Construction of Refractive-Index Profiles of Planar Dielectric Waveguides from the Distribution of Effective Indexes

KIN SENG CHIANG

Abstract—**From a set of effective indexes, a continuous effective-index function is defined, which is then used to construct a refractive-index profile by numerically solving a WKB equation. The method applies to synthesis of profiles from prescribed propagation constants. Profiles for spatial image transmission are presented as an example. The method can also be applied to recovery of smooth profiles from measured effective indexes. Effects on the recovery of profiles due to errors in measured effective indexes are demonstrated and procedures are given to alleviate those effects.**

I. Introduction

It is USEFUL for the design of waveguides to know the exact refractive-index profiles from prescribed propagation constants that characterize the propagation properties of the waveguides. Yukon and Bendow [1] have applied inverse-scattering theory to model refractive-index profiles from an arbitrary set of propagation constants. Unfortunately, they obtained profiles of multiple peaks or, at best, of symmetrical shape which are not easily realized with present fabrication technology.

Construction of refractive-index profile from the distribution of effective indexes (normalized propagation constants) is not only important for waveguide prescriptions but also useful for the evaluation of existing waveguides, if the profile of the waveguide can be accurately recovered from the measured effective indexes. Tien *et al.* [2] recovered the profile of a solid-solution waveguide by setting up simultaneous equations for different modes based on the WKB integral, and evaluating the locations of turning points at which the refractive indexes are equal to the corresponding effective indexes. Vassell [3] presented a direct method bsaed on the evaluation of the inverted WKB integral. Approximating the profile by straight line segments, White and Heidrich [4] and, more recently, Čtyroký *et al.* [5] constructed the profile by computing the locations of turning points recursively. Their method, however, is accurate only for highly multimode waveguides as the number of straight line segments is equal to the number of guided modes.

In this paper, a different idea is described for the construction of refractive-index profiles from the distribution of effective indexes. A continuous effective-index function is first formed from the given set of discrete effective indexes. The peak refractive index is then obtained from the newly formed effective-index function. With the knowledge of the peak refractive index, the refractive-index profile can be constructed from the effective-index function by using an efficient recursive formula. The construction of effective-index function is the key of the method that makes possible the construction of smooth profiles based on a small number of modes. As we shall see later, interpolation technique is the simplest way to construct the effective-index function while the least-squares method is the more practical one, especially for recovery of profiles.

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The author is with the School of Electrical Engineering & Computer Science, University of New South Wales, Kensington, New South Wales 2033, Australia.

II. Numerical Procedures

Consider a step-asymmetrical refractive-index profile which is continuous and monotonically decreasing for $x \geqq 0$ with peak index at the surface boundary $x = 0$. Profiles of this kind are typical in diffused optical waveguides. According to the WKB method, the characteristic equation for the mth-order mode is given by

$$k \int_0^{x_t(m)} [n^2(x) - N^2(m)]^{1/2}\, dx = m\pi + \phi_a + \phi_s \quad (1)$$

and

$$n(x_t(m)) = N(m) \quad (2)$$

where $n(x)$ is the refractive-index profile, $N(m) = \beta_m/k$ is the effective index of the mth-order mode with m the mode order and β_m the corresponding propagation constant, $k = 2\pi/\lambda$ is the free-space wavenumber with λ the wavelength and $x_t(m)$ is the turning point at which the refractive index is equal to the effective index as given by (2). The phase change at the surface boundary $2\phi_a$ is given by

$$2\phi_a = 2 \tan^{-1} \left\{ r_a \left[\frac{N^2(m) - n_a^2}{n_0^2 - N^2(m)} \right]^{1/2} \right\} \quad (3)$$

where n_0 is the peak index, and n_a is the refractive index of the air superstrate, i.e., $n_a = 1$ and $r_a = 1$ for TE modes and $r_a = (n_0/n_a)^2$ for TM modes. The phase change $2\phi_s$ at the turning point is given by $\pi/2$, i.e., $\phi_s = 0.25\pi$, for both TE and TM modes if the variation of refractive index at that point is sufficiently slow [6].

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Given a refractive-index profile $n(x)$, (1) uniquely defines an effective-index function $N(m)$. It happens that when m takes integer values 0, 1, 2, $\cdots$, the corresponding effective indexes represent physical guided modes. Conversely, an "admissible" effective-index function also uniquely defines a physical profile. Since an infinite number of effective-index functions can be constructed from the same set of physical effective indexes, an infinite number of profiles that provide the same set of effective indexes can be found. There exists one-to-one correspondence between refractive-index profile $n(x)$ and effective-index function $N(m)$, but the correspondence does not hold between $n(x)$ and a set of discrete effective indexes. The above observation leads to the following procedures for the construction of profiles: 1) to construct an effective-index function $N(m)$ from the prescribed effective indexes, and 2) to determine the corresponding profile $n(x)$ from the WKB equation.

