Quadrotor Control

Aircraft Dynamics
UNIVERSITY OF COLORADO **BOULDER**

Example

$$A = \begin{bmatrix} -1.3256 & -0.5415 & -0.4313 \\ -0.5415 & -1.4162 & -0.0523 \\ -0.4313 & -0.0523 & -0.6582 \end{bmatrix}$$

$$\mathbf{v}_i = \begin{bmatrix} 0.6985 \\ 0.6703 \\ 0.2506 \end{bmatrix}, \begin{bmatrix} 0.4916 \\ -0.7040 \\ 0.5126 \end{bmatrix}, \begin{bmatrix} -0.5200 \\ 0.2348 \\ 0.8212 \end{bmatrix}$$

$$\lambda_i = \{-2, -1, -.4\}$$

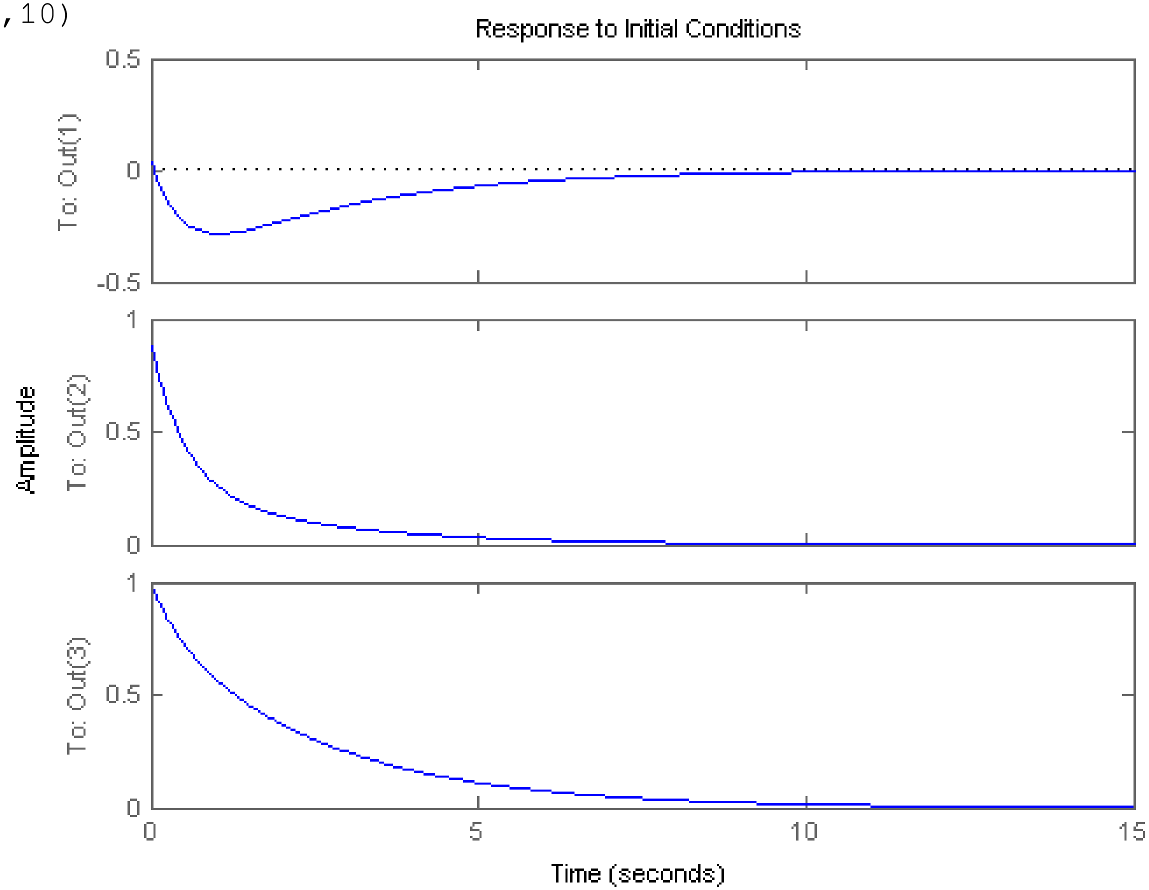
For a state space model the eigenvalues λ_i of the matrix A are the poles of the system.

