Integer Arithmetic

Computer Organization and Assembly Languages

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with slides by Kip Irvine

Announcements

- Assignment #2 is due next week.
- Assignment #3 box filter is online now, due on 12/4.

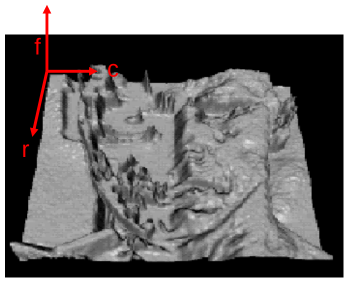
Assignment #3 Box Filter

Assignment #3 Box Filter

Assignment #3 Box Filter

What is an image

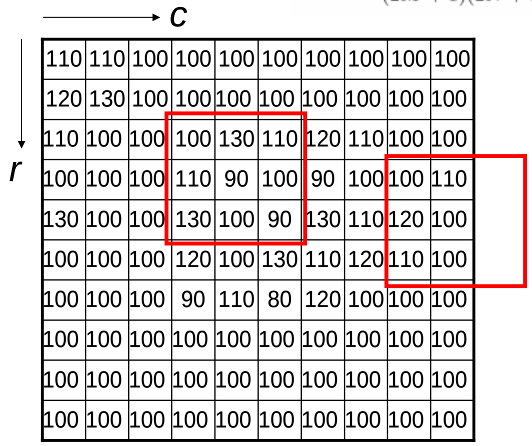
- We can think of an **image** as a function, f: $R^2 \rightarrow R$:
  - $f(r, c)$ gives the **intensity** at position (r, c)
  - defined over a rectangle, with a finite range:
    - f: $[0,h-1] \times [0,w-1] \rightarrow [0,255]$



A digital image

- The image can be represented as a matrix of integer values

$$k(r,c) = \frac{1}{(2M+1)(2N+1)} \sum_{r'=-M}^{M} \sum_{c'=-N}^{N} f(r+r', c+c')$$



110	110	100	100	100	100	100	100	100	100
120	130	100	100	100	100	100	100	100	100
110	100	100	100	130	110	120	110	100	100
100	100	100	110	90	100	90	100	100	110
130	100	100	130	100	90	130	110	120	100
100	100	100	120	100	130	110	120	110	100
100	100	100	90	110	80	120	100	100	100
100	100	100	100	100	100	100	100	100	100
100	100	100	100	100	100	100	100	100	100
100	100	100	100	100	100	100	100	100	100

Assignment #3 Box Filter

```
unsigned int c_blur(unsigned char *img_in,
  unsigned char *img_out, int knl_size, int w, int h)
{
  for each row r
    for each column c
      calculate k(r, c)
      save it
}
```

- Memory layout
- Only integer arithmetic operations
- MD5/CRC32 checksum

Assignment #3 Box Filter

```
for (int i = 0; i < height; i++) {
   for (int j = 0; j < width; j++) {
      pixel = 0;  pixel_num = 0;
      for (int y=-knl_size/2; y<=knl_size/2; y++) {
        for (int x=-knl_size/2; x<=knl_size/2; x++) {
          int x_s = j + x;
          int y_s = i + y;
          /* make sure that it's in the image */
          if (x_s>=0 && x_s<w && y_s>=0 && y_s<h) {
            pixel += img_in[y_s * w + x_s];
            pixel_num++;
          }
        }
      }
      pixel = pixel / pixel_num;
      img_out[i * width + j] = (unsigned char)pixel;
   }
}
```

Chapter 7 Integer Arithmetic Overview

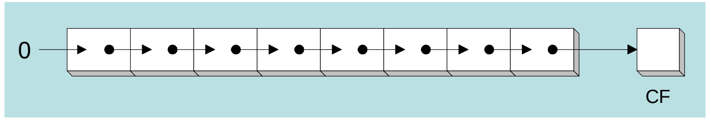
- Shift and Rotate Instructions
- Shift and Rotate Applications
- Multiplication and Division Instructions
- Extended Addition and Subtraction
- ASCII and Packed Decimal Arithmetic

