Quantum Computing: Principles, Applications & Quantum Computers

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Abstract

When the current technological landscape is examined, it becomes clear that quantum computing has emerged as a powerful player in the problem solving game of computer science. It is something with a growing public, private, and academic market that will continue to mature as hardware advances. What stems from such a progression is the need for literature to elucidate the complexity of quantum computing. This report tries to address this demand by educating and informing its audience on what quantum computing is, where it comes from, and what is happening surrounding its emergence in the world. We discuss a brief history of the field, provide explanations on what principles underlie the science, and offer examples of algorithmic applications and real world instances of quantum technologies.

Keywords: quantum mechanics, classical mechanics, quantum computing, classical computing, qubit(s), superposition, states/quantum states, quantum gates/circuits entanglement, quantum computers, quantum parallelism, quantum error correction

Quantum Computing: Principles, Information Processing & Quantum Computers

Think of computation as the manipulation of the states of a physical system to solve problems (Kanamori & Yoo, 2020). Thus, we can define quantum computing as a revolutionary approach to computation that leverages the principles of quantum mechanics to more efficiently solve problems (Feng et al., 2023). Quantum computing basically deals with the manipulation of quantum systems to process information, the physical details of which are dependent on the quantum computer's hardware design (Abhijith et al., 2022). Compared to classical computing, quantum computing offers the potential to dramatically reduce execution time (Abhijith et al., 2022). The prospective benefits have given rise to public and private investment in the form of research and development and economic contributions from governments and enterprises.

While the mathematical basis of quantum computing, the programming model, and most quantum algorithms have been published decades ago (starting in the 20th century), they have been of interest only to a small dedicated community (Abhijith et al., 2022). This paper aims to help address this niche nature and bring a basic understanding of quantum computing to a broader audience. Specifically, we intend to convey the fundamental principles underlying quantum computing and how they form the basis for quantum information processing and the development of quantum computers, as well as discuss topical examples of real-world quantum computing technologies and events. It is paramount for the wider computer science community to understand what quantum computing really is because its consequences will radically change the way we consider and apply computation in our lives.

Brief History

Quantum computing initially entered the broader conversation in scientific communities after American theoretical physicist Richard Feynman delivered a speech titled "There's Plenty of Room at the Bottom: An Invitation to Enter a New Field of Physics" in 1959 at CalTech, which discussed the possibility of manipulating and arranging individual atoms and molecules to create new materials and devices. Following this talk, from 1960-1980, many researchers, like Alexander Holevo, Charles H. Bennett, Roman S. Ingarden, Yuri Manin, etc. released publications about using quantum mechanics to solve computing problems that classical systems could not.

Then came 1980, when American physicist Paul Benioff published a paper titled “The computer as a physical system: A microscopic quantum mechanical Hamiltonian model of computers as represented by Turing machines,” describing a quantum mechanical model of a computer. Shortly after, in 1981, Richard Feynman reentered the discourse by giving a speech titled “Simulating physics with computers,” in which he explained how certain quantum mechanical effects cannot be simulated efficiently on classical computers and that we must develop quantum hardware devices to solve these types of complex problems. A famous quote arose from this event encapsulating the general sentiment and push for the development of quantum computing systems, “nature isn't classical, dammit, and if you want to make a simulation of nature, you'd better make it quantum mechanical, and by golly it's a wonderful problem, because it doesn't look so easy. Thank you.”

In the proceeding years, British physicist David Deutsch came on the scene, publishing a paper titled “Quantum Theory, the Church-Turning Principles and the Universal Quantum Computer” which finally outlined and formalized a concept for a universal quantum computer capable of simulating any physical process given enough resources. Nearly ten years after that, in 1994, American professor Peter Shor shocked the field by describing a polynomial time quantum algorithm for factoring integers, which pushed the field of quantum computing to come into its own (Rieffel, 2000). Since the middle of the 1990s, the field of quantum computing has been growing at an ever faster rate.

Principles and Fundamentals

To understand the larger picture of quantum computing, we must start with the basics. At its heart is what’s known as the quantum computing programming model. “The quantum programming model is fundamentally different from traditional computer programming. It is also dominated by physics and algebraic notations that at times present unnecessary entry barriers for mainstream computer scientists and other more mathematically trained scientists ” (Abhijith et al., 2022). In essence, the model involves quantum mechanical changes to computation.

