End-Term Report: Finsearch_A36_OptionPricingModels

Evaluation of Option Pricing Models & Their Accuracy

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1. Introduction to Option Pricing Models

In the dynamic world of finance, the **accurate valuation of options is paramount** for investors, traders, and financial institutions. An option is a powerful financial derivative that bestows upon its holder the **right, but not the obligation**, to either buy or sell an underlying asset at a pre-determined price (known as the strike price) on or before a specified expiry date. The ability to accurately ascertain an option's fair value is crucial for making informed trading decisions, managing risk, and ensuring market efficiency.

This report delves into two foundational and widely adopted methods for option valuation: the **Black-Scholes Model (BSM)** and the **Monte Carlo Simulation (MCS)**. While both models aim to determine an option's fair value, they employ fundamentally different approaches, each with its unique strengths, weaknesses, and applicability. This document will detail their methodologies, assumptions, and practical utility, culminating in a comprehensive overview of our project, Finsearch_A36_OptionPricingModels, which specifically evaluated the effectiveness of the Black-Scholes Model using a synthetic dataset.

2. The Black-Scholes Model (BSM)

2.1 Origin and Core Principles

The **Black-Scholes Model**, developed by Fisher Black and Myron Scholes in 1973, represents a **landmark achievement in financial economics**. It provides a theoretical framework for pricing European-style options, which are options that can only be exercised at their expiration date. The BSM is renowned for being a **closed-form solution**, meaning it uses a direct mathematical formula to calculate a **single, precise price** for an option. This formulaic nature is the primary reason for its speed and efficiency in computation.

The model relies on several key inputs to derive an option's price: the current stock price, the option's strike price, the time remaining until expiration, the prevailing risk-free interest rate, and the volatility of the underlying asset.

2.2 Fundamental Assumptions

A critical aspect of understanding the Black-Scholes Model is its **rigid set of assumptions**, which, while simplifying the complex realities of financial markets, also introduce limitations:

- **European Option**: The model is exclusively designed for European-style options, meaning they can only be exercised at their expiration. This is a significant limitation as many real-world options, particularly in the US market, are American-style and can be exercised at any time up to expiration.

- **No Dividends**: The model assumes that the underlying asset will not pay any dividends during the option's life. This is often unrealistic for dividend-paying stocks and requires adjustments to the model for practical application.
- **Random Market Movements**: It presumes that market movements are random and follow a **geometric Brownian motion**, implying that asset prices move continuously and that price changes are normally distributed.
- **Constant Risk-Free Interest Rate and Volatility**: Both the risk-free interest rate and the volatility of the underlying asset are assumed to remain constant throughout the option's life. In reality, these parameters are dynamic and fluctuate significantly.
- **No Transaction Costs or Taxes**: The model assumes a frictionless market where there are no transaction costs (like brokerage fees) or taxes involved in buying or selling the option or the underlying asset.
- **Continuous Trading**: It assumes that trading can occur continuously, and assets are perfectly divisible.

2.3 Strengths and Weaknesses

Strengths: The Black-Scholes Model's primary advantages lie in its **high efficiency and speed**. For simple European options, it offers a quick and clear valuation, which has established it as an **industry standard**. Its analytical elegance and straightforward application for a specific type of option make it widely used.

Weaknesses: Despite its widespread use, the BSM's **rigid and often unrealistic assumptions** are its major drawbacks. These assumptions limit its applicability, as it **cannot be used for American options** or for more complex, **path-dependent options** (where the payoff depends on the price path of the underlying asset over time, not just its terminal price).

3. The Monte Carlo Simulation (MCS)

3.1 Introduction and Versatility

The **Monte Carlo Simulation** is a powerful computational method that offers a stark contrast to the closed-form approach of Black-Scholes. It is particularly **useful for valuing options that are difficult or impossible to price with traditional analytical methods**. The core principle of MCS revolves around **simulating numerous future stock price paths** for the underlying asset. By generating thousands or even millions of these random paths, the Monte Carlo method models the full distribution of potential outcomes for the option's value. The final option price is then determined by taking the **average of the discounted payoffs** from all these simulated paths.

The method's **flexibility is a significant advantage**. It can effectively handle:

- **Path-dependent options** such as Asian, Barrier, and Lookback options.
- **Multi-asset derivatives**, like basket options.
- Payoffs influenced by **multiple stochastic factors**, including varying interest rates, volatility, or commodity prices.