A. Construction of Effective-Index Function

A natural way to construct $N(m)$ is to interpolate the discrete effective indexes $N(0)$, $N(1)$, $\cdots$, $N(\nu - 1)$ by using the Gregory–Newton interpolation formula [7], in which $N(0)$ is used as the baseline and forward finite difference is employed. $N(m)$ constructed in this way is in general a polynomial of order $\nu - 1$ where ν is the number of prescribed effective indexes. With $N(m)$, we can obtain the peak index n_0. From (2), we have $n_0 = n(0) = N(m_0)$, where $m = m_0$ can be solved by putting $x_t = 0$ into (1). With $N(m_0) = n_0$, it is obvious that $\phi_a = \pi/2$ from (3), i.e., $\phi_a + \phi_s = 0.75\pi$. $m = m_0$, must be equal to -0.75 to make the left-hand side of (1) vanish. The peak index n_0 is therefore obtained by evaluating $N(m)$ at $m = -0.75$, i.e., $n_0 = N(-0.75)$. In White and Heidrich's method, n_0 is determined by a much more complicated procedure [4].

To see the general behavior of $N(m)$, we differentiate (2) with respect to m

$$\frac{dN(m)}{dm} = \frac{dn}{dx_t} \cdot \frac{dx_t}{dm}, \qquad x > 0, \qquad m > -0.75. \tag{4}$$

We have $dn/dx_t < 0$ for $x > 0$ since $n(x)$ is monotonically decreasing for $x > 0$. For physically realizable profile, x_t must increase with mode order m, i.e., $dx_t/dm > 0$ for $m > -0.75$. We thus have

$$\frac{dN(m)}{dm} < 0, \qquad m > -0.75. \tag{5}$$

That is, $N(m)$ must be monotonically decreasing for $m > -0.75$ with $N(-0.75) = n_0$ as the absolute maximum. It can also be shown from (4) and (1) that if dN/dm is continuous, dn/dx will also be continuous. $N(m)$ so constructed may not satisfy inequality (5) due to bad distribution of the given effective indexes and/or due to the way we construct $N(m)$. Unfortunately, there are no general rules available that can predict which types of effective-index spectra yield physical profiles and which do not. Inequality (5) is, nevertheless, a necessary condition (not sufficient) for $N(m)$ to yield physical profiles.

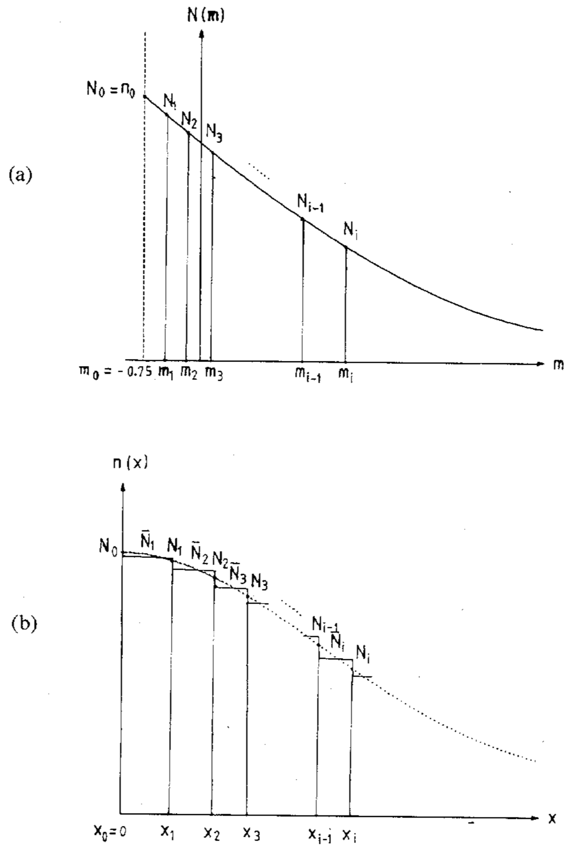

Fig. 1. (a) The effective-index function $N(m)$ is sampled into N_i's with $N_0 = N(-0.75) = n_0$. (b) Step-wise approximation is used to construct the refractive-index profile by computing x_i's at which $n(x_i) = N_i$.

B. Construction of Refractive-Index Profile

In principle, once $N(m)$ is defined, the corresponding $n(x)$ is also defined from (1). However, it is very difficult to determine $n(x)$ by inverting the WKB integral since the right-hand side of (1) is also a function of $N(m)$ (i.e., $\phi_a(N(m))$—unless ϕ_a is approximated by $\pi/2$ as in [2]–[5]). Instead of looking for analytic expression for $n(x)$, we construct $n(x)$ point-wise by using a very simple algorithm as described below.

