No Wind

$$\vec{x} = \begin{bmatrix} x_E \\ y_E \\ z_E \\ \phi \\ \theta \\ \psi \\ u \\ v \\ w \\ p \\ q \\ r \end{bmatrix} \begin{matrix} \left.\begin{matrix} \\ \\ \\ \end{matrix}\right\} \vec{p}_E^{\,E} = \vec{p}_E \\ \left.\begin{matrix} \\ \\ \\ \end{matrix}\right\} \vec{\Theta} \\ \left.\begin{matrix} \\ \\ \\ \end{matrix}\right\} \vec{V}_B \\ \left.\begin{matrix} \\ \\ \\ \end{matrix}\right\} \vec{\omega}_B^{\,E} = \vec{\omega}_B \end{matrix}$$

$$\dot{\vec{p}}_E = R_B^E \vec{V}_B^E$$

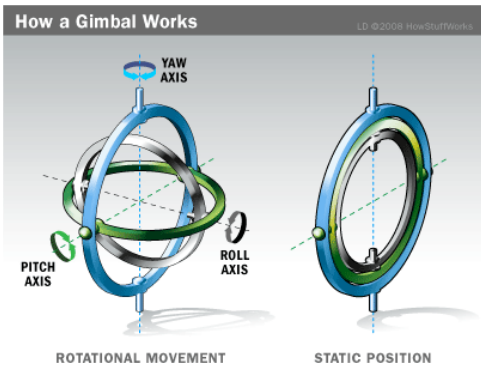
$$\dot{\vec{\Theta}} = T \vec{\omega}_B$$

attitude influence

$$\dot{\vec{V}}_B^E = \frac{\vec{f}_B}{m} - \vec{\omega} \times \vec{V}_B^E$$

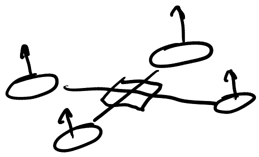
$$\dot{\vec{\omega}}_B = I_B^{-1}\left[\vec{G}_B - \vec{\omega} \times I_B \vec{\omega}_B\right]$$

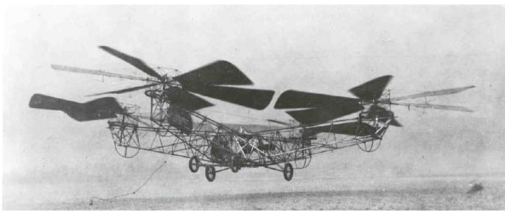
$$T = \begin{bmatrix} 1 & \sin\phi \, \underline{\tan\theta} & \cos\phi \, \underline{\tan\theta} \\ 0 & \cos\phi & -\sin\phi \\ 0 & \sin\phi \, \underline{\sec\theta} & \cos\phi \, \underline{\sec\theta} \end{bmatrix}$$

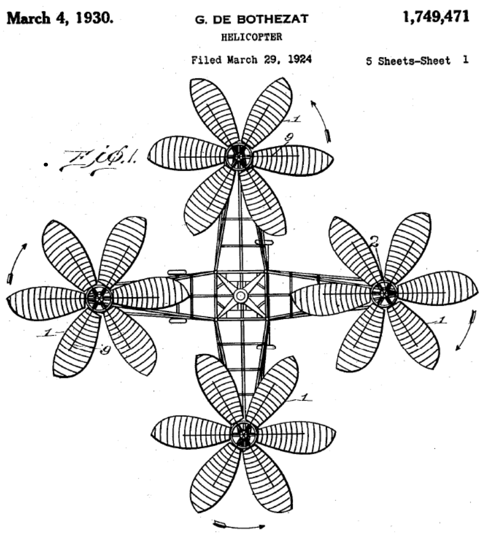


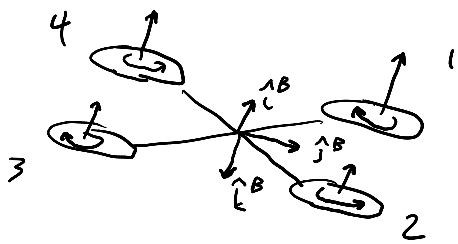
Multicopter

Quadrotor









$${}^c\vec{f}_B = \begin{bmatrix} 0 \\ 0 \\ Z_c \end{bmatrix}$$

$${}^c\vec{G} = \begin{bmatrix} L_c \\ M_c \\ N_c \end{bmatrix}$$

$$\vec{f} = {}^g\vec{f} + {}^a\vec{f} + {}^c\vec{f}$$
$$\vec{G} = {}^a\vec{G} + {}^c\vec{G}$$