- Scenarios involving **early exercise** (though this often requires more advanced techniques like Least Squares Monte Carlo) or embedded optionality in corporate finance projects.

3.2 Mathematical Foundation

For many Monte Carlo pricing implementations, the underlying assumption is that the asset price follows **Geometric Brownian Motion (GBM)**. This stochastic differential equation is defined as: **$dS = \mu S\, dt + \sigma S\, dW$** Where:

- S = asset price
- μ = drift rate (expected return)
- σ = volatility
- W = standard Wiener process (Brownian motion)

The **closed-form solution for the terminal price (ST) at maturity (T)**, which is essential for simulating paths, is given by: **ST=S0·e((r−2σ2)T+σTZ)**, where **Z ~ N(0,1)** This formula allows for the generation of terminal asset prices for each simulated path by drawing a random value Z from the standard normal distribution.

3.3 How Monte Carlo Simulation Works

The Monte Carlo simulation process is systematic and involves several distinct steps:

1. **Input Parameters**: Gather necessary values such as the initial asset price (S0), strike price (K), risk-free rate (r), volatility (σ), time to maturity (T), and the desired number of simulations (N).
2. **Simulate Price Paths**: Generate a large number of possible future asset prices until expiration using the Geometric Brownian Motion formula. For each simulation, a random Z value is drawn, leading to a unique ST.
3. **Calculate Payoff**: For each individual simulated path, the option's payoff at expiration is calculated. For a call option, this is max(ST - K, 0), and for a put option, it is max(K - ST, 0).
4. **Average Payoffs**: Once payoffs for all simulated paths are determined, their mean (average) is computed.
5. **Discount to Present Value**: The calculated average payoff is then discounted back to its present value using the risk-free rate to arrive at the estimated option price. The formula for a European call option pricing is C = e^{-rT} × Mean Payoff.

An example provided illustrates pricing a European Call option with specific parameters (S0=100, K=105, r=5%, σ=20%, T=1 year) using N=1,000,000 simulations, stabilizing the estimate at approximately $8.02, which matched the Black-Scholes benchmark.

3.4 Strengths (Benefits) and Weaknesses (Limitations)

Strengths (Benefits):

- **Versatility**: Works exceptionally well for **exotic and path-dependent options** that Black-Scholes cannot handle.

- **Flexibility**: Can easily **incorporate complex features** like stochastic volatility, changing interest rates, and price jumps.
- **Simplicity in Concept**: The underlying logic of simulating and averaging is straightforward.
- **Parallelizable**: The simulations are independent, making it **easily distributed across multiple processors**, speeding up computation.

Weaknesses (Limitations):

- **Computationally Intensive**: MCS is significantly **slower than Black-Scholes** because it requires millions of simulations to achieve high accuracy.
- **Not Exact**: The result is a **statistical estimate**, not a precise value, with accuracy dependent on the number of simulations.
- **Computationally Expensive**: Time-consuming for complex multi-asset models.
- **Requires Quality Random Number Generation (RNG)**: Poor RNG can skew results.

3.5 Accuracy and Variance Reduction

Accuracy in Monte Carlo simulations is inherently **statistical**, not deterministic. It depends on how well the finite set of simulated payoffs represents the theoretical probability distribution of outcomes.

Sources of Error:

- **Statistical Sampling Error**: Caused by the finite sample size and measured by the **standard error (SE)** ($SE = \sigma_{payoff} / \sqrt{N}$).
- **Model Error**: Occurs when underlying assumptions (e.g., constant volatility, lognormal distribution) do not hold true in real markets.
- **Numerical Error**: Related to floating-point precision issues in calculations.

MCS provides not just a point estimate but also a **confidence range**, typically expressed as a confidence interval (e.g., 95% confidence level: Price $\in$ [Estimate - 1.96 × SE, Estimate + 1.96 × SE]) which quantifies the statistical reliability.

To improve accuracy without linearly increasing the number of simulations (N), **Variance Reduction Techniques** are critical:

- **Antithetic Variates**: Generate paired simulations with Z and -Z to cancel out symmetric random noise.
- **Control Variates**: Use a related problem with a known analytical solution (e.g., Black-Scholes for European options) to adjust results.
- **Stratified Sampling**: Divide the random number space into strata and sample proportionally.
- **Quasi-Random Sequences**: Use low-discrepancy sequences (Sobol, Halton) for faster convergence than purely random draws.

