

Longitudinal Modes

$$x(t) = \sum_i k_i \vec{v}_i e^{\lambda_i t}$$

Dynamics of Flight, Eq. (4.9,18)

$$\dot{x}_{lon} = A_{lon} x_{lon} + c_{lon}$$

$$x_{lon} = \begin{pmatrix} \Delta u \\ \Delta w \\ \Delta q \\ \Delta \theta \end{pmatrix} \quad c_{lon} = \begin{pmatrix} \frac{\Delta X_c}{m} \\ \frac{\Delta Z_c}{m - Z_{\dot{w}}} \\ \frac{\Delta M_c}{I_y} + \frac{M_{\dot{w}}}{I_y} \frac{\Delta Z_c}{(m - Z_{\dot{w}})} \\ 0 \end{pmatrix}$$



$$A_{lon} = \begin{pmatrix} \frac{X_u}{m} & \frac{X_w}{m} & 0 & -g \cos \theta_0 \\ \frac{Z_u}{m - Z_{\dot{w}}} & \frac{Z_w}{m - Z_{\dot{w}}} & \frac{Z_q + mu_0}{m - Z_{\dot{w}}} & \frac{-mg \sin \theta_0}{m - Z_{\dot{w}}} \\ \frac{1}{I_y} \left[M_u + \frac{M_{\dot{w}} Z_u}{m - Z_{\dot{w}}} \right] & \frac{1}{I_y} \left[M_w + \frac{M_{\dot{w}} Z_w}{m - Z_{\dot{w}}} \right] & \frac{1}{I_y} \left[M_q + \frac{M_{\dot{w}} (Z_q + mu_0)}{m - Z_{\dot{w}}} \right] & \frac{-M_{\dot{w}} mg \sin \theta_0}{I_y (m - Z_{\dot{w}})} \\ 0 & 0 & 1 & 0 \end{pmatrix}$$

$$h_0 = 40k ft$$

$$V_0 = 774 ft/s$$

$$\gamma_0 = \theta_0 = \alpha_0 = 0$$

$$\omega_n = \sqrt{a^2 + b^2}$$

$$\lambda_{1,2} = -0.37 \pm 0.89i$$

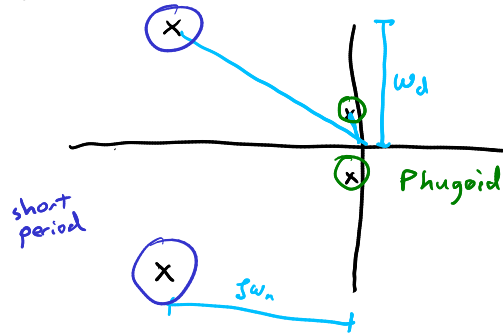
$$\omega_n = 0.96, \zeta = 0.38$$

$$\lambda_{3,4} = -0.0033 \pm 0.067i$$

$$\omega_n = 0.067$$

$$\zeta = 0.049$$

$$A_{lon} = \begin{pmatrix} -0.006868 & 0.01395 & 0 & -32.2 \\ -0.09055 & -0.3151 & 773.98 & 0 \\ 0.0001187 & -0.001026 & -0.4285 & 0 \\ 0 & 0 & 1 & 0 \end{pmatrix}$$



Eigenvectors

$$A \vec{v}_i = \vec{v}_i \lambda_i$$

$$(A - \lambda_i I) \vec{v}_i = 0$$

$$A = \begin{bmatrix} 0 & 1 \\ -2 & -3 \end{bmatrix}$$

$$|A - \lambda_i I| = 0 = \begin{vmatrix} 0 - \lambda & 1 \\ -2 & -3 - \lambda \end{vmatrix} = \lambda^2 + 3\lambda + 2 = 0$$

$$\lambda_1 = -1, \lambda_2 = -2$$

$$(A - \lambda_i I) \vec{v}_i = 0$$

$$\rightarrow \begin{bmatrix} 1 & 1 \\ -2 & -2 \end{bmatrix} \vec{v}_1 = 0$$

$$\vec{v}_1 = \begin{bmatrix} 1 \\ -1 \end{bmatrix} \propto$$

$$\vec{v}_2 = \begin{bmatrix} 1 \\ -2 \end{bmatrix}$$

$$\text{let } \vec{v}_1[1] = 1 \\ \vec{v}_1[2] = -1$$

$$\vec{v}_1[1] + \vec{v}_1[2] = 0$$

$$\vec{v}_{1,2} = \begin{bmatrix} 0.02 \pm 0.06i \\ 0.9996 \\ -0.0001 \pm 0.001i \\ 0.0011 \mp 0.0004i \end{bmatrix} \begin{matrix} \Delta u \\ \Delta w \\ \Delta q \\ \Delta \theta \end{matrix}$$

