Lateral Stability Derivatives
Sideslip, Coordinated Turn

Table 5.2
Summary—Lateral Derivatives

	C_y	C_l	C_n
β	$*-a_F \frac{S_F}{S}\left(1-\frac{\partial\sigma}{\partial\beta}\right)$	N.A.	$*a_F V_V\left(1-\frac{\partial\sigma}{\partial\beta}\right)$
$\hat{p}$	$*-a_F \frac{S_F}{S}\left(2\frac{z_F}{b}-\frac{\partial\sigma}{\partial\hat{p}}\right)$	N.A.	$*a_F V_V\left(2\frac{z_F}{b}-\frac{\partial\sigma}{\partial\hat{p}}\right)$
$\hat{r}$	$*a_F \frac{S_F}{S}\left(2\frac{l_F}{b}+\frac{\partial\sigma}{\partial\hat{r}}\right)$	$*a_F \frac{S_F}{S}\frac{z_F}{b}\left(2\frac{l_F}{b}+\frac{\partial\sigma}{\partial\hat{r}}\right)$	$*-a_F V_V\left(2\frac{l_F}{b}+\frac{\partial\sigma}{\partial\hat{r}}\right)$

*means contribution of the *tail only*, formula for wing-body not available; $V_F/V = 1$.
N.A. means no formula available.

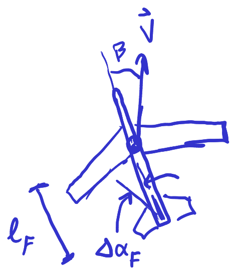
β derivatives

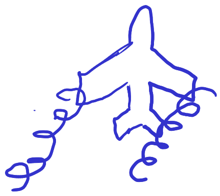
C_{l_β}: Dihedral Effect

1) Wing Height
2) Dihedral Angle
3) Vertical Tail
4) Wing Sweep

C_{n_β}: Weathervane Derivative Sign? (+)

Main Component: Vertical Tail





$$\alpha_F = -\beta + \sigma \quad \leftarrow \text{sidewash}$$

$$\rightarrow C_{L_F} = a_F(-\beta+\sigma)$$

$$N_F = -C_{L_F}\tfrac{1}{2}\rho V_F^2 S_F \ell_F$$

$$C_{n_F} = -C_{L_F}\frac{S_F\,\ell_F}{S\,b}\left(\frac{V_F}{V}\right)^2$$

$$= -V_V C_{L_F}\left(\frac{V_F}{V}\right)^2$$

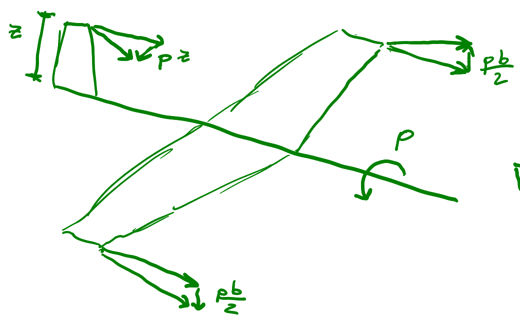
$$V_V \equiv \frac{S_F\,\ell_F}{S\,b}$$

$$C_{n_\beta} \approx \frac{\partial C_{n_F}}{\partial\beta}\Big|_0 = -V_V\left(\frac{V_F}{V}\right)^2\frac{\partial C_{L_F}}{\partial\beta}\Big|_0 = \boxed{V_V\, a_F\left(\frac{V_F}{V}\right)^2\left(1-\frac{\partial\sigma}{\partial\beta}\right)}$$

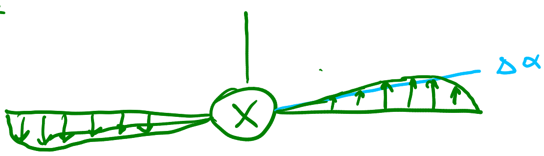
C_{y_β} (usually small) (similar derivation to C_{n_β})

p derivatives

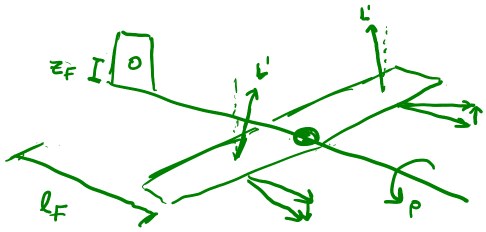
C_{l_p} roll damping derivative (−)



(From rear of aircraft)