4. Comparison of the Two Models

The choice between Black-Scholes and Monte Carlo fundamentally depends on the option type and the required level of complexity. The table below summarises their key differences:

Feature	Black-Scholes Model	Monte Carlo Simulation	Source
Methodology	Closed-form mathematical formula	Random sampling and simulation of many price paths	
Type of Option Best For	European options	Versatile; handles European, American, and Exotic options	
Computational Speed	Extremely fast and efficient	Computationally intensive and much slower	
Flexibility & Complexity	Very rigid; limited by assumptions	Highly flexible; can accommodate complex features	
Key Assumptions	Constant volatility, no dividends, continuous trading	Fewer inherent assumptions; can model non-constant variables	
Result	A single, precise value	A statistical estimate (an average of many simulations)	
Example Use Case	Standard call/put on non-dividend stock	Asian option (depends on average price)	

Black-Scholes is ideal for quickly and efficiently pricing simple European options, maintaining its status as an industry standard for its speed and elegance. Conversely, Monte Carlo is invaluable for complex, path-dependent, or exotic options with non-standard features that Black-Scholes cannot handle. Its flexibility is a powerful asset in modern financial engineering, where realism often takes precedence over sheer computational speed.

5. Project Implementation and Evaluation: Finsearch_A36_OptionPricingModels

5.1 Project Goal and Context

Our project, titled **Finsearch_A36_OptionPricingModels**, aimed to **evaluate the effectiveness of the Black-Scholes Model (BSM) in pricing options**. The core objective was to implement the BSM formula from scratch and then benchmark its predictions against simulated market prices. This benchmarking process allowed us to identify specific areas where the model either aligned with or deviated from expected market behaviour.

The analysis was conducted using a **synthetic dataset mimicking real-world Nifty50 index options**. This dataset was **artificially generated**, drawing inspiration from the actual Nifty50 index option structure. A crucial aspect of the data generation phase involved **obtaining data from the NSE (National Stock Exchange) site and meticulously cleaning it** to fit our specific parameter needs. This careful preparation of the dataframe was deemed essential for achieving accurate and bona fide predictions that could be compared meaningfully to real-world scenarios.

5.2 Group Contributions

This project involved a **comprehensive workflow** that distinctly highlighted the **interdisciplinary nature of financial modelling**, combining theoretical understanding, practical coding, rigorous data analysis, and effective communication. Each member of Group A36 contributed significantly to specific phases of the project:

- **Data Generation & Structuring**: This critical phase involved the creation of the synthetic dataset. Key activities included the **design of expiry and volatility parameters** and the thorough **cleaning of data obtained from the NSE site** to ensure it met the precise input requirements of our model. This meticulous data preparation was fundamental for generating accurate and genuine predictions comparable to real-world market conditions.
- **BSM Model Coding & Integration**: This phase focused on the technical implementation, involving **coding the mathematical formulas of the Black-Scholes Model** from the ground up. It also encompassed the integration of various financial parameters into the model to facilitate a robust comparison with real-world scenarios.
- **Visualization & Error Analysis**: This contribution focused on translating complex data into understandable insights. It involved **developing clear visual representations of the data** and conducting a **detailed analysis of the model's pricing errors**, helping to pinpoint where and why deviations occurred.
- **Report Writing & Video Script Preparation**: This phase involved the crucial task of **compiling all findings and analyses into this comprehensive end-term report**. Additionally, it included the preparation of the script for the accompanying project video, ensuring clear and concise communication of our work.

5.3 Technologies Used

The primary technology utilised for the implementation and analysis throughout this project was **Jupyter Notebook**, which accounted for **100.0%** of the project's codebase. This indicates a strong reliance on Python for all computational, analytical, and visualisation tasks.

6. Project Findings and Analysis

Our rigorous evaluation of the Black-Scholes Model yielded several key insights into its performance and limitations when applied to synthetic Nifty50-inspired options data.

6.1 Overall Performance Metrics

The analysis revealed that the **Black-Scholes model performs with very high predictive accuracy on average across the dataset**. This high level of accuracy is quantitatively supported by an impressive **R^2 Score of 0.9922**. However, despite this strong overall performance, it was observed that some errors persist, particularly based on specific option characteristics, indicating that while the model is robust, it is not perfect.