Since symmetric about $\hat{i}^B - \hat{k}^B$ plane and $\hat{i}^B - \hat{j}^B$ plane

$$I_{xy} = I_{yz} = I_{xz} = 0$$

$$I_B = \begin{bmatrix} I_x & 0 & 0 \\ 0 & I_y & 0 \\ 0 & 0 & I_z \end{bmatrix}$$

$$\begin{pmatrix} \dot{x}_E \\ \dot{y}_E \\ \dot{z}_E \end{pmatrix} = \begin{pmatrix} c_\theta c_\psi & s_\phi s_\theta c_\psi - c_\phi s_\psi & c_\phi s_\theta c_\psi + s_\phi s_\psi \\ c_\theta s_\psi & s_\phi s_\theta s_\psi + c_\phi c_\psi & c_\phi s_\theta s_\psi - s_\phi c_\psi \\ -s_\theta & s_\phi c_\theta & c_\phi c_\theta \end{pmatrix} \begin{pmatrix} u^E \\ v^E \\ w^E \end{pmatrix}$$

$$\begin{pmatrix} \dot{\phi} \\ \dot{\theta} \\ \dot{\psi} \end{pmatrix} = \begin{pmatrix} 1 & \sin\phi\tan\theta & \cos\phi\tan\theta \\ 0 & \cos\phi & -\sin\phi \\ 0 & \sin\phi\sec\theta & \cos\phi\sec\theta \end{pmatrix} \begin{pmatrix} p \\ q \\ r \end{pmatrix}$$

$$\begin{pmatrix} \dot{u}^E \\ \dot{v}^E \\ \dot{w}^E \end{pmatrix} = \overbrace{\begin{pmatrix} rv^E - qw^E \\ pw^E - ru^E \\ qu^E - pv^E \end{pmatrix}}^{\vec{\omega}_B \times \vec{V}_B^E} + \overbrace{g \begin{pmatrix} -\sin\theta \\ \cos\theta\sin\phi \\ \cos\theta\cos\phi \end{pmatrix} + \frac{1}{m}\underbrace{\begin{pmatrix} X \\ Y \\ Z \end{pmatrix}}_{{}^a\vec{f}_B} + \frac{1}{m}\underbrace{\begin{pmatrix} 0 \\ 0 \\ Z_c \end{pmatrix}}_{{}^c\vec{f}_B}}^{\vec{f}_B}$$

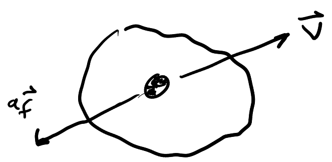
$$I_B = \begin{bmatrix} I_x & 0 & 0 \\ 0 & I_y & 0 \\ 0 & 0 & I_z \end{bmatrix}$$

$$I_B^{-1} = \begin{bmatrix} 1/I_x & 0 & 0 \\ 0 & 1/I_y & 0 \\ 0 & 0 & 1/I_z \end{bmatrix}$$

$$\begin{pmatrix} \dot{p} \\ \dot{q} \\ \dot{r} \end{pmatrix} = \overbrace{\begin{pmatrix} \frac{I_y - I_z}{I_x} qr \\ \frac{I_z - I_x}{I_y} pr \\ \frac{I_x - I_y}{I_z} pq \end{pmatrix}}^{I_B^{-1}(\vec{\omega}_B \times I\vec{\omega}_B)} + \begin{pmatrix} \frac{1}{I_x} L \\ \frac{1}{I_y} M \\ \frac{1}{I_z} N \end{pmatrix} + \begin{pmatrix} \frac{1}{I_x} L_c \\ \frac{1}{I_y} M_c \\ \frac{1}{I_z} N_c \end{pmatrix}$$

