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Analog between optical waveguide system and quantum-mechanical tunneling

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The analog between an optical waveguide coupler and the quantum-mechanical double-well problem is established. Well-known results of the quantum-mechanical system are applied to the waveguide system. This way of thinking of the waveguide system provides a quick understanding of the coupling mechanism and provides new interpretations of results obtained from Maxwell's equations. Just as ray opticals give insight into certain types of optics problems, we believe that the tunneling interpretation will give new insight into the field of guided light waves.

I. INTRODUCTION

It is, of course, neither new nor surprising that there is a close analog between some aspects of optics and quantum mechanics. It is, however, helpful to look at a current problem in a slightly different way and discover that it is identical to one which has already been solved in another branch of physics. The problem, which we describe here, is that of coupling light energy from one optical waveguide to another.

The current thrust toward integrated optical circuits has been primarily motivated by the extreme care needed in alignment of the current beam circuits and their sensitivity to vibrations. (Anyone who has done an experiment using the usual lens, mirror, polarizer, etc., will attest to the difficulties produced by the startup of a nearby motor, for example.) A great impetus could be given to the commercial use of optics if the circuitry for a given process could be integrated into a solid. Such a circuit would not be subject to nearly as great an extent to ambient temperature gradients, temperature changes, or acoustic and vibrational effects as the laboratory beam circuits. One of the many circuit elements which needs to be developed before integrated optical circuits can be realized is the waveguide coupler. (For example, efficient couplers are needed to couple lasers to the guides, modulators, etc.)

The waveguides in integrated optics are dielectrics embedded in or surrounded by dielectric substrates. The guide dielectric need have an index of refraction only 1% or so larger than the substrate and dimensions of the order of a few microns in order to guide the light wave. The description of the modes which propagate in the guide are developed either from Maxwell's equations or ray optics.[1,2] The work which has been done so far in the coupling of guides has also started with Maxwell's equations[2]; however, a new insight into the problem can be obtained by casting the problem into the one analogous to the quantum-mechanical potential well problem. Not only are solutions to such problems readily available (every undergraduate physical science, or engineering major must have solved at least one such problem!) but a more complete understanding of the coupling process is often obtained. For example, Campi and Harrison[3] show the usefulness of using the waveguide–quantum analog to demonstrate tunneling through an obstruction in a rectangular waveguide.

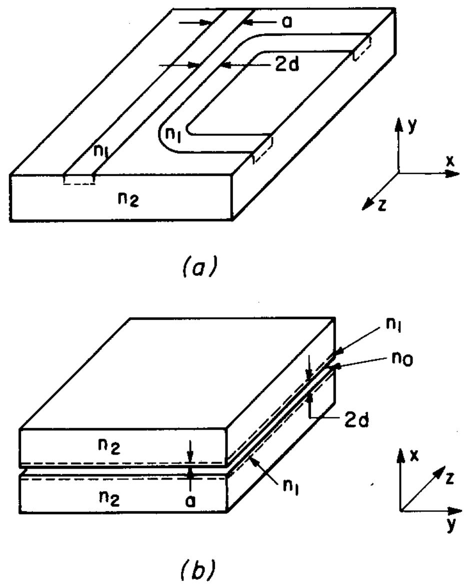


Fig. 1. Possible physical realizations of the coupled waveguide system. (a) The coupling between the two guides by bringing one guide (gradually) closer to the other. Once the guides are close together, the energy of one guide may "leak" into the other. (b) The coupling in this case is achieved by placing one guide plate very close to the other over a distance sufficiently long enough to transfer the energy between the two.