In this section, we explain the abstract fundamental ideas underpinning the quantum computing programming model and quantum computers through a mathematical and conceptual understanding based on relevant literature we sourced and reviewed.

Qubit

The fundamental information carrying unit in quantum computers is called a qubit, also known as a quantum bit (Abhijith et al., 2022). It can be [conceptualized] as the quantum mechanical generalization of a bit, 0 or 1, in classical computers (Abhijith et al., 2022). In a quantum system, one-bit information can be encoded using two independent and mutually exclusive states of a microscopic object, such as a photon, electron, ion, etc. (Kanamori & Yoo, 2020). In a classical system, one-bit information is typically encoded in electrical circuits using transistors to manipulate electrical currents.

Because qubits leverage quantum mechanical effects to perform computations, they are governed by the laws of quantum mechanics. The state of any quantum system is always represented by a normalized vector in a complex vector space, usually called a Hilbert space (Abhijith et al., 2022). Therefore, we can use *Dirac notation* to mathematically describe the overall state of a single qubit as a two-dimensional vector, $|\psi\rangle$:

$$|\psi\rangle = \alpha|0\rangle + \beta|1\rangle \tag{1}$$

Here, α and β are complex numbers such that, |α|^2 + |β|^2 = 1 (normalized so the total probability of all the outcomes of a measurement sum to one). In the *Dirac notation*, the |0⟩ and |1⟩ are shorthands for the vectors encoding the two *basis states* of a two dimensional vector space. *Basis states* represent the classical binary states a qubit can be measured upon observation. Another way to think about it is the symbol |0⟩ refers to the classical binary state (10) and the symbol |1⟩ refers to the classical binary state (01). Understand also that $|\alpha|$^2 is the probability of getting the state |0⟩ as the result of the measurement on the qubit $|\psi\rangle$ while $|\beta|$^2 is the probability of getting |1⟩ (Kanamori & Yoo, 2020). Essentially, the statement reads: the state of a qubit is equal to the probability of getting the classical value "0" or "1" upon measurement of the qubit.

All of that logic is just for a single qubit, but a typical quantum computer's processing unit is outfitted with multiple qubits, so it is necessary to know how to construct the combined state of a system of qubits given the states of the individual qubits (Abhijith et al., 2022).We can describe a system of two qubits by employing a mathematical technique called the tensor product, denoted by the symbol, ⊗:

$$|\psi 1\rangle \otimes |\psi 2\rangle = |00\rangle + |01\rangle + |10\rangle + |11\rangle \tag{2}$$

Here, there are two independent qubits combining through the tensor product to produce the full state of a system of two qubits. If you notice, we input two qubits and get 2^2 basis states. We can progress further by describing the full state of a system of three qubits:

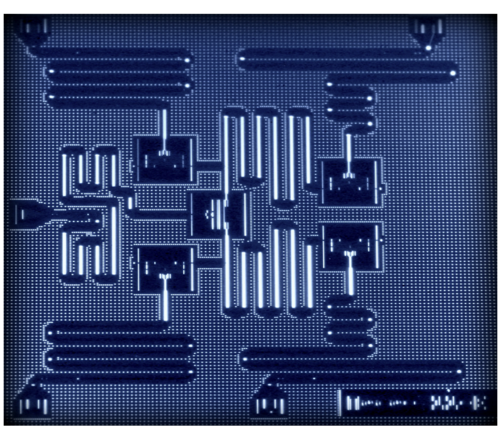
$$|\psi 1\rangle \otimes |\psi 2\rangle \otimes |\psi 3\rangle = |000\rangle + |001\rangle + |010\rangle + |011\rangle + |100\rangle + |101\rangle + |110\rangle + |111\rangle \tag{3}$$

A pattern begins to appear, evidenced by the fact that with a linear increase in *n* qubits, you increase the possible basis states by an order of 2^n.

