Maticové operácie

Klasické násobenie

C=A.B
A,B,C - matice 2×2

$$\begin{pmatrix} c_{1,1} & c_{1,2} \\ c_{2,1} & c_{2,2} \end{pmatrix} = \begin{pmatrix} a_{1,1} & a_{1,2} \\ a_{2,1} & a_{2,2} \end{pmatrix} . \begin{pmatrix} b_{1,1} & b_{1,2} \\ b_{2,1} & b_{2,2} \end{pmatrix}$$

$$c_{1,1} = a_{1,1}.b_{1,1} + a_{1,2}.b_{2,1}$$

$$c_{1,2} = a_{1,1}.b_{1,2} + a_{1,2}.b_{2,2}$$

$$c_{2,1} = a_{2,1}.b_{1,1} + a_{2,2}.b_{2,1}$$

$$c_{2,2} = a_{2,1}.b_{1,2} + a_{2,2}.b_{2,2}$$

$$c_{i,j} = \sum_{k=1}^{2} a_{i,k}.b_{k,j}, \quad i,j = 1,2.$$

Klasické násobenie

$$c_{1,1} = a_{1,1}.b_{1,1} + a_{1,2}.b_{2,1}$$

$$c_{1,2} = a_{1,1}.b_{1,2} + a_{1,2}.b_{2,2}$$

$$c_{2,1} = a_{2,1}.b_{1,1} + a_{2,2}.b_{2,1}$$

$$c_{2,2} = a_{2,1}.b_{1,2} + a_{2,2}.b_{2,2}$$

8 súčinov

4 súčty

Klasické násobenie

C=A.B
A,B,C - matice 4×4

$$\begin{pmatrix} c_{1,1} & c_{1,2} & c_{1,3} & c_{1,4} \\ c_{2,1} & c_{2,2} & c_{2,3} & c_{2,4} \\ c_{3,1} & c_{3,2} & c_{3,3} & c_{3,4} \\ c_{4,1} & c_{4,2} & c_{4,3} & c_{4,4} \end{pmatrix} =$$

$$= \begin{pmatrix} a_{1,1} & a_{1,2} & a_{1,3} & a_{1,4} \\ a_{2,1} & a_{2,2} & a_{2,3} & a_{2,4} \\ a_{3,1} & a_{3,2} & a_{3,3} & a_{3,4} \\ a_{4,1} & a_{4,2} & a_{4,3} & a_{4,4} \end{pmatrix} \cdot \begin{pmatrix} b_{1,1} & b_{1,2} & b_{1,3} & b_{1,4} \\ b_{2,1} & b_{2,2} & b_{2,3} & b_{2,4} \\ b_{3,1} & b_{3,2} & b_{3,3} & b_{3,4} \\ b_{4,1} & b_{4,2} & b_{4,3} & b_{4,4} \end{pmatrix}$$

$$c_{i,j} = \sum_{k=1}^{4} a_{i,k}.b_{k,j}, \quad i,j = 1,2,3,4.$$

Násobenie po blokoch

C=A.B

A,B,C - matice $2n \times 2n$

$A_{i,j}, B_{i,j}, C_{i,j},\ i,j \in \{1,2\}$ - matice $n \times n$

$$\begin{pmatrix} C_{1,1} & C_{1,2} \\ C_{2,1} & C_{2,2} \end{pmatrix} = \begin{pmatrix} A_{1,1} & A_{1,2} \\ A_{2,1} & A_{2,2} \end{pmatrix} . \begin{pmatrix} B_{1,1} & B_{1,2} \\ B_{2,1} & B_{2,2} \end{pmatrix}$$

$$C_{1,1} = A_{1,1}.B_{1,1} + A_{1,2}.B_{2,1}$$

$$C_{1,2} = A_{1,1}.B_{1,2} + A_{1,2}.B_{2,2}$$

$$C_{2,1} = A_{2,1}.B_{1,1} + A_{2,2}.B_{2,1}$$

$$C_{2,2} = A_{2,1}.B_{1,2} + A_{2,2}.B_{2,2}$$

Násobenie komplexných čísel

$$(a + bi).(c + di) = (ac - bd) + i(ad + bc).$$

4 súčiny, 2 súčty/rozdiely.

