

# **Partial Persistent Data-structure**

## **Pointer Machine Model**

**PROJECT MEMBERS:** ANANNYO DEY, SOUMYAJIT RUDRA SARMA , DEBASMIT ROY, KANKO GHOSH AND KUSHAL DAS

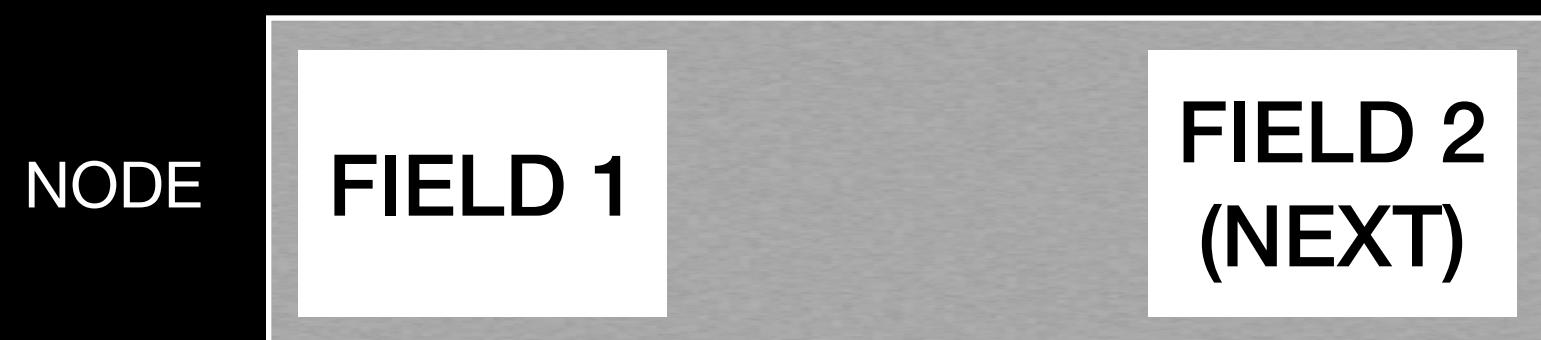
# Ephemeral Linked List

## Operations:

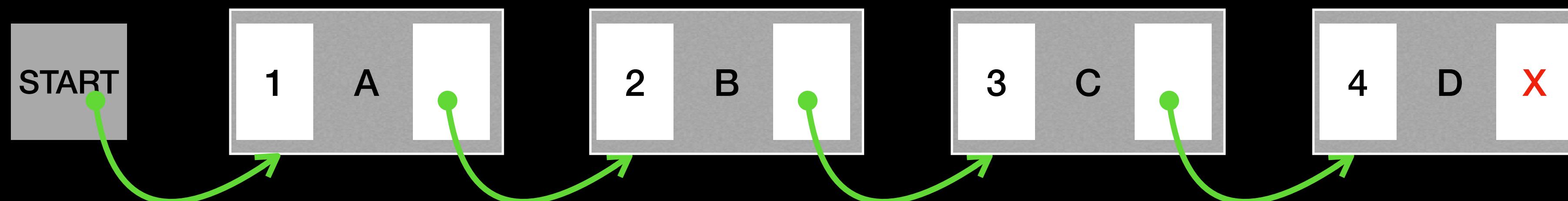
- `start = init()` : To initiate linked list and “start” pointer holds the starting position
- `add(x, y)` : Add new node x after y
- `remove(x)` : Remove node x
- `iterate_over_LL()` : Iterate over the whole linked list
- `update(f_i,x,val)` : Update the i-th field in node x to new value ‘val’

# Ephemeral Linkedlist

## Node structure



LINKED LIST

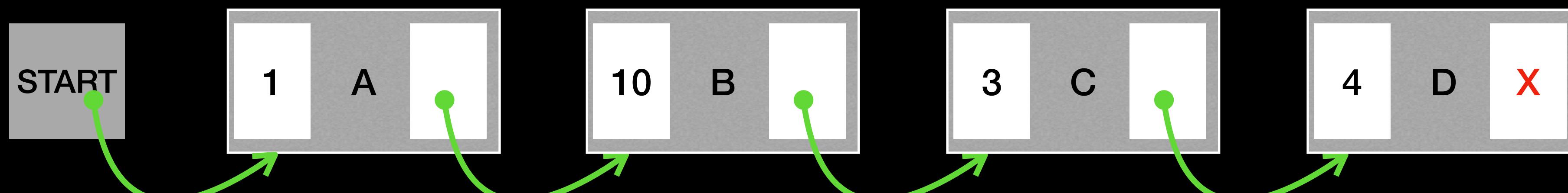


HERE IN-DEGREE OF A NODE IS  $\leq 1$  AND OUT-DEGREE OF A NODE IS  $\leq 1$

# SOME OPERATIONS

update(f1,B,10)

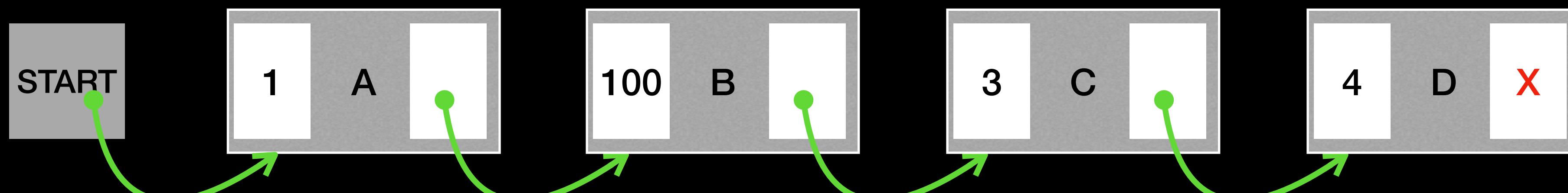
LINKED LIST



# SOME OPERATIONS

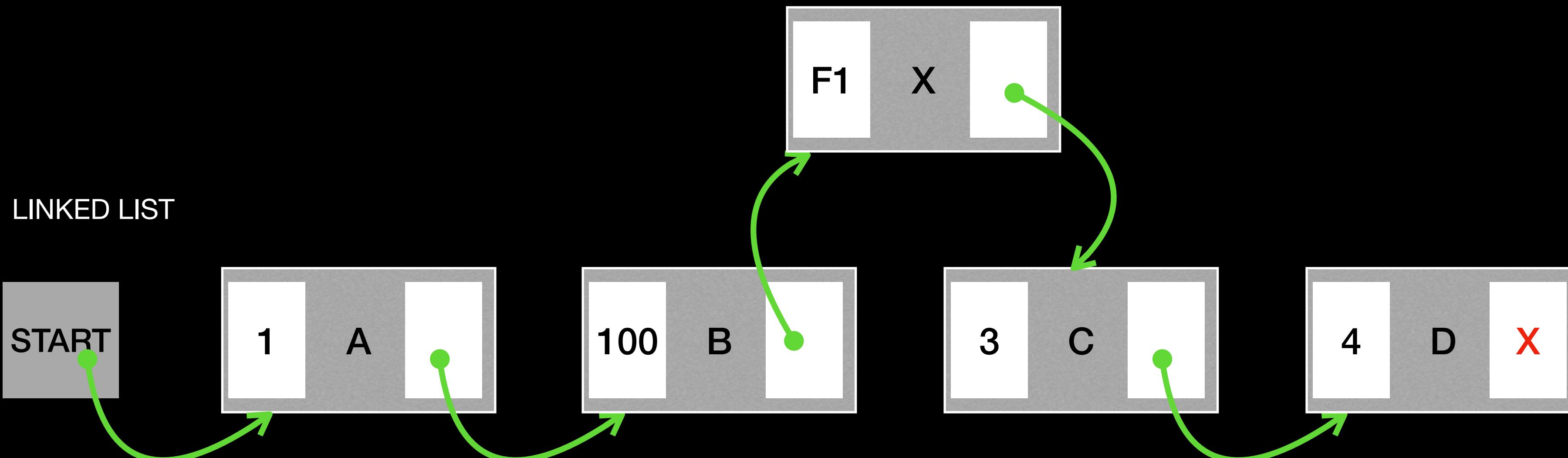
update(f1,B,100)

LINKED LIST



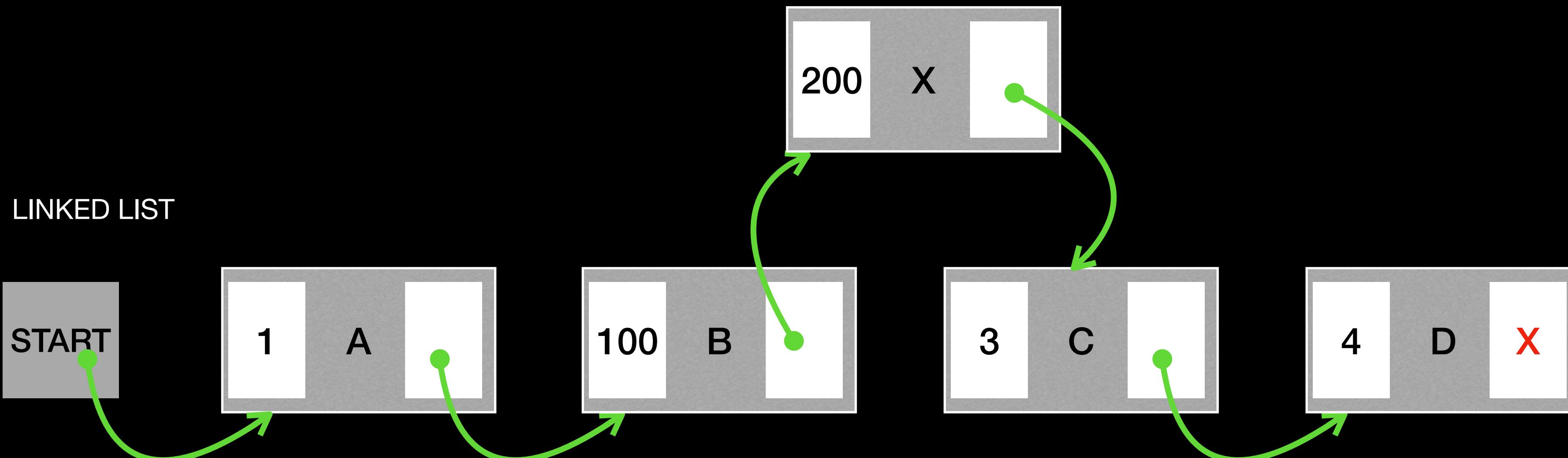
# SOME OPERATIONS

add(X,B)



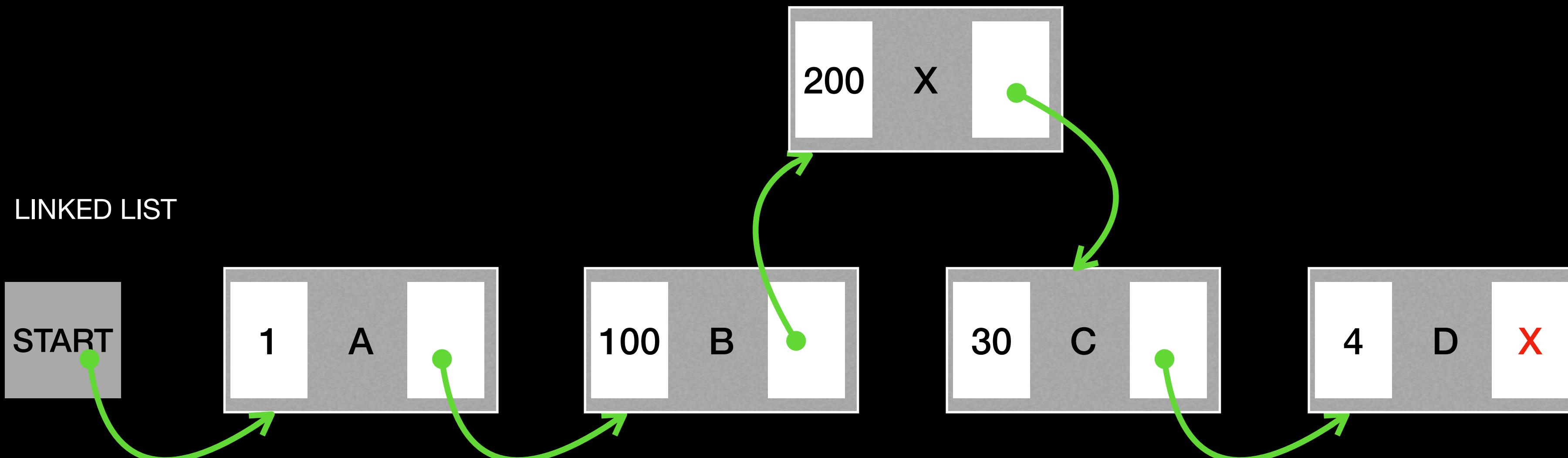
# SOME OPERATIONS

update(f1,X,200)



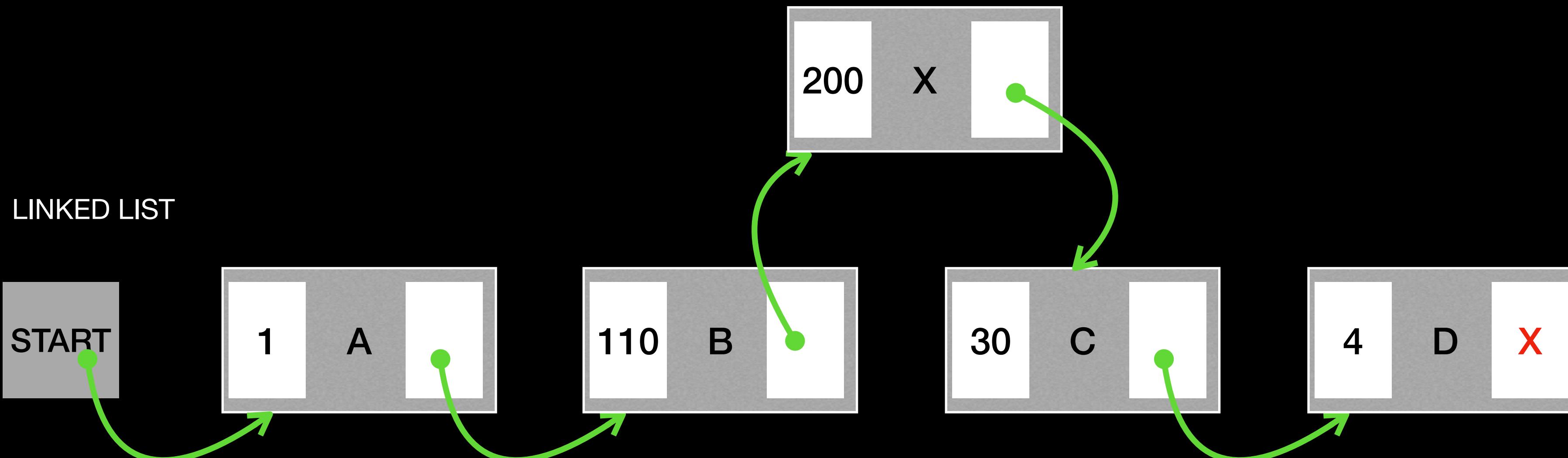
# SOME OPERATIONS

update(f1,C,30)



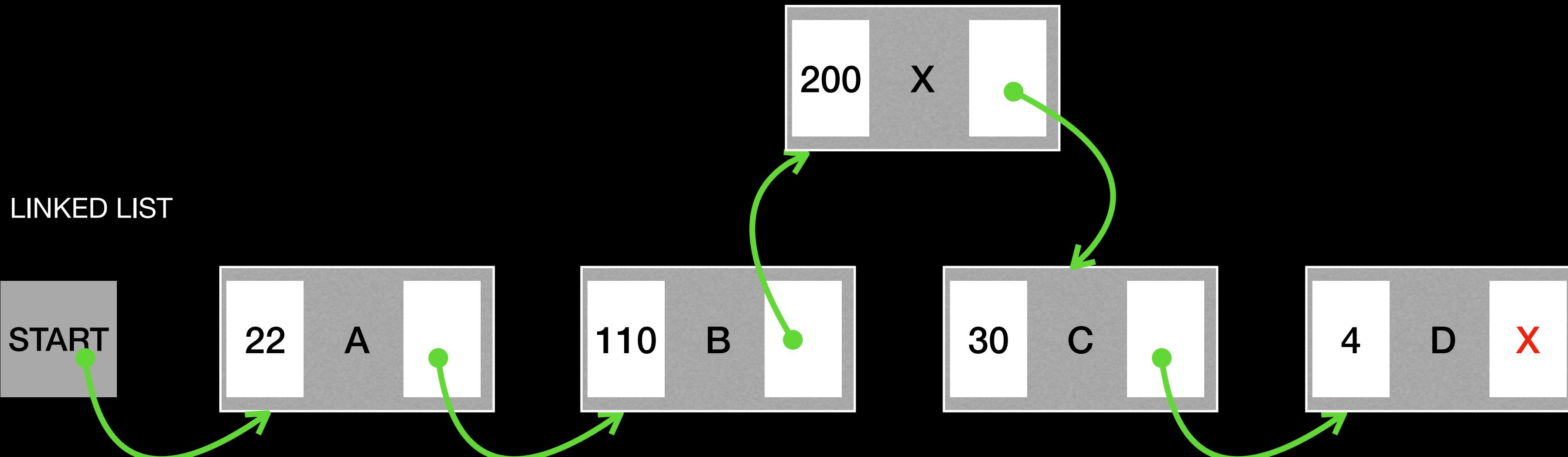
# SOME OPERATIONS

update(f1,B,110)



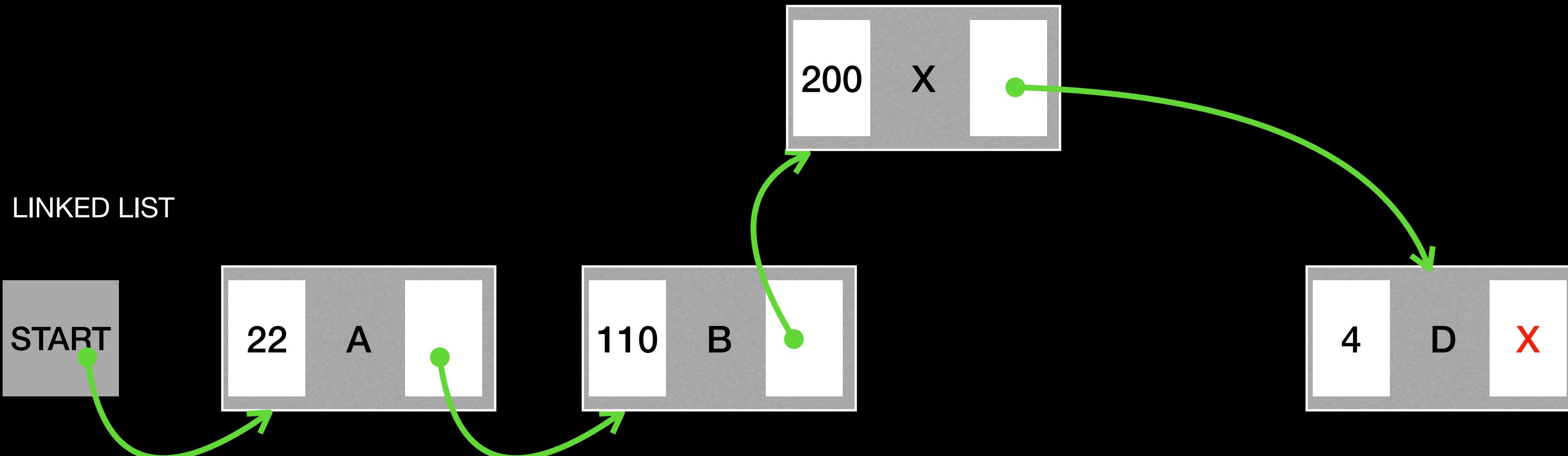
# SOME OPERATIONS

update(f1,A,22)



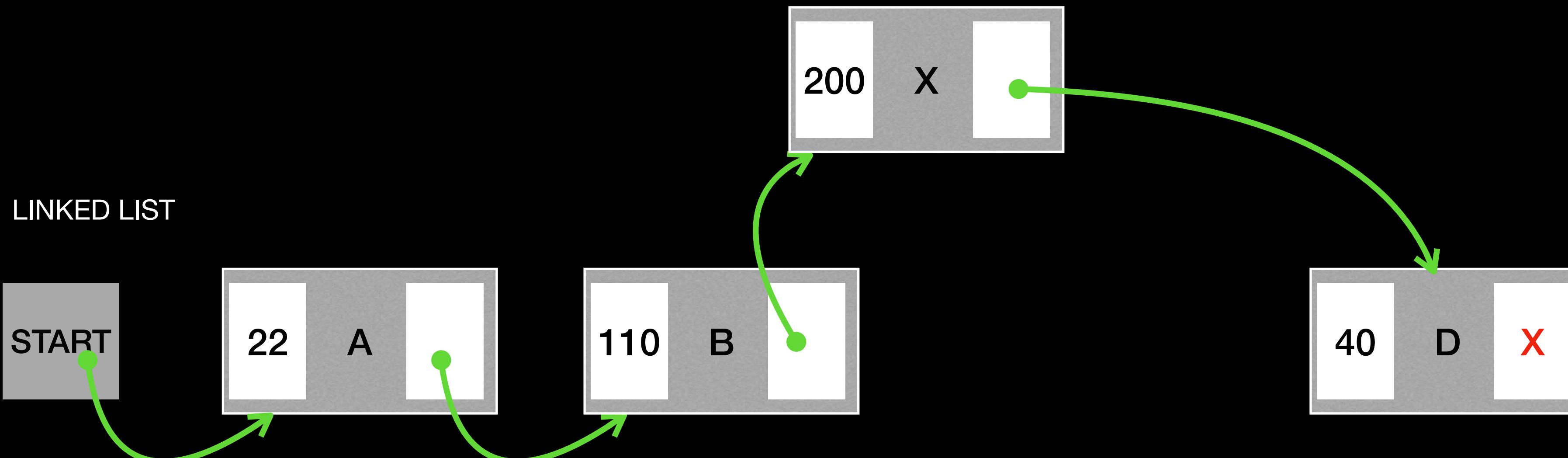
# SOME OPERATIONS

`remove(C)`



# SOME OPERATIONS

update(f1,D,40)



# Partial Persistent Linked List

## Operations:

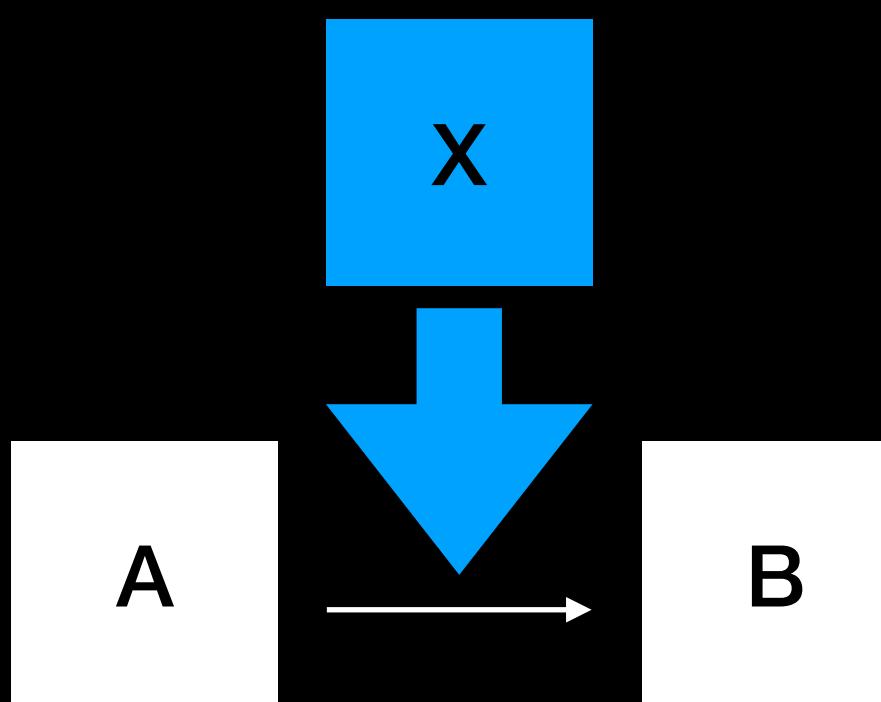
- `start = init()` : To initiate linked list and “start” pointer holds the starting position in v0
- `create_node(x, a)`: Allocate a new node x with `f1 = a` and `f2 = NULL`, and set its default version to current version
- `add(x, y, a)` : Add new node x with `value = a` after y and update the version
- `remove(x)` : Remove node x and update the version
- `iterate_over_LL(v)` : Iterate over the whole linked list in version v
- `update(f_i,x,val)` : Update the i-th field in node x to new value ‘val’ and update the version

# Interesting thing!

## add(x,y) and remove(x) are not Elementary operations

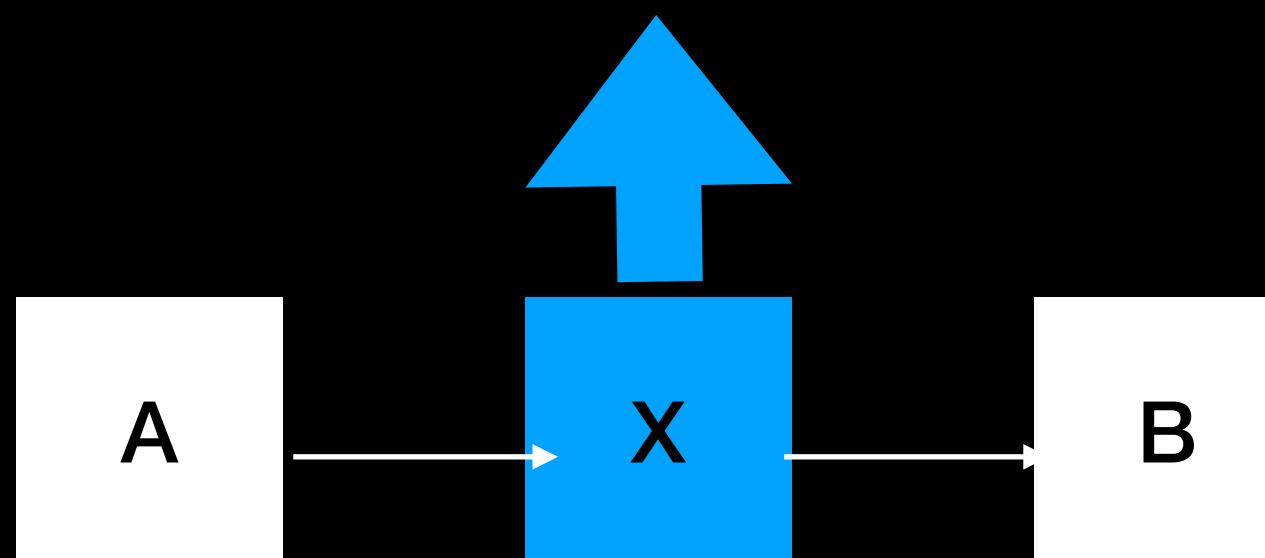
add(X,C,123) consists of

Create node X with value 123
Modify f2 of A to X : update(f2, A, X)
Modify f2 of X to C : update(f2, X, C)
Set BP of X to A: X.bp = A
Set BP of B to X: B.bp = X
Set f1 of X(optional)



remove(x) consists of

Modify F2 of Parent C (i.e., X) -> F2 of C (successor of C)
update(f2, A, B)
Set BP of B to A: B.bp = A
If all shared reference of X is removed Then free up the memory associated with X

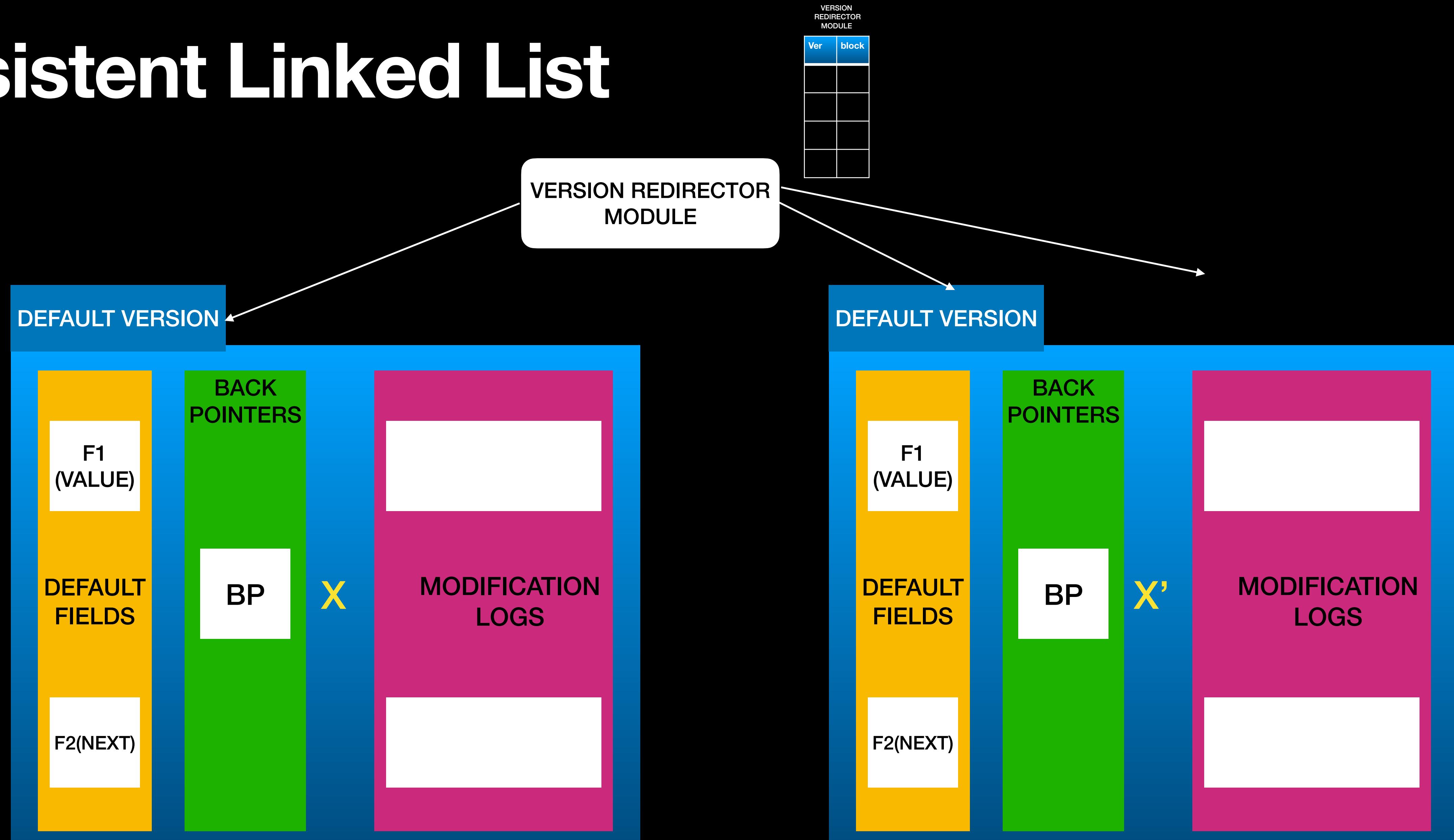


# Elementary Operations:

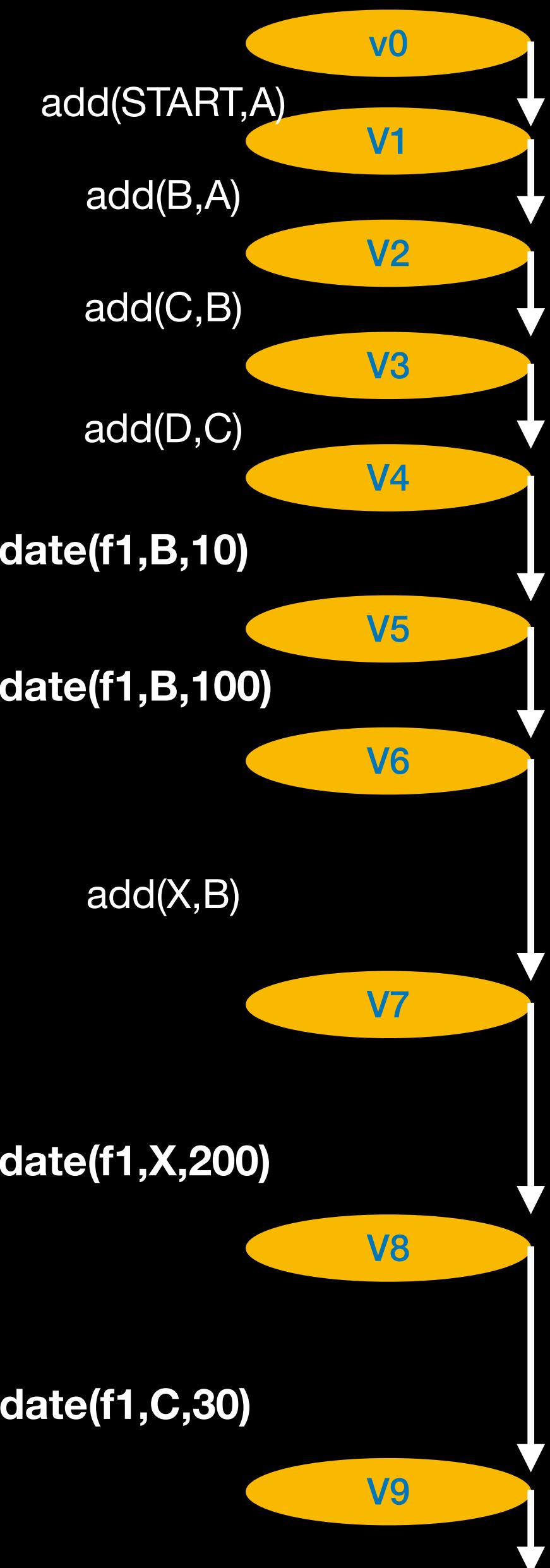
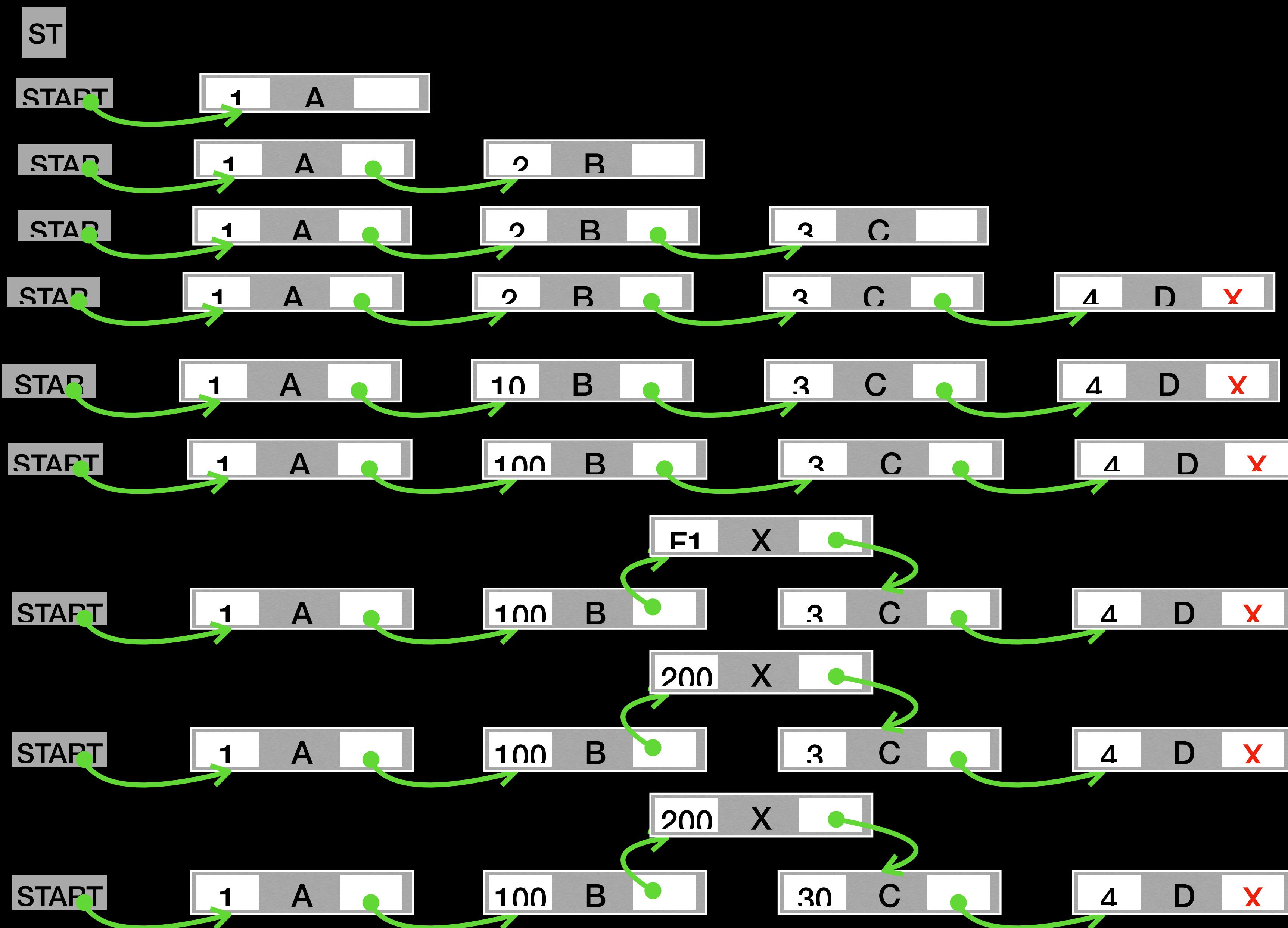
- `start = init()`
- `create_node(x, a)`
- `iterate_over_LL(v)`
- `update(f_i,x,val)`

# Persistent Linked List

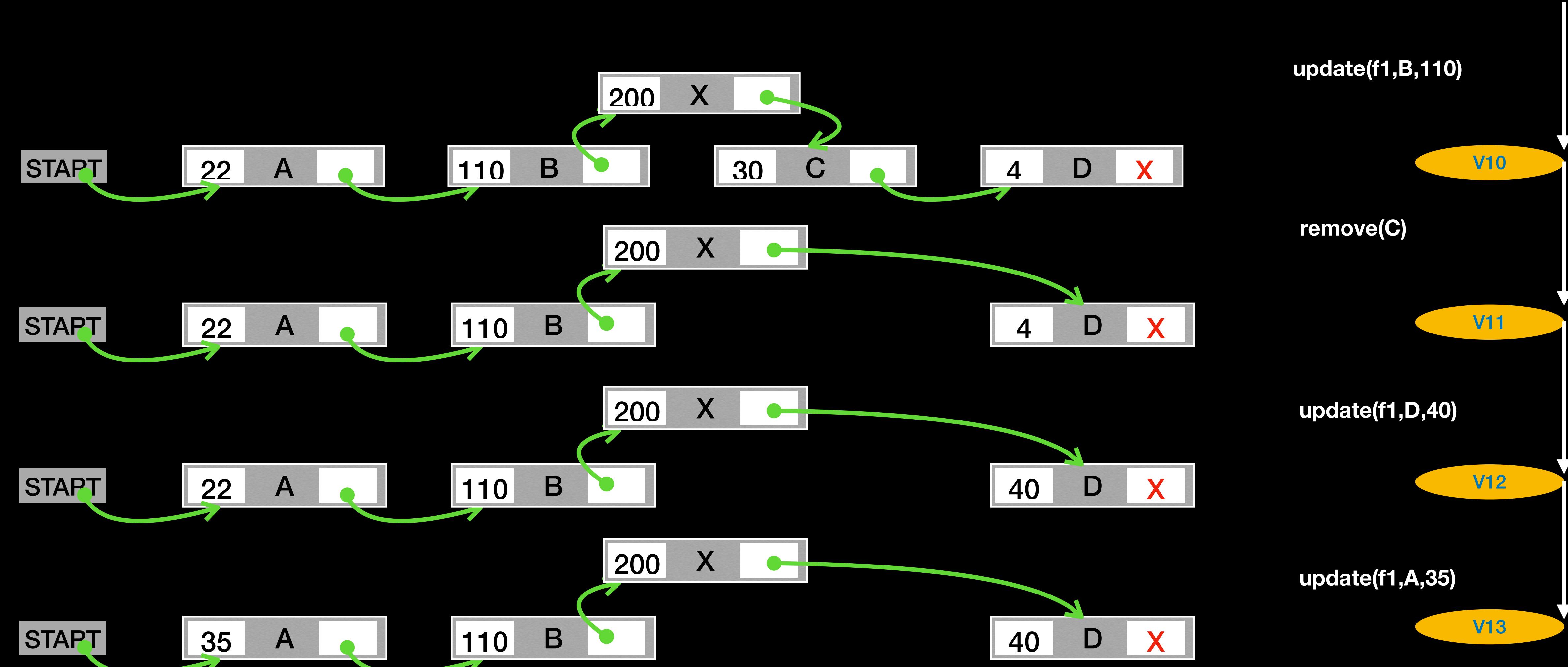
## Node



# Ephemeral List and Corresponding Versions



# Ephemeral List and Corresponding Versions (contd.)



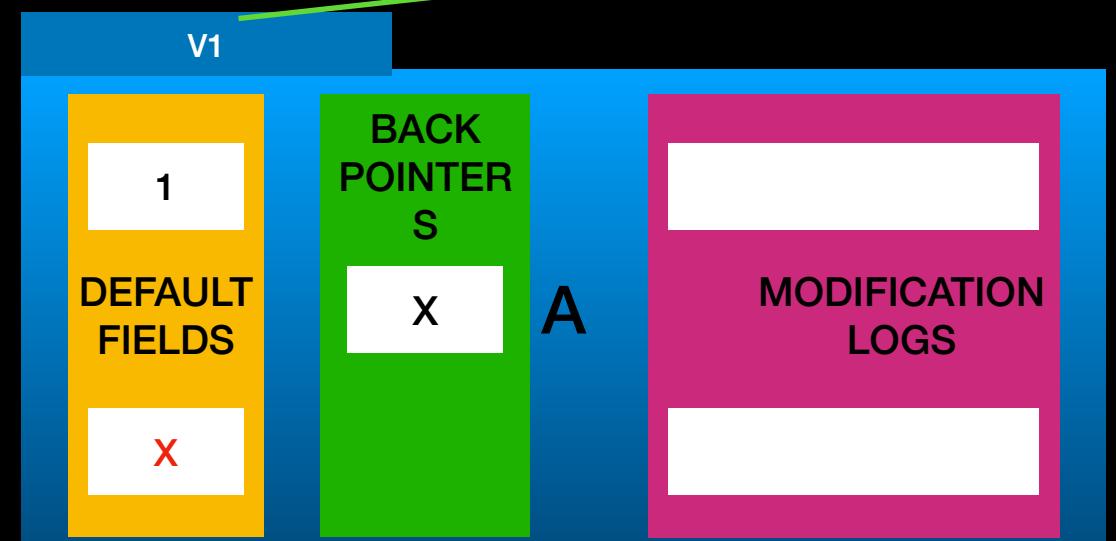
# IMPLEMENTATION IN PERSISTENT POINTER MACHINE MODEL



Current Version: v1

## START MODULE

add(A,START,1)



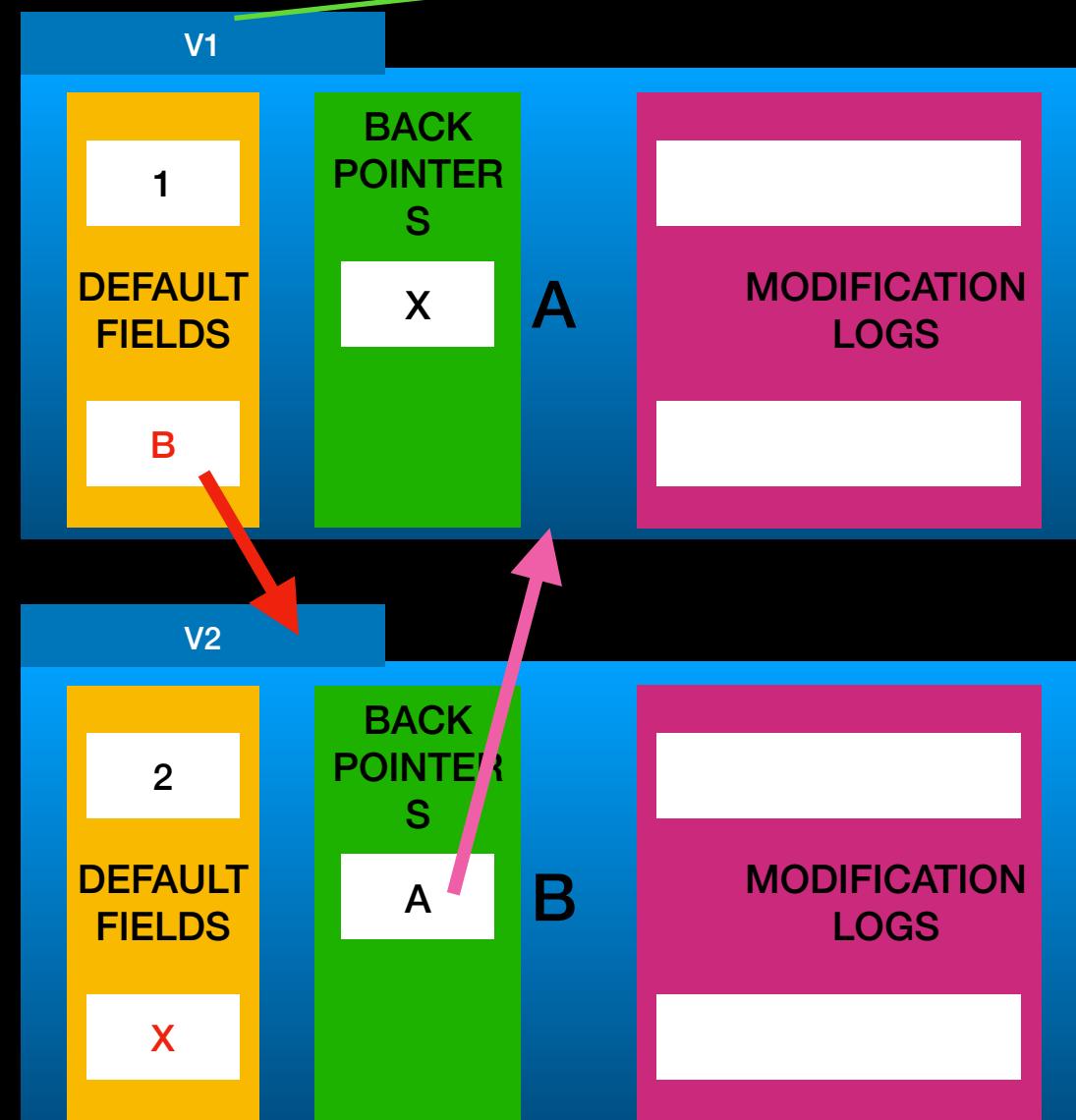
VERSION  
REDIRECTOR  
MODULE

Ver	block
1	A

Current Version: v2

# START MODULE

add(B,A,2)



VERSION  
REDIRECTOR  
MODULE

Ver	block
1	A

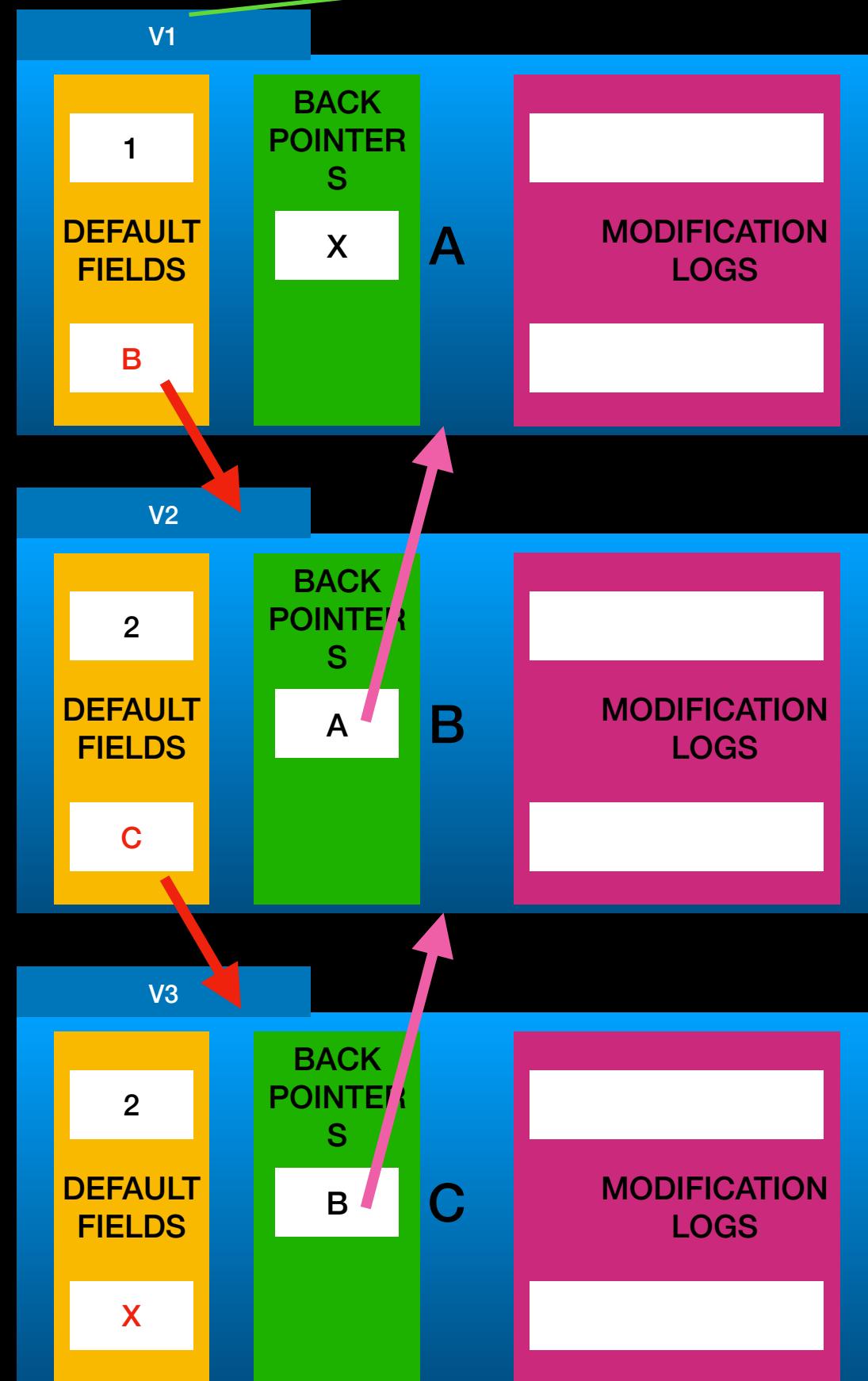
VERSION  
REDIRECTOR  
MODULE

Ver	block
2	B

Current Version: v3

# START MODULE

add(C,B,3)



v0

VERSION  
REDIRECTOR  
MODULE

Ver	block
1	A

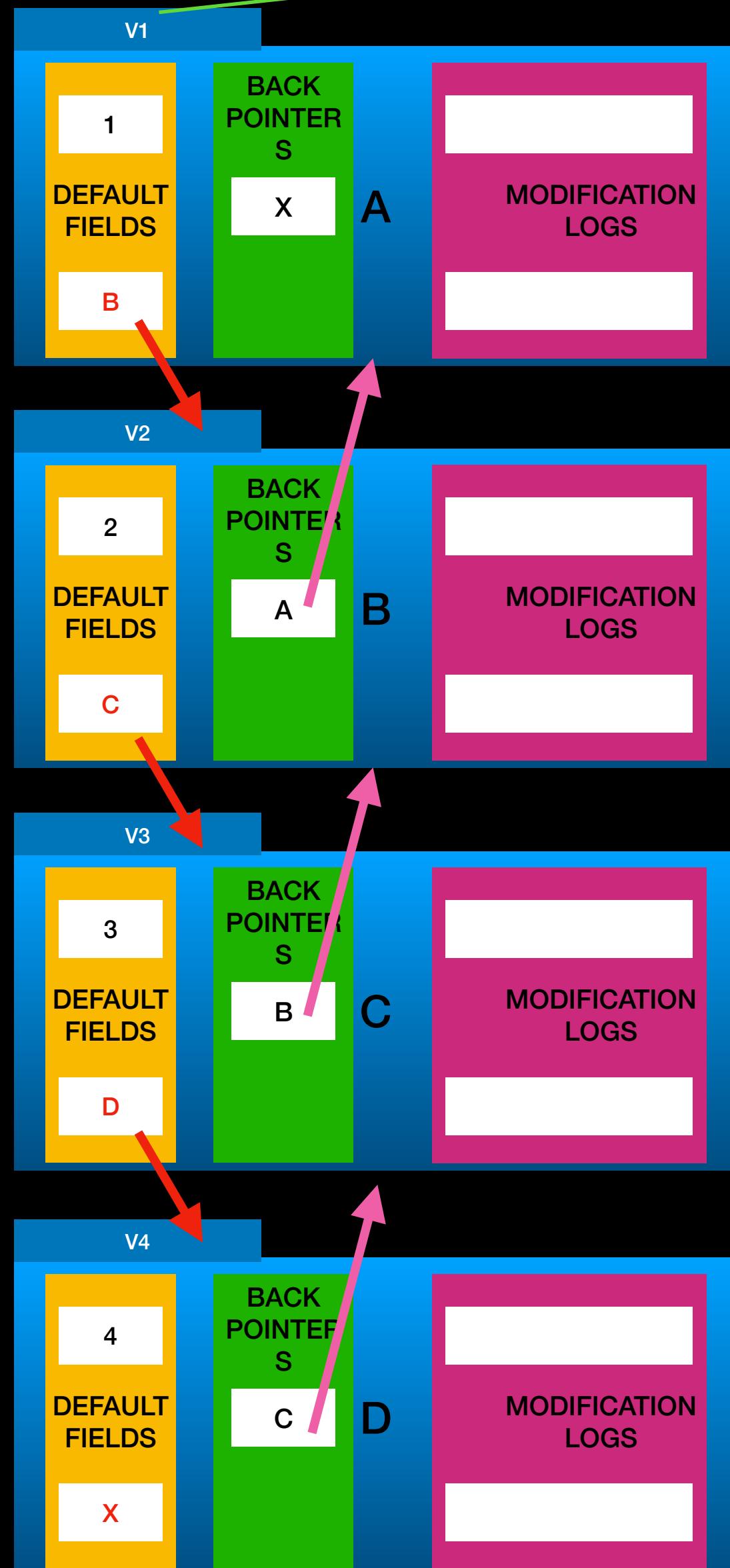
Ver	block
2	B

Ver	block
3	C

Current Version: v4

# START MODULE

add(D,C,4)



v0

VERSION  
REDIRECTOR  
MODULE

Ver	block
1	A
2	B
3	C
4	D

VERSION  
REDIRECTOR  
MODULE

Ver	block
2	R
3	C
4	X

VERSION  
REDIRECTOR  
MODULE

Ver	block
1	A
2	B
3	C
4	D

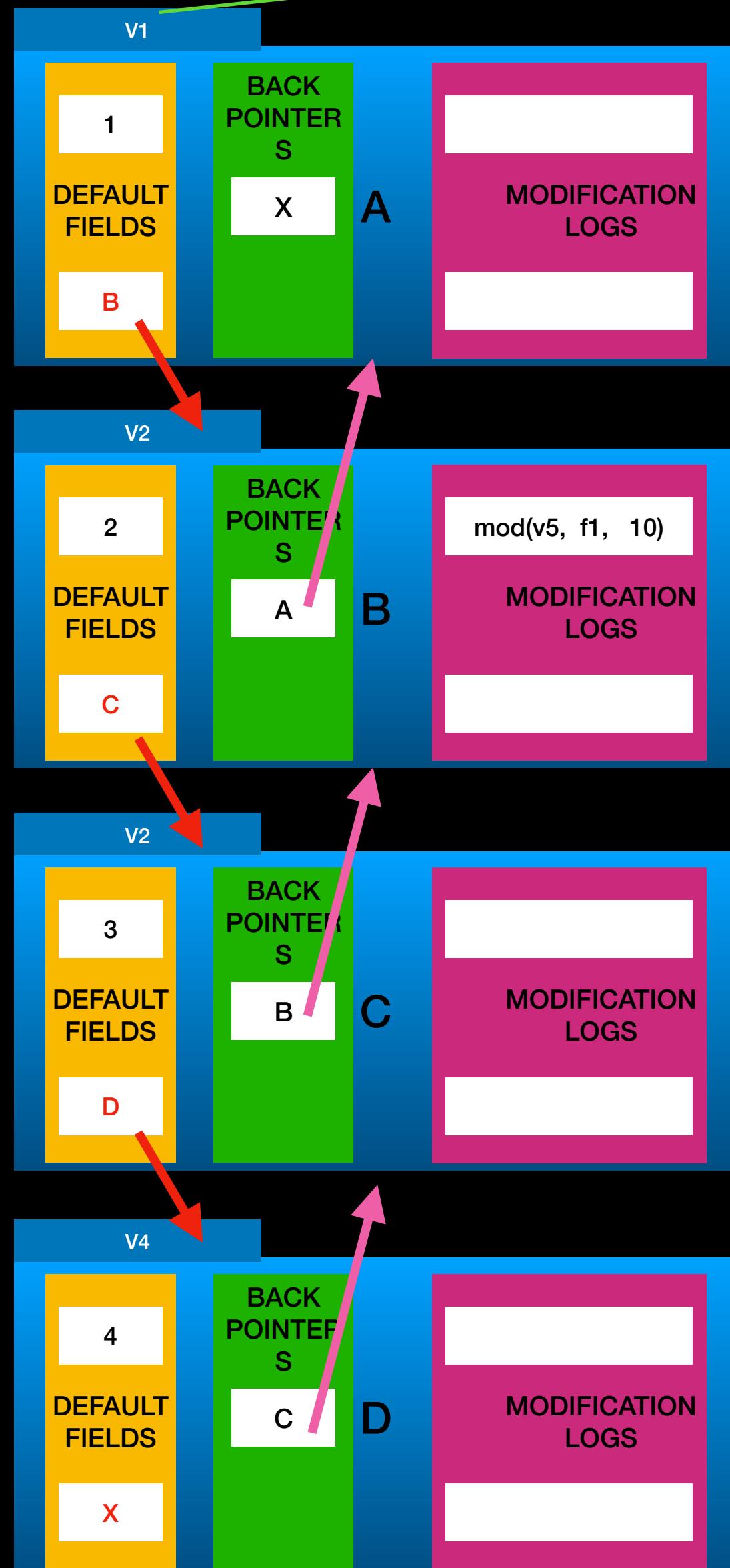
VERSION  
REDIRECTOR  
MODULE

Ver	block
1	A
2	B
3	C
4	X

Current Version: v5

# START MODULE

update(f1,B,10)



VERSION  
REDIRECTOR  
MODULE

Ver	block
1	A
2	
3	
4	

VERSION  
REDIRECTOR  
MODULE

Ver	block
2	R
3	
4	

VERSION  
REDIRECTOR  
MODULE

Ver	block
3	C
4	

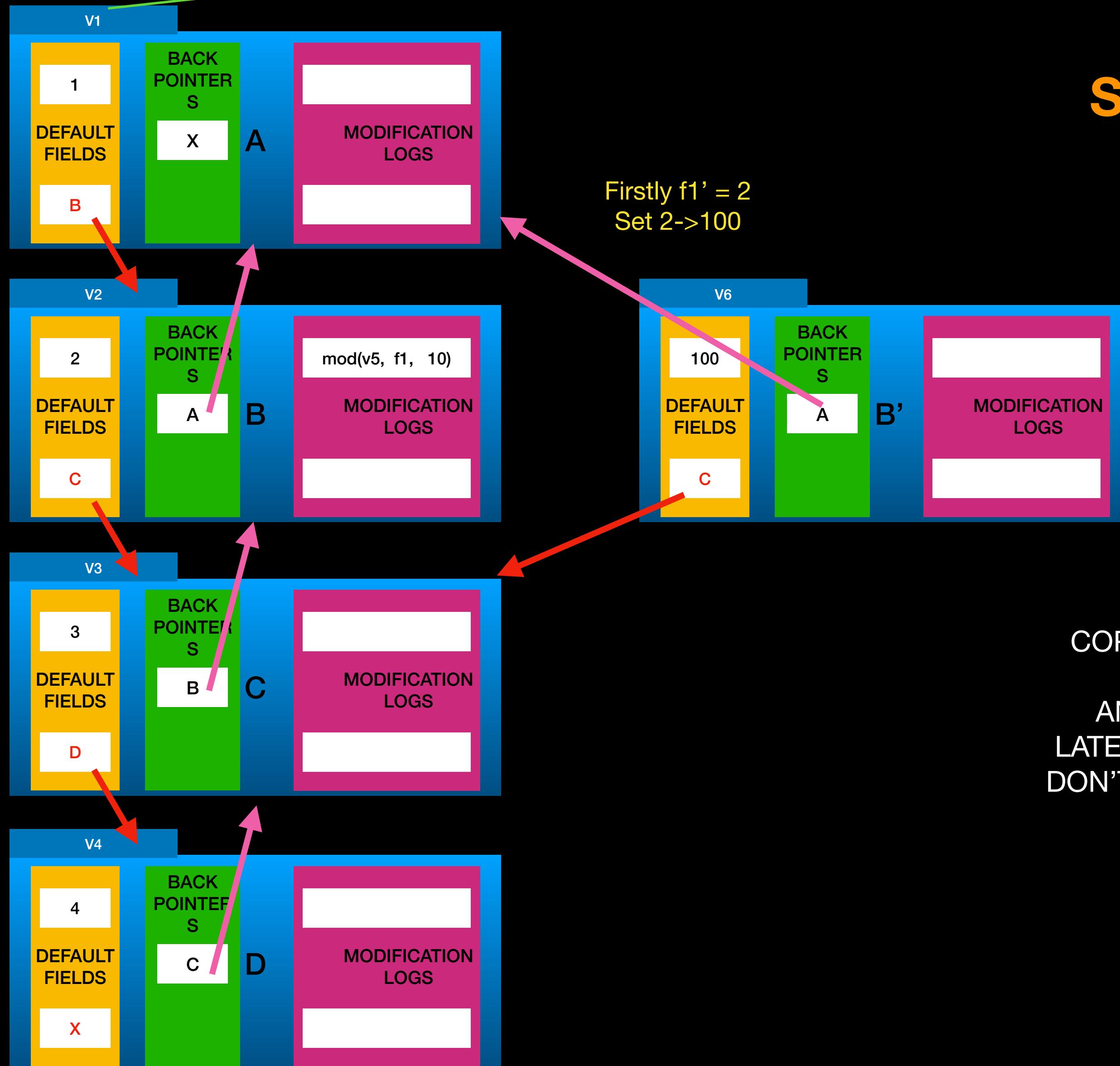
VERSION  
REDIRECTOR  
MODULE

Ver	block
4	D
5	

Current Version: v6

# START MODULE

update(f1,B,100)



VERSION  
REDIRECTOR  
MODULE

Ver	block
1	A

VERSION  
REDIRECTOR  
MODULE

Ver	block
2	B
6	B'

VERSION  
REDIRECTOR  
MODULE

Ver	block
3	C

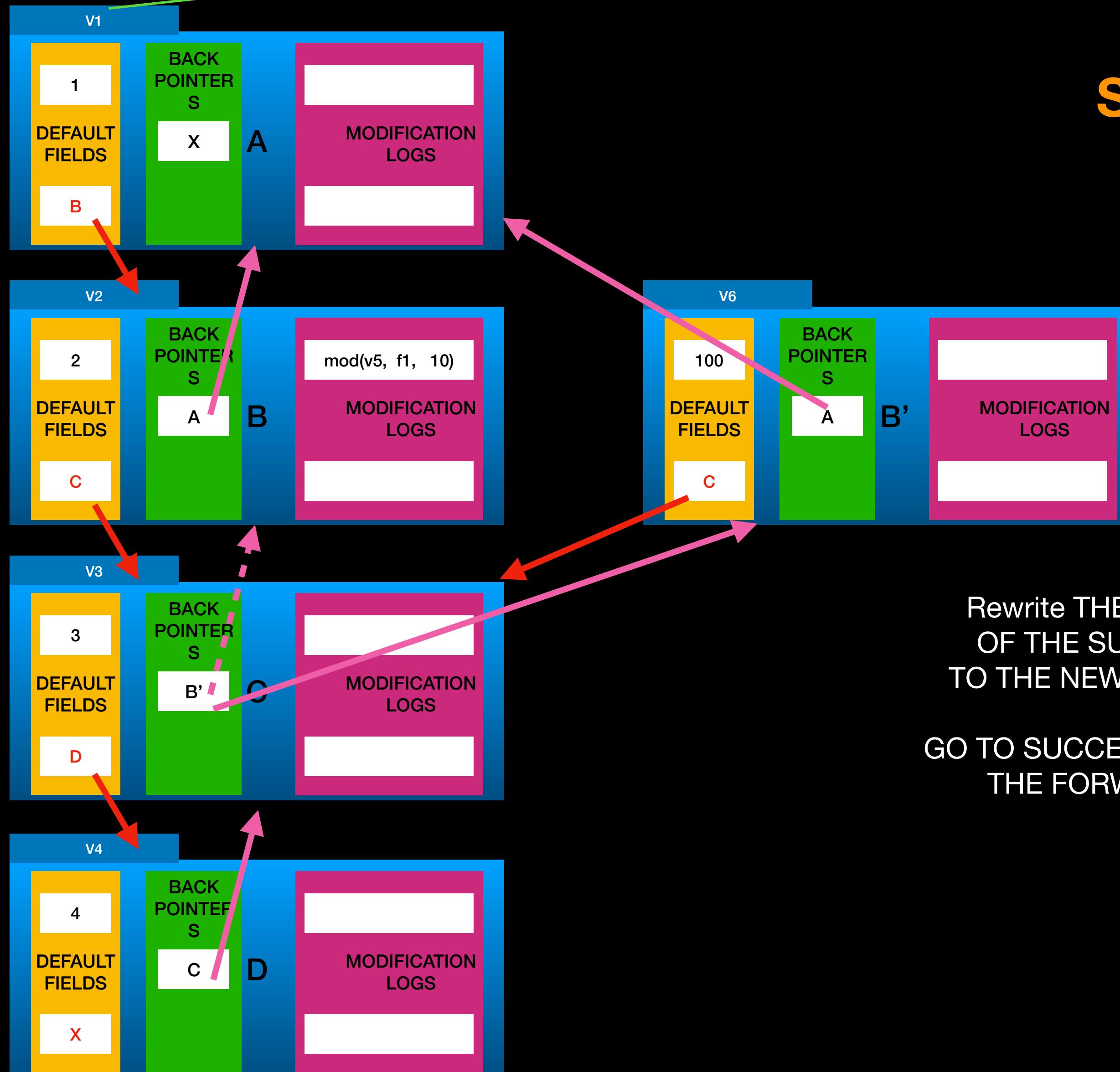
VERSION  
REDIRECTOR  
MODULE

Ver	block
4	D

Current Version: v6

# START MODULE

update(f1,B,100)



## STEP 2

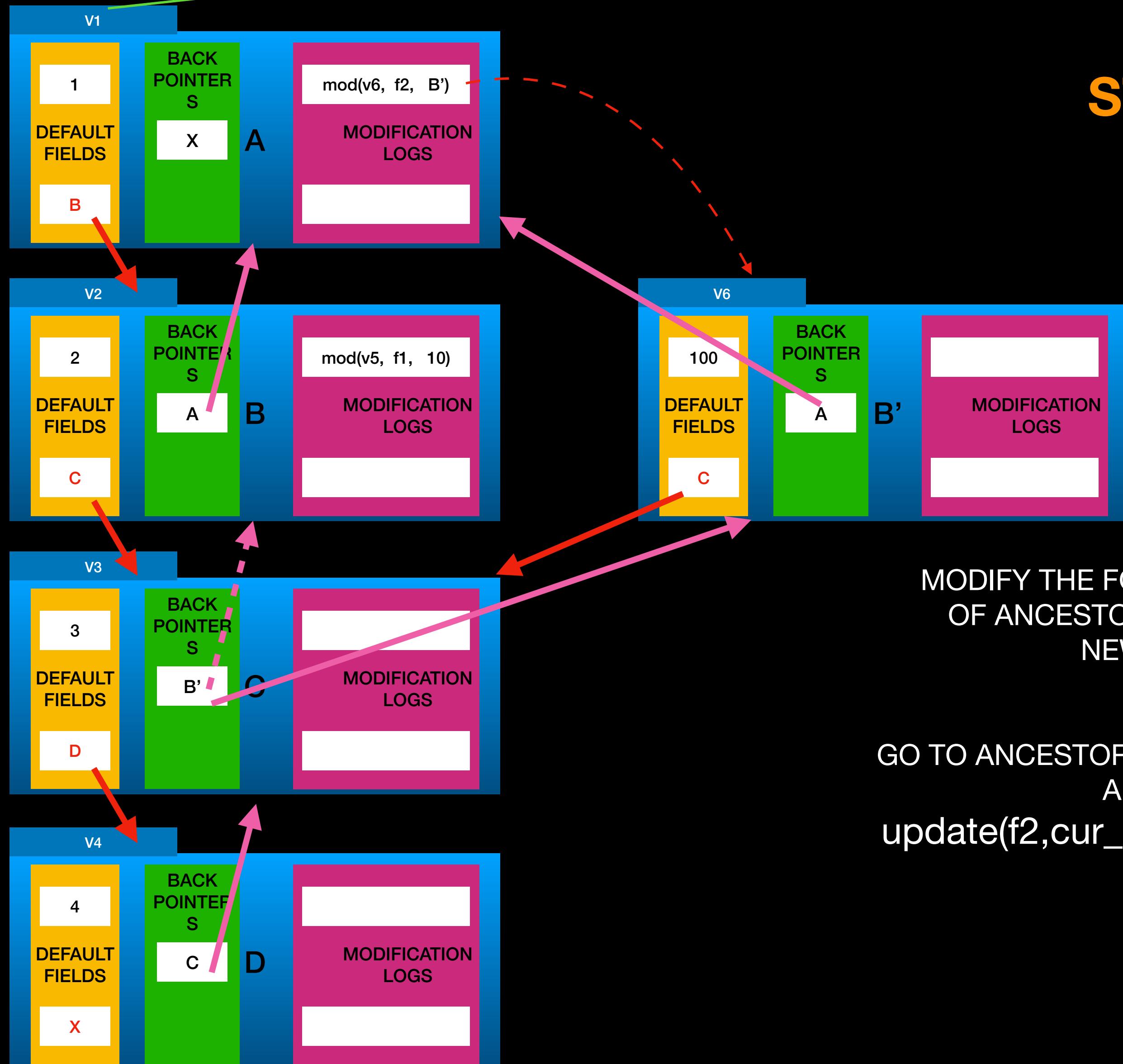
Rewrite THE BACK POINTERS  
OF THE SUCCESSOR NODE  
TO THE NEWLY CREATED NODE

GO TO SUCCESSOR NODES USING  
THE FORWARD POINTERS  
IN O(1)

Current Version: v6

# START MODULE

update(f1,B,100)



## STEP 3

MODIFY THE FORWARD POINTERS  
OF ANCESTOR NODES TO THE  
NEW NODE

GO TO ANCESTOR NODES RECURSIVELY  
AND DO  
update(f2,cur\_node,NEW\_NODE)

VERSION REDIRECTOR MODULE	
Ver	block
1	A

VERSION REDIRECTOR MODULE	
Ver	block
2	B
6	B'