To further conceptualize a qubit, we can look at a real quantum processor comprising five physical qubits (Figure 1). Quantum computers use various hardware designs to create qubits, but for this example, we present an implementation of superconducting qubits. To give some background, they are constructed using superconducting materials, like aluminum and/or niobium (Wright, 2023). A typical superconducting qubit is a tiny loop or line of metal that behaves like an atom—an inherently quantum object (Wright, 2023). It achieves this behavior by supercooling the metal to extremely low temperatures, around -273 celsius, just shy of absolute zero. This is done via an auxiliary apparatus situated around the qubits in a quantum processing unit, normally known as a dilution refrigerator. When a superconducting material is cooled to extremely low temperatures, they begin to exhibit quantum properties that allow for the realization of a qubit.

Figure 1

5 Qubit Processing Unit



Layout of IBM's five superconducting quantum bit device from 2015. (Credit: IBM Research)

Note. Each little square is a qubit with the light blue resonators to control electronics. Wright, K. (2023, September 28). *What's a qubit? 3 ways scientists build quantum computers*. Scientific American.

Superposition

Unlike classical bits, which have discrete states that you can measure to receive a deterministic result, measuring a qubit changes its state yielding a probabilistic result. What this suggests is that when a physical system has multiple *possible* states upon measurement, its most comprehensive state can be described as a combination of all these possibilities (Logunova, 2023). When looking back at the definition of a qubit, we can see that a qubit holds the value "0" and "1" simultaneously until measurement occurs. If you conduct a series of measurements, you will get probabilistic results that vary between the values "0" and "1." This abstract idea of a quantum system existing in multiple states at the same time is called *superposition*. In other words, we can say that a qubit (a type of quantum system), until measured, exists in a *superposition* of "0" and "1" at the same time.

In a more precise, mathematical sense, *superposition* refers to the fact that any linear combination of two quantum states, once normalized, will also be a valid quantum state, meaning any quantum state can be expressed as a linear combination of a few basis states (Abhijith et al., 2022). This is just like earlier in (1), (2), and (3), where the state of any *n* qubit system can be written as a normalized linear combination of the 2^*n* basis states formed by the tensor products of $|0\rangle$'s and $|1\rangle$'s (Abhijith et al., 2022).

Entanglement

Closely related to superposition, the idea of *entanglement* can be conceptualized as a physical phenomenon that occurs when the states of a quantum system (e.g. qubit) become correlated with each other, regardless of physical distance. It is a very abstract concept to fully grasp. When you measure a qubit in an entangled system of qubits, the measurement outcomes for one qubit become correlated with the outcomes for the other qubits without any direct interaction. Therefore, in an *entangled* system of qubits, the collective state of all qubits is a *superposition* of different possible configurations, signifying that each qubit in the *entangled* system contributes to the overall superposition, allowing the whole system to represent multiple possibilities simultaneously. This property of quantum mechanics separates quantum computing from its classical counterpart to such an extent "that without the existence of such states quantum computers would be no more powerful than their classical counterparts" (Abhijith et al., 2022).

Mathematically, *entangled* states are the states of a quantum system that cannot be expressed as a tensor product of its individual states (Abhijith J., et al., 2022). "For a system of *n* qubits, this means that an *entangled* state cannot be written as a tensor product of *n* single qubit states" (Abhijith et al., 2022). The state of entangled qubits is described by a joint quantum state that cannot be factored into independent states for each qubit. For example, you can write this as

$$|\psi\rangle = \alpha(|000\rangle + |111\rangle) \tag{4}$$

$$|\psi\rangle = \alpha(|0\rangle\otimes|0\rangle + |1\rangle\otimes|1\rangle) \tag{5}$$

The fact that they cannot be separated into independent states is a mathematical representation of the correlation between entangled qubits.

Quantum Gates/Circuits

To actually manipulate a qubit's states requires the next concept, quantum gates and circuits.

A *quantum gate* is an operation that transforms one quantum state into another. Quantum computers encode information in qubits, but they encode algorithms in quantum gates (Fang et al., 2023). It is similar to how classical computers encode algorithms in logic gates using boolean logic, however, instead of AND, OR, NOT they are XOR, NAND, cNOT, etc. Different hardware devices realize quantum gates differently, with some common methods being microwave pulses and shining light (Wright, 2023).

