Dynamics $\vec{f}$ $\vec{G}$

Translational Dynamics

Newtons 2nd Law

$$\vec{f} = m\vec{a}$$

$$\vec{f} = m\underline{\frac{d}{dt}}\vec{V}^E$$

$$\frac{d}{dt}\vec{V}_B^E = \dot{\vec{V}}_B^E + \vec{\omega}_B \times \vec{V}_B^E$$

$$m\left(\dot{\vec{V}}_B^E + \tilde{\omega}_B \vec{V}_B^E\right) = \vec{f}_B$$

$$\boxed{\dot{\vec{V}}_B^E = \frac{\vec{f}_B}{m} - \tilde{\omega}_B \vec{V}_B^E} \quad *$$

Rotational Dynamics

Euler's 2nd Law

angular momentum

$$\frac{d}{dt}\vec{h} = \vec{G}$$

moments

$$\vec{h} = I\vec{\omega}$$

moment of inertia

$$\mathbf{I} = \begin{pmatrix} \int (y^2+z^2)\,dm & -\int xy\,dm & -\int xz\,dm \\ -\int xy\,dm & \int (x^2+z^2)\,dm & -\int yz\,dm \\ -\int xz\,dm & -\int yz\,dm & \int (x^2+y^2)\,dm \end{pmatrix}$$

$$= \begin{pmatrix} I_x & -I_{xy} & -I_{xz} \\ -I_{yx} & I_y & -I_{yz} \\ -I_{zx} & -I_{zy} & I_z \end{pmatrix}$$

$\dot{\vec{\omega}}_B$

$$\frac{d}{dt}\vec{h}_B = \dot{\vec{h}}_B + \tilde{\omega}_B \vec{h}_B = \vec{G}_B$$

$$I\dot{\vec{\omega}}_B + \tilde{\omega}_B I\vec{\omega}_B = \vec{G}_B$$

$$\boxed{\dot{\vec{\omega}}_B = I^{-1}\left[\vec{G}_B - \tilde{\omega}_B I\vec{\omega}_B\right]}$$

$$\dot{\vec{x}} = \begin{bmatrix} \dot{p}_E^E \\ \dot{\vec{\Theta}} \\ \dot{\vec{V}}_B^E \\ \dot{\vec{\omega}}_B^E \end{bmatrix} \quad *$$

$$\dot{\vec{p}}_E = R_B^E \vec{V}_B^E$$

$$\dot{\vec{\Theta}} = T\vec{\omega}_B$$