Gauss:

$$(ad + bc) = (a + b).(c + d) - ac - bd.$$

3 súčiny, 5 súčtov/rozdielov.

Rýchle násobenie matíc

Strassen
C=A.B
A,B,C - matice $N \times N = 2^n \times 2^n$
ak treba, doplnia sa matice A a B nulami na rozmer $2^n \times 2^n$

Rýchle násobenie matíc

$$\begin{pmatrix} C_{1,1} & C_{1,2} \\ C_{2,1} & C_{2,2} \end{pmatrix} = \begin{pmatrix} A_{1,1} & A_{1,2} \\ A_{2,1} & A_{2,2} \end{pmatrix} \cdot \begin{pmatrix} B_{1,1} & B_{1,2} \\ B_{2,1} & B_{2,2} \end{pmatrix}$$

Princíp: zrýchliť násobenie po blokoch.

Rýchle násobenie matíc

$$I = (A_{1,1} - A_{1,2}).(B_{2,1} + B_{2,2})$$
$$II = (A_{1,1} + A_{1,2}).(B_{1,1} + B_{2,2})$$
$$III = (A_{1,1} - A_{2,1}).(B_{1,1} + B_{1,2})$$
$$IV = (A_{1,1} + A_{1,2}).B_{2,2}$$
$$V = A_{1,1}.(B_{1,2} - B_{2,2})$$
$$VI = A_{2,2}.(B_{2,1} - B_{1,1})$$
$$VII = (A_{2,1} + A_{2,2}).B_{1,1}$$

$$C_{1,1} = I + II - IV + VI$$
$$C_{1,2} = IV + V$$
$$C_{2,1} = VI + VII$$
$$C_{2,2} = II - III + V - VII$$

rekurzívny postup

7 súčinov, 18 súčtov/rozdielov

Rýchle násobenie matíc

```
matica MatProd(matica A, matica B, int N)
{
  // N je rozmer matice, vráti A.B
  if (N == 1)
    return A.B;
  else
  {
    rozlož matice A a B
```

$$A = \begin{pmatrix} A_{1,1} & A_{1,2} \\ A_{2,1} & A_{2,2} \end{pmatrix}, B = \begin{pmatrix} B_{1,1} & B_{1,2} \\ B_{2,1} & B_{2,2} \end{pmatrix}$$

$I = \texttt{MatProd}((A_{1,1} - A_{1,2}), (B_{2,1} + B_{2,2}), N/2);$
$II = \texttt{MatProd}(A_{1,1} + A_{1,2}), (B_{1,1} + B_{2,2}), N/2);$
$III = \texttt{MatProd}((A_{1,1} - A_{2,1}), (B_{1,1} + B_{1,2}), N/2);$

Rýchle násobenie matíc

$$
\begin{aligned}
&IV = \texttt{MatProd}((A_{1,1} + A_{1,2}), B_{2,2}, N/2);\\
&V = \texttt{MatProd}(A_{1,1}, (B_{1,2} - B_{2,2}), N/2);\\
&VI = \texttt{MatProd}(A_{2,2}, (B_{2,1} - B_{1,1}), N/2);\\
&VII = \texttt{MatProd}((A_{2,1} + A_{2,2}), B_{1,1}, N/2);\\
&C_{1,1} = I + II - IV + VI;\\
&C_{1,2} = IV + V;\\
&C_{2,1} = VI + VII;\\
&C_{2,2} = II - III + V - VII;\\
&\texttt{return} \begin{pmatrix} C_{1,1} & C_{1,2} \\ C_{2,1} & C_{2,2} \end{pmatrix};
\end{aligned}
$$

}
}

Rýchle násobenie matíc - zložitosť

7 krát rekurzívne volanie funkcie MatProd
18 súčtov/rozdielov
$f(n)$ - počet násobení čísel pri rozmere $N \times N, N = 2^n$
$g(n)$ - počet súčtov/rozdielov čísel pri rozmere $N \times N, N = 2^n$

$f(0) = 1$
$f(n) = 7f(n-1), \quad n \geq 1$
$f(n) = 7^n = N^{2,81}$

$g(n) = 7g(n-1) + 18.2^{n-1}.2^{n-1}$
po vyriešení rekurencie a úprave
$g(n) = \mathcal{O}(N^{2,81})$