Shift and Rotate Instructions

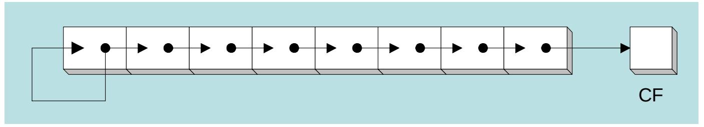
- Logical vs Arithmetic Shifts
- SHL Instruction
- SHR Instruction
- SAL and SAR Instructions
- ROL Instruction
- ROR Instruction
- RCL and RCR Instructions
- SHLD/SHRD Instructions

Logical vs arithmetic shifts

- A logical shift fills the newly created bit position with zero:

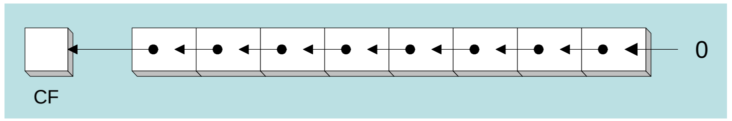


- An arithmetic shift fills the newly created bit position with a copy of the number’s sign bit:



SHL instruction

- The SHL (shift left) instruction performs a logical left shift on the destination operand, filling the lowest bit with 0.



- Operand types: **SHL *destination,count***

```
SHL reg,imm8
SHL mem,imm8
SHL reg,CL
SHL mem,CL
```

Fast multiplication

Shifting left 1 bit multiplies a number by 2

```
mov dl,5
shl dl,1
```

Before: 0 0 0 0 0 1 0 1 = 5

After: 0 0 0 0 1 0 1 0 = 10

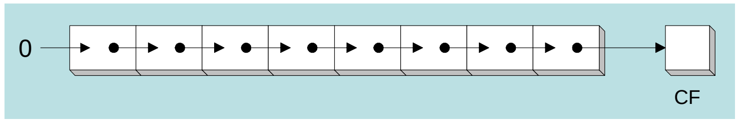
Shifting left *n* bits multiplies the operand by 2^n

For example, $5 * 2^2 = 20$

```
mov dl,5
shl dl,2             ; DL = 20
```

SHR instruction

- The SHR (shift right) instruction performs a logical right shift on the destination operand. The highest bit position is filled with a zero.

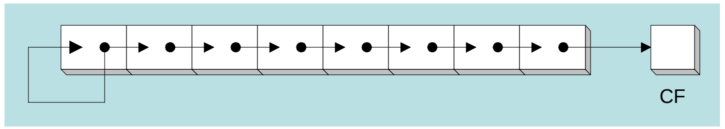


Shifting right n bits divides the operand by 2^n

```
mov dl,80
shr dl,1             ; DL = 40
shr dl,2             ; DL = 10
```

SAL and SAR instructions

- SAL (shift arithmetic left) is identical to SHL.
- SAR (shift arithmetic right) performs a right arithmetic shift on the destination operand.

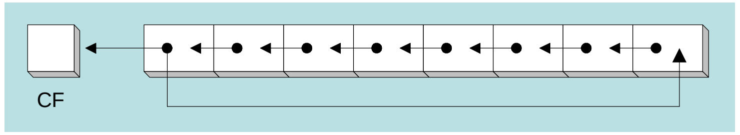


An arithmetic shift preserves the number's sign.

```
mov dl,-80
sar dl,1             ; DL = -40
sar dl,2             ; DL = -10
```

ROL instruction

- ROL (rotate) shifts each bit to the left
- The highest bit is copied into both the Carry flag and into the lowest bit
- No bits are lost

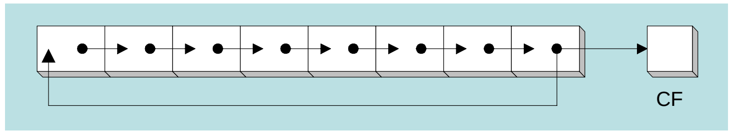


```
mov al,11110000b
rol al,1                ; AL = 11100001b

mov dl,3Fh
rol dl,4                ; DL = F3h
```

ROR instruction

- ROR (rotate right) shifts each bit to the right
- The lowest bit is copied into both the Carry flag and into the highest bit
- No bits are lost



```
mov al,11110000b
ror al,1                ; AL = 01111000b

mov dl,3Fh
ror dl,4                ; DL = F3h
```

Your turn . . .