II. COUPLING BETWEEN TWO IDENTICAL THIN-FILM WAVEGUIDES

In order to cast the optical problem into one identical with the coupled potential well problem in quantum mechanics, we assume the light source to be monochromatic (i.e., constant energy sources). We choose the z direction to be the direction of propagation and the wave number of the light wave to be ζ. In Figs. 1(a) and 1(b) two possible physical realizations of the coupled guides are shown. If we assume the mode which propagates is a TE mode (i.e., $E_z \equiv 0$) and that there is no variation in the y direction, then we may write

$$E_y = E_y(x) \exp i(\zeta z - \omega t).$$

Maxwell's equations quickly yield the equation[2]

$$\frac{d^2}{dx^2} E_y + \beta^2 E_y = 0 \qquad (1)$$

for a perfect dielectric medium (i.e., $\nabla \cdot E = 0$), where $\beta^2 = \omega^2\mu\epsilon - \zeta^2$ (in mks units, for example). A plane wave propagating along the z axis would give $\zeta_0 = \omega(\epsilon\mu)^{1/2} = \omega(n/c)$, where c is the velocity of light and n is the index of refraction. The wave function of a particle of mass m would have the same form of the light wave with a wave vector $k = (2m\mathcal{E})^{1/2}/\hbar$. By analog, we will associate ζ^2 with the longitudinal energy. Equation (1) shows that the transverse component of electric field can take two forms: a propagating solution when $\zeta < \omega(n/c)$ and a decaying solution when $\zeta > \omega(n/c)$. For the case of a two-dimensional well, a particle will propagate along the axis of the well with a longitudinal energy $k_l = (\mathcal{E}_l 2m)^{1/2}/2\hbar$, where $\mathcal{E}_l$ is the longitudinal energy. The transverse energy of the particle would be $\mathcal{E} - \mathcal{E}_l = \mathcal{E} - (\hbar^2k^2/2m)$, where $\mathcal{E}$ is the total energy. In units where $\hbar^2/2m = 1$ we would write the transverse energy as $\mathcal{E} - k^2$. The analog expression for the transverse energy in the waveguide system would be $\omega^2(n/c)^2 - \zeta^2$. In both the quantum-mechanical and the waveguide system, it is the transverse "energy" which determines how far the wave will penetrate into the barrier (i.e., into the region where $\beta^2 < 0$). In the two-well system, if the barrier which separates the two wells is thin enough, there is a finite probability that a wave, originally confined to one of the wells, will tunnel into the other.

If we choose the index of refraction of the waveguide system such that $\beta^2 > 0$ within the guide and $\beta^2 < 0$ everywhere else, the light wave will be confined to the guiding structure. It is convenient to write $\omega^2\mathcal{E}\mu = (n_i - k_0)^2$, where n_i is the index of refraction of the ith dielectric region and $k_0 = 2\pi/\lambda_0$ with λ_0 the free-space wavelength.

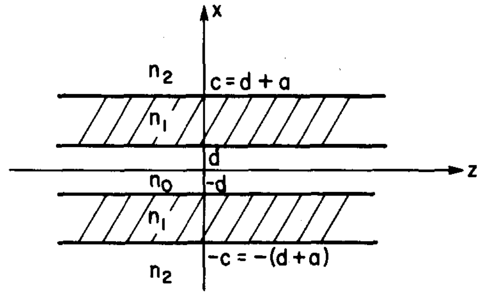
An idealization of the coupled waveguide system is shown in Fig. 2. Two guides of index of refraction n_1 are embedded in a substrate of index of refraction n_2. The two guides are separated by a dielectric [such as the air gap in Fig. 1(b)] of index of refraction n_0. In terms of the (transverse) potential energy we may write

$$V(x) = \begin{cases} V_0 = [\mathcal{E}_1 - \zeta^2 - (n_0k_0)^2], & |x| < d, \\ V_1 = [\mathcal{E}_1 - \zeta^2 - (n_1k_0)^2], & c > |x| > d, \\ V_2 = [\mathcal{E}_1 - \zeta^2 - (n_2k_0)^2], & c < |x|, \end{cases} \qquad (2)$$

where k_0 is the free-space wave vector and $\mathcal{E}_1$ is the total transverse "energy" of the wave. The potential energy diagram shows two wells, separated by a barrier of thickness $2d$.