Quantum computers conduct certain computations by performing a series of quantum gate operations on quantum states (Fang et al., 2023). Think of this series of quantum gate operations as a quantum circuit. These quantum circuits, like classical circuits, are the building blocks of a quantum computer's physical architecture.

Parallel Processing

All of the quantum mechanical effects discussed thus far culminate in a radical potential breakthrough in computational power provided by quantum computers called *quantum parallel processing*. In classical computers, to achieve an exponential decrease in execution time requires an exponential increase in the number of processors, and hence an exponential increase in the amount of physical space needed. Think of GPUs doing parallel tasks/data processing to render graphics or run machine learning algorithms. Quantum computers, on the other hand, only need a linear increase in processing power to achieve an exponential decrease in time, a physical fact that stems from the nature of how qubits behave (Rieffel, 2000). Quantum computers and algorithms (see section 3) maneuver to exploit the phenomenon of superposition and entanglement to enhance the parallel efficiency of their computations. The main problem many have with this explanation is that the computational value seems to be lost once a qubit is measured. While it may appear counterintuitive, the quantum speedup occurs during the computation itself. The final measurement provides a probabilistic result, yes, but all the efficiency comes from how quantum computers utilize quantum parallelism during the computation phase of the problem, i.e. when a quantum computer explores and processes a large number of possibilities in parallel (simultaneously) by manipulating the quantum states of qubits through quantum gates that utilize superposition and entanglement to perform operations.

Algorithms and Applications

In order for all of the different parts of a quantum computer to run and be applicable, there are many different algorithms that have been created in order to aid the usages of quantum computers. A quantum algorithm is an algorithm that can only be carried out on a quantum computer, and it is used by searching the database in order to solve the problem. There are many different types of quantum algorithms, but the two that will be forced on are the Shor Algorithm and Grover Search Algorithm.

Although quantum computing started development in the 1960s, it wasn’t until Peter Shor in the 1990s that caused the rapid development in the field. The Shor Factoring Algorithm was introduced in 1994 by Peter Shor, and as mentioned earlier, it was key pioneering work done for quantum computing. The purpose of the Shor Factoring Algorithm is for factorization to be performed in polynomial time, as opposed to exponential time which is achieved using a classical algorithm (Abhijith et al., 2022) How the algorithm works is by selecting a random integer smaller than the number to be factored. Then the greatest common divisor (GCD) is found classically. Once the GCD of the integer had been found, the system then determines whether the target number had been factored accidentally. For smaller numbers, it is a possibility. For larger numbers, a supercomputer could be needed. As for numbers that are believed to be cryptographically secure, which means the number is extremely hard for a third party to find, a quantum computer will be needed.

The Grover Search Algorithm was introduced in 1996 by Lov Grover, an Indian-American computer scientist, to address the fundamental problem of searching an unsorted database (Abhijith et al., 2022). This algorithm works by starting in a state of superposition for all *N* elements. Then the diffusion operator, which is a quantum operation, is left to amplify the amplitude of the states that correspond to the marked element, which is the element being searched for. Finally, the algorithm measures the state of the system. This collapses the superposition and shows the marked element.

Real World Applications

Topical Examples Available Today

Quantum computing has charmed many organizations due to its revolutionary problem-solving capabilities with promising business and government applications. Different governments have been increasing the funding for quantum computing research and development not only for the advancement of computing technology but also for their national security [1] [2](Kanamori & Yoo, 2020). There are places like IBM, Google, Amazon, Microsoft, Nasa, and many others exploring the research and development of quantum hardware devices to make theoretical and algorithmic abstractions into a practical reality.

For instance, since the Canadian company D-wave unveiled a commercial annealer-based quantum computer in 2012, quantum computing has attracted much more increasing attention from enterprises ("D-Wave: Quantum Computing Applications," 2019; Robert Hackett, 2019). Microsoft has released internet cloud access to their quantum computer simulators and the real quantum hardware supplied by Honeywell, IonQ, and QCI via their service Azure Quantum (Kanamori & Yoo, 2020); Amazon started a quantum computing service via AWS, called Amazon Bracket, where users can remotely use the quantum computer hardware of their partners: D-wave, IonQ[1], and Rigetti (Kanamori & Yoo, 2020); JPMorgan Chase and Goldman Sachs' research teams have found that quantum computing could significantly reduce the time to calculate option pricing and risk-assessment calculations (Kanamori & Yoo, 2020); Microsoft