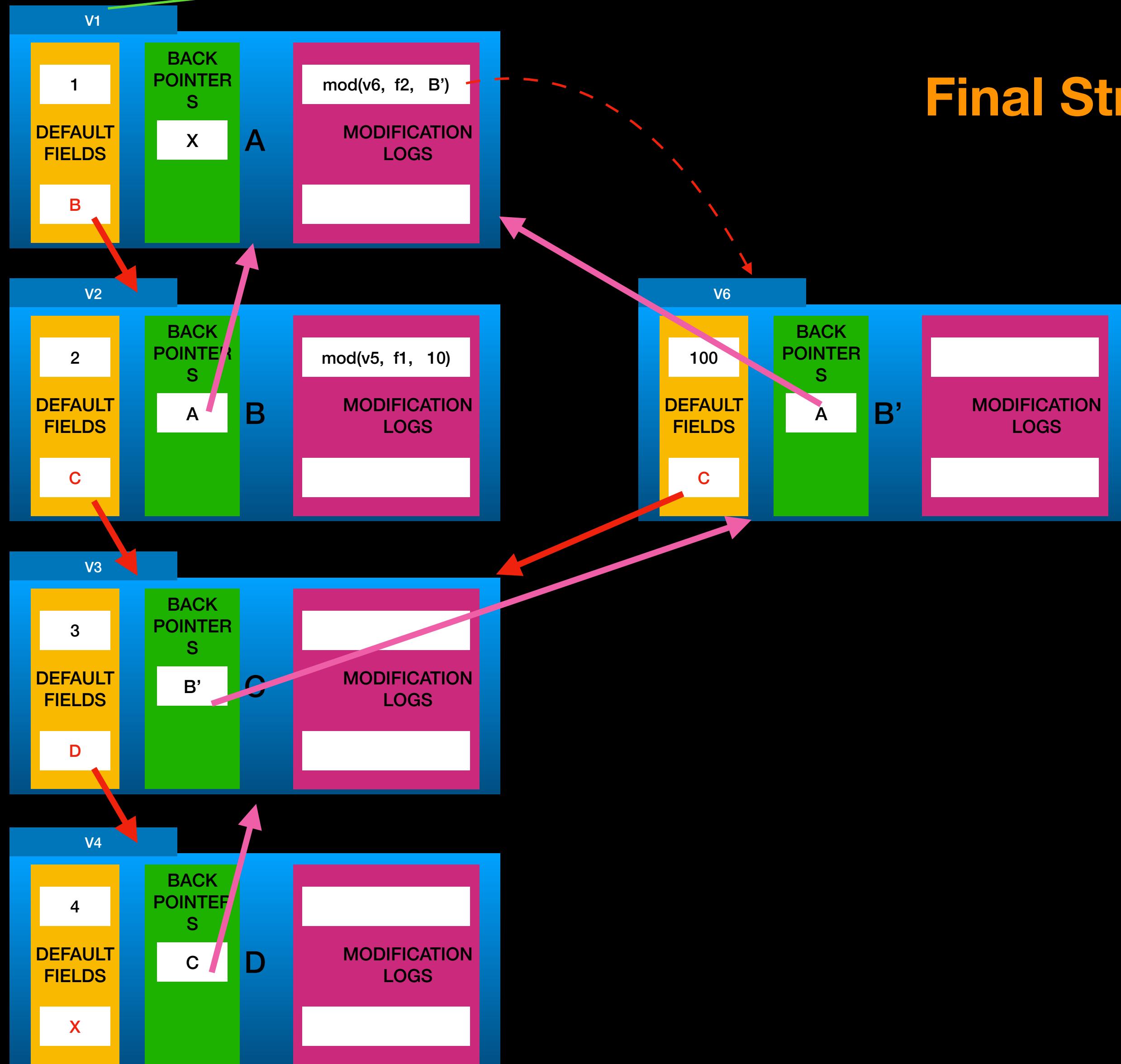
VERSION REDIRECTOR MODULE	
Ver	block
3	C

VERSION REDIRECTOR MODULE	
Ver	block
4	D

Current Version: v6

# START MODULE

update(f1,B,100)



## Final Structure of v6

VERSION REDIRECTOR MODULE	
Ver	block
1	A

VERSION REDIRECTOR MODULE	
Ver	block
2	B
6	B'

VERSION REDIRECTOR MODULE	
Ver	block
3	C

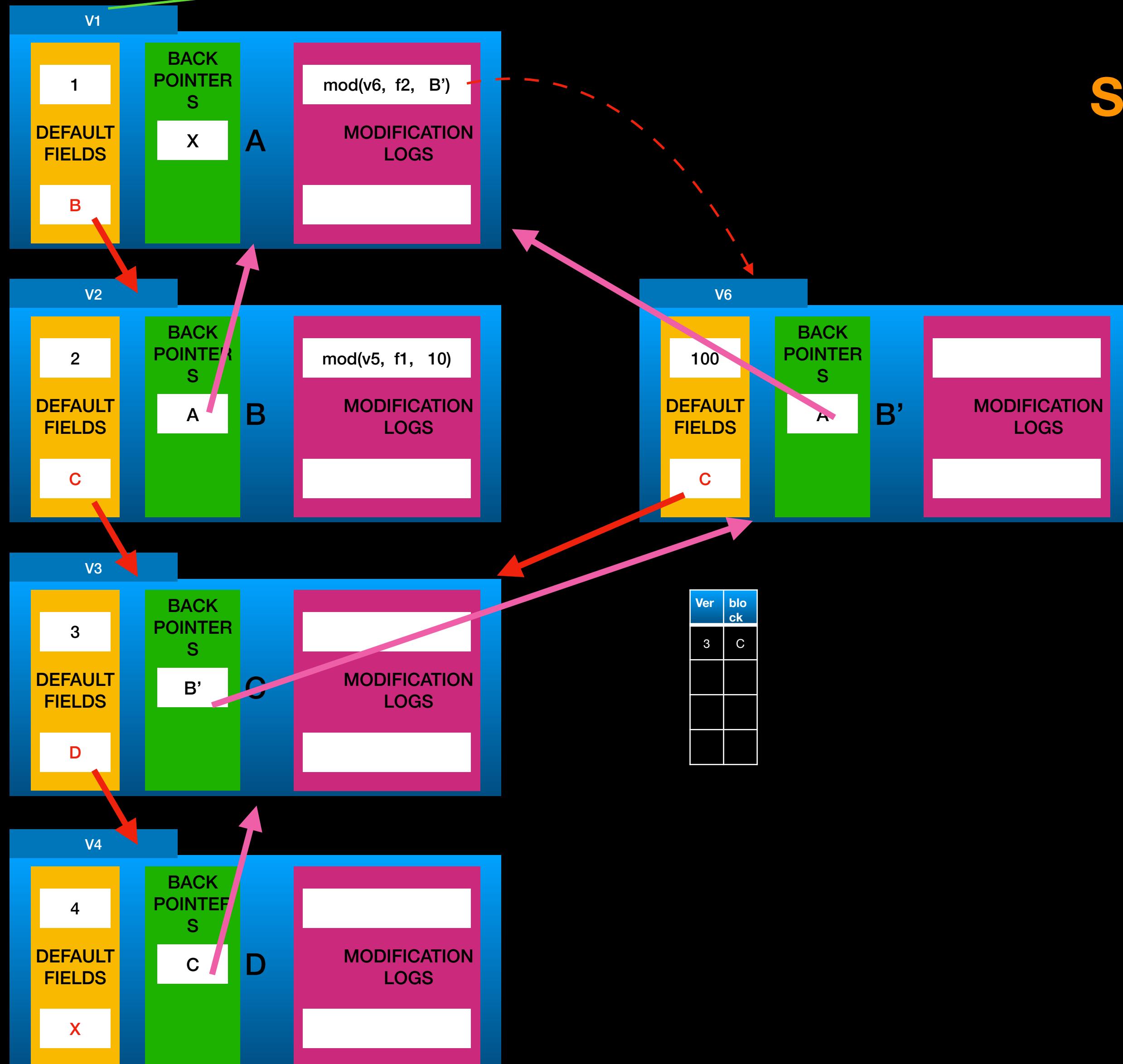
VERSION REDIRECTOR MODULE	
Ver	block
4	D

Current Version: v7

# START MODULE

v0

## STEP 1



`add(X, B, _)` consists of

Create node X
Modify f2 of B' to X
Modify f2 of X to C + Set BP of X to B'
Update BP of C to X
Set f1 of X

VERSION  
REDIRECTOR  
MODULE

Ver	block
1	A

VERSION  
REDIRECTOR  
MODULE

Ver	block
2	B
6	B'

VERSION  
REDIRECTOR  
MODULE

Ver	block
3	C

VERSION  
REDIRECTOR  
MODULE

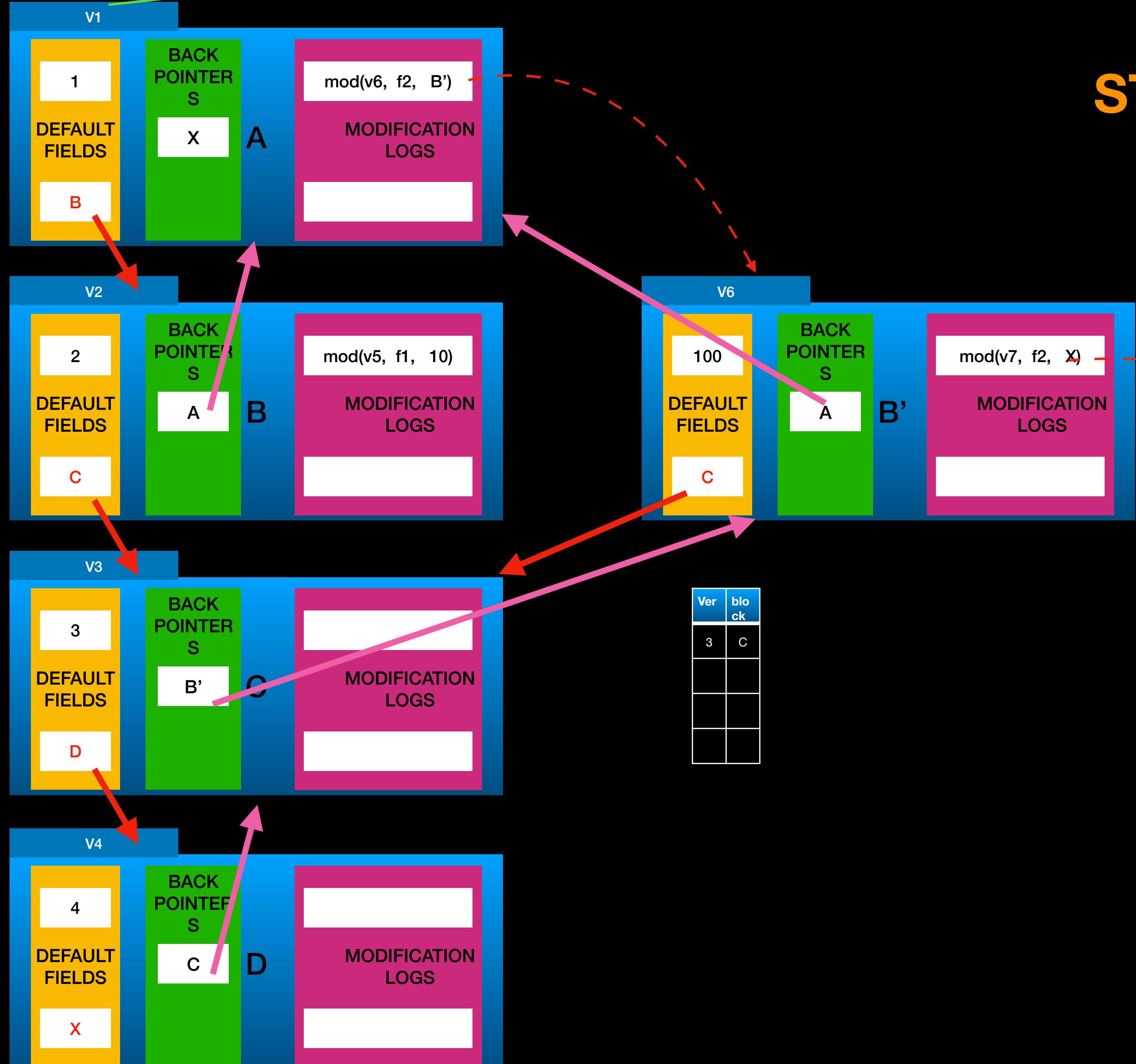
Ver	block
7	X

Current Version: v7

# START MODULE

v0

## STEP 2



add(X,B,\_) consists of

Create node X
Modify f2 of B' to X
Modify f2 of X to C + Set BP of X to B'
Update BP of C to X
Set f1 of X

VERSION  
REDIRECTOR  
MODULE

Ver	block
1	A

VERSION  
REDIRECTOR  
MODULE

Ver	block
2	B
6	B'

VERSION  
REDIRECTOR  
MODULE

Ver	block
3	C

VERSION  
REDIRECTOR  
MODULE

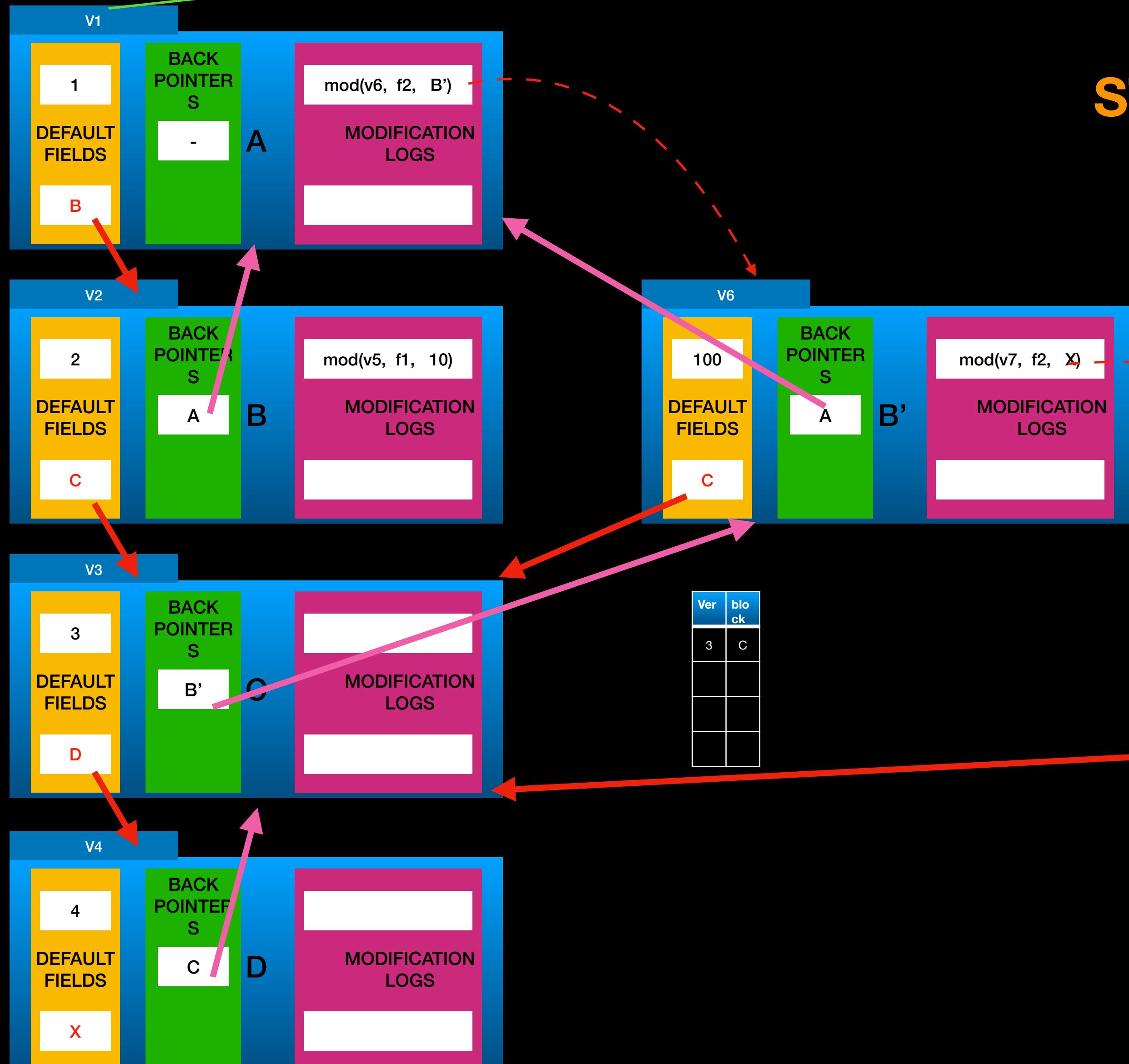
Ver	block
7	X

Current Version: v7

# START MODULE

v0

## STEP 3



add(X,B,\_) consists of

Create node X
Modify f2 of B' to X
Modify f2 of X to C + Set BP of X to B'
Update BP of C to X
Set f1 of X

VERSION  
REDIRECTOR  
MODULE

Ver	block
1	A

VERSION  
REDIRECTOR  
MODULE

Ver	block
2	B
6	B'

VERSION  
REDIRECTOR  
MODULE

Ver	block
7	X

VERSION  
REDIRECTOR  
MODULE

Ver	block
4	D

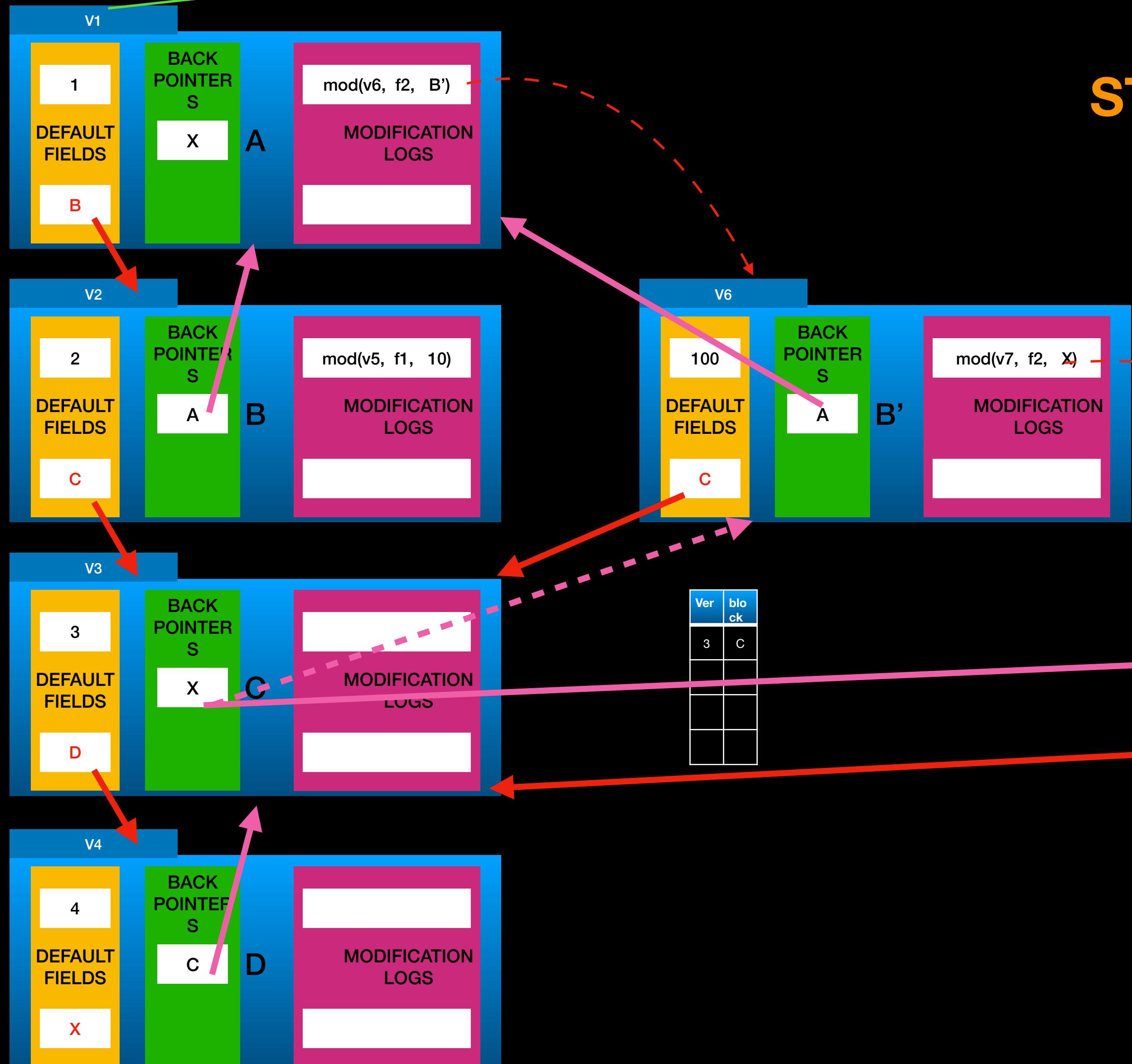
add(X,B,\_)

Current Version: v7

# START MODULE

v0

## STEP 4



add(X,B,\_) consists of

Create node X
Modify f2 of B' to X
Modify f2 of X to C + Set BP of X to B'
Update BP of C to X
Set f1 of X

VERSION  
REDIRECTOR  
MODULE

Ver	block
1	A

VERSION  
REDIRECTOR  
MODULE

Ver	block
2	B
6	B'

VERSION  
REDIRECTOR  
MODULE

Ver	block
7	X

VERSION  
REDIRECTOR  
MODULE

Ver	block
4	D

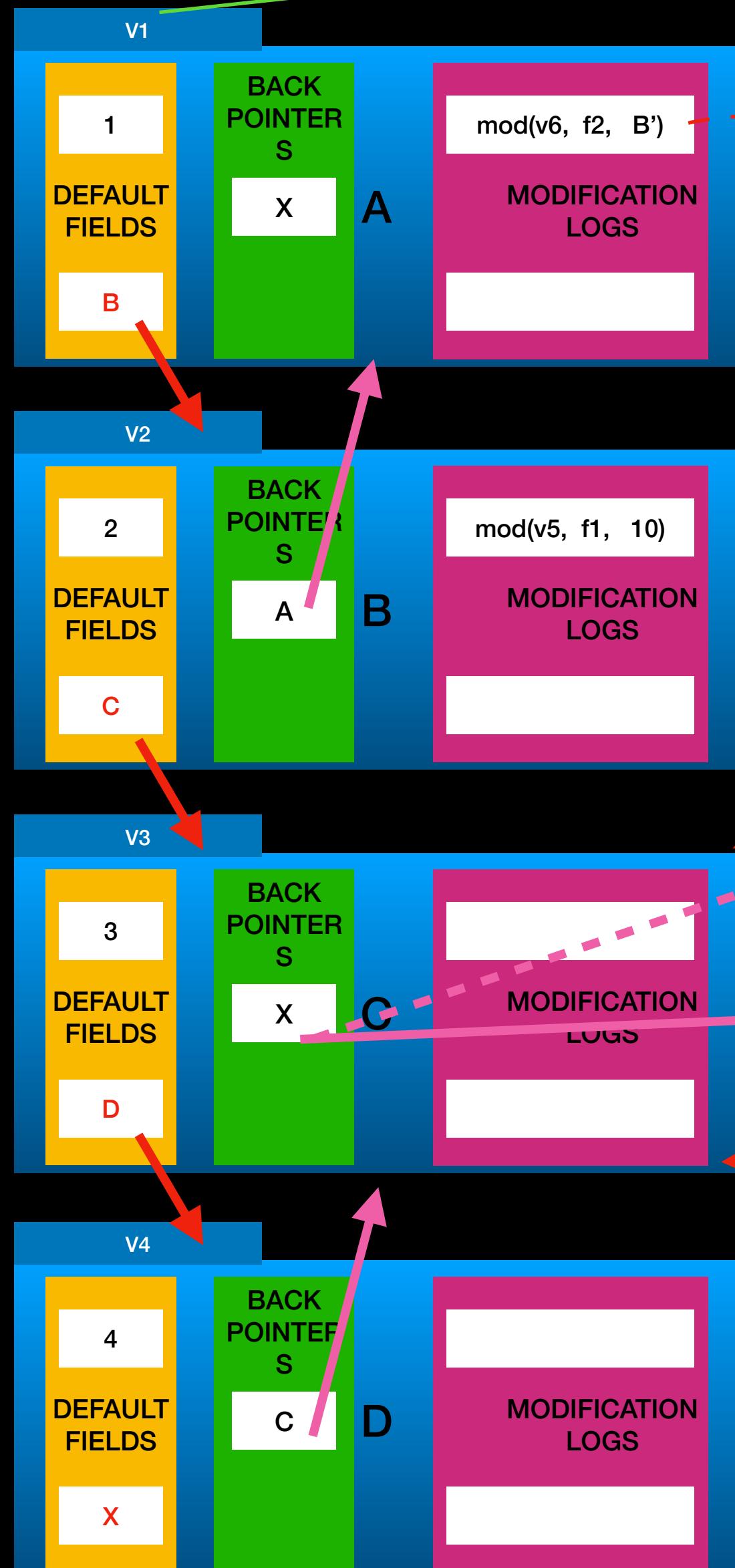
add(X,B,\_)

Current Version: v7

# START MODULE

v0

**STEP 5**



add(X,B,\_) consists of

Create node X
Modify f2 of B' to X
Modify f2 of X to C + Set BP of X to B'
Update BP of C to X
Set f1 of X

VERSION  
REDIRECTOR  
MODULE

Ver	block
1	A

VERSION  
REDIRECTOR  
MODULE

Ver	block
2	B
6	B'

VERSION  
REDIRECTOR  
MODULE

Ver	block
7	X

VERSION  
REDIRECTOR  
MODULE

Ver	block
4	D

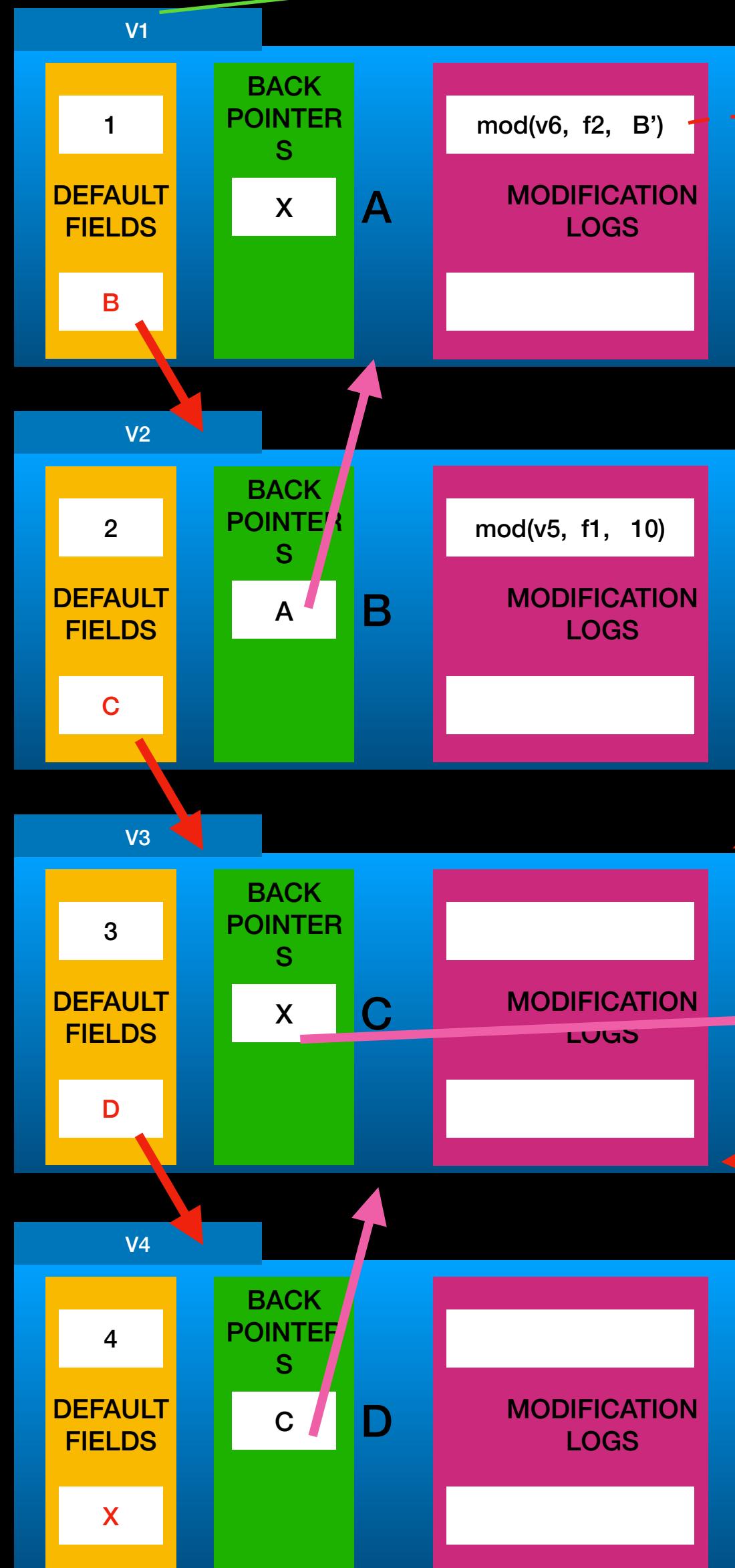
add(X,B,\_)

Current Version: v7

# START MODULE

v0

$\text{add}(X, B, \_)$



## Final Structure of v7

Ver	block
3	C



VERSION REDIRECTOR MODULE	
Ver	block
1	A

VERSION REDIRECTOR MODULE	
Ver	block
2	B
6	B'

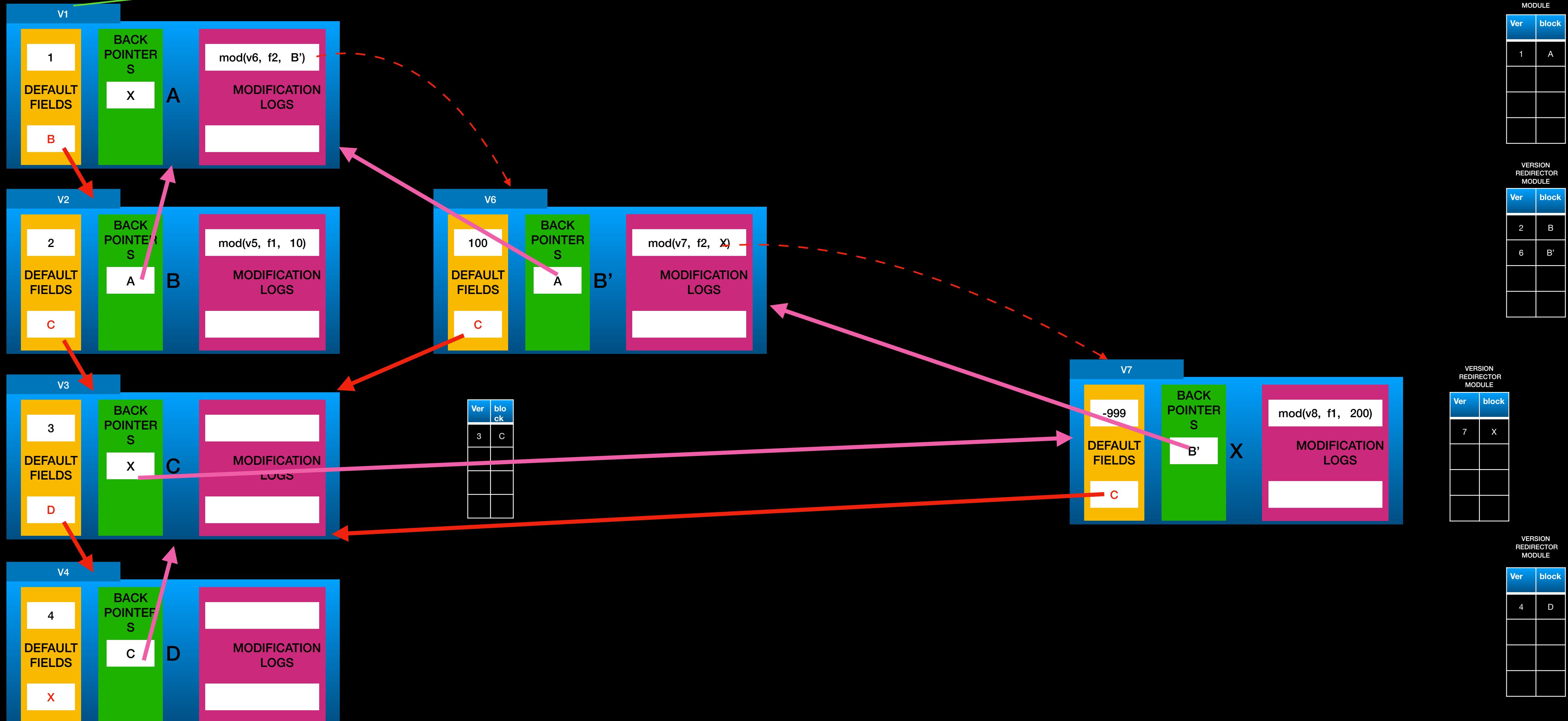
VERSION REDIRECTOR MODULE	
Ver	block
7	X

VERSION REDIRECTOR MODULE	
Ver	block
4	D

Current Version: v8

# START MODULE

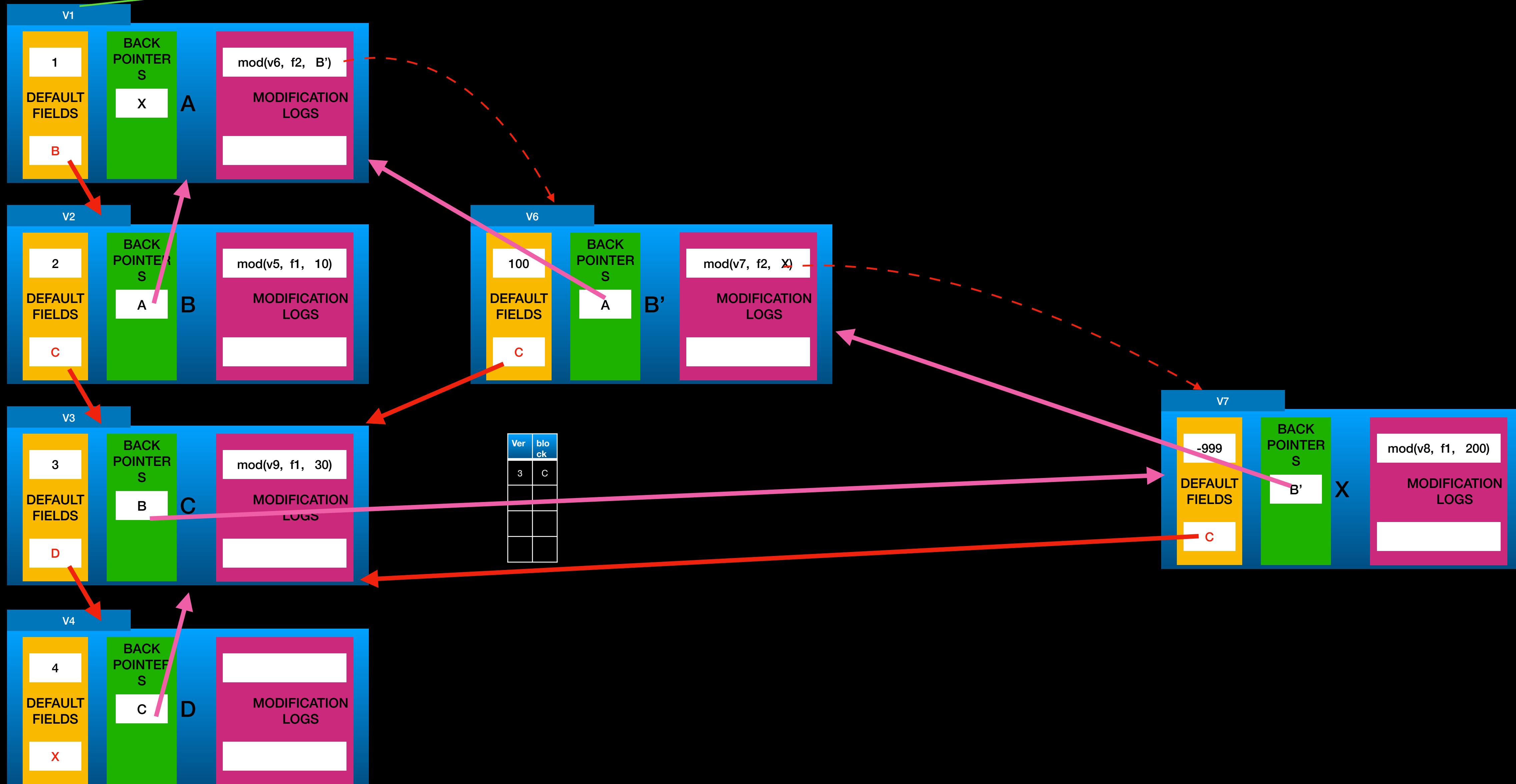
update(f1, x, 200)



Current Version: v9

# START MODULE

update(f1, C, 30)



VERSION  
DIRECTOR  
MODULE

Ver	block
1	A

VERSION  
DIRECTOR  
MODULE

Ver	block
2	B
6	B'

VERSION  
DIRECTOR  
MODULE

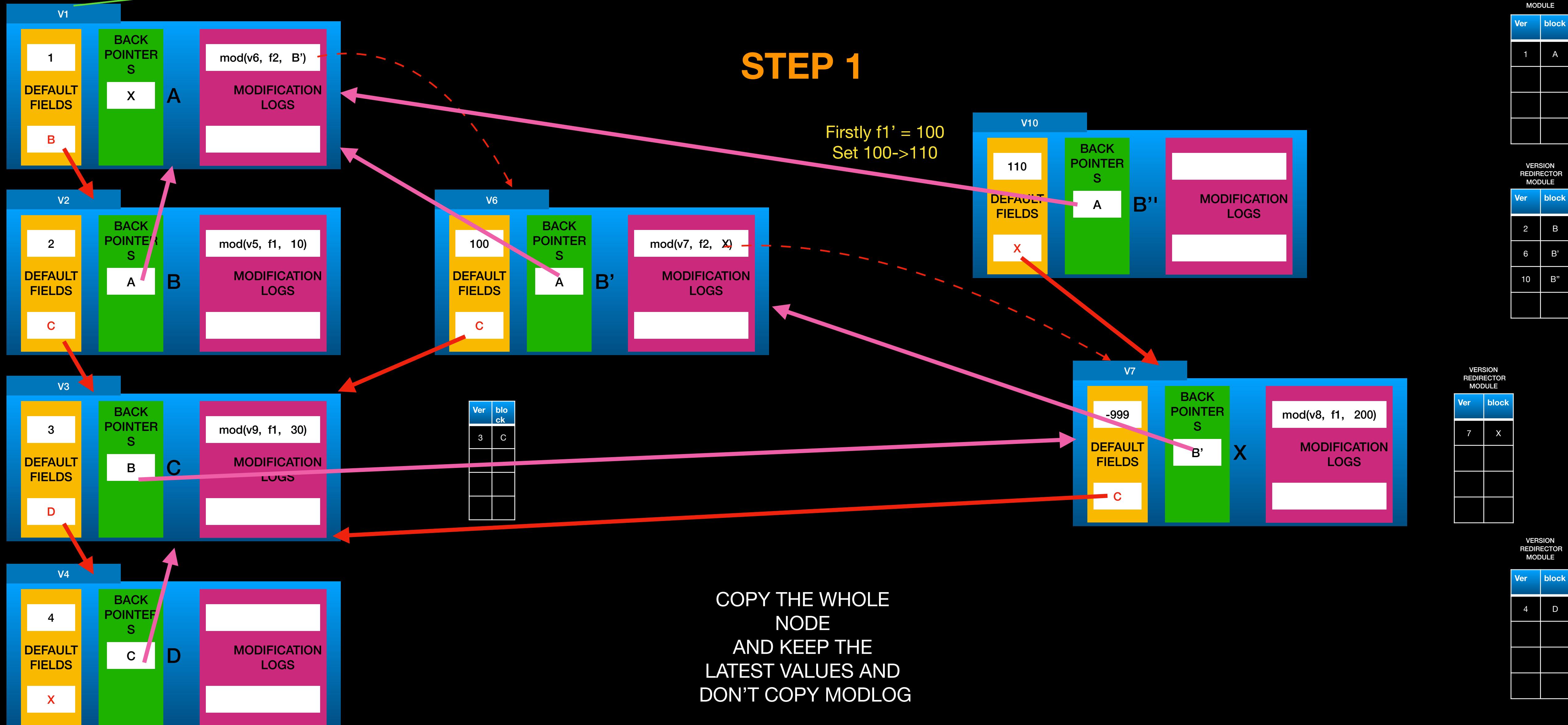
Ver	block
3	C

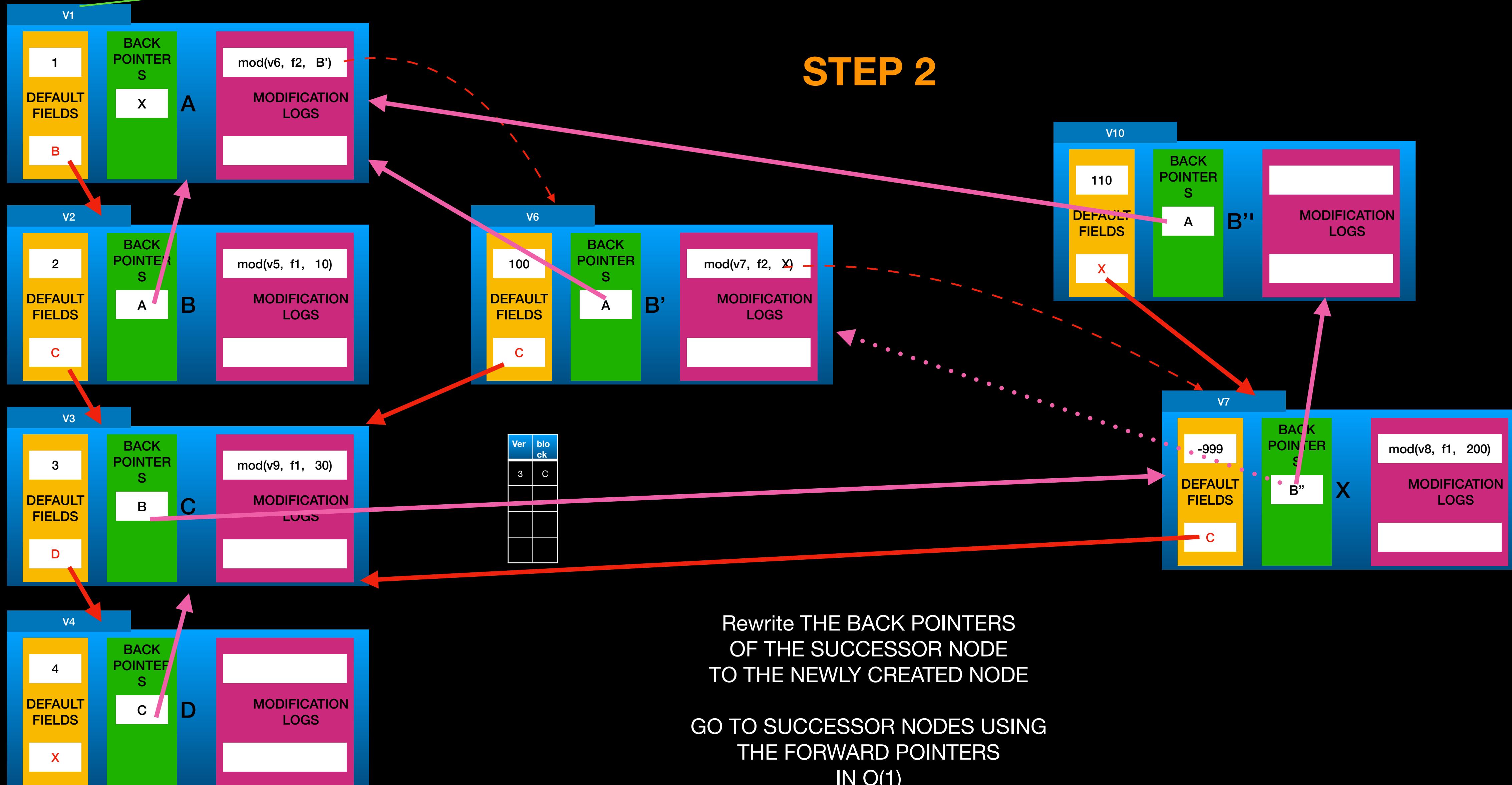
VERSION  
DIRECTOR  
MODULE

Ver	block
4	D

## START MODULE

update(f1, B,110)





VERSION REDIRECTOR MODULE	
Ver	block
1	A

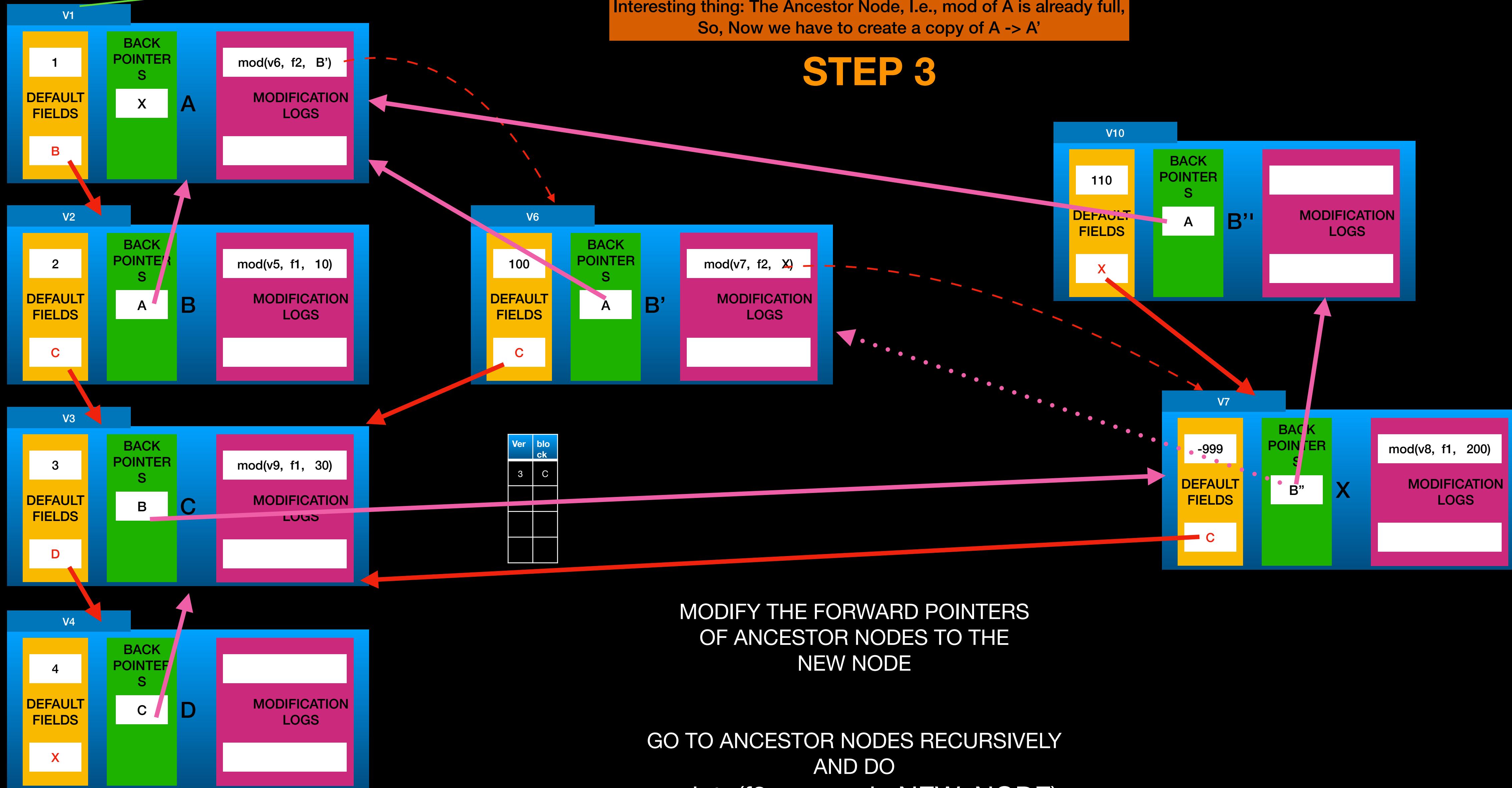
VERSION REDIRECTOR MODULE	
Ver	block
2	B
6	B'
10	B''

VERSION REDIRECTOR MODULE	
Ver	block
3	C

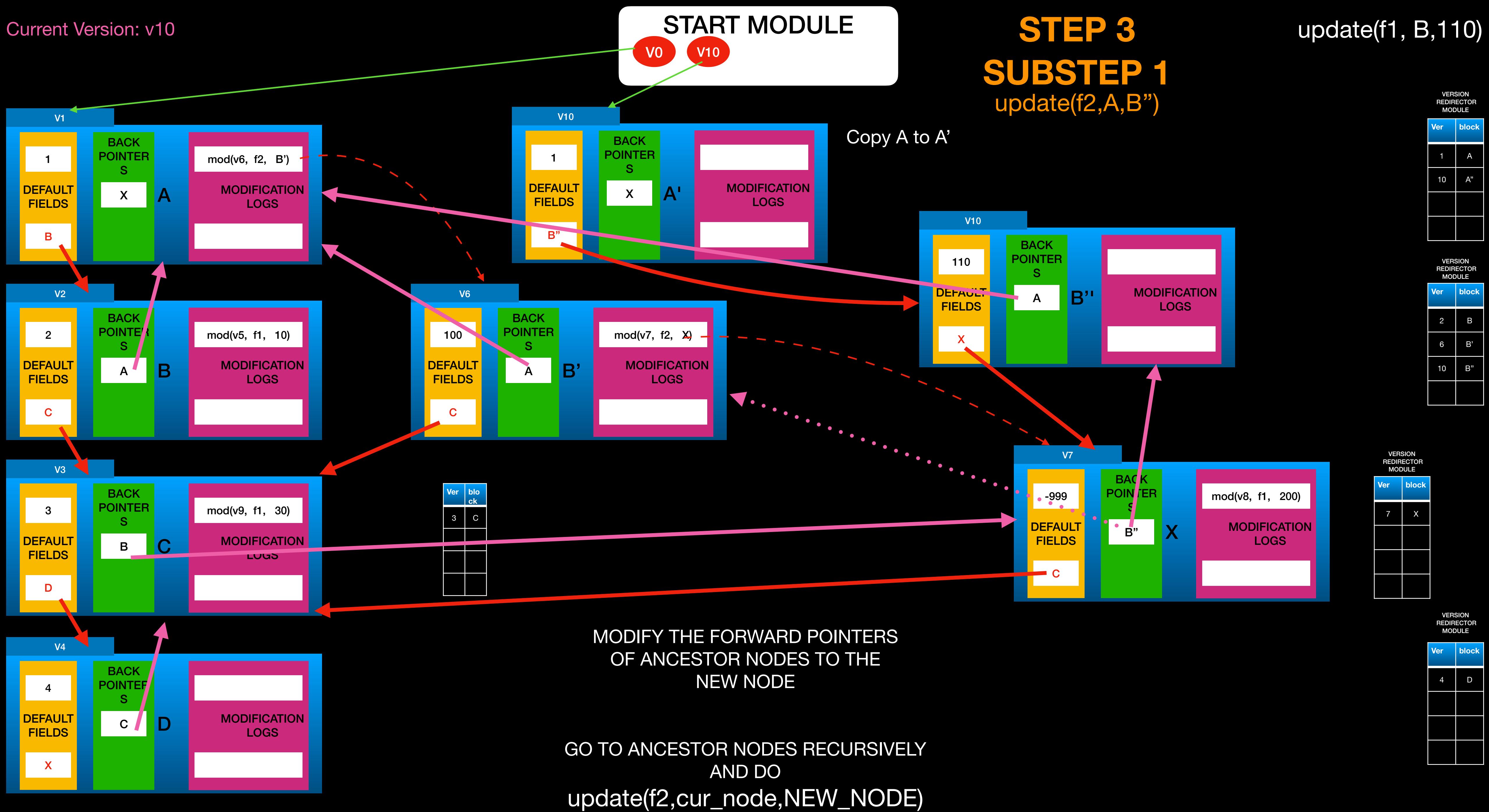
VERSION REDIRECTOR MODULE	
Ver	block
7	X

## START MODULE

update(f1, B,110)



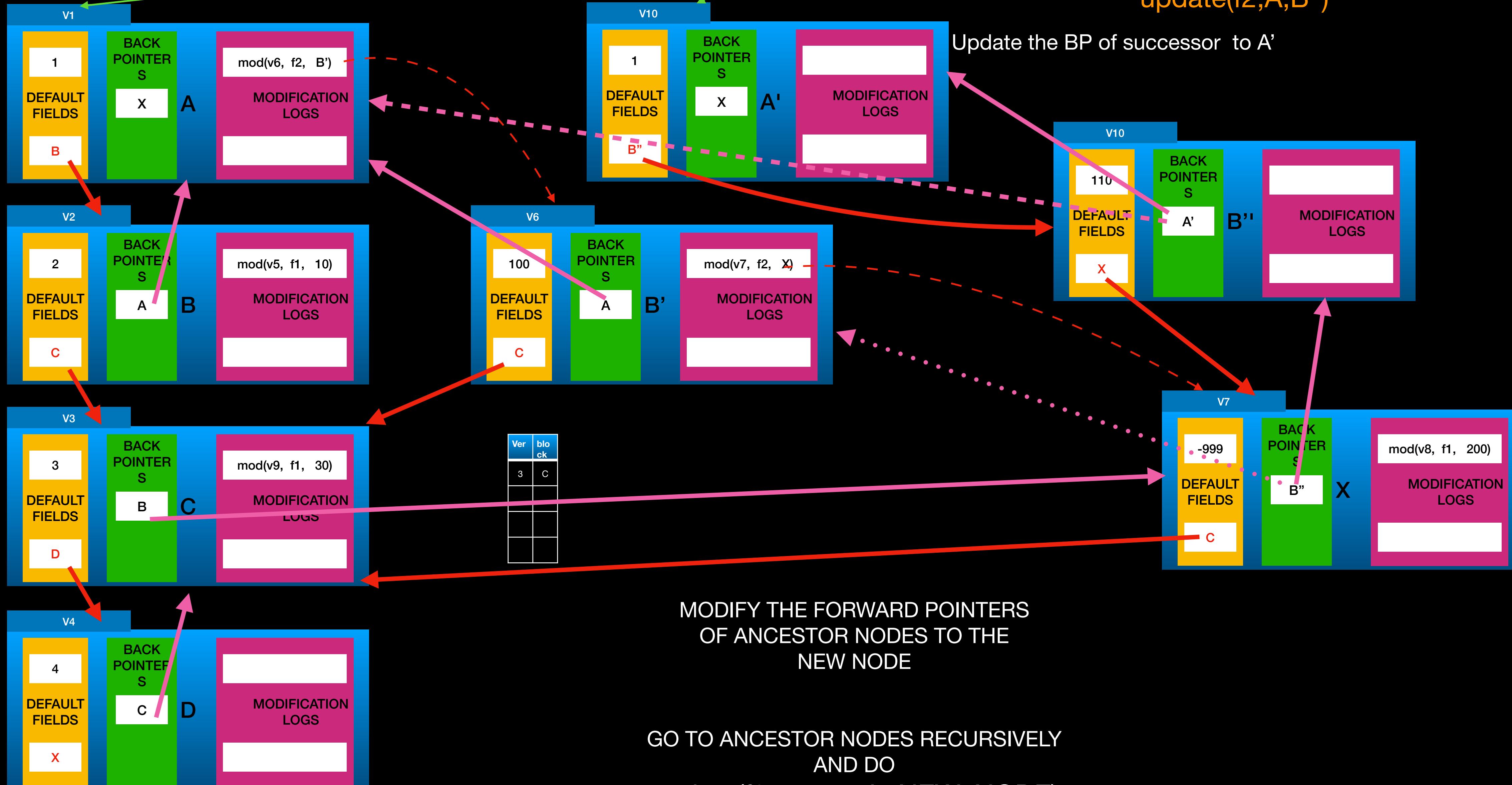
Current Version: v10



# Current Version: v10

# START MODULE

V0 V10



# STEP 3

# SUBSTEP 2

## update(f2,A,B")

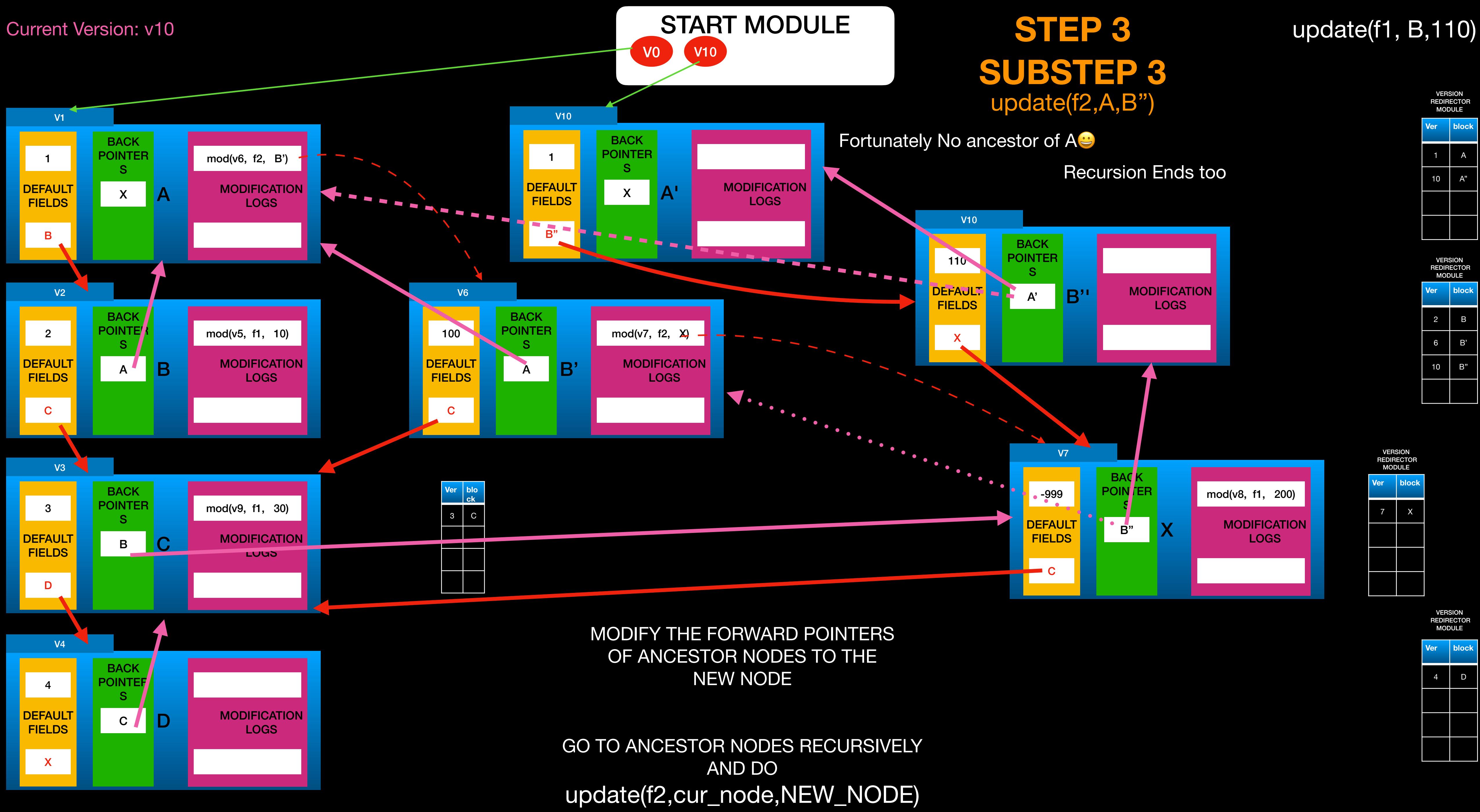
**update(f1, B,110)**

VERSION REDIRECTOR MODULE	
Ver	block
1	A
10	A''

VERSION REDIRECTOR MODULE	
Ver	block
2	B
6	B'
10	B''

MODULE	
Ver	block
4	D

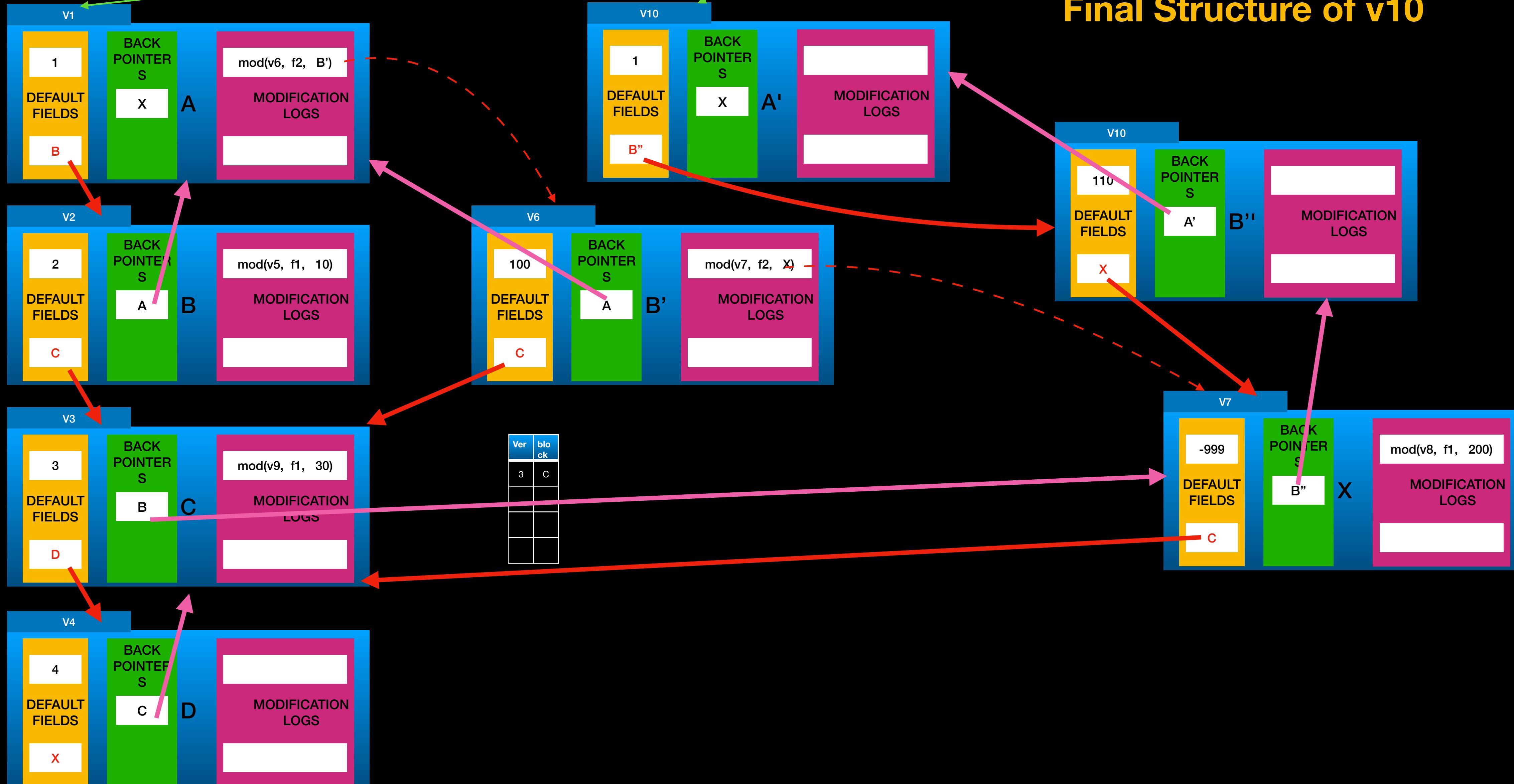
# Current Version: v10



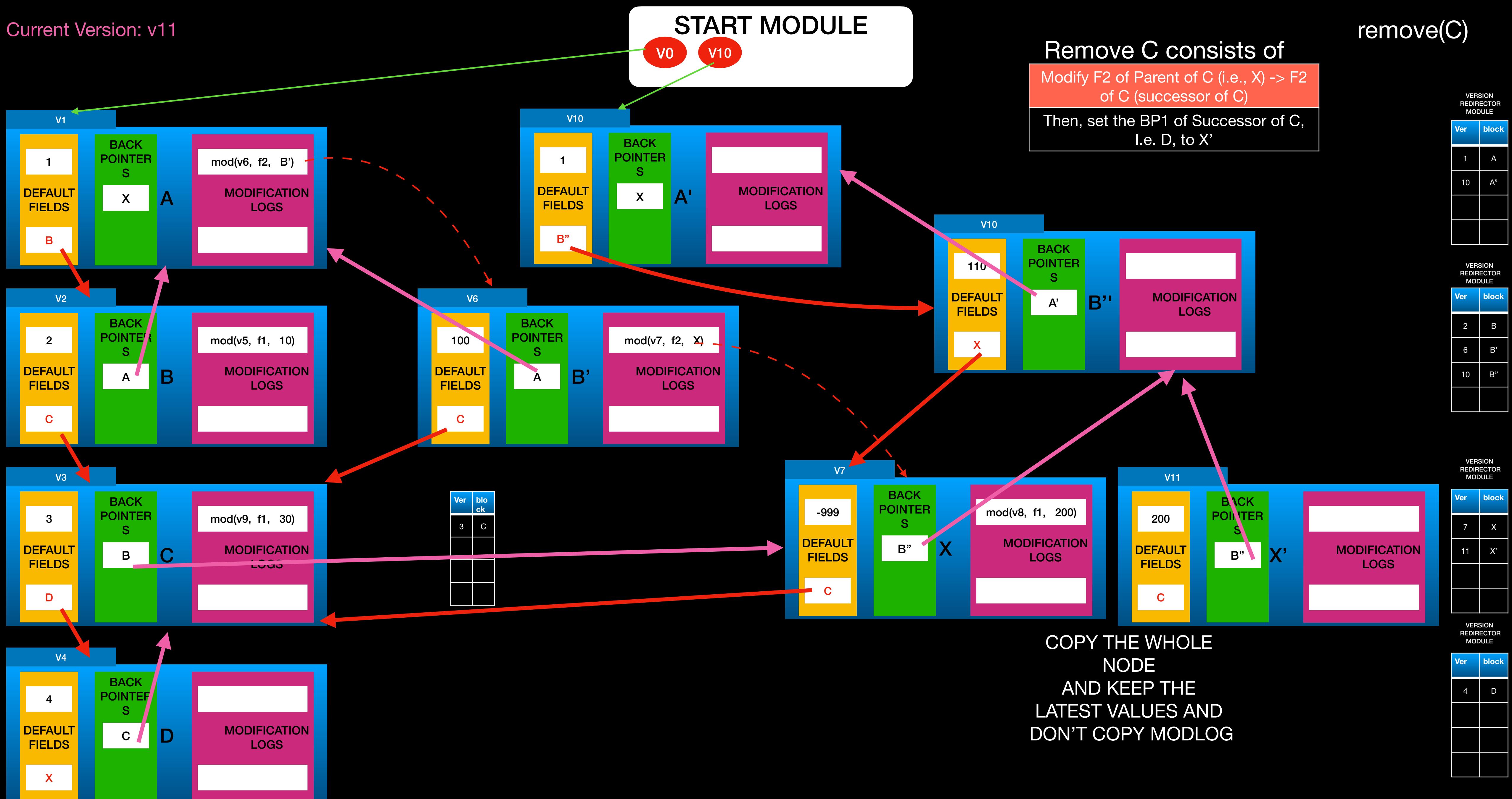
Current Version: v10

## START MODULE

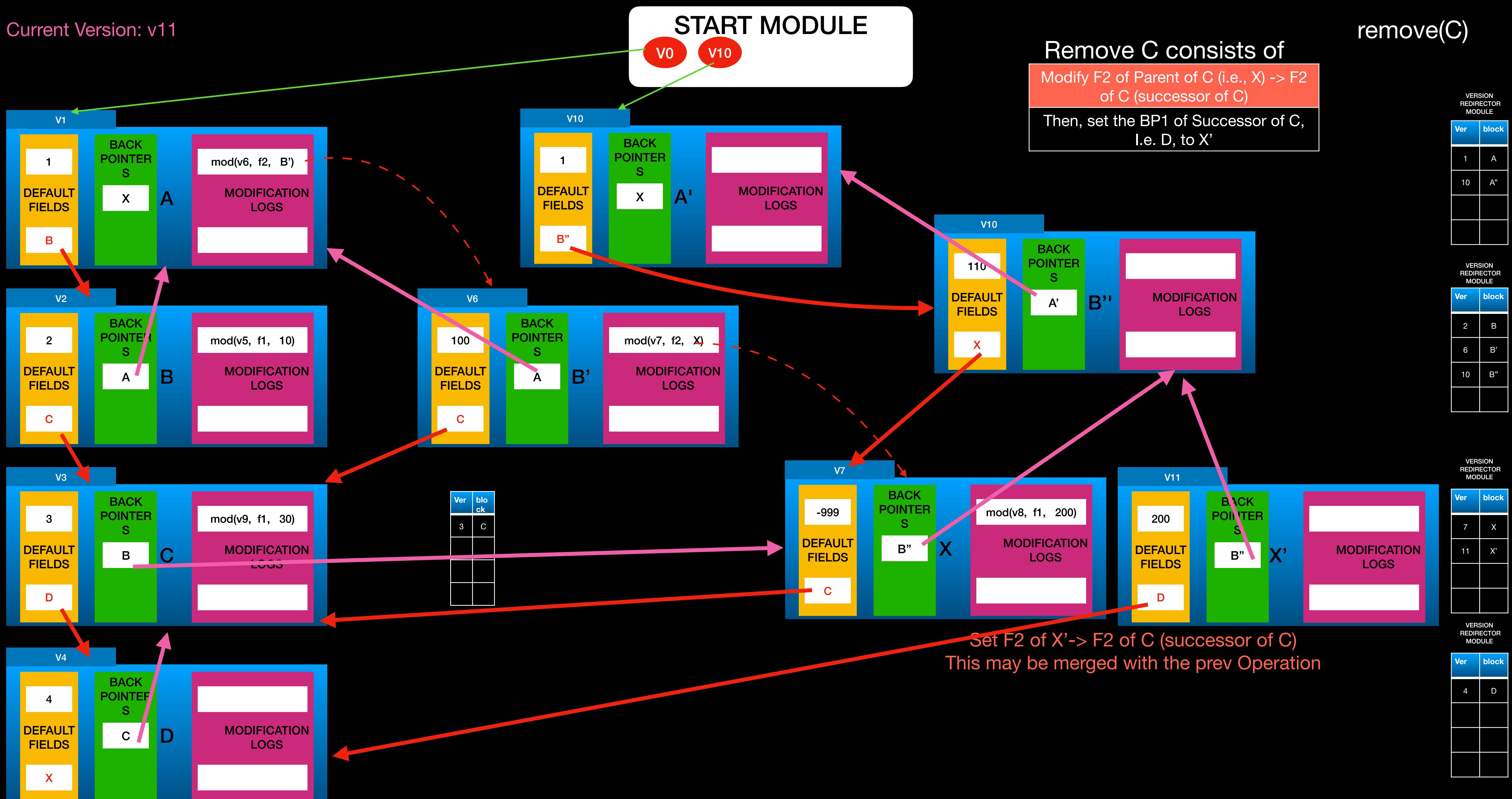
update(f1, B,110)