The Schrödinger equation for such a potential well system may be written

$$\left[-\frac{\hbar^2}{2m}\frac{d^2}{dx^2} + V(x)\right]\psi = \mathcal{E}_1\psi, \qquad (3)$$



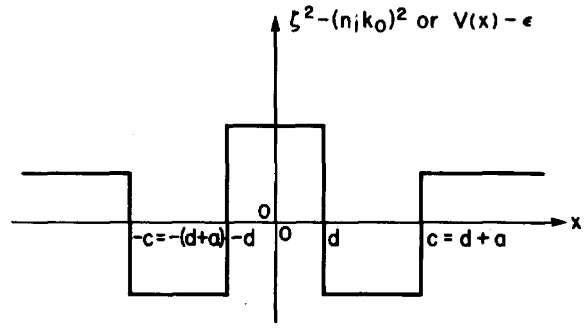


Fig. 2. (a) The geometry of two coupled waveguides. The guide index of refraction n_1 need only be 1% greater than the substrate index n_2. The guides are separated by a dielectric (such as air) of index of refraction n_0. (b) The analogous quantum-mechanical potential well system, which gives identical wave equations.

where $\mathcal{E}_1$ is the energy corresponding to the x coordinate of motion (i.e., $\mathcal{E}_1 = \mathcal{E} - \hbar^2k^2/2m$). If we choose units such that $2m/\hbar^2 = 1$, the Schrödinger and Maxwell equations become

$$\left[\frac{d^2}{dx^2} + \mathcal{E}_1 - V(x)\right]\psi = \left[\frac{d^2}{dx^2} + \mathcal{E} - V(x) - k^2\right]\psi = 0,$$
$$\left[\frac{d^2}{dx^2} + (n_ik_0)^2 - \zeta^2\right]E_y = 0. \qquad (4)$$

Clearly, $(n_ik_0)^2$ is analogous to the difference between the total energy $\mathcal{E}$ and the transverse potential energy $V(x)$. If we use the potential, defined by Eq. (2), the two equations in Eq. (4) are identical.

For the TE modes the boundary conditions for the transverse components of **E** and **H** lead to the conclusion that E_y and its first derivative with respect to x must be continuous at each of the boundaries between the different dielectric regions. These are, of course, the boundary conditions for the one-dimensional potential well problem.

The solutions in the five regions shown in Fig. 1 are as follows:

$$E_{y\mathrm{I}}, \psi_{\mathrm{I}} = A\exp(K_1x), \qquad K_1 = [\zeta^2 - (n_2k_0)^2]^{1/2}, (V_2 - \epsilon_1)^{1/2}, \qquad x \leq c,$$
$$E_{y\mathrm{II}}, \psi_{\mathrm{II}} = B_1\exp(iKx) + B_2\exp(-iKx), \qquad K = [(n_1k_0)^2 - \zeta^2]^{1/2}, (\epsilon_1 - V_1)^{1/2}, \qquad -c \leq x \leq -d,$$

$$E_{y\text{III}}, \psi_{\text{III}} = C_1 \exp(K_2 x) + C_2 \exp(-K_2 x), \quad K_2 = [\zeta^2 - (n_0 k_0)]^{1/2}, (V_0 - \epsilon_1)^{1/2}, \quad -d \leqslant x \leqslant d,$$

$$E_{y\text{IV}}, \psi_{\text{IV}} = D_1 \exp(iKx) + D_2 \exp(-iKx), \quad d \leqslant x \leqslant c,$$

$$E_{y\text{V}}, \psi_{\text{V}} = F \exp(-K_1 x), \quad x \geqslant c. \tag{5}$$

Due to the symmetrical properties of the potential well, the steady state solutions can be classified according to their parity; more particularly, since the bound states in one dimension are nondegenerate, each bound state in the symmetric potential must be either of even or odd parity. The even solutions will have $A = F$, $B_1 = D_2$; $B_2 = D_1$, and $C_1 = C_2$. Applying the appropriate boundary conditions allows us to calculate

$$\cot Ka = -\frac{1 - (K^2/K_1K_2)\coth K_2 d}{K/K_1 + (K/K_2)\coth K_2 d}, \tag{6}$$

where $a = c - d$. In a similar manner, the antisymmetric or odd modes are characterized by the equation

$$\cot Ka = -\frac{1 - (K^2/K_1K_2)\tanh K_2 d}{K/K_1 + (K/K_2)\tanh K_2 d}. \tag{7}$$