We first sample $N(m)$ in descending order $N_0 > N_1 > N_2 > \cdots$ (not necessarily equally spaced) with $N_0 = N(-0.75) = n_0$ as shown in Fig. 1(a). Each sample N_i corresponds to a turning point x_i as shown in Fig. 1(b). When x_i's are computed, the refractive-index profile is obtained. To determine x_i's we use step-wise approximation to the profile. This is valid provided that x_i and x_{i+1} are sufficiently close to each other. The index of step i, $\bar{N}_i$ is given by the average value of N_i and N_{i-1}, i.e., $\bar{N}_i = (N_i + N_{i-1})/2$, for $i = 1, 2, 3, \cdots$ as shown in Fig. 1(b). The WKB integral can then be replaced by a finite sum. Equation (1) can be written as

$$I = k \int_0^{x_i} (n^2 - N_i^2)^{1/2}\, dx = \alpha_i \tag{6}$$

with

$$I \cong k[(\bar{N}_1^2 - N_i^2)^{1/2}(x_1 - x_0) + (\bar{N}_2^2 - N_i^2)^{1/2}(x_2 - x_1) + \cdots + (\bar{N}_i^2 - N_i^2)^{1/2}(x_i - x_{i-1})]$$
$$= k\left\{x_i(\bar{N}_i^2 - N_i^2)^{1/2} + \sum_{j=1}^{i-1} x_j[(\bar{N}_j^2 - N_i^2)^{1/2} - (\bar{N}_{j+1}^2 - N_i^2)^{1/2}]\right\}, \qquad x_0 = 0$$

and

$$\alpha_i = m_i\pi + \phi_a(N(m_i)) + 0.25\pi, \qquad \text{for} \quad i = 1, 2, 3, \cdots. \tag{7}$$

Equation (6) actually gives an algorithm that allows x_i's to be computed in a recursive way. Rearrange (6) and have

$$x_i = \frac{\alpha_i - \sum_{j=1}^{i-1} \{x_j[(\bar{N}_j^2 - N_i^2)^{1/2} - (\bar{N}_{j+1}^2 - N_i^2)^{1/2}]\}}{k(\bar{N}_i^2 - N_i^2)^{1/2}}, \qquad \text{for} \quad i = 1, 2, 3, \cdots. \tag{8}$$

Equation (8) is similar in form to those recursive formulas given in [4] and [5] but the idea behind it is different. The refractive indexes that appear in (8) are not the measured (or prescribed) effective indexes; instead they are obtained from the effective-index function. The profile constructed by (8) can be as smooth as desired depending on the number of samples chosen, while the profile constructed in [4] and [5] is bound to be piecewise linear, whose smoothness depends on the number of modes available.

III. Other Types of Profiles

It is very easy to extend the method to obtain symmetrical pofiles. The WKB method gives the following characteristic equation:

$$k\int_{-x_t(m)}^{x_t(m)} [n^2(x) - N^2(m)]^{1/2}\,dx = m\pi + 0.5\pi. \tag{9}$$

The phase changes at both sides of the profile are equal to $\pi/2$ under WKB approximation, i.e., $\phi_a + \phi_s = 0.5\pi$. The left-hand side of (9) vanishes at $m = -0.5$. The peak index n_0 is thus given by $N(-0.5)$. All the discussions in the previous sections apply here with trivial modifications. The x_i calculated from (8) should be divided by two and α_i in (8) should be replaced by $(m_i + 0.5)\pi$.

We can also seek asymmetrical profiles by breaking up (9) into two parts with the introduction of a skew factor s, where $0 \leq s < 1$

$$k\int_0^{x_t(m)} [n^2(x) - N^2(m)]^{1/2}\,dx = 0.5(1 + s)(m\pi + 0.5\pi) \tag{10a}$$

and

$$k\int_{-x_t(m)}^{0} [n^2(x) - N^2(m)]^{1/2}\,dx = 0.5(1 - s)(m\pi + 0.5\pi). \tag{10b}$$

Expressing the characteristic equation in the above forms suggests that, applying the algorithm (8) twice, we can compute the two halves of the profile ($x > 0$ and $x < 0$) independently based on the same effective-index function. For $s = 0$, we have a symmetrical profile. When s takes values between 0 and 1, we obtain asymmetrical profiles that are of shapes between the symmetrical one and the step asymmetrical one. It should be noted that if s is very close to 1, (10a) and (10b) will no longer be accurate because the variation of the profile at one side (here $x < 0$) will be so large that the phase changes at turning points will no longer be $\pi/2$ as presumed. This is also manifested by the fact that with $s = 1$ in (10a) and (10b), the accurate equation (1) is not recovered. We therefore should not take s too close to 1 for synthesis of asymmetrical profies.