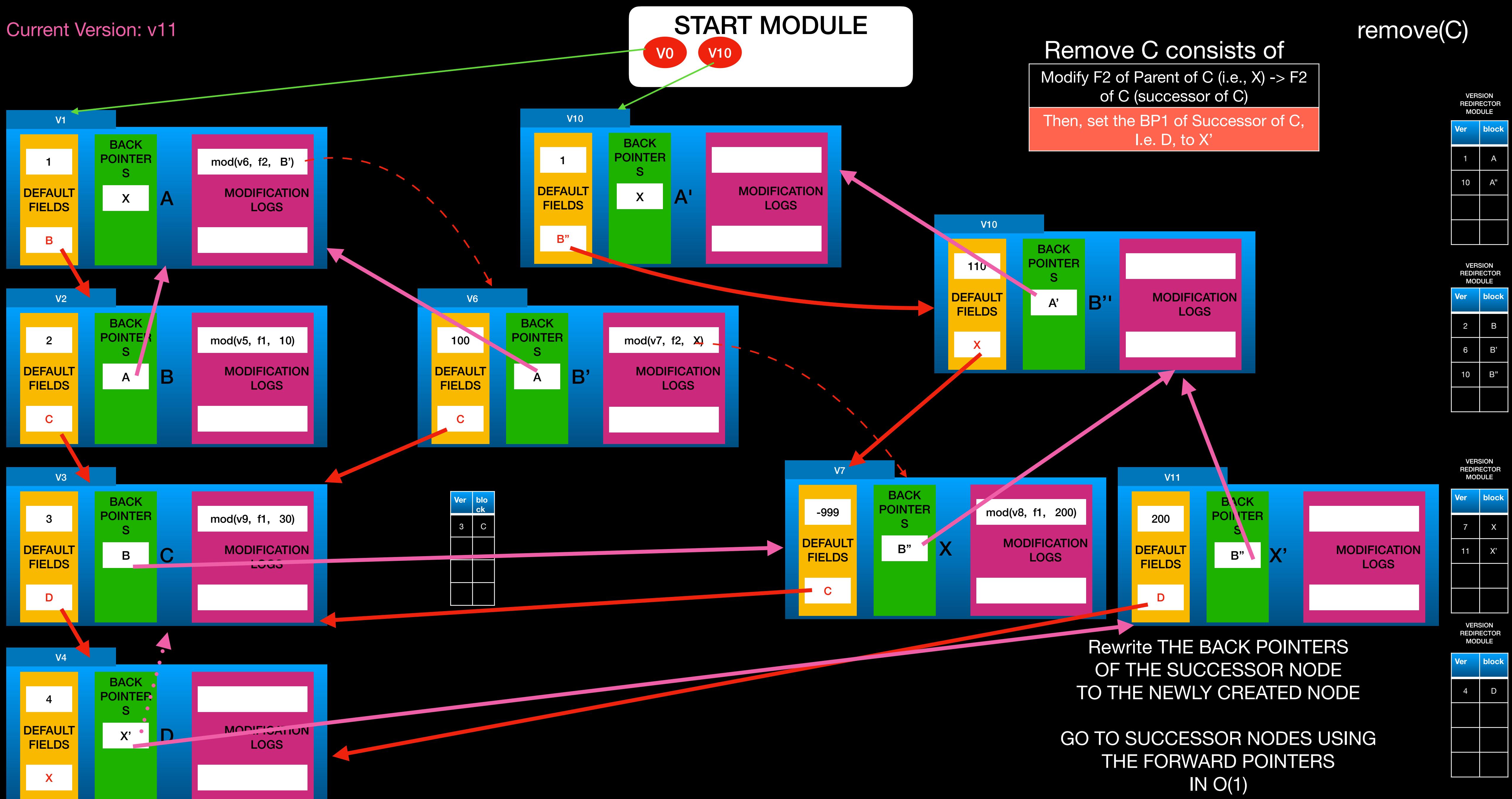
Current Version: v11



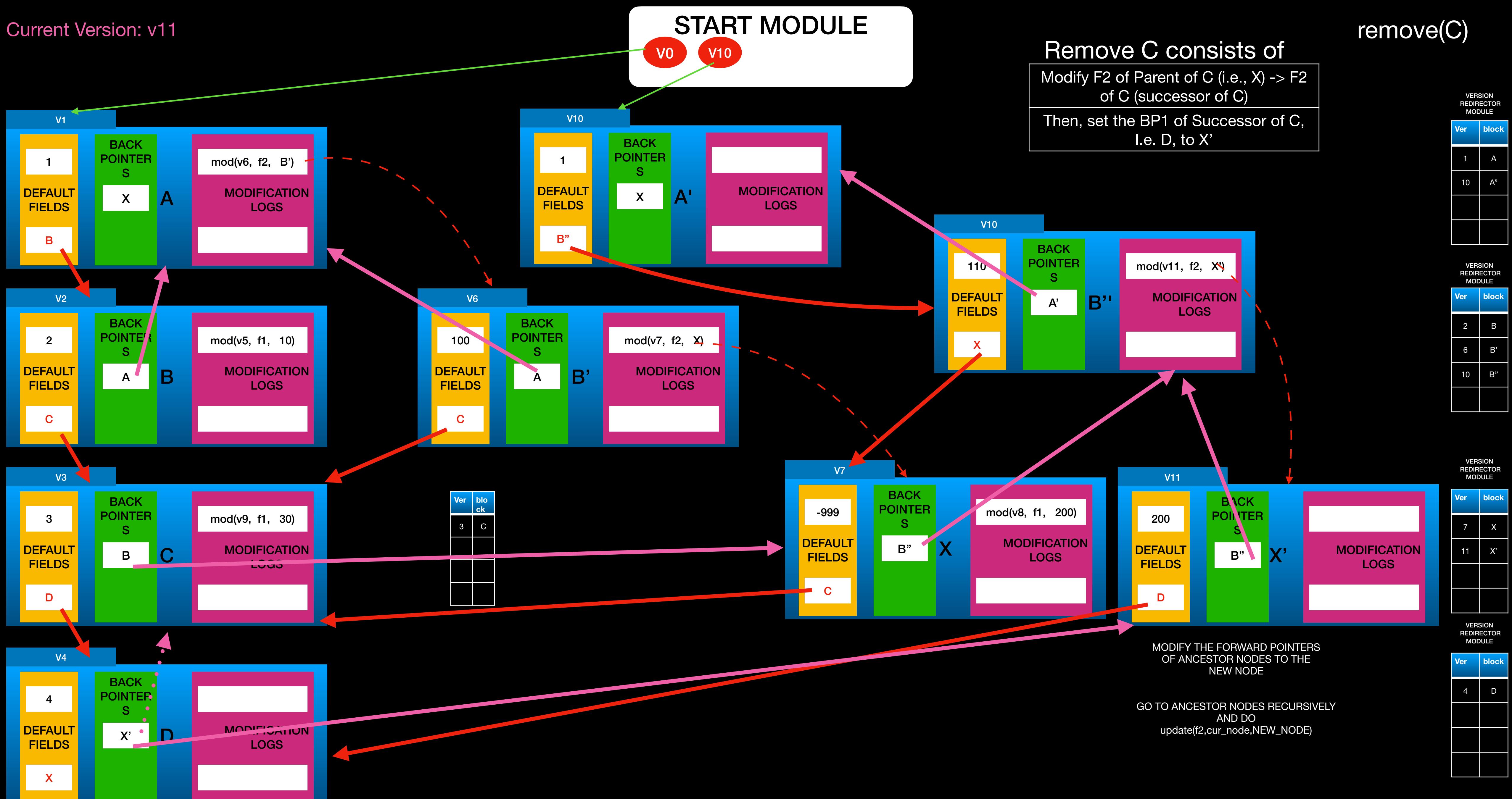
Current Version: v11



Current Version: v11



Current Version: v11

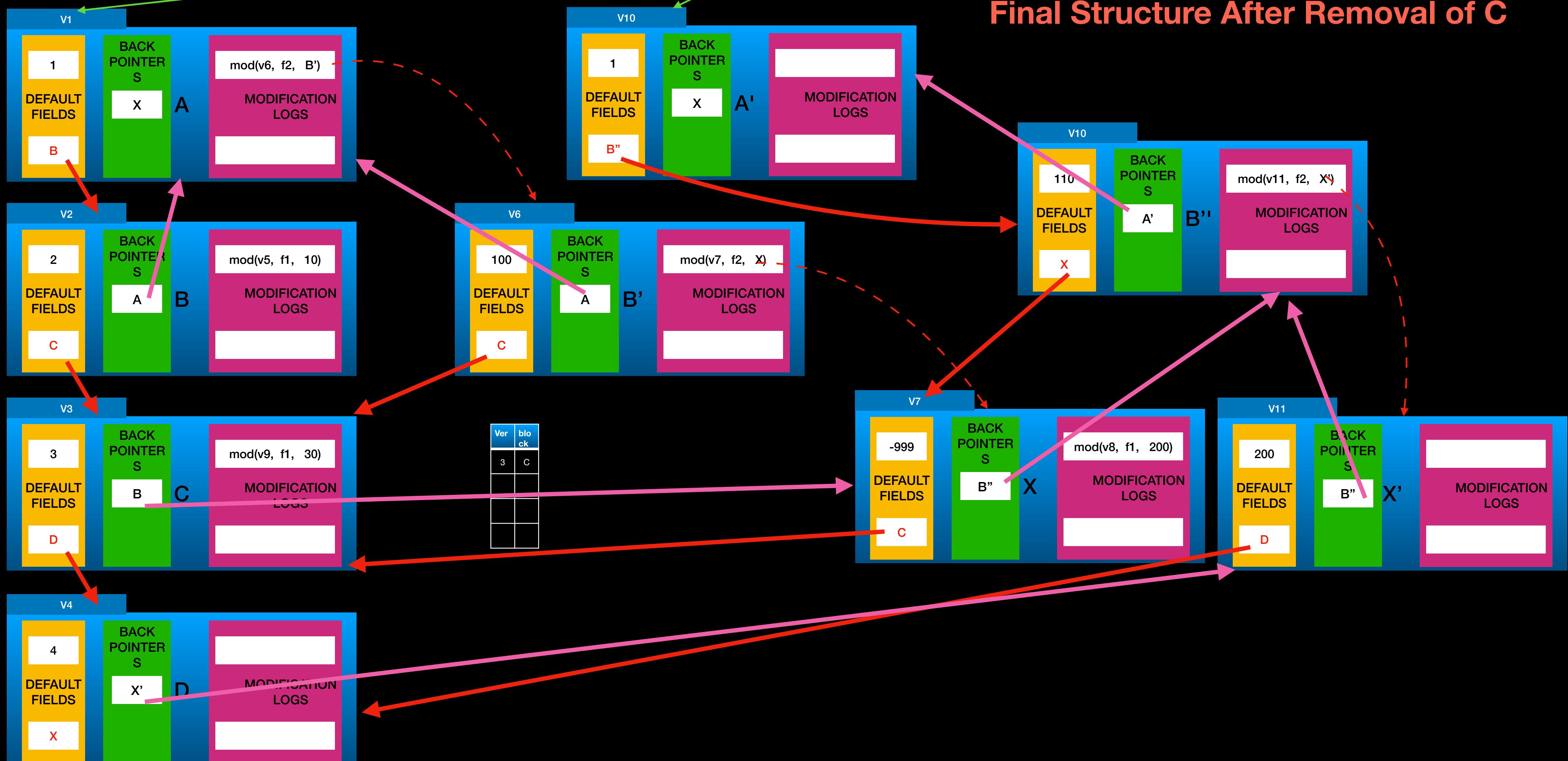


Current Version: v11

# START MODULE

V0 V10

remove(C)



VERSION REDIRECTOR MODULE	
Ver	block
1	A
10	A'
2	B
6	B'
10	B''
7	X
11	X'

VERSION REDIRECTOR MODULE	
Ver	block
2	B
6	B'
10	B''

VERSION REDIRECTOR MODULE	
Ver	block
3	C
4	D
7	X
11	X'

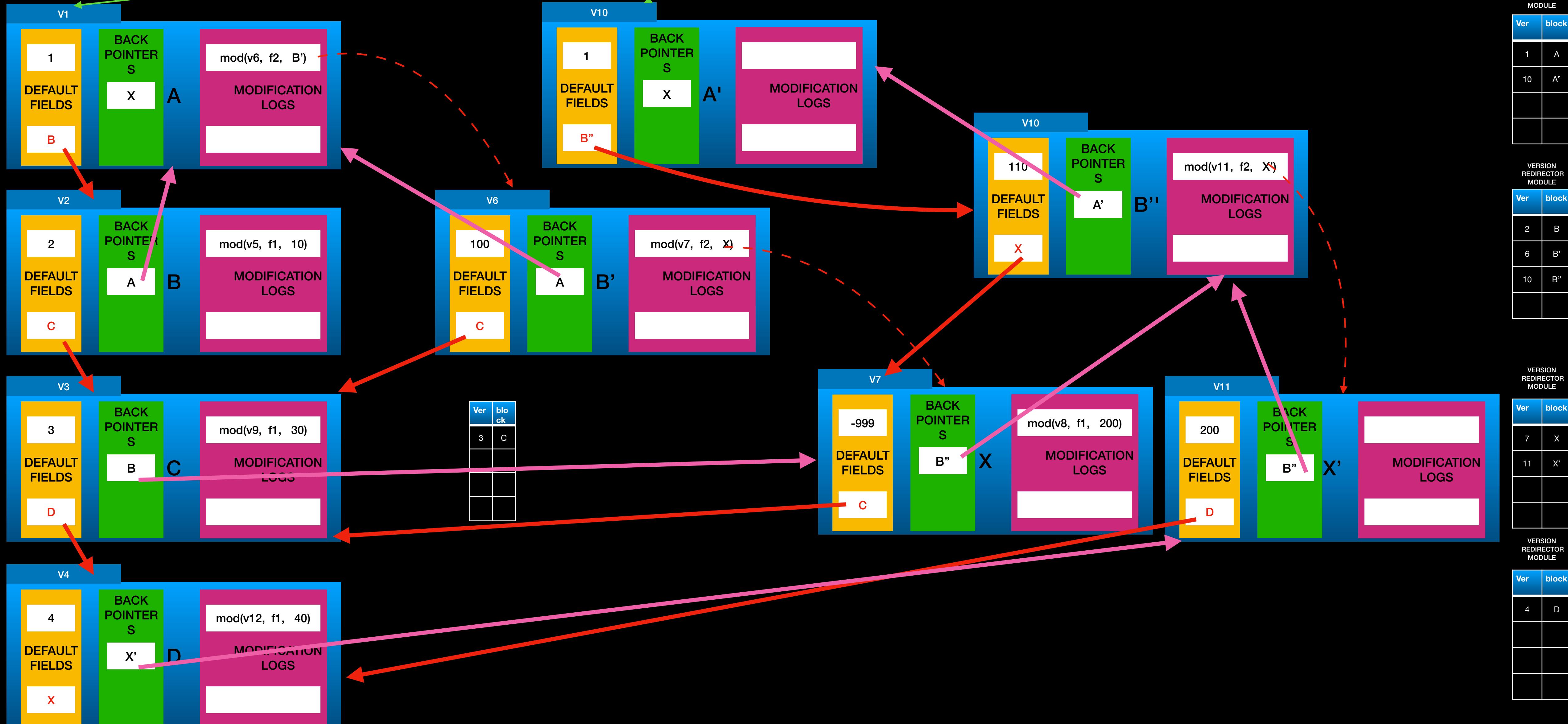
VERSION REDIRECTOR MODULE	
Ver	block
4	D

Current Version: v12

# START MODULE

V0 V10

update(f1,D,40)

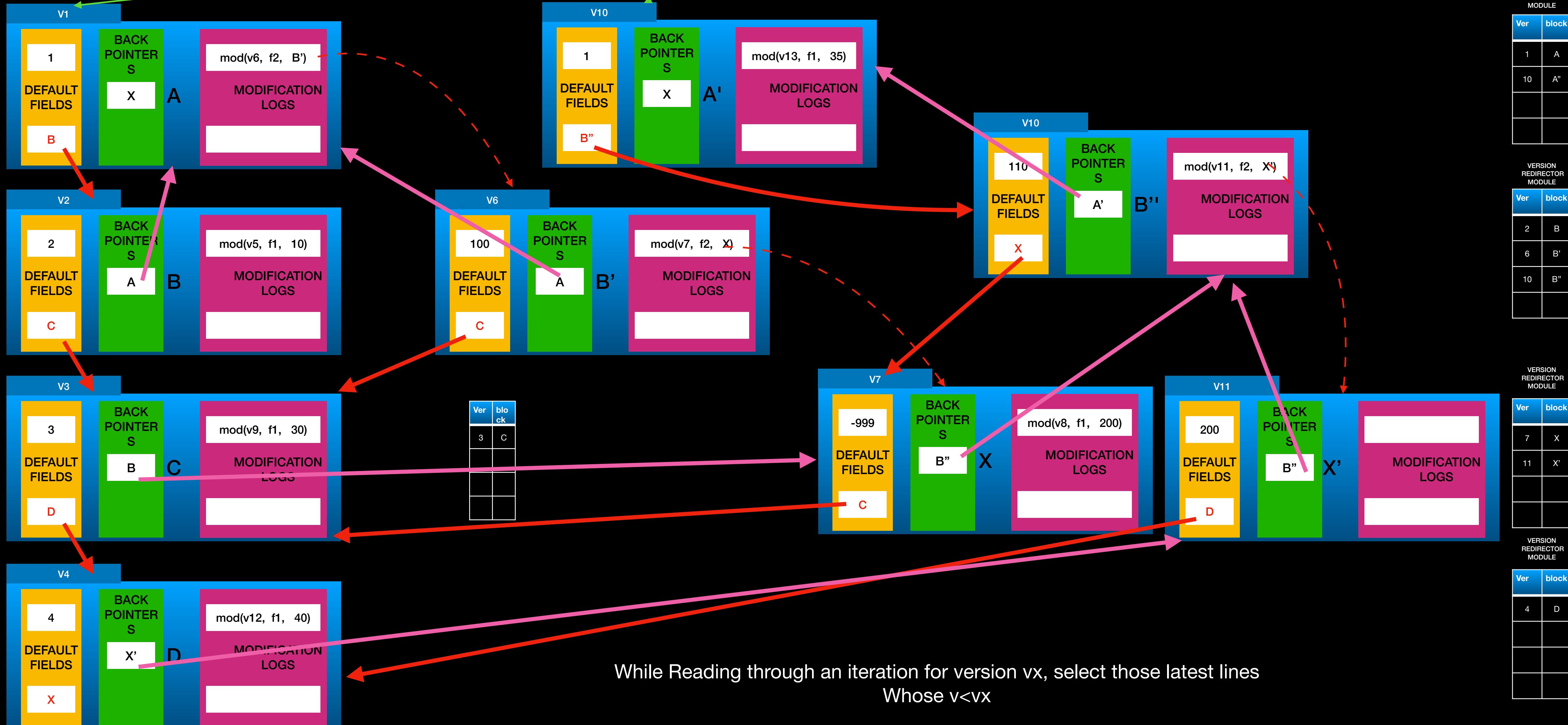


Current Version: v13

# START MODULE

V0 V10

update(f1,A,35)



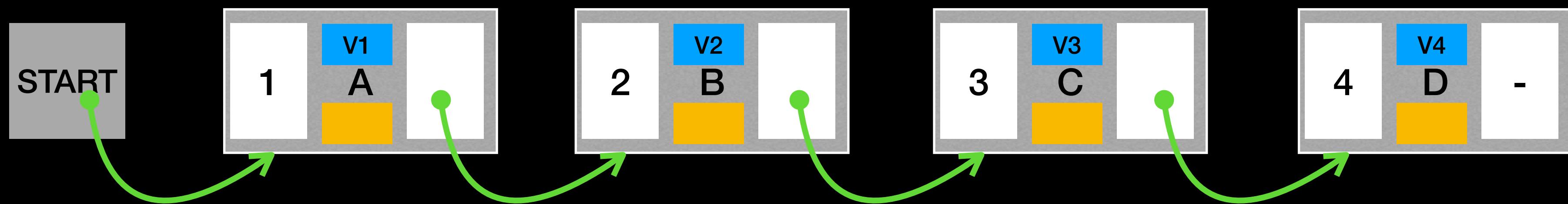
# **iterate\_LL\_at\_v(vx)**

To print the list at v\_x

- Start from start module
- Choose those lines whose version **IS JUST LESS THAN OR EQUAL TO v\_x** (as, suppose if we are traversing for v5 , either v0,v1,v2,v3, v4 or v5 can be on that path, lines>v5 can't be on that).
- The word “JUST LESS” is written because if a NODE has two version lines in that, e.g. v2 and v4 and we are searching for v5, then we should prefer v4 line over v2.

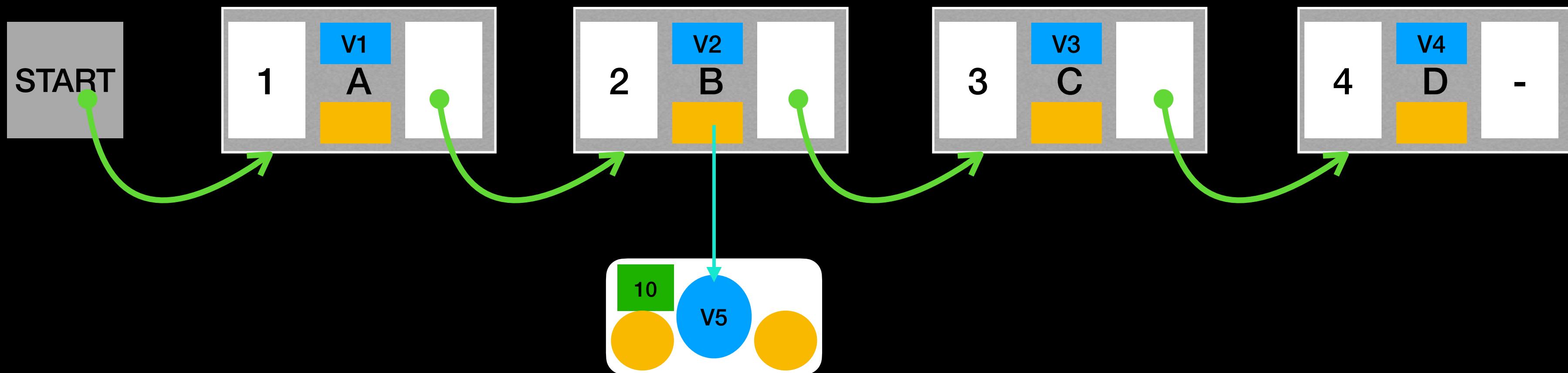
# Why such approach is better ? 😭

## Normal Implementation (using balanced BST [may use just LL])



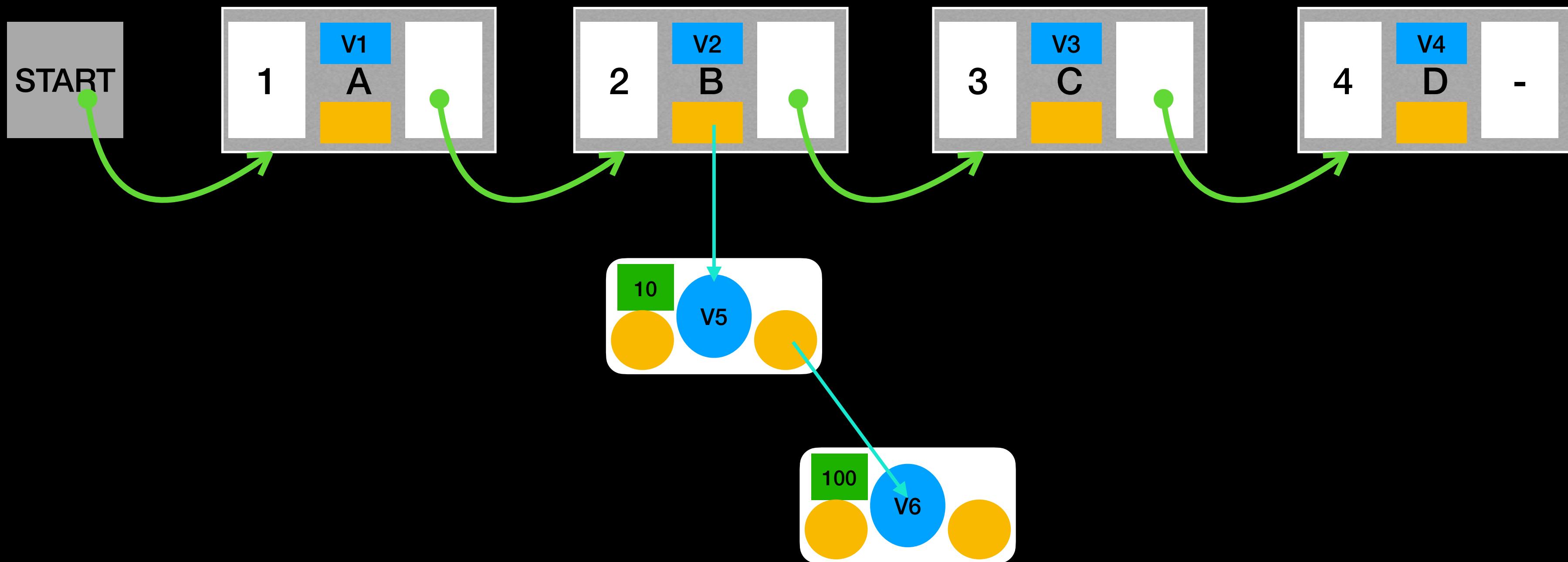
# Why such approach is better ? 😭

## Normal Implementation (using balanced BST)



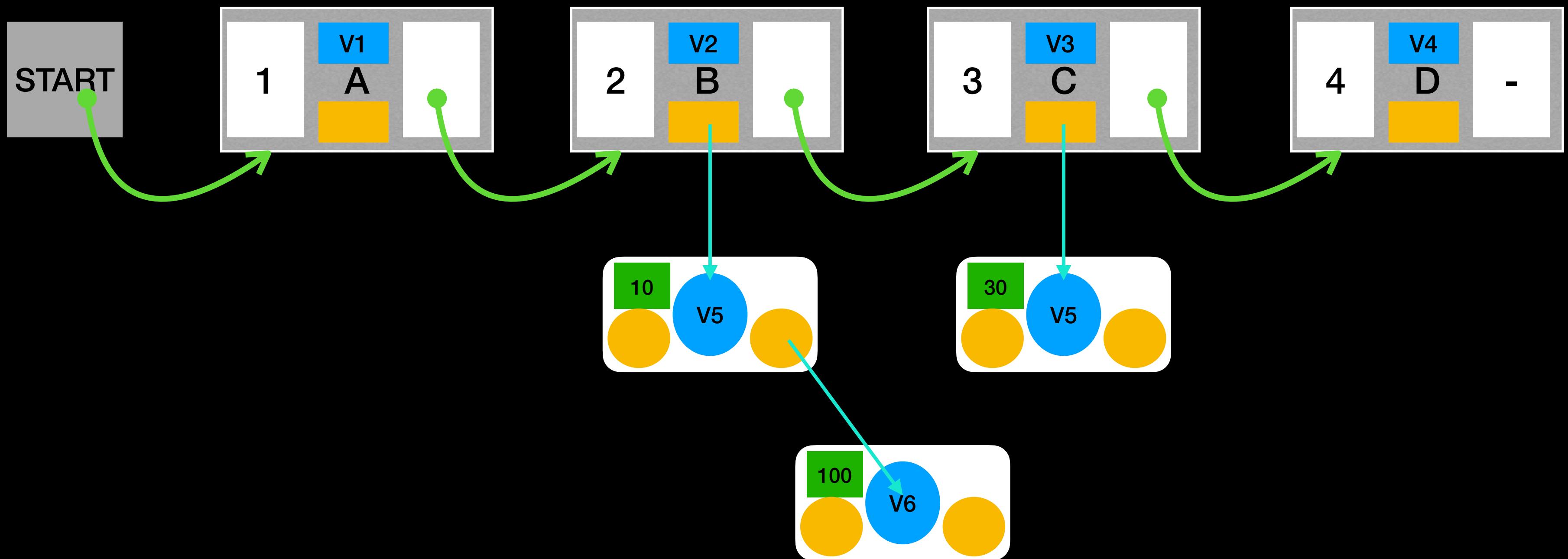
# Why such approach is better ? 😭

## Normal Implementation (using balanced BST)



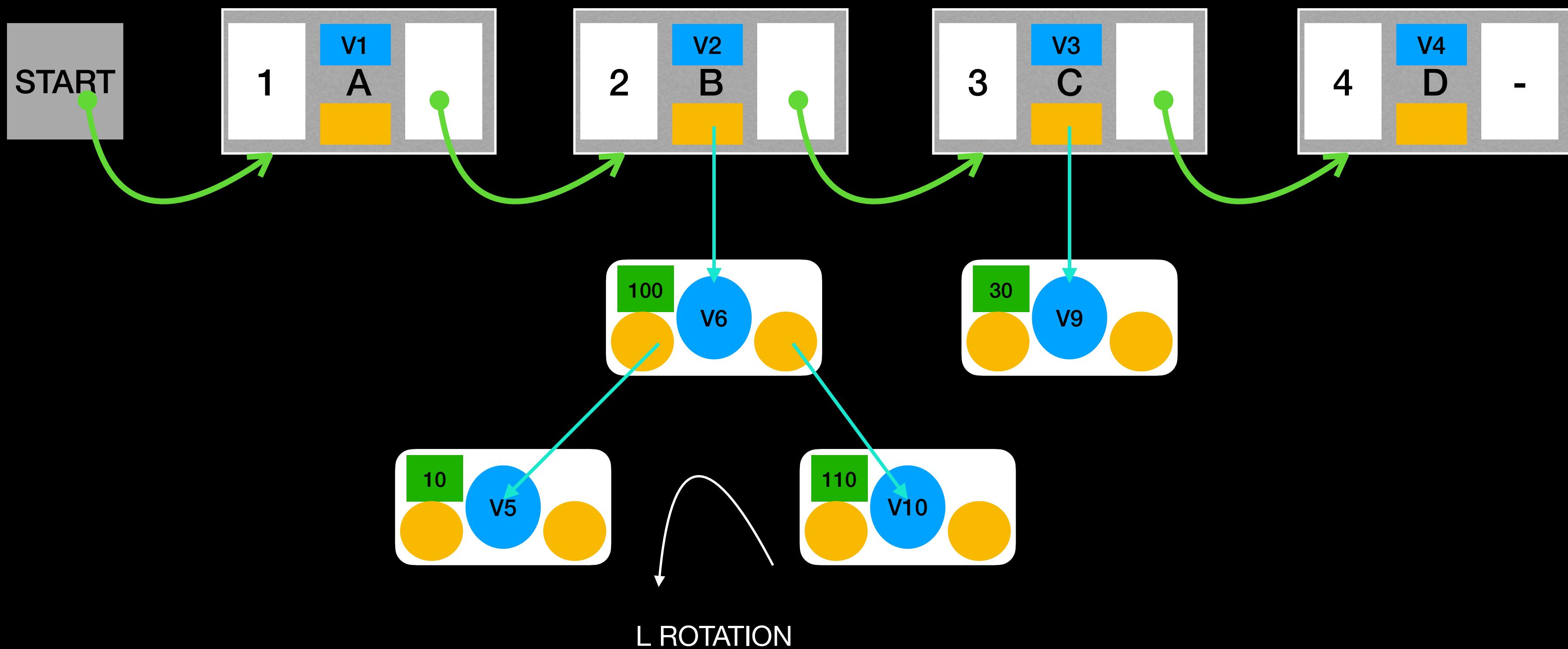
# Why such approach is better ? 😭

## Normal Implementation (using balanced BST)



# Why such approach is better ? 😭

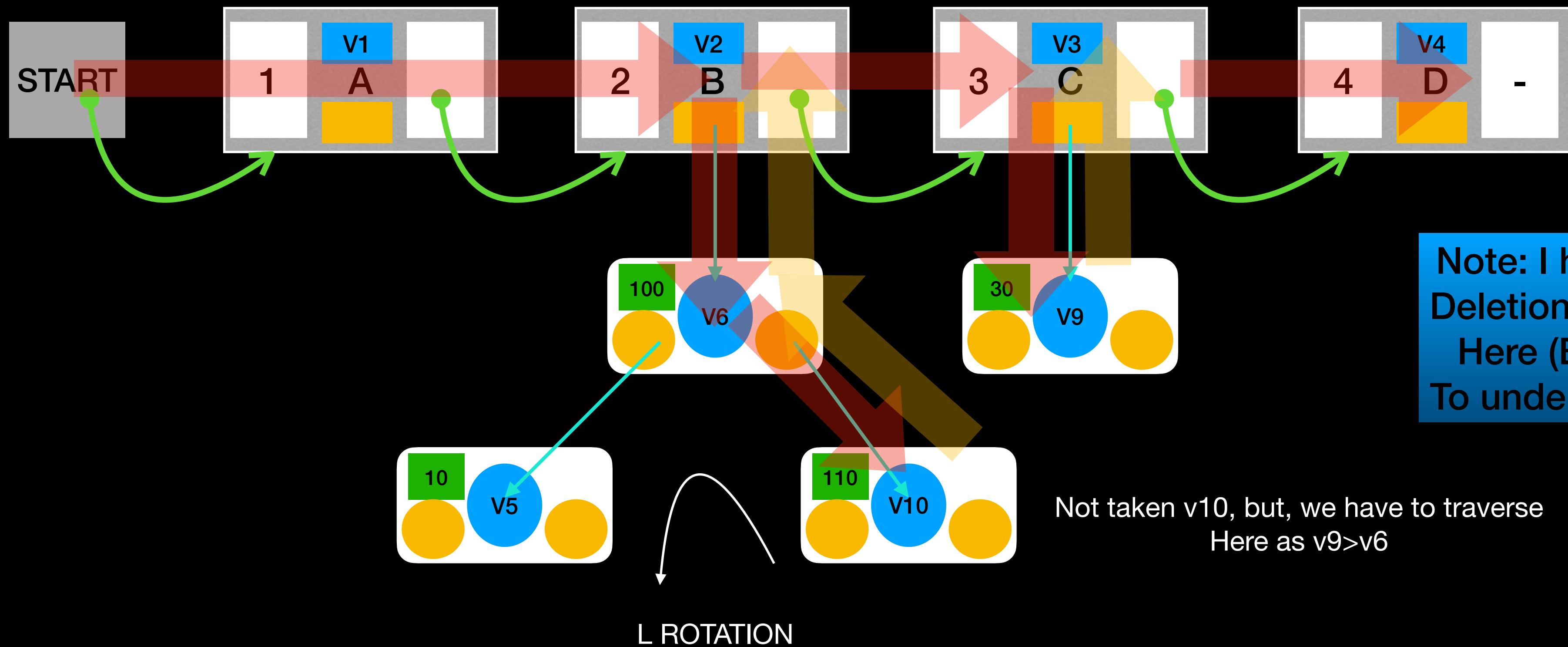
## Normal Implementation (using balanced BST)



# Why such approach is better ? 😊

## Normal Implementation (using balanced BST) (Reading v9)

Output: 1 -> 100 -> 30 -> 4

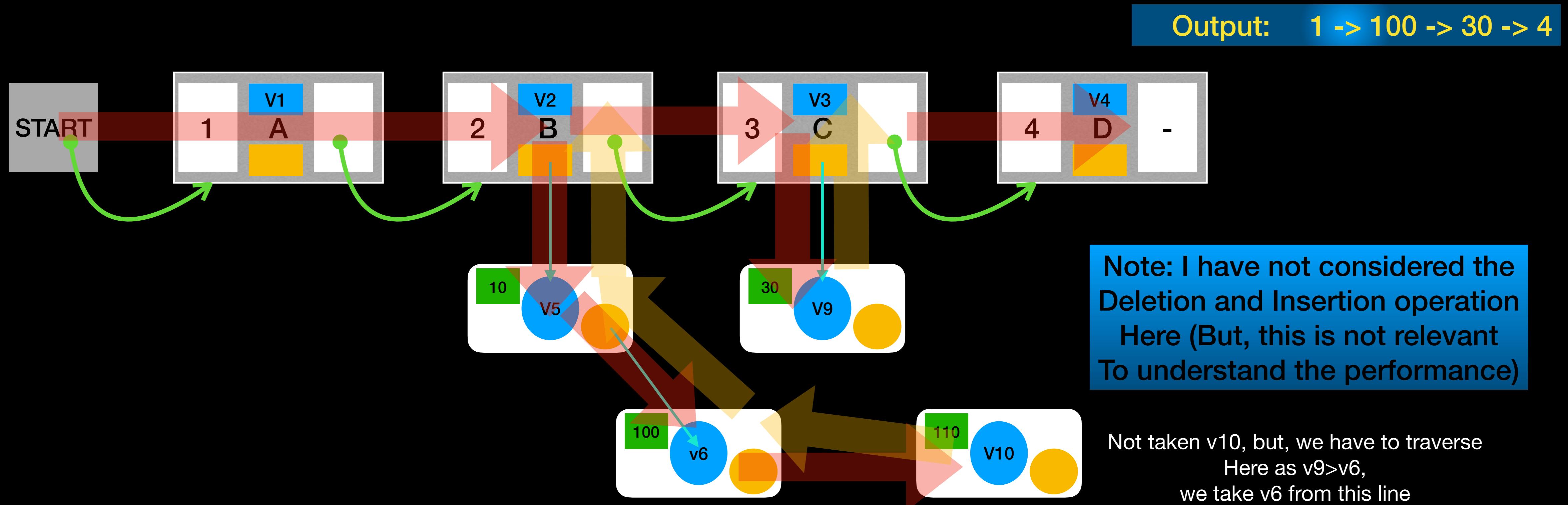


So, the time complexity of reading list v9 ... n \* O(log m)  
n is length of LL and m is #mods per node

Also the creation of the AVL of version to node  
Takes O(log m) each time

# Why such approach is better ? 😊

## Normal Implementation (Using List) (Reading v9)



So, the time complexity of reading list v9 ...  $n * O(m)$   
n is length of LL and m is #mods per node

The creation of the List of version to node  
Takes O(1) each time

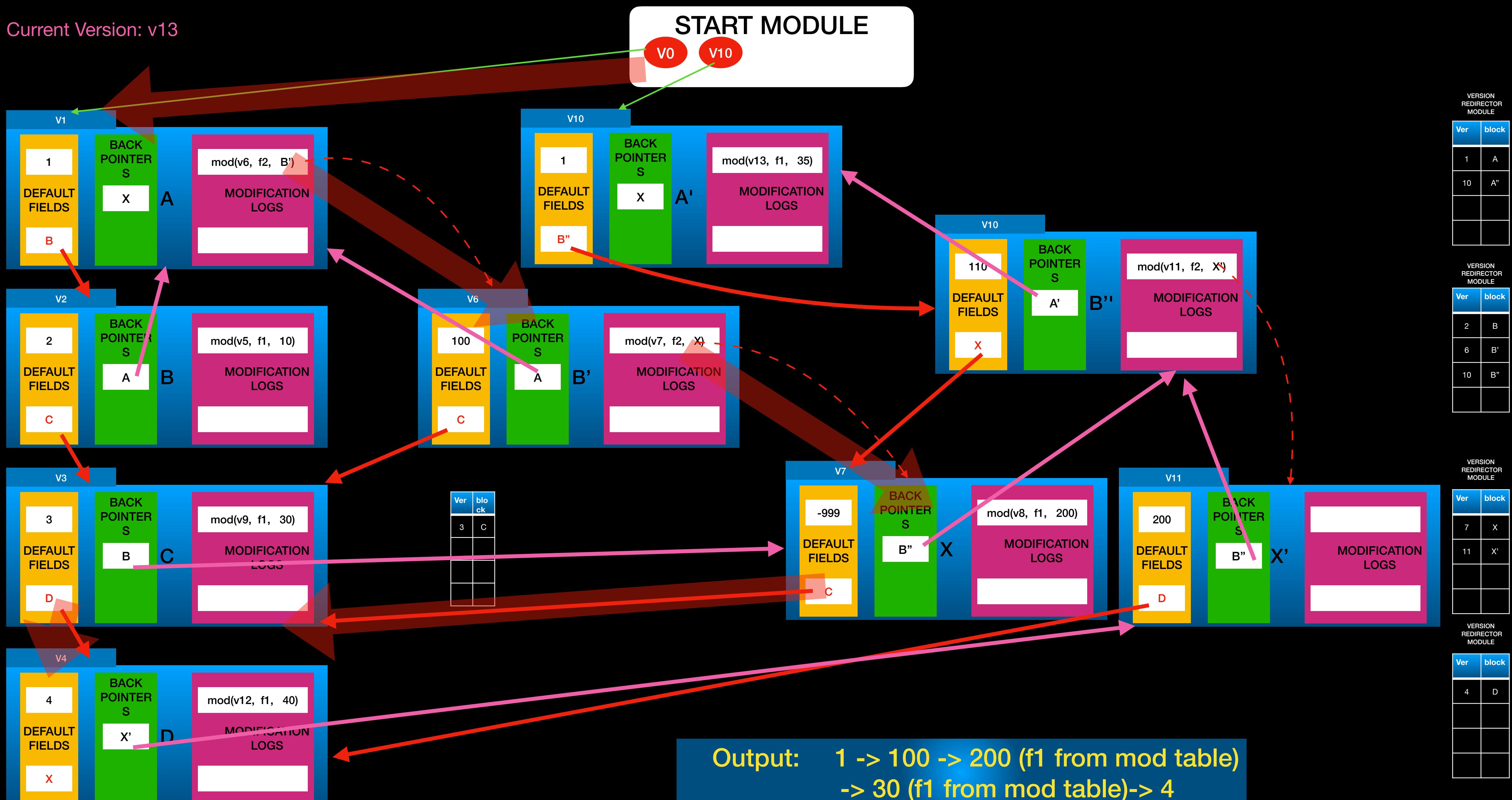
# Why such approach is better ? 😭

## Pointer Machine (Reading v9)

We are taking  $O(2k \cdot n) \rightarrow O(n)$  time for iterating over the Linked List  
Here k is the in-degree (Here 1) of the DS

We are taking  $O(2)$  amortised time for modifying/adding each node to the Linked List  
Here k is the in-degree (Here 1) of the DS

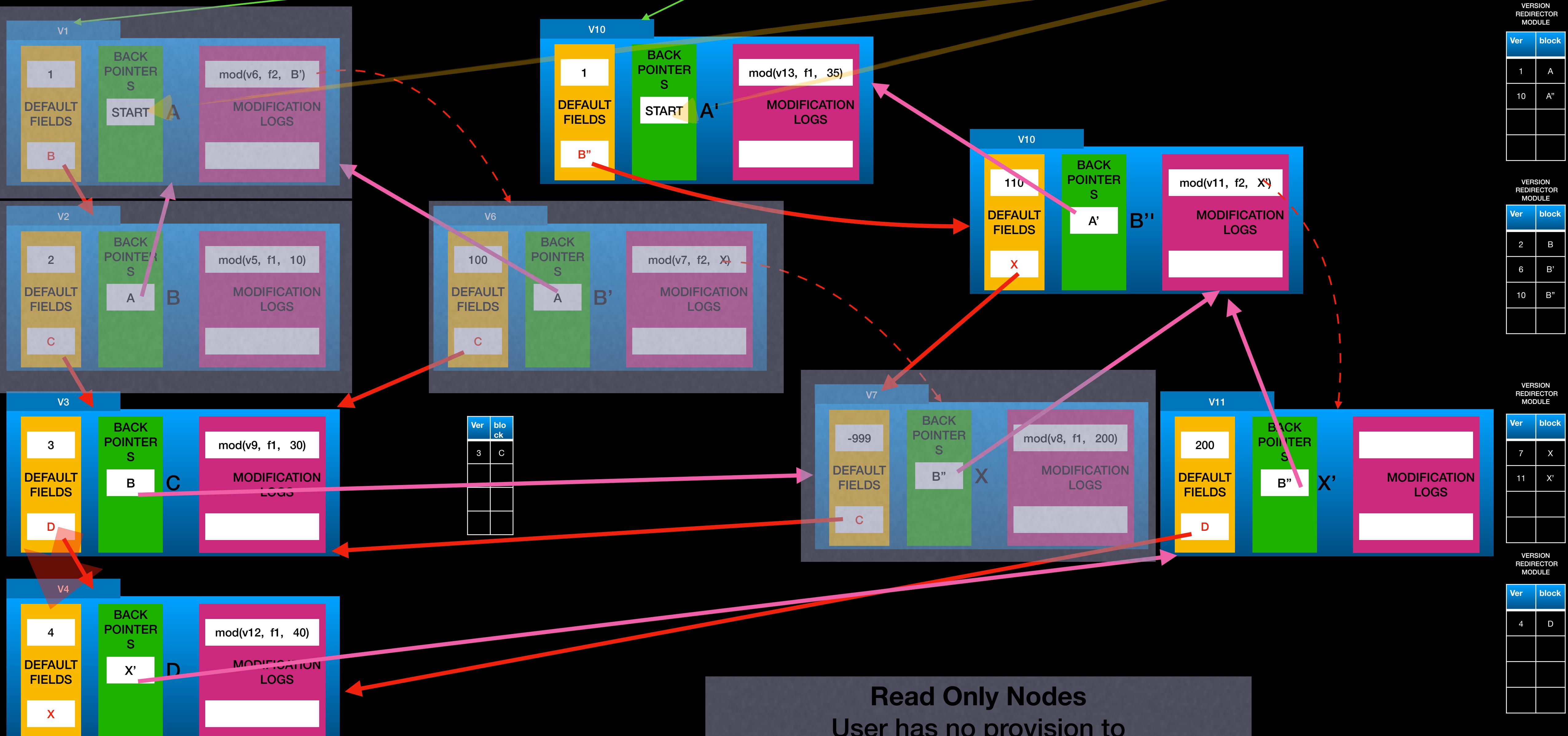
Current Version: v13



# FINAL NOTE

## START MODULE

Nothing At BP Means pointing to address of Start Module



# **Full Persistent Data-structure**

## **Pointer Machine Model**

**PROJECT MEMBERS:** ANANNYO DEY, SOUMYAJIT RUDRA SARMA , DEBASMIT ROY, KANKO GHOSH AND KUSHAL DAS

# Full Persistent Linked List

## Operations:

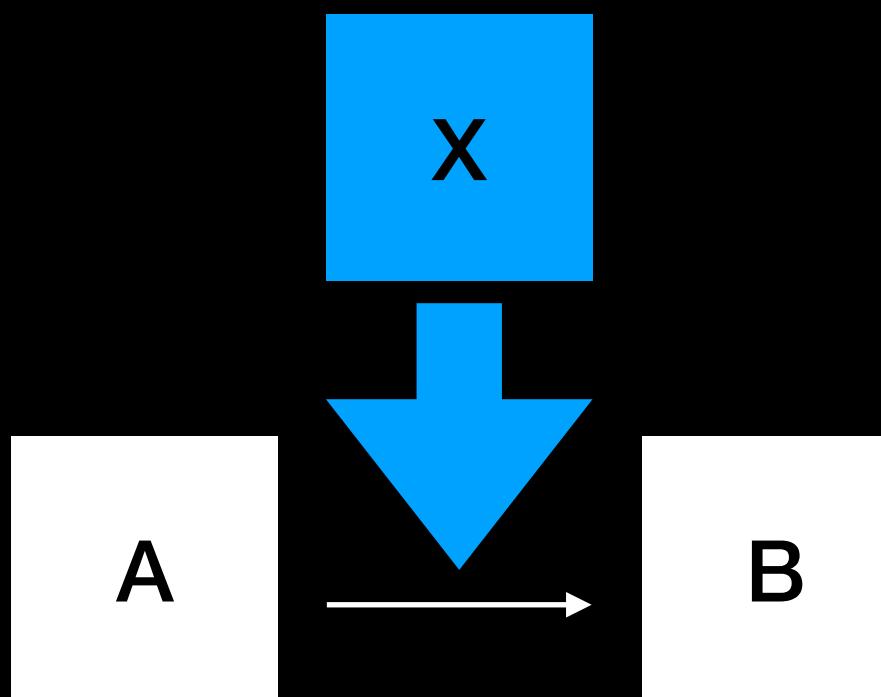
- `start = init()` : To initiate linked list and “start” pointer holds the starting position in v0
- `add(x, y, a, v)` : Add new node x after y at version v with f1 = a and f2 = NULL, and update the version in version tree.
- `create_node(x, a)`: Allocate a new node x with value = a, and set its default version = current time
- `remove(x,v)` : Remove node x and update the version at version v and update the version in version tree.
- `iterate_over_LL(v)` : Iterate over the whole linked list in version v
- `update(x,f_i,val,v)` : Update the i-th field in node x to new value ‘val’ at version v and update the version in version tree.

# Interesting thing!

## $\text{add}(x,y,v)$ and $\text{remove}(x,v)$ are not Elementary operations

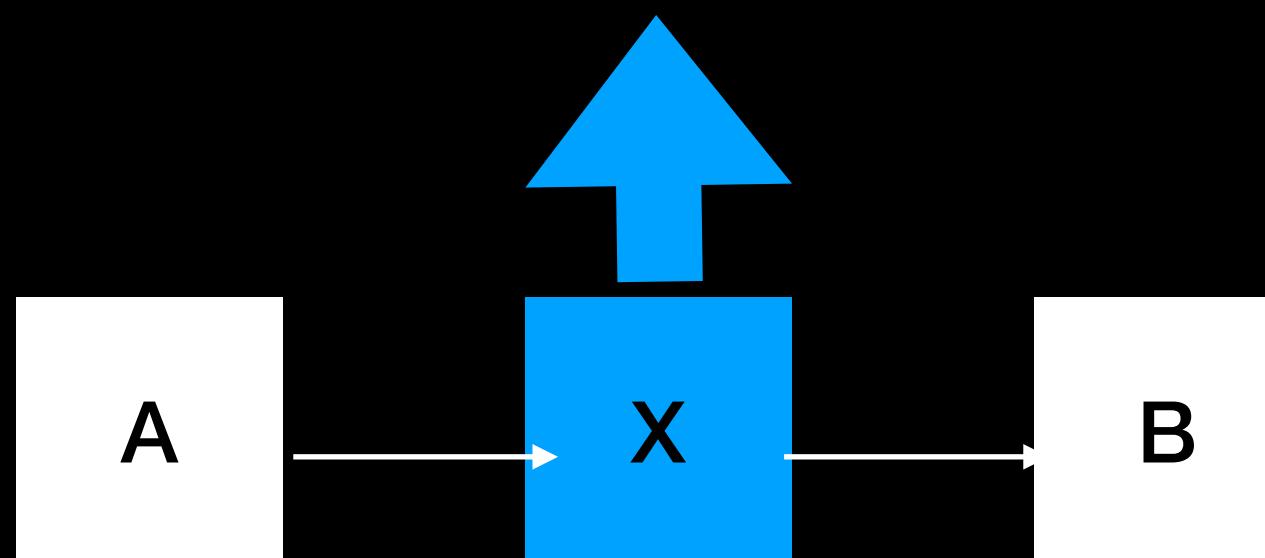
$\text{add}(X,C,123)$  consists of

Create node X with value 123
Modify f2 of A to X : update(f2, A, X,v)
Modify f2 of X to C : update(f2, X, C,v)
Modify BP1 of X to A: update(bp1, X, A,v)
Modify BP1 of B to X: update(bp1, B, X,v)
Set f1 of X(optional)
Add current ver to the respective position in version tree



$\text{remove}(x)$  consists of

Modify F2 of Parent C (i.e., X) -> F2 of C (successor of C after version v)
update(f2, A, B,v)
Modify BP1 of B to A: update(bp1, B, A,v)
If all shared reference of X is removed Then free up the memory associated with X



# Elementary Operations:

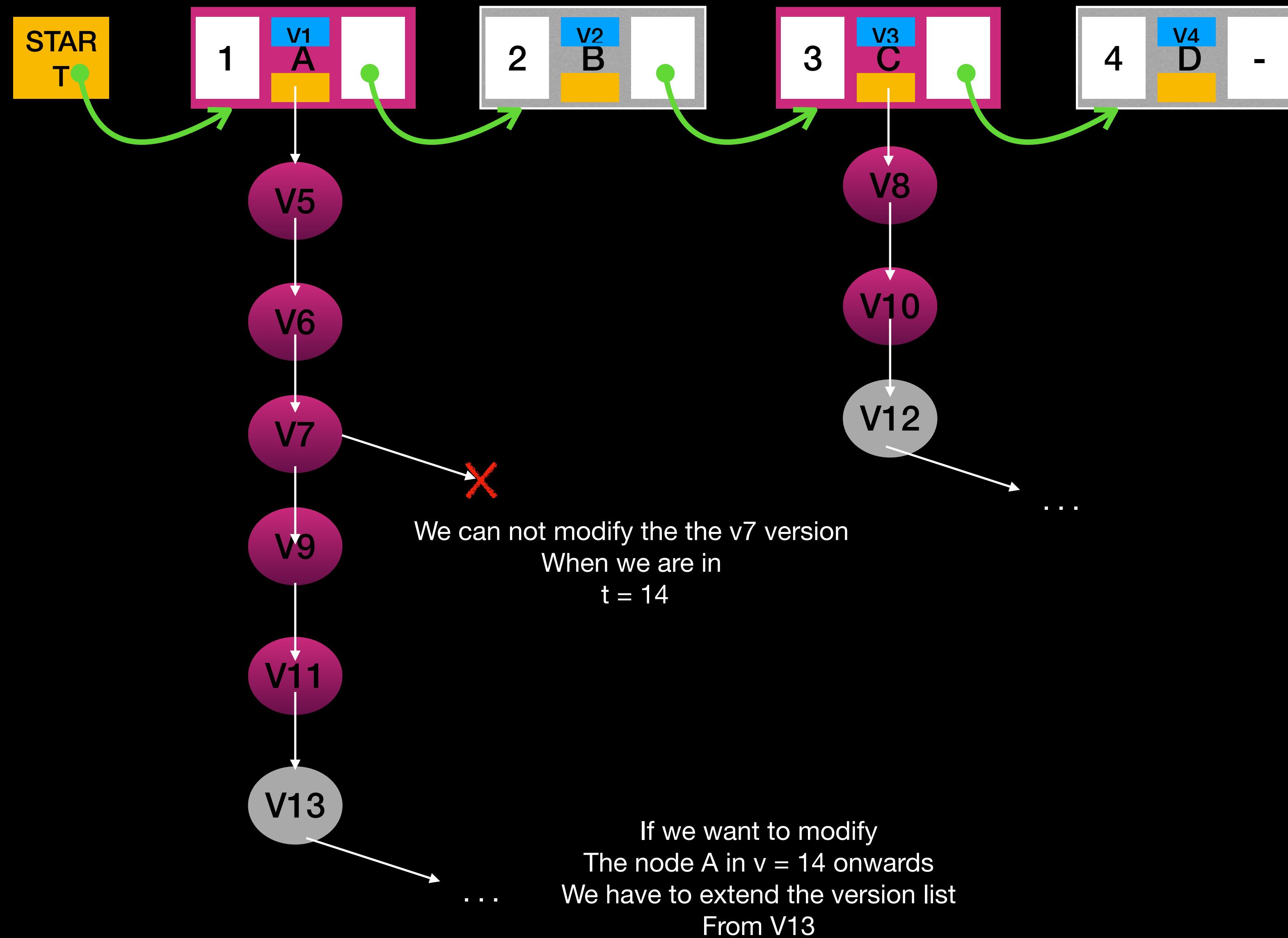
- `start = init()`
- `create_node(x, a)`
- `iterate_over_LL(v)`
- `update(x,f_i,val,v)`

# Why we need Full Persistent Data-structure?

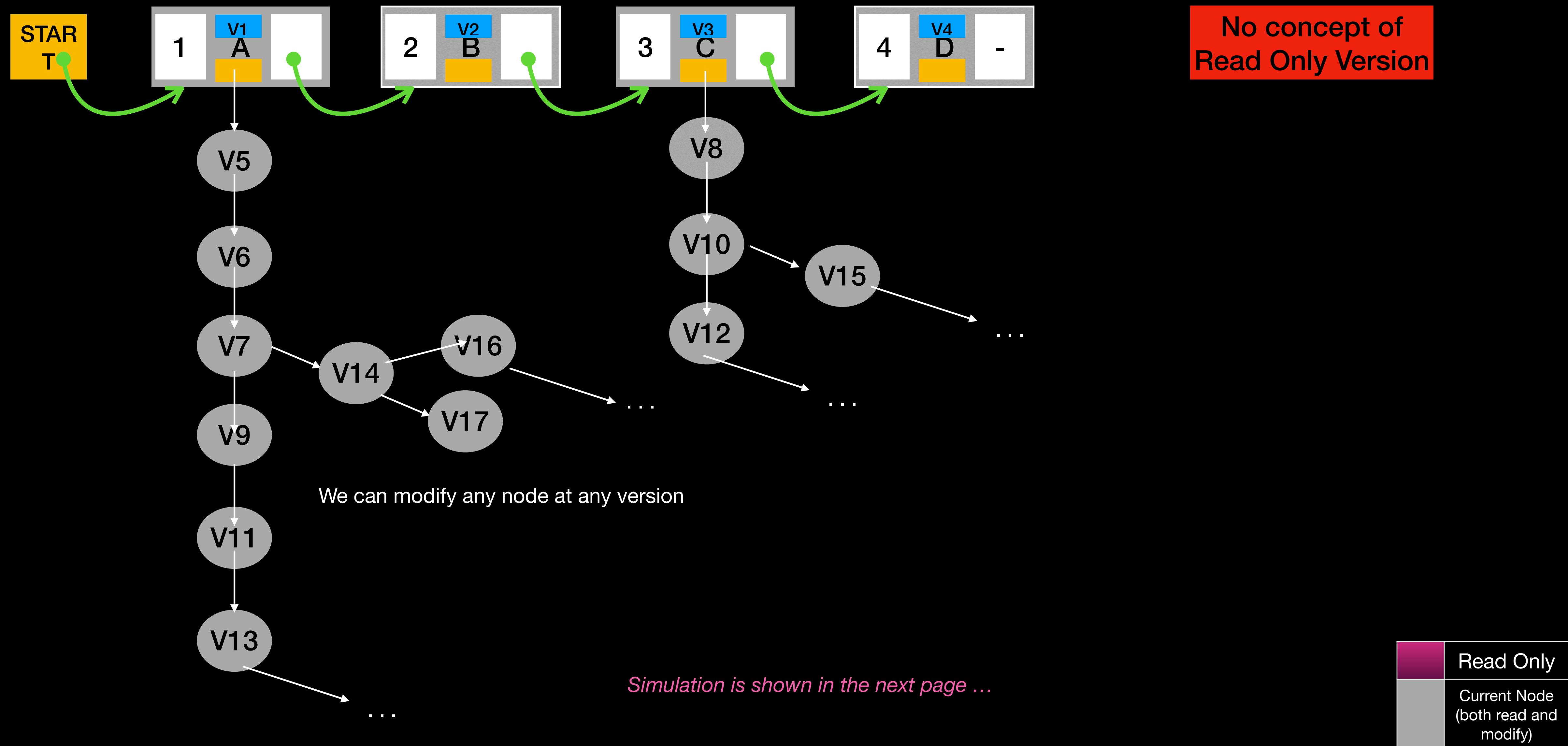
## Basic Idea

- Now we can modify any previous version.
- Branching of versions - is possible
- In Partial Persistent Data Structure we saw Linear Ordering of versions, previous versions were in read-only state. We could modify the latest version of any node.
- But, in Full Persistent Mode, we can branch the version order using Version Tree (*optimisation can be done using Order Maintenance List*) and modify any node at any version.

# Problem in Partial Persistent Mode



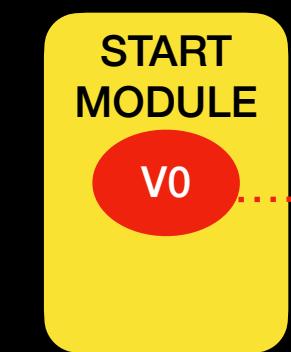
# Remedy in Full Persistent Mode



# Query: init\_LL()

Note: Here **VERSION** at each nodes/modules  
are not just numbers, rather they are  
the **ADDRESS** of corresponding version nodes in  
version tree

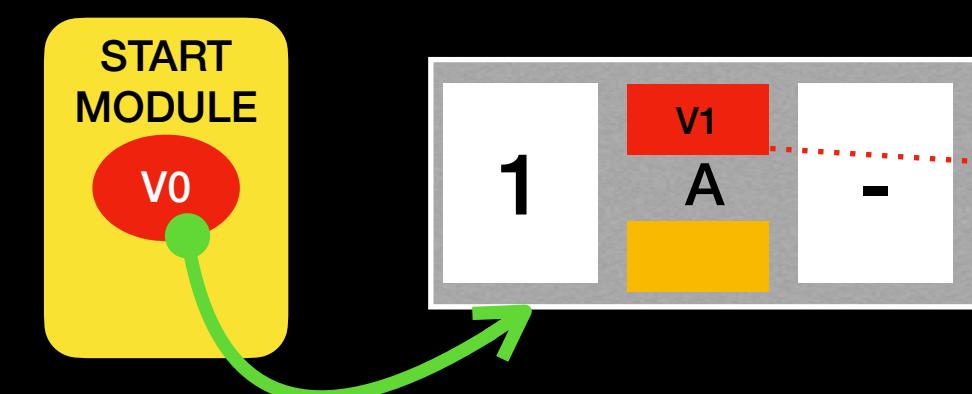
Current time, t = 0



Version tree



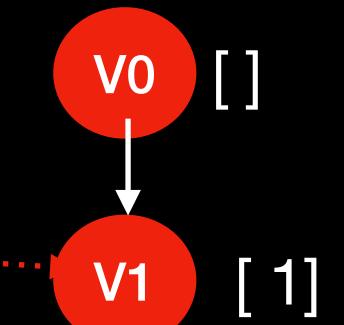
# Query: add(A,\_,v0,1)



Note: Here **VERSION** at each nodes/modules  
are not just numbers, rather they are  
the **ADDRESS** of corresponding version nodes in  
version tree

Current time, t = 1

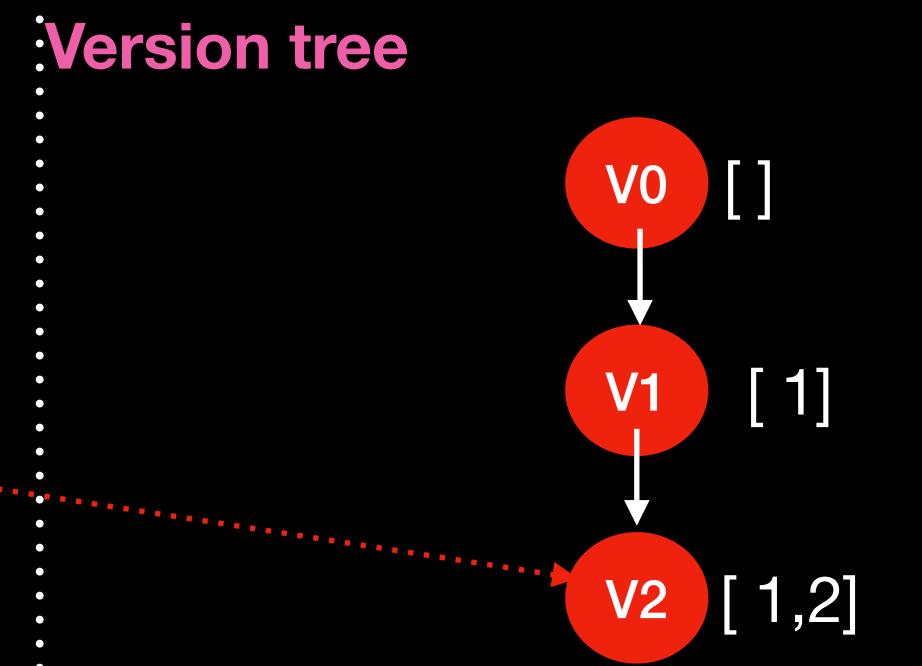
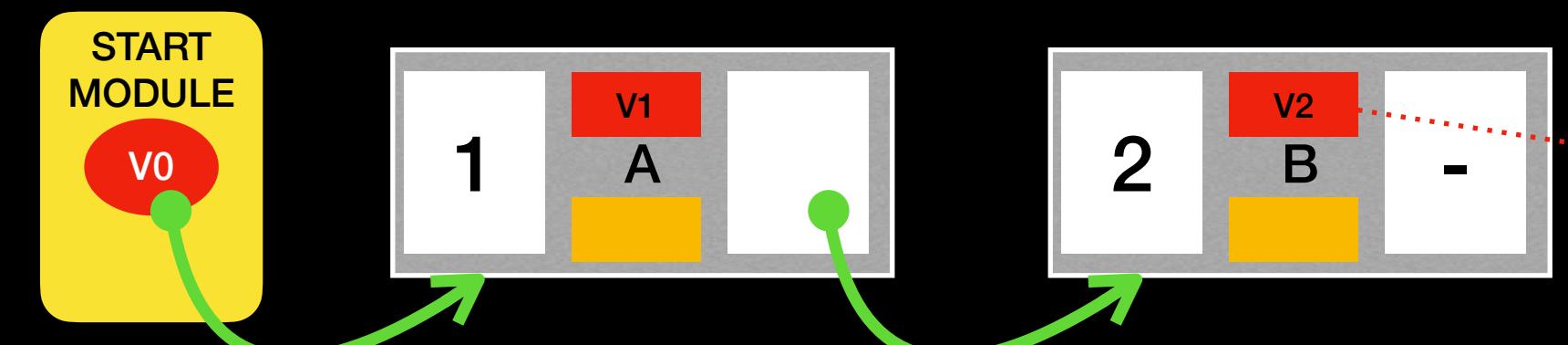
Version tree



# Query: add(B,A,v1,2)

Note: Here **VERSION** at each nodes/modules  
are not just numbers, rather they are  
the **ADDRESS** of corresponding version nodes in  
version tree

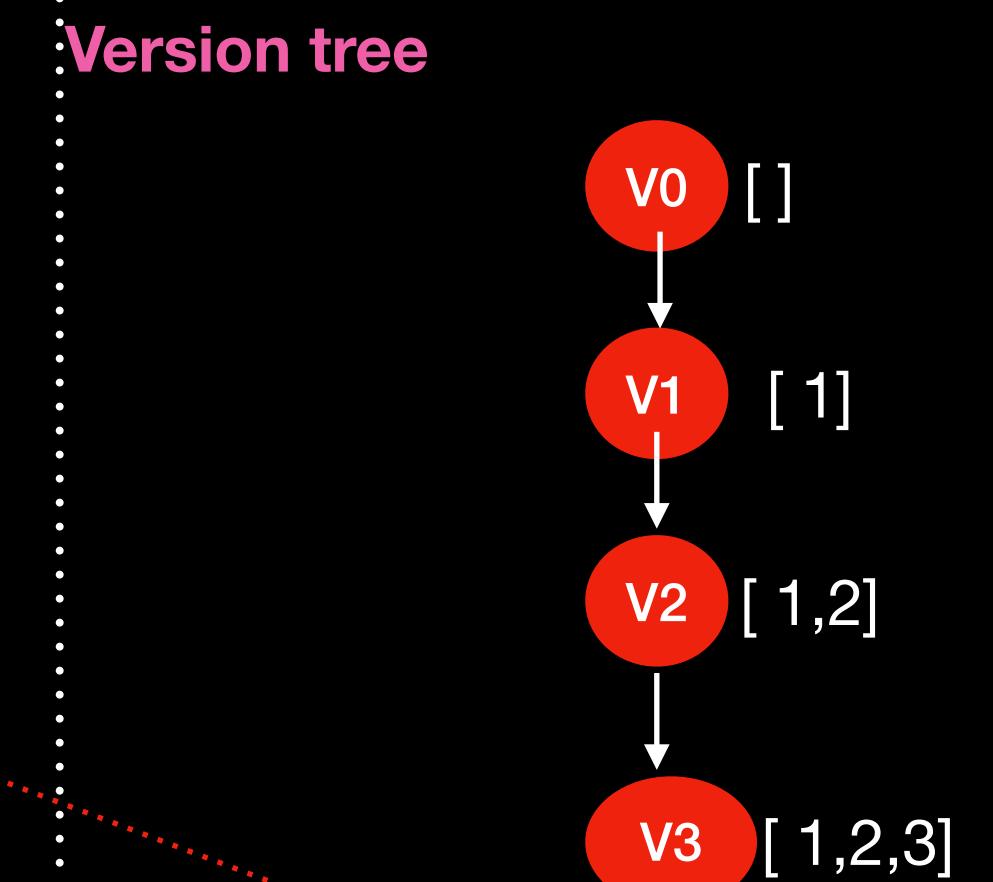
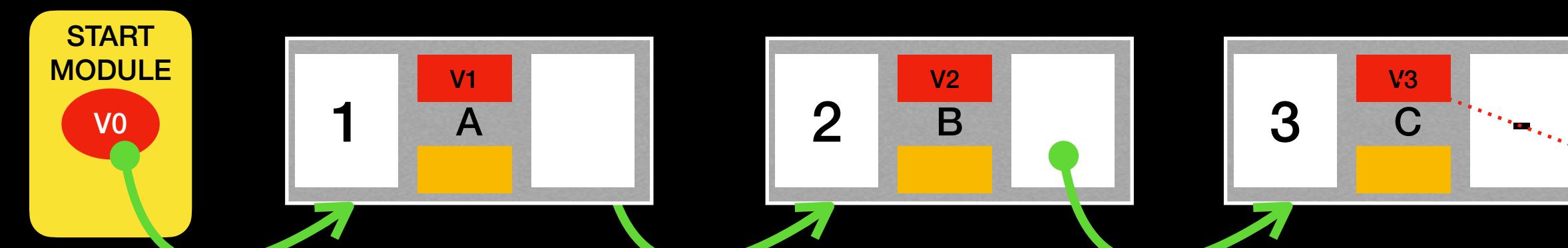
Current time, t = 2



# Query: add(C,B,v2,3)

Note: Here **VERSION** at each nodes/modules  
are not just numbers, rather they are  
the **ADDRESS** of corresponding version nodes in  
version tree

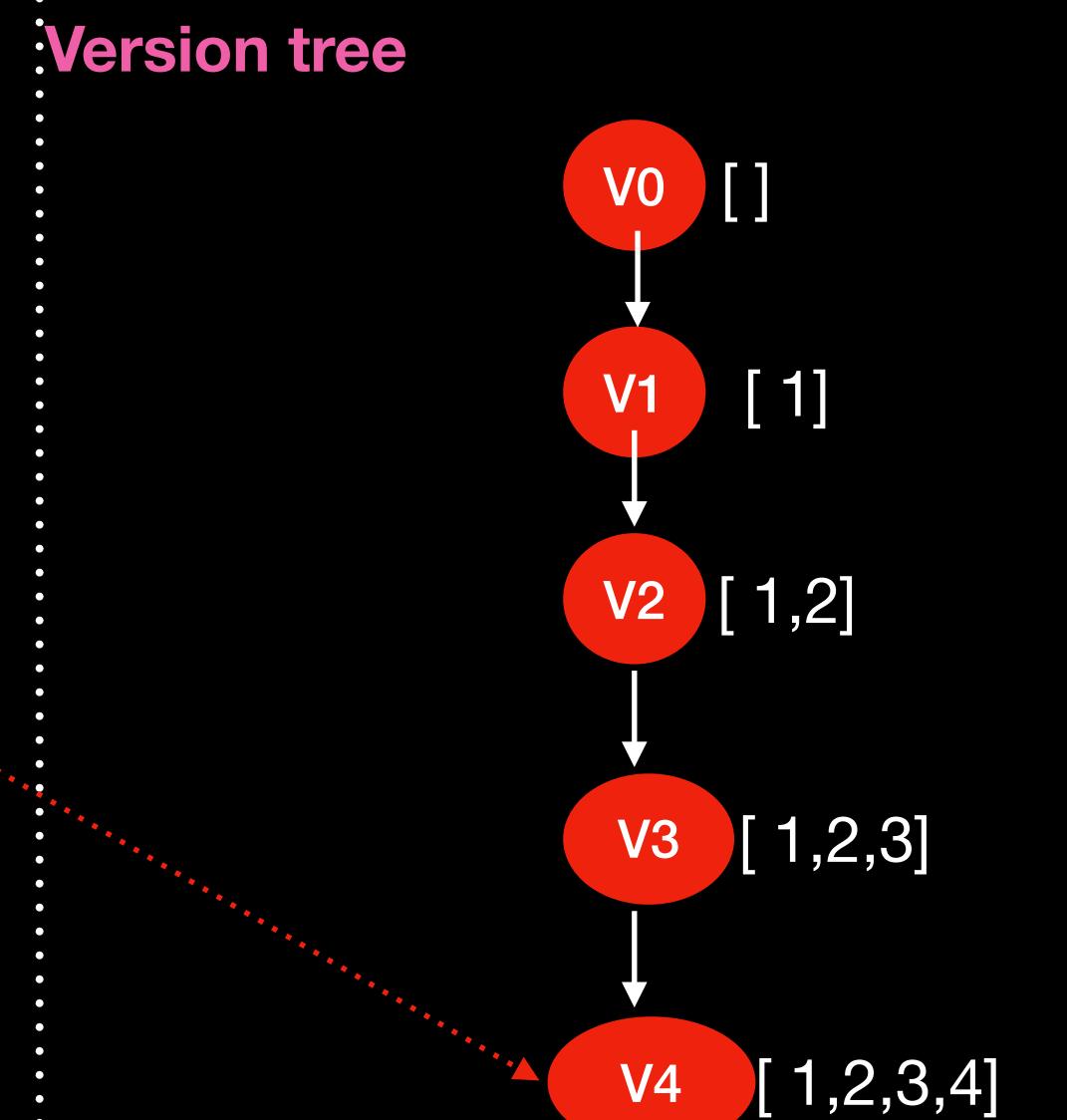
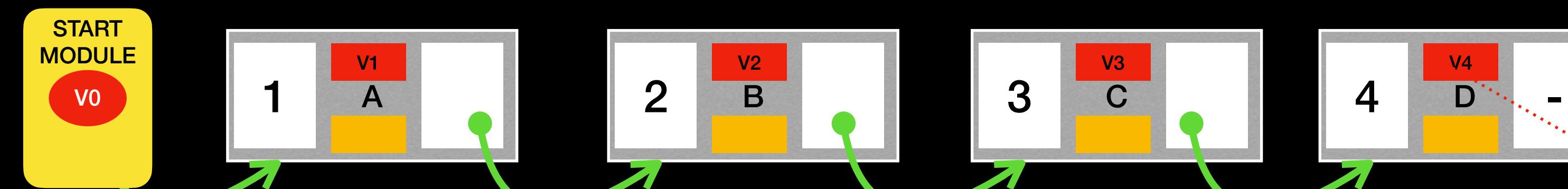
Current time, t = 3



