Cvičenia

Cvičenie 9.1. Definujete maticu koeficientov $\boldsymbol{A}$, stĺpcový vektor neznámych $\boldsymbol{x}$ a stĺpcový vektor konštantných členov $\boldsymbol{b}$ pre systémy

(a)

$$x_1 + x_2 = 1$$
$$2x_1 - x_2 = 0$$
$$A = \begin{pmatrix} 1 & 1 \\ 2 & -1 \end{pmatrix},\ x = \begin{pmatrix} x_1 \\ x_2 \end{pmatrix},\ b = \begin{pmatrix} 1 \\ 0 \end{pmatrix}.$$

(b)

$$x_2 = 1$$
$$x_1 = 0$$
$$A = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix},\ x = \begin{pmatrix} x_1 \\ x_2 \end{pmatrix},\ b = \begin{pmatrix} 1 \\ 0 \end{pmatrix}.$$

(c)

$$x_1 + x_2 + x_3 = 0$$
$$x_1 - x_2 + x_3 = 0$$
$$x_1 + x_2 - x_3 = 1$$
$$x_1 - x_2 - x_3 = -1$$
$$A = \begin{pmatrix} 1 & 1 & 1 \\ 1 & -1 & 1 \\ 1 & 1 & -1 \\ 1 & -1 & -1 \end{pmatrix},\ x = \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix},\ b = \begin{pmatrix} 0 \\ 0 \\ 1 \\ -1 \end{pmatrix}.$$

(d)

$$x_1 + x_2 + x_3 - 2x_4 = 1$$
$$A = \begin{pmatrix} 1 & 1 & 1 & -2 \end{pmatrix},\ x = \begin{pmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{pmatrix},\ b = \begin{pmatrix} 1 \end{pmatrix}$$

Cvičenie 9.2. Riešte pomocou inverznej matice systém rovníc

$$2x + 2y - 6z = 4$$
$$-x + y + 2z = 3$$
$$-3x + 5y + 3z = -1$$

Inverzná matica koeficientov bola spočítaná v cvičení 8.17a.

$$A=\begin{pmatrix}2&2&-6\\-1&1&2\\-3&5&3\end{pmatrix},\ x=\begin{pmatrix}x\\y\\z\end{pmatrix},\ b=\begin{pmatrix}4\\3\\-1\end{pmatrix},\ x=A^{-1}b=\begin{pmatrix}7/8&9/2&-5/4\\3/8&3/2&-1/4\\1/4&2&-1/2\end{pmatrix}\begin{pmatrix}4\\3\\-1\end{pmatrix}=\begin{pmatrix}73/4\\25/4\\15/2\end{pmatrix}$$

Cvičenie 9.3. Pomocou Gaussovej eliminačnej metódy riešte systémy lineárnych rovníc

(a)

$$\begin{aligned}x\ &+y\ &+z\ &=\ 2\\2x\ &-2y\ &-z\ &=\ 2\\3x\ &+y\ &-2z\ &=\ -2\end{aligned}$$

$$A'=\left(\begin{array}{ccc|c}1&1&1&2\\\boxed{2}&-2&-1&2\\3&1&-2&-2\end{array}\right)\sim\left(\begin{array}{ccc|c}1&1&1&2\\0&-4&-3&-2\\0&\boxed{-2}&-5&-8\end{array}\right)\sim\left(\begin{array}{ccc|c}1&1&1&2\\0&-4&-3&-2\\0&\boxed{4}&10&16\end{array}\right)\sim\left(\begin{array}{ccc|c}1&1&1&2\\0&-4&-3&-2\\0&0&7&14\end{array}\right)$$

$$7z=14\Rightarrow z=2,\ -4y-3z=-2\Rightarrow y=-1,\ x+y+z=2\Rightarrow x=1,\ \boldsymbol{x}=\begin{pmatrix}1\\-1\\2\end{pmatrix}.$$

