

ASEN 3728 Aircraft Dynamics

Written Homework 2

Due date listed on Gradescope.

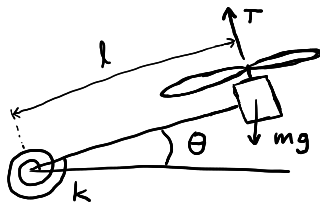
Question 1. A 10 kg quadrotor has state vector $\mathbf{x} = [0 \text{ m}, 0 \text{ m}, 0 \text{ m}, 0 \text{ rad}, -\pi/6 \text{ rad}, 0 \text{ rad}, 11 \text{ m/s}, 4 \text{ m/s}, 1 \text{ m/s}, -0.1 \text{ rad/s}, 0.2 \text{ rad/s}, 0.4 \text{ rad/s}]$. There is wind with velocity $\mathbf{W}_B^E = [2, 2, 0]^T \text{ m/s}$. Each rotor applies a control force of magnitude 30 N to the aircraft, and $\nu = 10^{-2} \text{ kg/m}$. Considering forces due to gravity, aerodynamic drag, and the rotor control, what is $\dot{\mathbf{V}}_B^E$?

Question 2. Consider a 25 kg quadrotor operating with the state:

$$\mathbf{x} = \begin{bmatrix} -1200 \text{ m} & 3075 \text{ m} & -1550 \text{ m} \dots \\ 0 \text{ rad} & \pi/3 \text{ rad} & \pi/2 \text{ rad} \dots \\ 10 \text{ m/s} & -2 \text{ m/s} & 3 \text{ m/s} \dots \\ 0.02 \text{ rad/sec} & -0.04 \text{ rad/sec} & 0.05 \text{ rad/sec} \end{bmatrix}^T$$

- (A) What is the time rate of change of the inertial z-position of this aircraft?
- (B) What is the magnitude of the angular velocity vector?
- (C) Given the air-relative velocity vector in body-coordinates $\mathbf{V}_B = (10, -1, -2) \text{ m/s}$, what is the inertial wind vector expressed in inertial coordinates?
- (D) Assume that the quadrotor is symmetric about the $\mathbf{i}^B - \mathbf{k}^B$ and $\mathbf{j}^B - \mathbf{k}^B$ planes, $I_x = 10 \text{ kg} \cdot \text{m}^2$, and $I_y = I_z$. If the angular acceleration around the x-axis is $\dot{p}^E = 0.50 \text{ rad/s}^2$, what is the aerodynamic moment acting along the x-axis of the aircraft?

Question 3. The test rig in the diagram below is used to measure the thrust of a rotor, T . The rotor, which has mass m , is mounted at the end of a massless rod of length l , which has a torsional spring with stiffness k at its base and an drag moment coefficient μ .



The equation of motion is

$$\ddot{\theta} = \frac{-mgl \cos(\theta) - k\theta + Tl - \mu|\dot{\theta}|\dot{\theta}}{ml^2}. \quad (1)$$

- (A) Suppose that the rig comes to rest at an equilibrium point where the angle is θ_0 . Find an expression for the thrust T_0 in terms of θ_0 and the other parameters.
- (B) In order to measure the thrust with this rig, which of the parameters m , l , μ , and k must be known?
- (C) Linearize the equation of motion about the equilibrium point where the angle is θ_0 .
- (D) Write the state space equations for the linearized system with control input ΔT and output $\Delta\theta$.