# Query: add(D,C,v3,4)

Note: Here **VERSION** at each nodes/modules  
are not just numbers, rather they are  
the **ADDRESS** of corresponding version nodes in  
version tree

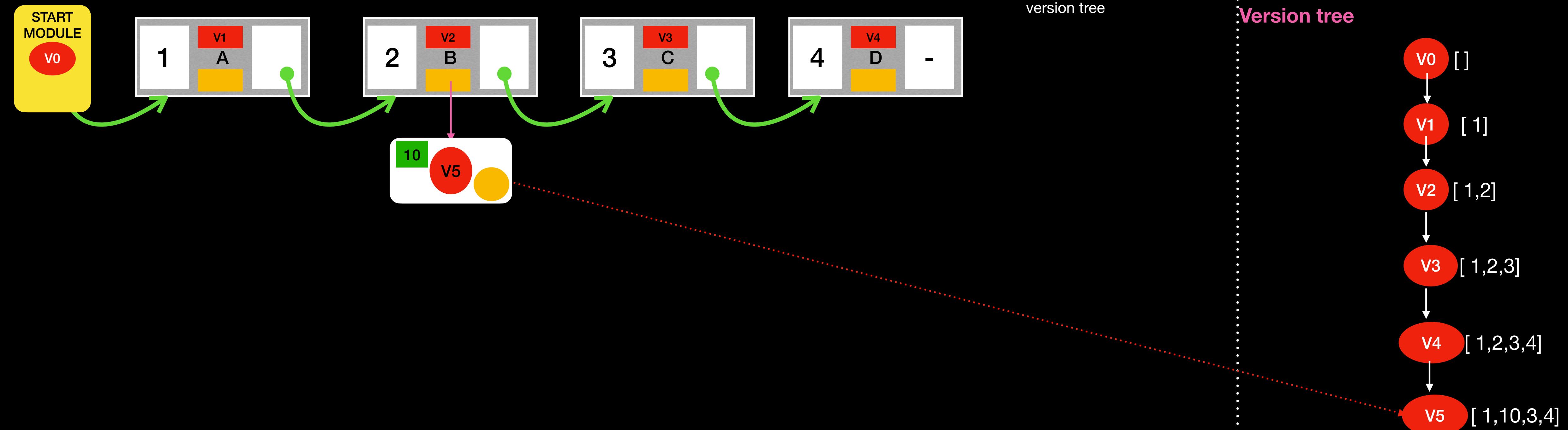
Current time, t = 4



# Query: update(B,f1,10,v4)

Note: Here **VERSION** at each nodes/modules  
are not just numbers, rather they are  
the **ADDRESS** of corresponding version nodes in  
version tree

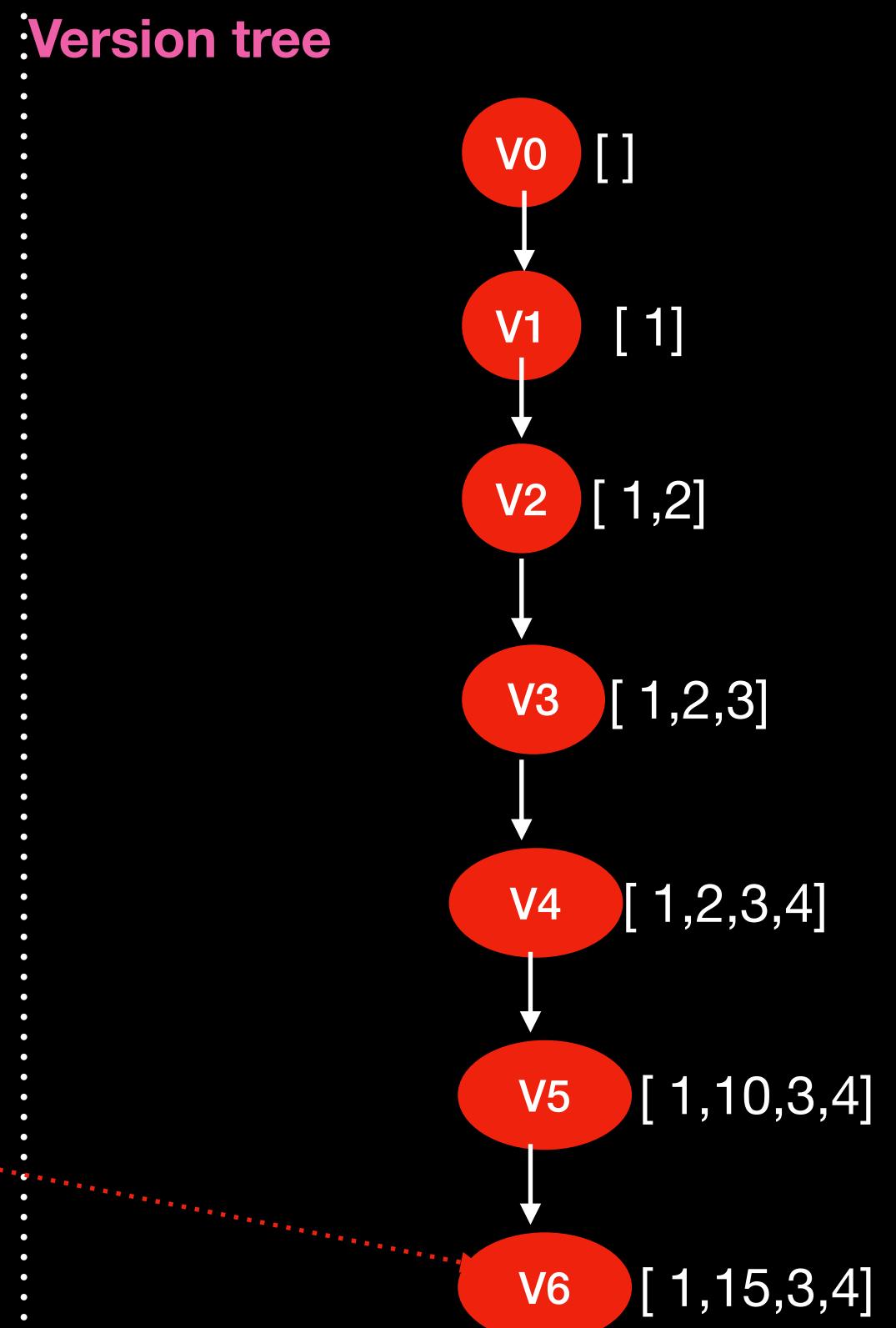
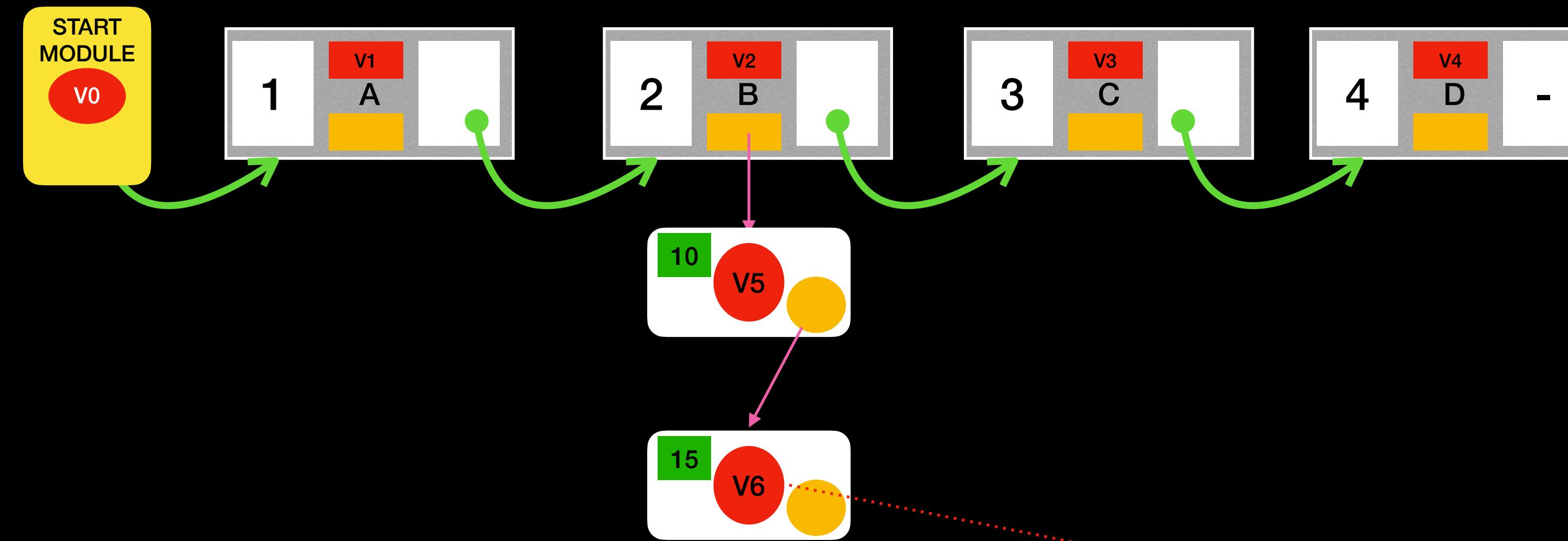
Current time, t = 5



# Query: update(B,f1,15,v5)

Note: Here **VERSION** at each nodes/modules  
are not just numbers, rather they are  
the **ADDRESS** of corresponding version nodes in  
version tree

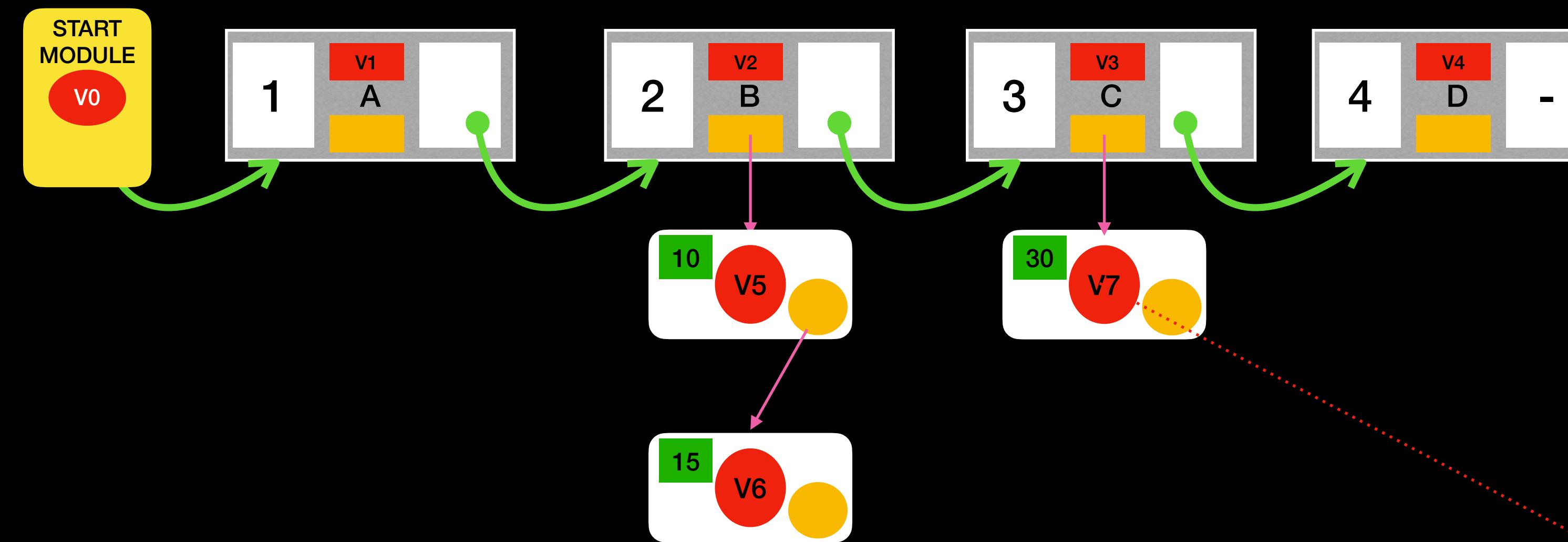
Current time, t = 6



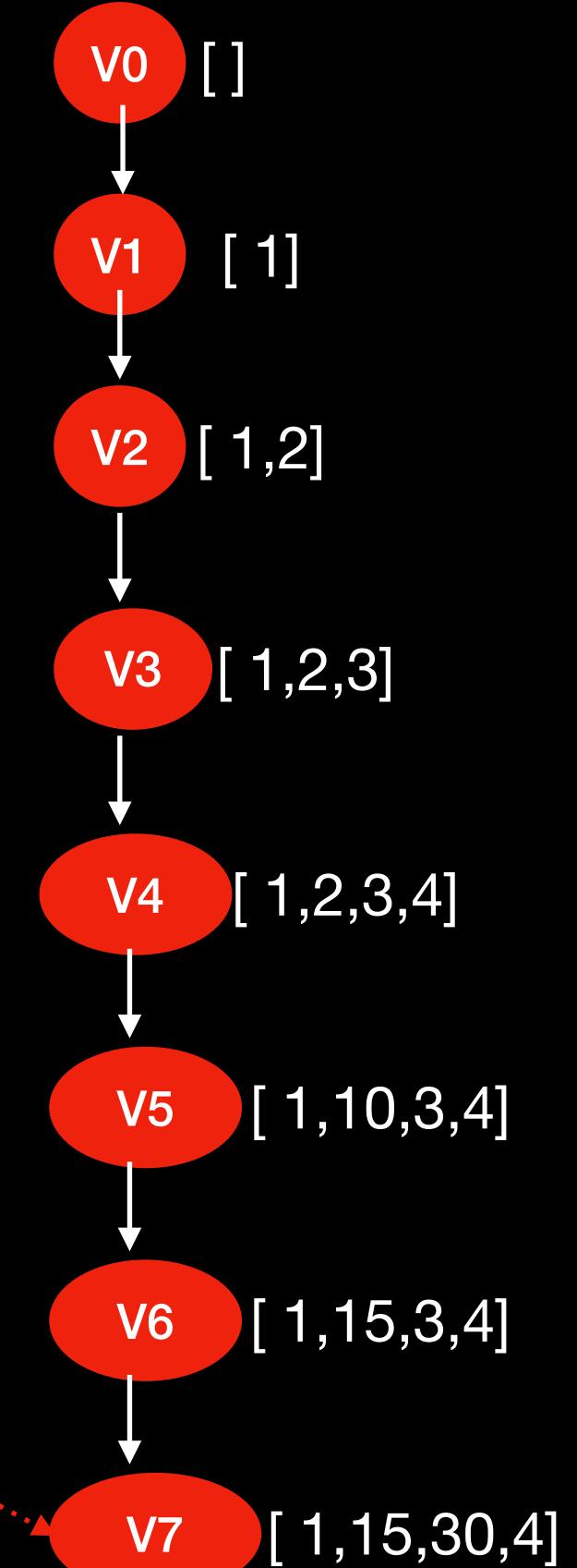
# Query: update(C,f1,30,v6)

Note: Here **VERSION** at each nodes/modules  
are not just numbers, rather they are  
the **ADDRESS** of corresponding version nodes in  
version tree

Current time, t = 7

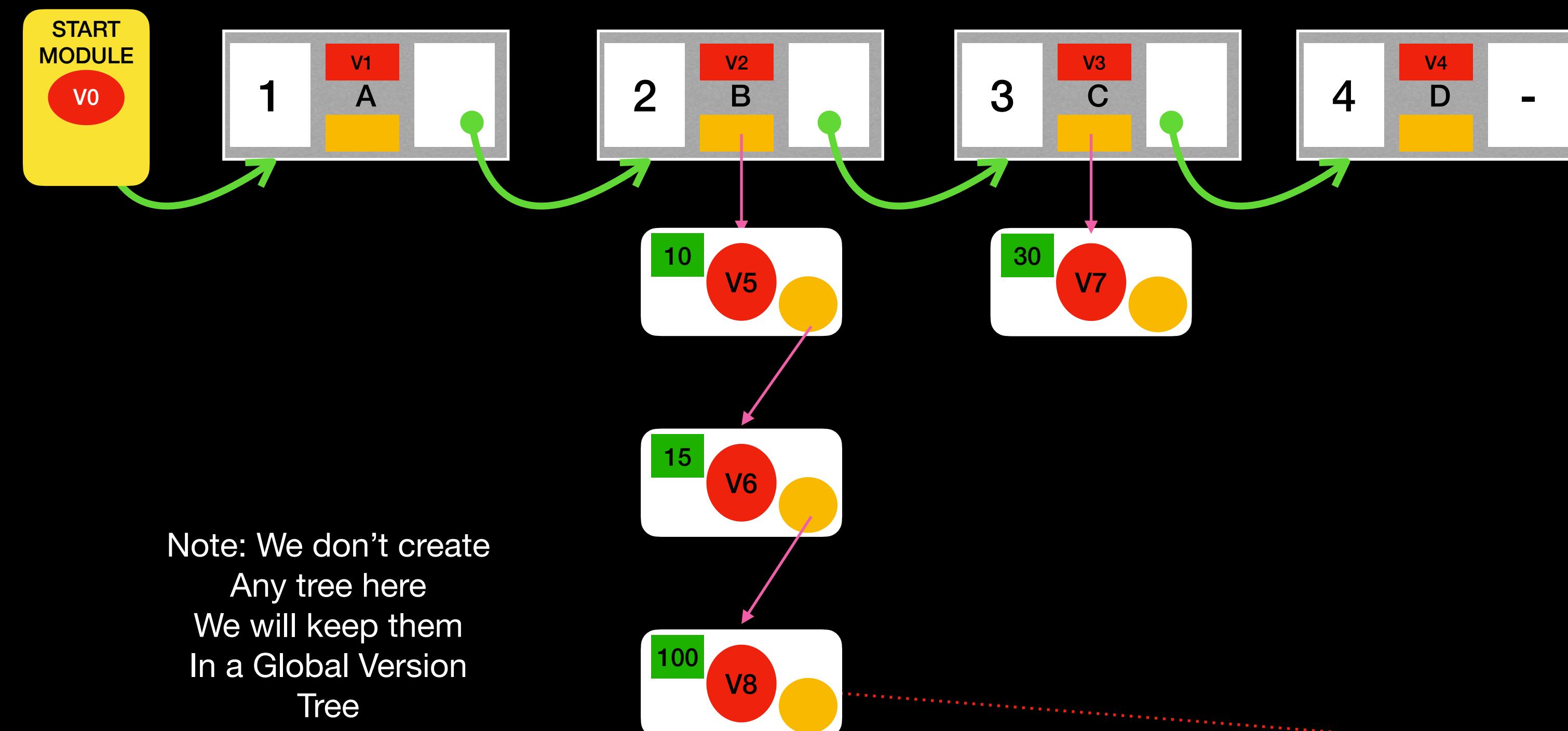


**Version tree**

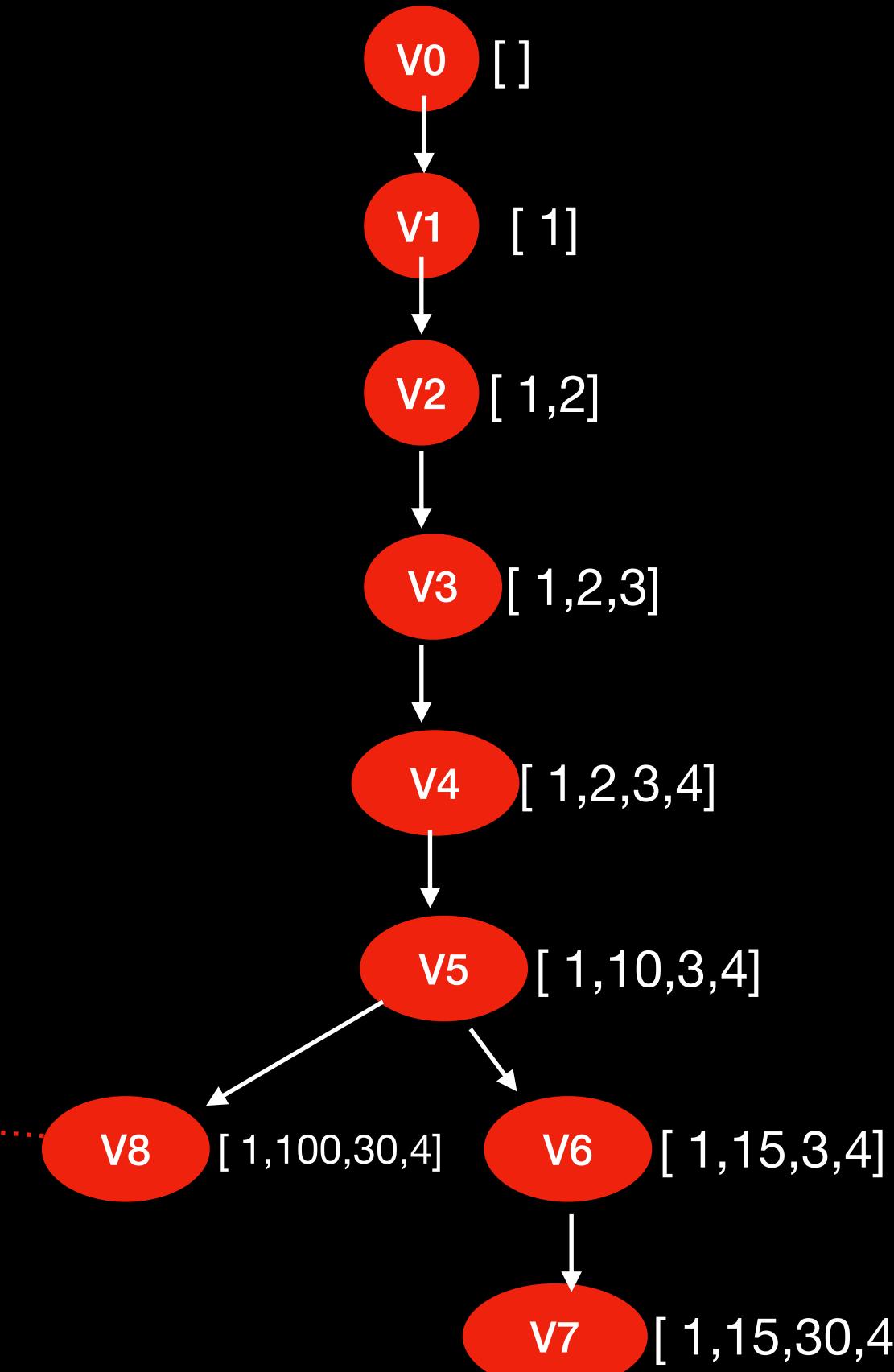


# Query: update(B,f1,100,v5)

Current time, t = 8



Version tree

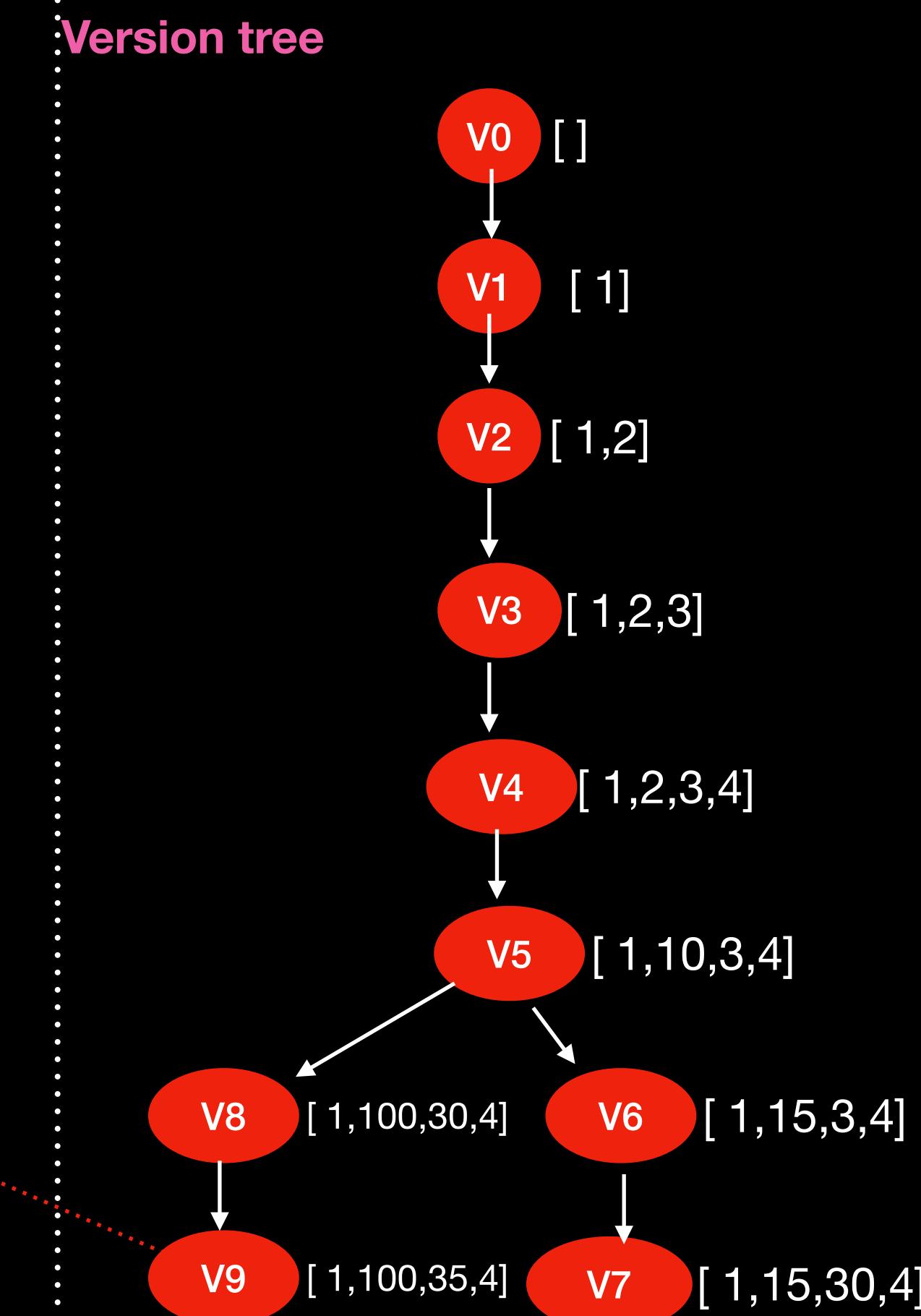
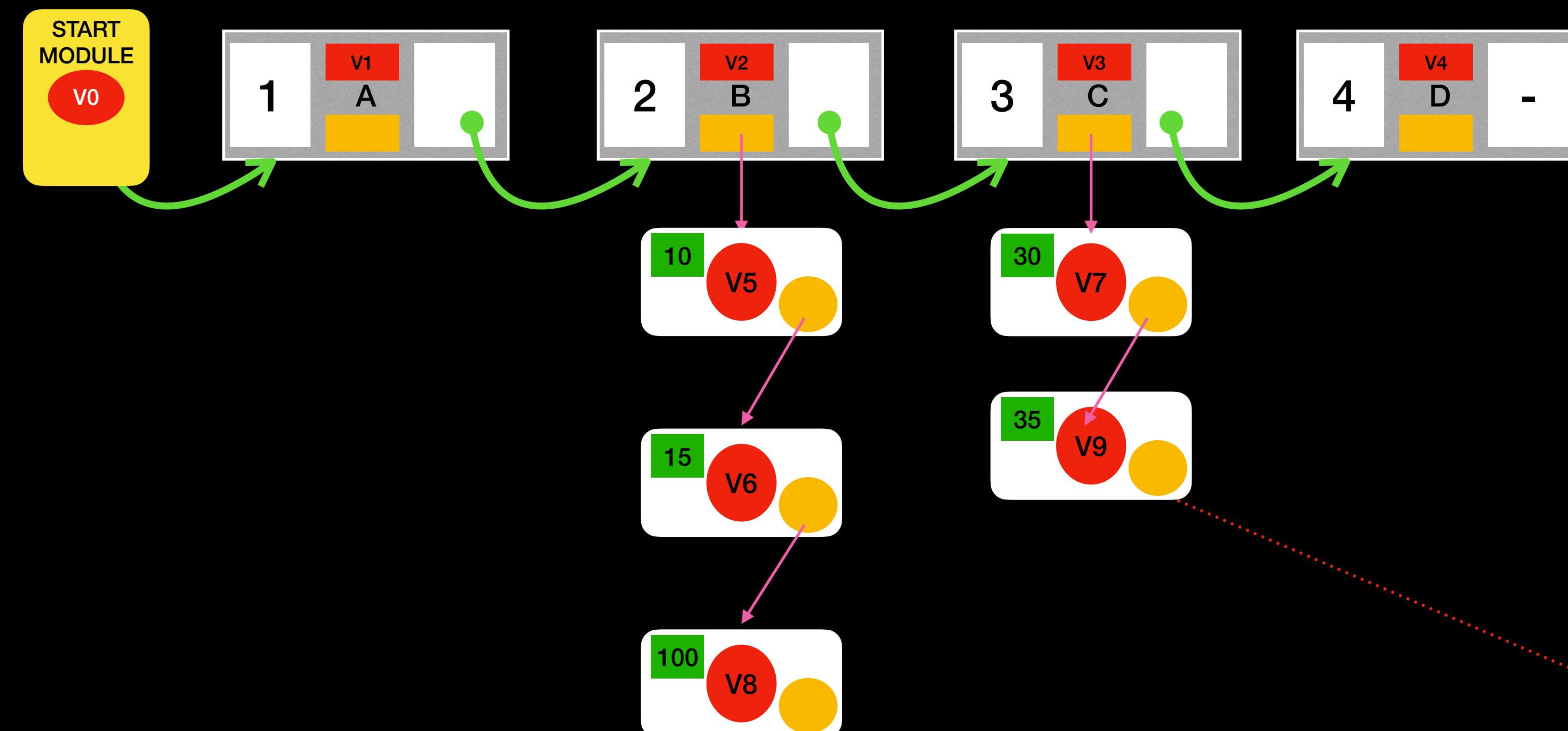


How to do branching ?

1. Create new Update Module with version = v8 and updated fields
2. Just add v8 node to v5 node in v-tree

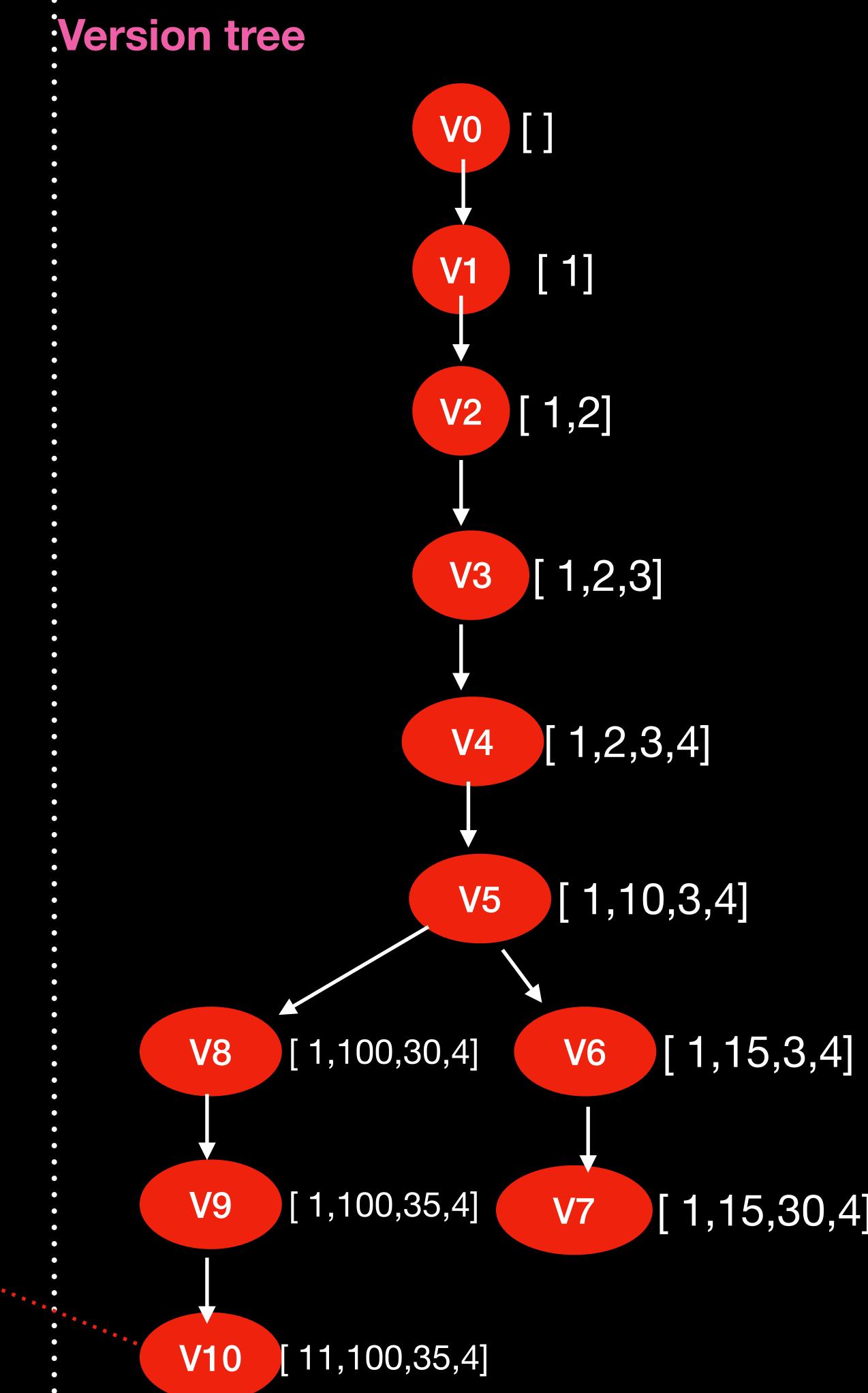
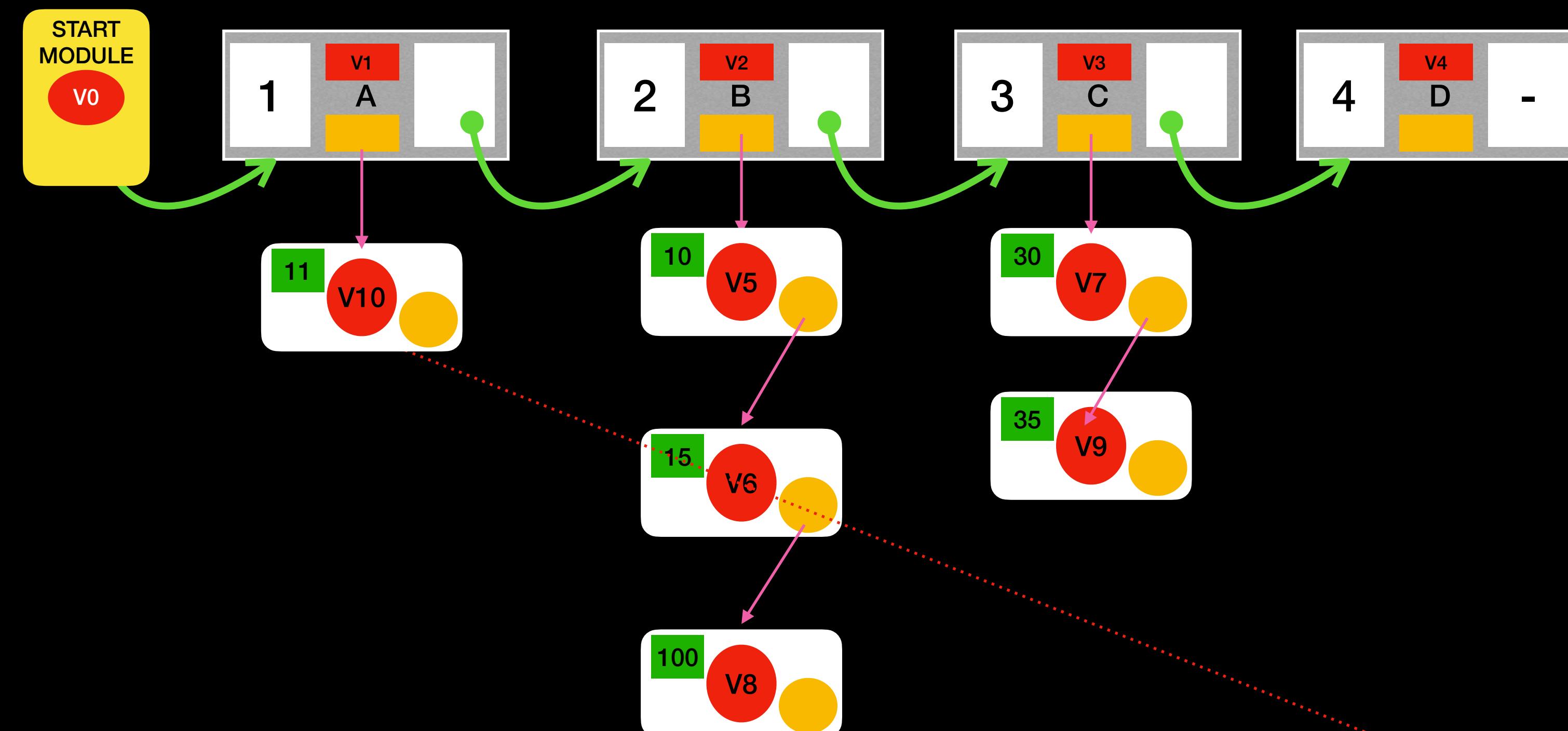
# Query: update(C,f1,35,v8)

Current time, t = 9



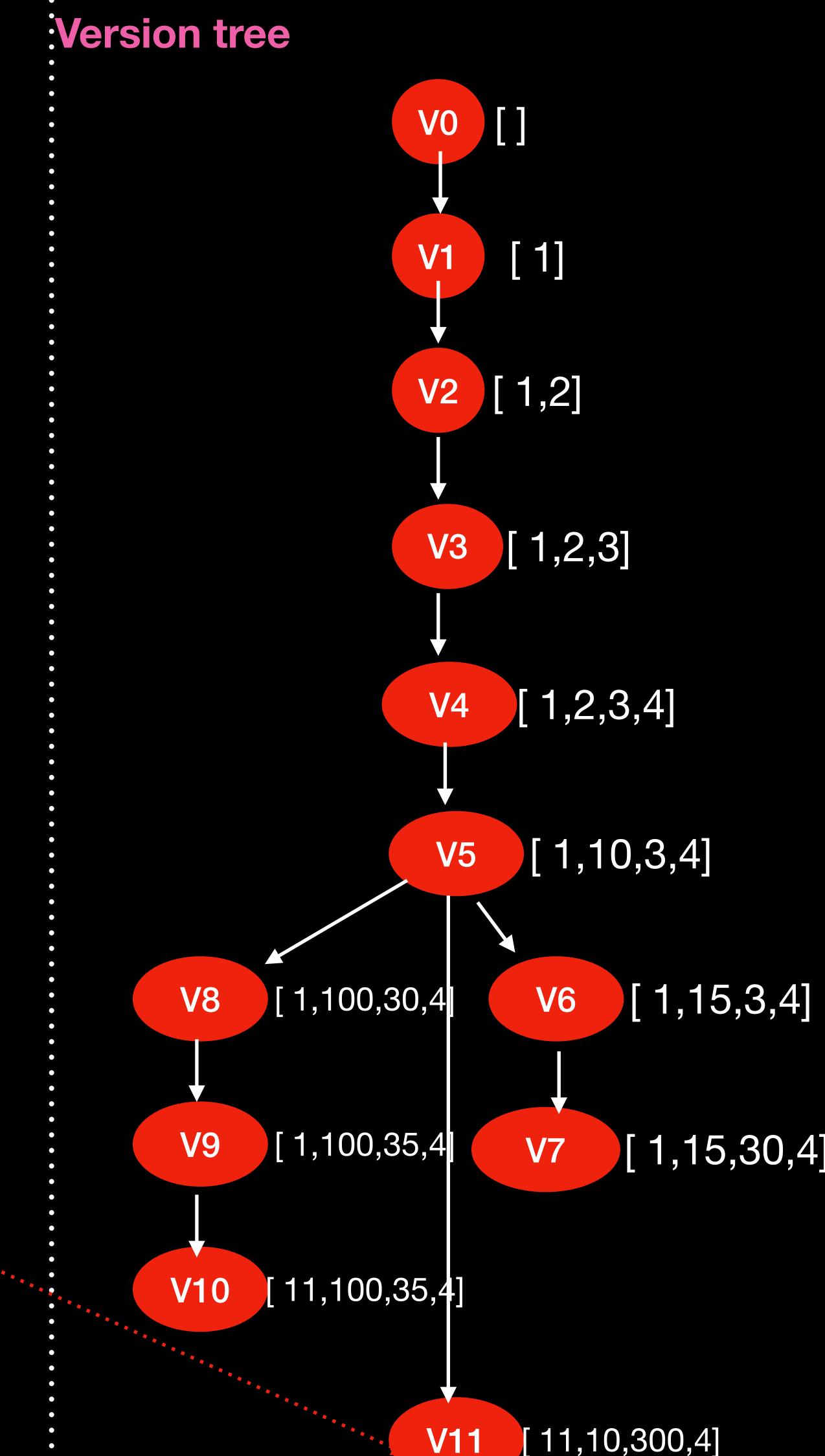
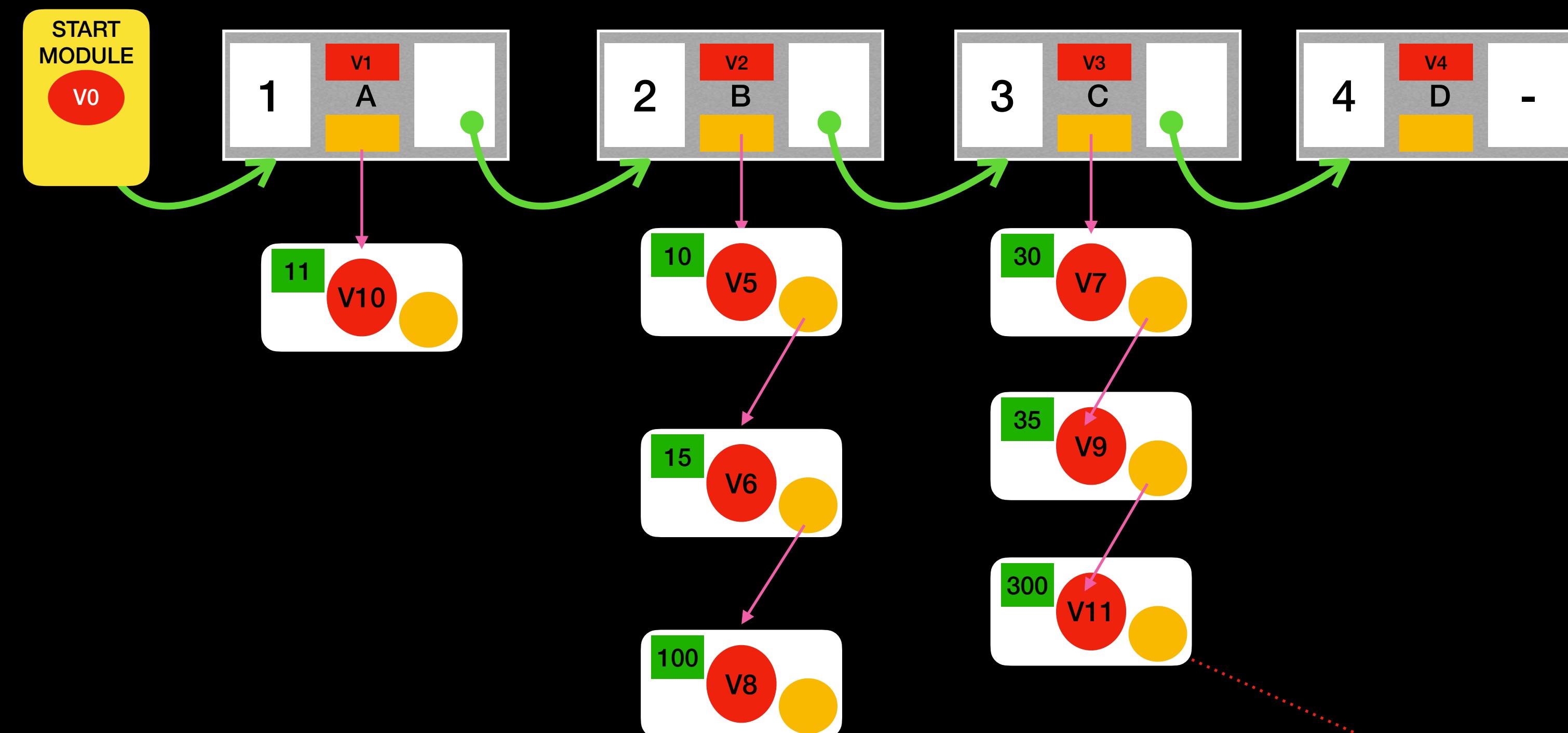
# Query: update(A,f1,11,v9)

Current time, t = 10



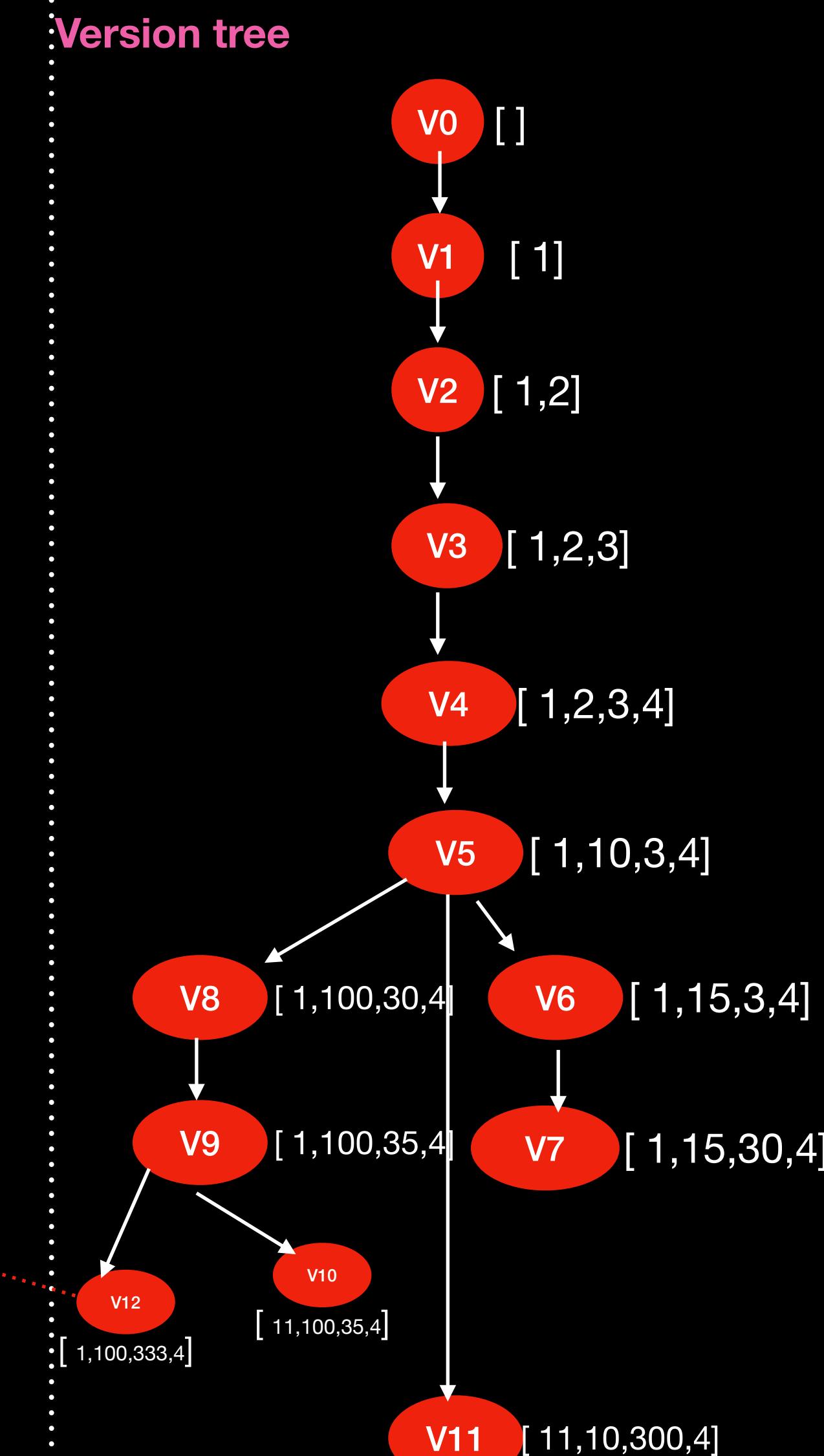
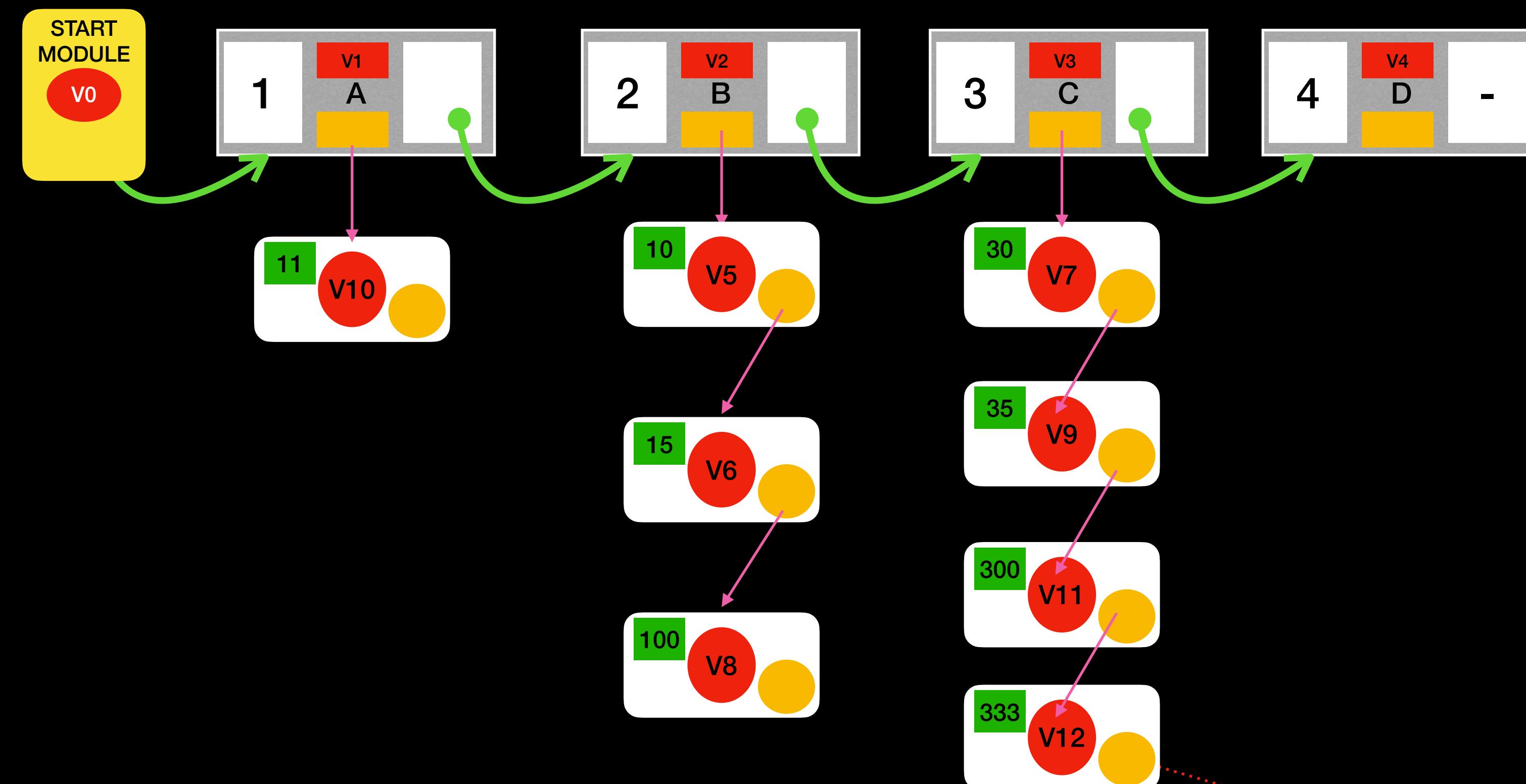
# Query: update(C,f1,300,v5)

Current time, t = 11



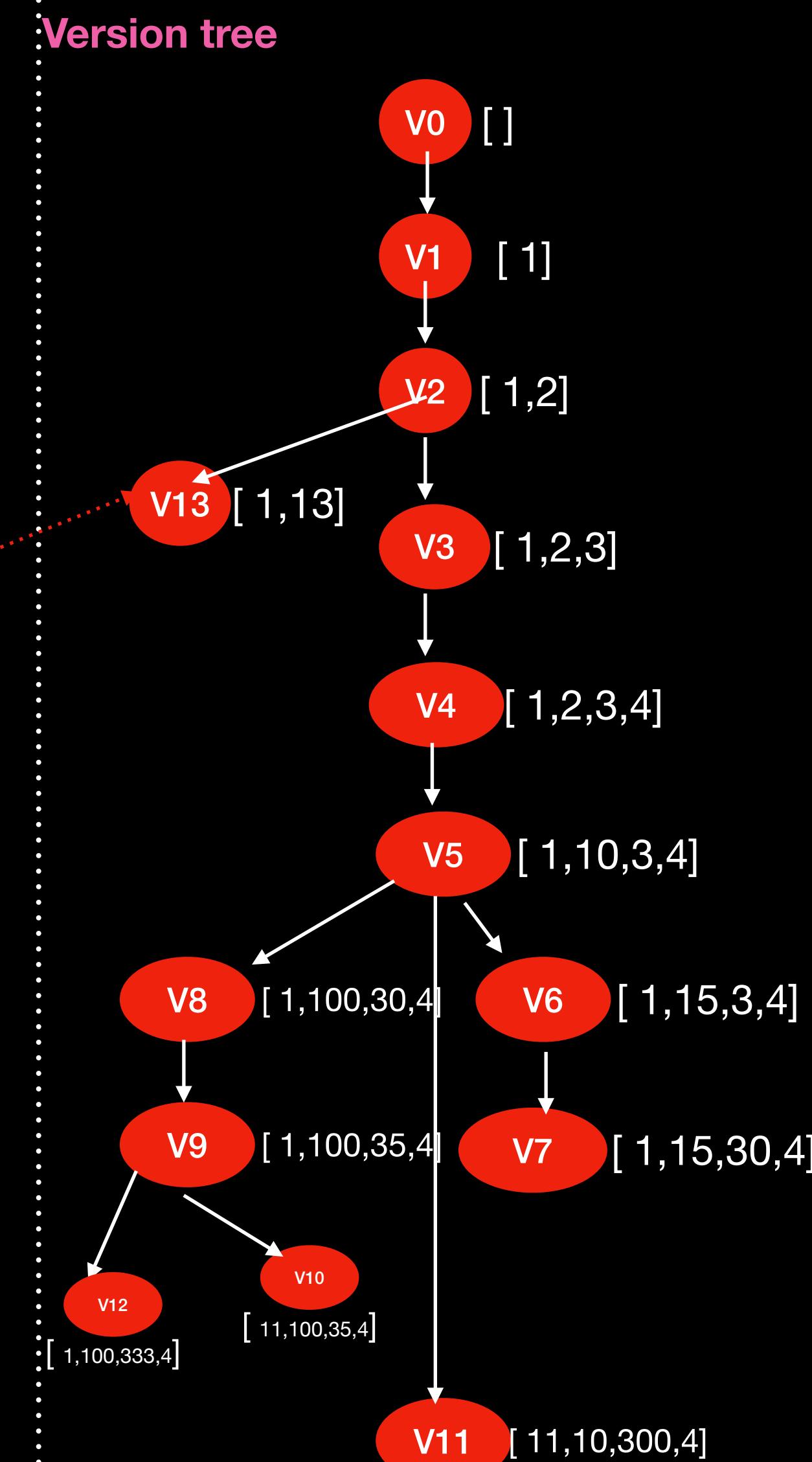
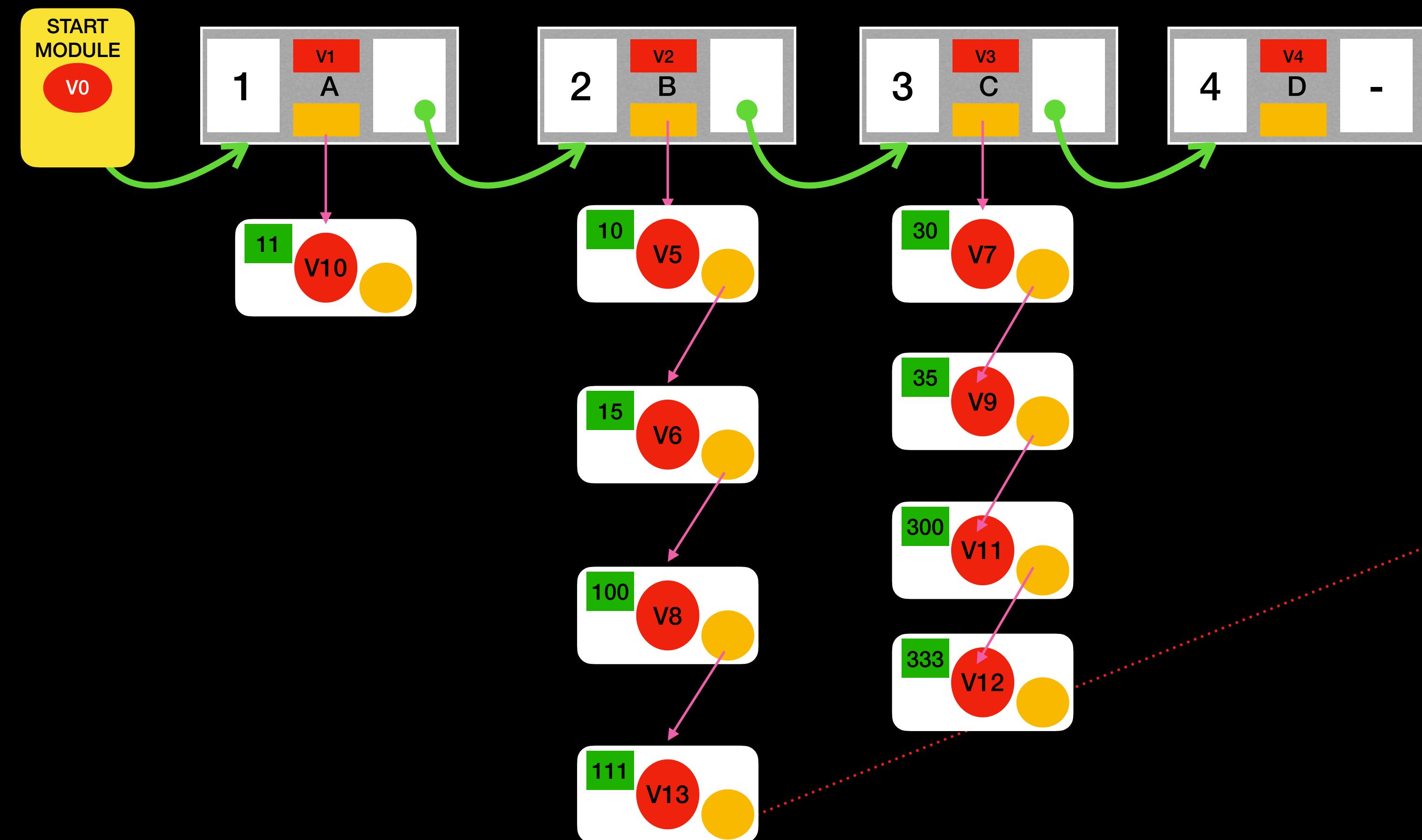
# Query: update(C,f1,333,v9)

Current time, t = 12



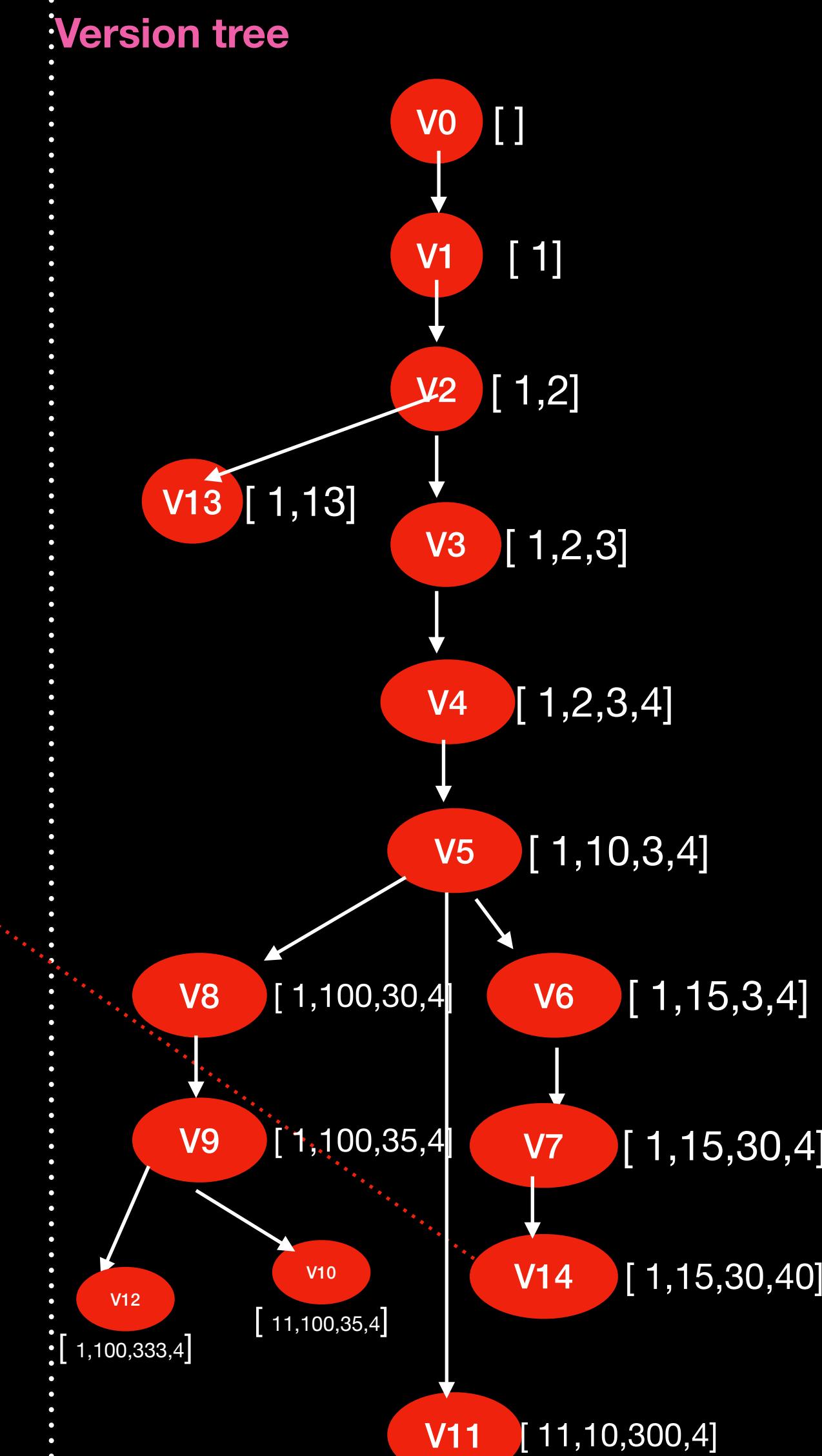
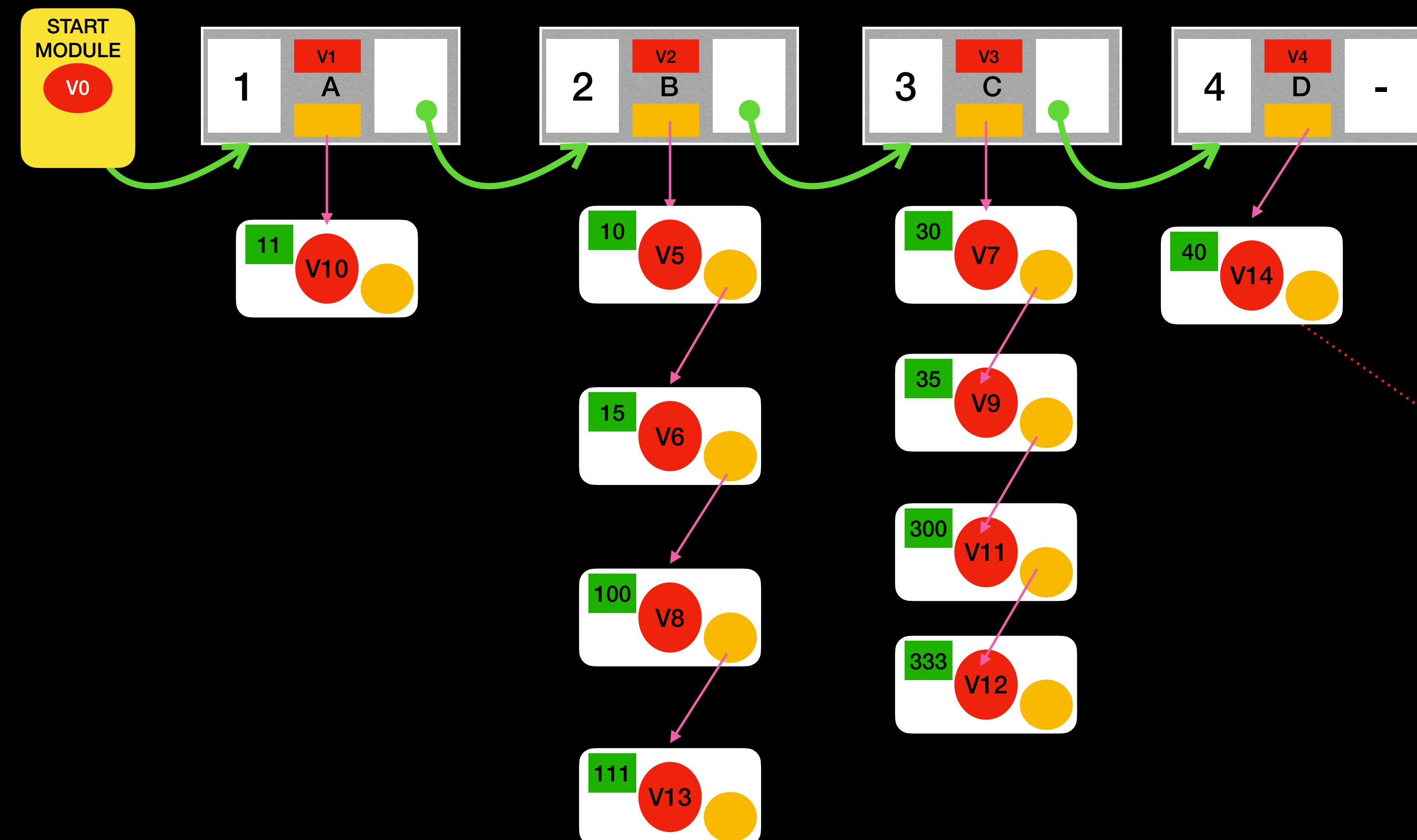
# Query: update(B,f1,111,v2)