The problem of interest is to find the change in energy (or wave velocity in the direction of propagation) due to the presence of coupling between the two guides. The modes of the uncoupled guides may be found by letting $K_2 d \to \infty$ (whereupon the even and odd modes become degenerate). As coupling is introduced, the degenerate mode splits into an even and an odd mode. For the case of weak coupling, the change in energy may be found by letting $K = K_0 + \Delta K$ (where the subscript zero stands for the uncoupled value). We note that since both $K^2 + K_1^2$ and $K^2 + K_2^2$ are equal to a constant,

$$\Delta K_1 = -(K_0/K_{10})\Delta K,$$
$$\Delta K_2 = -(K_0/K_{20})\Delta K \tag{8}$$

(where again the zero subscripts stand for the uncoupled values). After a bit of algebra we arrive at the value of ΔK:

$$\Delta K = \frac{\mp 2(K_{20}/K_0)(K_0^2 + K_{10}^2)\exp(-2K_{20}d)}{(K_0^2 + K_{10}K_{20})(K_{10} + K_{20})(K_0^{-2} + K_{20}^{-2} + K_{10}^{-2} - K_{10}^{-1}K_{20}^{-1}) + a[K_0^2 + K_{10}^2 + K_{20}^2 + (K_{10}K_{20}/K_0)^2]}. \tag{9}$$

For the case in which the guides are embedded in a dielectric of index n_2 (i.e., $n_0 = n_2$ or $K_{10} = K_{20}$) the splitting reduces to the simple form

$$\Delta K = \frac{\mp 2K_0 K_{10}^2 \exp(-2K_{10}d)}{\gamma^2(2 + K_{10}a)}, \tag{10}$$

where $\gamma^2 = K_0^2 + K_{10}^2 = (n_1^2 - n_2^2)k_0^2$. Since $\beta^2 + \zeta^2 = (n_i k_0)^2$, we see that the change in the propagation wave vector is given by

$$-\Delta\zeta = (K_0/\zeta_0)\Delta K. \tag{11}$$

Just as in the case of the quantum-mechanical double-well problem, the physical significance of this splitting is best appreciated by considering the system to be such that the light wave (or particle) is isolated in the right-hand guide (well) at, say, $z = 0$. Let the lowest value of the wave vector ζ_0 (energy) correspond to the field (wave function) $(E_y)_0$ and the highest one, ζ_1, correspond to $(E_y)_1$. Then the field will be characterized at $z = 0$ by $(E_y)_0 + (E_y)_1$ to first approximation. This eigenvector for other positions is written

$$E_y(z) = (E_y)_0 \exp(i\zeta_0 z) + (E_y)_1 \exp(i\zeta_1 z)$$
$$= \exp\left[i\left(\frac{\zeta_0 + \zeta_1}{2}\right)z\right]\left[[(E_y)_0 + (E_y)_1]\cos\left(\frac{\Delta\zeta z}{2}\right) - i[(E_y)_0 - (E_y)_1]\sin\left(\frac{\Delta\zeta z}{2}\right)\right].$$

Just as in the case of the quantum-mechanical double-well problem,[4] the light shuttles back and forth between the two guides. In the case of the light coupler, the energy is transferred from one guide to the other as a function of distance along the guides. The rate of the buildup of the electric field within the second well is just $\Delta\zeta/2$ per unit length of guide. To prevent the transferral of the light back to the first guide, the coupling would have to be gradually weakened as $\Delta\zeta z$ approaches π. Such weakening of the coupling could be affected by gradually increasing the separation, d, between the two guides, as is shown in Fig. 1(a).

We see that the quantum analog provides an easily visualized interpretation and understanding of the coupling system. The application of familiar quantum-mechanical results to the guide system produces a great savings in time and effort in arriving at useful results.

[1]P. K. Tien, Appl. Opt. **10**, 2395 (1971).

[2]For example, see N. S. Kapany and J. J. Burke, *Optical Waveguides* (Academic, New York, 1972), p. 56.

[3]M. Campi and M. Harrison, Am. J. Phys. **35**, 133 (1967).

[4]For example, see E. Merzbacher, *Quantum Mechanics* (Wiley, New York, 1961), p. 73.