IV. Profiles for Spatial Image Transmission

As an illustration of the method, we model a few profiles in which the rays, and hence the image, will be focused at multiples of length L. The propagation constants must satisfy

$$\beta_i - \beta_j = (i - j)2\pi/L$$

i.e.,

$$N(i) - N(j) = (i - j)\lambda/L, \qquad i, j = 0, 1, 2, \cdots.$$

The difference between the effective indexes of two adjacent modes is equal to the constant λ/L. To be specific, consider a profile that supports five TE modes with the following effective indexes:

TE_0:	1.6100
TE_1:	1.5900
TE_2:	1.5700
TE_3:	1.5500
TE_4:	1.5300.

Following the numerical procedures described earlier, we constructed a few profiles that supported the given set of effective indexes at $\lambda = 632.8$ nm ($L = 31.64$ μm) as shown in Fig. 2. Fig. 2(a) shows the step asymmetrical profile that can be closely matched by a second-order polynomial. In Fig. 2(b), we have a symmetrical profile that fits remarkably the theoretical hyperbolic secant profile [9]. Asymmetrical profiles corresponding to skew factors 0.1, 0.2, and 0.4 are shown in Fig. 2(c).

Finally, we should remark that as far as synthesis of profiles is concerned, it is only allowable to prescribe the effective indexes for either TE or TM modes.

