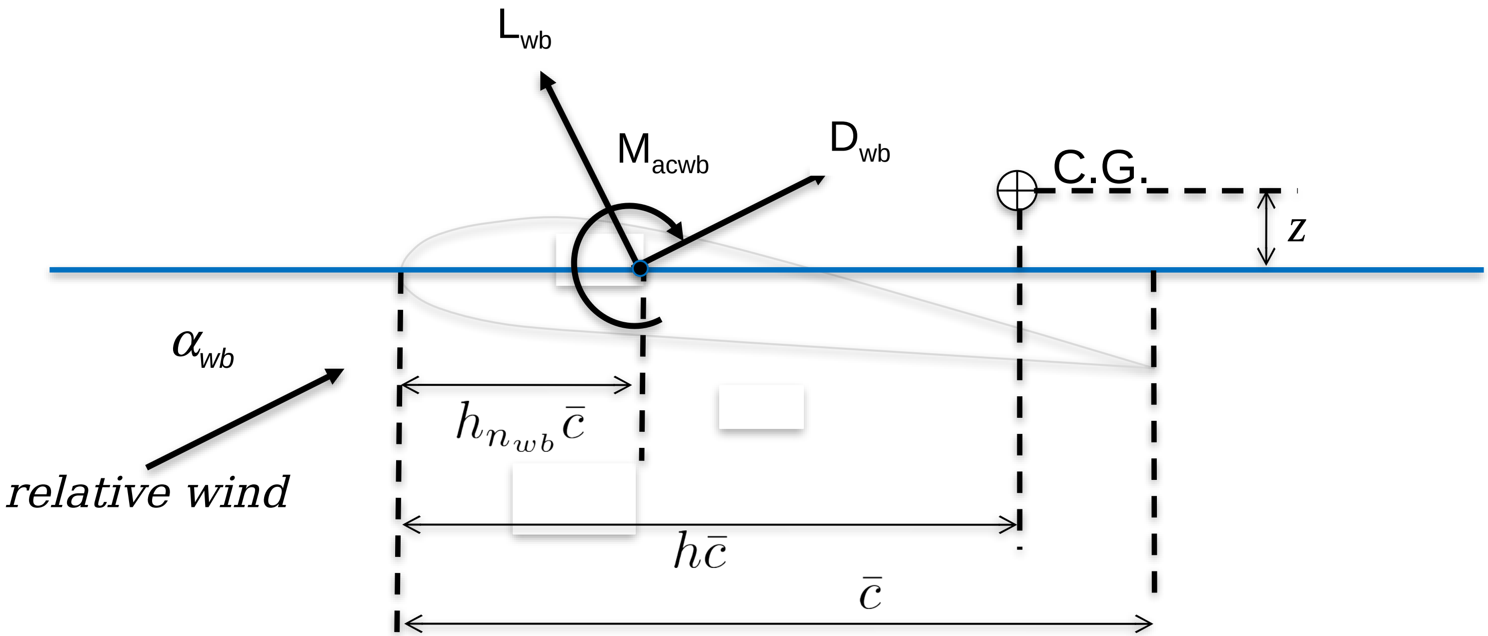
Longitudinal Trim

Aircraft Dynamics
UNIVERSITY OF COLORADO **BOULDER**

Wing / Body / Nacelle



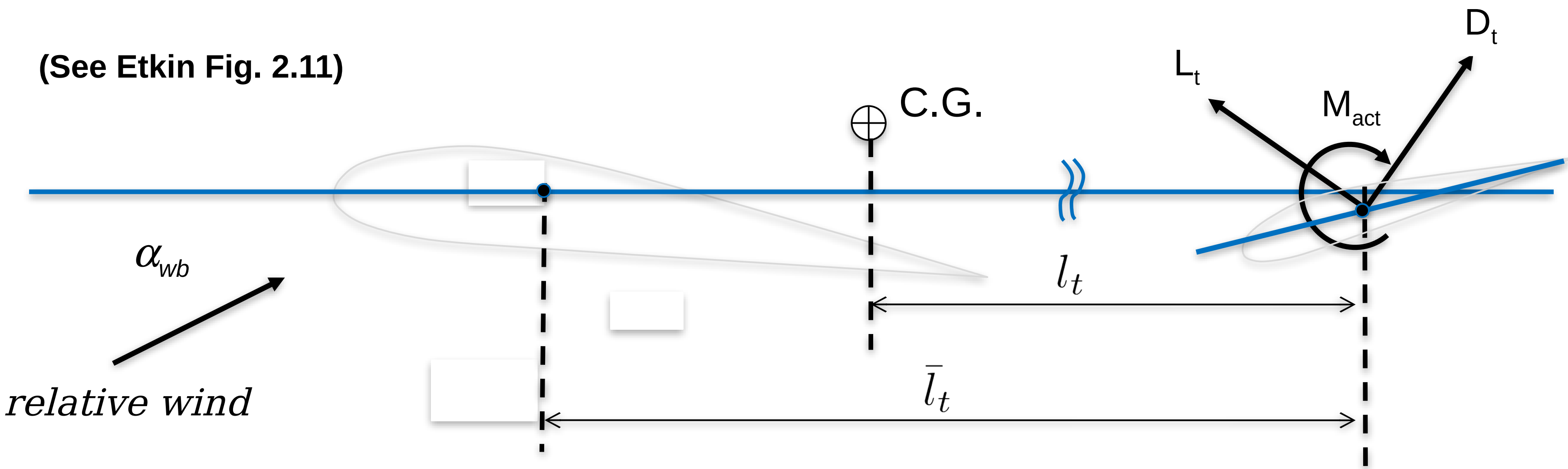
$$M_{wb} = M_{ac_{wb}} + (L\cos\alpha_{wb} + D\sin\alpha_{wb})\,(h - h_{n_{wb}})\,\bar{c} + (L\sin\alpha_{wb} - D\cos\alpha_{wb})\,z$$

Non-dimensionalize and ignore several terms

$$\begin{aligned} C_{m_{wb}} &= C_{m_{ac_{wb}}} + C_{L_{wb}}\,(h - h_{n_{wb}}) \\ &= C_{m_{ac_{wb}}} + C_{L_{\alpha_{wb}}}\alpha_{wb}\,(h - h_{n_{wb}}) \\ &= C_{m_{ac_{wb}}} + a_{wb}\alpha_{wb}\,(h - h_{n_{wb}}) \end{aligned}$$

Tail

(See Etkin Fig. 2.11)