Current time, t = 13



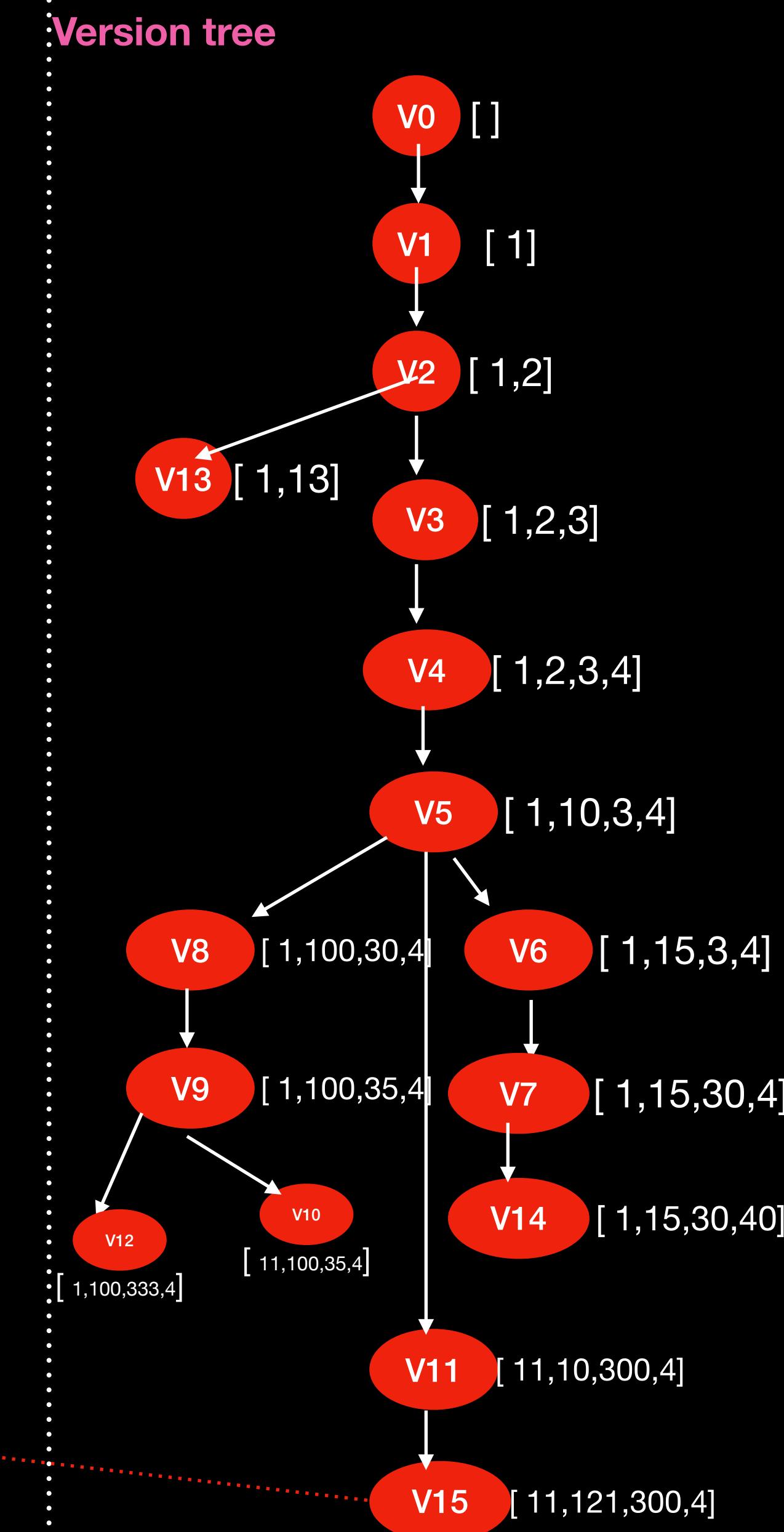
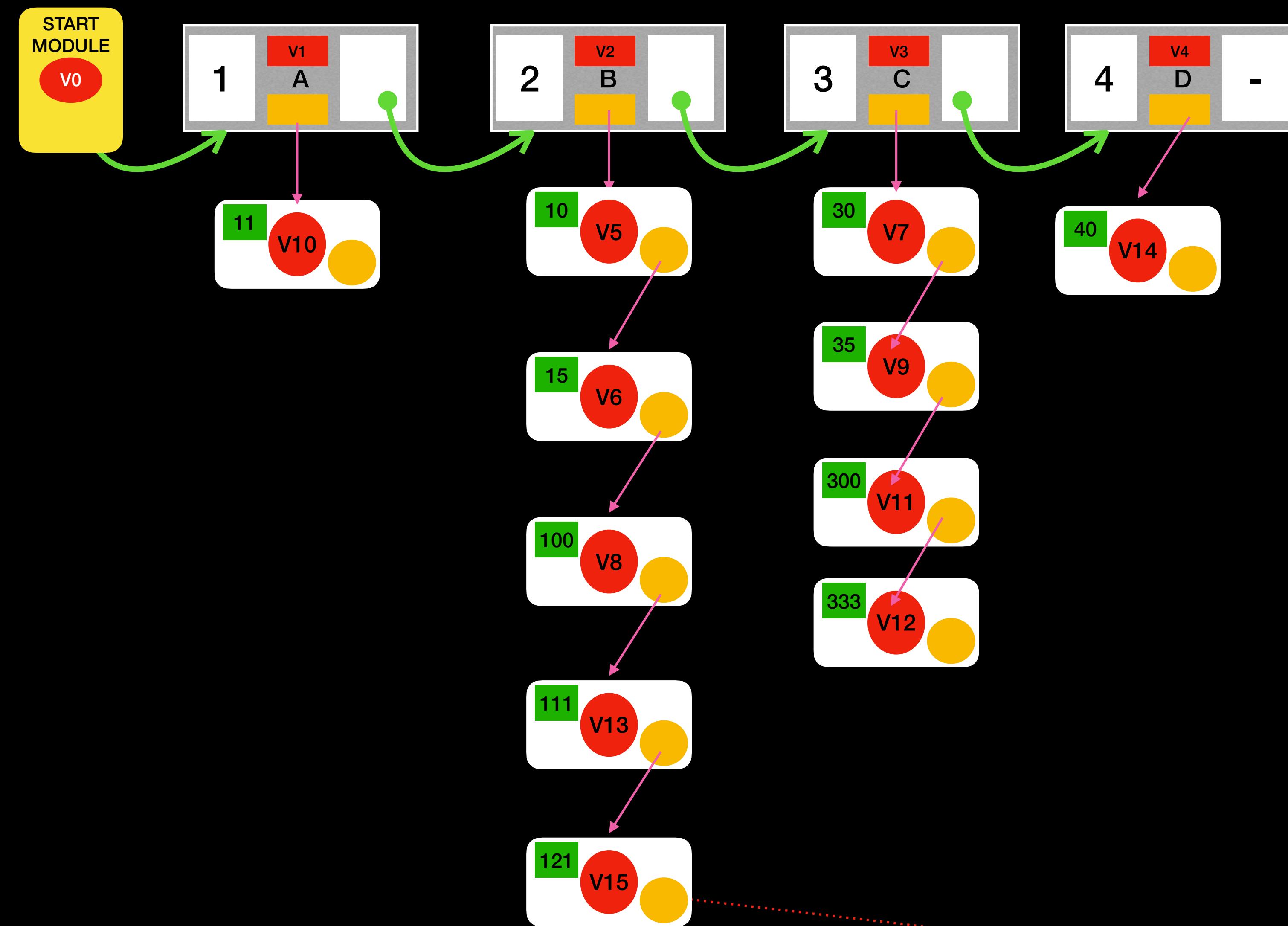
# Query: update(D,f1,40,v7)

Current time, t = 14

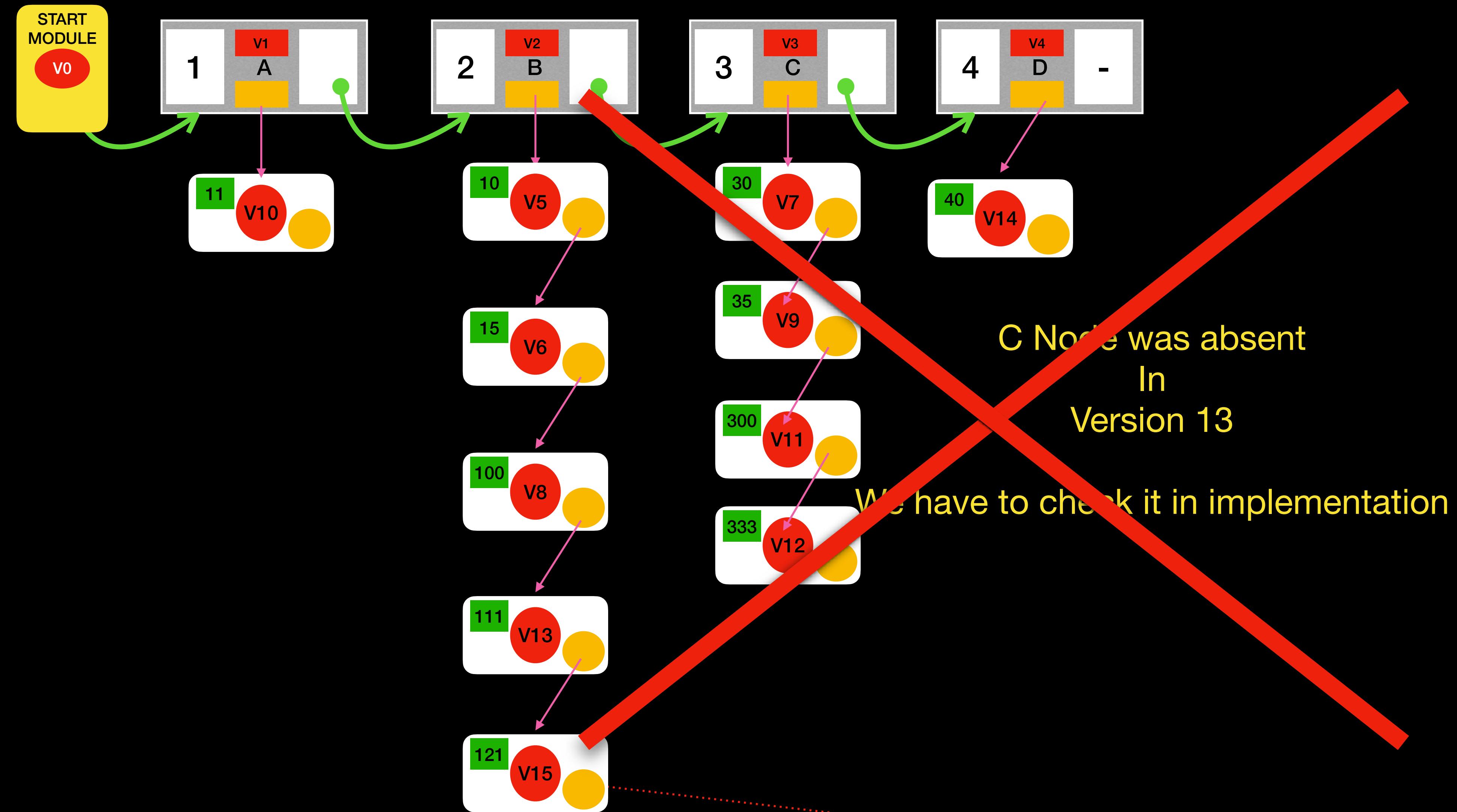


# Query: update(B,f1,121,v11)

Current time, t = 15

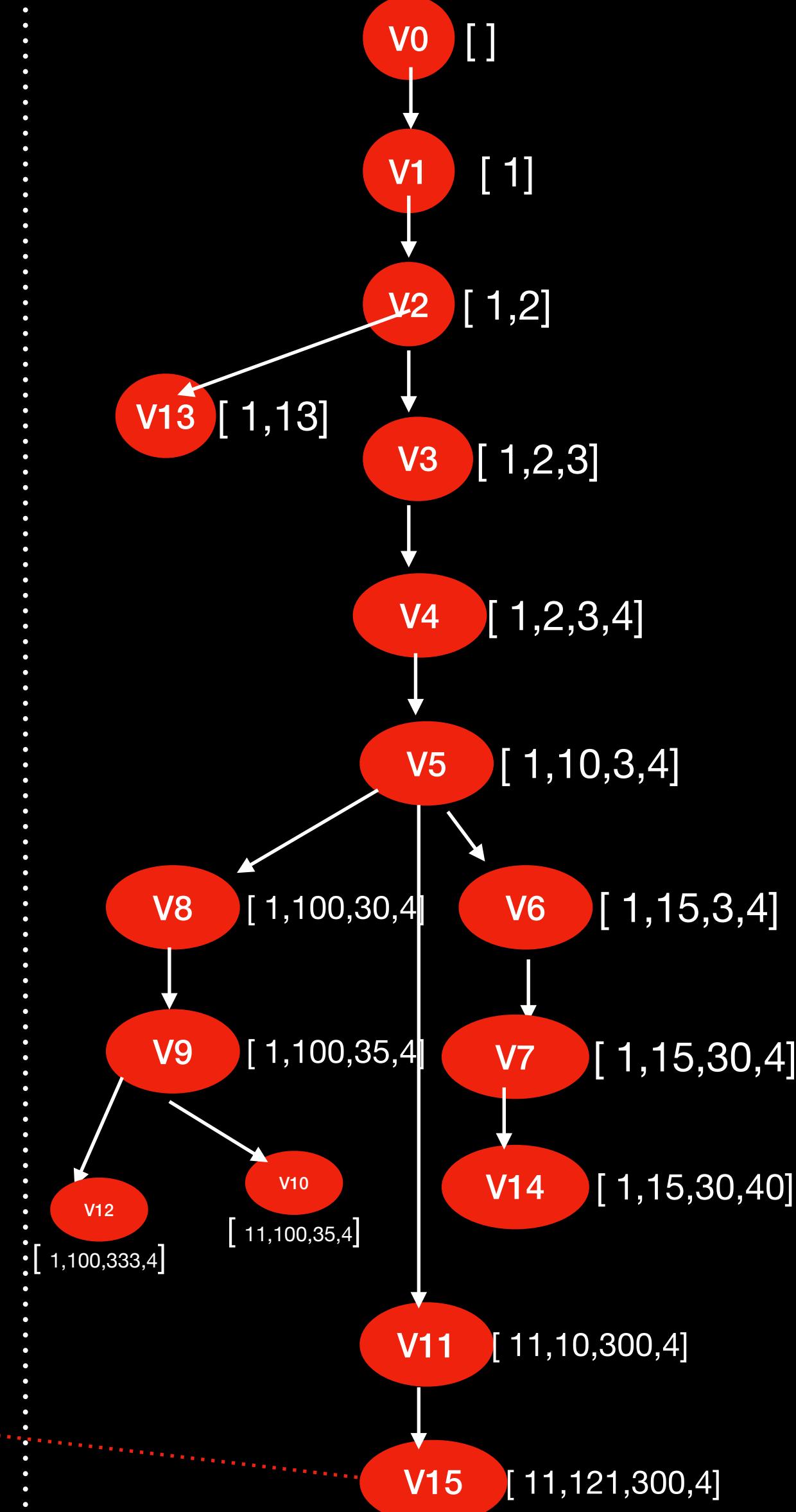


# INVALID Query: update(C,f1,3000,v13)



Current time, t = 15

Version tree



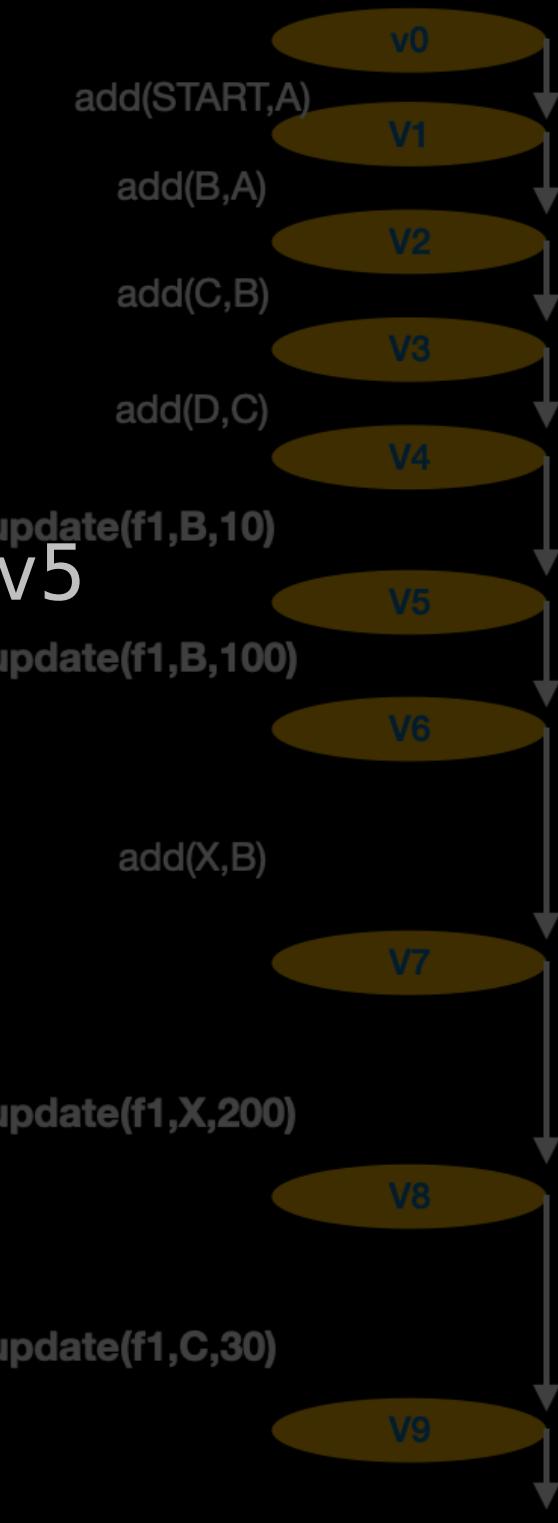
# Ok cool !! How to iterate through list in version v

Iteration in Partial Mode vs Full Mode

**iterate\_LL\_at\_v(vx)**

## Partial Mode

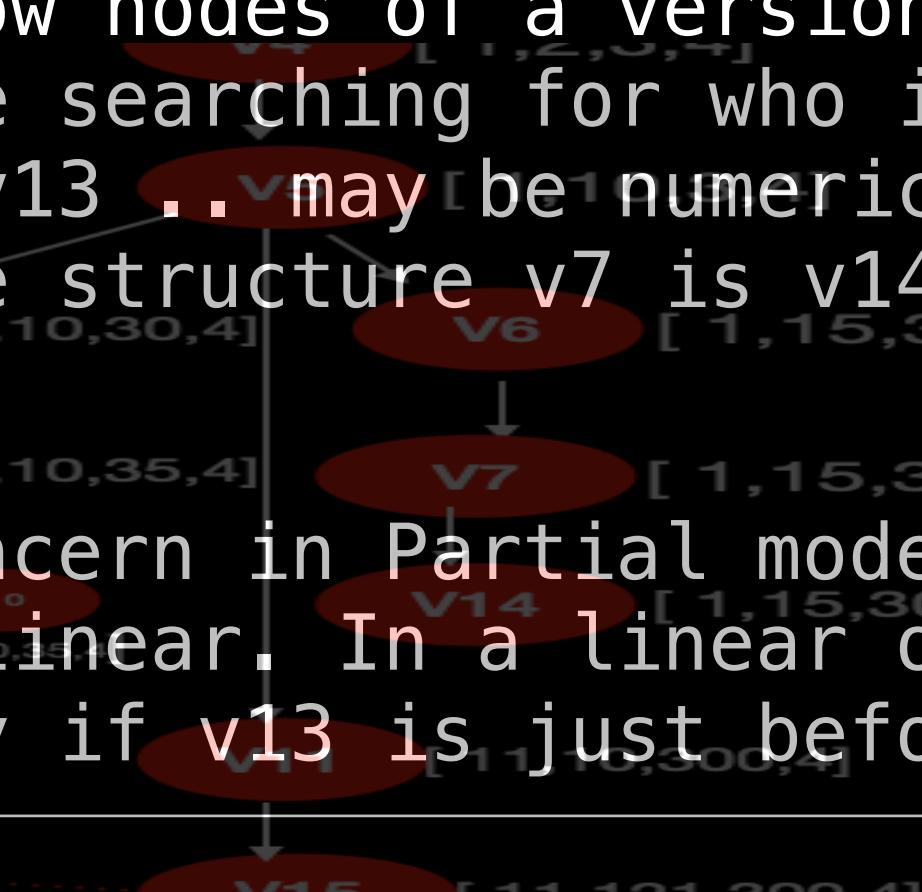
1. Start from start module
2. Choose those lines whose version **IS JUST LESS THAN OR EQUAL TO vx** (as, suppose if we are traversing for v5 , either v0,v1,v2,v3, v4 or v5 can be on that path, lines>v5 can't be on that).



3. The word “JUST LESS” is written because if a NODE has two version lines in that, e.g. v2 and v4 and we are searching for v5, then we should prefer v4 line over v2.

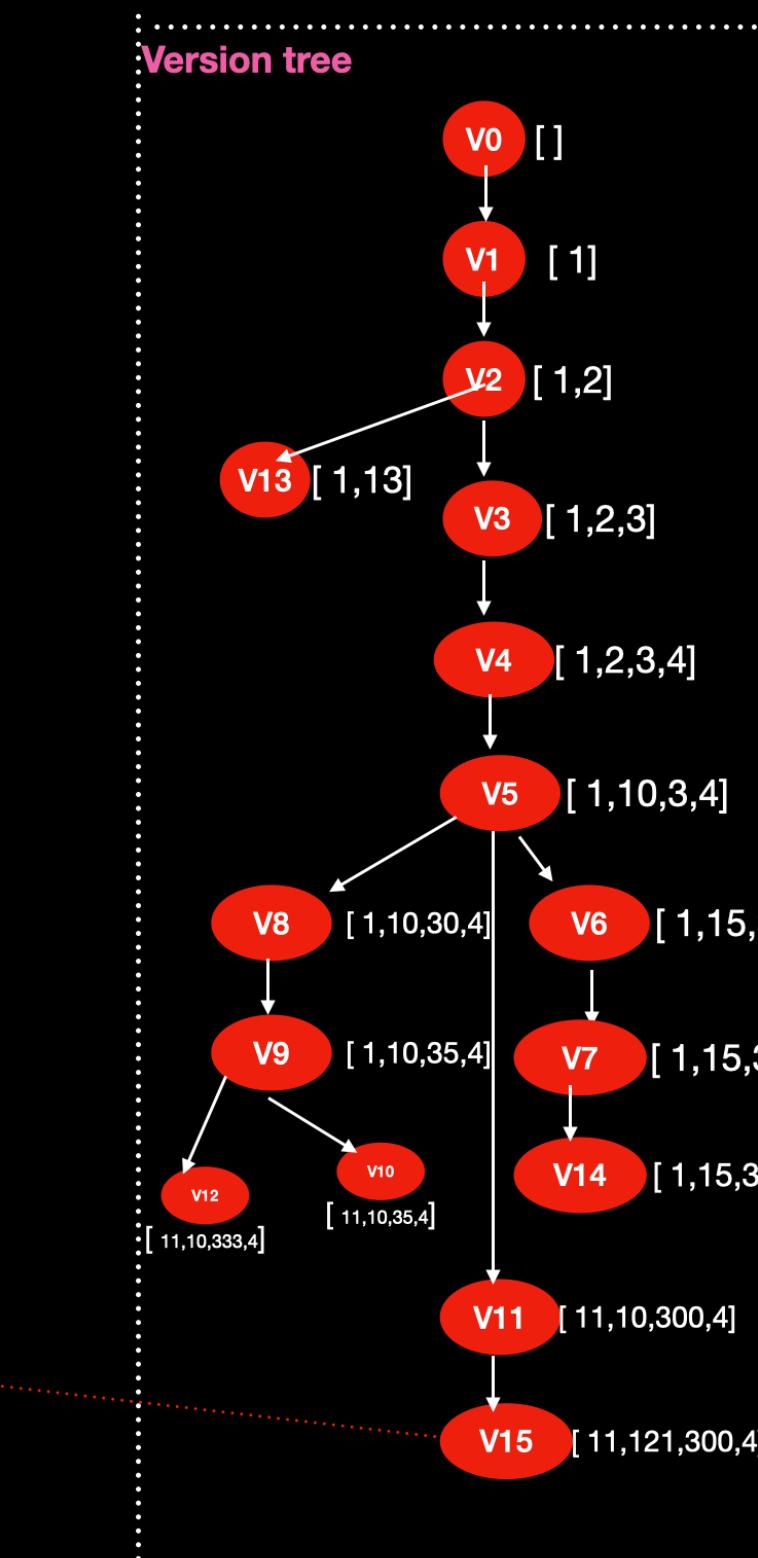
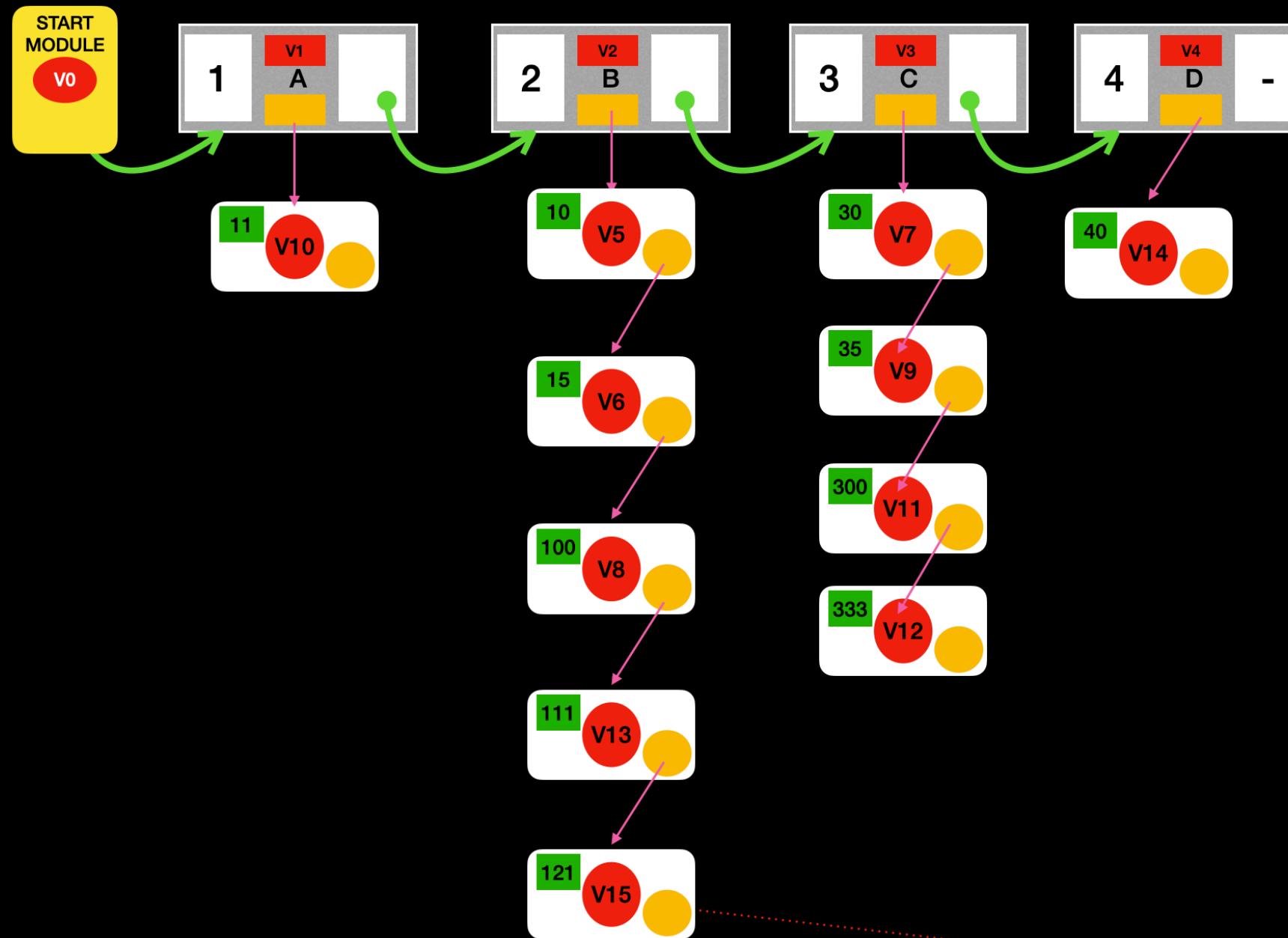
## Full Mode

1. Start from start module
2. Choose those lines whose VERSION IS **IS NEAREST/LOWEST ANCESTOR OR EQUAL TO THAT OF vx**
3. Here, we are searching for LOWEST ANCESTOR Because, versions are now not just number. The versions are now nodes of a version tree. E.g. suppose we are searching for who is before v14, here v11,v12,v13 .. may be numerically less than 14 but, in the structure v7 is v14's parent.
4. It was not a concern in Partial mode because the versions were linear. In a linear order v13<v14 if and only if v13 is just before v14.



# Question!!

How to choose NEAREST/LOWEST ANCESTOR from all possible ancestors ?



Suppose, we are searching for v14  
Status in node B.,

Note that, v2, v5, v6 are possible ancestors.  
But, v6 is Nearest Ancestor of v14

How Do I Find that?

**ALL POSSIBLE ANCESTORS MUST BE IN A LINEAR ORDER.**  
**AND, WE KNOW THAT THE VERSION WITH HIGHEST MAGNITUDE IS THE MOST RECENT IN LINEAR ORDER**

Hence, we can say

**Full Persistent Data-structure**

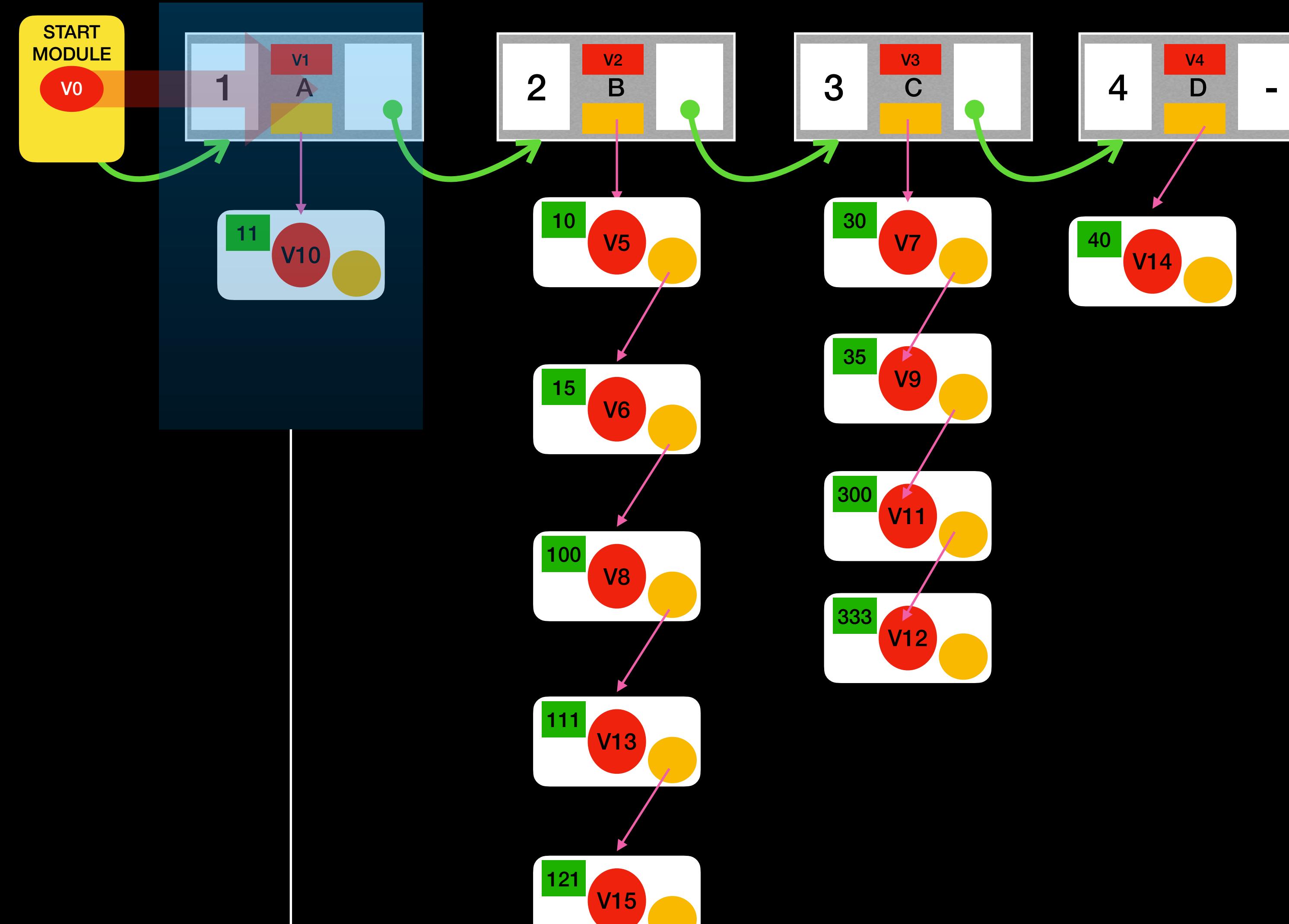
is a general form of

**Partial Persistent Data-structure**



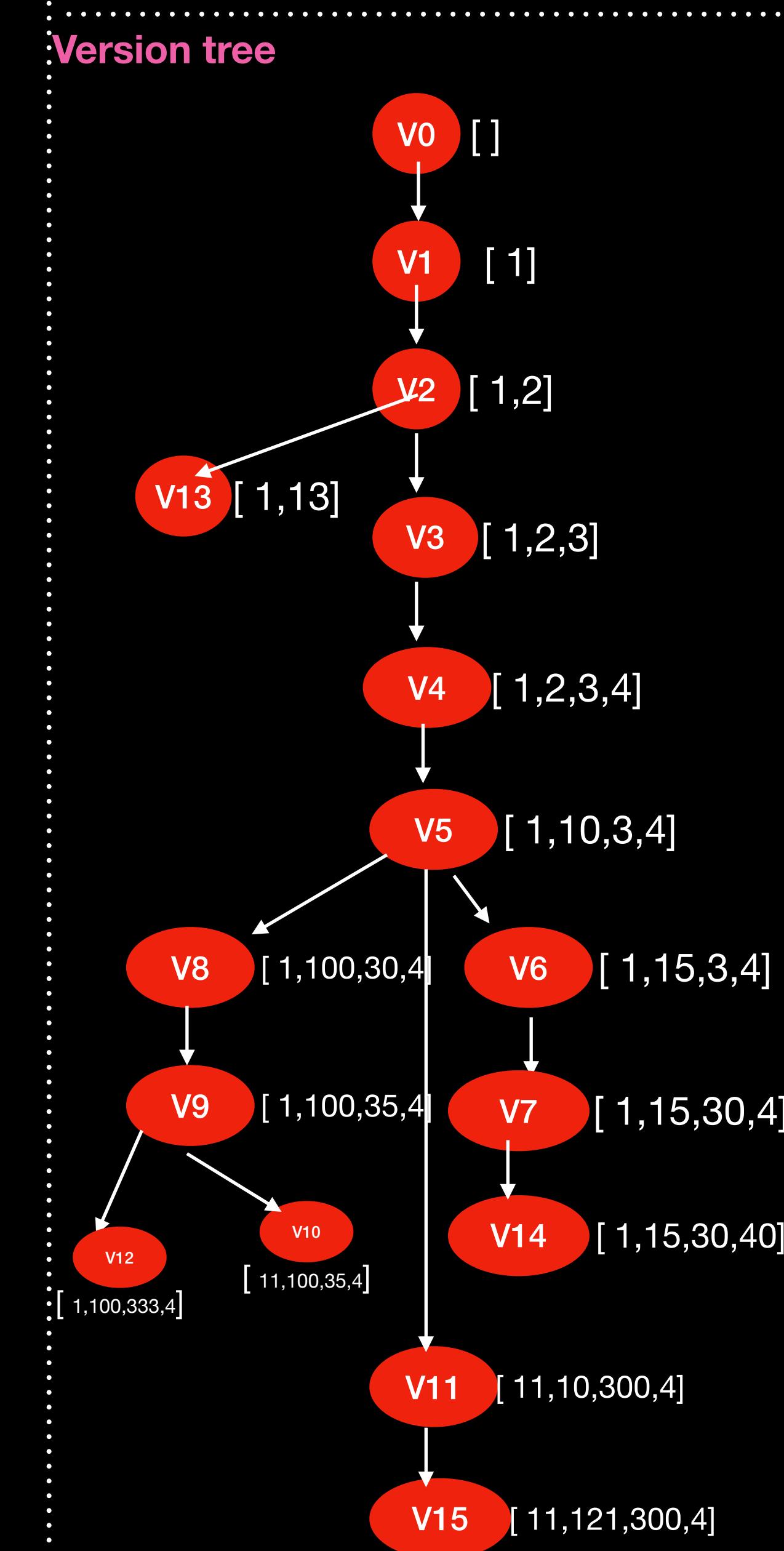
**iterate\_LL\_at\_v(v12)**

# iterate\_LL\_at\_v(v12)



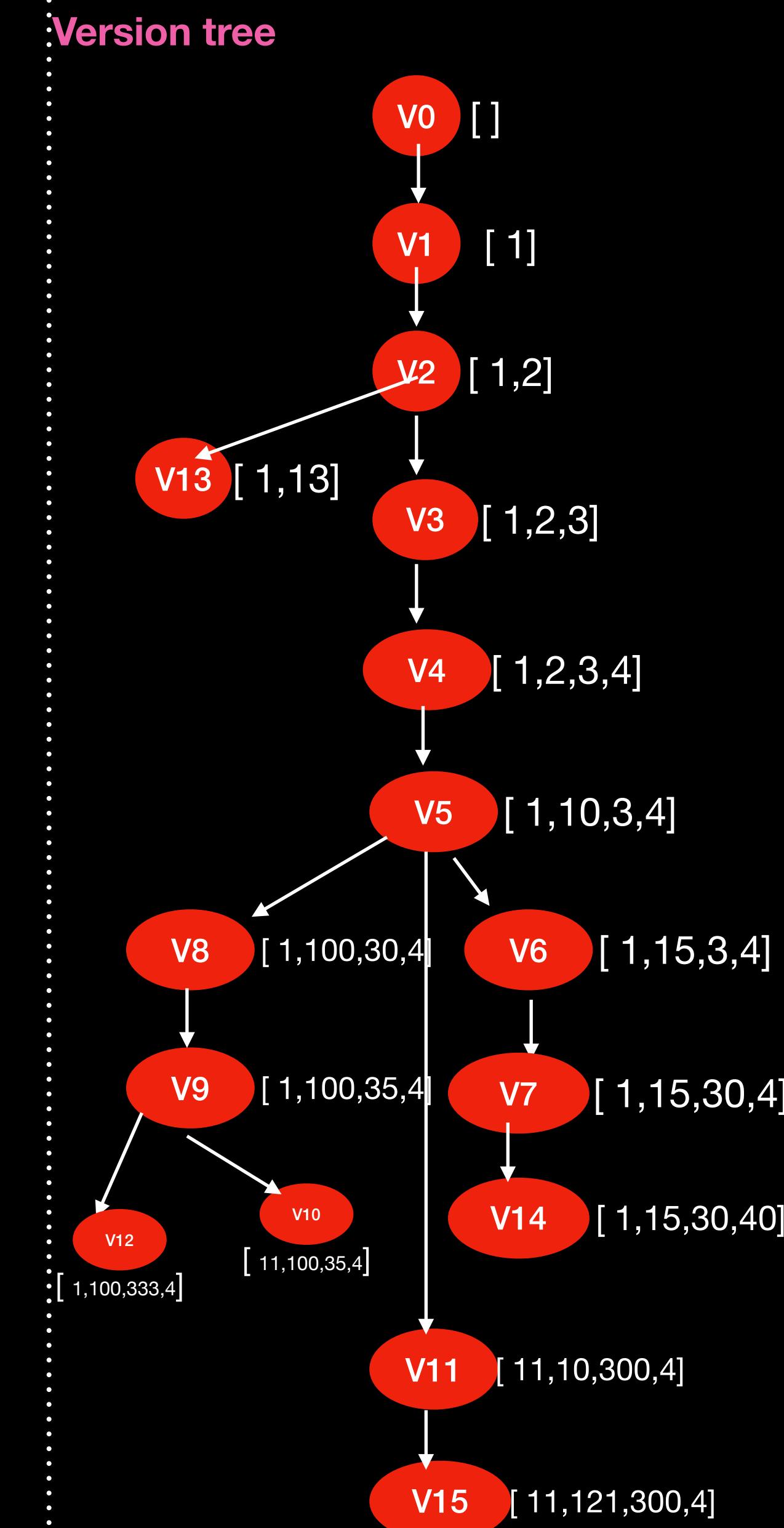
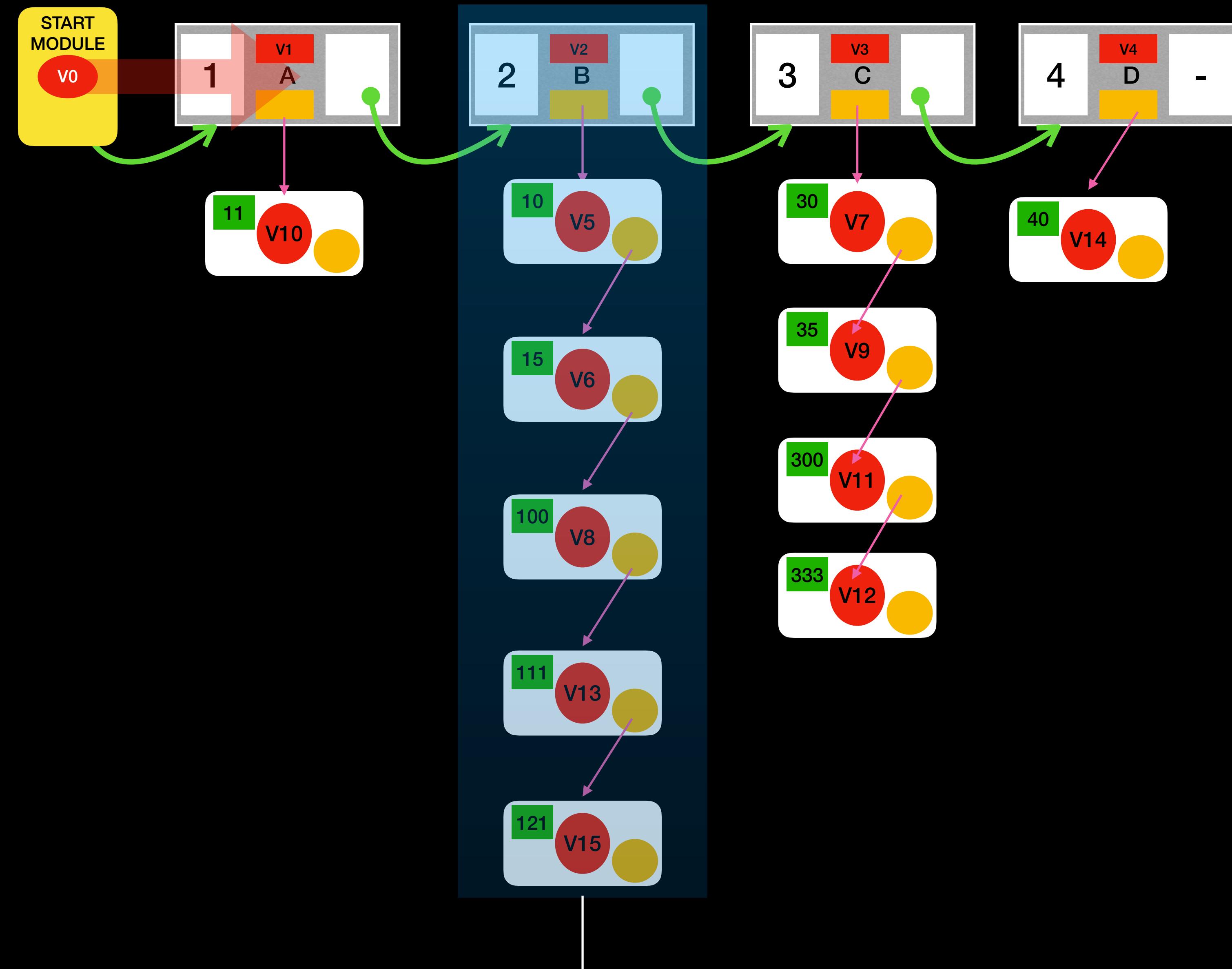
OUTPUT : 1(v1)

Current time, t = 15



# iterate\_LL\_at\_v(v12)

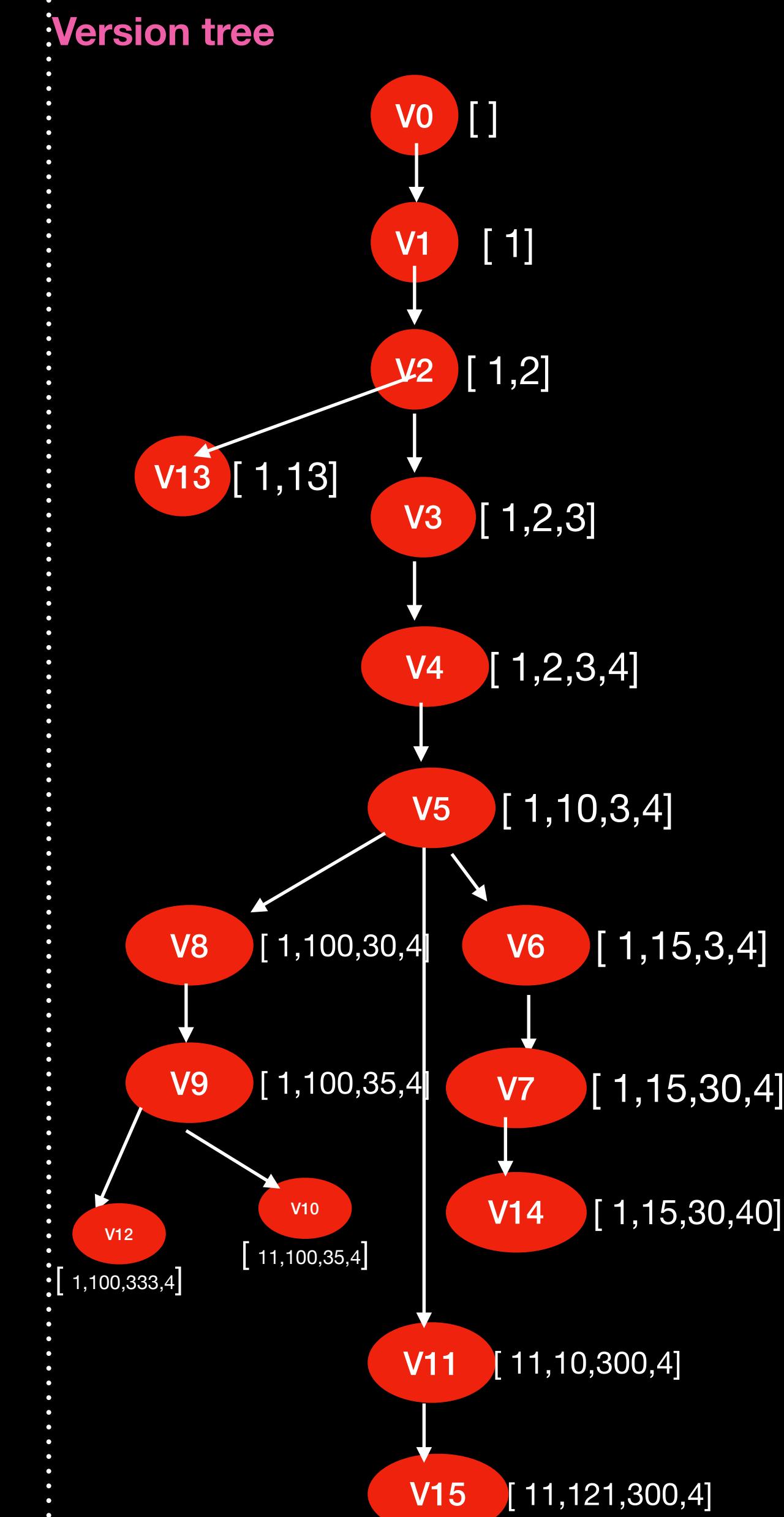
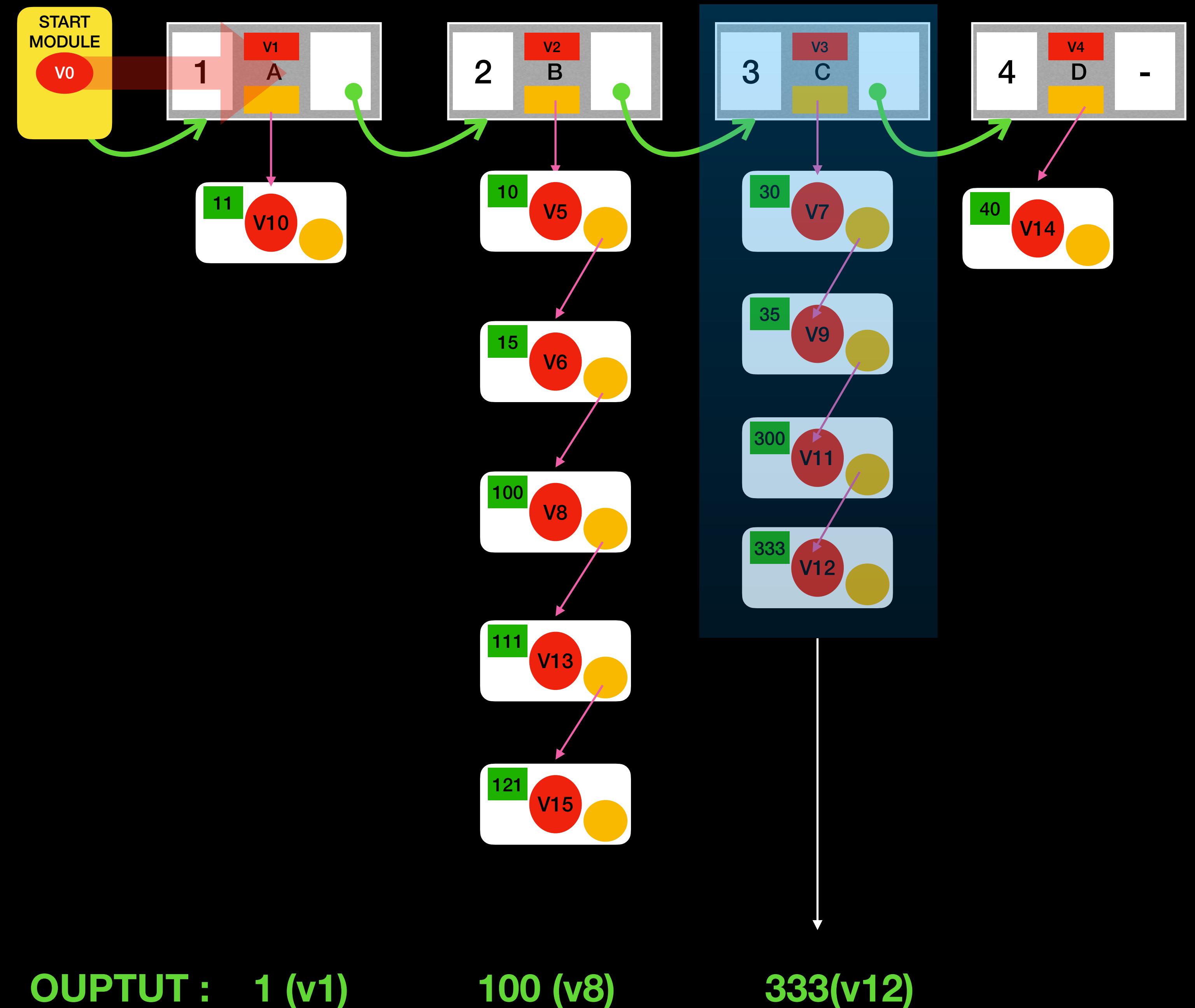
Current time, t = 15



OUTPUT : 1 (v1)

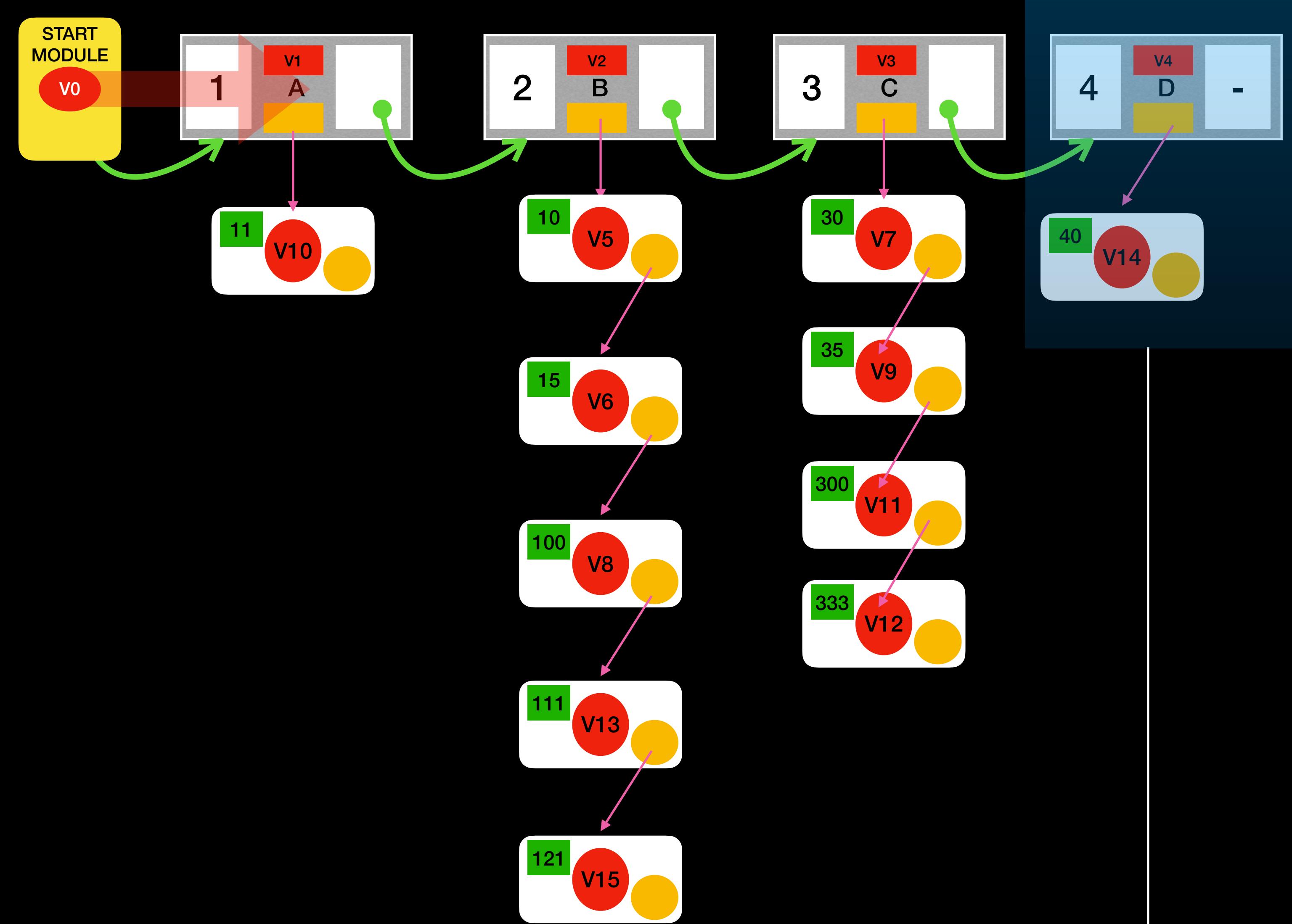
# iterate\_LL\_at\_v(v12)

## Current time, t = 15



# iterate\_LL\_at\_v(v12)

Current time, t = 15

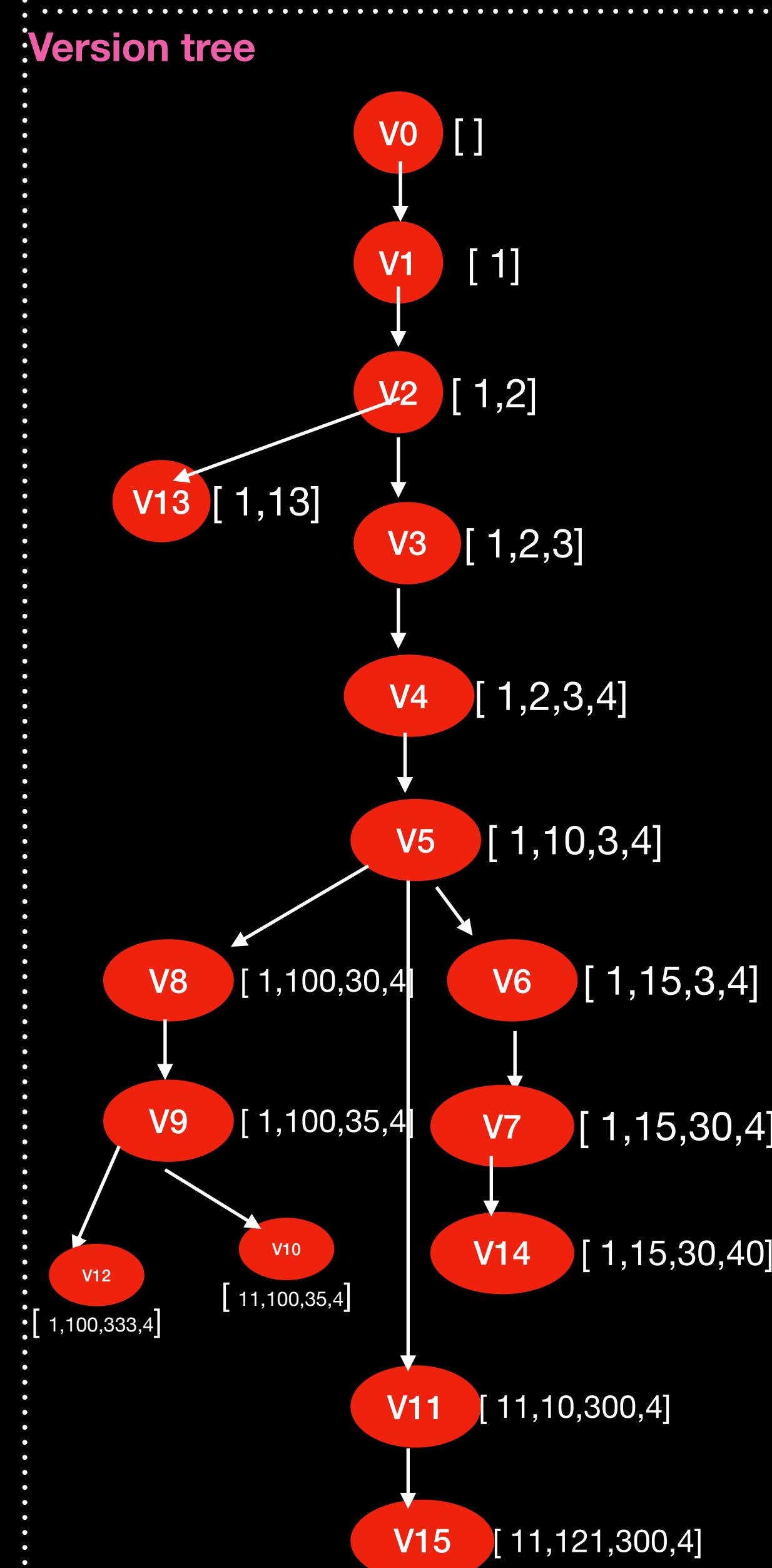


OUTPUT : 1 (v1)

100 (v8)

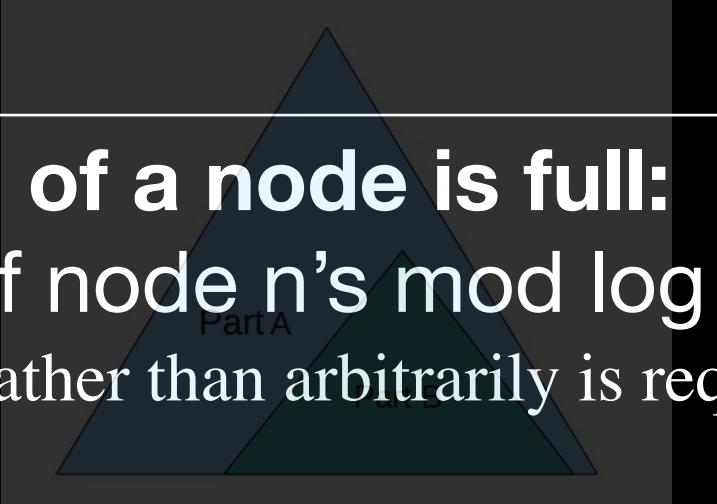
333(v12)

4 (v4)



# Implementation Using Pointer Machine

# Basic Difference In Structure in Pointer machine

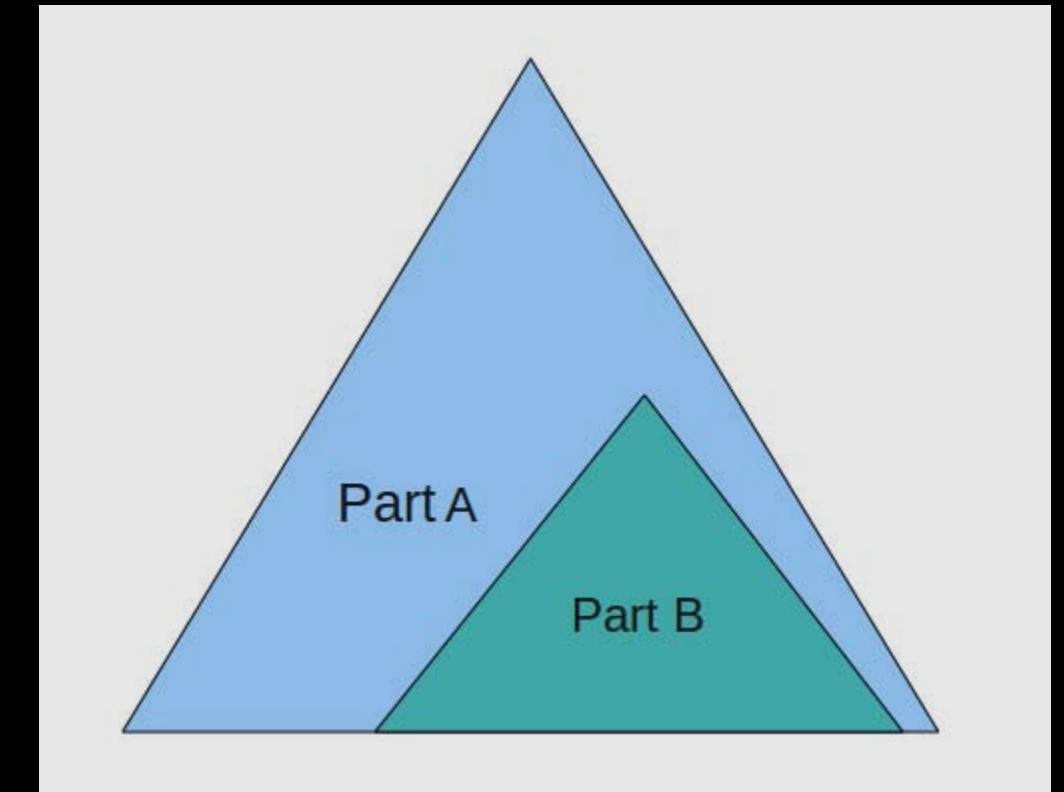
Partial Persistent PM	Full Persistent PM
Versions are Just Numbers.	Versions are reference to Nodes Of Version Maintenance Data structure (typically, V Tree)
Here we allow upto $2 * p$ modification in each node	Here we allow upto $2(d + p + 1)$ modification in each node. Additionally we now also version <b>back-pointers</b> .
We create a copy of the the node, when the Mod-log of a node is full, we create a copy of the node $\rightarrow$ node' with the latest values of fields and BPs. And, Don't copy any thing to Mod_log in node'.	<b>When the Mod-log of a node is full:</b> Split the contents of node n's mod log into two parts. Partitioning into subtrees rather than arbitrarily is required.  From the 'old' mod entries in node n, compute the latest values of each field and write them into the data and back pointer section of node m

# Major Difference

**We split the mod-log of older node into 1:1 or 2:1 partition**

**Transfer the 50% or 33% recent mods to the newly created node**

**Set the fields of newly created node, according to the latest values from the previous 50% or 66% Mods left in the older node.**



Why?

To reduce the pressure on a node of particular version.

As, we can modify a particular node of a specific version multiple number of time in Full Persistent Strategy.

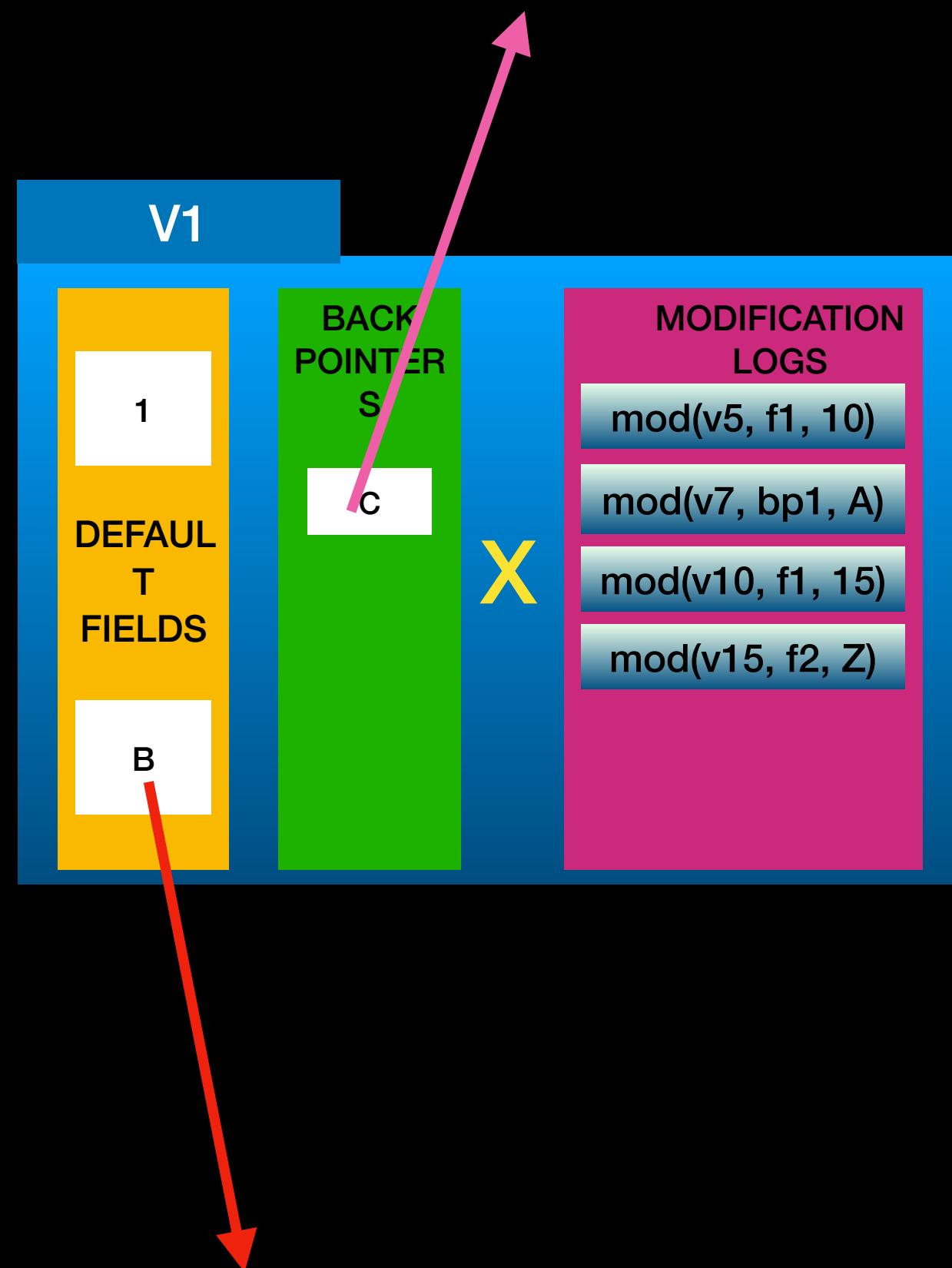
**Bad Way (We just create copy with 1:0 split):**

A particular node of a specific version with Full Mod-Log will create a empty-mod copy how many times we tried to modify that specific version.

**Good Way (We just create copy with 1:1 / 2:1 split):**

That particular node of a specific version with Full Mod-Log is now not Full anymore it has 50% or 33% space in Mod-log free to keep the upcoming modification.

