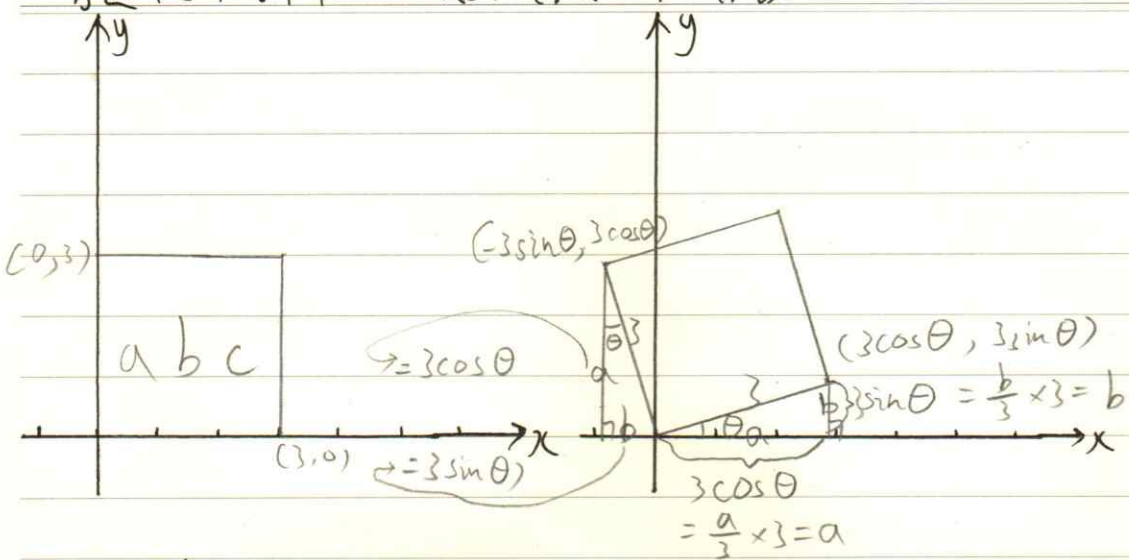
旋转矩阵 Rotation Matrix



$$\begin{bmatrix} x' \\ y' \end{bmatrix} = \begin{bmatrix} A & B \\ C & D \end{bmatrix} \times \begin{bmatrix} x \\ y \end{bmatrix}$$

$$\begin{bmatrix} 3\cos\theta \\ 3\sin\theta \end{bmatrix} = \begin{bmatrix} A & B \\ C & D \end{bmatrix} \times \begin{bmatrix} 3 \\ 0 \end{bmatrix}$$

$$\begin{bmatrix} -3\sin\theta \\ 3\cos\theta \end{bmatrix} = \begin{bmatrix} A & B \\ C & D \end{bmatrix} \times \begin{bmatrix} 0 \\ 3 \end{bmatrix}$$

$$\therefore \begin{cases} 3\cos\theta = A\times 3 + B\times 0 & ① \\ -3\sin\theta = A\times 0 + B\times 3 & ② \end{cases} \qquad \therefore \begin{cases} 3\sin\theta = C\times 3 + D\times 0 & ④ \\ 3\cos\theta = C\times 0 + D\times 3 & ⑤ \end{cases}$$

由①得：$3\cos\theta = 3A$

$A = \cos\theta$ ③

由④得：$3\sin\theta = 3C$

$C = \sin\theta$

③⇒②：$-3\sin\theta = 3B$

$B = -\sin\theta$

④→⑤：$3\cos\theta = 3D$

$D = \cos\theta$

$$\therefore \begin{cases} A = \cos\theta \\ B = -\sin\theta \end{cases} \qquad \therefore \begin{cases} C = \sin\theta \\ D = \cos\theta \end{cases}$$

∴旋转矩阵为 $\begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix}$，推广后 $\begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix}$。