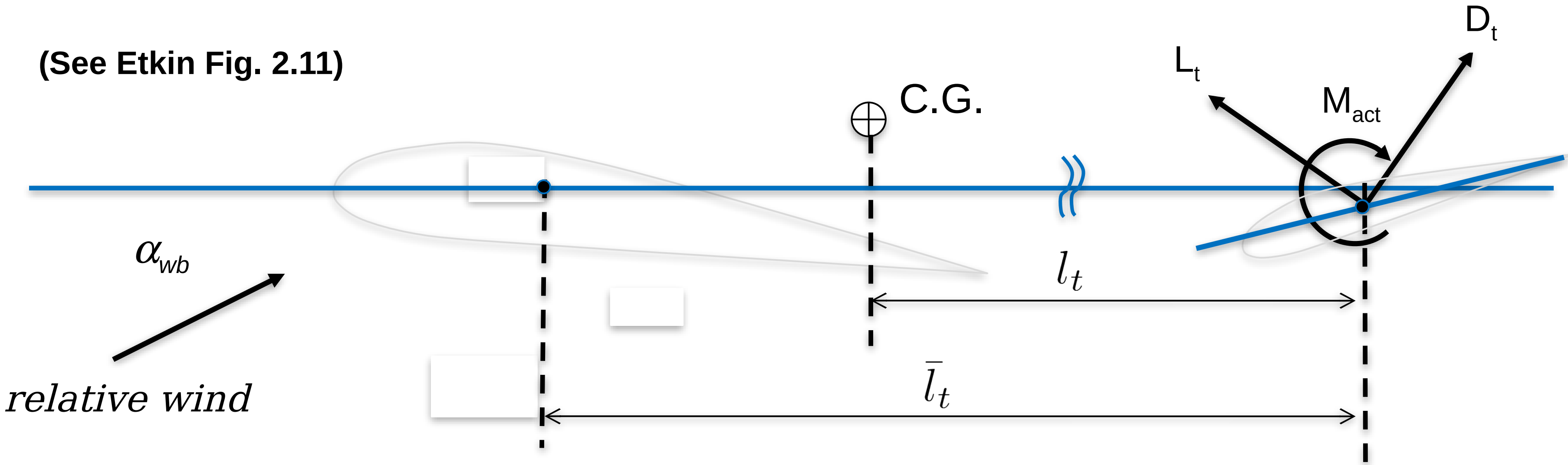


$$\begin{aligned} L &= L_{wb} + L_t \\ &= C_{L_{wb}}\left(\tfrac{1}{2}\rho V^2 S\right) + C_{L_t}\left(\tfrac{1}{2}\rho V^2 S_t\right) \\ &= C_L\left(\tfrac{1}{2}\rho V^2 S\right) \end{aligned} \quad\Longrightarrow\quad C_L = C_{L_{wb}} + \frac{S_t}{S} C_{L_t}$$

$$M_t = -l_t L_t = -l_t C_{L_t}\left(\tfrac{1}{2}\rho V^2 S_t\right) \quad\Longrightarrow\quad C_{m_t} = -\frac{l_t}{\bar{c}}\frac{S_t}{S} C_{L_t} = -V_H C_{L_t}$$

Tail

(See Etkin Fig. 2.11)



$$\bar{V}_H = \frac{\bar{l}_t}{\bar{c}} \frac{S_t}{S} \longrightarrow V_H = \bar{V}_H - \frac{S_t}{S}\left(h - h_{n_{wb}}\right)$$

$$C_{m_t} = -\bar{V}_H C_{L_t} + C_{L_t}\frac{S_t}{S}\left(h - h_{n_{wb}}\right)$$

CG can change during flight so more convenient to label relative to wing-body mean aerodynamic center

Total Pitch Moment

$$C_m = \underbrace{C_{m_{ac_{wb}}} + C_L\left(h - h_{n_{wb}}\right)}^{\text{wing-body}} \underbrace{- \bar{V}_H C_{L_t}}^{\text{tail}} \underbrace{+ C_{m_p}}^{\text{engine}}$$

$$C_{m_\alpha} = \frac{\partial C_{m_{ac_{wb}}}}{\partial \alpha} + C_{L_\alpha}\left(h - h_{n_{wb}}\right) - \bar{V}_H \frac{\partial C_{L_t}}{\partial \alpha} + \frac{\partial C_{m_p}}{\partial \alpha}$$

Neutral Point

For stability we need an increase in angle of attack to cause a negative moment

- Need negative $C_{m\alpha}$

Center of gravity location that gives $C_{m\alpha} = 0$ is called the “neutral point” h_n

$$C_{m_\alpha} = \frac{\partial C_{m_{ac_{wb}}}}{\partial \alpha} + C_{L_\alpha}(h - h_{n_{wb}}) - \bar{V}_H \frac{\partial C_{L_t}}{\partial \alpha} + \frac{\partial C_{m_p}}{\partial \alpha}$$

Neutral Point

For stability we need an increase in angle of attack to cause a negative moment

- Need negative $C_{m\alpha}$

Center of gravity location that gives $C_{m\alpha} = 0$ is called the “neutral point” h_n

$$0 = \frac{\partial C_{m_{ac_{wb}}}}{\partial \alpha} + C_{L_\alpha}(h_n - h_{n_{wb}}) - \bar{V}_H \frac{\partial C_{L_t}}{\partial \alpha} + \frac{\partial C_{m_p}}{\partial \alpha}$$