Example

Matlab:

```
sys = ss(A,eye(3),eye(3),0)
initial(sys,[0.05; 0.9; 1.0],10)
```

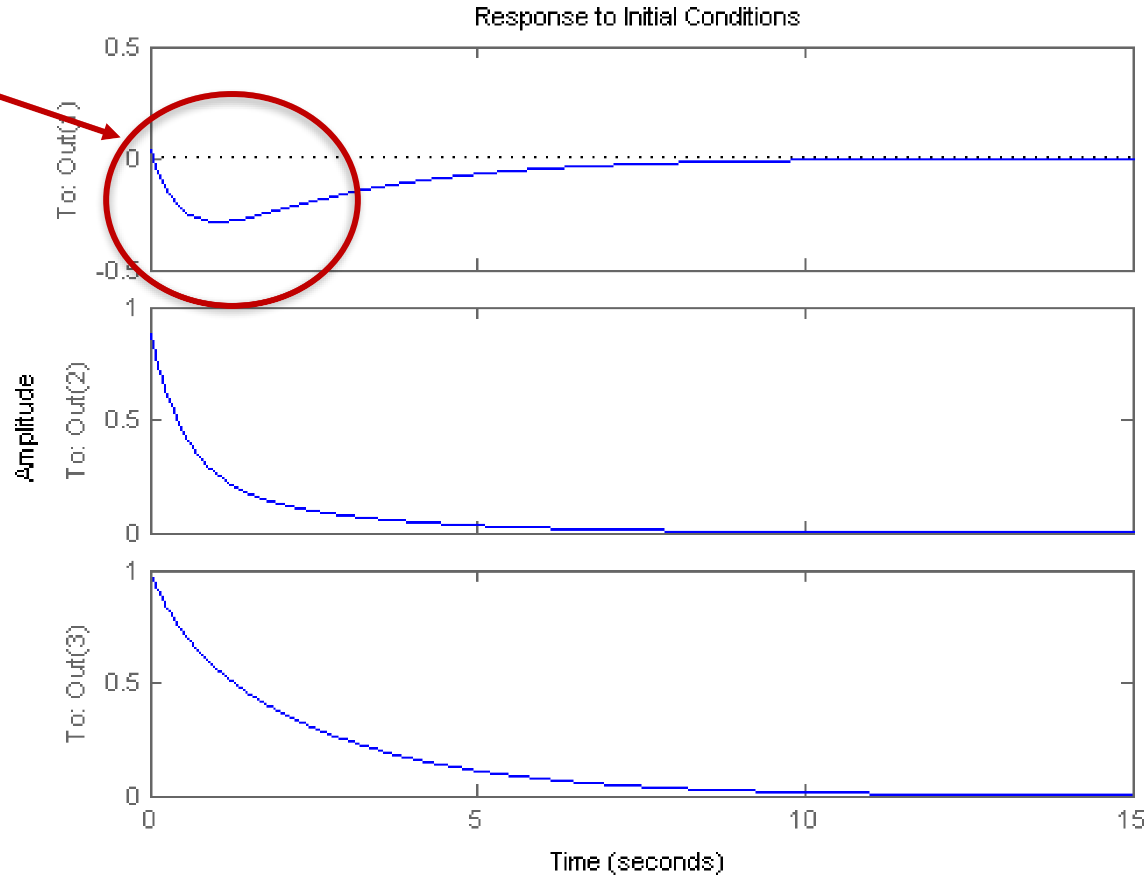
$$\mathbf{x}(0) = \begin{bmatrix} 0.05 \\ 0.9 \\ 1.0 \end{bmatrix}$$



Example

What is happening here if all poles are negative and real (exponential decay)