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[1] United States Information Technology Report - Q4 2023 (2023)

has released the quantum development kit (QDK) and t[2]he quantum programming toolkit Q# for Visual Studio ("Quantum Development Kit | Microsoft," 2019), which allows users to simulate quantum circuits on a classical computer (Kanamori & Yoo, 2020). This product and others, like Qiskit, Cirq, Quipper, LanQ, Silq, ProjectQ, etc. are forming the market for quantum programming languages; ExxonMobil and Volkswagen are exploring quantum solutions to optimization problems; In January 2019, IBM unveiled the first commercial general-purpose multi-qubit gate-based quantum computer called "IBM Q System one" and have been collaborating with more than 100 organizations, including the companies mentioned above, across industries and made their 5-qubit and 20-qubit quantum computers available via their cloud service called "IBM Q Experience" (Kanamori & Yoo, 2020). We encourage readers to seek out the vast market expanse of quantum computing applications offered by a multitude of companies.

A notable benchmark in the field of quantum computing is a term known as "quantum supremacy" that refers to a criterion where a quantum computer can outperform a classical computer at a specific task for all intents and purposes (intractably). At one point, Google claimed their quantum computer had demonstrated "quantum supremacy," stating it went beyond classical computers by demonstrating the ability to execute an algorithm in 200 seconds that a traditional supercomputer would need 10,000 years to do (Preskill, 2012). However, IBM claimed that Google's quantum computer did not reach quantum supremacy because the same task could be done with an ideal algorithm on a classical computer in 2.5 days (Pednault, Gunnels, Maslov, & Gambetta, 2019). As of yet, there are many claims with varying veracity on the true scope of who has truly achieved quantum supremacy. Lots of big market players are vying for the top spot and will continue to compete like Google and IBM. Such activity is something to keep an eye on for those in a computer science role because once "quantum supremacy" is reached at scale, it will disrupt the whole sector.

Challenges

The development of economically viable quantum computers comes with some tricky challenges because quantum systems are inherently very prone to interference from noise. External influences, like thermal energy, various particles, signals, etc. interfere with a quantum

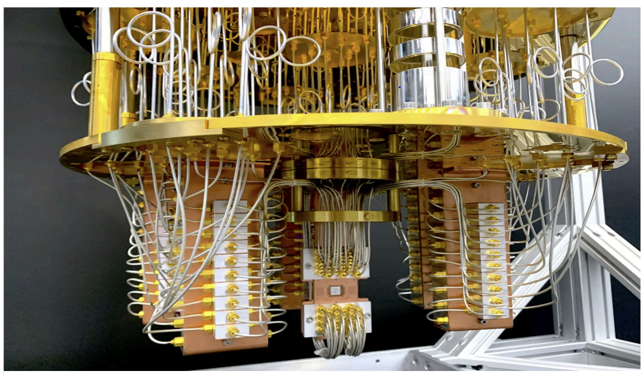
[2] Swallow, J. G., & Joneckis, L. G. (2020). Executive Summary.

state similarly to measurement. If a quantum computer is not sufficiently isolated from external influences then its qubits will be corrupted and yield error-filled results. The process of preventing these errors is known as quantum error correction. It is an area of interest vital to the long-term success of quantum computing as it deals with the scalability and reliability of quantum technologies. Unfortunately, a cost-effective and dependable mode of quantum error correction has not been fully realized yet (Fang et al., 2023).

What Does A Quantum Computer Look Like?

Figure 4.1

IBM "Quantum System One"



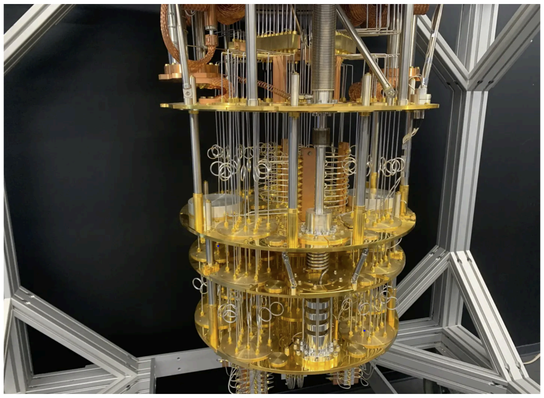
IBM takes PopSci inside their quantum computer world. *Charlotte Hu*

Note. This is IBM's "Quantum System One" quantum computer that was commercially released in 2019 via their cloud service "IBM Q Experience." It is an integrated superconducting qubit gate-based quantum computer consisting of a cooling apparatus, quantum processing unit, container, and auxiliary electronic equipment.