Indicate the hexadecimal value of AL after each shift:

```
mov al,6Bh
shr al,1                a. 35h
shl al,3                b. A8h
mov al,8Ch
sar al,1                c. C6h
sar al,3                d. F8h
```

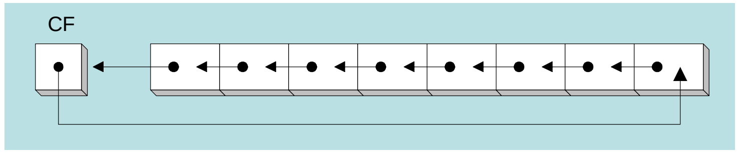
Your turn . . .

Indicate the hexadecimal value of AL after each rotation:

```
mov al,6Bh
ror al,1                a. B5h
rol al,3                b. ADh
```

RCL instruction

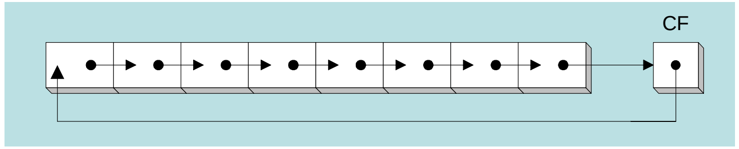
- RCL (rotate carry left) shifts each bit to the left
- Copies the Carry flag to the least significant bit
- Copies the most significant bit to the Carry flag



```
clc                  ; CF = 0
mov bl,88h           ; CF,BL = 0 10001000b
rcl bl,1             ; CF,BL = 1 00010000b
rcl bl,1             ; CF,BL = 0 00100001b
```

RCR instruction

- RCR (rotate carry right) shifts each bit to the right
- Copies the Carry flag to the most significant bit
- Copies the least significant bit to the Carry flag



```
stc                  ; CF = 1
mov ah,10h           ; CF,AH = 00010000 1
rcr ah,1             ; CF,AH = 10001000 0
```

Your turn . . .

Indicate the hexadecimal value of AL after each rotation:

```
stc
mov al,6Bh
rcr al,1                a. B5h
rcl al,3                b. AEh
```

SHLD instruction

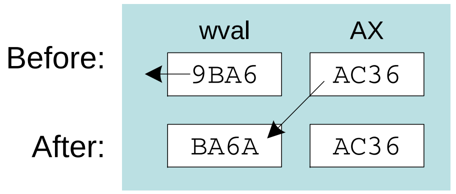
- Syntax:

  SHLD *destination, source, count*
- Shifts a destination operand a given number of bits to the left
- The bit positions opened up by the shift are filled by the most significant bits of the source operand
- The source operand is not affected

SHLD example

Shift wval 4 bits to the left and replace its lowest 4 bits with the high 4 bits of AX:

```
.data
wval WORD 9BA6h
.code
mov  ax,0AC36h
shld wval,ax,4
```



SHRD instruction

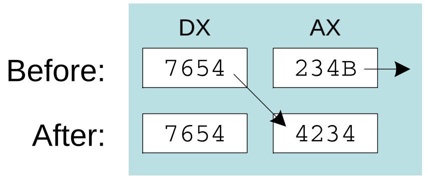
- Syntax:

  SHRD *destination, source, count*
- Shifts a destination operand a given number of bits to the right
- The bit positions opened up by the shift are filled by the least significant bits of the source operand
- The source operand is not affected

SHRD example

Shift AX 4 bits to the right and replace its highest 4 bits with the low 4 bits of DX:

```
mov  ax,234Bh
mov  dx,7654h
shrd ax,dx,4
```



Your turn . . .

Indicate the hexadecimal values of each destination operand:

```
mov  ax,7C36h
mov  dx,9FA6h
shld dx,ax,4       ; DX = FA67h
shrd dx,ax,8       ; DX = 36FAh
```

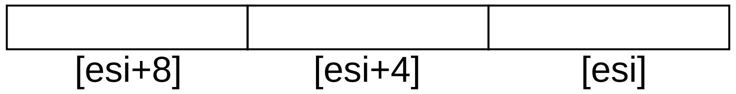
Shift and rotate applications

- Shifting Multiple Doublewords
- Binary Multiplication
- Displaying Binary Bits
- Isolating a Bit String