$$\mathbf{x}(0) = \begin{bmatrix} 0.05 \\ 0.9 \\ 1.0 \end{bmatrix}$$



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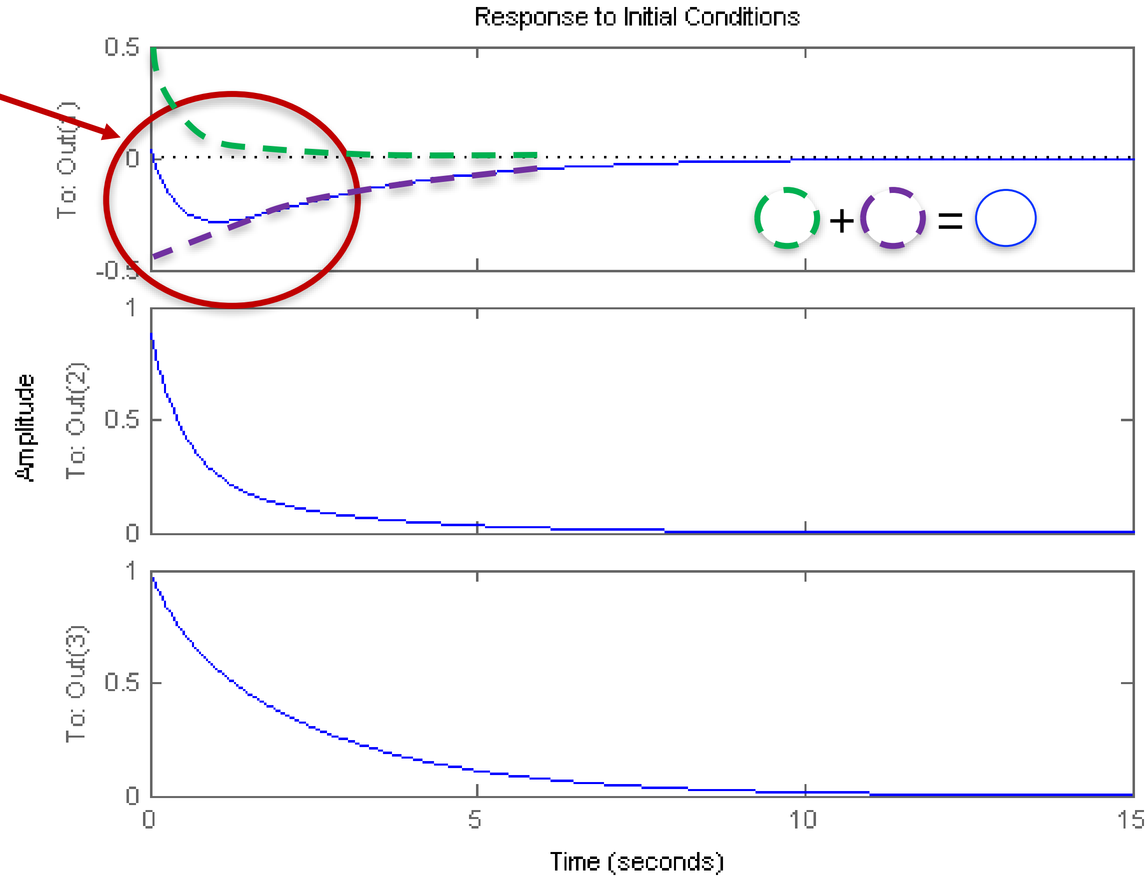
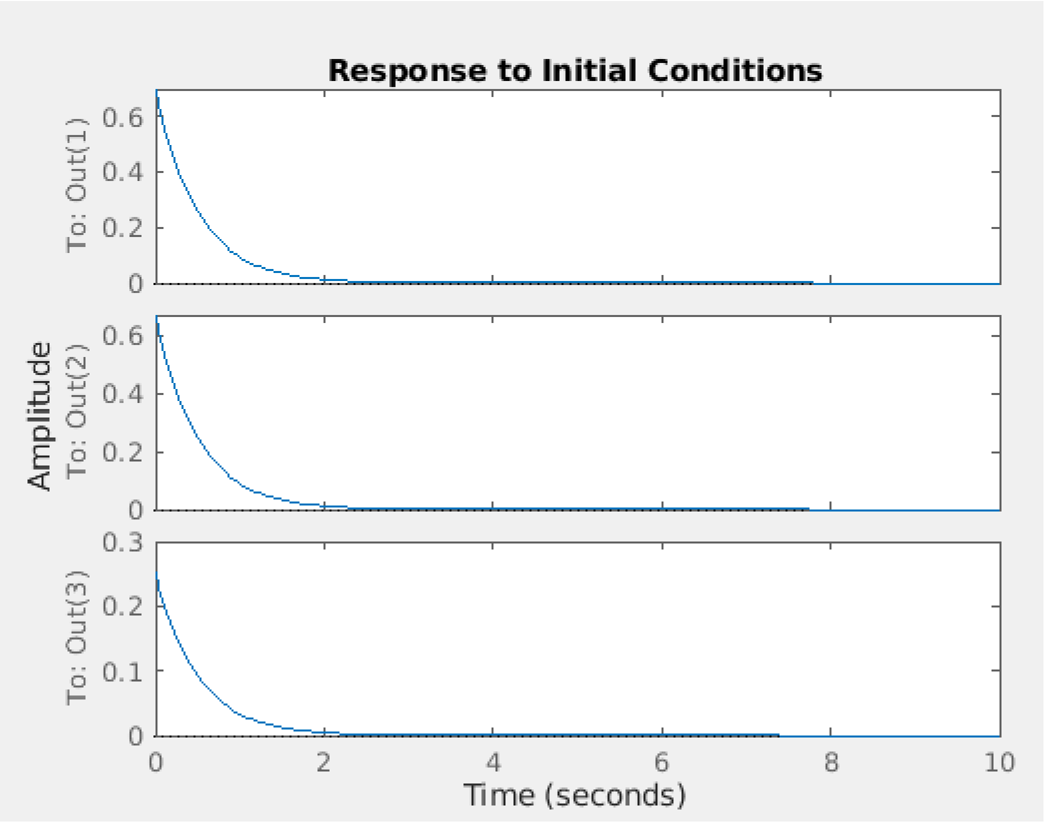