C_{n_p} Wing Effects



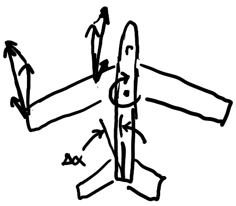
Tail Effect $\Delta\alpha_F = -\frac{p z_F}{u_0} + p\frac{\partial\sigma}{\partial p} = -\hat{p}\left(2\frac{z_F}{b} - \frac{\partial\sigma}{\partial\hat{p}}\right)$

$$(\Delta C_n)_{tail} = -\Delta C_{Y_F}\frac{S_F}{S}\frac{l_F}{b} = a_F V_V \hat{p}\left(2\frac{z_F}{b} + \frac{\partial\sigma}{\partial\hat{p}}\right)$$

$$(C_{n_p})_{tail} = a_F V_V\left(2\frac{z_F}{b} + \frac{\partial\sigma}{\partial\hat{p}}\right)$$

C_{Y_p} (usually small) (similar derivation to $(C_{n_p})_{tail}$)

<u>r-derivatives</u>



$$\Delta\alpha_F = \frac{r l_F}{u_0} + r\frac{\partial\sigma}{\partial r}$$

$$= \hat{r}\left(2\frac{l_F}{b} + \frac{\partial\sigma}{\partial\hat{r}}\right)$$

$$(C_{Y_r})_{tail} = a_F\frac{S_F}{S}\left(2\frac{l_F}{b} + \frac{\partial\sigma}{\partial\hat{r}}\right)$$

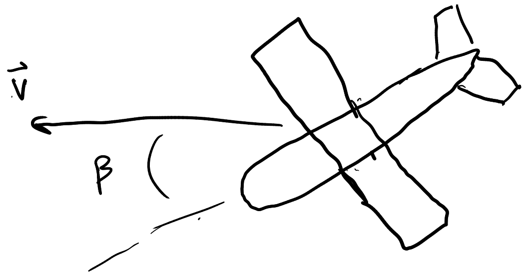
$$(C_{l_r})_{tail} = a_F\frac{S_F}{S}\frac{z_F}{b}\left(2\frac{l_F}{b} + \frac{\partial\sigma}{\partial\hat{r}}\right)$$

$$(C_{n_r})_{tail} = -a_F V_V\left(2\frac{l_F}{b} + \frac{\partial\sigma}{\partial\hat{r}}\right)$$

← yaw damping derivative

2 steady flight conditions (not "level")

Sideslip



$$\Delta Y + mg\,\Delta\phi = 0$$

$$\Delta L = 0$$
$$\Delta N = 0$$

can incorporate 1-engine out

$$\Delta Y = Y_v \Delta v + \cancelto{0}{Y_p p} + \cancelto{0}{Y_r r} + Y_{\delta_a}\delta_a + Y_{\delta_r}\delta_r$$

$$\underbrace{\begin{bmatrix} Y_{\delta_r} & 0 & mg \\ L_{\delta_r} & L_{\delta_a} & 0 \\ N_{\delta_r} & N_{\delta_a} & 0 \end{bmatrix}}_{\text{invert}} \begin{bmatrix} \delta_r \\ \delta_a \\ \Delta\phi \end{bmatrix} = -\begin{bmatrix} Y_v \\ L_v \\ N_v \end{bmatrix} \Delta v$$

βu_0

Steady Sideslip

For Piper Cherokee

inv

$$\begin{bmatrix} 280.7 & 0 & 2400 \\ 755.7 & -3821.9 & 0 \\ -3663.5 & 359 & 0 \end{bmatrix} \begin{bmatrix} \delta_r \\ \delta_a \\ \phi \end{bmatrix} = \begin{bmatrix} 2.991 \\ 102.93 \\ -19.394 \end{bmatrix} \cancel{v} \tag{7.8,4}$$

βu_0

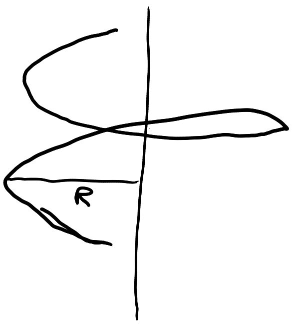
It is convenient to express the sideslip as an angle instead of a velocity. To do so we recall that $\beta \doteq v/u_0$, with u_0 given above as 112.3 fps. The solution of (7.8,4) is found to be

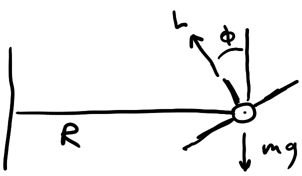
$$\delta_r/\beta = .303$$
$$\delta_a/\beta = -2.96$$
$$\phi/\beta = .104$$

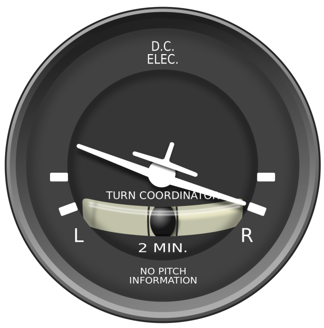
We see that a positive sideslip (to the right) of say 10° would entail $\boxed{\text{left rudder of } 3°}$ and $\boxed{\text{right aileron of } 29.6°}$. Clearly the main control action is the aileron displacement, without which the airplane would, as a result of the sideslip to the right, roll to the left. The $\boxed{\text{bank angle is seen to be only } 1°}$ to the right so the sideslip is almost flat.