Wright, K. (2023, September 28). *What's a qubit? 3 ways scientists build quantum computers*. Scientific American.

Figure 4.2

Dilution Refrigerator



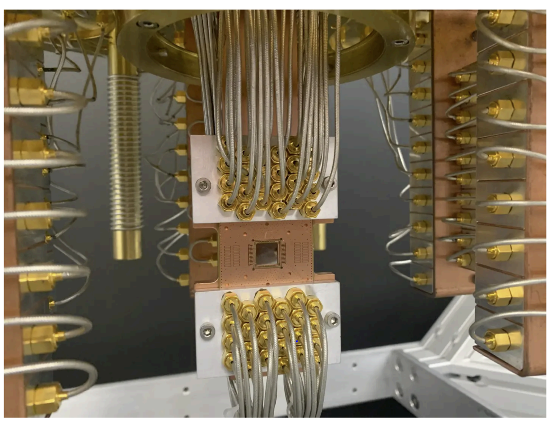
The dilution refrigerator. *Charlotte Hu*

Note. This is the cooling apparatus called a dilution refrigerator that pumps coolant down through multiple golden slabs that reach a temperature around -273 degrees celsius at the quantum processing unit.

Wright, K. (2023, September 28). *What's a qubit? 3 ways scientists build quantum computers*. Scientific American.

Figure 4.3

Quantum Processing Unit



The quantum processor inside the chandelier. *Charlotte Hu*

Note. The small chip in the middle of the image is the quantum processing unit. It is cooled by the dilution refrigerator and consists of multiple superconducting qubits made of aluminum and niobium.

Wright, K. (2023, September 28). *What's a qubit? 3 ways scientists build quantum computers*. Scientific American.

Figure 4.4

Control Electronics

The control electronics behind the quantum computer. *Charlotte Hu*

Note. These are the auxiliary electronic devices that handle the cloud-based user inputs as well as controlling the conditions of the overall system to ensure the quantum computer works error-free.

Wright, K. (2023, September 28). *What's a qubit? 3 ways scientists build quantum computers*. Scientific American.

Figure 4.5

Dilution Refrigerator Container

The container that holds the dilution refrigerator inside. *Charlotte Hu*

Note. This is an image of the air and water-tight container encasing the "Quantum System One" to make sure it is insulated from external environmental influence that could corrupt the integrity of the quantum system. Part of quantum error correction

Wright, K. (2023, September 28). *What's a qubit? 3 ways scientists build quantum computers*. Scientific American.

Conclusion

Quantum computing is the usage of quantum mechanics for computation and is significantly important due to its impact on computer science and daily life. Quantum computing is a large and vastly complicated topic with many moving components and there is a rich history to how quantum computing got its start, involving many scientists, mathematicians, and computer scientists. Some key historical figures are Richard Feynman, who discussed the need for quantum computers for complex problems, David Deutsch who is credited for describing a quantum computer, and Peter Shor for creating one of the most influential algorithms for a quantum computer.

Due to the complexity of quantum computing, there are many important and necessary parts that contribute to the functionality of quantum computing. To start, the most fundamental aspect of a quantum computer is the qubit, which is the information carrying unit. There are many different aspects of a qubit, such as superposition and entanglement. By exploiting these qualities of a qubit, quantum computers are able to conduct quantum parallelism/parallel processing and explore quantum gates and quantum circuits.

In order for quantum computers to run and be effective, there are certain algorithms that are used. Two of the algorithms that were discussed were the Shor Factoring algorithm and the Grover Search Algorithm. The Shor Factoring Algorithm is used to factor complex scenarios in polynomial time and the Grover Search Algorithm is used for searching unstructured databases. These algorithms, and many more, amplify the usages and effectiveness of quantum computers, but still make use of classical computing.