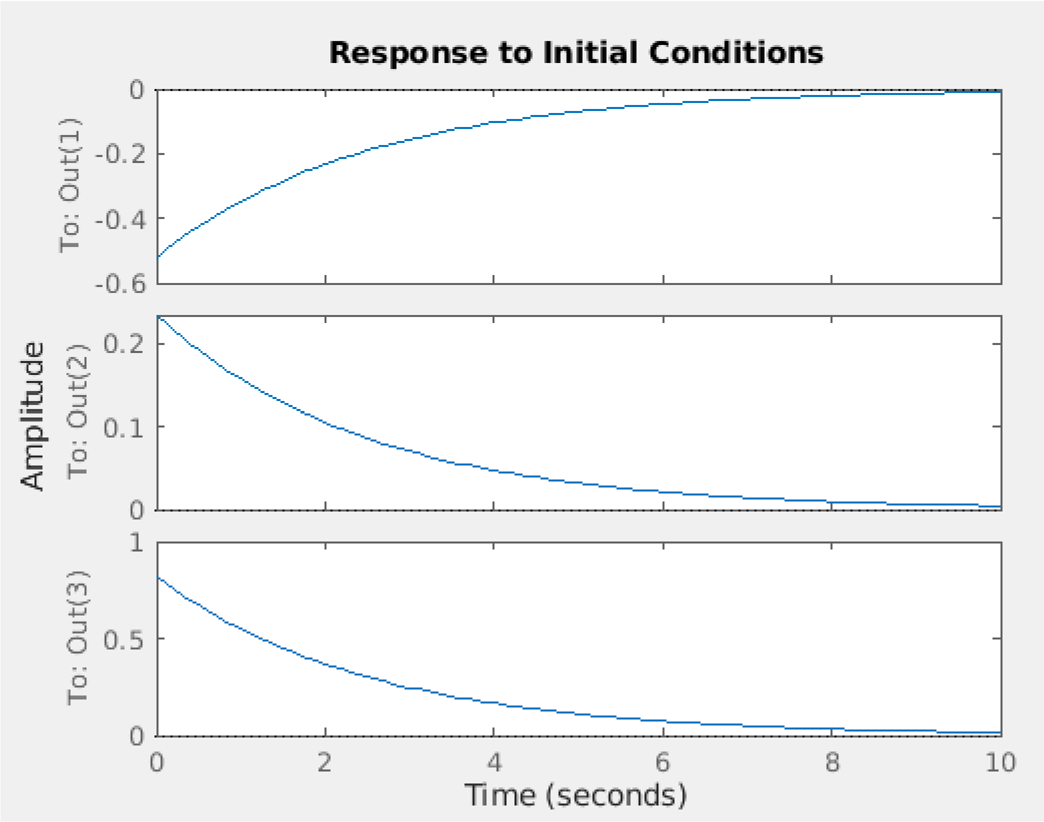
Example

$$\mathbf{x}(0) = \mathbf{v}_1 = \begin{bmatrix} 0.6985 \\ 0.6703 \\ 0.2506 \end{bmatrix}$$



Example

$$\mathbf{x}(0) = \mathbf{v}_3 = \begin{bmatrix} -0.5200 \\ 0.2348 \\ 0.8212 \end{bmatrix}$$