(b)

$$\begin{aligned}2x\ &+2y\ &+z\ &=\ 4\\x\ &-y\ &-z\ &=\ 2\\3x\ &+y\ &&=\ 6\end{aligned}$$

$$A'=\left(\begin{array}{ccc|c}2&2&1&4\\\boxed{1}&-1&-1&2\\3&1&0&6\end{array}\right)\sim\left(\begin{array}{ccc|c}2&2&1&4\\0&4&3&0\\0&\boxed{4}&3&0\end{array}\right)\sim\left(\begin{array}{ccc|c}2&2&1&4\\0&4&3&0\\\cancel{0}&\cancel{0}&\cancel{0}&\cancel{0}\end{array}\right)\sim\left(\begin{array}{cccc}2&2&1&4\\0&4&3&0\end{array}\right)$$

$$z=t,\ y=-\frac{3}{4}t,\ x=2+\frac{1}{4}t,\ \boldsymbol{x}=\begin{pmatrix}2+(1/4)t\\-(3/4)t\\t\end{pmatrix}=\begin{pmatrix}2\\0\\0\end{pmatrix}+t\begin{pmatrix}1/4\\-3/4\\1\end{pmatrix}.$$

(c)

$$\begin{aligned}2x_1\ &-x_2\ &+5x_3\ &+3x_4\ &=\ 5\\x_1\ &+x_2\ &+4x_3\ &+3x_4\ &=\ 7\\x_1\ &&+3x_3\ &+2x_4\ &=\ 4\\&x_2\ &+x_3\ &+x_4\ &=\ 3\end{aligned}$$

$$A'=\left(\begin{array}{cccc|c}2&-1&5&3&5\\1&1&4&3&7\\1&0&3&2&4\\0&1&1&1&3\end{array}\right)\sim\left(\begin{array}{cccc|c}1&0&3&2&4\\\boxed{1}&1&4&3&7\\\boxed{2}&-1&5&3&5\\0&1&1&1&3\end{array}\right)\sim\left(\begin{array}{cccc|c}1&0&3&2&4\\0&1&1&1&3\\\cancel{0}&\cancel{-1}&\cancel{-1}&\cancel{-1}&\cancel{-3}\\\cancel{0}&\cancel{-1}&\cancel{-1}&\cancel{-1}&\cancel{-3}\end{array}\right)$$

$$\sim \left(\begin{array}{cccc|c} 1 & 0 & 3 & 2 & 4 \\ 0 & 1 & 1 & 1 & 3 \end{array}\right)$$

$x_4 = l$, $x_3 = k$, $x_2 = 3 - k - l$, $x_1 = 4 - 3k - 2l$, kde $k, l \in \mathbb{R}$,

$$x = \begin{pmatrix} 4-3k-2l \\ 3-k-l \\ k \\ l \end{pmatrix} = \begin{pmatrix} 4 \\ 3 \\ 0 \\ 0 \end{pmatrix} + k \begin{pmatrix} -3 \\ -1 \\ 1 \\ 0 \end{pmatrix} + l \begin{pmatrix} -2 \\ -1 \\ 0 \\ 1 \end{pmatrix},$$

(d)

$$\begin{array}{rrcr} 2x & -3y & = & -4 \\ x & +2y & = & 5 \\ -4x & +6y & = & 8 \end{array}$$

$$A' = \left(\begin{array}{cc|c} 2 & -3 & -4 \\ 1 & 2 & 5 \\ -4 & 6 & 8 \end{array}\right) \sim \left(\begin{array}{cc|c} 2 & -3 & -4 \\ 0 & -7 & -14 \\ \cancel{0} & \cancel{0} & \cancel{0} \end{array}\right) \sim \left(\begin{array}{cc|c} 2 & -3 & -4 \\ 0 & 1 & 2 \end{array}\right)$$

$y = 2$, $2x - 3 \cdot 2 = -4 \Rightarrow x = 1$, $x = \begin{pmatrix} 1 \\ 2 \end{pmatrix}$.