Shifting multiple doublewords

- Programs sometimes need to shift all bits within an array, as one might when moving a bitmapped graphic image from one screen location to another.
- The following shifts an array of 3 doublewords 1 bit to the right:

```
mov esi,0
shr array[esi + 8],1 ; high dword
rcr array[esi + 4],1 ; middle dword,
rcr array[esi],1     ; low dword,
```



Binary multiplication

- We already know that SHL performs unsigned multiplication efficiently when the multiplier is a power of 2.
- Factor any binary number into powers of 2.
  - For example, to multiply EAX * 36, factor 36 into 32 + 4 and use the distributive property of multiplication to carry out the operation:

```
EAX * 36
= EAX * (32 + 4)
= (EAX * 32)+(EAX * 4)
```

```
mov eax,123
mov ebx,eax
shl eax,5
shl ebx,2
add eax,ebx
```

Your turn . . .

Multiply AX by 26, using shifting and addition instructions. *Hint:* 26 = 16 + 8 + 2.

```
mov ax,2             ; test value

mov dx,ax
shl dx,4             ; AX * 16
push dx              ; save for later
mov dx,ax
shl dx,3             ; AX * 8
shl ax,1             ; AX * 2
add ax,dx            ; AX * 10
pop dx               ; recall AX * 16
add ax,dx            ; AX * 26
```

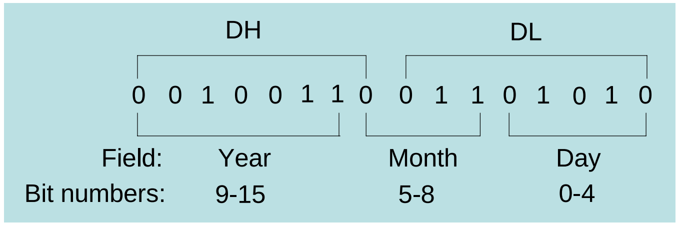
Displaying binary bits

Algorithm: Shift MSB into the Carry flag; If CF = 1, append a "1" character to a string; otherwise, append a "0" character. Repeat in a loop, 32 times.

```
    mov ecx,32
    mov esi,offset buffer
L1: shl eax,1
    mov BYTE PTR [esi],'0'
    jnc L2
    mov BYTE PTR [esi],'1'
L2: inc esi
    loop L1
```

Isolating a bit string

- The MS-DOS file date field packs the year (relative to 1980), month, and day into 16 bits:



Isolating a bit string

```
mov al,dl        ; make a copy of DL
and al,00011111b ; clear bits 5-7
mov day,al       ; save in day variable
```

```
mov ax,dx        ; make a copy of DX
shr ax,5         ; shift right 5 bits
and al,00001111b ; clear bits 4-7
mov month,al     ; save in month variable
```

```
mov al,dh        ; make a copy of DX
shr al,1         ; shift right 1 bit
mov ah,0         ; clear AH to 0
add ax,1980      ; year is relative to 1980
mov year,ax      ; save in year
```

Multiplication and division instructions

- MUL Instruction
- IMUL Instruction
- DIV Instruction
- Signed Integer Division
- Implementing Arithmetic Expressions

MUL instruction

- The MUL (unsigned multiply) instruction multiplies an 8-, 16-, or 32-bit operand by either AL, AX, or EAX.
- The instruction formats are:

```
MUL r/m8
MUL r/m16
MUL r/m32
```

Implied operands:

Multiplicand	Multiplier	Product
AL	*r/m8*	AX
AX	*r/m16*	DX:AX
EAX	*r/m32*	EDX:EAX

MUL examples

100h * 2000h, using 16-bit operands:

```
.data
val1 WORD 2000h
val2 WORD 100h
.code
mov ax,val1
mul val2  ; DX:AX=00200000h, CF=1
```

The Carry flag indicates whether or not the upper half of the product contains significant digits.

12345h * 1000h, using 32-bit operands:

```
mov eax,12345h
mov ebx,1000h
mul ebx   ; EDX:EAX=0000000012345000h, CF=0
```

Your turn . . .