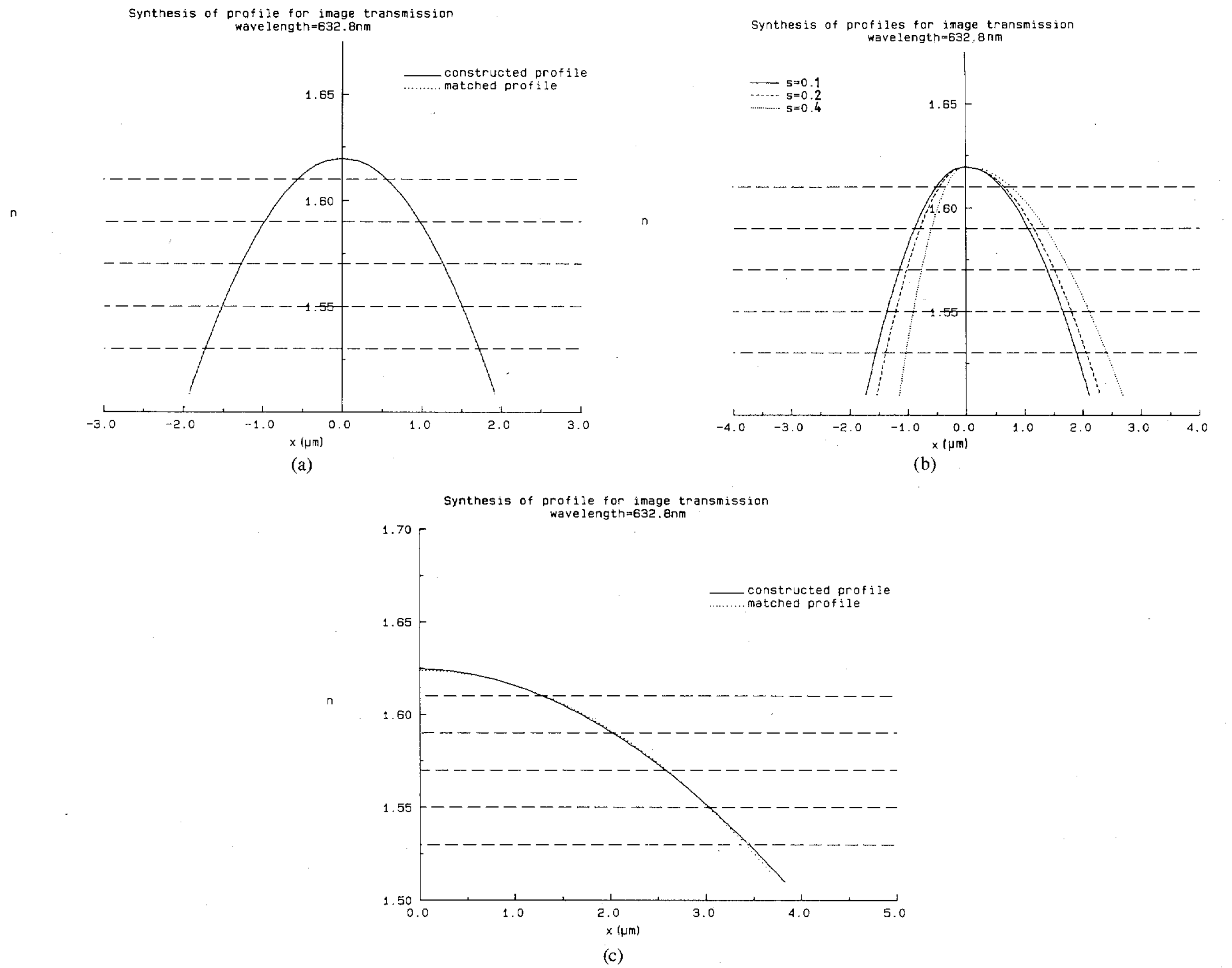

Fig. 2. Profiles for image transmission: (a) step-asymmetrical profile matched closely by $n(x) = 1.624 - 0.008057x^2$; (b) symmetrical profile matched perfectly by the theoretical profile $n(x) = 1.62 \operatorname{sech}(0.1986x)$; (c) asymmetrical profiles corresponding to $s = 0.1$, 0.2, and 0.4.

V. Recovery of Refractive-Index Profiles

It seems natural to apply the method to the evaluation of existing waveguides since the required effective indexes can be accurately obtained from measurements, for example, by use of a prism coupler [10]. However, two questions must be answered before the method can be used practically. How accurately does the recovered profile corresponding to the effective-index function constructed as described previously match the real profile? How significantly will the recovered profile be affected by errors in measured effective indexes?

To answer the first question, we tried four different step asymmetrical profiles: Gaussian, second-order polynomial, triangular, and exponential profiles, whose effective indexes for TE modes were accurately calculated by a finite-element program developed by the author. It was found that the recovered triangular and exponential profiles were acceptable while the recovered Gaussian and second-order polynomial profiles were excellent. The recovered Gaussian and triangular profiles as well as the exact ones are shown in Fig. 3(a) and (b), where the horizontal dashed lines mark the exact effective indexes from which the effective-index functions are formed. A few observations can be made from the study: 1) in all cases, the recovered profiles fit the exact ones very well in the range between the first and last effective indexes; 2) for those "round-top" profiles, like Gaussian and second-order polynomial functions, the estimated peak indexes can be very accurate (around 1 percent in the examples—all percentage errors mentioned in this paper are relative to the index difference $n(0) - n(\infty)$); 3) if the substrate is of the type $n(x) = n_s$ for $x_s \geq x$ as shown in Fig. 3(b), i.e., dn/dx is discontinuous at $x = x_s$, the WKB formula given by (1) will no longer hold near x_s. The mode near cutoff should not therefore be used to construct the effective-index function, otherwise the overall accuracy of the recovered profile will be degraded, especially in the re-

