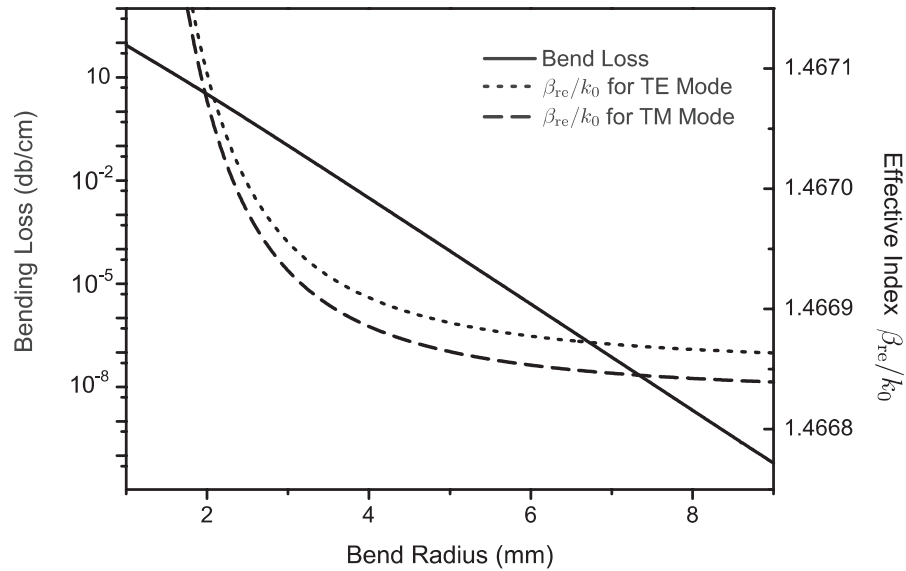
Figure 3. The bending loss (solid line) and the effective refractive index (dotted line for the TE mode, and dashed line for the TM mode) as the bend radius R varies. The difference in the bending losses for the TE and TM modes is too small to be seen in the figure.

Table 2
Bending loss and phase enhancement for a 90° circularly bent SiO_2 waveguide

R (mm)	TE mode		TM mode	
	Bending loss (db)	Phase enhancement	Bending loss (db)	Phase enhancement
1.0	13.4	2.300	13.7	2.309
3.0	4.99×10^{-2}	0.7545	5.15×10^{-2}	0.7583
5.0	7.03×10^{-5}	0.4419	7.70×10^{-5}	0.4438
7.0	8.08×10^{-8}	0.3140	9.19×10^{-8}	0.3153
9.0	8.73×10^{-11}	0.2437	1.03×10^{-10}	0.2448

than that for the TE mode. The phase enhancements shown in Table 2 are calculated by $(\beta_{re} - \beta_{straight})\pi R/2$, where $\beta_{straight}$ is the propagation constant for the corresponding straight waveguide. From this table one can see that the bending can cause a considerable phase enhancement when R is not large enough. Our calculation shows that a total length of 19.52 mm (for the TM mode) can cause a phase enhancement of π for such a bent SiO_2 waveguide with a bend radius of 3 mm. This effect has to be taken into account when designing phase-based PLCs involving bent waveguides.

Next, in order to validate our method with the existing experimental data, we consider a ridge waveguide structure proposed in Austin [19] that has been previously analyzed by method of line [20], finite element method [21], finite difference method [22], and WKB method [5]. The waveguide structure is shown in Figure 4, which consists of a rib
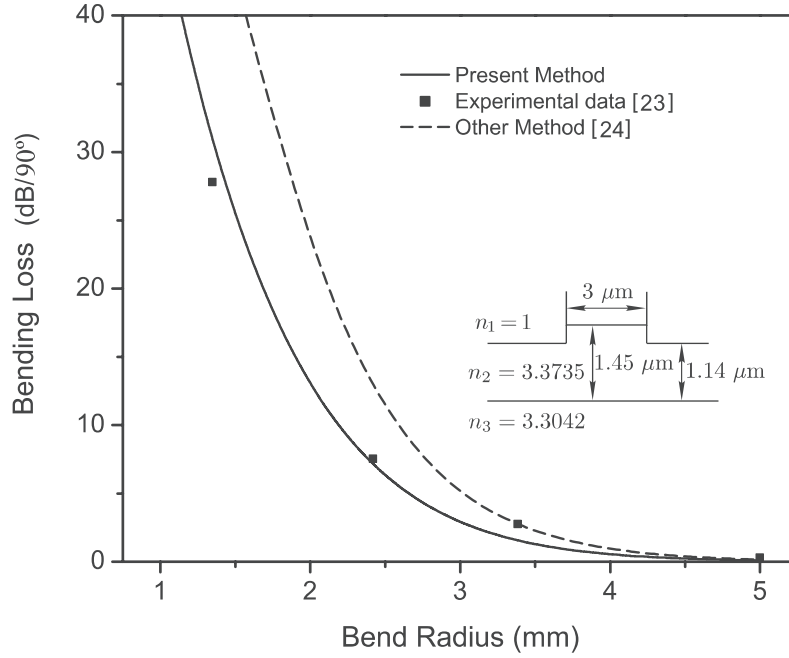
Figure 4. Bending loss of a ridge GaAs-AlGaAs waveguide as the bend radius varies for the quasi-TE case.

GaAs guiding layer (with the refractive index of 3.3735) on a thick $Al_{0.15}Ga_{0.85}As$ layer (with the refractive index of 3.3042) [23]. The wavelength is 1.52 μm. As we know, the weakly guided ridge waveguide can be accurately modeled by the effective index method (EIM) [16, 17] as a 2-D planar waveguide structure in the xy-plane if the modes of the waveguide are operated in the far-from-cutoff region. For quasi-TE (or -TM) modes, we first project the 3-D refractive index profile onto the xy-plane with the effective refractive index n_{TE} (or n_{TM}) found by "averaging" in the z-direction with the EIM for each fixed point (x, y). Then we find the propagation constant for this 2-D bent waveguide structure using the TM (or TE) boundary condition. The comparison of our numerical results with the results of an experiment [23] and other numerical method [24] is shown in Figure 4 for the quasi-TE case. From this figure one can see that our results agree well with the experimental results.

Summary

In the present article, we have introduced a simple and effective numerical method to investigate the bending radiation loss and phase enhancement of a bent planar optical waveguide. The wave field is approximated in terms of Airy functions with the assumption that the bend radius is much larger than the width of the core layer. The radiation far field in the outer cladding layer has been correctly represented and used to derive a correct eigenvalue equation. The complex propagation constant has been determined effectively by solving the eigenvalue equation with the Newton–Raphson iterative method (only a few iterations suffice). The bending loss of the bent waveguide is obtained directly from the imaginary part of the propagation constant. The results have been compared

with some previous experimental and numerical results and have been shown to be highly accurate and effective. The enhancement of the phase of the lightwave in a bent waveguide (associated with the difference in the real part of the propagation constant for a bent waveguide and the corresponding straight waveguide) due to the bending has also been studied.

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