What will be the hexadecimal values of (E)DX, (E)AX, and the Carry flag after the following instructions execute?

```
mov ax,1234h
mov bx,100h
mul bx
```

DX = 0012h, AX = 3400h, CF = 1

```
mov eax,00128765h
mov ecx,10000h
mul ecx
```

EDX = 00000012h, EAX = 87650000h, CF = 1

IMUL instruction

- IMUL (signed integer multiply) multiplies an 8-, 16-, or 32-bit signed operand by either AL, AX, or EAX
- Preserves the sign of the product by sign-extending it into the upper half of the destination register

Example: multiply 48 * 4, using 8-bit operands:

```
mov  al,48
mov  bl,4
imul bl     ; AX = 00C0h, OF=1
```

OF=1 because AH is not a sign extension of AL.

DIV instruction

- The DIV (unsigned divide) instruction performs 8-bit, 16-bit, and 32-bit division on unsigned integers
- A single operand is supplied (register or memory operand), which is assumed to be the divisor
- Instruction formats:

DIV *r/m8*

DIV *r/m16*

DIV *r/m32*

Default Operands:

Dividend	Divisor	Quotient	Remainder
AX	*r/m8*	AL	AH
DX:AX	*r/m16*	AX	DX
EDX:EAX	*r/m32*	EAX	EDX

DIV examples

Divide 8003h by 100h, using 16-bit operands:

```
mov dx,0          ; clear dividend, high
mov ax,8003h      ; dividend, low
mov cx,100h       ; divisor
div cx            ; AX = 0080h, DX = 3
```

Same division, using 32-bit operands:

```
mov edx,0         ; clear dividend, high
mov eax,8003h     ; dividend, low
mov ecx,100h      ; divisor
div ecx           ; EAX=00000080h,DX= 3
```

Your turn . . .

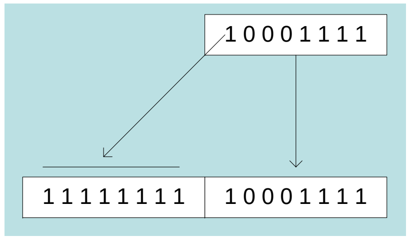
What will be the hexadecimal values of DX and AX after the following instructions execute?

```
mov dx,0087h
mov ax,6000h
mov bx,100h
div bx
```

DX = 0000h, AX = 8760h

Signed integer division

- Signed integers must be sign-extended before division takes place
  - fill high byte/word/doubleword with a copy of the low byte/word/doubleword's sign bit
- For example, the high byte contains a copy of the sign bit from the low byte:



CBW, CWD, CDQ instructions

- The CBW, CWD, and CDQ instructions provide important sign-extension operations:
  - CBW (convert byte to word) extends AL into AH
  - CWD (convert word to doubleword) extends AX into DX
  - CDQ (convert doubleword to quadword) extends EAX into EDX
- For example:

```
mov eax,0FFFFFF9Bh      ; -101 (32 bits)
cdq         ; EDX:EAX = FFFFFFFFFFFFFF9Bh
            ; -101 (64 bits)
```

IDIV instruction

- IDIV (signed divide) performs signed integer division
- Uses same operands as DIV

Example: 8-bit division of –48 by 5

```
mov al,-48
cbw            ; extend AL into AH
mov bl,5
idiv bl        ; AL = -9,  AH = -3
```

IDIV examples

Example: 16-bit division of –48 by 5

```
mov  ax,-48
cwd              ; extend AX into DX
mov  bx,5
idiv bx      ; AX = -9,  DX = -3
```

Example: 32-bit division of –48 by 5

```
mov  eax,-48
cdq         ; extend EAX into EDX
mov  ebx,5
idiv ebx    ; EAX = -9,  EDX = -3
```

Your turn . . .