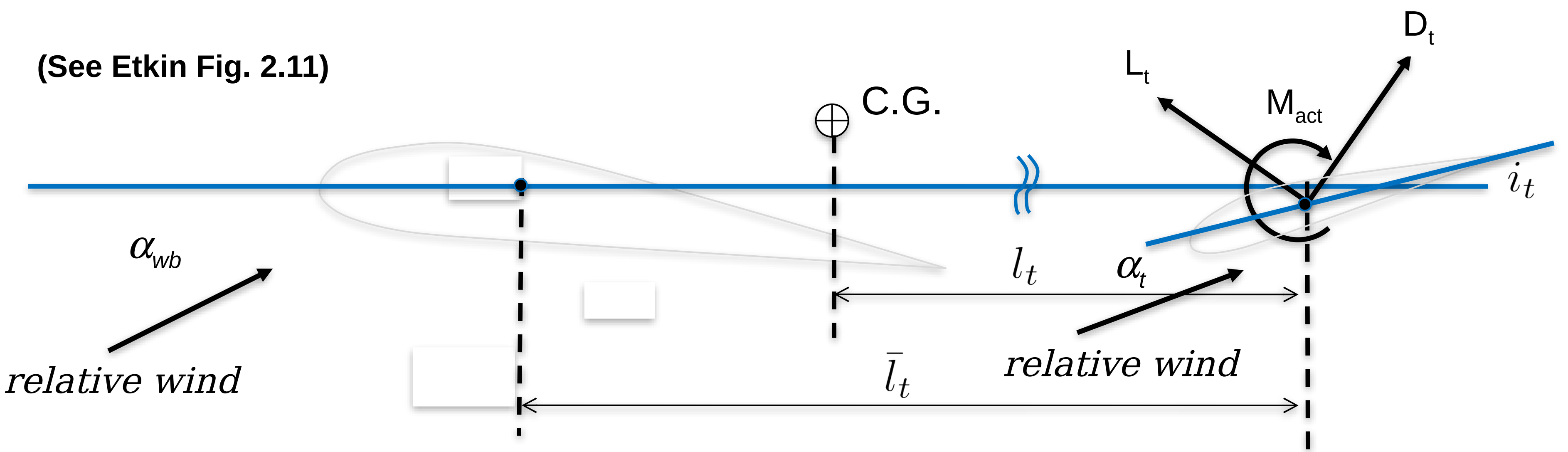
$$h_n = h_{n_{wb}} - \frac{1}{C_{L_\alpha}} \left(\frac{\partial C_{m_{ac_{wb}}}}{\partial \alpha} - \bar{V}_H \frac{\partial C_{L_t}}{\partial \alpha} + \frac{\partial C_{m_p}}{\partial \alpha} \right)$$

$$C_{m_\alpha} = C_{L_\alpha} (h - h_n)$$

$$\text{static margin} = K_n = h_n - h \qquad K_n > 0 \rightarrow C_{m_\alpha} < 0 \rightarrow \text{static stability}$$

Linear Lift

(See Etkin Fig. 2.11)



$$C_{L_{wb}} = a_{wb}\alpha_{wb}$$
$$C_{L_t} = a_t\alpha_t$$
$$C_{m_p} = C_{m0_p} + \frac{\partial C_{m_p}}{\partial \alpha}\alpha$$

$$\alpha_t = \alpha_{wb} - i_t - \left(\epsilon_0 + \frac{\partial \epsilon}{\partial \alpha}\alpha_{wb}\right)$$

Linear Lift

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$$\alpha_t = \alpha_{wb} - i_t - \left(\epsilon_0 + \frac{\partial \epsilon}{\partial \alpha}\alpha_{wb}\right)$$