Another Example

$$A = \begin{bmatrix} -0.0129 & -0.0059 & 0 & -0.0507 \\ -0.104 & -0.8185 & 1 & 0 \\ 0.2867 & -12.6811 & -1.424 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix}$$

$$\mathbf{v}_i = \begin{bmatrix} 0.0032 \mp 0.0021i \\ -0.0222 \mp 0.2608i \\ 0.9321 \\ -0.0755 \mp 0.2389i \end{bmatrix}, \begin{bmatrix} -0.0473 \pm 0.5548i \\ -0.0001 \pm 0.0056i \\ -0.0054 \pm 0.0622i \\ 0.8288 \end{bmatrix}$$

$$\lambda_i = \{-1.1212 \pm 3.5472i, -0.0065 \pm 0.0752i\}$$

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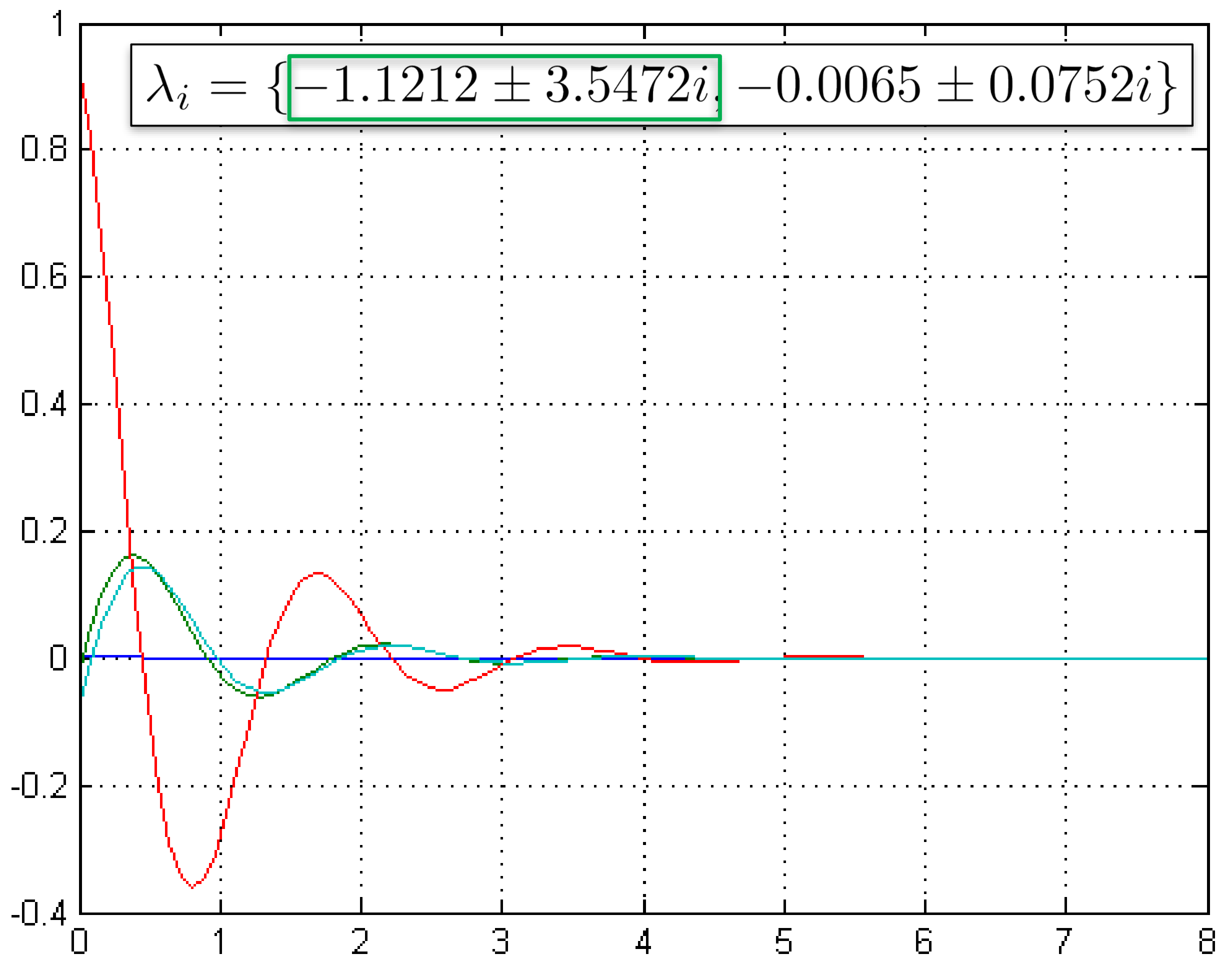
For a state space model the eigenvalues λ_i of the matrix A are the poles of the system, and

$$\lambda_i = -\omega_n \zeta + \omega_n \sqrt{1-\zeta^2} i = -\sigma + \omega_d i$$