What will be the hexadecimal values of DX and AX after the following instructions execute?

```
mov ax,0FDFFh       ; -513
cwd
mov bx,100h
idiv bx
```

DX = FFFFh (–1), AX = FFFEh (–2)

Divide overflow

- *Divide overflow* happens when the quotient is too large to fit into the destination.

```
mov ax, 1000h
mov bl, 10h
div bl
```

It causes a CPU interrupt and halts the program. (divided by zero cause similar results)

Implementing arithmetic expressions

- Some good reasons to learn how to implement expressions:
  - Learn how compilers do it
  - Test your understanding of MUL, IMUL, DIV, and IDIV
  - Check for 32-bit overflow

Example: **var4 = (var1 + var2) * var3**

```
mov eax,var1
add eax,var2
mul var3
jo  TooBig      ; check for overflow
mov var4,eax    ; save product
```

Implementing arithmetic expressions

Example: **eax = (-var1 * var2) + var3**

```
mov eax,var1
neg eax
mul var2
jo  TooBig      ; check for overflow
add eax,var3
```

Example: **var4 = (var1 * 5) / (var2 – 3)**

```
mov eax,var1      ; left side
mov ebx,5
mul ebx           ; EDX:EAX = product
mov ebx,var2      ; right side
sub ebx,3
div ebx           ; final division
mov var4,eax
```

Implementing arithmetic expressions

Example: **var4 = (var1 * -5) / (-var2 % var3);**

```
mov  eax,var2     ; begin right side
neg  eax
cdq               ; sign-extend dividend
idiv var3         ; EDX = remainder
mov  ebx,edx      ; EBX = right side
mov  eax,-5       ; begin left side
imul var1         ; EDX:EAX = left side
idiv ebx          ; final division
mov  var4,eax     ; quotient
```

Sometimes it's easiest to calculate the right-hand term of an expression first.

Your turn . . .

Implement the following expression using signed 32-bit integers:

```
eax = (ebx * 20) / ecx
```

```
mov eax,20
mul ebx
div ecx
```

Your turn . . .

Implement the following expression using signed 32-bit integers. Save and restore ECX and EDX:

```
eax = (ecx * edx) / eax
```

```
push ecx
push edx
push eax            ; EAX needed later
mov  eax,ecx
mul  edx            ; left side: EDX:EAX
pop  ecx            ; saved value of EAX
div  ecx            ; EAX = quotient
pop  edx            ; restore EDX, ECX
pop  ecx
```

Your turn . . .

Implement the following expression using signed 32-bit integers. Do not modify any variables other than var3:

```
var3 = (var1 * -var2) / (var3 – ebx)
```

```
mov eax,var1
mov edx,var2
neg edx
mul edx        ; left side: edx:eax
mov ecx,var3
sub ecx,ebx
div ecx        ; eax = quotient
mov var3,eax
```

Extended addition and subtraction

- ADC Instruction
- Extended Addition Example
- SBB Instruction

ADC instruction

- ADC (add with carry) instruction adds both a source operand and the contents of the Carry flag to a destination operand.
- Example: Add two 32-bit integers (FFFFFFFFh + FFFFFFFFh), producing a 64-bit sum:

```
mov edx,0
mov eax,0FFFFFFFFh
add eax,0FFFFFFFFh
adc edx,0 ;EDX:EAX = 00000001FFFFFFFEh
```

Extended addition example

- Add two integers of any size
- Pass pointers to the addends and sum
- ECX indicates the number of words

```
L1:
  mov eax,[esi] ; get the first integer
  adc eax,[edi] ; add the second integer
  pushfd        ; save the Carry flag
  mov [ebx],eax ; store partial sum
  add esi,4     ; advance all 3 pointers
  add edi,4
  add ebx,4
  popfd         ; restore the Carry flag
  loop L1       ; repeat the loop
  adc word ptr [ebx],0 ; add leftover carry
```

Extended addition example

```
.data
op1 QWORD 0A2B2A40674981234h
op2 QWORD 08010870000234502h
sum DWORD 3 dup(?)
    ; = 0000000122C32B0674BB5736
.code
...
mov  esi,OFFSET op1 ; first operand
mov  edi,OFFSET op2 ; second operand
mov  ebx,OFFSET sum ; sum operand
mov  ecx,2          ; number of doublewords
call Extended_Add
...
```

SBB instruction

- The SBB (subtract with borrow) instruction subtracts both a source operand and the value of the Carry flag from a destination operand.
- The following example code performs 64-bit subtraction. It sets EDX:EAX to 0000000100000000h and subtracts 1 from this value. The lower 32 bits are subtracted first, setting the Carry flag. Then the upper 32 bits are subtracted, including the Carry flag:

```
mov edx,1        ; upper half
mov eax,0        ; lower half
sub eax,1        ; subtract 1
sbb edx,0        ; subtract upper half
```