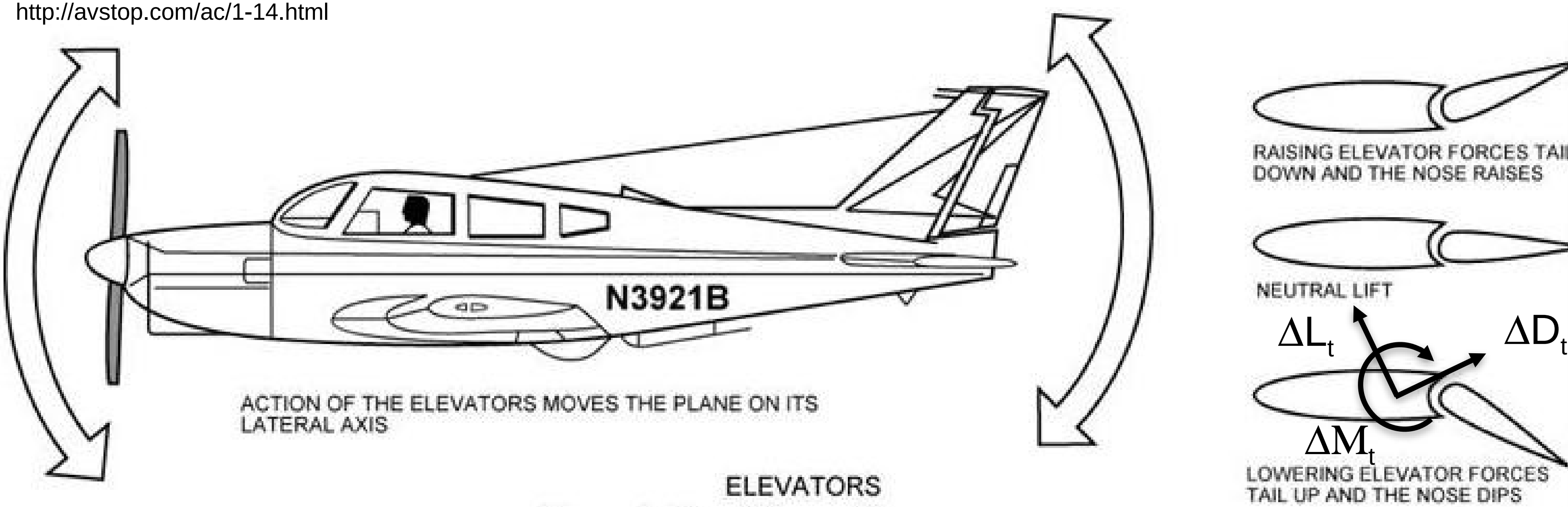
$$\begin{aligned}
C_L &= C_{L_{wb}} + \frac{S_t}{S}C_{L_t} = a_{wb}\alpha_{wb} + \frac{S_t}{S}a_t\alpha_t \\
&= \alpha_{wb}\boxed{a_{wb}\left[1 + \frac{a_t S_t}{a_{wb}S}\left(1 - \frac{\partial \epsilon}{\partial \alpha}\right)\right]}\boxed{- a_t\frac{S_t}{S}\left(i_t + \epsilon_0\right)} \\
&= \boxed{C_{L_0}} + \boxed{a}\alpha_{wb} \\
&= C_{L_\alpha}\alpha \equiv a\alpha
\end{aligned}$$

Defines body coordinate system so zero angle of attack creates no lift

Angle of attack α_{wb} of wing and angle of attack α_t of tail differ from angle of attack α by constant offsets

Longitudinal Control (Elevator)

http://avstop.com/ac/1-14.html



$$C_L = C_{L_\alpha}\alpha + C_{L_{\delta_e}}\delta_e \qquad C_{L_{\delta_e}} = \frac{\partial C_{L_t}}{\partial \delta_e}\frac{S_t}{S} = a_e\frac{S_t}{S}$$

$$C_m = C_{m_0} + C_{m_\alpha}\alpha + C_{m_{\delta_e}}\delta_e \qquad C_{m_{\delta_e}} = -a_e\bar{V}_H + C_{L_{\delta_e}}\left(h - h_{n_{wb}}\right)$$

Summary

$$\boxed{\begin{aligned} C_L &= C_{L_\alpha}\alpha + C_{L_q}\hat{q} + C_{L_{\delta_e}}\delta_e \\ C_D &= C_{D_{min}} + K\left(C_L - C_{L_{min}}\right)^2 \\ C_m &= C_{m_0} + C_{m_\alpha}\alpha + C_{m_q}\hat{q} + C_{m_{\delta_e}}\delta_e \end{aligned}}$$

$$C_{L_\alpha} = a = a_{wb}\left[1 + \frac{a_t S_t}{a_{wb} S}\left(1 - \frac{\partial\epsilon}{\partial\alpha}\right)\right]$$

$$C_{m_0} = C_{m_{ac_{wb}}} + C_{m0_p} + a_t \bar{V}_H\left(\epsilon_0 + i_t\right)\left[1 - \frac{a_t S_t}{aS}\left(1 - \frac{\partial\epsilon}{\partial\alpha}\right)\right]$$

$$h_n = h_{n_{wb}} + \frac{a_t}{a}\bar{V}_H\left(1 - \frac{\partial\epsilon}{\partial\alpha}\right) - \frac{1}{a}\frac{\partial C_{m_p}}{\partial\alpha}$$

$$C_{m_\alpha} = C_{L_\alpha}\left(h - h_n\right)$$

Direct dependence on CG location h

$$C_{L_{\delta_e}} = \frac{\partial C_{L_t}}{\partial\delta_e}\frac{S_t}{S} = a_e\frac{S_t}{S}$$

$$C_{m_{\delta_e}} = -a_e\bar{V}_H + C_{L_{\delta_e}}\left(h - h_{n_{wb}}\right)$$