Orientation

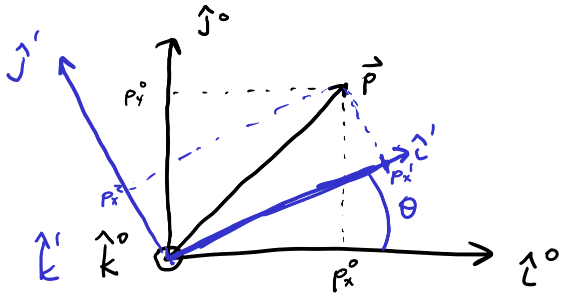
"1" ϕ roll

"2" θ pitch 3-2-1

"3" ψ yaw

know $\vec{V}^{E}_{B}$ (Frame: E; Coordinate: B) want $\vec{V}^{E}_{E}$

$\vec{p}_0 = \begin{bmatrix} p_x^0 \\ p_y^0 \\ p_z^0 \end{bmatrix}$ $\vec{p}_1 = \begin{bmatrix} p_x^1 \\ p_y^1 \\ p_z^1 \end{bmatrix}$



frame F_0

F_1

$\vec{p} = p_x^0 \hat{i}^0 + p_y^0 \hat{j}^0 + p_z^0 \hat{k}^0$

$\vec{p} = p_x' \hat{i}' + p_y' \hat{j}' + p_z' \hat{k}'$

→ want p_x' in terms of $\vec{p}_0$

$p_x' = \vec{p} \cdot \hat{i}'$

$= p_x^0 \hat{i}^0 \cdot \hat{i}' + p_y^0 \hat{j}^0 \cdot \hat{i}' + p_z^0 \hat{k}^0 \cdot \hat{i}'$

$= \underline{\begin{bmatrix} \hat{i}^0 \cdot \hat{i}' & \hat{j}^0 \cdot \hat{i}' & \hat{k}^0 \cdot \hat{i}' \end{bmatrix}} \begin{bmatrix} p_x^0 \\ p_y^0 \\ p_z^0 \end{bmatrix}$

$p_x' = \begin{bmatrix} \cos\theta & \sin\theta & 0 \end{bmatrix} \vec{p}_0$

$$\vec{p}_1 = \begin{bmatrix} p_x' \\ p_y' \\ p_z' \end{bmatrix} = \begin{bmatrix} \hat{i}^0 \cdot \hat{i}' & \hat{j}^0 \cdot \hat{i}' & \hat{k}^0 \cdot \hat{i}' \\ \hat{i}^0 \cdot \hat{j}' & \hat{j}^0 \cdot \hat{j}' & \hat{k}^0 \cdot \hat{j}' \\ \hat{i}^0 \cdot \hat{k}' & \hat{j}^0 \cdot \hat{k}' & \hat{k}^0 \cdot \hat{k}' \end{bmatrix} \begin{bmatrix} p_x^0 \\ p_y^0 \\ p_z^0 \end{bmatrix}$$

$\vec{p}_1 = R_0^1 \, \vec{p}_0$

Direction Cosine Matrix Rotation Matrix

$$R_3(\theta) = \begin{bmatrix} \cos\theta & \sin\theta & 0 \\ -\sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

(3: axis; θ: axis)

$$\begin{bmatrix} c\theta & s\theta \\ -s\theta & c\theta \end{bmatrix}$$

$$R_1(\theta) = \begin{bmatrix} 1 & 0 & 0 \\ 0 & c\theta & s\theta \\ 0 & -s\theta & c\theta \end{bmatrix} \qquad R_2(\theta) = \begin{bmatrix} c\theta & 0 & -s\theta \\ 0 & 1 & 0 \\ s\theta & 0 & c\theta \end{bmatrix}$$

Properties of DCMS

$\vec{p}_2 = R^2_1 \vec{p}_1 \qquad \vec{p}_1 = R^1_0 \vec{p}_0 \qquad \vec{p}_2 = R^2_1 \vec{p}_1 = \vec{p}_2 = R^2_1 \underbrace{R^1_0 \vec{p}_0}$

Chaining $\qquad R^2_0 = R^2_1 R^1_0 \qquad \boxed{R^C_A = R^C_B R^B_A}$

Inverse $\qquad \vec{p}_2 = R^2_1 \vec{p}_1$

$$\underbrace{R^{2^{-1}}_1 \vec{p}_2 = \cancel{R^{2^{-1}}_1 R^2_1} \vec{p}_1 = R^1_2 \vec{p}_2}$$

$$R^1_2 = (R^2_1)^{-1} \qquad R^B_A = (R^A_B)^{-1}$$

since DCMs are orthonormal

$$(R^B_A)^{-1} = R^{B\,T}_A \qquad \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$\boxed{R^B_A = R^{A\,T}_B}$$

Euler Angles

Any rotation can be described by 3 Euler angles about 3 n-repeated axes

E.g. 3-1-3 rotation through α, β, γ
3-2-1 rotation through ψ, θ, ϕ

$$\vec{p}_B = R^B_E \vec{p}_E = \underbrace{R_1(\phi)}_{\text{roll}} \underbrace{R_2(\theta)}_{\text{pitch}} \underbrace{R_3(\psi)}_{\text{yaw}} \vec{p}_E$$

$$R^B_E = \begin{pmatrix} c_\theta c_\psi & c_\theta s_\psi & -s_\theta \\ s_\phi s_\theta c_\psi - c_\phi s_\psi & s_\phi s_\theta s_\psi + c_\phi c_\psi & s_\phi c_\theta \\ c_\phi s_\theta c_\psi + s_\phi s_\psi & c_\phi s_\theta s_\psi - s_\phi c_\psi & c_\phi c_\theta \end{pmatrix}$$

$\vec{p}^{\,E}_{pilot} = \vec{p}^{\,E}_{A/C} + \vec{p}^{\,B}_{pilot}$ ← abstract vectors

$$\boxed{\vec{p}^{\,E}_{pilot\,E} = \vec{p}^{\,E}_{A/C\,E} + R^E_B \vec{p}^{\,B}_{pilot\,B}}$$