$$\vec{v}_{3,4} = \begin{bmatrix} -0.9983 \\ -0.057 \pm 0.0097i \\ -0.0001 \mp 0.000i \\ 0.0001 \pm 0.0021i \end{bmatrix}$$

$$x(t) = \sum_i k_i \vec{v}_i e^{\lambda_i t}$$

$$x(0) = \underline{\underline{Re(\vec{v}_i)}} = \sum_i k_i \vec{v}_i e^{\lambda_i \cdot 0} = 0.5 \vec{v}_1 + 0.5 \vec{v}_2$$

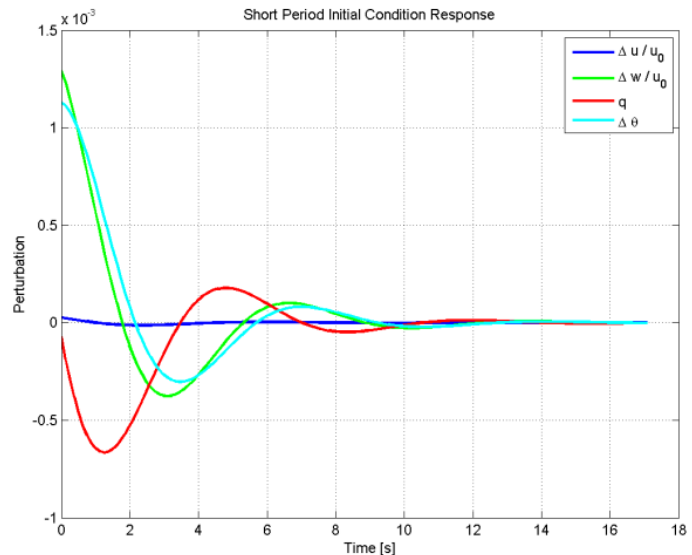
$$x(t) = 0.5 \vec{v}_1 e^{\lambda_1 t} + 0.5 \vec{v}_2 e^{\lambda_2 t}$$

$$\lambda_{1/2} = -0.372 \pm 0.888i$$

$$\zeta = 0.387$$

$$\omega_n = 0.962$$

$$x(0) = Re(v_1) = \begin{pmatrix} 0.0211 \\ 0.9996 \\ -0.0001 \\ 0.0011 \end{pmatrix}$$



$$z_1 = r_1 e^{i\phi_1} = r_1 \angle \phi_1$$

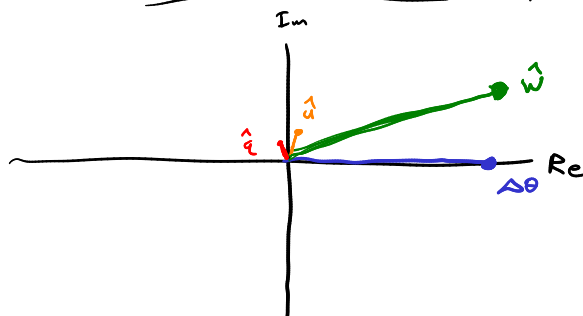
$$z_2 = r_2 e^{i\phi_2}$$

$$\frac{z_1}{z_2} = \frac{r_1}{r_2} e^{i(\phi_1 - \phi_2)}$$

$$\vec{v}_{1,2} / \vec{v}_{1,2}[u] = \vec{v}'_{1,2} = \begin{bmatrix} 0.02 \pm 0.06i / " " \\ 0.9996 / " " \\ -0.0001 \pm 0.001i / 0.0011 - 0.001i \\ 0.0011 \mp 0.0004i / 0.0011 - 0.0004i \end{bmatrix} = \begin{bmatrix} " " \\ " " \\ 1.0 \\ 1.0 \end{bmatrix}$$

$$\hat{v}_{1,2} = \begin{bmatrix} 0.016 \pm 0.024i \\ 1.02 \pm 0.36i \\ -0.0066 \pm 0.016i \\ 1.0 \end{bmatrix} \begin{matrix} \hat{u} = \frac{\Delta u}{u_0} \\ \hat{w} = \frac{\Delta w}{u_0} \approx \alpha \\ \hat{q} = \frac{\Delta q z \bar{z}}{u_0} \\ \Delta \theta \end{matrix}$$

Phasor Plot (\hat{v}_1 , Short Period)