## Problem with 1:0 Approach (What was in Partial Persistent Mode)

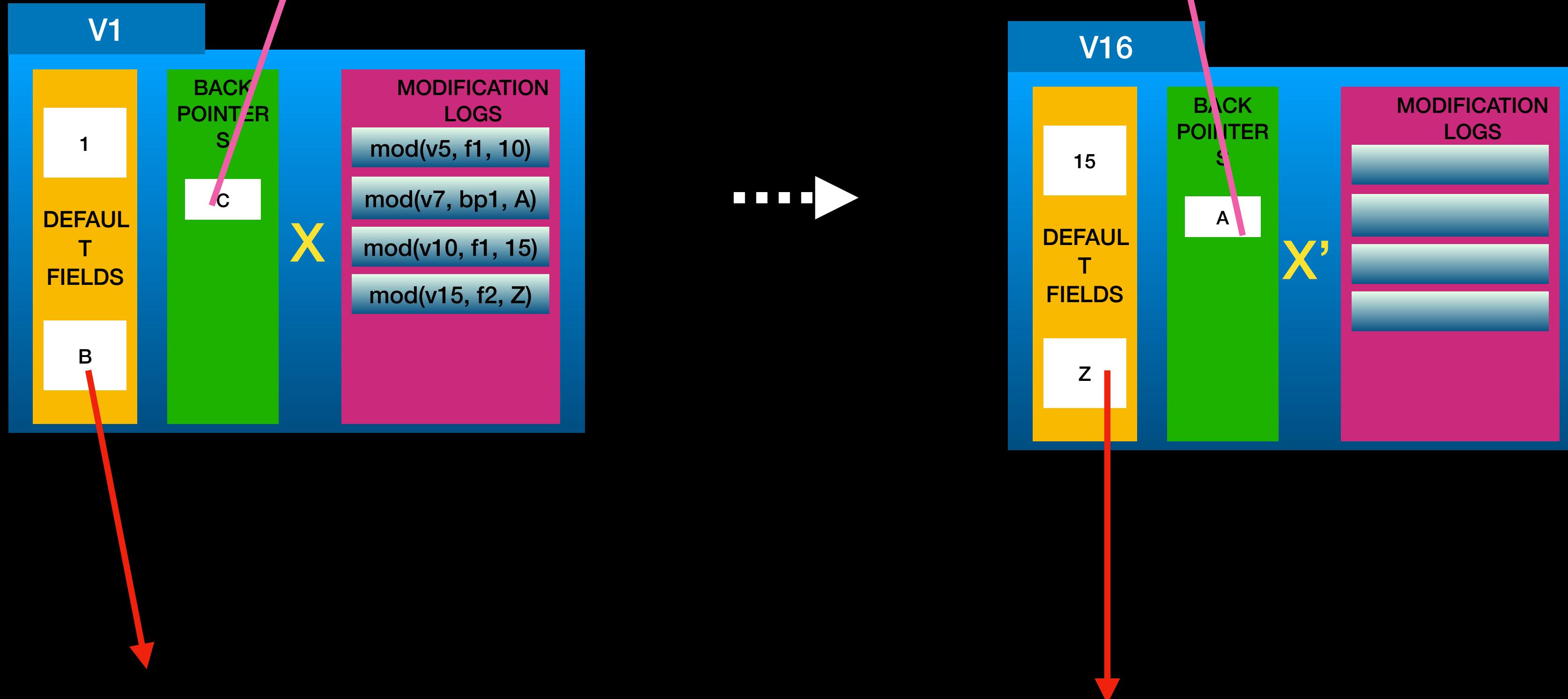


Suppose Node in any version ( cur  $t \geq 15$ ) looks like this

**NOTE: THIS IS NOT DONE IN FULL PERSISTENT STRATEGY**

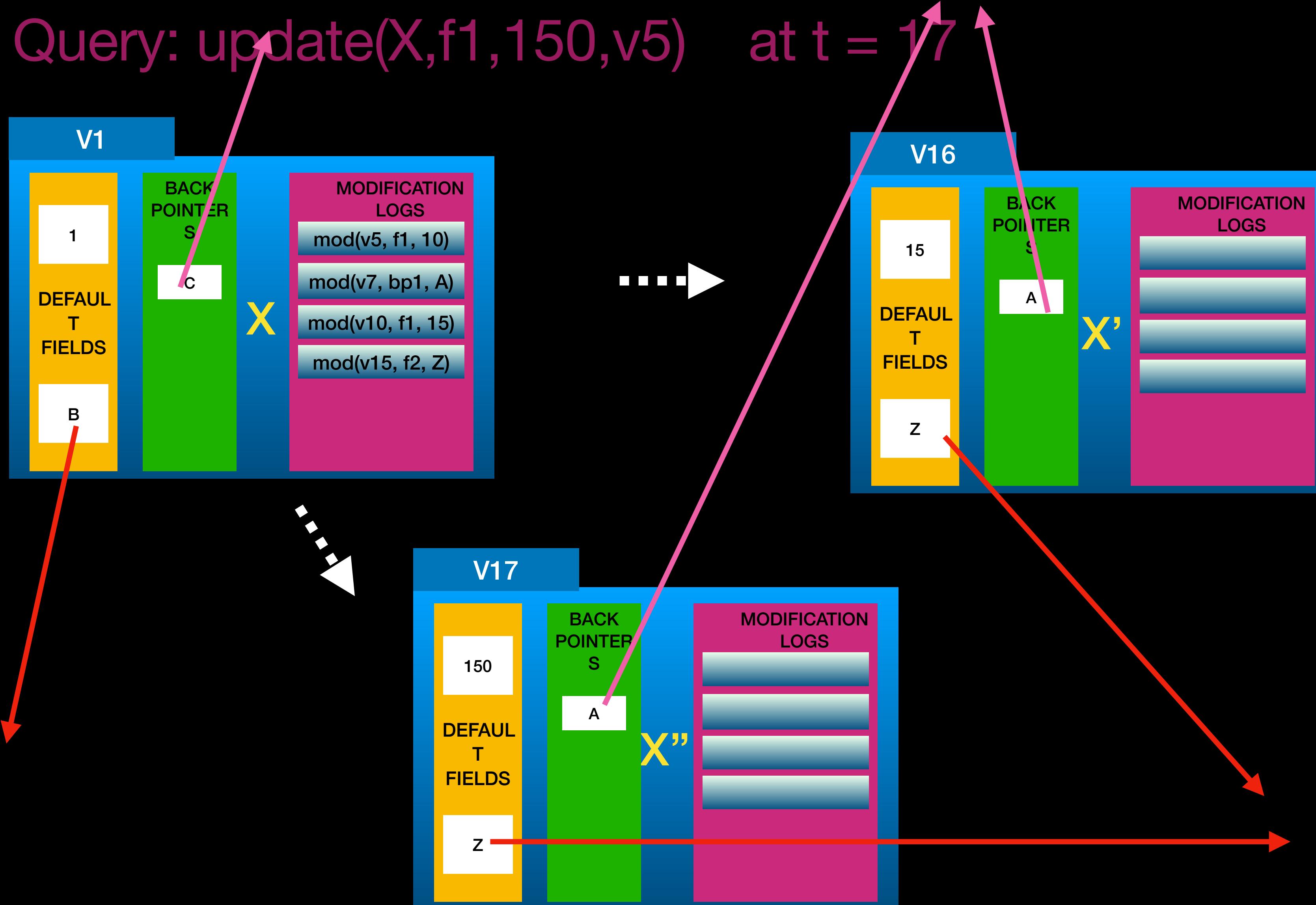
## Problem with 1:0 Approach (What was in Partial Persistent Mode)

Query:  $\text{update}(X, f1, 15, v5)$  at  $t = 16$



**NOTE: THIS IS NOT DONE IN FULL PERSISTENT STRATEGY**

## Problem with 1:0 Approach (What was in Partial Persistent Mode)



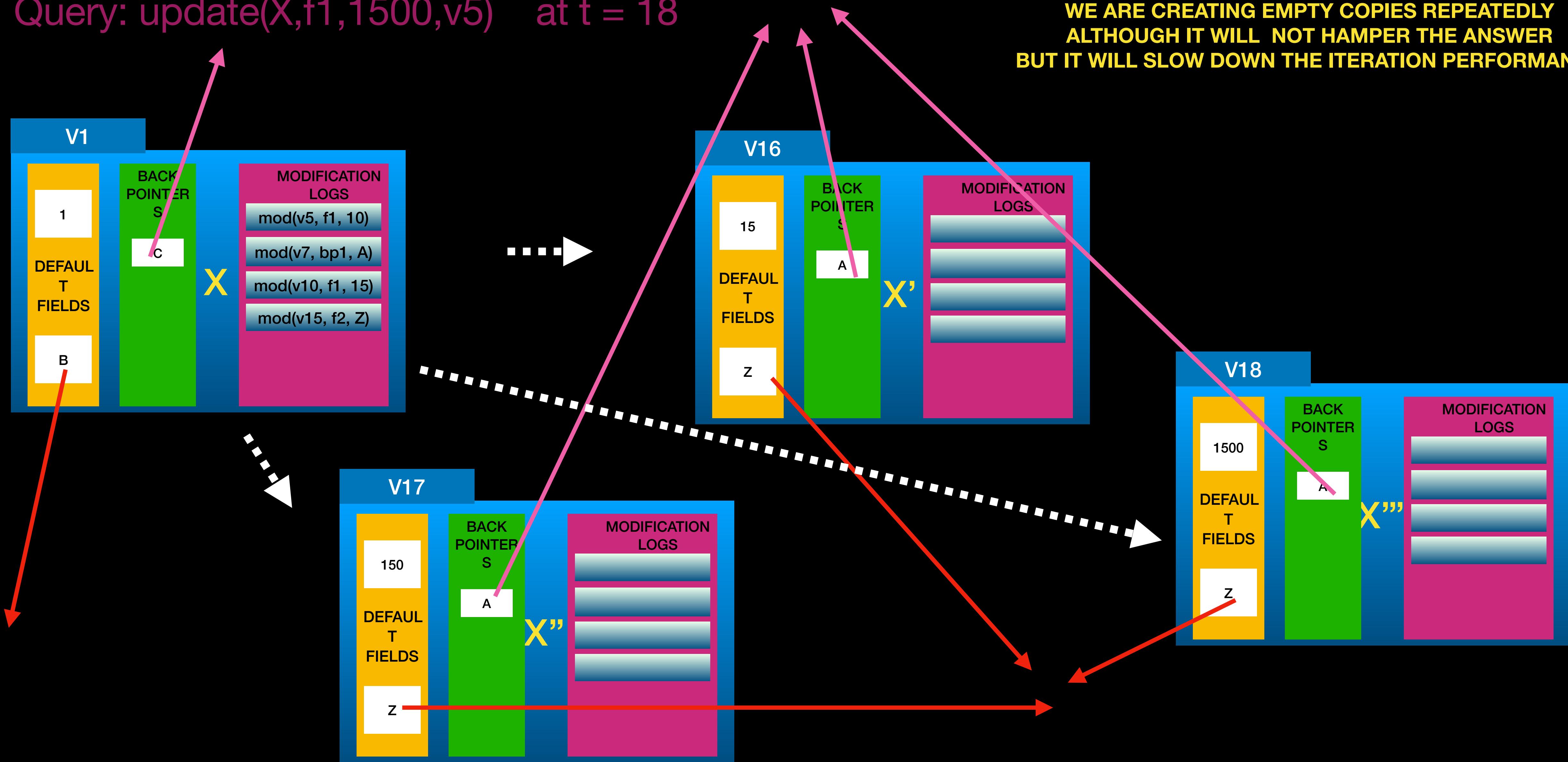
**NOTE: THIS IS NOT DONE IN FULL PERSISTENT STRATEGY**

# Problem with 1:0 Approach (What was in Partial Persistent Mode)

Query: `update(X, f1, 1500, v5)` at  $t = 18$

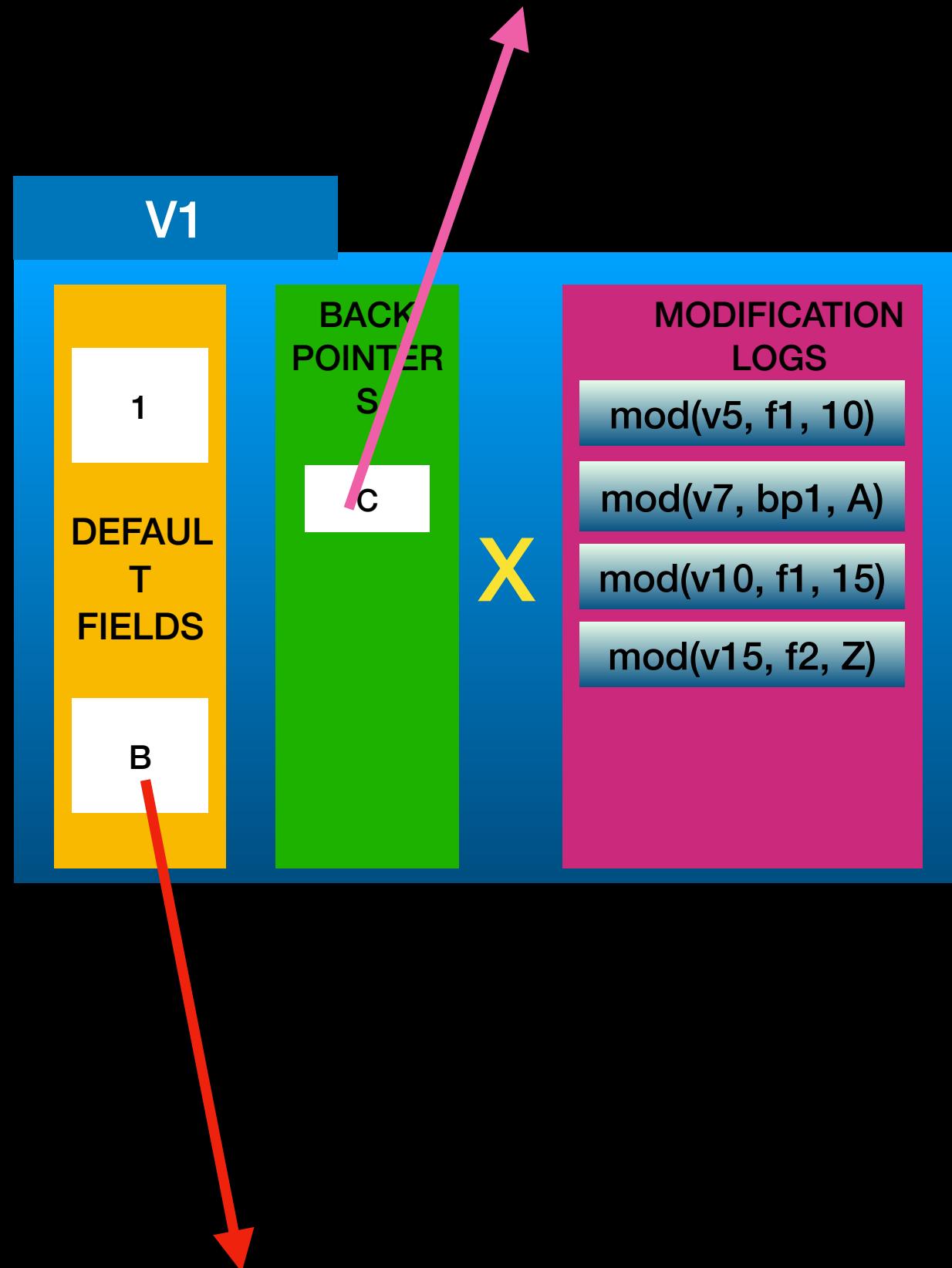
CONCERN:

WE ARE CREATING EMPTY COPIES REPEATEDLY  
ALTHOUGH IT WILL NOT HAMPER THE ANSWER  
BUT IT WILL SLOW DOWN THE ITERATION PERFORMANCE



NOTE: THIS IS NOT DONE IN FULL PERSISTENT STRATEGY

# OPTIMISATION with 1:1 OR 2:1 Approach



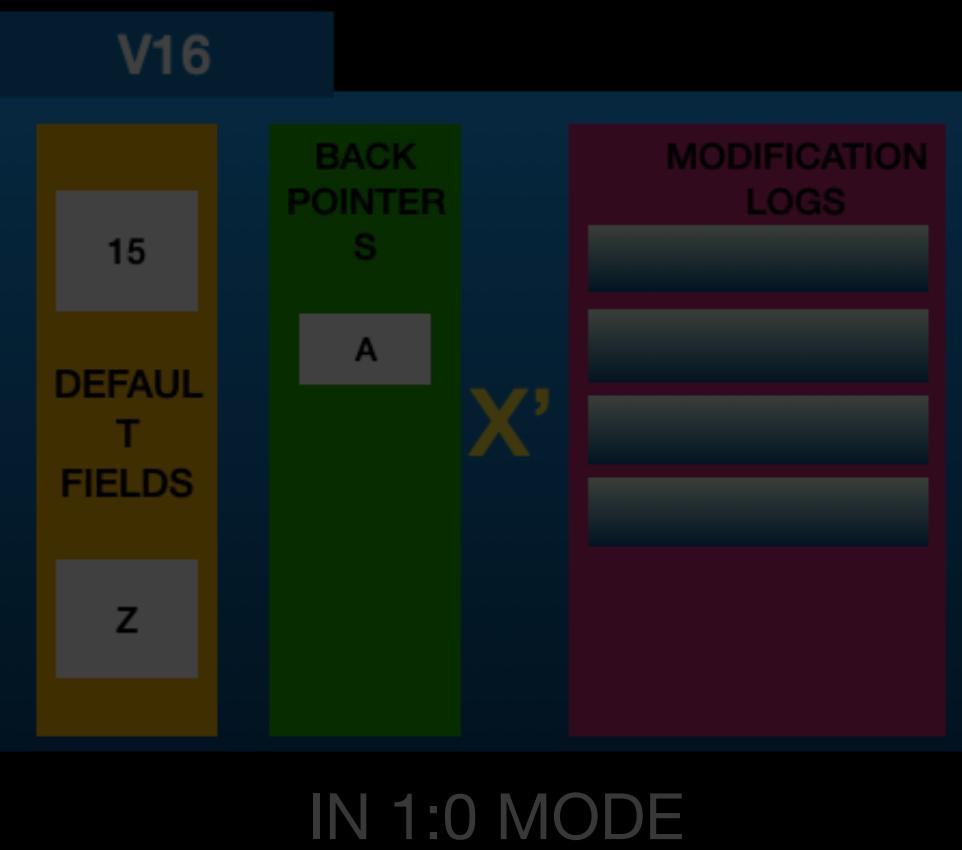
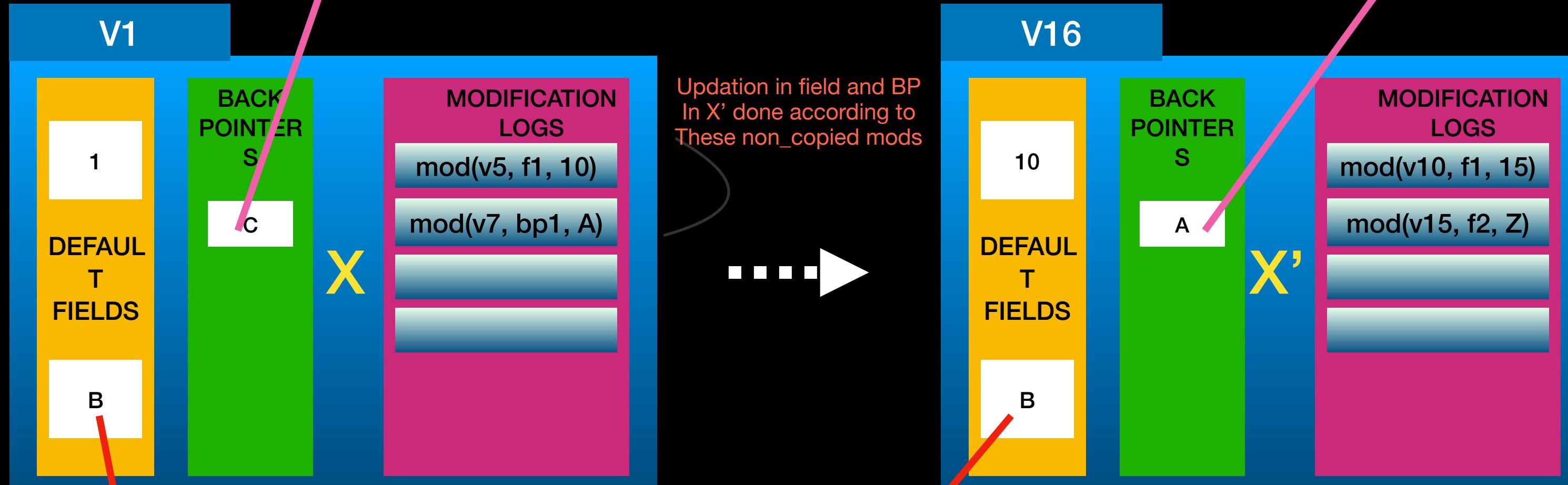
Suppose Node in any version ( cur t >=15) looks like this

*WE ARE GOING TO PRESENT 1:1 SPLIT*

**NOTE: THIS IS DONE IN FULL PERSISTENT STRATEGY**

# OPTIMISATION with 1:1 Approach

Query: update(X,f1,15,v5) at t = 16

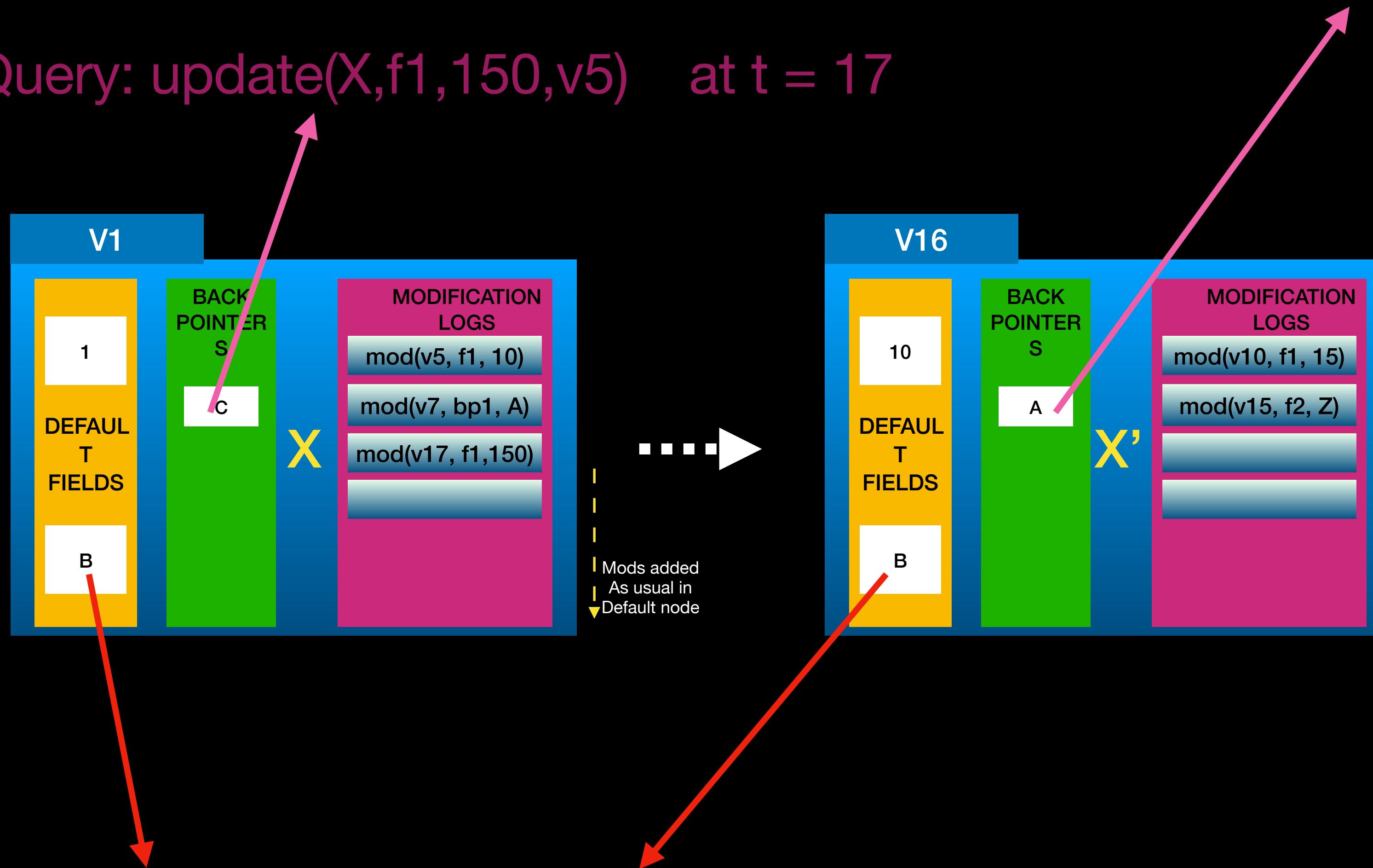


IN 1:0 MODE

**NOTE: THIS IS DONE IN FULL PERSISTENT STRATEGY**

# OPTIMISATION with 1:1 Approach

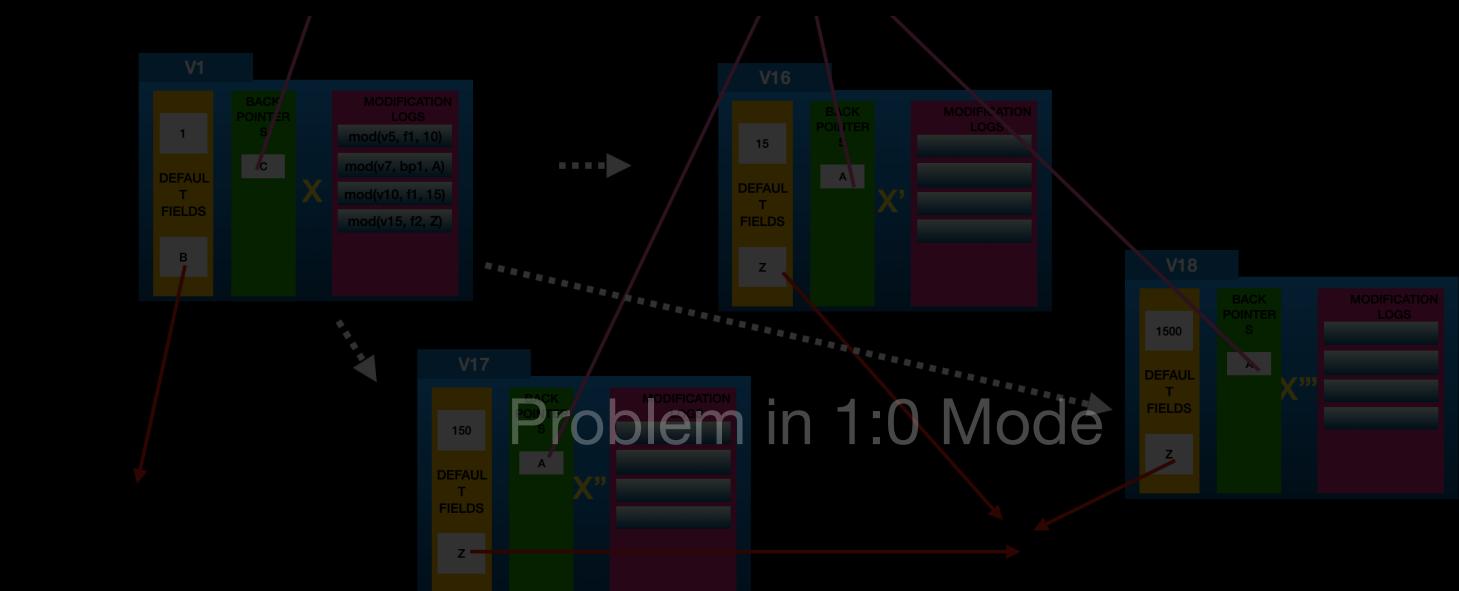
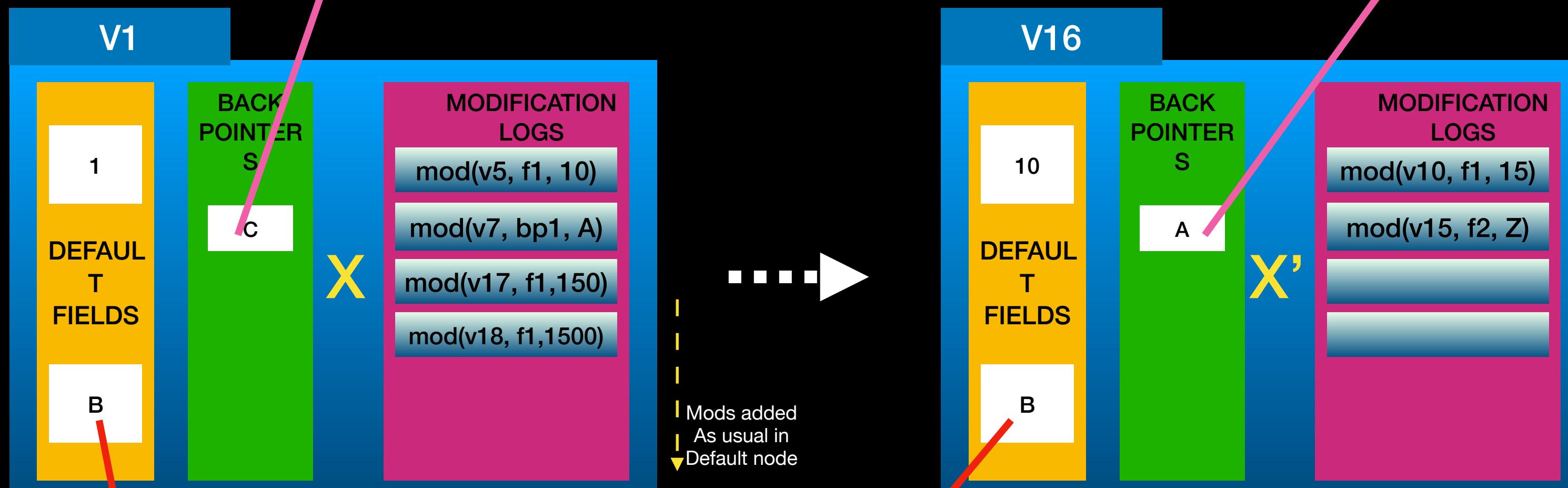
Query: update(X,f1,150,v5) at t = 17



**NOTE: THIS IS DONE IN FULL PERSISTENT STRATEGY**

# OPTIMISATION with 1:1 Approach

Query: update(X,f1,1500,v5) at t = 18



**CONCERN:**

**HERE WE ARE NOT CREATING SUCCESSIVE EMPTY\_MOD NODE-copies**

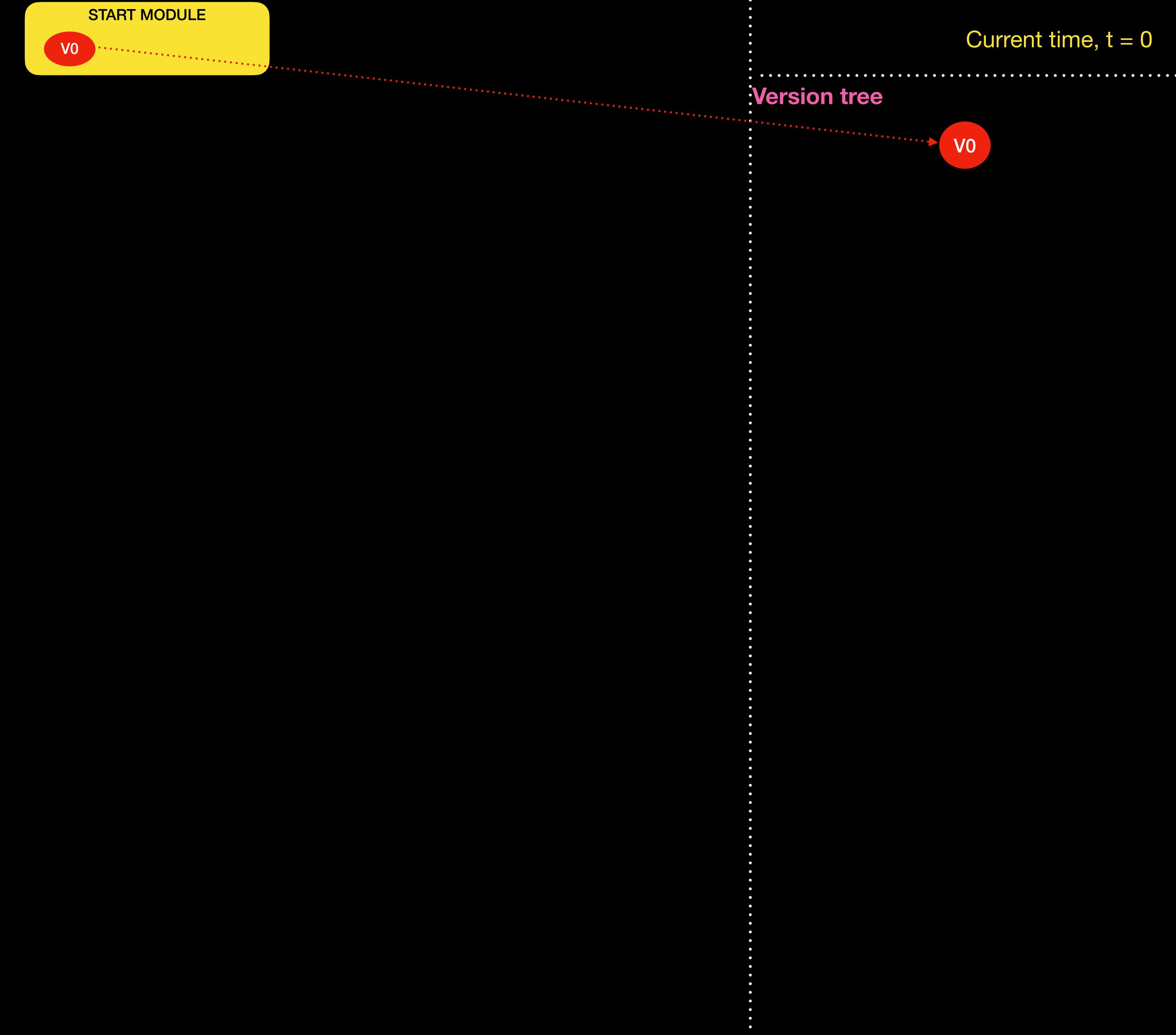


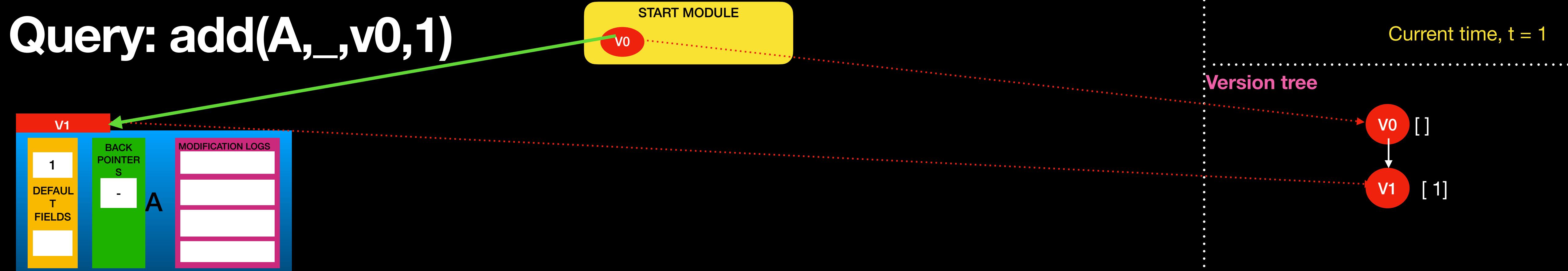
**NOTE: THIS IS DONE IN FULL PERSISTENT STRATEGY**

# Simulation Using Full Persistent Model



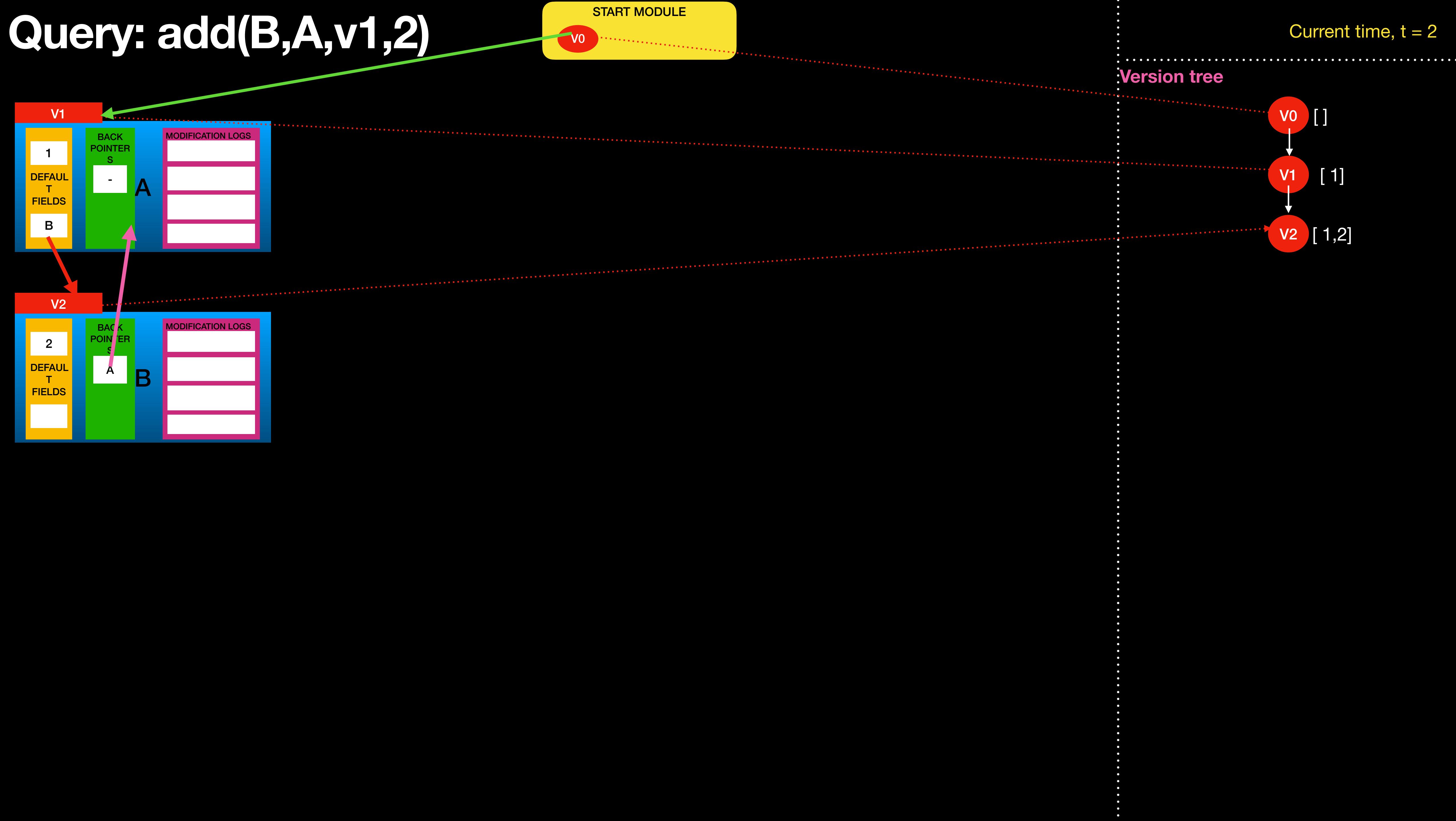
Query: init\_LL()

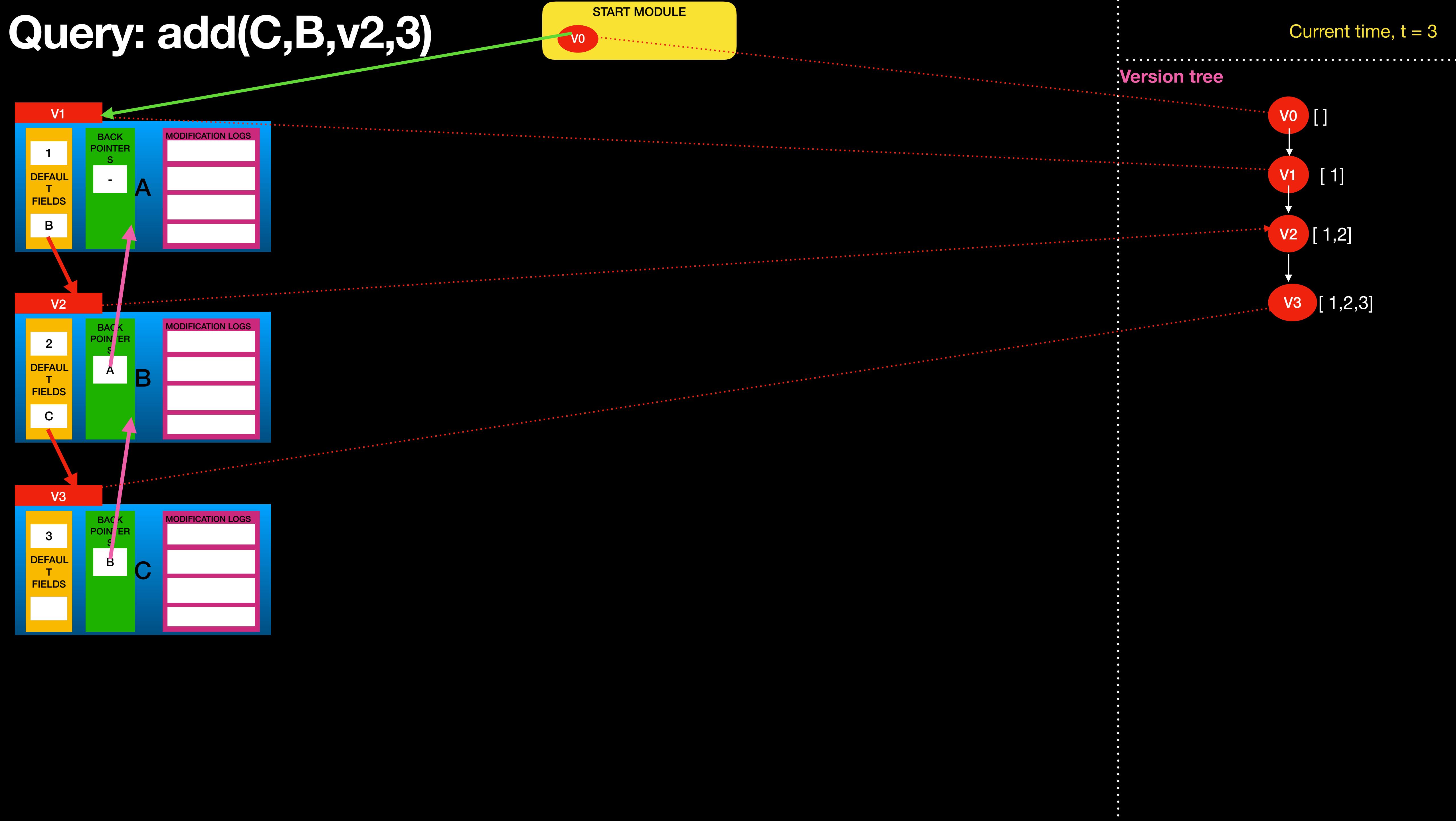


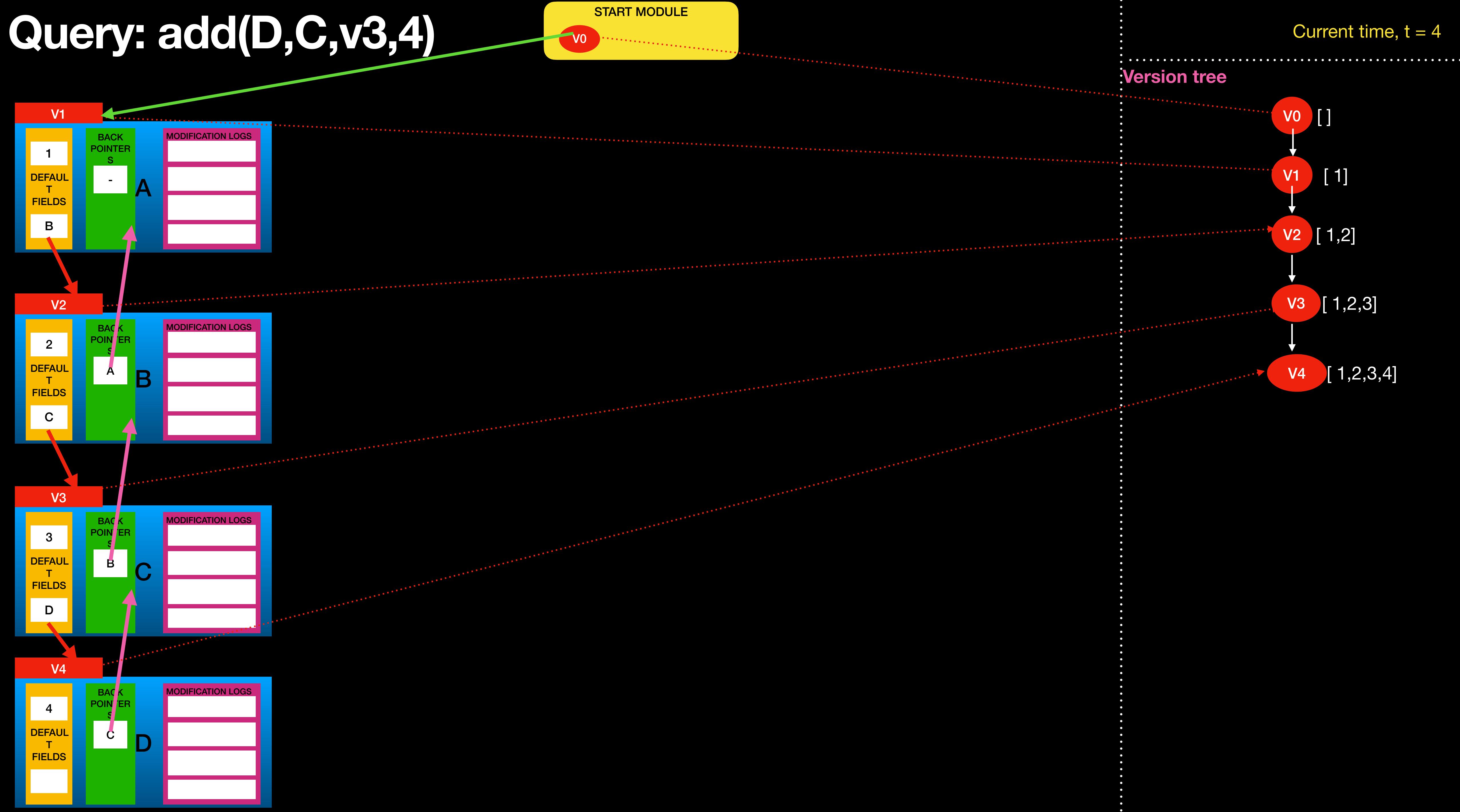


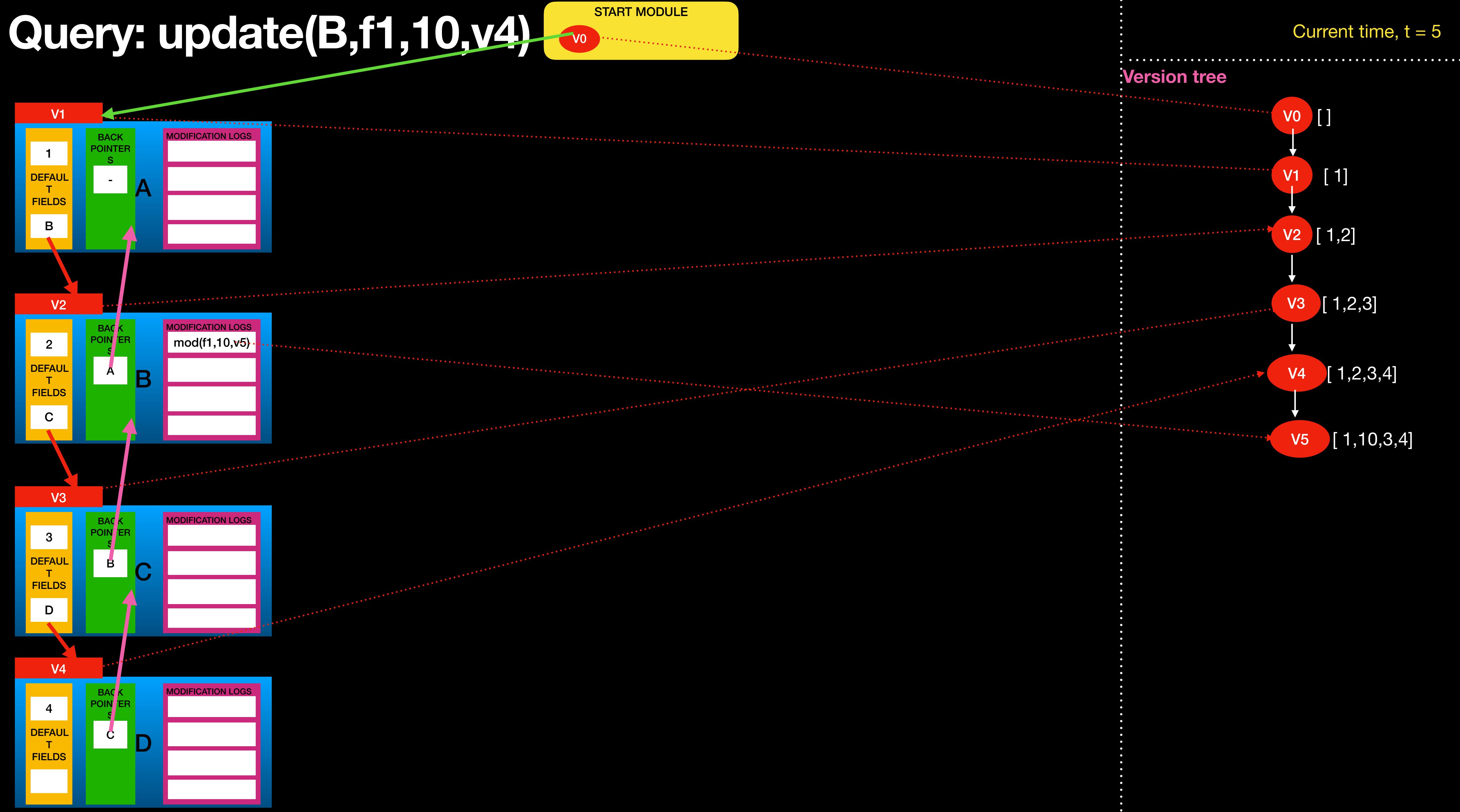
**add(X,W,v90,1000)  
at time = 95  
Includes**

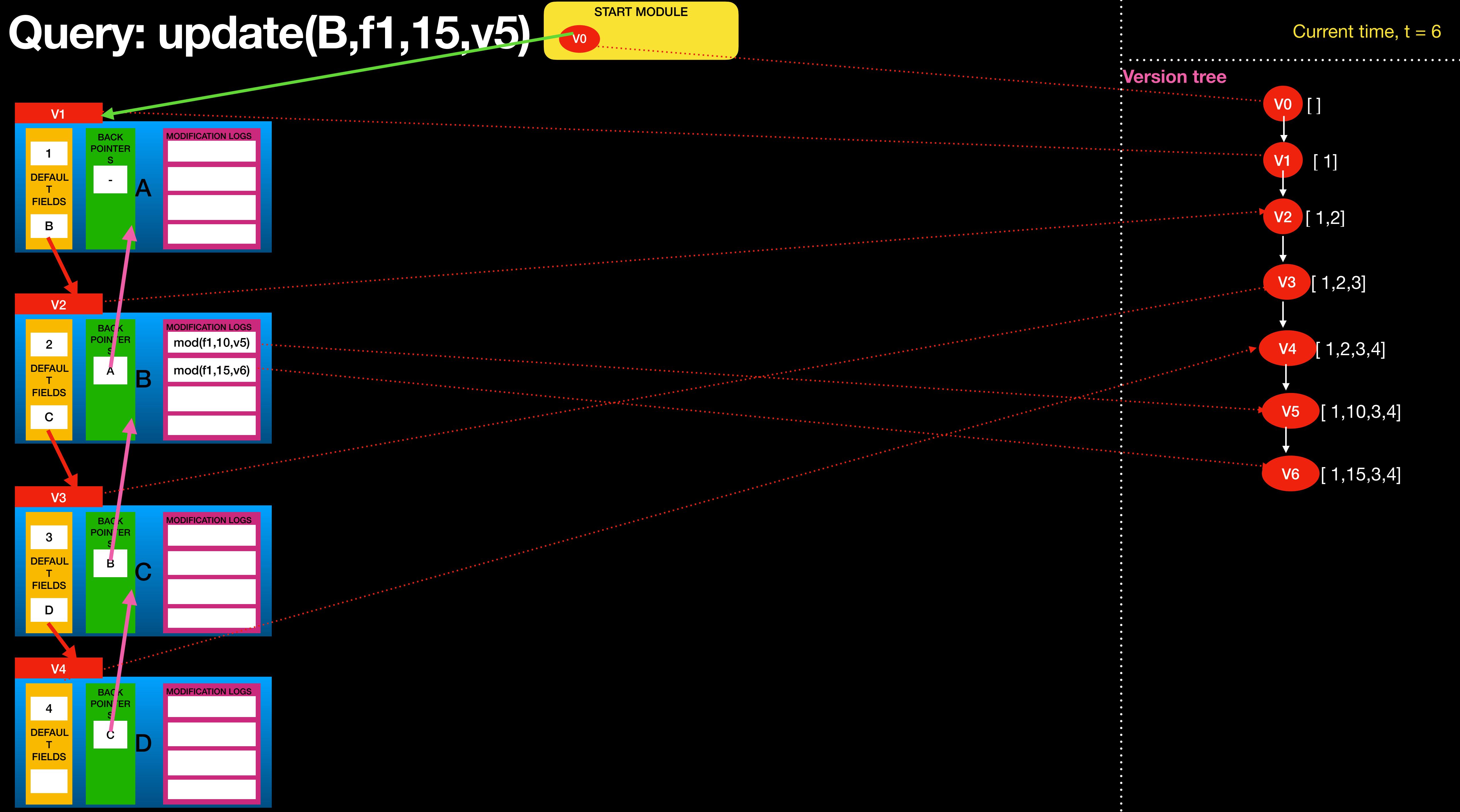
Create new node X with f1 = 1000, defaultVersion = v95
update(W,f2,X,v90)
update(X, bp, W, v90) [here, update means set at default level]
Add v95 under v90 in version tree

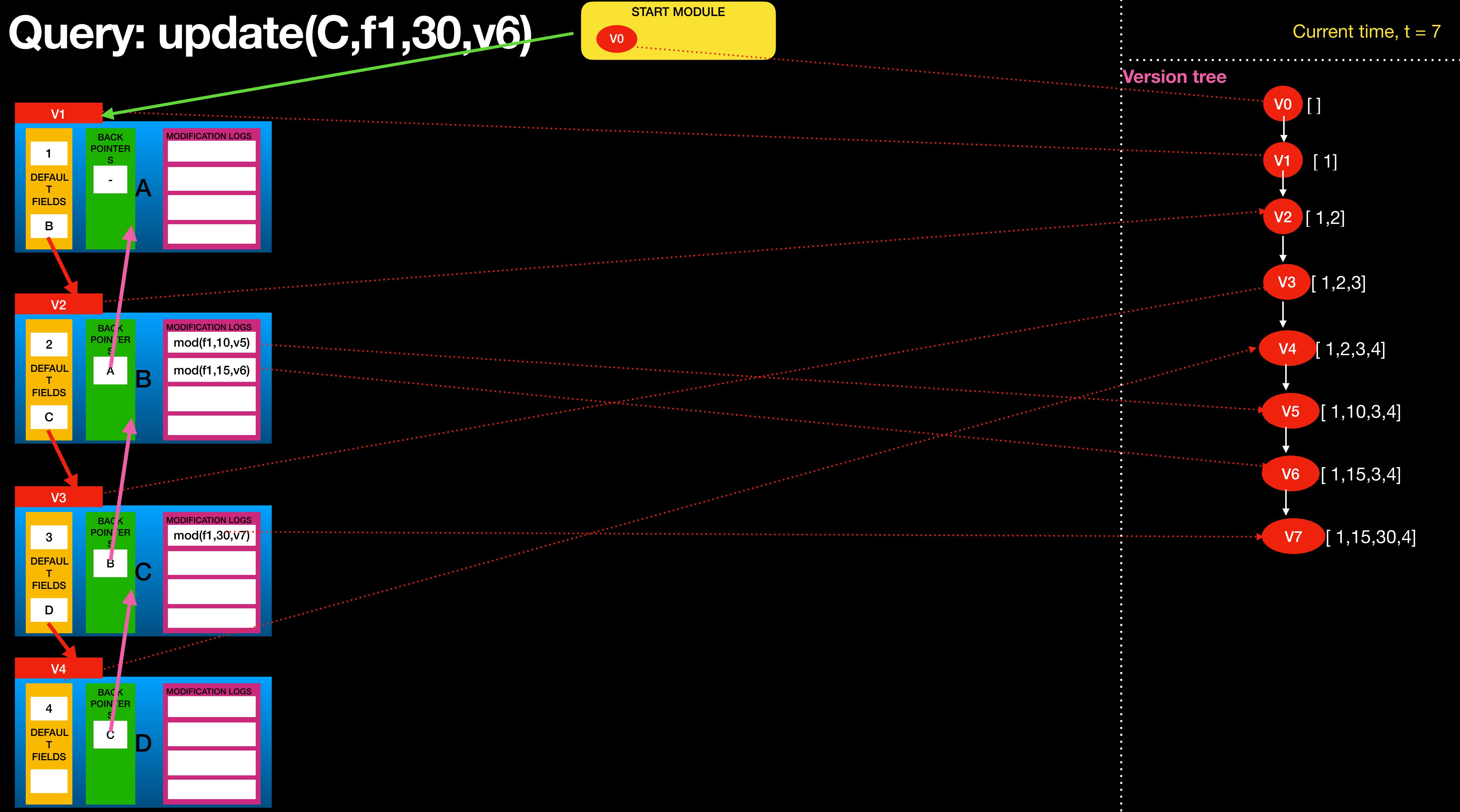


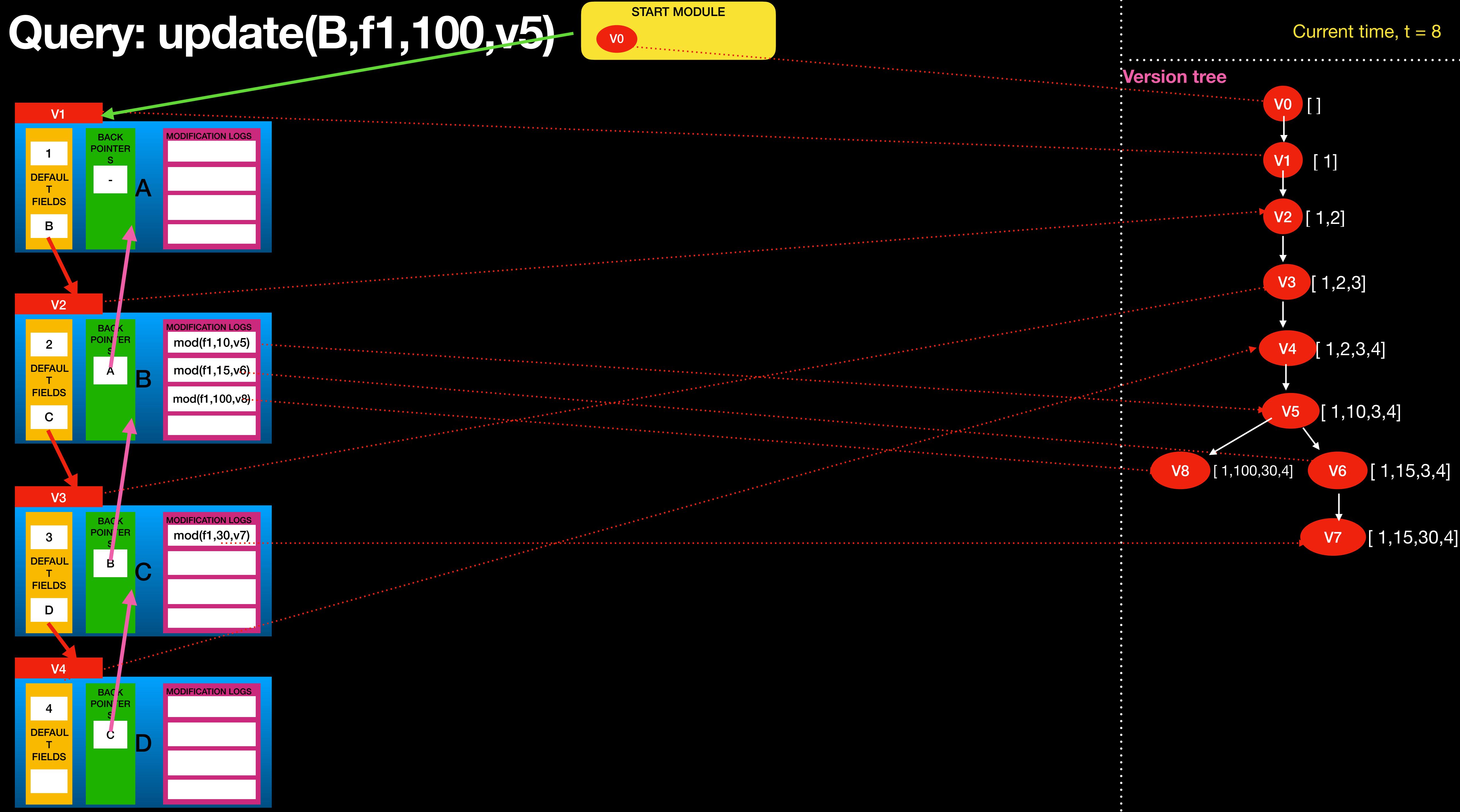


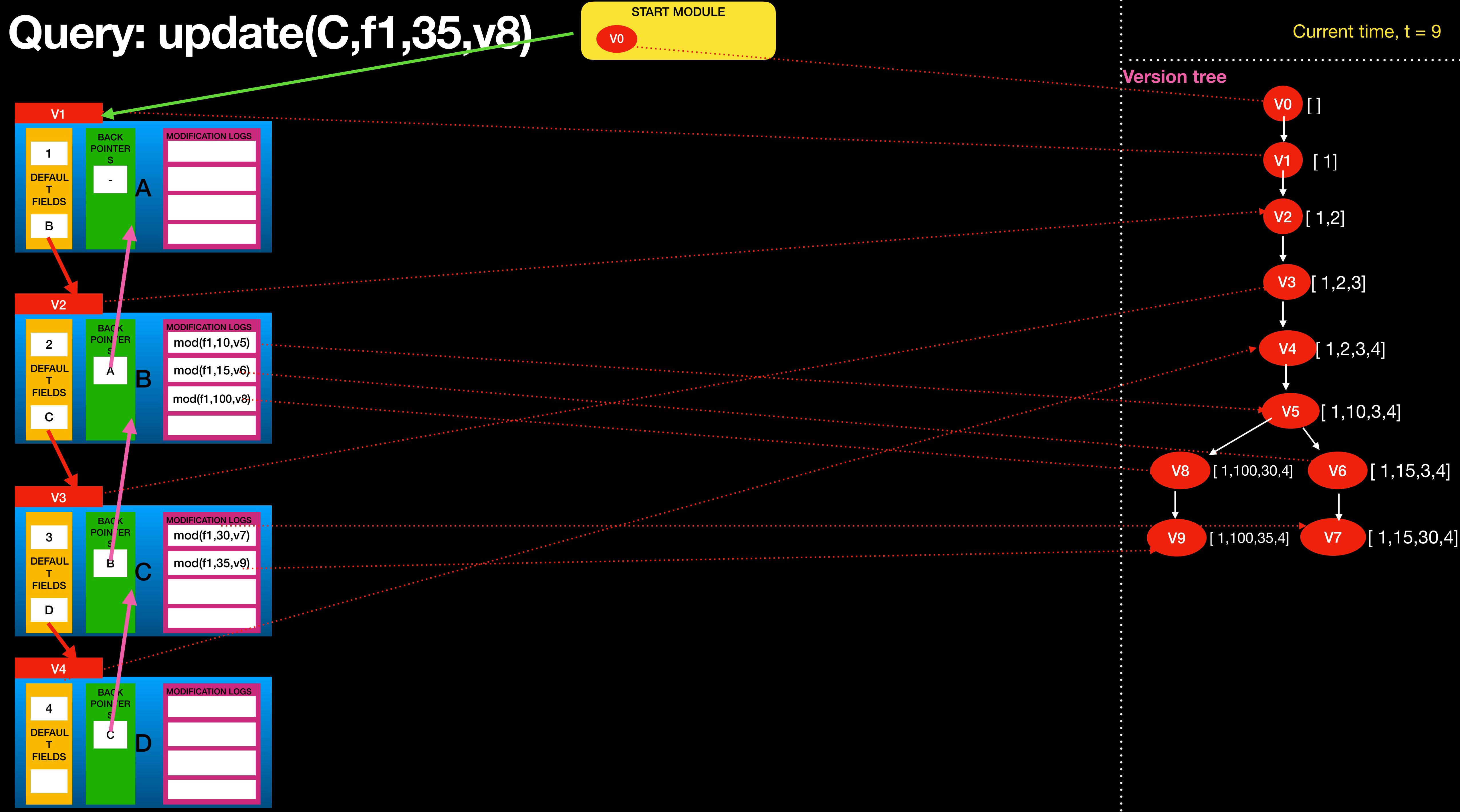


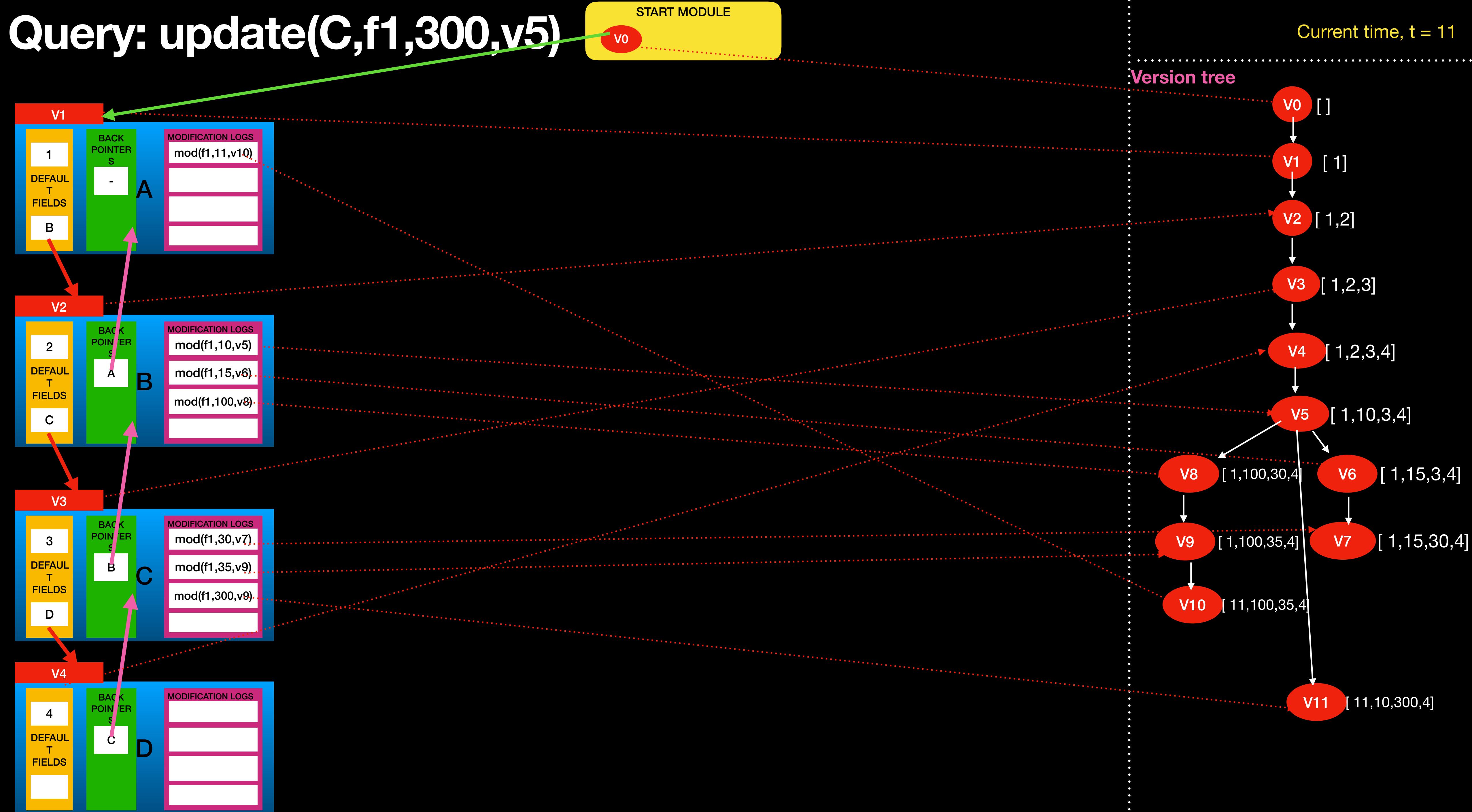


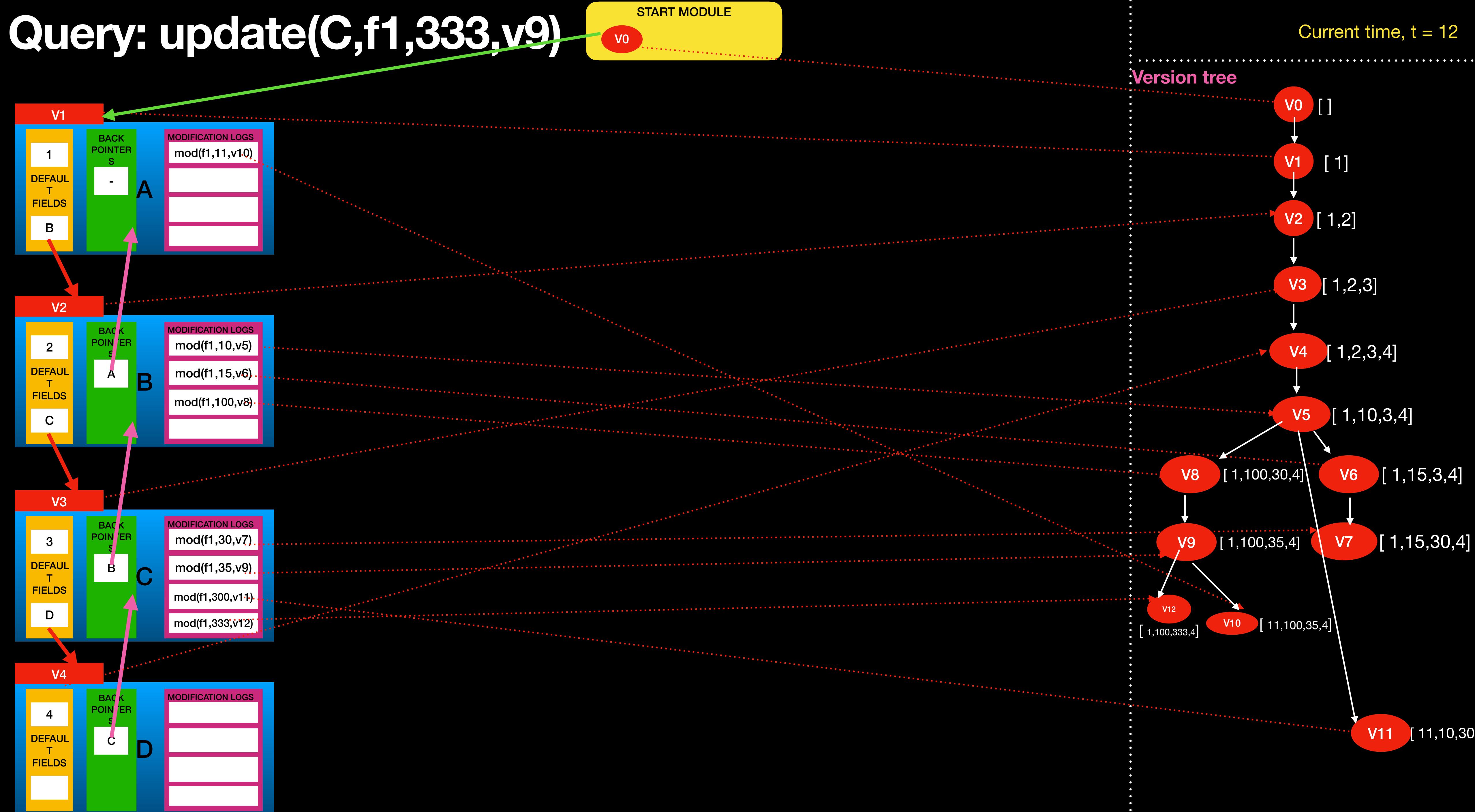










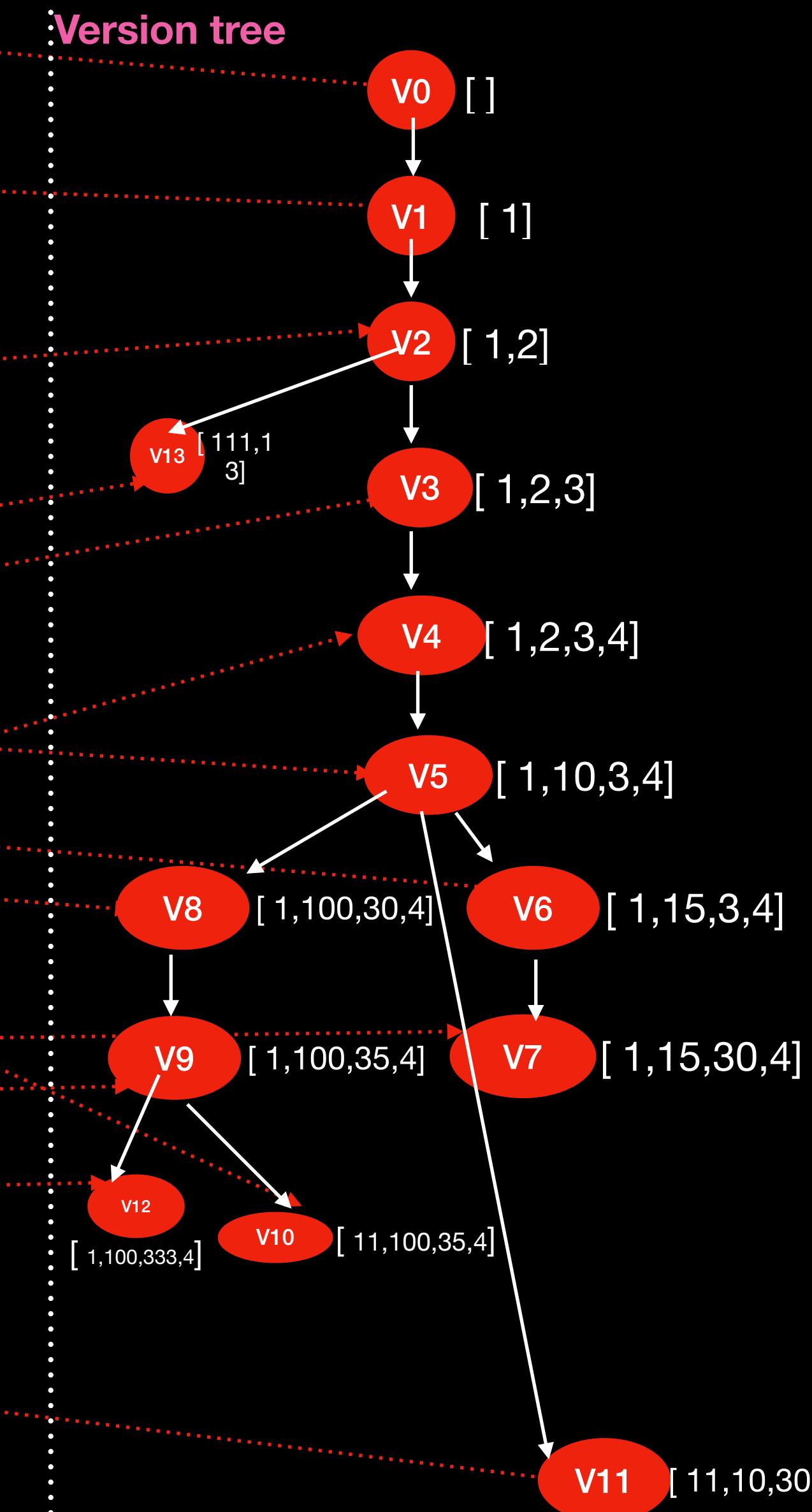
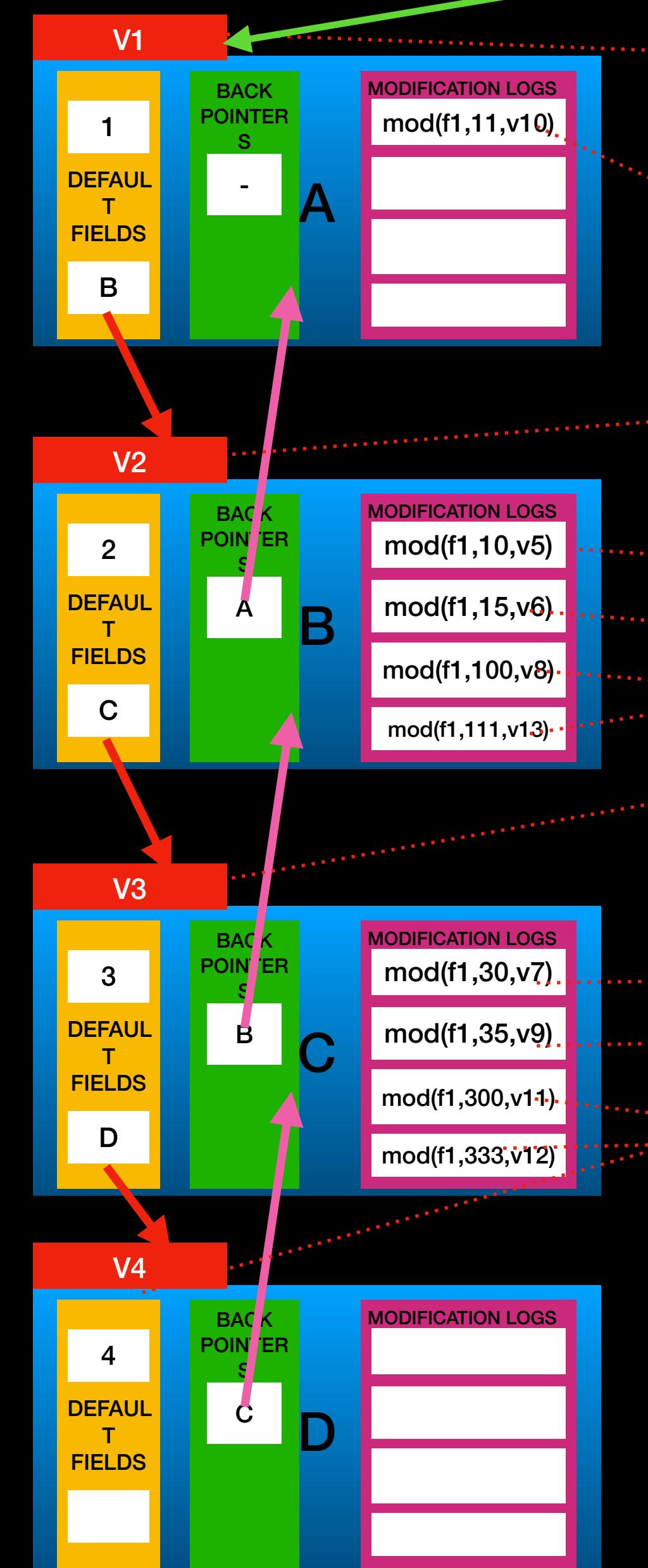


