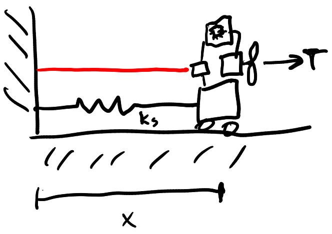
Longitudinal Control

State Space Review



$k_s \Leftrightarrow C_{m_\alpha}$

$T_0 = 0$

$T = \Delta T$

$$^{s}f = -k_s(x - x_0)$$

$$m\ddot{x} = -k_s(x - x_0)$$

$$m\Delta\ddot{x} = -k_s \Delta x + \Delta T$$

$\vec{u} = [\Delta T]$

ΔX_E, ΔY_E, ΔZ_E ; u, v, w

$$\rightarrow \quad \dot{\vec{x}} = \begin{bmatrix} \Delta\dot{x} \\ \Delta\ddot{x} \end{bmatrix} = \overbrace{\begin{bmatrix} 0 & 1 \\ -\frac{k_s}{m} & 0 \end{bmatrix}}^{A} \overbrace{\begin{bmatrix} \Delta x \\ \Delta\dot{x} \end{bmatrix}}^{\vec{x}} + \overbrace{\begin{bmatrix} 0 \\ \frac{1}{m} \end{bmatrix}}^{B} \vec{u}$$

$$\Delta x = \overbrace{\begin{bmatrix} 1 & 0 \end{bmatrix}}^{C} \begin{bmatrix} \Delta x \\ \Delta\dot{x} \end{bmatrix} + \overbrace{[0]}^{D} \vec{u}$$

$$|A - \lambda I| = 0 = \begin{vmatrix} -\lambda & 1 \\ -\frac{k_s}{m} & -\lambda \end{vmatrix} = \lambda^2 + \frac{k_s}{m} = 0$$

$$\lambda = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \sqrt{\frac{k_s}{m}}\, i$$

$$\lambda^2 + \underbrace{2\zeta\omega_n}_{0}\lambda + \underbrace{\omega_n^2}_{\frac{k_s}{m}} = 0$$

$$\Delta T = -k_p \Delta x - k_d \Delta\dot{x}$$

$$\vec{u} = -K\vec{x} \qquad K = [k_p \quad k_d] \qquad \vec{u} = [-k_p \quad -k_d]\vec{x}$$

$$\dot{\vec{x}} = A\vec{x} - BK\vec{x}$$

$$\dot{\vec{x}} = \underbrace{(A - BK)}_{A^{cl}}\vec{x}$$

$$A^{cl} = \begin{bmatrix} 0 & 1 \\ -\frac{(k_s + k_p)}{m} & -\frac{k_d}{m} \end{bmatrix}$$

Longitudinal Control

Dynamics of Flight, Eq. (4.9,18) $\qquad \dot{\mathbf{x}}_{lon} = \mathbf{A}_{lon}\mathbf{x}_{lon} + \mathbf{c}_{lon}$

$$\mathbf{x}_{lon} = \begin{pmatrix} \Delta u \\ \Delta w \\ \Delta q \\ \Delta\theta \end{pmatrix} \qquad \mathbf{c}_{lon} = \begin{pmatrix} \dfrac{\Delta X_c}{m} \\ \dfrac{\Delta Z_c}{m - Z_{\dot{w}}} \\ \dfrac{\Delta M_c}{I_y} + \dfrac{M_{\dot{w}}}{I_y}\dfrac{\Delta Z_c}{(m - Z_{\dot{w}})} \\ 0 \end{pmatrix}$$

$$\mathbf{A}_{lon} = \begin{pmatrix} \dfrac{X_u}{m} & \dfrac{X_w}{m} & 0 & -g\cos\theta_0 \\ \dfrac{Z_u}{m - Z_{\dot{w}}} & \dfrac{Z_w}{m - Z_{\dot{w}}} & \dfrac{Z_q + mu_0}{m - Z_{\dot{w}}} & \dfrac{-mg\sin\theta_0}{m - Z_{\dot{w}}} \\ \dfrac{1}{I_y}\left[M_u + \dfrac{M_{\dot{w}}Z_u}{m - Z_{\dot{w}}}\right] & \dfrac{1}{I_y}\left[M_w + \dfrac{M_{\dot{w}}Z_w}{m - Z_{\dot{w}}}\right] & \dfrac{1}{I_y}\left[M_q + \dfrac{M_{\dot{w}}(Z_q + mu_0)}{m - Z_{\dot{w}}}\right] & \dfrac{-M_{\dot{w}}mg\sin\theta_0}{I_y(m - Z_{\dot{w}})} \\ 0 & 0 & 1 & 0 \end{pmatrix}$$