(e)

$$\begin{array}{rrrcr} 3x & -2y & +z & = & -4 \\ x & +y & +2z & = & 2 \end{array}$$

$$A' = \left(\begin{array}{ccc|c} 3 & -2 & 1 & -4 \\ 1 & 1 & 2 & 2 \end{array}\right) \sim \left(\begin{array}{ccc|c} 3 & -2 & 1 & -4 \\ 0 & -5 & -5 & +10 \end{array}\right) \sim \left(\begin{array}{ccc|c} 3 & -2 & 1 & -4 \\ 0 & 1 & 1 & 2 \end{array}\right)$$

$z = k$, $y = 2 - k$, $3x - 2(2 - k) + k = -4 \Rightarrow x = -k$

$$x = \begin{pmatrix} -k \\ 2-k \\ k \end{pmatrix} = \begin{pmatrix} 0 \\ 2 \\ 0 \end{pmatrix} + k \begin{pmatrix} -1 \\ -1 \\ 1 \end{pmatrix}$$

(f)

$$\begin{array}{rrrcr} x_1 & -x_2 & +2x_3 & = & 0 \\ 2x_1 & +3x_2 & -x_3 & = & 0 \end{array}$$

$$A' = \left(\begin{array}{ccc|c} 1 & -1 & 2 & 0 \\ 2 & 3 & -1 & 0 \end{array}\right) \sim \left(\begin{array}{ccc|c} 1 & -1 & 2 & 0 \\ 0 & 5 & -5 & 0 \end{array}\right)$$

$z = k$, $y = k$, $x = -k$

$$x = k \begin{pmatrix} -1 \\ 1 \\ 1 \end{pmatrix}$$

Cvičenie 9.4. Vypočítajte determinanty matíc:

(a)

$$\boldsymbol{A}=\begin{pmatrix}0&1&0\\1&0&0\\0&0&1\end{pmatrix},\quad |\boldsymbol{A}|=\begin{vmatrix}0&1&0\\1&0&0\\0&0&1\end{vmatrix}\begin{matrix}0&1\\1&0\\0&0\end{matrix}=-1$$

(b)

$$\boldsymbol{A}=\begin{pmatrix}1&1&-1\\0&0&0\\1&0&1\end{pmatrix},\quad |\boldsymbol{A}|=\begin{vmatrix}1&1&-1\\0&0&0\\1&0&1\end{vmatrix}=0$$

(c)

$$\boldsymbol{A}=\begin{pmatrix}2&1&0&-1\\4&2&5&-1\\3&1/2&-1&2\\0&0&1&1\end{pmatrix}$$

$$|\boldsymbol{A}|=\begin{vmatrix}2&1&0&-1\\4&2&5&-1\\3&1/2&-1&2\\0&0&1&1\end{vmatrix}=-\begin{vmatrix}-1&1&0&2\\-1&2&5&4\\2&1/2&-1&3\\1&0&1&0\end{vmatrix}=\begin{vmatrix}-1&2&0&1\\\boxed{-1}&4&5&2\\2&3&-1&1/2\\1&0&1&0\end{vmatrix}=\begin{vmatrix}-1&2&0&1\\0&2&5&1\\0&\boxed{7}&-1&5/2\\0&2&1&1\end{vmatrix}$$

$$=\frac{7}{2}\begin{vmatrix}-1&2&0&1\\0&2&5&1\\0&-2&2/7&-5/7\\0&-2&-1&-1\end{vmatrix}=\frac{7}{2}\begin{vmatrix}-1&2&0&1\\0&2&5&1\\0&0&37/7&2/7\\0&0&4&0\end{vmatrix}=-\frac{7}{2}\begin{vmatrix}-1&2&1&0\\0&2&1&5\\0&0&2/7&37/7\\0&0&0&4\end{vmatrix}$$

$$=-\frac{7}{2}(-1)2\frac{2}{7}4=8$$

(d)

$$\boldsymbol{A}=\begin{pmatrix}\sin\alpha&-\cos\alpha\\\cos\alpha&\sin\alpha\end{pmatrix},\quad |\boldsymbol{A}|=\begin{vmatrix}\sin\alpha&-\cos\alpha\\\cos\alpha&\sin\alpha\end{vmatrix}=\sin^2\alpha+\cos^2\alpha=1$$

(e)

$$\boldsymbol{A}=\begin{pmatrix}1&2&3\\3&1&2\\2&3&1\end{pmatrix},\quad |\boldsymbol{A}|=\begin{vmatrix}1&2&3\\3&1&2\\2&3&1\end{vmatrix}\begin{matrix}1&2\\3&1\\2&3\end{matrix}=(1+8+27)-(6+6+6)=36-18=18$$