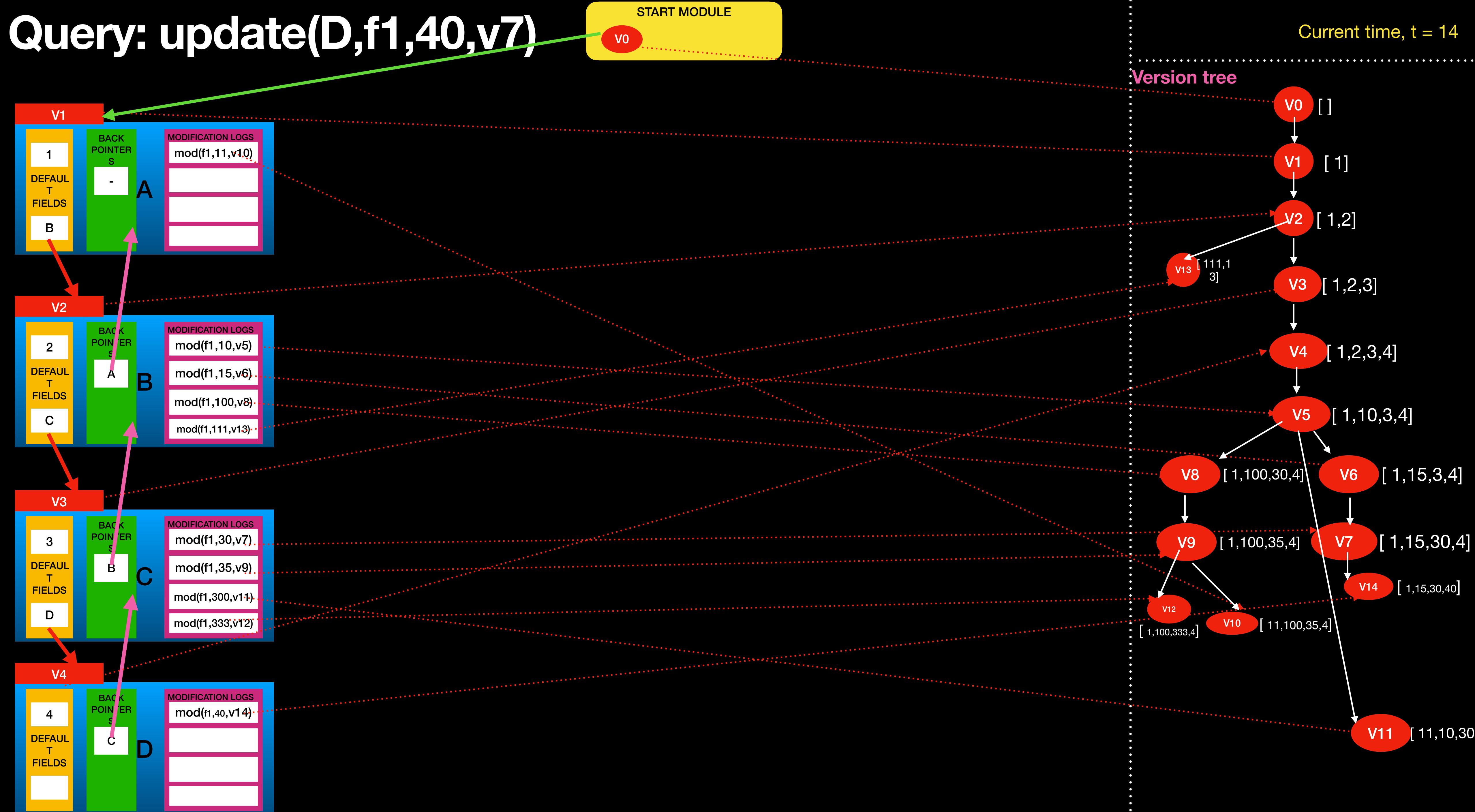
# Query: update(B,f1,111,y2)

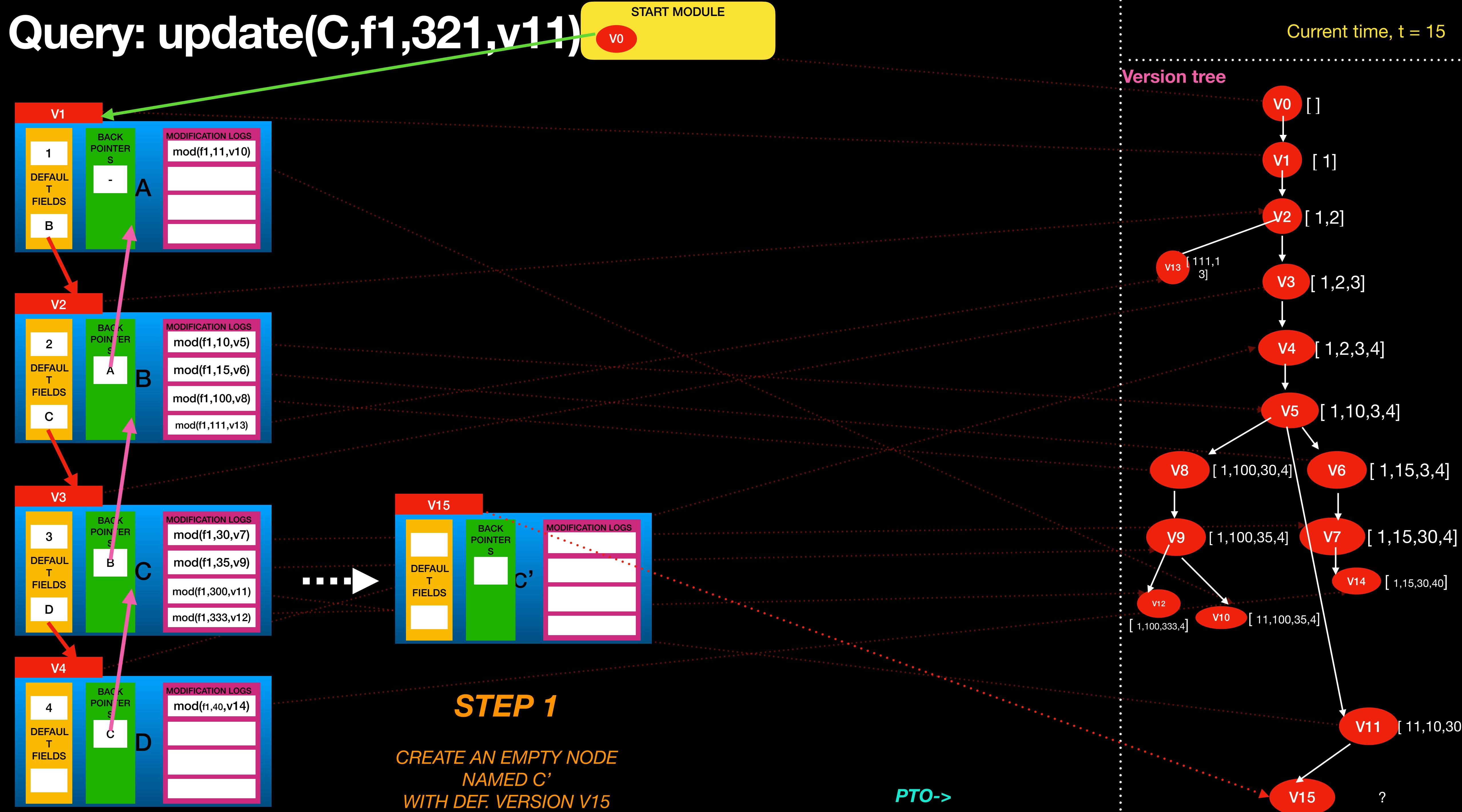
**START MODUL**

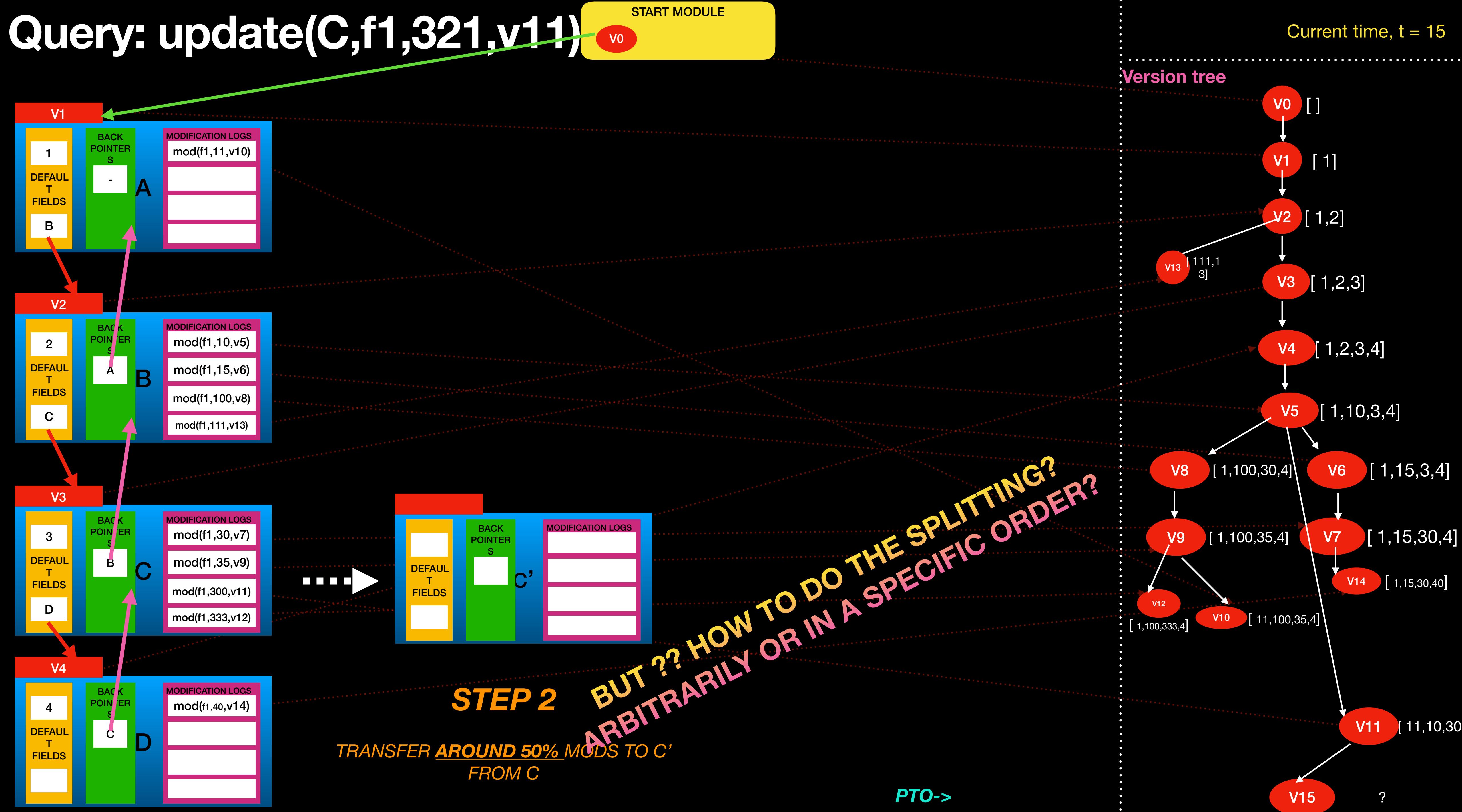
1

## Current time, t = 13











# HOW TO DO THE SPLITTING? ARBITRARILY OR IN A SPECIFIC ORDER?

mod(f1,30,v7)
mod(f1,35,v9)
mod(f1,300,v11)
mod(f1,333,v12)
V15

mod(f1,30,v7)
mod(f1,35,v9)
mod(f1,333,v12)
mod(f1,300,v11)
V15

Toposort

TOPOLOGICALLY SORT THESE MODS ACCORDING TO THE VERSION ORDER  
(if Vy is SUCCESSOR of Vx, then Vy is considered Greater - hence Vy will go to right subtree)  
Thus, you create an Ascending Order

I am still thinking!!  
Here. Any one help!!  
Thinking for an optimisation



## Who will go to New Copy??

Choose  
A Candidate Mod as pivot

Which has most number of mods who are strictly successors of that pivot  
In Topological Order

Then transfer that pivot along with its successors to the new copy.

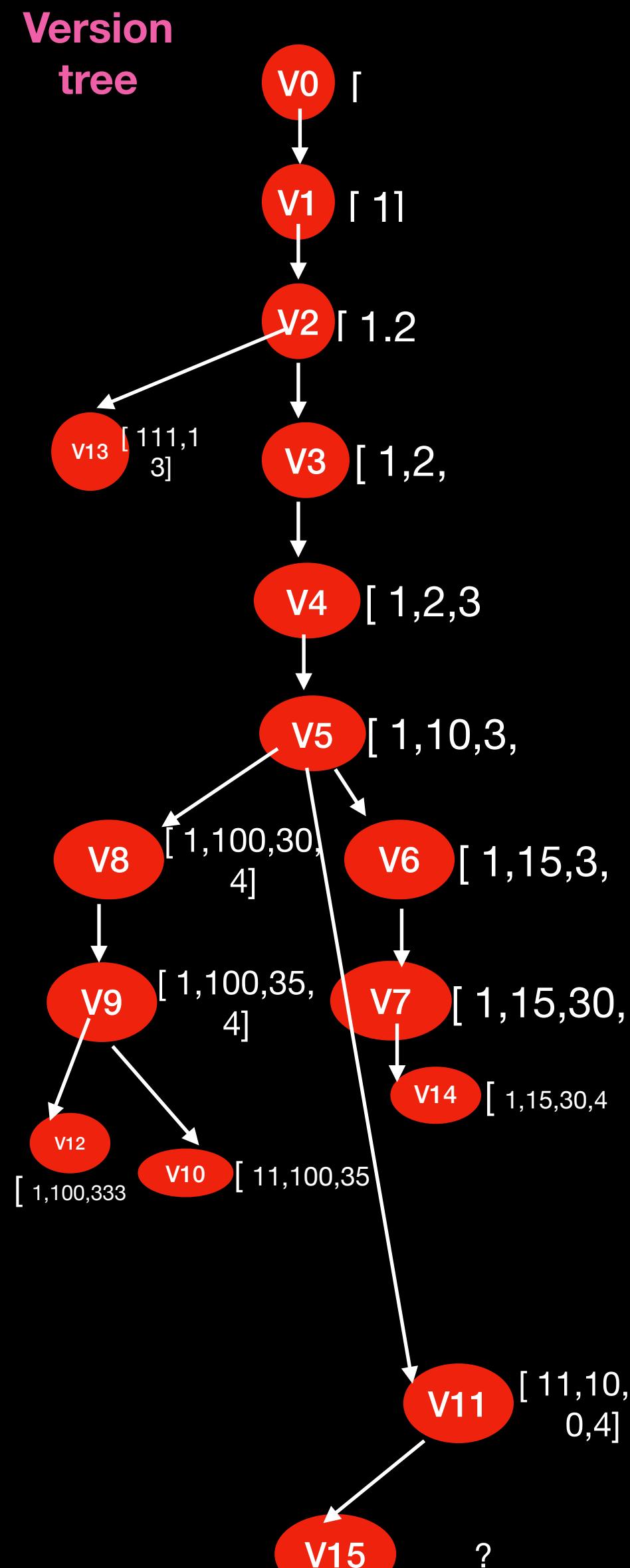
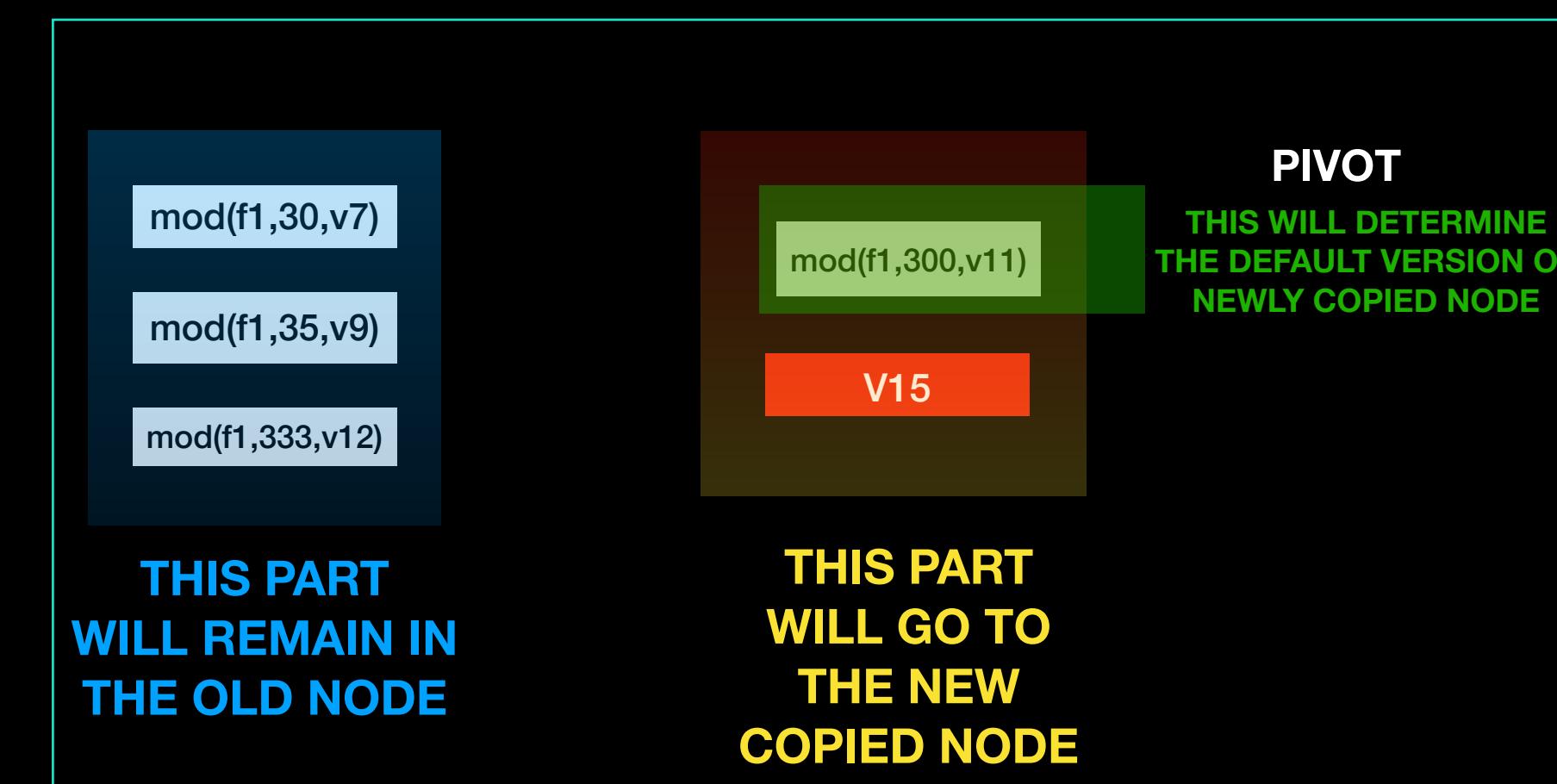
Option 1

mod(f1,30,v7)
mod(f1,35,v9)
mod(f1,333,v12)
mod(f1,300,v11)
V15

Option 2

mod(f1,30,v7)
mod(f1,35,v9)
mod(f1,333,v12)
mod(f1,300,v11)
V15

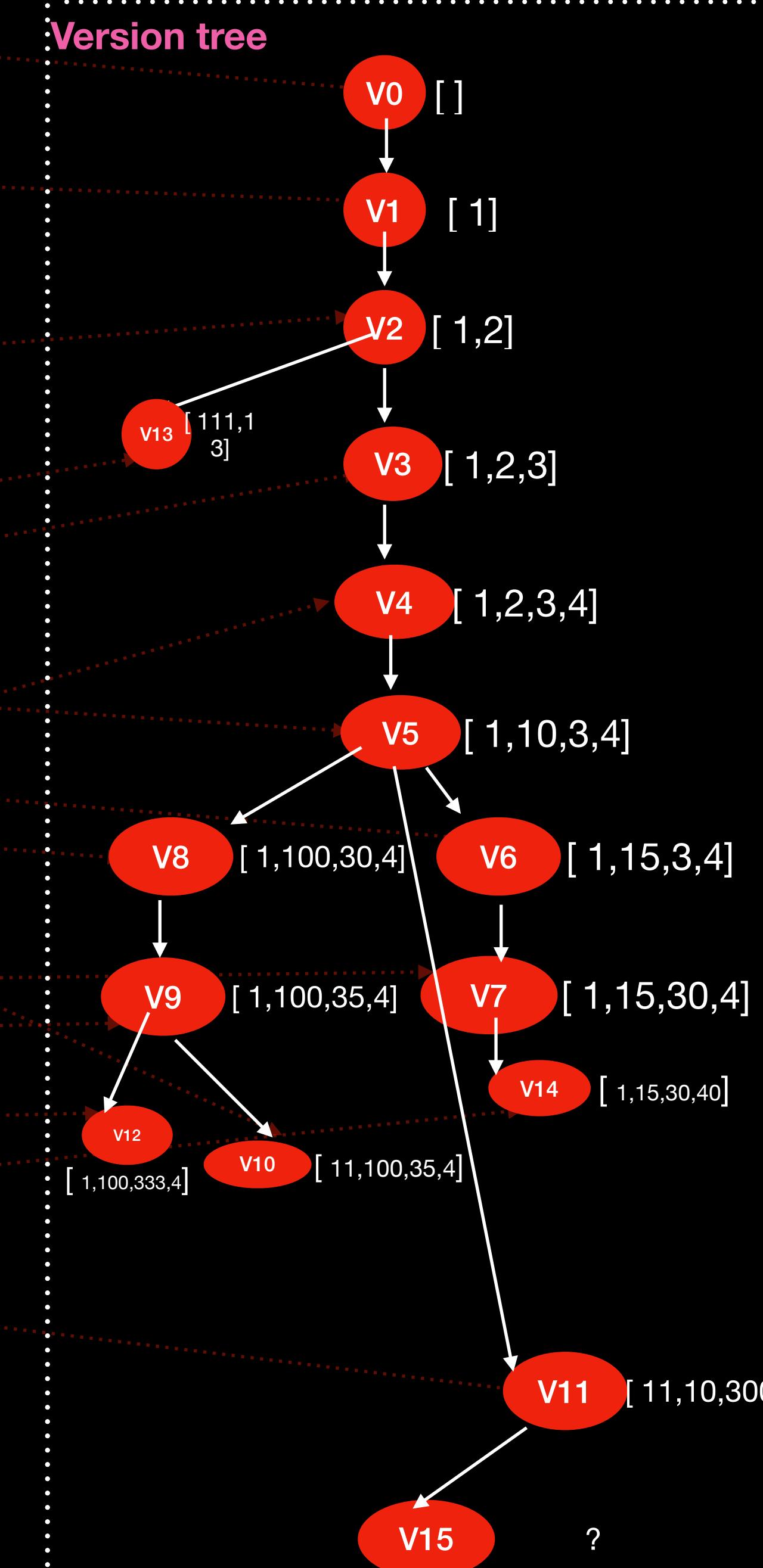
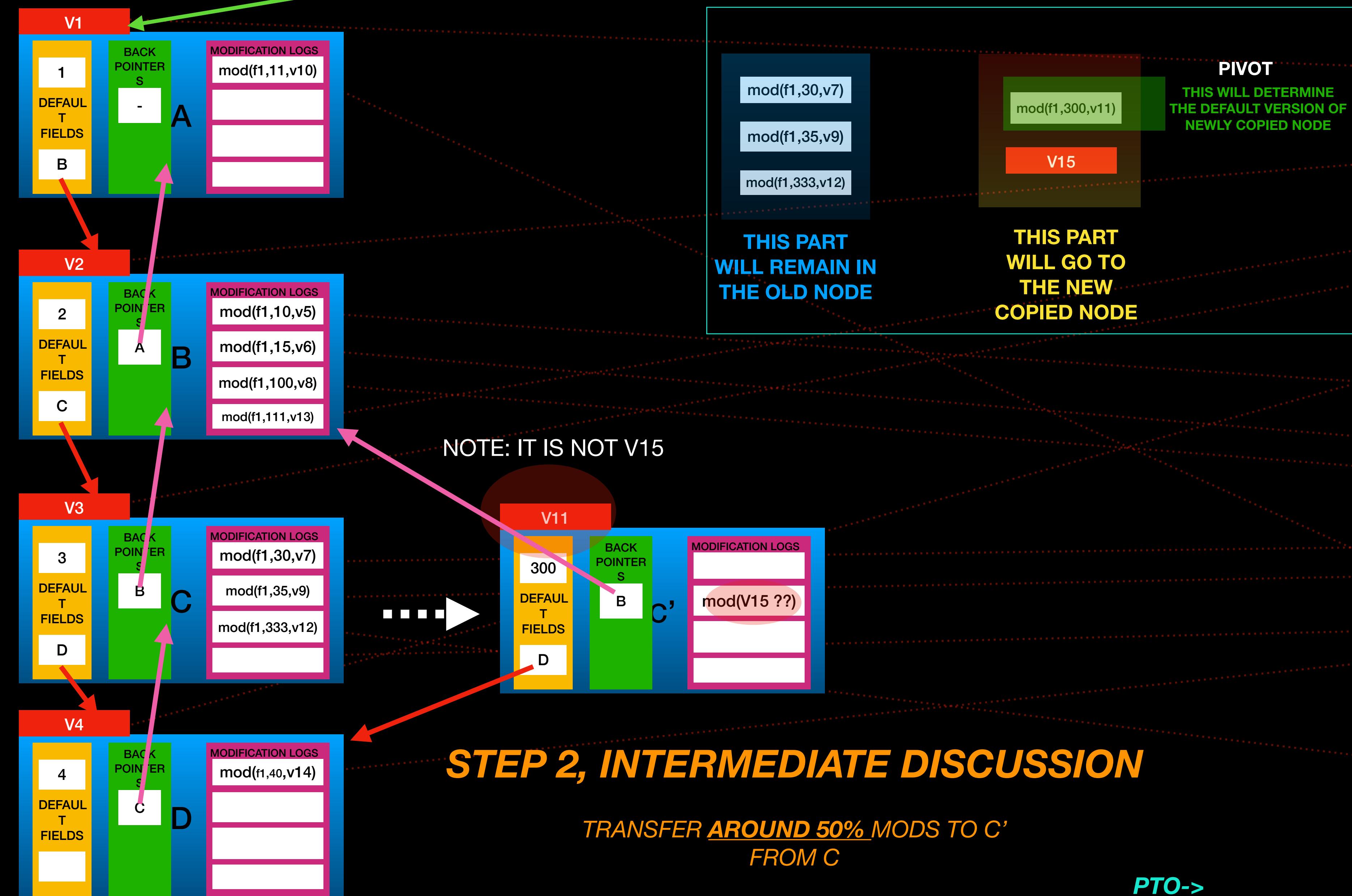
Suppose  
Option 2

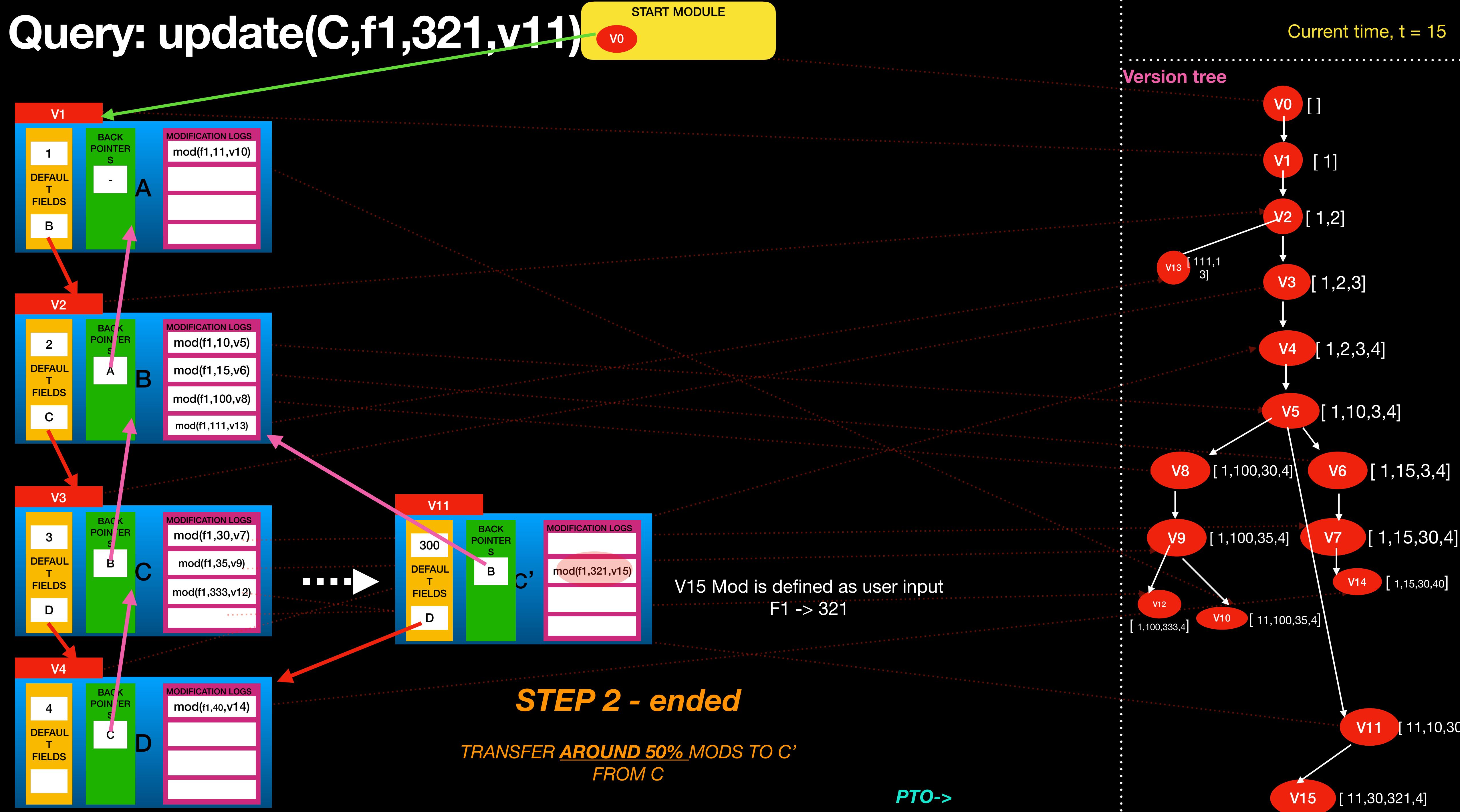


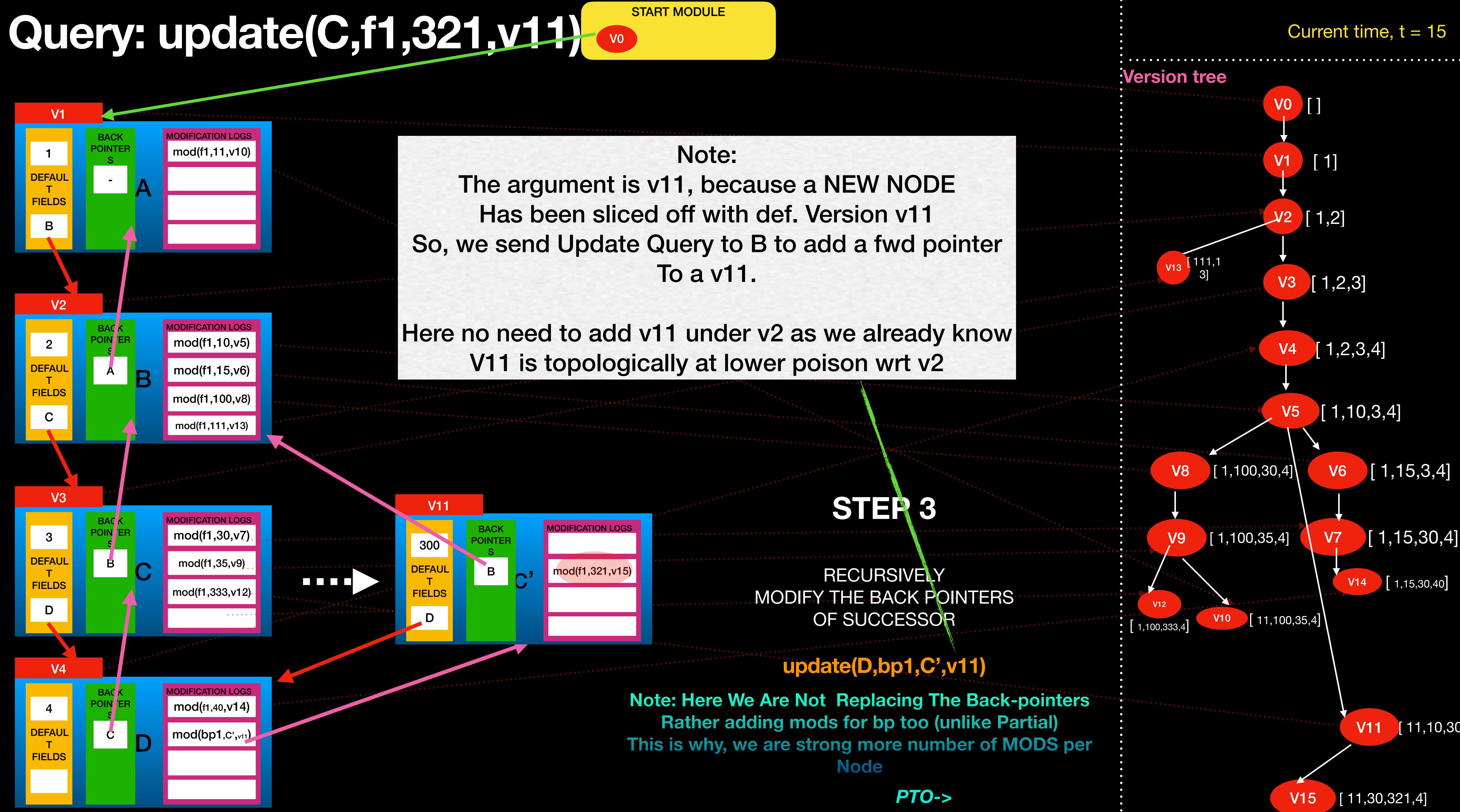
STEP 2, INTERMEDIATE DISCUSSION

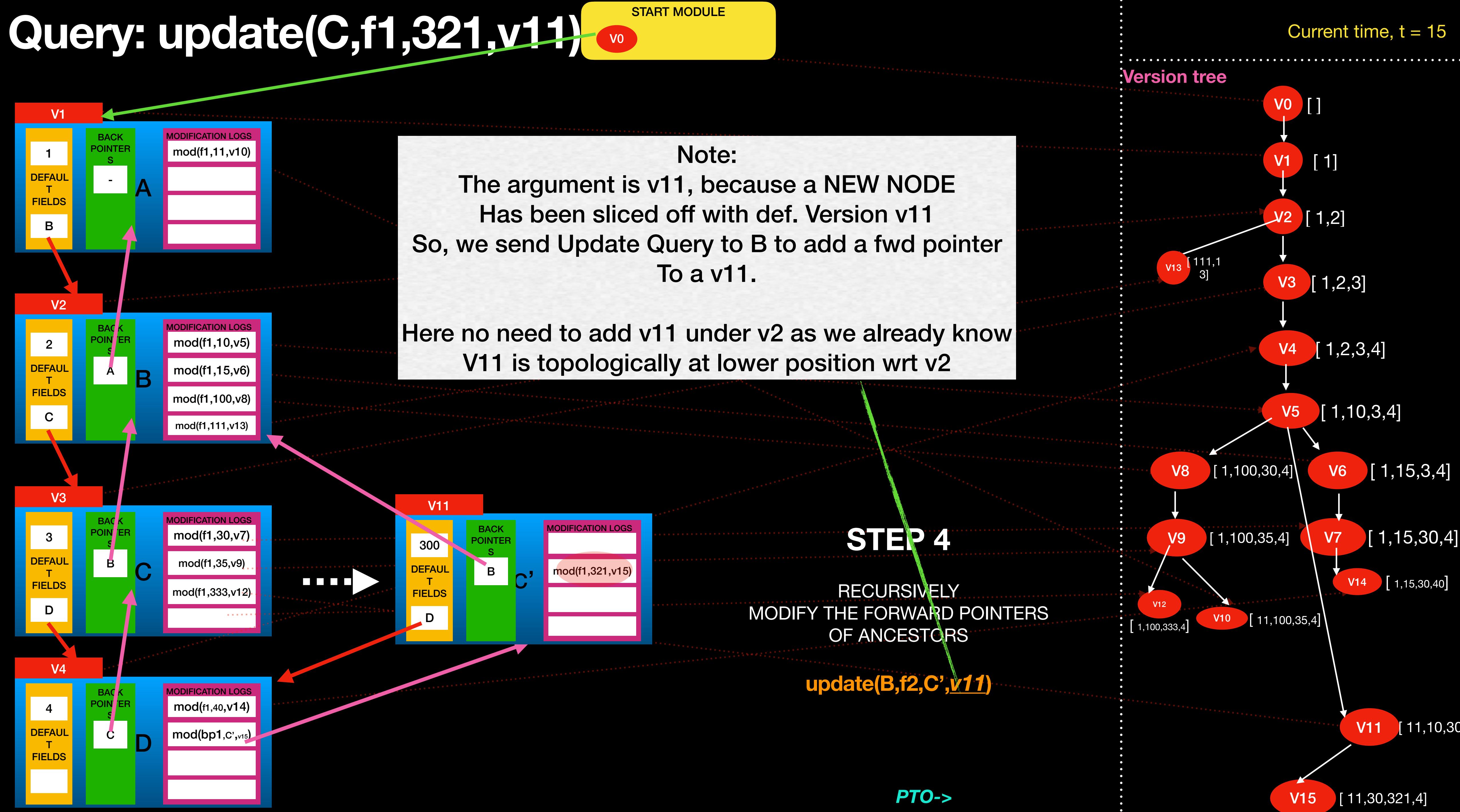
# Query: update(C,f1,321,y11)

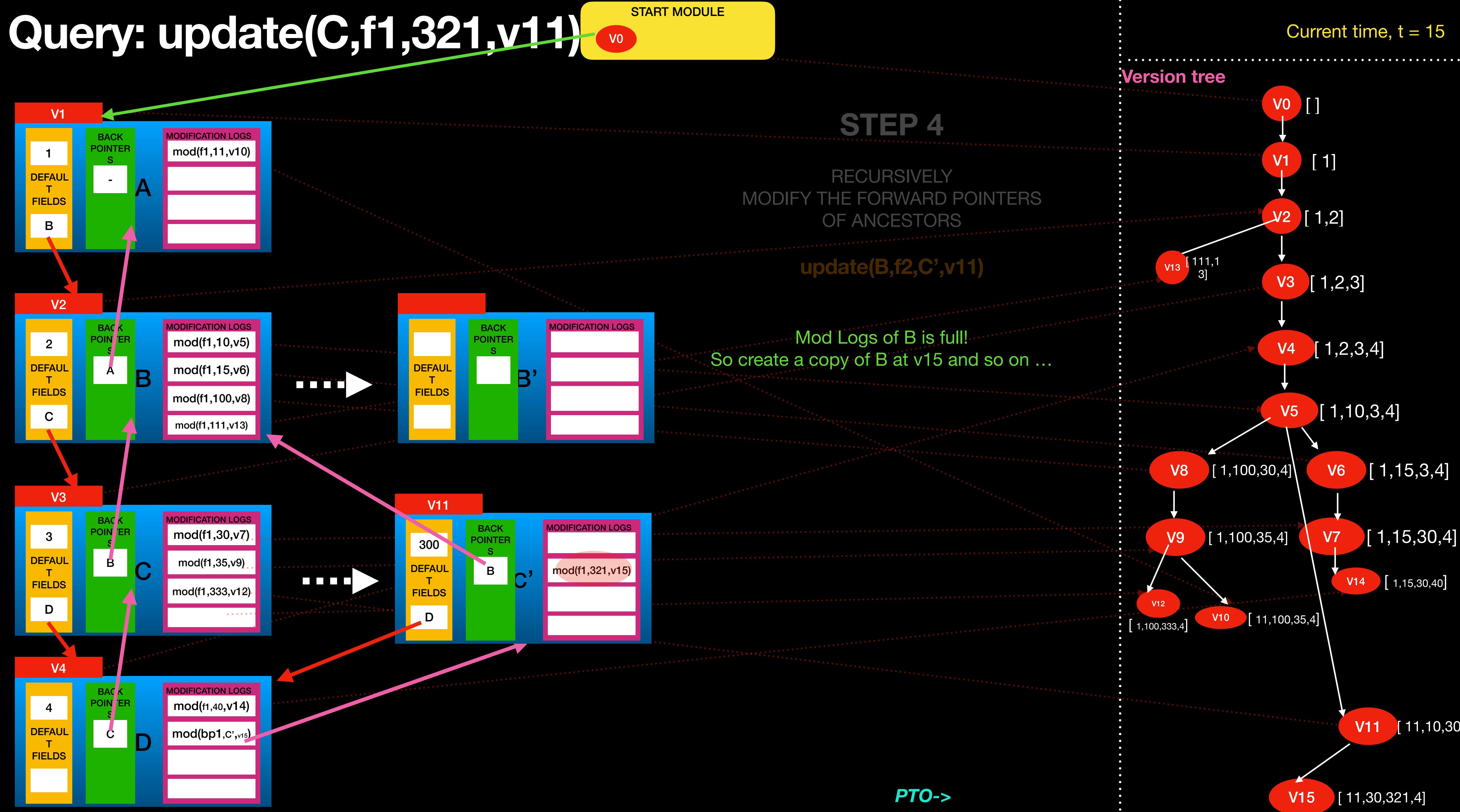
## Current time, t = 15













## HOW TO DO THE SPLITTING? ARBITRARILY OR IN A SPECIFIC ORDER?

TOPOLOGICALLY SORT THESE MODS ACCORDING TO THE VERSION ORDER  
(if Vy is SUCCESSOR of Vx, then Vy is considered Greater - hence Vy will go to right subtree)  
Thus, you create an Ascending Order

**I am still thinking!!**  
**Open problem!!**  
**Here. Any one help!!**  
**Thinking for an optimisation**

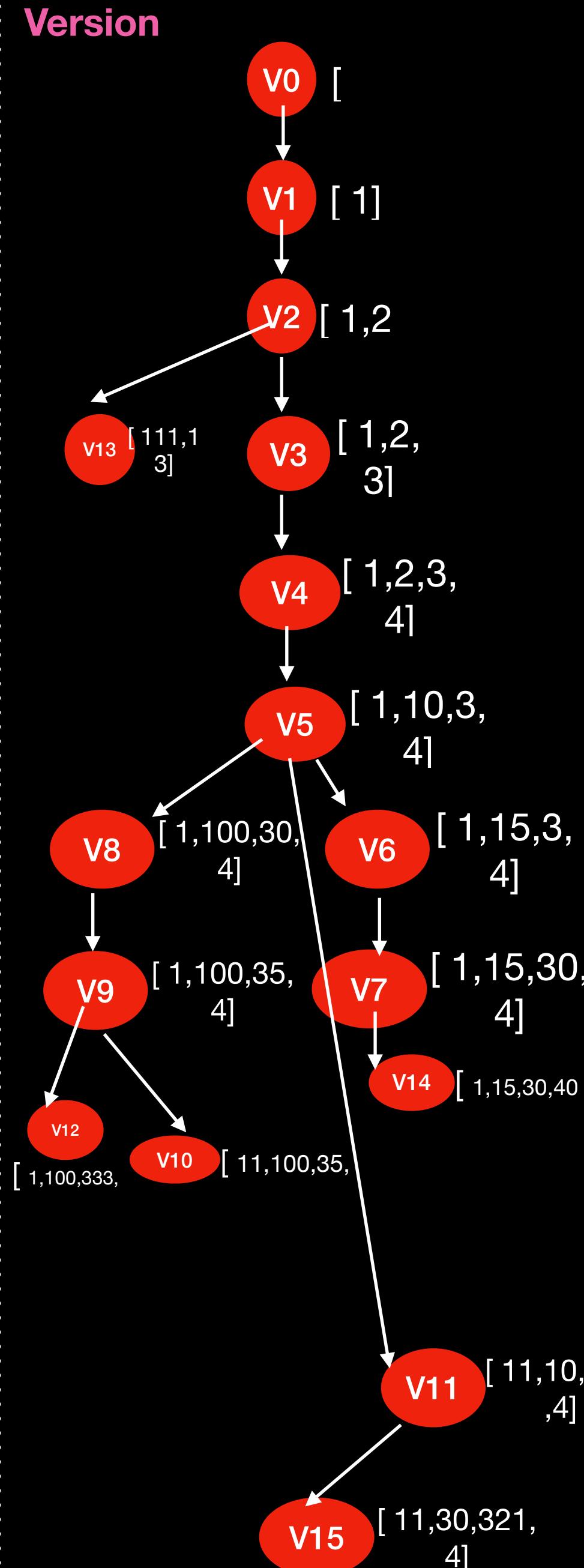
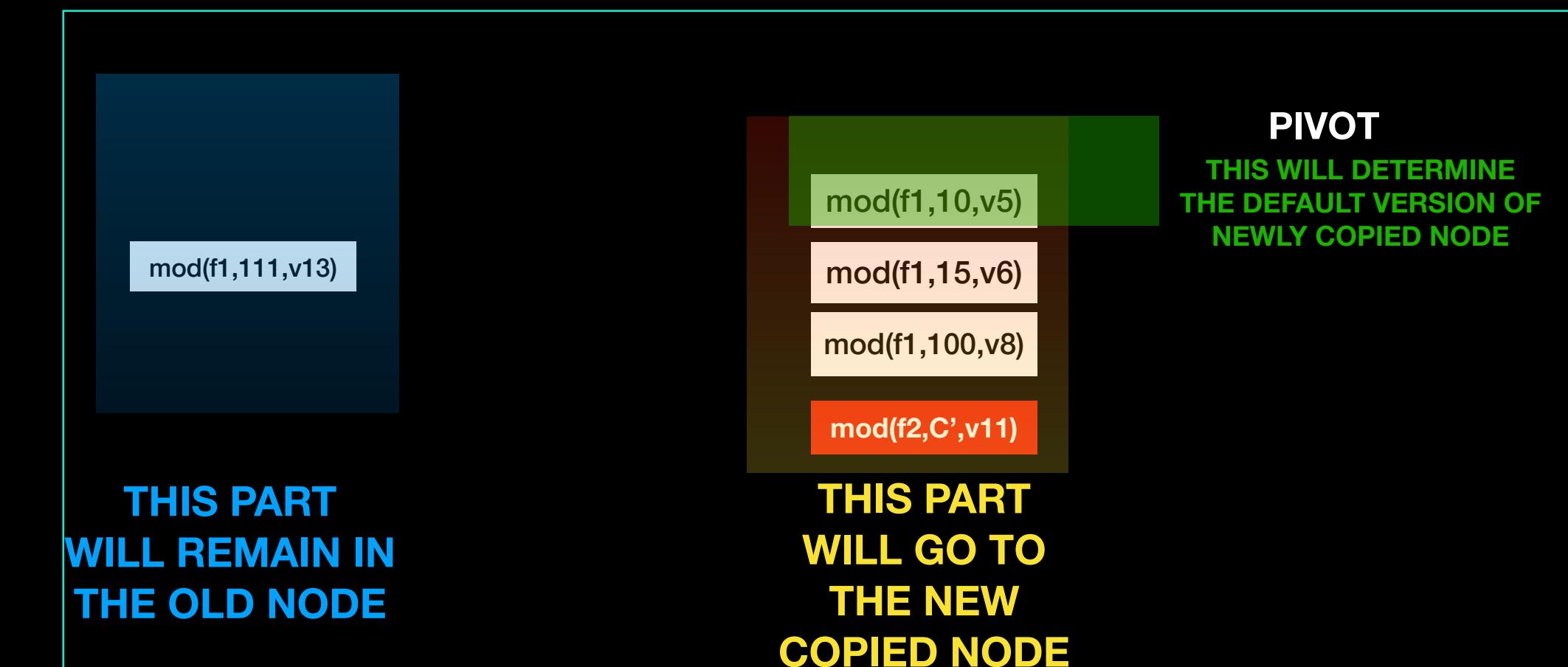
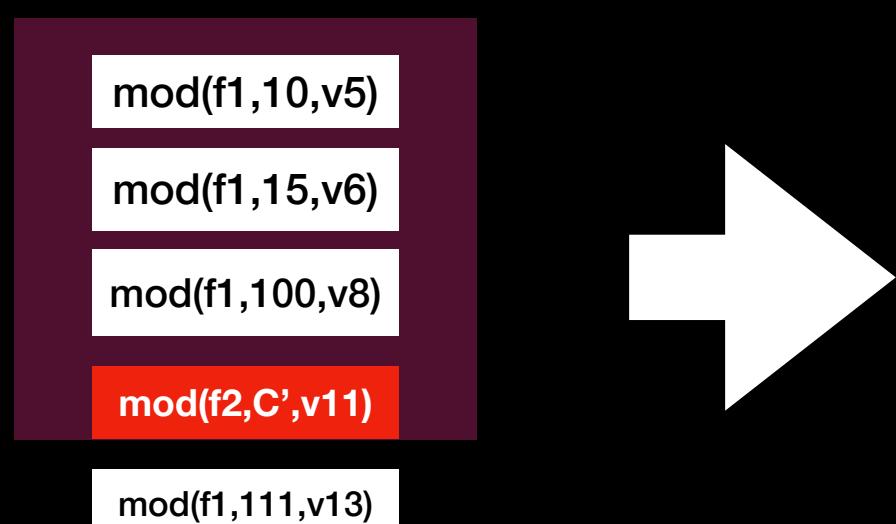
### Who will go to New Copy??

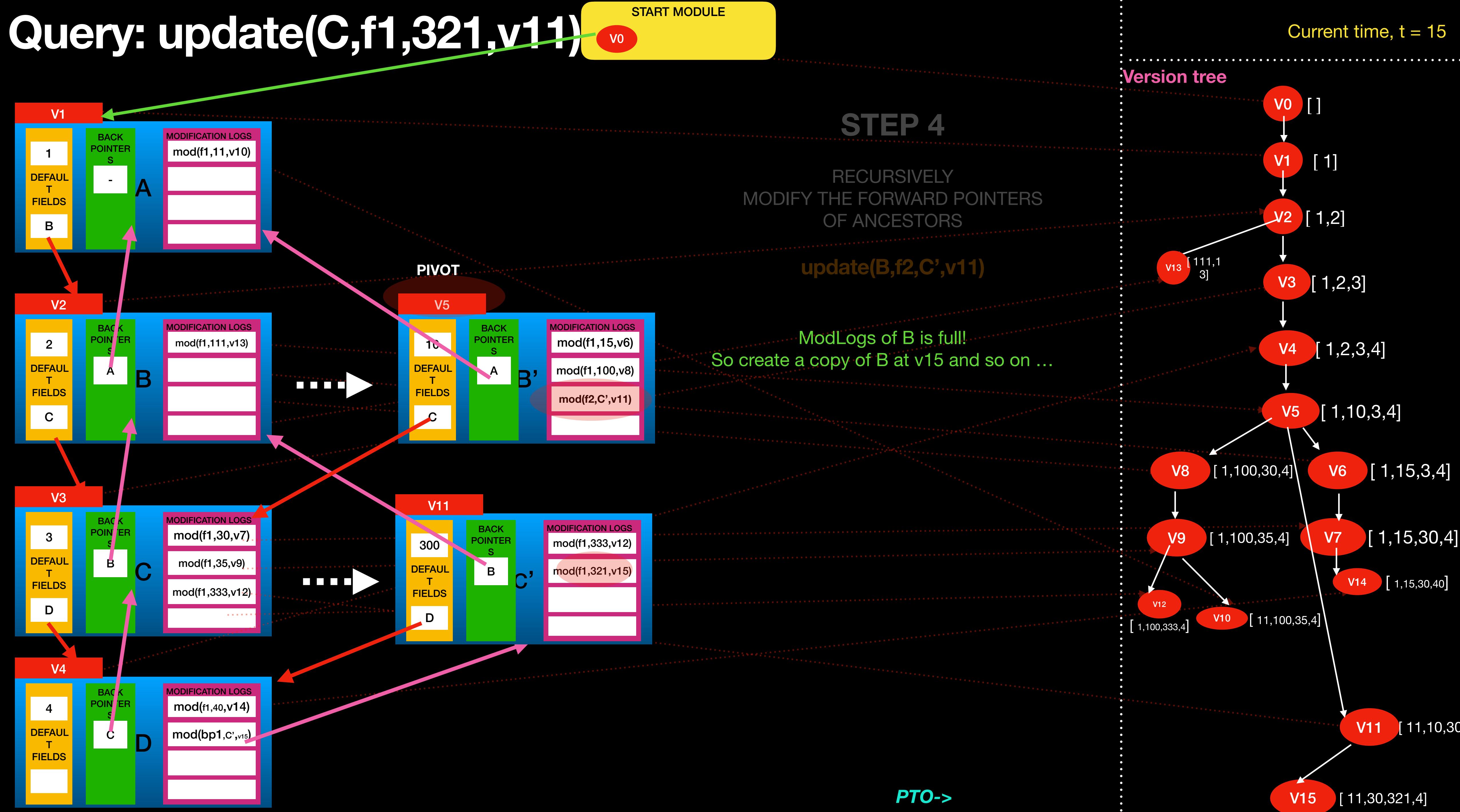
Choose  
A Candidate Mod as pivot

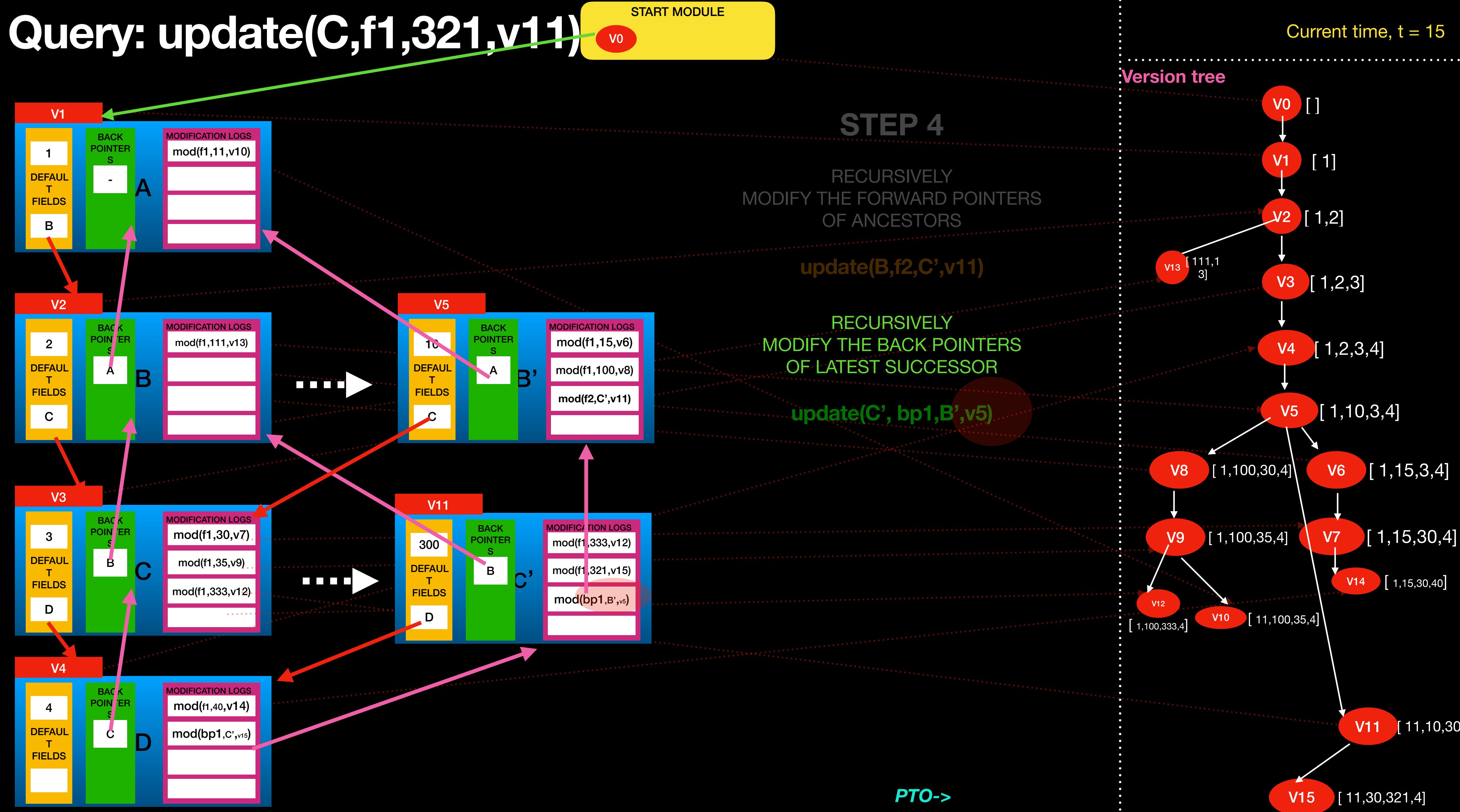


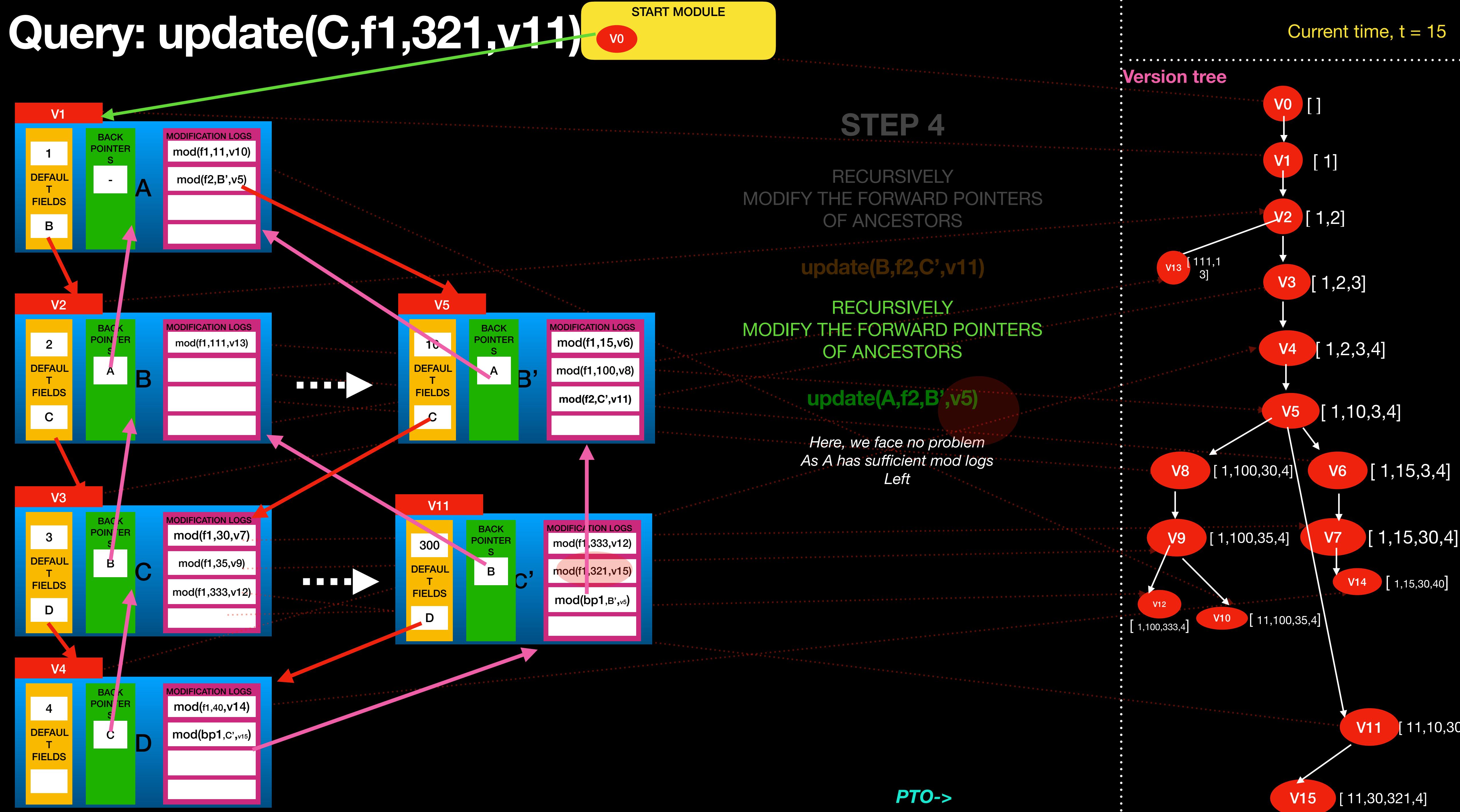
Which has most number of mods who are strictly successors of that pivot  
In Topological Order

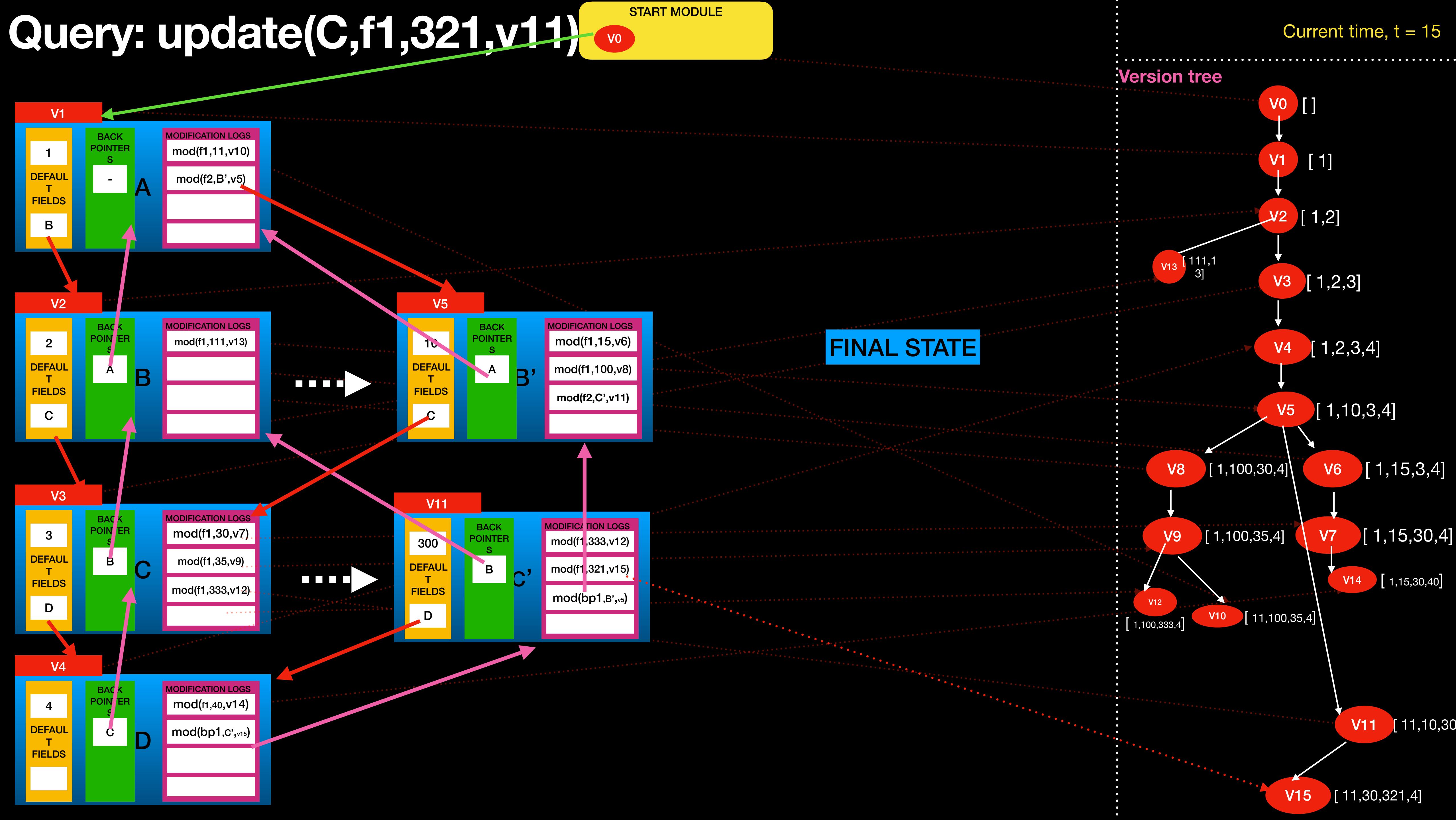
Then transfer that pivot along with its successors to the new copy.