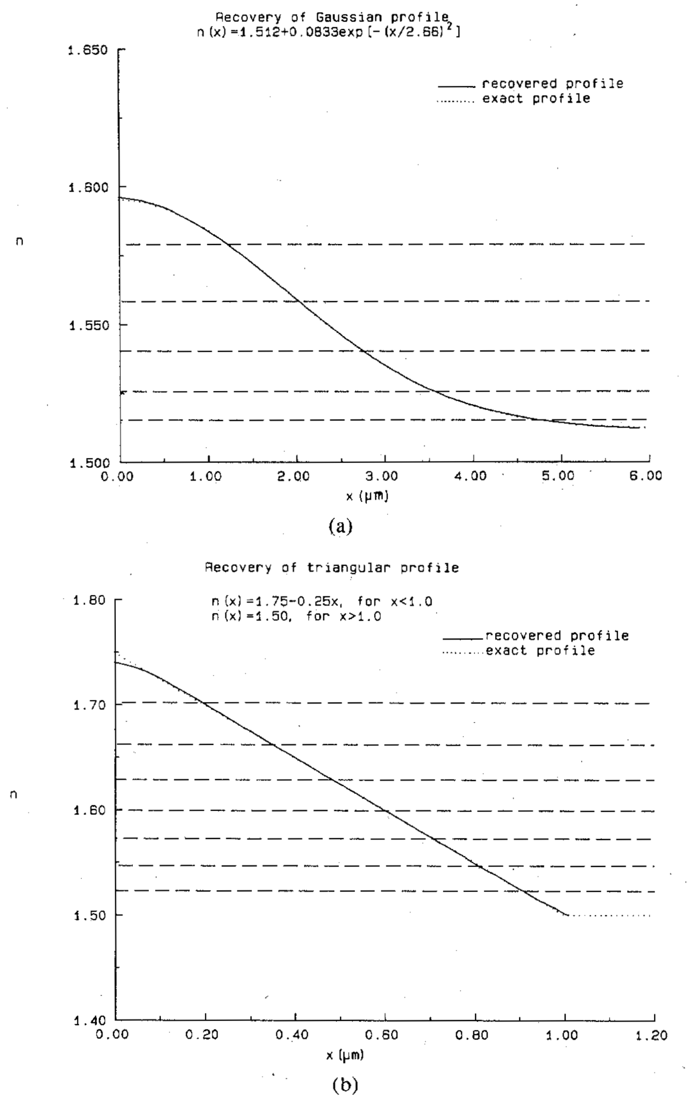

Fig. 3. Recovery of profiles based on effective-index functions constructed by interpolating exact effective indexes: (a) Gaussian profile; (b) triangular profile.

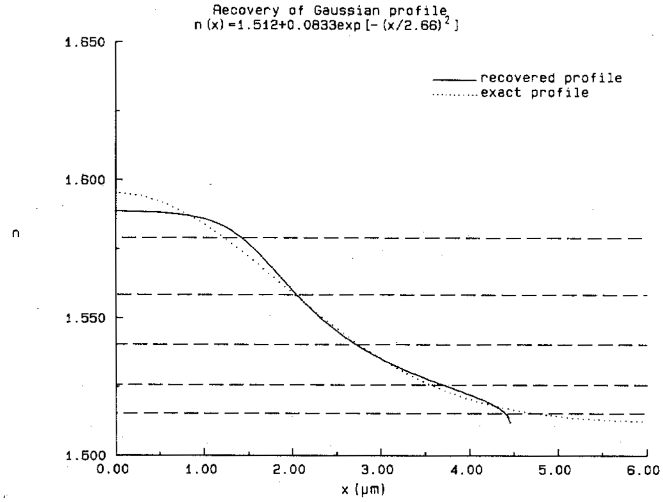

Fig. 4. Recovery of the Gaussian profile with -0.5 percent errors in the effective indexes for the even-order modes and $+0.5$ percent errors for the odd-order modes based on a fourth-order effective-index polynomial constructed by interpolating all the effective indexes.

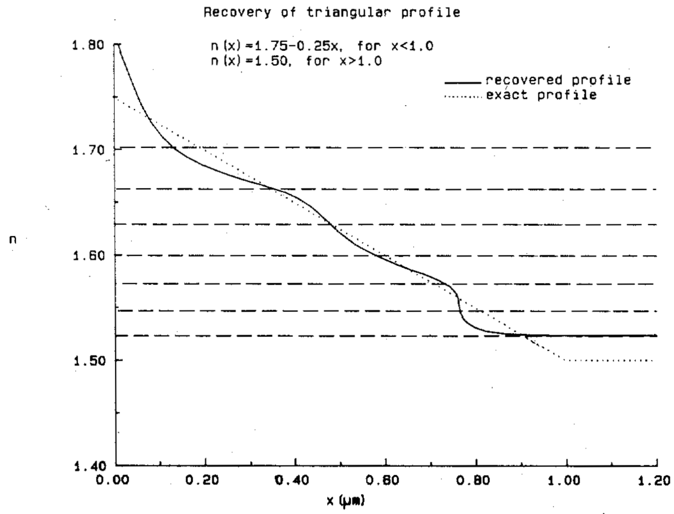

Fig. 5. Recovery of the triangular profile with $+0.5$ percent errors in the effective indexes of the even-order modes and -0.5 percent errors for the odd-order modes based on a sixth-order effective-index polynomial constructed by interpolating all the effective indexes.

gion near x_s. In obtaining the triangular profile in Fig. 3(b), the highest-order mode, which is very close to cutoff, was discarded. From the results of the studied examples, we can then conclude that the profiles constructed by the method do match the exact profiles of various kinds with good accuracy.

To study how sensitive the recovered profile is to errors in measured effective indexes, we artificially introduced "errors" to the exact effective indexes and then reconstructed the profile. It is obvious that if the errors in all effective indexes are of same sign and more or less of same amount, the recovered profile will preserve the shape of the real profile but shift up or down by roughly the same amount and still be acceptable. However, if the errors are of alternate signs, the effective-index function formed by interpolating the data may be significantly "twisted" and the oscillating nature of the errors may be largely amplified in the recovered profile. As demonstrations, we introduced -0.5 percent errors to the effective indexes for the even-order modes and $+0.5$ percent errors to those for the odd-order modes for the Gaussian profile, and in the next case, $+0.5$ percent for the even-order modes and -0.5 percent for the odd-order modes for the triangular profile. The recovered profiles are shown in Figs. 4 and 5, respectively, from which it is clear that the small oscillating errors in the measured effective indexes have very detrimental effects on the recovery of profiles.

