ASEN 3728 Aircraft Dynamics
Written Homework 5

Due date listed on Gradescope.

Question 1. Consider the following longitudinal dynamics of an F-16 flying at some trim condition with airspeed $V_a = 502.0$ ft/s, where $\Delta\mathbf{x} = [\Delta u, \Delta w, \Delta q, \Delta\theta]^T$:

$$\Delta\dot{\mathbf{x}} = \begin{bmatrix} \Delta\dot{u} \\ \Delta\dot{w} \\ \Delta\dot{q} \\ \Delta\dot{\theta} \end{bmatrix} = \begin{bmatrix} -0.020 & 0.016 & -0.65 & -32.17 \\ -0.13 & -1.019 & 454.21 & 0 \\ 0 & -0.0050 & -1.38 & 0 \\ 0 & 0 & 1.0 & 0 \end{bmatrix} \begin{bmatrix} \Delta u \\ \Delta w \\ \Delta q \\ \Delta\theta \end{bmatrix}$$

(a) Determine the natural frequency and damping ratio of the short period mode and the phugoid mode.

(b) Determine the eigenvectors (mode shapes) for the short period and phugoid modes. Normalize each eigenvector so that the term corresponding to the pitch angle is 1.0.

(c) Draw the phasor plots in terms of $\hat{u}$, $\hat{w}$, q, and $\Delta\theta$ for each mode. Label each component

Question 2. The control matrix $\mathbf{B}_{\delta_e}$ from the elevator angle to the state derivative for the F-16 from Problem 3 is

$$\mathbf{B}_{\delta_e} = \begin{bmatrix} -.244 \\ -1.46 \\ -.20 \\ 0 \end{bmatrix}$$

(a) Calculate the natural frequency and damping ratio of the short period mode approximation. To create the short period approximation use the terms in the longitudinal matrix from Problem 3.

(b) Consider the control law $\Delta\delta_e = -k_1\Delta q - k_2\Delta\theta$ where $k_1 = -5.0$ and $k_2 = -0.005$. Using the short period mode approximation, calculate the new eigenvalues of the short period mode with this control law.

(c) What are the eigenvalues of the full closed loop state space model using the control law from Part 2? How do they compare to the approximation from Part 2?

Question 3. There have been several historical flight emergencies in which a multi-engine airliner has suffered a complete loss of hydraulic fluid, but the pilots have attempted to fly the aircraft using only throttle controls. In this case, the average throttle across all engines, δ_t, is the relevant control input for the longitudinal dynamics, and managing the phugoid mode is an important challenge.

The phugoid mode for a large airliner can be approximated with

$$\begin{bmatrix} \Delta\dot{u} \\ \Delta\dot{\theta} \end{bmatrix} = \begin{bmatrix} -0.0025 & -30 \\ 0.0001 & 0 \end{bmatrix} \begin{bmatrix} \Delta u \\ \Delta\theta \end{bmatrix} + \begin{bmatrix} 10 \\ 0 \end{bmatrix} \Delta\delta_t.$$

(a) If the airspeed is $u_0 = 500\text{ft/s}$, what is the natural frequency of the phugoid mode predicted by the Lanchester approximation?

(b) Why is the phugoid mode more challenging to manage than the short period mode in this situation?

(c) Find the values of the natural frequency, ω_n, and damping ratio, ζ, of the uncontrolled (open-loop) system expressed in the matrix equation above. How does the natural frequency compare to the Lanchester approximation?

(d) Consider the control law $\Delta\delta_t = -k_u\Delta u - k_\theta\Delta\theta$. Write down the closed-loop controlled approximation of the phugoid mode in matrix form in terms of the gains.

(e) Choose one of the control gains, k_u or k_θ, and propose a value that will make the closed-loop damping ratio close to 0.5 (leave the other gain at 0).

(f) How would you qualitatively describe the control law calculated above to a pilot? Indicate your answer by circling the correct italicized words in the sentence below:

When the (*airspeed* | *pitch angle*) increases, (*increase* | *reduce*) the throttle.