Key evaluation metrics were computed to quantify the model's performance:

- **Root Mean Squared Error (RMSE)**: **37.49**
- **Mean Absolute Error (MAE)**: **26.67**
- **Mean Absolute Percentage Error (MAPE)**: **4.27%**

These metrics collectively confirm the model's high average accuracy but also highlight the presence of discernible pricing errors in certain conditions.

6.2 Key Visual & Analytical Insights

Our visual and analytical explorations provided a deeper understanding of the BSM's behaviour:

- **Scatter Plot: Market vs. BSM Price**: The scatter plot vividly demonstrated a **strong clustering of data points around the ideal diagonal line**, which represents perfect agreement between market and BSM prices. This clustering was particularly pronounced for **In-The-Money (ITM) and At-The-Money (ATM) options**. However, notable **deviations were clearly visible for Out-of-The-Money (OTM) options**, where the BSM showed a tendency to either overprice or underprice significantly.
- **Error Distribution (Histogram & Boxplot)**: An in-depth analysis of the pricing error distribution offered granular insights into BSM's accuracy across different option types and moneyness:
    - For **Call options**, the average pricing error was **+11.49 for Out-of-The-Money (OTM)** options and **+12.46 for In-The-Money (ITM)** options. Crucially, a significant deviation was observed for **At-The-Money (ATM) calls, with an average error of -44.53**, which indicates occasional **overpricing** by the model in these instances.
    - **Put options**, in contrast, exhibited **tighter error ranges with minimal average error**: **+2.33 for In-The-Money (ITM)** and **-3.50 for Out-of-The-Money (OTM)**.
    - Visualisations consistently suggested that **ATM calls showed the most instability** and largest pricing discrepancies when valued using the BSM.
- **Error vs. Time to Expiry**: This analysis revealed a crucial trend: **shorter-dated options (those with a low time-to-expiry) consistently had a wider spread in pricing error**. This indicates that the BSM appears **less reliable as the option's expiry date nears**, a phenomenon consistent with real-world market inefficiencies that often emerge closer to expiration.
- **Correlation Heatmap**: The correlation analysis highlighted the variables most strongly associated with pricing errors. The **highest correlation of pricing error was observed with Implied Volatility and Strike Price**. This finding strongly suggests the Black-Scholes Model's inherent **sensitivity to these specific variables** and underscores how inaccuracies in their estimation can significantly impact the model's output. **Spot Price and Time to Expiry showed a moderate correlation with pricing errors**.

7. Conclusion – Strategic Takeaways

Our comprehensive evaluation confirms that the **Black-Scholes Model is robust but inherently imperfect**. While it demonstrated very high predictive accuracy on average for our synthetic dataset, its **limitations become particularly evident under specific market conditions and for certain option characteristics**. These insights from our project strikingly mirror real-world limitations of the BSM.

The Black-Scholes Model's limitations are most pronounced and its predictions less reliable when:

- **Volatility is high and variable**. The BSM's assumption of constant volatility is a major source of its deviations in volatile markets.

- **Options are deep Out-of-The-Money (OTM) or very near expiry**. The model tends to overprice or underprice OTM options and struggles with the dynamics of options approaching expiration, where market inefficiencies are more pronounced.

These findings **validate the critical need for more flexible and sophisticated financial models** that can account for complexities that BSM overlooks. Such models include:

- **Monte Carlo Simulations**: As discussed, these are invaluable for pricing complex, path-dependent, or exotic options and can incorporate non-constant volatility and interest rates.
- **Stochastic Volatility Models**: These models overcome one of BSM's major assumptions by allowing volatility to change randomly over time.
- **Volatility Surface Calibration**: This involves creating a 3D plot of implied volatilities across different strike prices and maturities, providing a more realistic representation of market volatility than a single constant value.

In summary, while the Black-Scholes Model remains an industry standard for its speed and elegance in pricing simple European options, its rigid assumptions necessitate caution and a clear understanding of its boundaries. For the intricate and evolving landscape of modern financial derivatives, particularly those with complex features or those subject to dynamic market conditions, **more adaptable tools like Monte Carlo simulations are indispensable**. The successful execution of this project, involving data generation, model implementation, detailed error analysis, and comprehensive reporting, truly highlighted the **interdisciplinary nature of financial modelling**, seamlessly blending theoretical financial principles with practical coding, rigorous data analysis, and effective communication.