Coordinated Turn

- angular velocity vector is constant and aligned with inertial z
- No aerodynamic forces in Y direction







$$\omega = \frac{u_0}{R}$$

$$a_n = \omega^2 R = \frac{u_0^2}{R}$$

$$\vec{\omega}_E = \begin{bmatrix} 0 \\ 0 \\ \omega \end{bmatrix}$$

$$\vec{\omega}_B = R_E^B \vec{\omega}_E = \begin{bmatrix} p \\ q \\ r \end{bmatrix} = \begin{bmatrix} -\sin\Theta \\ \sin\phi\cos\Theta \\ \cos\phi\cos\Theta \end{bmatrix} \omega$$

Since there is increased need for lift due to bank angle, involves both lat and lon dynamics

$$L\cos\phi = mg\cos\Theta$$
$$L\sin\phi = m\frac{u_0^2}{R}$$
$$= m\omega u_0$$

$$\rightarrow \quad \boxed{\tan\phi} = \frac{L\sin\phi}{L\cos\phi} = \frac{m\omega u_0}{mg\cos\Theta} = \boxed{\frac{\omega u_0}{g\cos\Theta}}$$

Assume: no wind, $\Theta = \gamma$, $v \ll u$, $\cos\Theta = 1$, $\sin\Theta = \Theta$, $V = u = u_0$

From EOM:

$$Z = -mg\cos\phi - mqu \qquad \begin{bmatrix} p \\ q \\ r \end{bmatrix} = \begin{bmatrix} -\Theta\omega \\ \sin\phi\,\omega \\ \cos\phi\,\omega \end{bmatrix}$$

load factor "g's" $\quad n = -\frac{Z}{mg} = \cos\phi + \frac{qu}{g}$

$$= \cos\phi + \frac{\omega u_0 \sin\phi}{g}$$
$$= \cos\phi + \tan\phi\sin\phi$$
$$\boxed{n = \sec\phi}$$

$$\rightarrow \quad n = \frac{L}{W}$$
$$\Delta C_L = \frac{L - mg}{\frac{1}{2}\rho V^2 S} = (n-1)C_W$$

Coordinated Turn

$$C_Y = 0 \qquad = C_{Y_\beta}\beta + C_{Y_p}\hat{p} + C_{Y_{\delta_r}}\delta_r + C_{Y_{\delta_a}}\delta_a$$
$$C_l = 0 \qquad \text{-- . . . --}$$
$$C_n = 0 \qquad \text{. . .}$$
$$C_m = 0 \qquad \text{. . . .}$$
$$C_L = (n-1)C_W \qquad \text{. . .}$$

$$\boxed{\begin{aligned} &\begin{bmatrix} C_{Y_\beta} & C_{Y_{\delta_r}} & 0 \\ C_{l_\beta} & C_{l_{\delta_r}} & C_{l_{\delta_a}} \\ C_{n_\beta} & C_{n_{\delta_r}} & C_{n_{\delta_a}} \end{bmatrix} \begin{bmatrix} \beta \\ \delta_r \\ \delta_a \end{bmatrix} = \begin{bmatrix} C_{Y_p} & C_{Y_r} \\ C_{l_p} & C_{l_r} \\ C_{n_p} & C_{n_r} \end{bmatrix} \begin{bmatrix} \Theta \\ -\cos\phi \end{bmatrix} \frac{\omega b}{2u_0} \\ &\begin{bmatrix} C_{m_\alpha} & C_{m_{\delta_e}} \\ C_{L_\alpha} & C_{L_{\delta_e}} \end{bmatrix} \begin{bmatrix} \Delta\alpha \\ \Delta\delta_e \end{bmatrix} = -\begin{bmatrix} C_{m_q} \\ C_{L_q} \end{bmatrix} \frac{\omega \bar{c} \sin\phi}{2u_0} + \begin{bmatrix} 0 \\ (n-1)C_W \end{bmatrix} \end{aligned}}$$