ASEN 3728 Aircraft Dynamics
Written Homework 7

Due date listed on Gradescope.

Question 1. Suppose that you are on the design team for what will become the Convair 880. The aircraft parameters are shown below:

Center of gravity and mass characteristics

W = 126,000 lb
CG at 25% MAC
I_x = 115,000 Slug·ft²
I_y = 2450,000 Slug·ft²
I_z = 4070,000 Slug·ft²
I_{xz} = 0

Reference geometry

S = 2,000 ft²
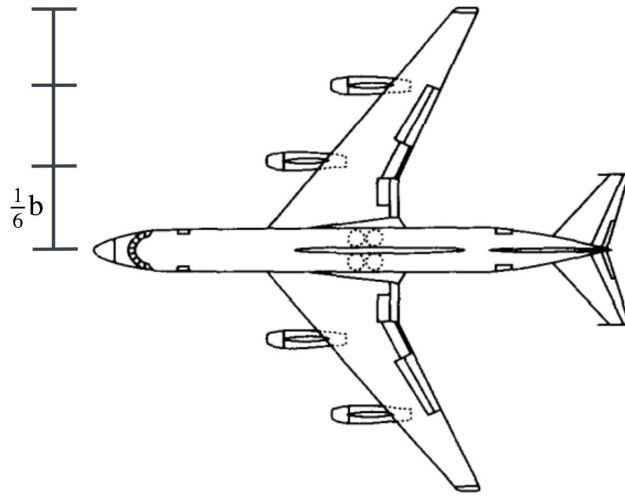b = 120 ft
$\bar{c}$ = 18.94 ft

$\frac{1}{6}$b

Table 1: Lateral stability derivatives for the Convair 880 at sea level.

C_{y_β}	C_{l_β}	C_{n_β}	C_{l_p}	C_{n_p}	C_{l_r}	C_{n_r}	$C_{l_{\delta_a}}$	$C_{n_{\delta_a}}$	$C_{y_{\delta_r}}$	$C_{l_{\delta_r}}$
-0.877	-0.196	0.139	-0.381	-0.049	0.198	-0.185	-0.038	0.017	0.216	0.0226

a) If the aircraft operates with $C_L = 0.68$ and $C_D = 0.08$ what is minimum thrust that this aircraft must apply to remain in trim assuming that thrust is directly opposed to drag.

b) Consider the case where the leftmost engine suddenly becomes non-responsive. The pilot wants to reduce the moment induced by the resulting asymmetry; however, the maximum thrust a single engine can contribute is 6000lb$_f$. What is the minimum yawing moment, ΔN_{thrust}, that can be achieved by adjusting the individual throttles while maintaining the same total thrust?

c) Use a linear approximation to estimate the rudder power $(C_{n_{\delta_r}})$ needed so that flight with no sideslip can be maintained with an absolute rudder deflection of 3° in this one-engine-out case.

d) Use a linear approximation to estimate the aileron deflection and sideslip angle **in degrees** that would be needed to attain a steady flight condition in this one-engine-out case without any rudder deflection.

Question 2. While flying near sea-level at $u_0 = 10$ m/s with $\theta_0 = 0$, the TTwistor aircraft from homework P3 has the lateral dynamics matrix shown below. For all parts, show work and/or describe any code used.

$$\mathbf{A}_{lat} = \begin{bmatrix} -0.2472 & -0.0671 & -9.7797 & 9.8100 \\ -0.7966 & -16.5375 & 1.8114 & 0 \\ 0.4607 & -0.3451 & -0.4586 & 0 \\ 0 & 1.0000 & 0 & 0 \end{bmatrix}$$

1. Find the time constant of the roll mode, time constant of the spiral mode, and natural frequency and damping ratio of the Dutch roll mode of the TTwistor using the full lateral matrix.

2. Calculate the time constant of the roll mode using the roll mode approximation. Compare this to the time constant found in part 1 (using % error). Is this a good approximation?

3. Calculate the natural frequency and damping ratio of the Dutch roll mode using the Dutch roll approximation. Compare this to the natural frequency and damping ratio found in part 1 (using % error).Is this a good approximation?

4. Calculate the time constant of the roll and spiral modes using the roll and spiral mode approximation. Compare this to the time constants found in part 1 (using % error). Is this a good approximation?

Question 3. In the Dutch Roll approximation, and under the assumption that off-diagonal elements in an aircraft's inertia matrix are negligible, the state space $\mathbf{A}_{lat}$ and $\mathbf{B}_{lat}$ matrices and state $\mathbf{x}_{lat}$ become:

$$\mathbf{x}_{dr} = \begin{bmatrix} \Delta v \\ \Delta r \end{bmatrix} \quad \mathbf{A}_{dr} = \begin{bmatrix} Y_v/m & -u_0 \\ N_v/I_z & N_r/I_z \end{bmatrix} \quad \mathbf{B}_{dr} = \begin{bmatrix} 0 & Y_{\delta_r}/m \\ N_{\delta_a}/I_z & N_{\delta_r}/I_z \end{bmatrix}$$

Consider implementing a yaw damper control law to improve the damping ratio of the Dutch Roll mode. This control law applies to the rudder only, and is $\Delta\delta_r = -k\Delta r$, $\Delta\delta_a = 0$.

1. Write an expression to calculate $\mathbf{u} = [\Delta\delta_a, \Delta\delta_r]^T$ in the form $\mathbf{u} = -\mathbf{K}\mathbf{x}_{dr}$, where $\mathbf{K}$ is a matrix of control gains. Then, find the closed-loop dynamics matrix $\mathbf{A}_{dr,cl}$ using $\mathbf{A}_{dr,cl} = \mathbf{A}_{dr} - \mathbf{B}_{dr}\mathbf{K}$

2. Write out the characteristic equation for the eigenvalues λ of $\mathbf{A}_{dr,cl}$, and then solve for k in terms of λ and variables in $\mathbf{A}_{dr}$ and $\mathbf{B}_{dr}$.

3. A large jet transport has the following $\mathbf{A}_{dr}$ and $\mathbf{B}_{dr}$ matrices:

$$\mathbf{A}_{dr} = \begin{bmatrix} -0.056 & -730 \\ 0.0012 & -0.15 \end{bmatrix} \quad \mathbf{B}_{dr} = \begin{bmatrix} 0 & 5.5 \\ 0.004 & -0.47 \end{bmatrix}$$

   What is the damping ratio ζ of the Dutch Roll mode without the yaw damper in place?

4. Find the transfer function, $G_{\Delta v\delta_r}(s)$, relating the rudder deflection to sideslip velocity using the numerical values above.

5. We wish to increase the damping ratio of the Dutch Roll mode using the yaw damper control law. If $\zeta = 0.7$ and $\omega_n = 0.9656$ rad/s, what value of k is required in the control law? (Note: A calculation using your result from Part 2 will likely give you a value of k that has a small imaginary part. Ignore the imaginary part in your answer, as it is an artifact of the way this problem was set up.)

6. The jet is capable of $\Delta\delta_r = \pm 25°$. Using the value of k found in the previous part, what is the largest Δr that the yaw damper can handle and still achieve $\zeta = 0.7$ for the Dutch Roll mode?