Aerodynamic Forces and Moments

$${}^a\vec{f} = -D\frac{\vec{v}}{|\vec{v}|}$$



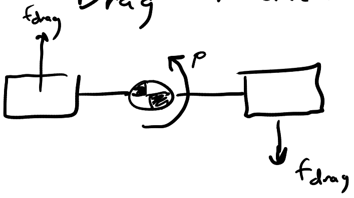
$$V_a = |\vec{v}|$$ airspeed

$$D = \overbrace{\frac{1}{2}\rho C_D A}^{\nu} V_a^2 = \nu V_a^2$$

density, coefficient of drag (shape), cross-sectional area

$${}^a\vec{f}_B = \begin{bmatrix} X \\ Y \\ Z \end{bmatrix} = -\nu V_a^2 \frac{\vec{v}_B}{V_a}$$
$$= -\nu V_a \begin{bmatrix} u \\ v \\ w \end{bmatrix}$$

Drag Moment



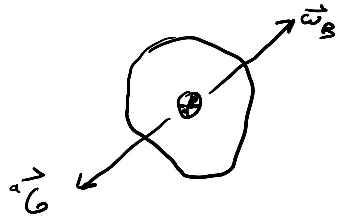
$$L_{drag} = -2 l f_{drag}$$

$$= -2l\left(\tfrac{1}{2}\rho\, C_D\, A (l_p)^2\right)\text{sign}(p)$$

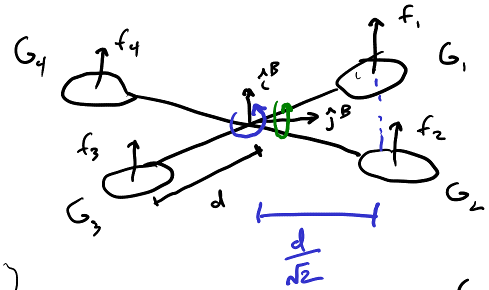
$$= -\mu |p| p$$

$$-p^2\,\text{sign}(p) = -|p|p$$

$${}^a\vec{G}_B = \begin{bmatrix} L \\ M \\ N \end{bmatrix} = -\mu\sqrt{p^2+q^2+r^2}\begin{bmatrix} p \\ q \\ r \end{bmatrix}$$



Control Forces + Moments



$${}^c\vec{f}_B = \begin{bmatrix} 0 \\ 0 \\ -f_1-f_2-f_3-f_4 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ Z_c \end{bmatrix}$$

$${}^c G_B = \begin{bmatrix} \frac{d}{\sqrt{2}}(-f_1-f_2+f_3+f_4) \\ \frac{d}{\sqrt{2}}(f_1-f_2-f_3+f_4) \\ \underbrace{G_1-G_2+G_3-G_4}_{k_m(f_1-f_2+f_3-f_4)} \end{bmatrix} = \begin{bmatrix} L_c \\ M_c \\ N_c \end{bmatrix}$$

$$G_i = k_G C_D \omega_r^2$$
$$f_i = k_f C_L \omega_r^2$$
$$k_m = \frac{k_G C_D}{k_f C_L}$$

$$G_i = k_m f_i$$

$$\begin{bmatrix} Z_c \\ L_c \\ M_c \\ N_c \end{bmatrix} = \begin{bmatrix} -1 & -1 & -1 & -1 \\ -\frac{d}{\sqrt{2}} & -\frac{d}{\sqrt{2}} & \frac{d}{\sqrt{2}} & \frac{d}{\sqrt{2}} \\ \frac{d}{\sqrt{2}} & -\frac{d}{\sqrt{2}} & -\frac{d}{\sqrt{2}} & \frac{d}{\sqrt{2}} \\ k_m & -k_m & k_m & -k_m \end{bmatrix}\begin{bmatrix} f_1 \\ f_2 \\ f_3 \\ f_4 \end{bmatrix}$$

invert

$$\begin{bmatrix} f_1 \\ f_2 \\ f_3 \\ f_4 \end{bmatrix} = \begin{bmatrix} \quad \end{bmatrix}\begin{bmatrix} Z_c \\ L_c \\ M_c \\ N_c \end{bmatrix}$$

	Multirotor	vs	Conv Helicopter
Aerodynamic Efficient			✓
Mechanical Control Complexity	✓		
Power System Complexity			
Turbine			✓
Electric	✓