From the above observations, it is clear that the interpolating effective-index function is extremely sensitive to oscillating errors in measured effective indexes. To alleviate the effects due to this kind of error, polynomials of order lower than that of the interpolating polynomial should be used as effective-index functions. This consideration immediately calls for the application of least-squares method to fit the mesured effective indexes [8]. That the least-squares polynomials do not in general fit all data can be justified since all data are subject to measurement errors and hence should not be fit exactly. (The interpolating polynomial that fits all data can be considered as a special least-squares polynomial.) It is, however, somewhat arbitrary to choose the order of polynomial to be used for the fitting of data. The deviation $d(i)$ defined as

$$d(i) = |\hat{N}(i) - N(i)|, \qquad i = 0, 1, 2, \cdots, \nu - 1$$

can be used as a criterion, where $N(i)$ is the measured effective index for the ith-order mode, $\hat{N}(i)$ is the corresponding value from the best-fit polynomial, and ν is the number of modes. In practive, we can use those polynomials (of order lower than that of the interpolating polynomial) with maximum $d(i)$ less than the accuracy of the measurement to construct profiles. Reliable result can usually be obtained by comparisons among those recovered profiles. Other criteria such as $\Sigma_i d^2(i)$ can also be used for the choice of the degree of polynomial. As an example, we recover the Gaussian profile with alternate errors in mesaured refractive indexes as described earlier, using the best-fit third-order, second-order, and first-order polynomials whose maximum $d(i)$'s are 0.4, 0.5, and 4 percent respectively. The profiles recovered from the third-order and second-order polynomials are both valid under our criterion since their maximum $d(i)$'s are within 0.5 percent (as set deliberately), and indeed both of them fit the exact profile remarkably. In fact, they are nearly as good as the one as shown in Fig. 3(a). The profile recovered from the first-order polynomial, which does not satisfy our criterion, does not resemble the exact profile and must be discarded. The least-squares method was extensively tested for the recovery of different profiles with various kinds of error in effective indexes and gave reliable results in all tested cases.

All discussions in this section apply to the recovery of symmetrical profiles. Profiles of asymmetrical type cannot be recovered because there is no way to predict the skew factor.

VI. Experimental Examples on Profile Recovery

Consider a Cu-diffused MgO film waveguide fabricated by Chung *et al.* [11]. The effecctive indexes for TE modes at five different wavelengths are given in Table I. Assuming that the profiles were Gaussian, Chung *et al.* [11] worked out the parameters of the profiles in a statistical way. We first employed the method based on interpolating effective-index functions to recover the profiles of the waveguide at the five wavelengths. The recovered profiles at $\lambda = 488.0$ nm, 496.5 nm, and 632.8 nm did appear very Gaussian, while the one at $\lambda = 0476.5$ nm exhibited fluctuations. We failed to recover the profile at $\lambda = 514.5$ nm because the interpolating effective-index function was not monotonically decreasing near $m = -0.75$ as required. Employing the least-squares fitting of measured effective indexes as described in the previous section, we recovered the profiles at different wavelengths based on effective-index polynomials of different orders. The accuracy of the measured effective indexes was not known but we observed that the maximum $d(i)$ of the best-fit first-order polynomial was significantly and intolerably larger than those of higher-order polynomials. In fact, it made little difference in all cases whether we used a second-order or third-order polynomial (or ever higher-order ones as long as the order was lower than that of the interpolating polynomial) as the effective-index function since the recovered profiles were nearly indistinguishable. A set of results is shown in Fig. 6, where the dotted lines represent the Gaussian functions based on the average values of the profile parameters given by [11] and the numbers in parentheses give the orders of the effective-index polynomials. The recovered profiles at the five wavelengths are very Gaussian and consistent with the results given by [11].

TABLE I
Measured Effective Indexes for TE Modes for a Cu-Diffused MgO Waveguide at Five Different Wavelengths

λ(nm) / m	476.5	488.0	496.5	514.5	632.8
0	1.75774	1.75613	1.75492	1.75266	1.74367
1	1.75587	1.75429	1.75310	1.75073	1.74113
2	1.75418	1.75251	1.75134	1.74889	1.73891
3	1.75258	1.75091	1.74976	1.74739	1.73683
4	1.75129	1.74959	1.74843	1.74606	1.73533
5	1.75016	1.74845	1.74737	1.74507	-
6	1.74925	1.74770	1.74649	1.74431	-
substrate index:	1.74800	1.74700	1.74600	1.74400	1.73500