There have been many applications of quantum computing in the modern world. From government involvement in funding quantum computing to leading companies creating their own quantum computers that can be accessed by the public through cloud platforms, quantum computing is a large field with many different applications. Unfortunately, due to the cost and technology required to power a quantum computer, there are several areas of quantum computing that have yet to be explored. The field is rich with activity and we encourage you to use the information detailed here to further individual and collective understanding and documentation of this marvelous technology.

References

Abhijith, J., et al. (2022). *Quantum algorithm implementations for beginners*. ACM Transactions on Quantum Computing, 3(4), 1-92. https://doi.org/10.1145/3517340

Almudever, C. G., Lao, L., Wille, R., & Guerreschi, G. G. (2020). *Realizing Quantum Algorithms on Real Quantum Computing Devices*. In 2020 Design, Automation & Test in Europe Conference & Exhibition (DATE) (pp. 864-872). Grenoble, France. https://doi.org/10.23919/DATE48585.2020.9116240

Djordjevic, I. (2012). *Quantum Information Processing and Quantum Error Correction*. Academic Press, 145-173. https://doi.org/10.1016/B978-0-12-385491-9.00005-8

Dowling, J. P., & Milburn, G. J. (2003). *Quantum Technology: The Second Quantum Revolution.* Philosophical Transactions: Mathematical, Physical and Engineering Sciences, 361(1809), 1655–1674. http://www.jstor.org/stable/3559215

Dyakonov, M. (2019). *When will useful quantum computers be constructed? Not in the foreseeable future, this physicist argues. Here's why: The case against: Quantum computing.* IEEE Spectrum, 56(3), 24-29. https://doi.org/10.1109/MSPEC.2019.8651931

Feng, G., Lu, D., Li, J., Xin, T., & Zeng, B. (2023). *Quantum computing: principles and applications*. Retrieved from Cornell University Library, arXiv.org.

Feynman, R. P. (1982). *Simulating physics with computers*. International Journal of Theoretical Physics, 21(6-7), 467–488. https://doi.org/10.1007/BF02650179

Huang, H. L., Xu, X. Y., Guo, C., et al. (2023). *Near-term quantum computing techniques: Variational quantum algorithms, error mitigation, circuit compilation, benchmarking and classical simulation*. Science China Physics, Mechanics & Astronomy, 66, 250302. https://doi.org/10.1007/s11433-022-2057-y

Kanamori, Y., & Yoo, S.-M. (2020). *Quantum Computing: Principles and Applications*. Journal of International Technology and Information Management, 29(2), Article 3. https://scholarworks.lib.csusb.edu/cgi/viewcontent.cgi?article=1410&context=jitim

Logunova, I. (2023, June 9). *What is quantum computing?* https://serokell.io/blog/what-is-quantum-computing

Preskill, J. (2012, November 10). *Quantum computing and the Entanglement Frontier*. arXiv.org. https://arxiv.org/abs/1203.5813

Swallow, J. G., & Joneckis, L. G. (2020). *Executive Summary. In Manufacturing for Quantum Systems* (Presentation) (pp. iii–viii). Institute for Defense Analyses. http://www.jstor.org/stable/resrep36522.2

Rieffel, E. G., & Polak, W. (2000, January 19). *An introduction to quantum computing for Non-Physicists*. arXiv.org. https://arxiv.org/abs/quant-ph/9809016

Wright, K. (2023, September 28). *What's a qubit? 3 ways scientists build quantum computers*. Scientific American. https://www.scientificamerican.com/article/whats-a-qubit-3-ways-scientists-build-quantum-computers/#:~:text=Google%27s%20qubits%20are%20made%20of,atom%E2%80%94an%20inherently%20quantum%20object

United States Information Technology Report - Q4 2023 (2023). London: Fitch Solutions Group Limited. Retrieved from ProQuest One Academic https://www.proquest.com/reports/united-states-information-technology-report-q4/docview/2873614211/se-2

Deutsch, D. (n.d.). *Rapid solution of problems by Quantum Computation*. royalsocietypublishing. https://royalsocietypublishing.org/doi/10.1098/rspa.1992.0167