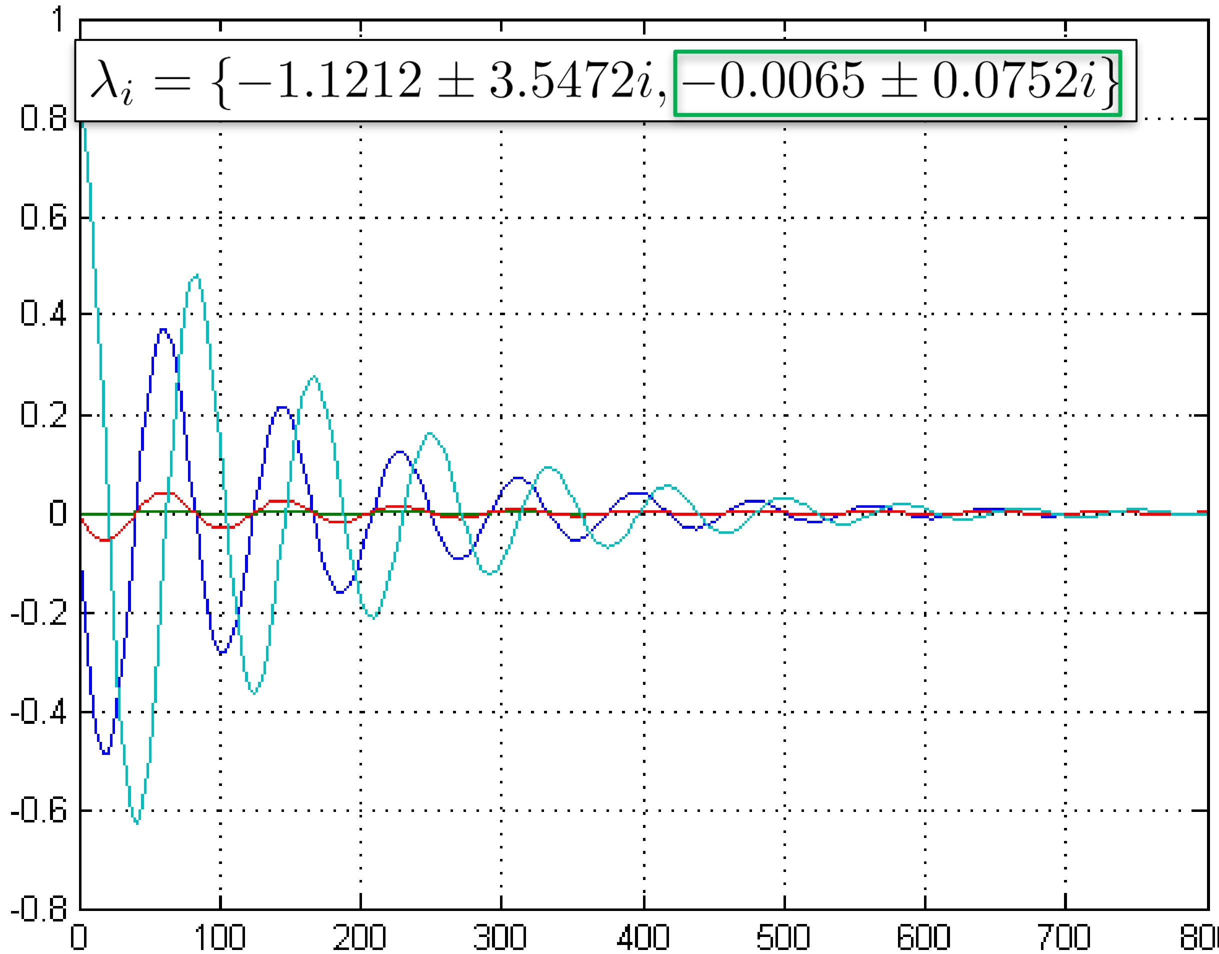
Another Example

$$\mathbf{x}(0) = \text{real}(\mathbf{v}_1)$$



Another Example

$$\mathbf{x}(0) = \mathrm{real}(\mathbf{v}_3)$$



Oscillations with (very) different natural frequency and damping ratio, different relative magnitude and phases

Linear Control Design Process

1) Derive Equations of Motion
2) Linearize and Separate Equations
3) Design Control Architecture
4) Choose Gain Values
5) Test in Linear Simulation
6) Test in Nonlinear Simulation

1) P(I)D Tuning
2) Pole Assignment
3) Root Locus
4) Optimal Control (LQR)

Pole Locations