$\vec{v} \begin{cases} \alpha \text{ and } \Delta \theta \text{ change in-phase} \\ \hat{u} \text{ relatively constant} \end{cases}$

$\lambda \begin{cases} \text{well-damped} \\ \text{high frequency} \end{cases}$

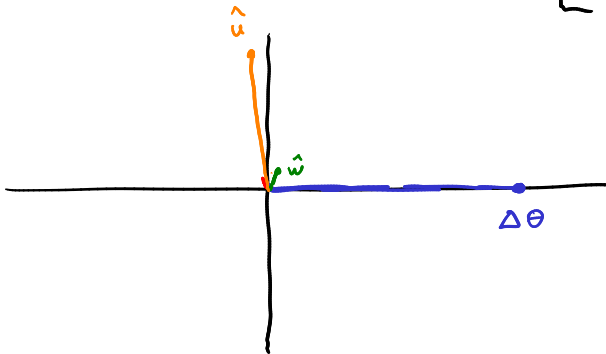
Phugoid Mode

phugoid = "flight"



side note:

$$\begin{aligned} z &= a + bi \\ &= r \angle \phi \\ &= r e^{i\phi} \\ r &= \sqrt{a^2 + b^2}; \phi = \text{atan2}(b, a) \end{aligned}$$

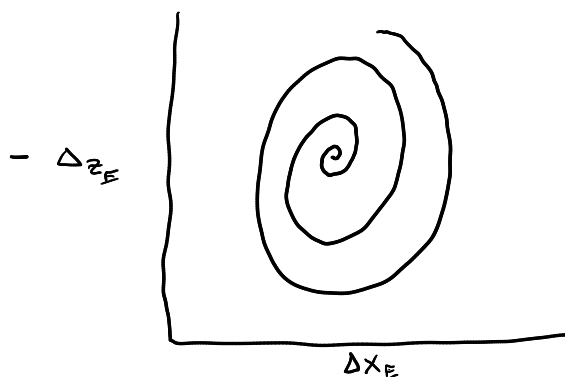
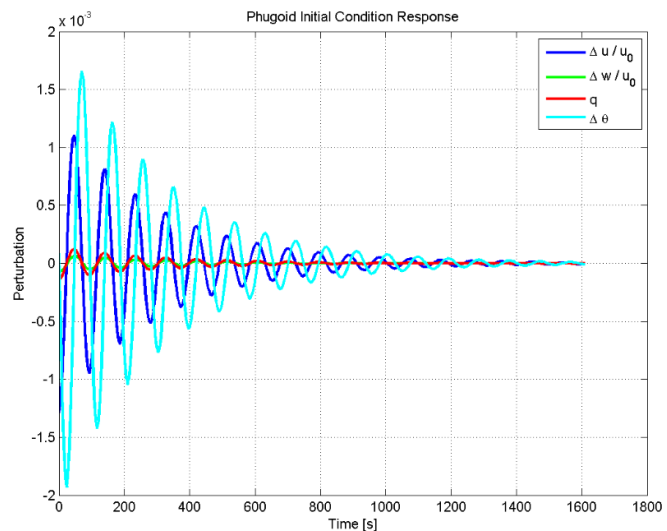


$$\hat{v}_{3,4} = \begin{bmatrix} 0.62 \angle 92^\circ \\ 0.036 \angle 83^\circ \\ 0.0012 \angle 93^\circ \\ 1.0 \angle 0 \end{bmatrix} \begin{matrix} \hat{u} \\ \hat{w} = \alpha \\ \hat{q} \\ \Delta\theta \end{matrix}$$

\vec{v} $\begin{cases} \text{Large } \hat{u}, \Delta\theta \text{ out-of-phase} \\ \text{oscillations (offset } \sim 90^\circ) \end{cases}$
 λ $\begin{cases} \text{small } \alpha \\ \text{low frequency} \\ \text{low damping} \end{cases}$

$$\begin{aligned} \lambda_{3/4} &= -3.29e-03 + 6.72e-02i \\ \zeta &= 0.0489 \leftarrow \text{poorly damped} \\ \omega_n &= 0.0673 \leftarrow \text{slow response} \end{aligned}$$

$$\mathbf{x}(0) = \text{Re}(\mathbf{v}_3) = \begin{pmatrix} -0.9983 \\ -0.0573 \\ -0.0001 \\ 0.0001 \end{pmatrix}$$



General Response

$$\mathbf{x}_0 = 0.8 \cdot \text{Re}(\mathbf{v}_{sp}) + 0.2 \cdot \text{Re}(\mathbf{v}_{ph})$$

