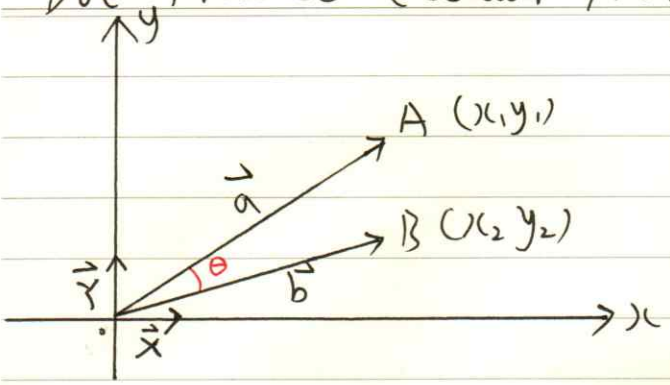
Dot Product (Scalar Product) 点积 (代数证明)



定义: $\vec{a} \cdot \vec{b} = \|\vec{a}\| \cdot \|\vec{b}\| \cdot \cos\theta$

解: 设 x 轴单位向量为 $\vec{X}$, y 轴单位向量为 $\vec{Y}$。

$\therefore \vec{a} = x_1\vec{X} + y_1\vec{Y}$; $\vec{b} = x_2\vec{X} + y_2\vec{Y}$

$\therefore \vec{a} \cdot \vec{b} = (x_1\vec{X} + y_1\vec{Y}) \cdot (x_2\vec{X} + y_2\vec{Y})$

$= x_1x_2\vec{X}^2 + y_1x_2\vec{X}\vec{Y} + x_1y_2\vec{X}\vec{Y} + y_1y_2\vec{Y}^2$

$\because \vec{X}, \vec{Y}$ 垂直,

$\therefore \vec{X} \cdot \vec{Y} = 1 \times 1 \times \cos 90^\circ = 0$,

$\therefore$ 原式 $= x_1x_2\vec{X}^2 + 0 + 0 + y_1y_2\vec{Y}^2$

又: $\vec{X}, \vec{Y}$ 是单位向量,

$\therefore \vec{X}^2 = \vec{X} \cdot \vec{X} = 1 \times 1 \times \cos 0^\circ = 1$

$\vec{Y}^2 = \vec{Y} \cdot \vec{Y} = 1 \times 1 \times \cos 0^\circ = 1$

$\therefore$ 原式 $= x_1 \cdot x_2 \cdot 1 + 0 + 0 + y_1y_2 \times 1$

$= x_1x_2 + y_1y_2$。

$\therefore$ 定义2: $\vec{a} \cdot \vec{b} = x_ax_b + y_ay_b$