δ_e = elevator + downward deflection

δ_t = throttle + more power

δ_p in book

dimensional control derivatives

$$\Delta X_c = X_{\delta_e}\delta_e + X_{\delta_t}\delta_t$$

$$\Delta Z_c = Z_{\delta_e}\delta_e + \boxed{Z_{\delta_t}}\delta_t \qquad \text{often zero}$$

$$\Delta M_c = M_{\delta_e}\delta_e + \boxed{M_{\delta_t}}\delta_t$$

$$\dot{\vec{x}}_{lon} = A_{lon}\vec{x}_{lon} + B_{lon}\vec{u}_{lon}$$

$$B_{lon} = \begin{bmatrix} \frac{X_{\delta_e}}{m} & \frac{X_{\delta_t}}{m} \\ \frac{Z_{\delta_e}}{m-Z_{\dot w}} & \frac{Z_{\delta_t}}{m-Z_{\dot w}} \\ \frac{M_{\delta_e}}{I_y}+\frac{M_{\dot w}}{I_y}\frac{Z_{\delta_e}}{(m-Z_{\dot w})} & \frac{M_{\delta_t}}{I_y}+\frac{M_{\dot w}Z_{\delta_e}}{I_y(m-Z_{\dot w})} \\ 0 & 0 \end{bmatrix} \qquad \vec{u}_{lon} = \begin{bmatrix} \delta_e \\ \delta_t \end{bmatrix}$$

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$$B_{lon} = \begin{bmatrix} -0.000187 & 9.66 \\ -17.85 & 0 \\ -1.158 & 0 \\ 0 & 0 \end{bmatrix}$$

γ

$$\gamma = \underbrace{\begin{bmatrix} 0 & \frac{-1}{774} & 0 & 1 \end{bmatrix}}_{C_\gamma} \begin{bmatrix} \Delta u \\ \Delta w \\ \Delta q \\ \Delta\theta \end{bmatrix} + \underbrace{\begin{bmatrix} 0 & 0 \end{bmatrix}}_{D_\gamma} \begin{bmatrix} \delta_e \\ \delta_t \end{bmatrix}$$

$\gamma = \Delta\theta - \Delta\alpha \longrightarrow C_\gamma$

$$\delta_e = 10^\circ$$

$$\delta_t = \frac{1}{6} \approx 0.05\,w$$

Longitudinal Stability Augmentation

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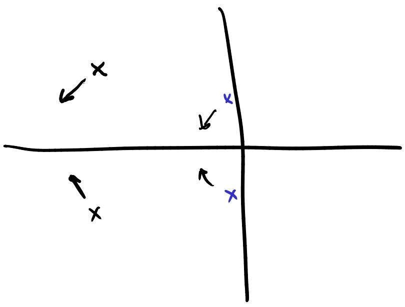
$$A_{sp} = \begin{bmatrix} -0.3151 & 773.98 \\ -0.0010 & -0.4285 \end{bmatrix}$$



$$|A-\lambda I| = (-0.3151-\lambda)(-0.4285-\lambda) - (-0.0010)(773.98)$$

$$\lambda = -0.372 \pm 0.889i$$

$$\omega_n = 0.964$$

$$\zeta = 0.386 \quad \times \quad 0.7$$

$$B_{sp} = \begin{bmatrix} -17.85 & 0 \\ -1.158 & 0 \end{bmatrix}$$

measured by rate gyro

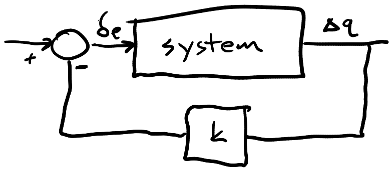
$$\delta_e = -k_s\Delta q$$

$$\begin{bmatrix} \delta_e \\ \delta_t \end{bmatrix} = -\underbrace{\begin{bmatrix} 0 & k_s \\ 0 & 0 \end{bmatrix}}_{K_{sp}} \underbrace{\begin{bmatrix} \Delta w \\ \Delta q \end{bmatrix}}_{\vec{x}_{sp}}$$

$$A^{cl} = A_{sp} - B_{sp}K_{sp} = A_{sp} - \begin{bmatrix} 0 & -17.85\,k_s \\ 0 & -1.158\,k_s \end{bmatrix}$$

$$= \begin{bmatrix} -0.3151 & 773.98 + 17.85\,k_s \\ -0.0010 & -0.4285 + 1.158\,k_s \end{bmatrix}$$

Matlab rlocus



$C_{\Delta q} = [0 \;\; 1]$ $\quad D_{\Delta q} = [0]$

$B_{\delta e} = \begin{bmatrix} -17.85 \\ -1.158 \end{bmatrix}$