VII. Conclusion

We have devised a very efficient and powerful method for the construction of refractive-index profiles from a set of effective indexes for guided modes (either TE or TM modes). The operation of the method relies on the appropriate construction of effective-index function. In principle, it is desirable in synthesis problems to employ interpolation technique to construct effective-index function because the corresponding profiles (from the symmetrical one to the highly asymmetrical one) are exact in the sense that they really support the prescribed effective indexes (under the accuracy of WKB approximation). In some cases, however, it may be more suitable to use the least-squares method to construct effective-index function, which may lead to smoother and hence more readily fabricated profiles by introducing some tolerable discrepancies in the prescribed effective indexes. As already demonstrated, it is necessary to employ least-squares fitting to construct effective-index function for evaluation of profiles because of the inevitable existence of errors in measured effective indexes. The present method can be accurately used for recovery of smooth profiles from a small number of measured effective indexes, but any small bumps and fluctuations in the profile can not be detected.

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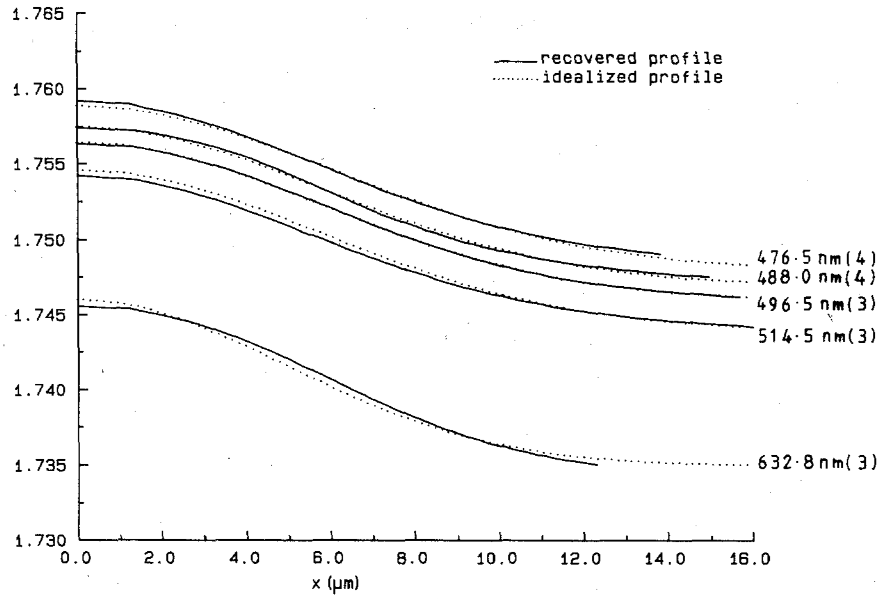

Fig. 6. The profiles of a Cu-diffused MgO wavelength at five different wavelengths are recovered based on effective-index polynomials constructed by least-squares method.

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References

[1] S. P. Yukon and B. Bendow, *J. Opt. Soc. Amer.*, vol. 70, pp. 172–179, 1980.
[2] P. K. Tein, S. Riva-Sanseverino, R. J. Martin, A. A. Ballman, and H. Brown, *Appl. Phys. Lett.*, vol. 24, pp. 503–506, 1974.
[3] M. O. Vassell, *J. Opt. Soc. Amer.*, vol. 65, pp. 1019–1021, 1975.
[4] J. W. White and P. F. Heidrich, *Appl. Opt.*, vol. 15, pp. 151–155, 1976.
[5] J. Čtyroký, J. Janta, and J. Schrofel, *Opt. Lett.*, vol. 7, pp. 552–554, 1982.
[6] R. H. Dicke and J. P. Wittke, *Introduction to Quantum Mechanics.* Reading, MA: Addison-Wesley, 1960, pp. 245–248.
[7] R. W. Hornbeck, *Numerical Methods.* Englewood Cliffs, NJ: Prentice-Hall, 1975, pp. 37–41.
[8] R. W. Hornbeck, *Numerical Methods.* Englewood Cliffs, NJ: Prentice-Hall, 1975, pp. 122–125.
[9] S. Kawakami and J. Nishizawa, *IEEE Trans. Microwave Theory Tech.*, vol. MTT-16, pp. 814–818, 1968.
[10] P. K. Tein and R. Ulrich, *J. Opt. Soc. Amer.*, vol. 60, pp. 1325–1337, 1970.
[11] P. S. Chung, S. P. Singh, and C. W. Pitt, *Microwaves, Optics and Acoustics*, vol. 2, pp. 201–203, 1978.

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Kin Seng Chiang was born in Canton, China, in 1957. He received the B.E. degree in electrical engineering from the University of New South Wales, Sydney, Australia, in 1982. He is currently a Ph.D. student at the same university with interest in numerical analysis of optical fibers and integrated optical waveguides.

Mr. Chiang is a member of the IREE Electro-optics Group, Australia.