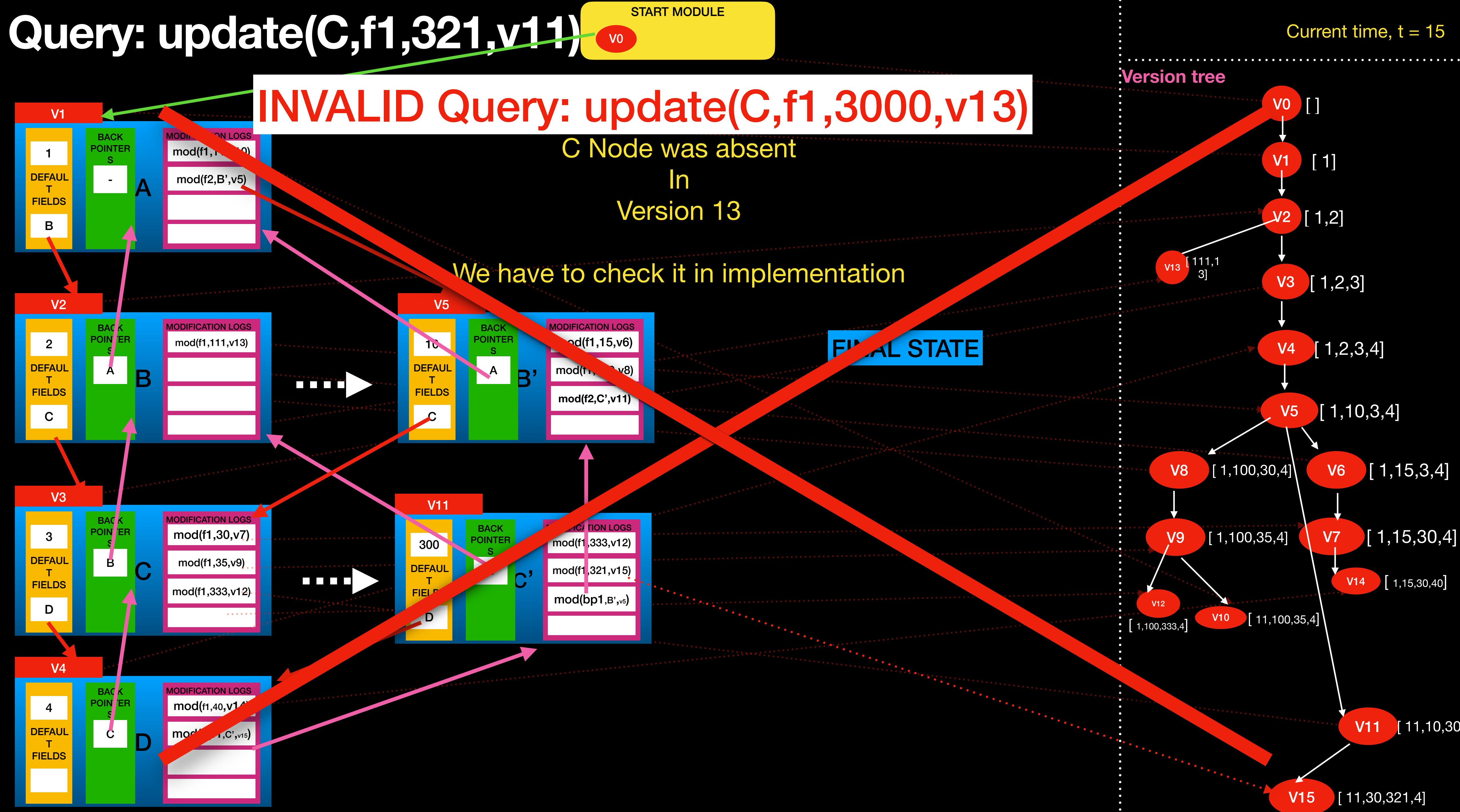












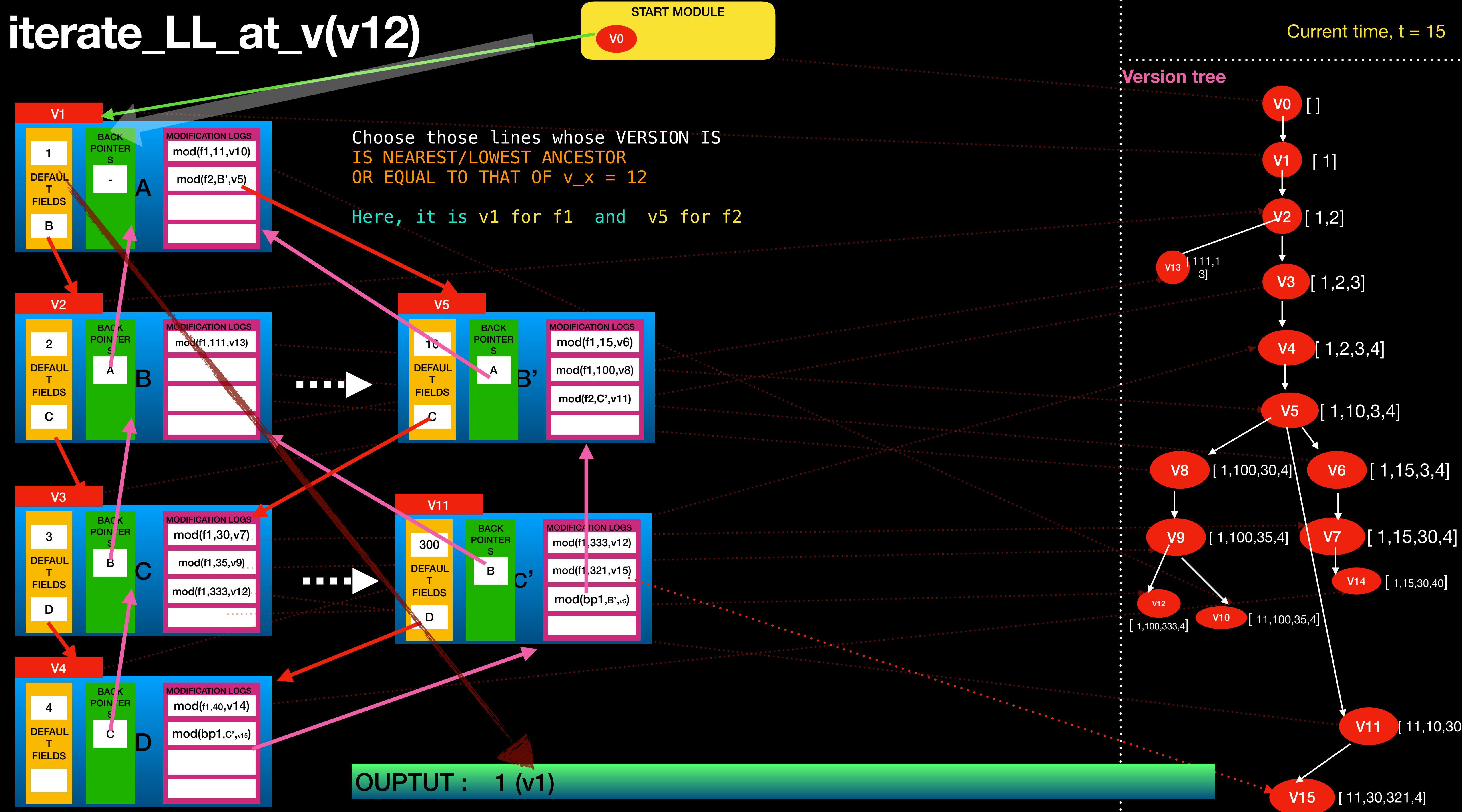
# Same Way we can show

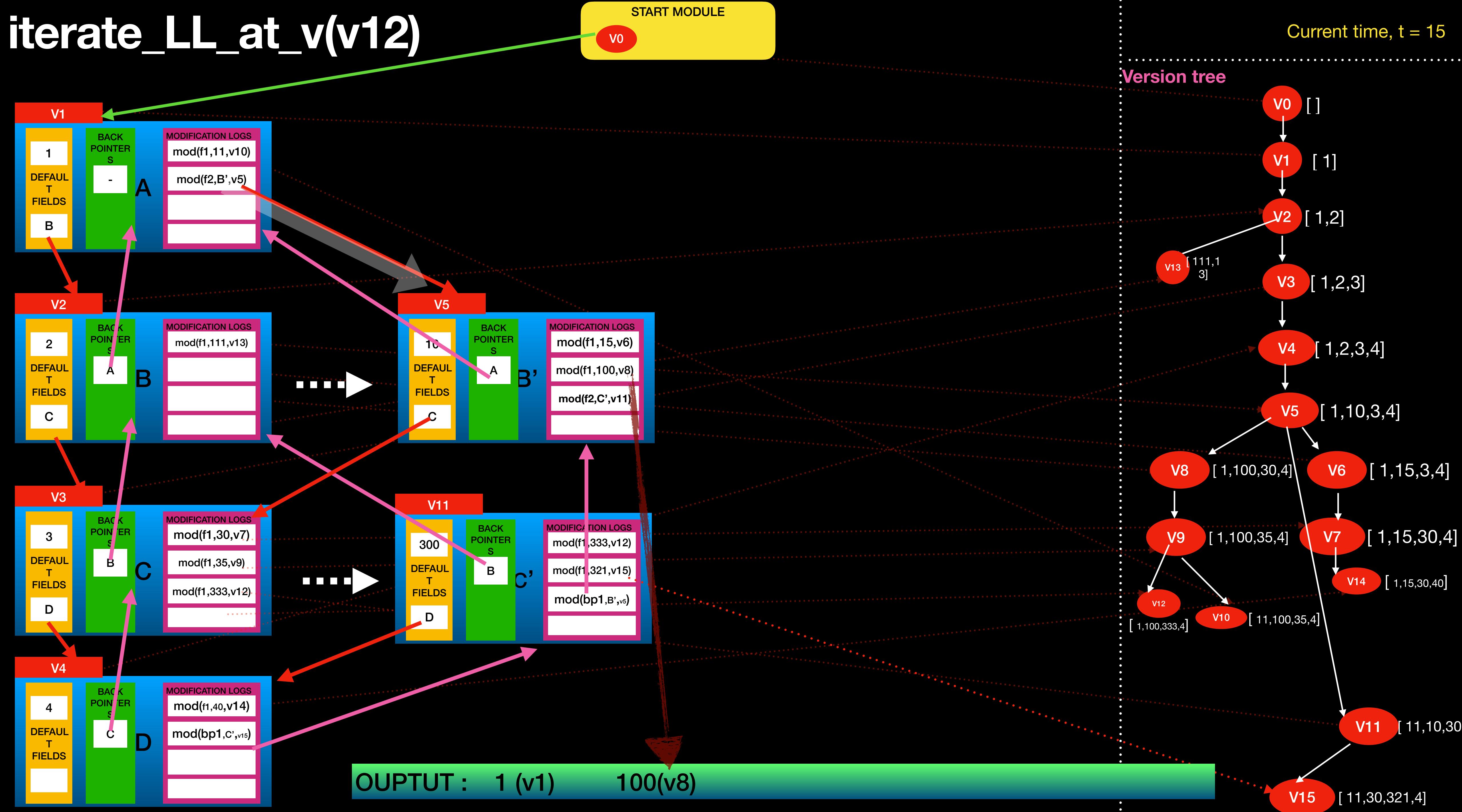
- > Deletion of Node
- > Insertion of Node

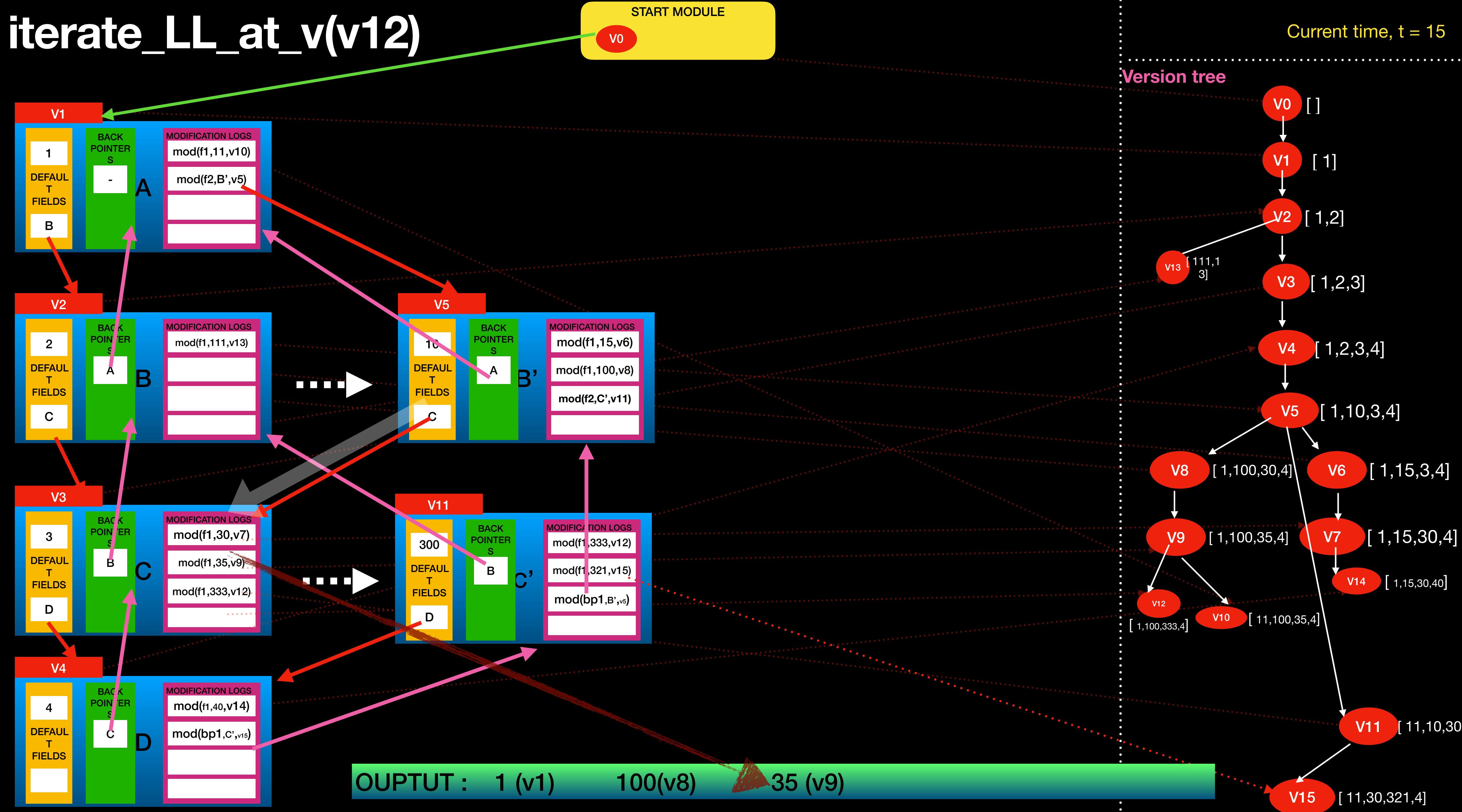
**remove(x,v)**  
**add(x, y, v)**

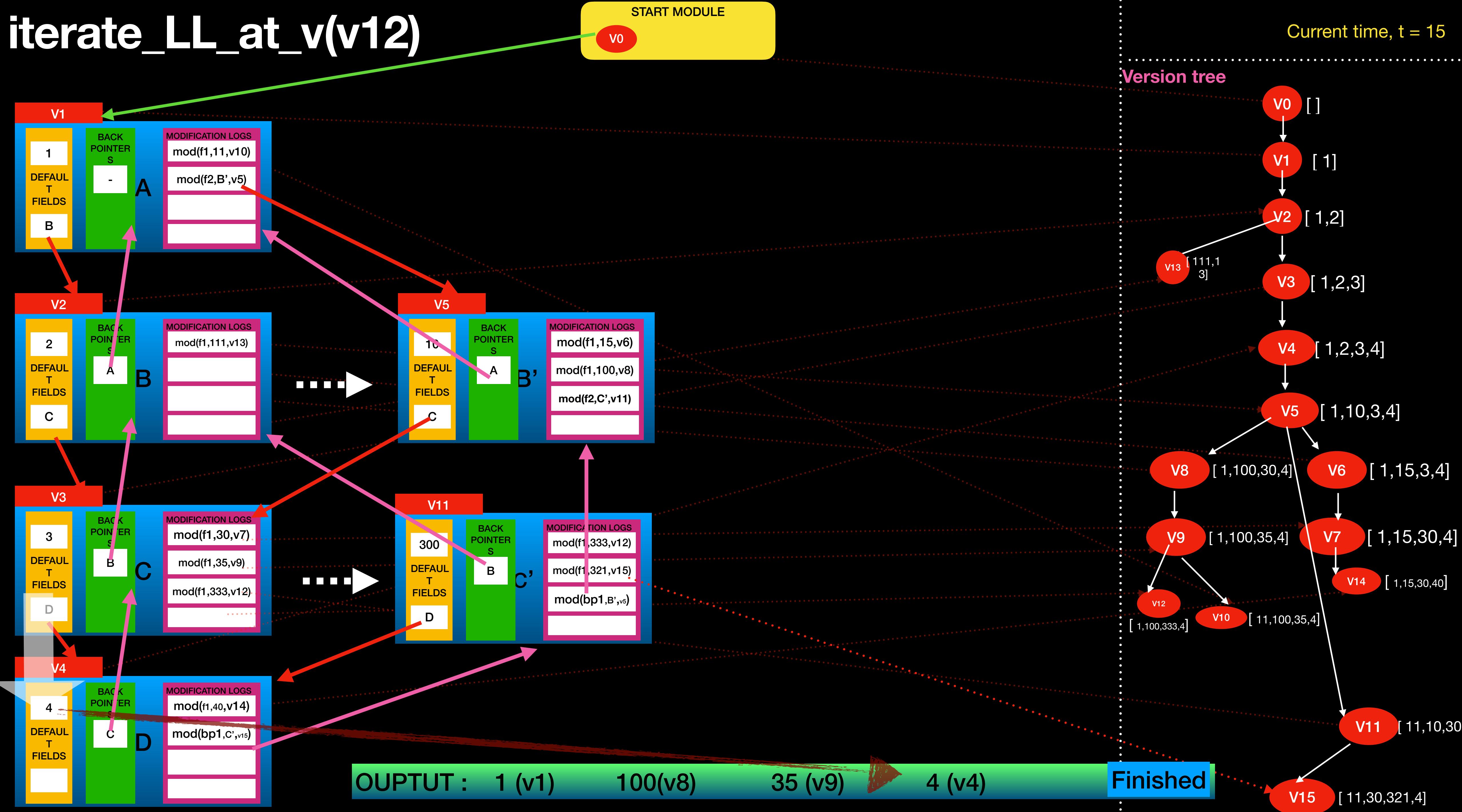
Shown in partial persistent mode

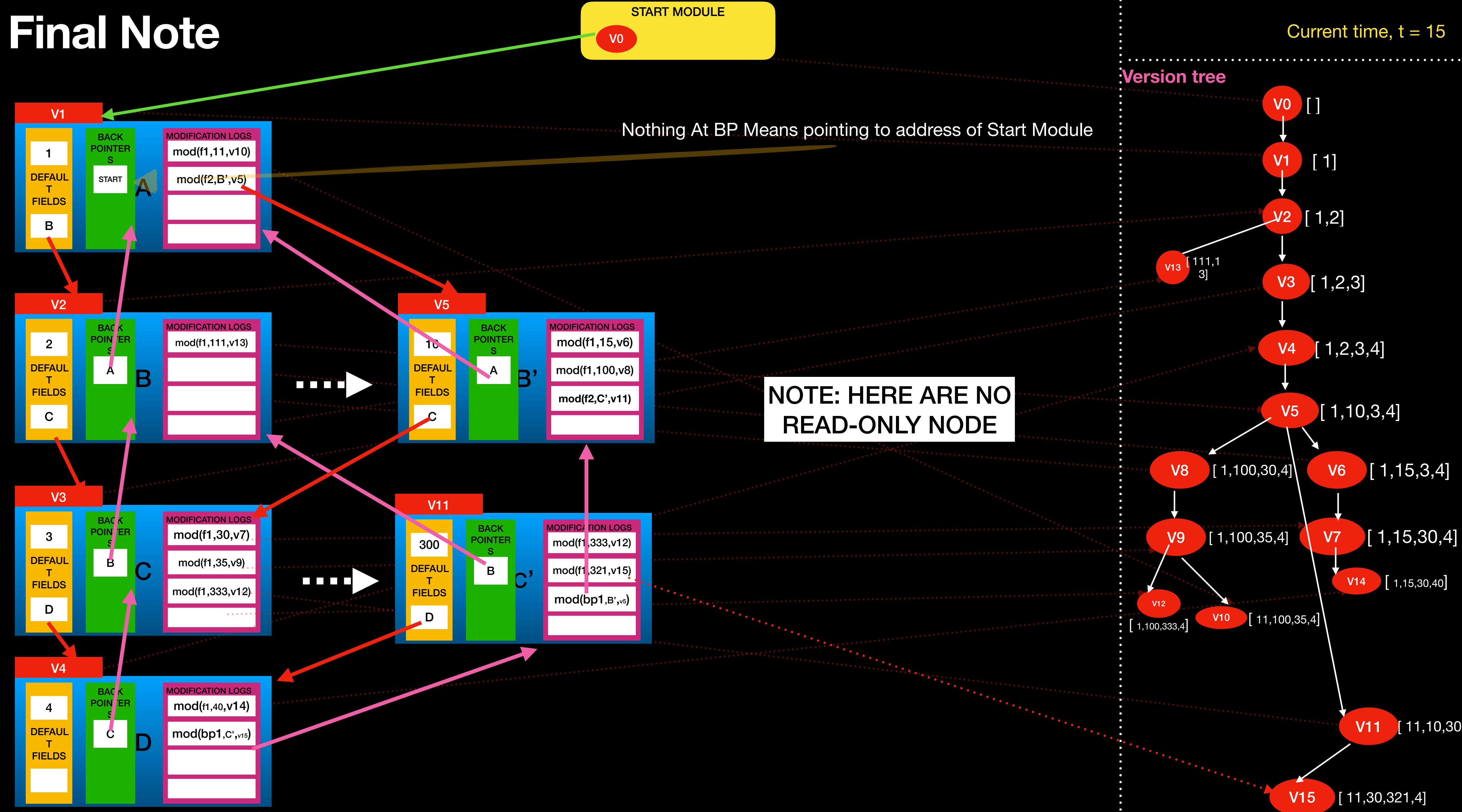
**iterate\_LL\_at\_v(v12)**











# **Persistent Stack**

**Main Concept in Persistent Stack**

**Kushal Das 30/12/2021**

# **Here I have used pointer machine to implement my fully persistent Stack**

I have taken some basic functions of stack

- Push
- Pop
- IsEmpty
- Print

I have taken the print function just to see what the stack was in a particular version.

In this I have implemented a list using pointer machine in which each node has:

- Data
- Next pointer
- Version
- Back pointer
- Modifications log
- Data
- Next and Back pointers
- Version

Then I implemented an **insert at an index** and **delete at an index** in any version functions.

Then for stack the index is 0 i.e. insert at start and delete at start.

# Persistent Data structure

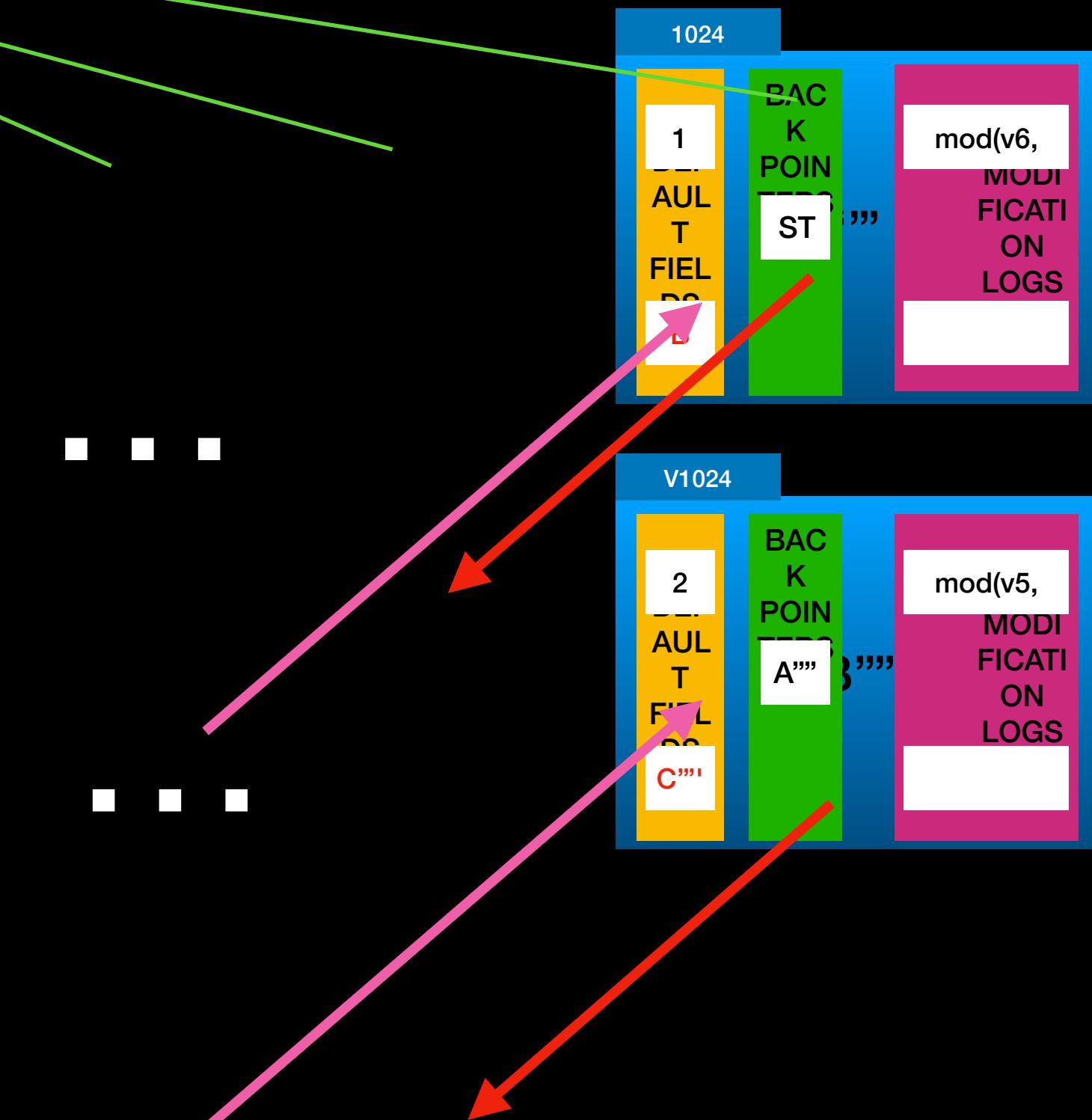
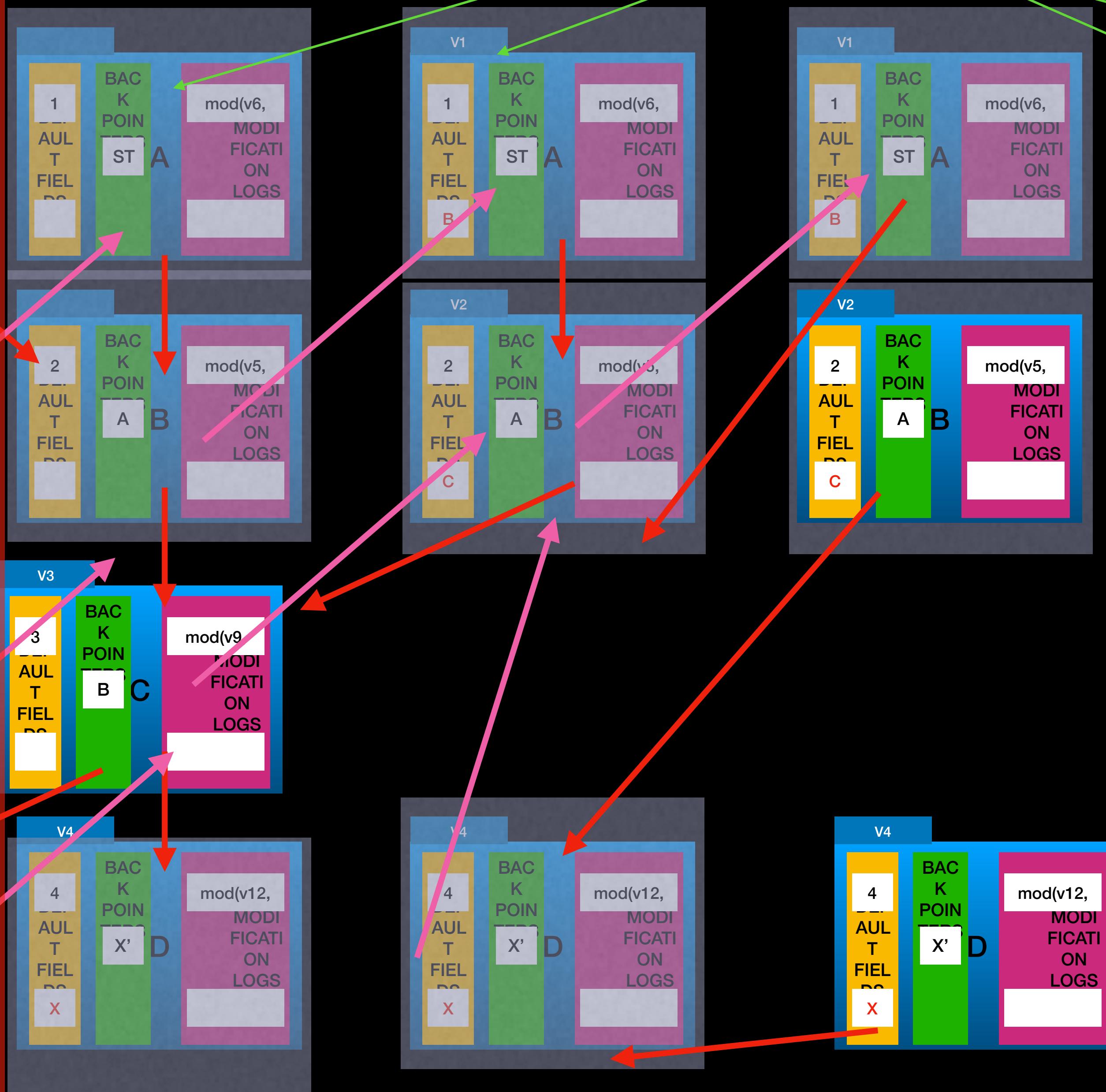
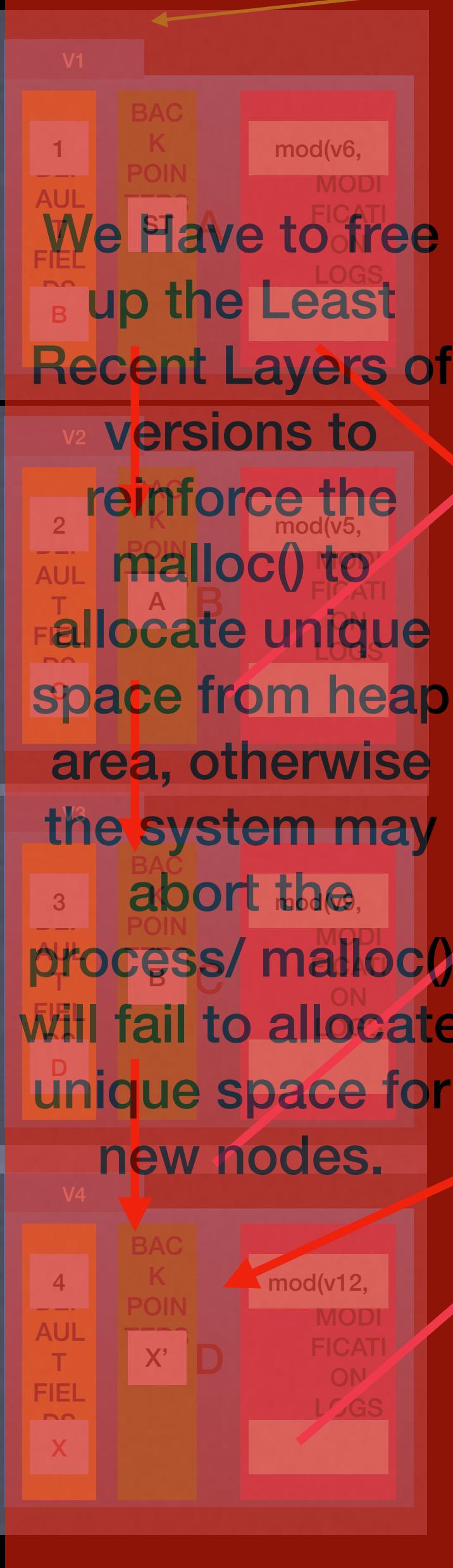
**PROJECT MEMBERS:** ANANNYO DEY, SOUMYAJIT RUDRA SARMA , DEBASMIT ROY, KANKO GHOSH AND KUSHAL DAS

# 1. Try To Keep A Limited Number of Latest Version in Persistence

Because, the PPDS are very memory consuming.

# START MODULE

Note: This is not an actual diagram  
Drawn to illustrate the Huge number of versions



We always try to keep latest 1024 versions In persistence

# Supporting Code

## Unit Test To Validate The Expected Result And Actual Result

```
1 #include "gtest/gtest.h"
2 #include <iostream>
3 #include <vector>
4 #include "src/lib/headers_preprocessors.h"
5 #include "src/lib/PPDS_LinkedList.h"
6 #include "src/lib/test_with_Vector_and_PDSLL.h"
7
8 using namespace std;
9
10 const int max_mod = 3;
11 const int max_size = 200;
12
13 TEST(VECT_AND_PDSLL_VALIDATION_TEST, COMPARE_VECTOR_VS_PDSLL) {
14
15     vector<int> vect(max_size, SENTINEL);
16     PPDS_LINKED_LIST<int> list(max_mod);
17     VECT_AND_PDSLL vp_test(max_mod, max_size);
18     vector<int> expectedResult, actualResult;
19
20     vp_test.randomized_Insert(list, vect);
21     expectedResult = vp_test.trimVect(vect);
22     actualResult = list.iterate_and_print_F1_at_ver_Vect(list.getCurTime());
23
24     EXPECT_EQ(expectedResult, actualResult); // random insert
25     expectedResult.clear(); actualResult.clear();
26
27
28     vp_test.randomized_Update(list, vect);
29     expectedResult = vp_test.trimVect(vect);
30     actualResult = list.iterate_and_print_F1_at_ver_Vect(list.getCurTime());
31
32     EXPECT_EQ(expectedResult, actualResult); // random update
33
34 }
35
36 }
```

max\_size = 200

```
1 Target //tests:VALIDATION_TEST up-to-date:
2   bazel-bin/tests/VALIDATION_TEST
3 INFO: Elapsed time: 2.258s, Critical Path: 2.05s
4 INFO: 3 processes: 1 internal, 2 darwin-sandbox.
5 INFO: Build completed successfully, 3 total actions
6 INFO: Running command line: external/bazel_tools/tools/test/test-setup.sh tests/VALIDATIONINFO: Build completed successfully, 3 total actions
7 exec ${PAGER:-/usr/bin/less} "$@" || exit 1
8 Executing tests from //tests:VALIDATION_TEST
9
10 Running main() from gmock_main.cc
11 [=====] Running 1 test from 1 test suite.
12 [-----] Global test environment set-up.
13 [-----] 1 test from VECT_AND_PDSLL_VALIDATION_TEST
14 [ RUN   ] VECT_AND_PDSLL_VALIDATION_TEST.COMPARE_VECTOR_VS_PDSLL
15 [      OK ] VECT_AND_PDSLL_VALIDATION_TEST.COMPARE_VECTOR_VS_PDSLL (208 ms)
16 [-----] 1 test from VECT_AND_PDSLL_VALIDATION_TEST (208 ms total)
17
18 [-----] Global test environment tear-down
19 [=====] 1 test from 1 test suite ran. (208 ms total)
20 [  PASSED ] 1 test.
```

max\_size = 2000

```
1 Target //tests:VALIDATION_TEST up-to-date:
2   bazel-bin/tests/VALIDATION_TEST
3 INFO: Elapsed time: 1.622s, Critical Path: 1.49s
4 INFO: 3 processes: 1 internal, 2 darwin-sandbox.
5 INFO: Build completed successfully, 3 total actions
6 INFO: Running command line: external/bazel_tools/tools/test/test-setup.sh tests/VALIDATIONINFO: Build completed successfully, 3 total actions
7 exec ${PAGER:-/usr/bin/less} "$@" || exit 1
8 Executing tests from //tests:VALIDATION_TEST
9
10 Running main() from gmock_main.cc
11 [=====] Running 1 test from 1 test suite.
12 [-----] Global test environment set-up.
13 [-----] 1 test from VECT_AND_PDSLL_VALIDATION_TEST
14 [ RUN   ] VECT_AND_PDSLL_VALIDATION_TEST.COMPARE_VECTOR_VS_PDSLL
15 [      OK ] VECT_AND_PDSLL_VALIDATION_TEST.COMPARE_VECTOR_VS_PDSLL (5 ms)
16 [-----] 1 test from VECT_AND_PDSLL_VALIDATION_TEST (5 ms total)
17
18 [-----] Global test environment tear-down
19 [=====] 1 test from 1 test suite ran. (5 ms total)
20 [  PASSED ] 1 test.
```

These Results Passed

# Supporting Code

max\_size = 3000

```
1 2 warnings generated.
2 Target //tests:VALIDATION_TEST up-to-date:
3   bazel-bin/tests/VALIDATION_TEST
4 INFO: Elapsed time: 2.383s, Critical Path: 2.16s
5 INFO: 3 processes: 1 internal, 2 darwin-sandbox.
6 INFO: Build completed successfully, 3 total actions
7 INFO: Running command line: external/bazel_tools/tools/test/test-setup.sh tests/VALIDATIONINFO: Build completed successfully, 3 total actions
8 exec ${PAGER:-/usr/bin/less} "$0" || exit 1
9 Executing tests from //tests:VALIDATION_TEST
10 -----
11 Running main() from gmock_main.cc
12 [=====] Running 1 test from 1 test suite.
13 [-----] Global test environment set-up.
14 [-----] 1 test from VECT_AND_PDSLL_VALIDATION_TEST
15 [ RUN      ] VECT_AND_PDSLL_VALIDATION_TEST.COMPARE_VECTOR_VS_PDSLL
16 Assertion failed: (0), function NodeAtIndex, file PPDS_LinkedList.h, line 385.
```

## Failed To Validate The Result

```
1 Current number of Nodes Are Used: 3511
2 Current Length of LL: 2999
3 Total Update Request: 6000
4 Current Memory Usage For Allocation Of Nodes: 646024 Byte
5 Is the LL stable? 0
```

*In-depth analysis of this failure ,  
we shall do after exam*

We can see that the length of the LL is not 3000 anymore

# 2. Application in JAVA

The Java programming language is not particularly functional. Despite this, the core JDK package `java.util.concurrent` includes `CopyOnWriteArrayList` and `CopyOnWriteArraySet` which are **partial persistent** structures, implemented using copy-on-write techniques

## Class declaration

```
public class CopyOnWriteArrayList  
    extends Object  
implements List, RandomAccess, Cloneable, Serializable
```

1. `CopyOnWriteArrayList` is a thread-safe variant of `ArrayList` where operations which can change the `ArrayList` (`add`, `update`, `set` methods) creates a clone of the underlying array.
2. `CopyOnWriteArrayList` is to be used in a Thread based environment where read operations are very frequent and update operations are rare.
3. Iterator of `CopyOnWriteArrayList` will never throw `ConcurrentModificationException`.
4. Any type of modification to `CopyOnWriteArrayList` will not reflect during iteration since the iterator was created.
5. List modification methods like `remove`, `set` and `add` are not supported in the iteration.

This method will throw `UnsupportedOperationException`.  
`null` can be added to the list.

```

1  /**
2   * Appends the specified element to the end of this list.
3   *
4   * @param e element to be appended to this list
5   * @return {@code true} (as specified by {@link Collection#add})
6   */
7  public boolean add(E e) {
8      synchronized (lock) {
9          Object[] es = getArray();
10         int len = es.length;
11         es = Arrays.copyOf(es, len + 1);
12         es[len] = e;
13         setArray(es);
14         return true;
15     }
16 }
17

```



```

1  /**
2   * Inserts the specified element at the specified position in this
3   * list. Shifts the element currently at that position (if any) and
4   * any subsequent elements to the right (adds one to their indices).
5   *
6   * @throws IndexOutOfBoundsException {@inheritDoc}
7   */
8  public void add(int index, E element) {
9      synchronized (lock) {
10         Object[] es = getArray();
11         int len = es.length;
12         if (index > len || index < 0)
13             throw new IndexOutOfBoundsException(outOfBounds(index, len));
14         Object[] newElements;
15         int numMoved = len - index;
16         if (numMoved == 0)
17             newElements = Arrays.copyOf(es, len + 1);
18         else {
19             newElements = new Object[len + 1];
20             System.arraycopy(es, 0, newElements, 0, index);
21             System.arraycopy(es, index, newElements, index + 1,
22                             numMoved);
23         }
24         newElements[index] = element;
25         setArray(newElements);
26     }
27 }
28

```



## Code Taken From underlying source code of “CopyOnWriteArrayList” class

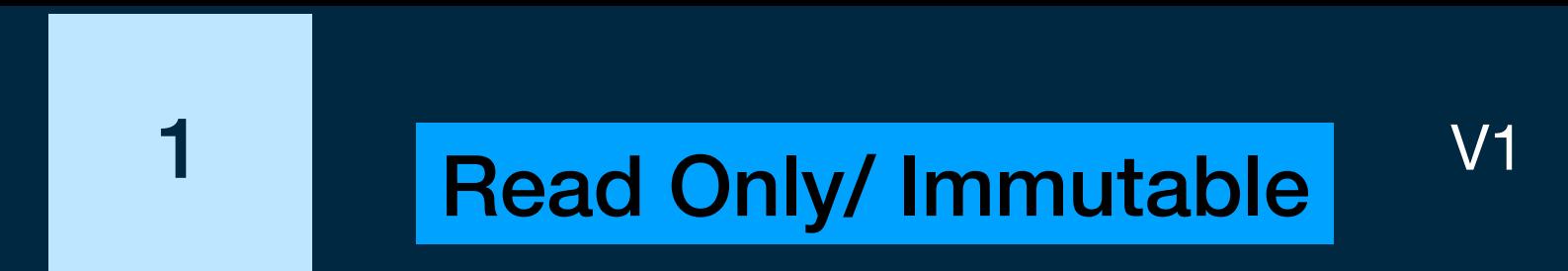
Note: We have tried to implement the same  
Functionality in C++.  
[Code Attached]

# Idea:

```
1     CopyOnWriteArrayList<String> list  
2         = new CopyOnWriteArrayList<>();  
3
```



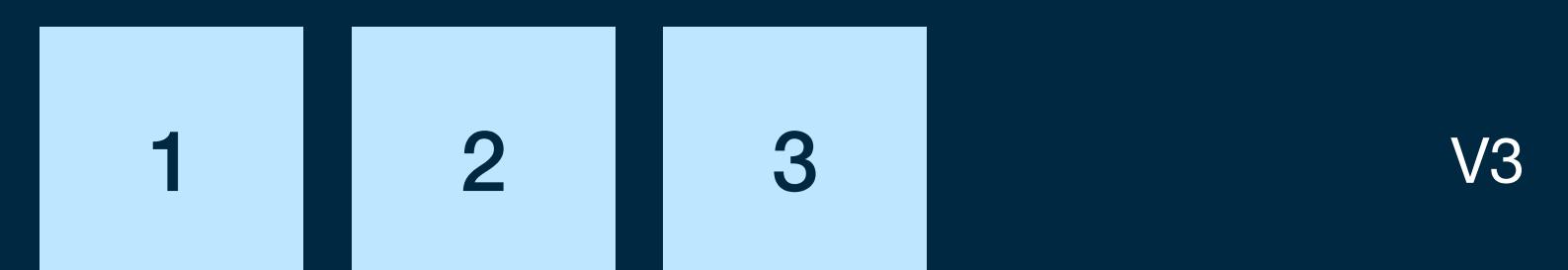
Array1:



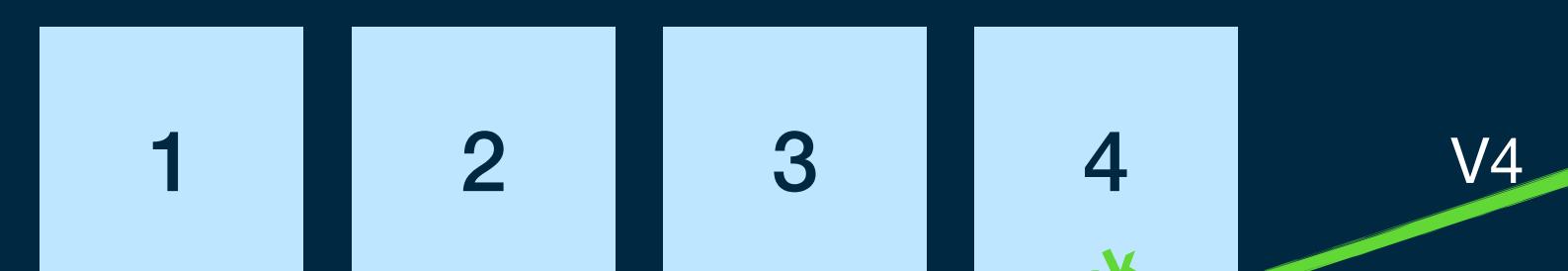
Array2 (Copy):



Array3 (Copy):



Array4 (Copy):

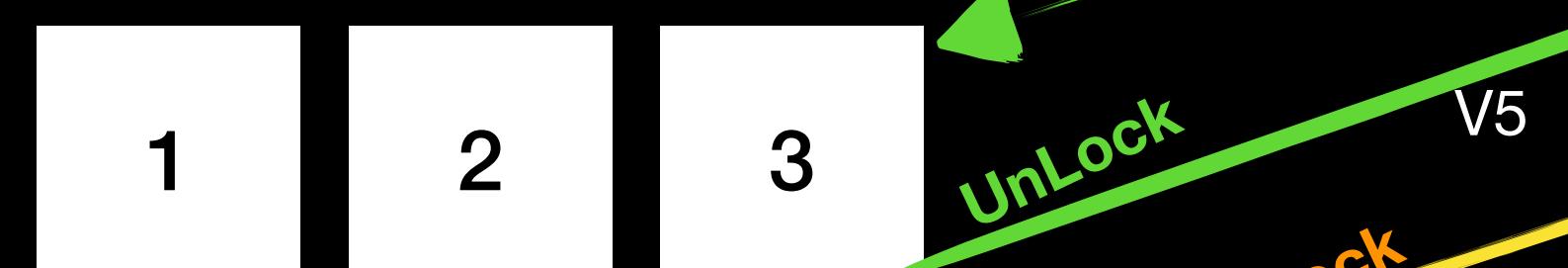


Thread 1: list.remove(4)

Thread Safe

Thread 2: list.add(5)

Array5 (Copy):



Thread Safe

Array6 (Copy):



UnLock

# 3. Application In Clojure

## Clojure

Like many programming languages in the Lisp family, Clojure contains an implementation of a linked list, but unlike other dialects its implementation of a Linked List has enforced persistence instead of being persistent by convention.<sup>[19]</sup> Clojure also has efficient implementations of persistent vectors, maps, and sets based on persistent hash array mapped tries. These data structures implement the mandatory read-only parts of the Java collections framework.<sup>[20]</sup>

The designers of the Clojure language advocate the use of persistent data structures over mutable data structures because they have value semantics which gives the benefit of making them freely shareable between threads with cheap aliases, easy to fabricate, and language independent.<sup>[21]</sup>

These data structures form the basis of Clojure's support for parallel computing since they allow for easy retries of operations to sidestep data races and atomic compare and swap semantics.<sup>[22]</sup>

*[Slides in Clojure are attached Separately ]*

# Persistent Data Structures

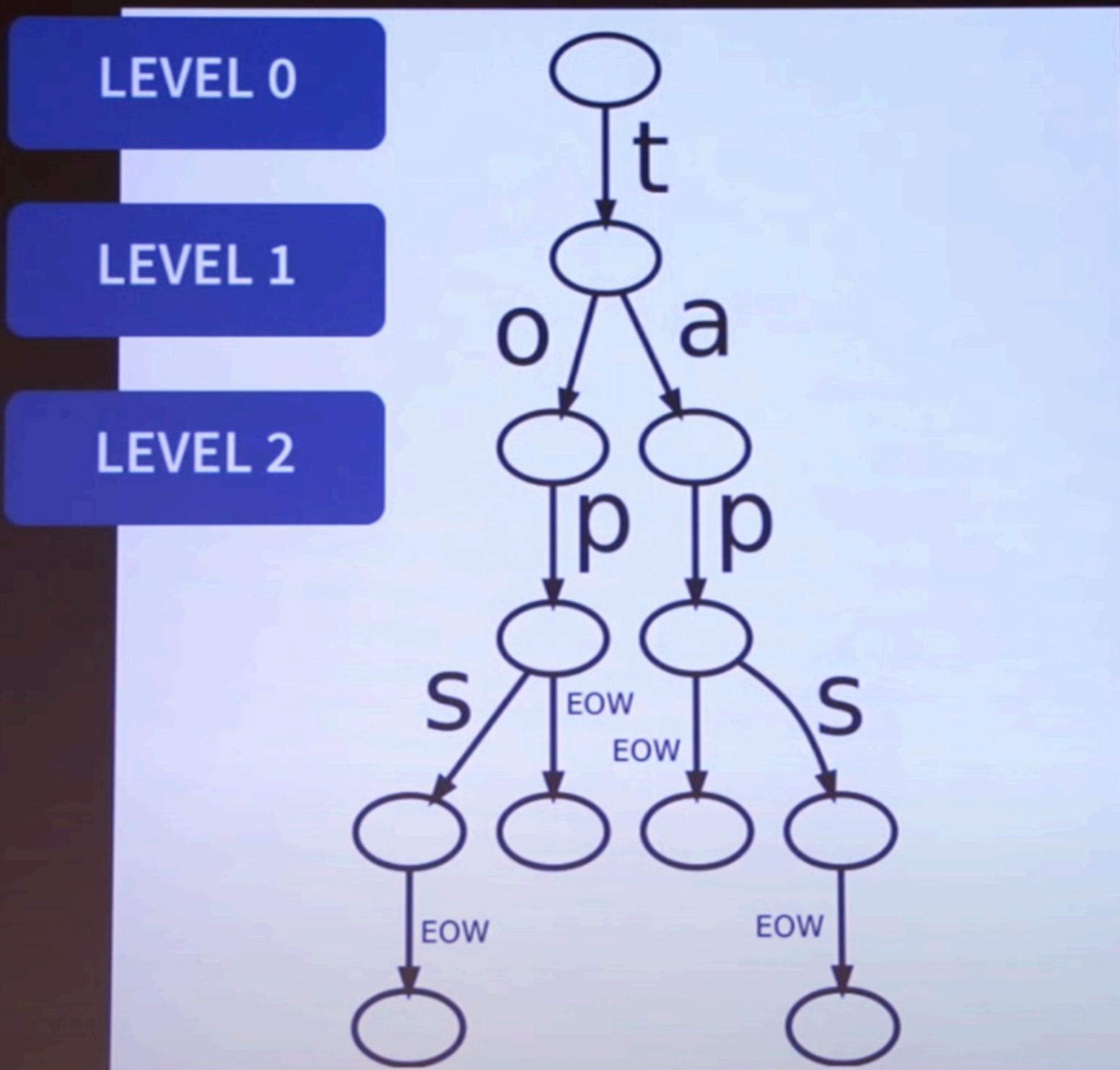
*In Clojure Language*

*A Better Presentation On Clojure Will be made after Exam*

- Composite values - immutable
- ‘Change’ is merely a function, takes one value and returns another, ‘changed’ value
- Collection maintains its performance guarantees
  - Therefore new versions are not full copies
  - Old version of the collection is still available after ‘changes’, with same performance
- Example - hash map/set and vector based upon array mapped hash tries (Bagwell)



# A Better Presentation On Clojure Will be made after Exam



00000 01011 01001 11001 10111 00010 11100

0

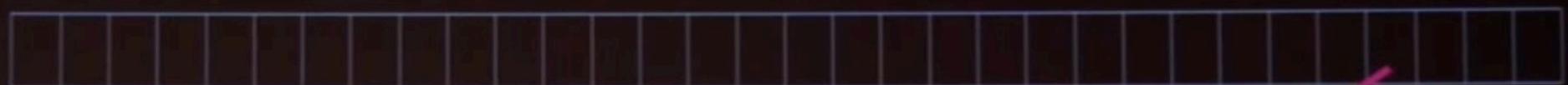
11

9

25

28

ArrayNode  
shift = 0



ArrayNode  
shift = 5



ArrayNode  
shift = 10



BitmapIndexedNode  
shift = 15

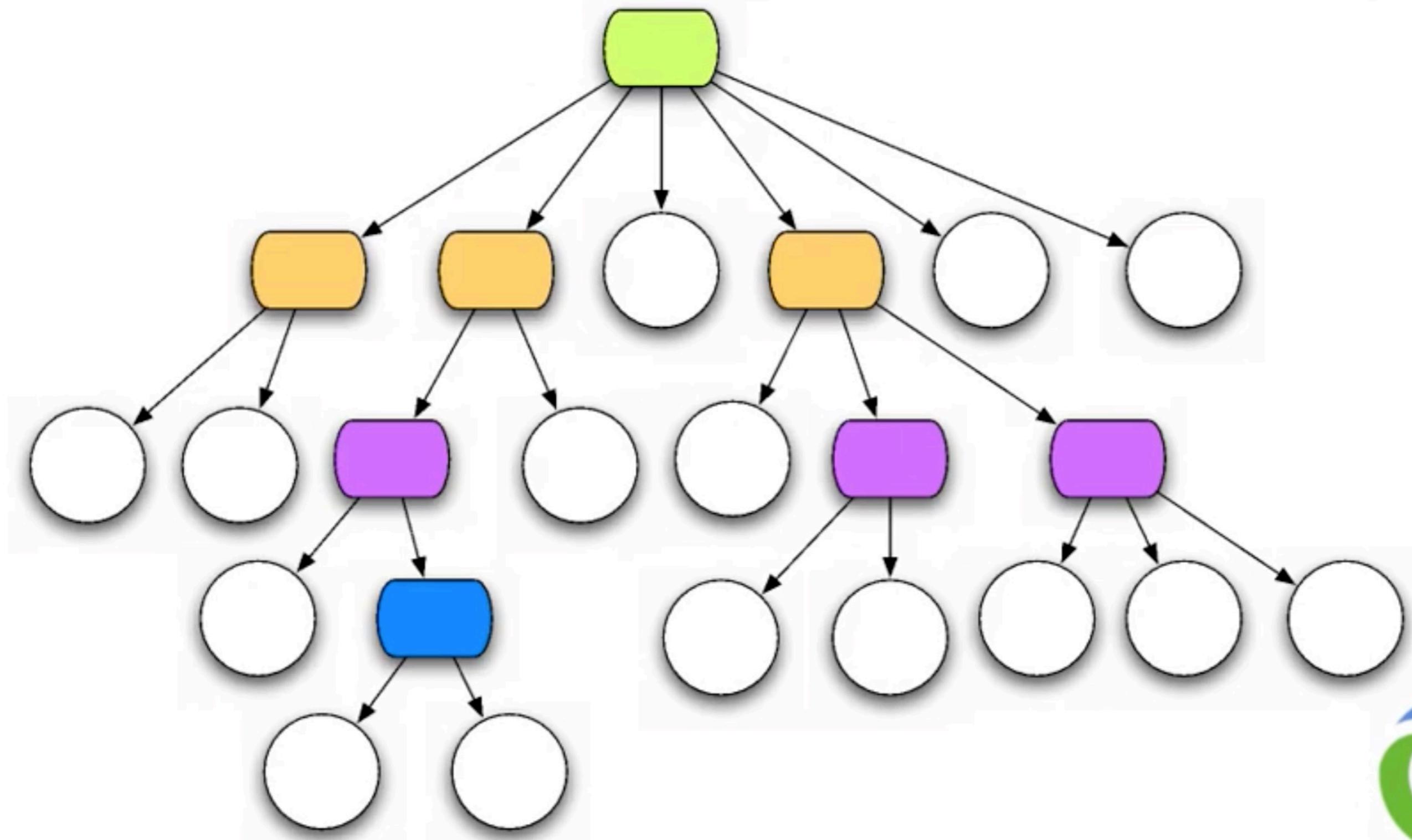


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... and then follow the AMT down

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# Bit-partitioned hash tries



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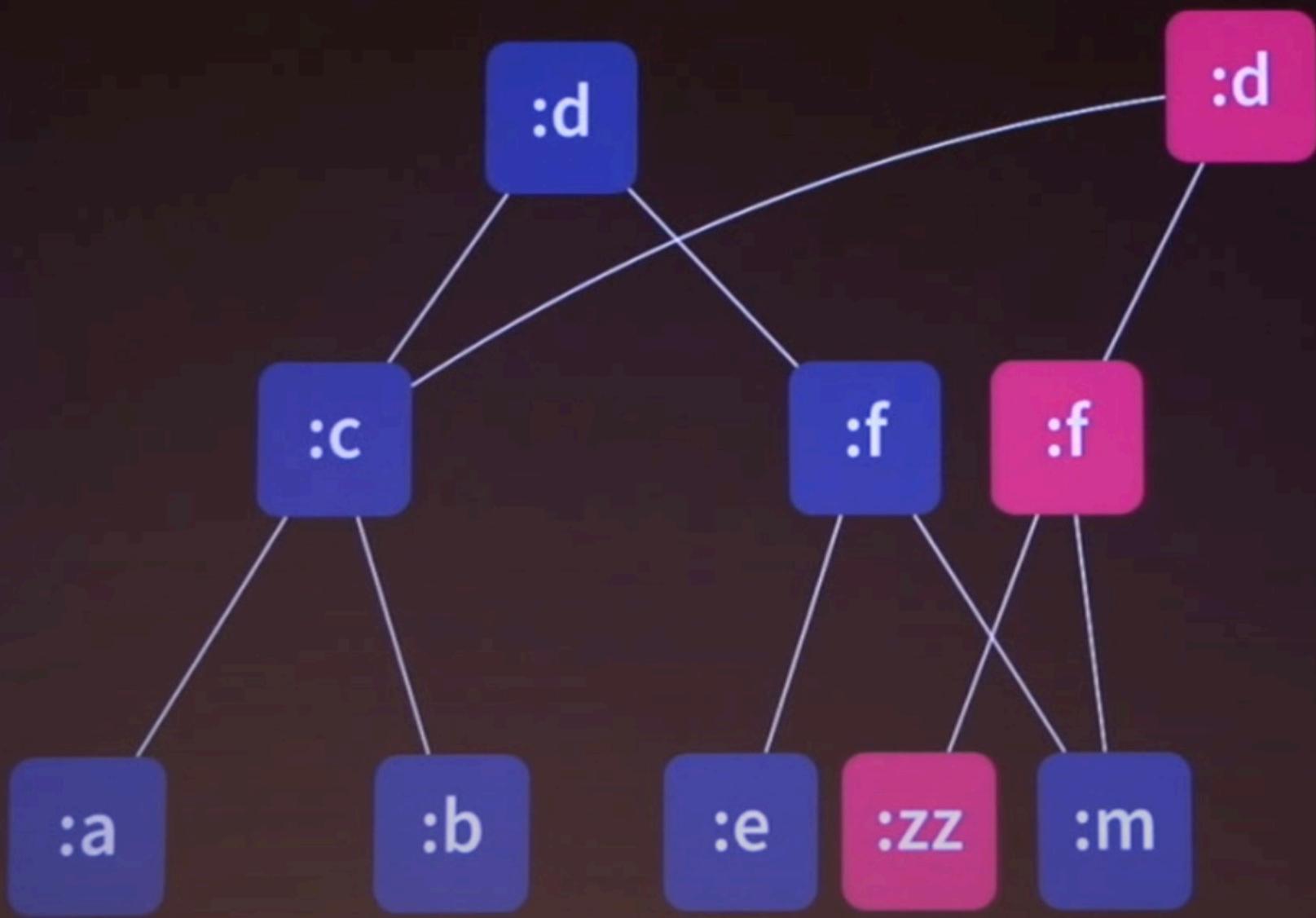
# Structural Sharing

- Key to efficient ‘copies’ and therefore persistence
- Everything is immutable so no chance of interference
- Thread safe
- Iteration safe

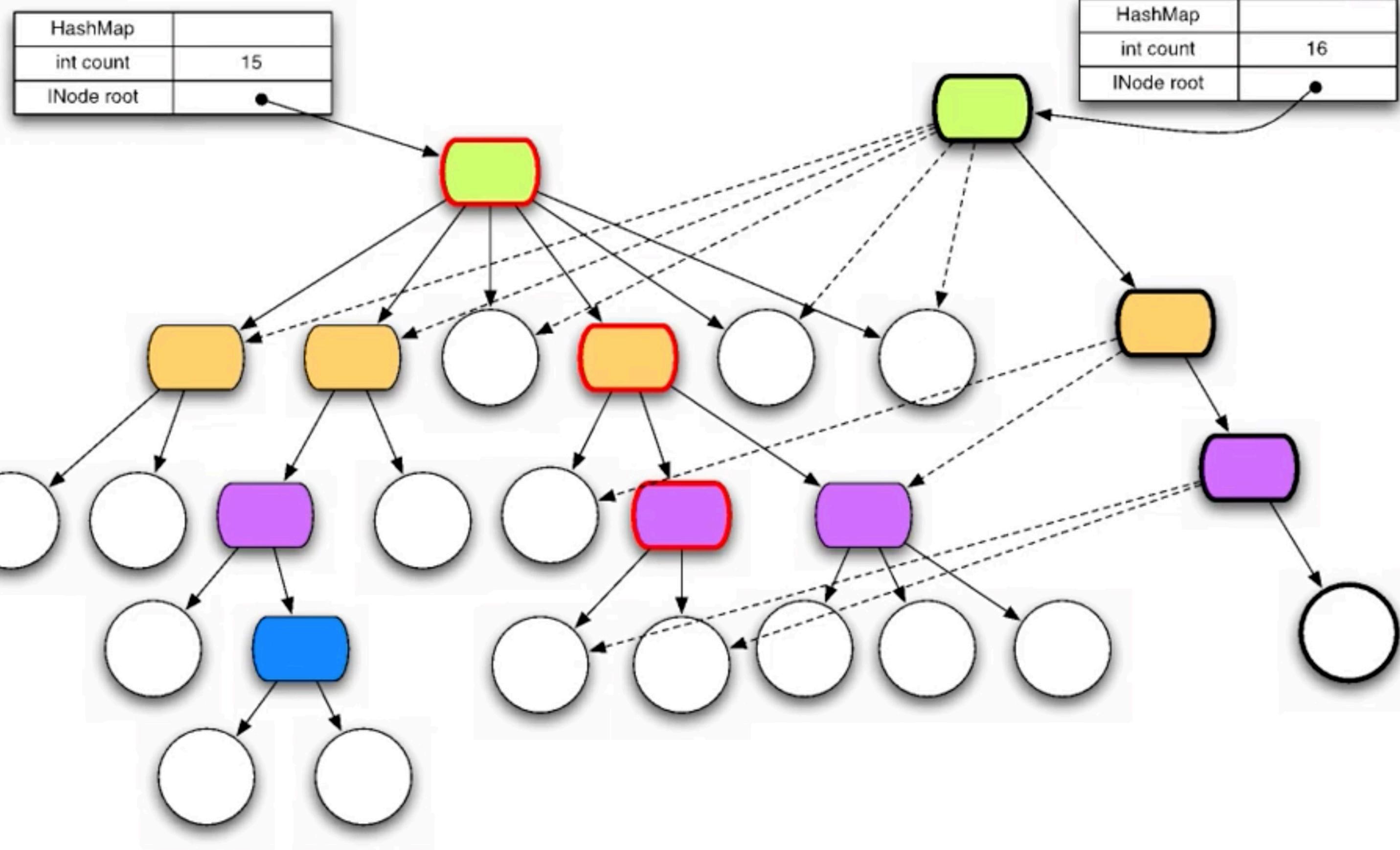


# STRUCTURAL SHARING

(assoc v 4 :zz)



# Path Copying



# REGULAR HASH TABLE?

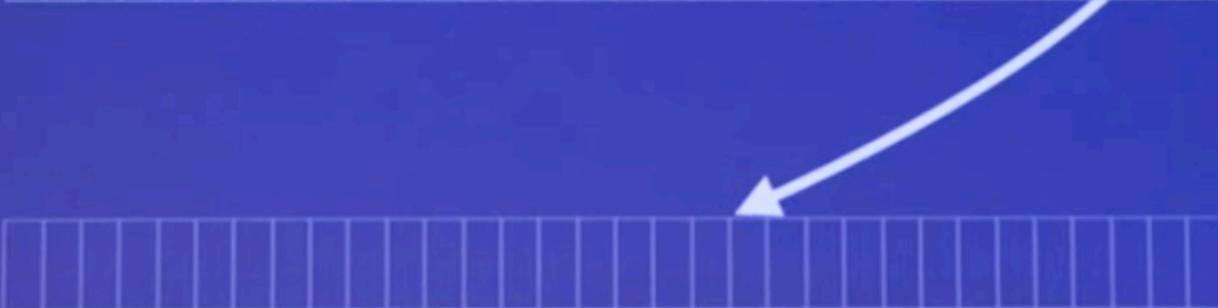
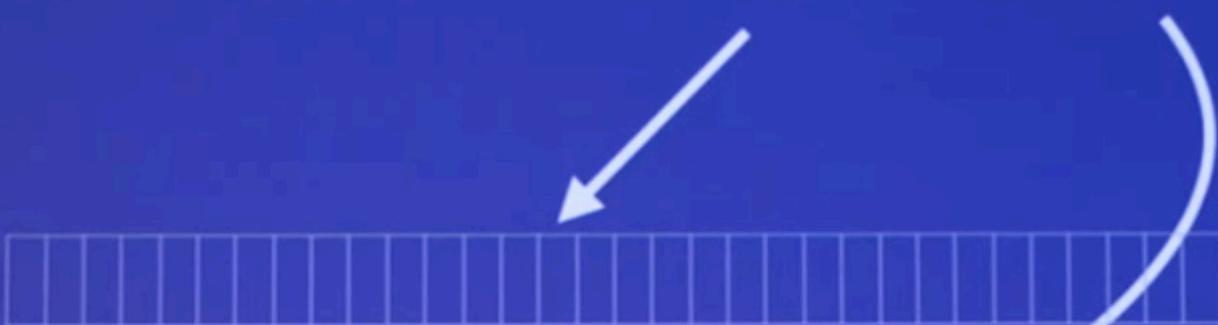
NEED ROOT RESIZING

NOT AMENABLE TO  
STRUCTURAL SHARING

# THE TAIL OPTIMIZATION

PersistentVector

count shift root tail



Slides are taken from the seminars:

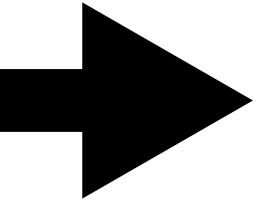
1. What Lies Beneath - A Deep Dive Into Clojure's Data Structures - Mohit Thatte
2. Persistent Data Structures and Managed References - Rich Hickey

## **Now lets come to Javascript**

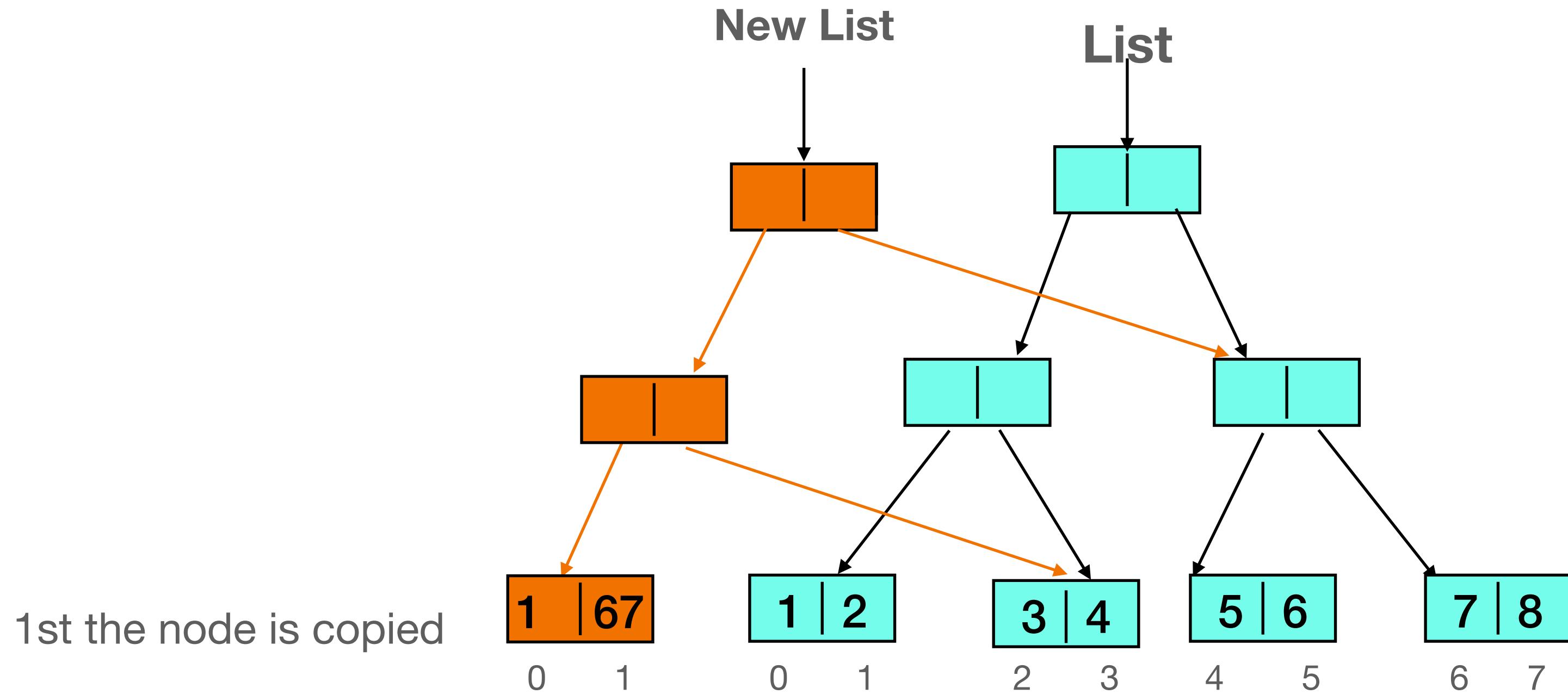
Suppose we want to make a list of some numbers

list=[1,2,3,4,5,6,7,8]

Now suppose we want to change the number at index 1 i.e. 2 to 67.

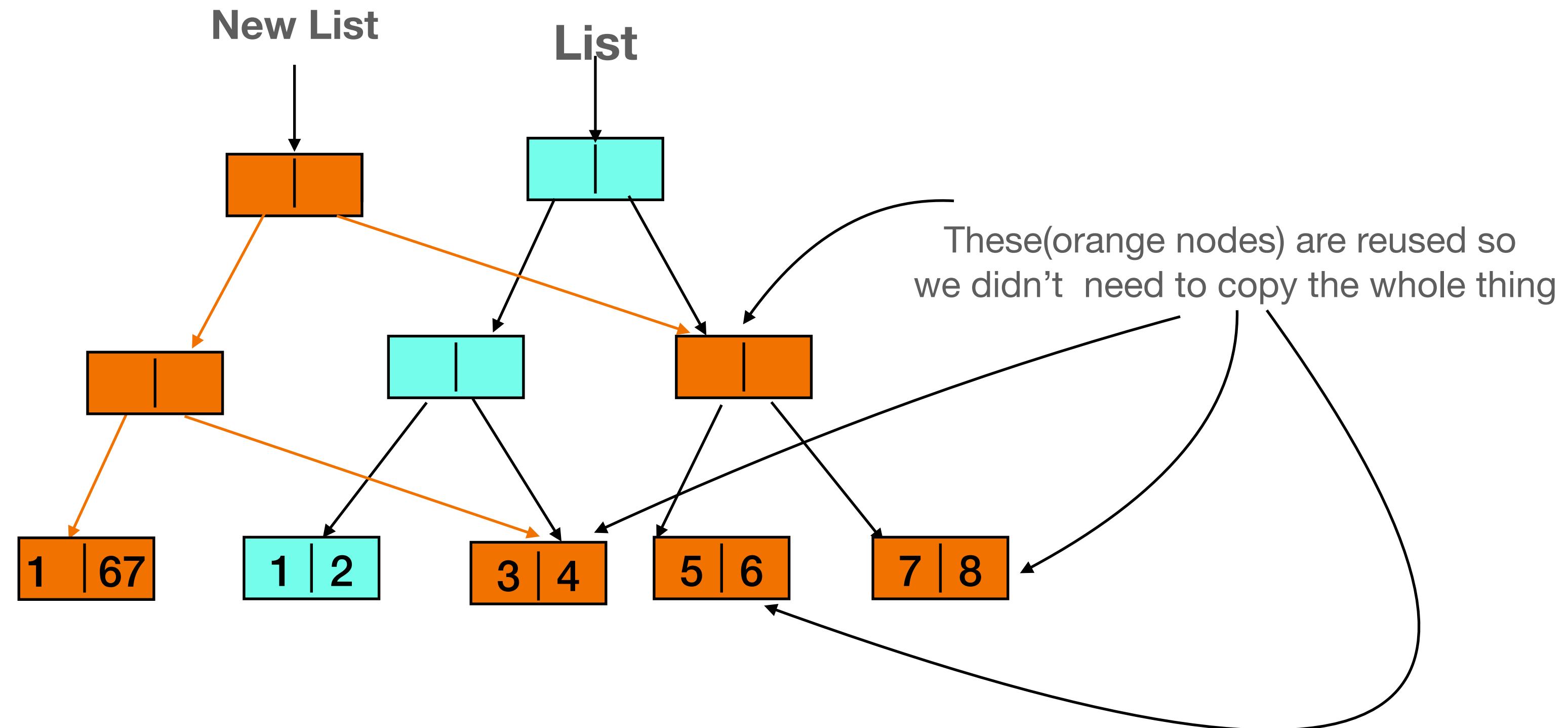
Ok so now lets break our list of numbers in size of 2     1,2  3,4  5,6  7,8

So now we will update the value 2 to 67



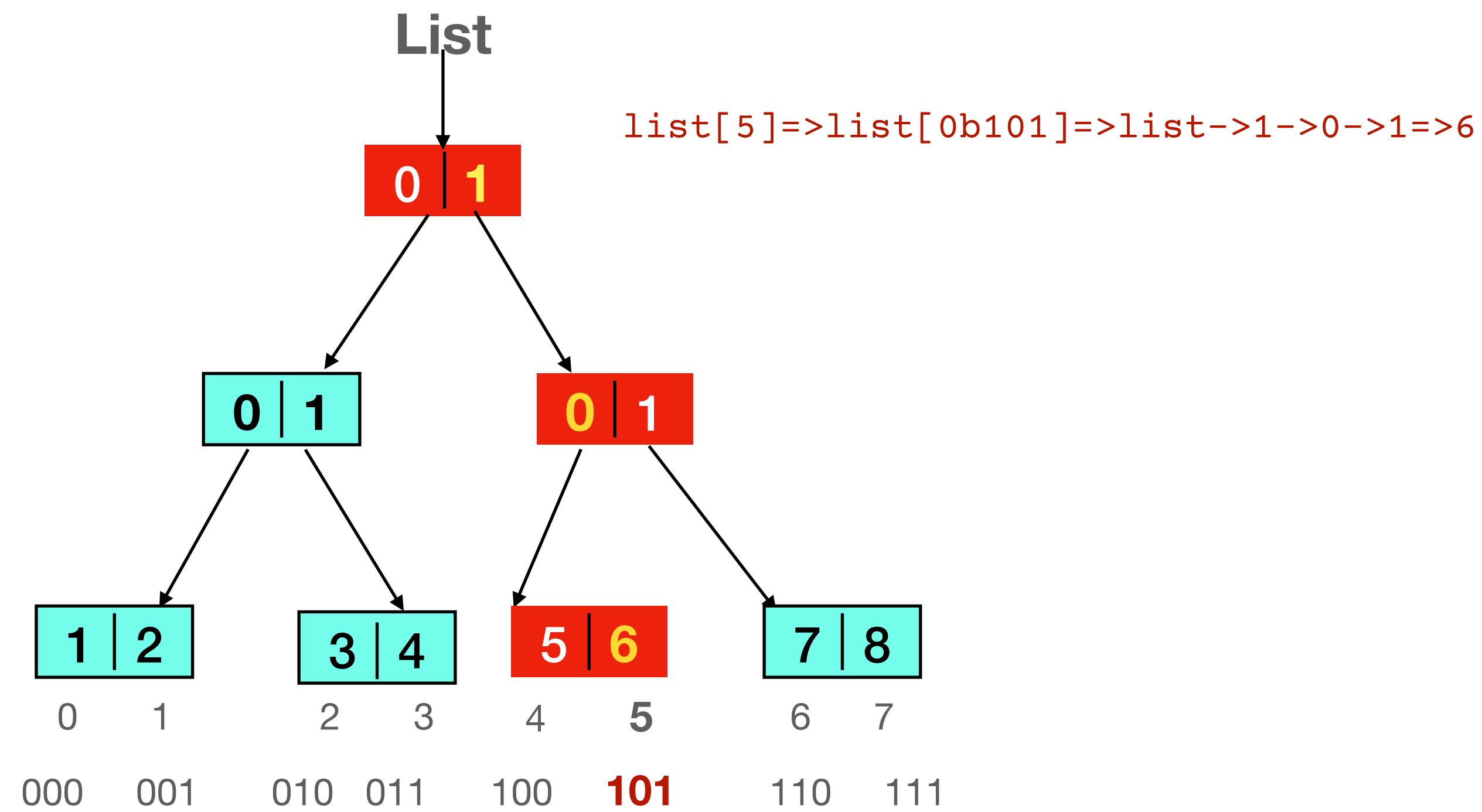
Then path copying happens

So now the list is



So now if we want to get a value at any index say 5  
We can do that easily using hashing 5 to its binary value **0b101**

We will go from 1 → 0 → 1 to get  
our value i.e. 5



In this way we can reach to any node quickly and return the value in a less time using hashing