Lateral / Directional Dynamics

More Accurate
Fewer Assumption

Big Picture

$\dot{\vec{p}} = R_B^E \vec{V}_B^E$
$\dot{\vec{\Theta}} = \ldots$
$\dot{\vec{V}}_B^E =$
$\dot{\vec{\omega}}_B =$

Nonlinear Rigid Body EOM

$C_{L_{trim}} = C_{L_\alpha} \alpha_{trim} + C_{L_{\delta e}} \delta e_{trim}$

$C_{m_{trim}} = C_{m_0} + C_{m_\alpha} \alpha_{trim} + C_{m_{\delta e}} \delta e_{trim}$

Trim

Linearization

Stability Derivatives
Control Derivatives

A_{lon}, B_{lon} $\qquad$ A_{lat} B_{lat}

S. P. mode $\qquad$ Phugoid mode

More Assumptions
Easier to Analyze

$$\Delta\dot{\phi} = \Delta p + \Delta r \tan\theta_0$$

$$\Delta\dot{\theta} = \Delta q$$

$$\Delta\dot{u} = -g\cos\theta_0 \Delta\theta + \frac{\Delta X}{m}$$

$$\Delta\dot{v} = -u_0 \Delta r + g\cos\theta_0 \Delta\phi + \frac{\Delta Y}{m}$$

$$\Delta\dot{w} = u_0 \Delta q - g\sin\theta_0 \Delta\theta + \frac{\Delta Z}{m}$$

$$\Delta\dot{p} = \Gamma_3 \Delta L + \Gamma_4 \Delta N$$

$$\Delta\dot{q} = \frac{\Delta M}{I_y}$$

$$\Delta\dot{r} = \Gamma_4 \Delta L + \Gamma_8 \Delta N$$

$$Y = \frac{1}{2}\rho V^2 S C_y(\beta, p, r, \delta_a, \delta_r)$$
$$L = \frac{1}{2}\rho V^2 S b C_\ell(\beta, p, r, \delta_a, \delta_r)$$
$$N = \frac{1}{2}\rho V^2 S b C_n(\beta, p, r, \delta_a, \delta_r)$$

$$\hat{p} = \frac{pb}{2u_0}$$

$$Y \approx \frac{1}{2}\rho V_a^2 S \left[C_{Y_0} + C_{Y_\beta}\beta + C_{Y_p}\frac{b}{2V_a}p + C_{Y_r}\frac{b}{2V_a}r + C_{Y_{\delta_a}}\delta_a + C_{Y_{\delta_r}}\delta_r\right]$$
$$L \approx \frac{1}{2}\rho V_a^2 Sb \left[C_{l_0} + C_{l_\beta}\beta + C_{l_p}\frac{b}{2V_a}p + C_{l_r}\frac{b}{2V_a}r + C_{l_{\delta_a}}\delta_a + C_{l_{\delta_r}}\delta_r\right]$$
$$N \approx \frac{1}{2}\rho V_a^2 Sb \left[C_{n_0} + C_{n_\beta}\beta + C_{n_p}\frac{b}{2V_a}p + C_{n_r}\frac{b}{2V_a}r + C_{n_{\delta_a}}\delta_a + C_{n_{\delta_r}}\delta_r\right]$$

For symmetric aircraft, $C_{Y_0} = C_{l_0} = C_{n_0} = 0$

$$\dot{\mathbf{x}}_{lat} = \mathbf{A}_{lat}\mathbf{x}_{lat} + \mathbf{c}_{lat}$$

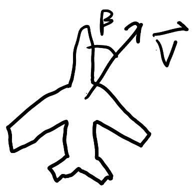
$$\mathbf{x}_{lat} = \begin{pmatrix} \Delta v \\ \Delta p \\ \Delta r \\ \Delta\phi \end{pmatrix} \qquad \mathbf{c}_{lat} = \begin{pmatrix} \dfrac{\Delta Y_c}{m} \\ \Gamma_3 \Delta L_c + \Gamma_4 \Delta N_c \\ \Gamma_4 \Delta L_c + \Gamma_8 \Delta N_c \\ 0 \end{pmatrix}$$

$$\mathbf{A}_{lat} = \begin{pmatrix} \dfrac{Y_v}{m} & \dfrac{Y_p}{m} & \left(\dfrac{Y_r}{m} - u_0\right) & g\cos\theta_0 \\ \Gamma_3 L_v + \Gamma_4 N_v & \Gamma_3 L_p + \Gamma_4 N_p & \Gamma_3 L_r + \Gamma_4 N_r & 0 \\ \Gamma_4 L_v + \Gamma_8 N_v & \Gamma_4 L_p + \Gamma_8 N_p & \Gamma_4 L_r + \Gamma_8 N_r & 0 \\ 0 & 1 & \tan\theta_0 & 0 \end{pmatrix}$$

$$\Gamma_1 = \frac{I_{xz}(I_x - I_y + I_z)}{\Gamma} \qquad \Gamma_4 = \frac{I_{xz}}{\Gamma} \qquad \Gamma_7 = \frac{I_x(I_x - I_y) + I_{xz}^2}{\Gamma}$$
$$\Gamma_2 = \frac{I_z(I_z - I_y) + I_{xz}^2}{\Gamma} \qquad \Gamma_5 = \frac{I_z - I_x}{I_y} \qquad \Gamma_8 = \frac{I_x}{\Gamma}$$
$$\Gamma_3 = \frac{I_z}{\Gamma} \qquad \Gamma_6 = \frac{I_{xz}}{I_y} \qquad \Gamma = I_x I_z - I_{xz}^2$$

Sideslip: β

$+\beta \Rightarrow$ wind coming from right



$$\beta = \sin^{-1}\frac{\Delta V}{V}$$
$$\beta \approx \frac{\Delta V}{u_0} = \hat{v}$$

$$L = \frac{1}{2}\rho V^2 S b C_\ell$$

$$C_{\ell_\beta} \equiv \frac{\partial C_\ell}{\partial \beta} = \frac{\partial C_\ell}{\partial \hat{v}}$$

$$L_v \equiv \frac{\partial L}{\partial v}\bigg|_0 = \frac{1}{2}\rho u_0^2 S b \frac{\partial C_\ell}{\partial v}\bigg|_0$$

$$= \frac{1}{2}\rho u_0^2 b S \frac{\partial C_\ell}{\partial \beta}\bigg|_0 \frac{\partial \beta}{\partial v}\bigg|_0$$

$$\beta = \hat{v} = \frac{\Delta v}{u_0}$$

$$\frac{\partial \beta}{\partial v} = \frac{1}{u_0}$$

$$\boxed{L_v = \frac{1}{2}\rho u_0 b S C_{\ell_\beta}}$$

$$C_{\ell_p} \equiv \frac{\partial C_\ell}{\partial \hat{p}}$$

$$\hat{p} = \frac{bp}{2u_0}$$

$$L_p \equiv \frac{\partial L}{\partial p}\bigg|_0 = \frac{1}{2}\rho u_0^2 b S \frac{\partial C_\ell}{\partial p}\bigg|_0$$

$$= \frac{1}{2}\rho u_0^2 b S \frac{\partial C_\ell}{\partial \hat{p}}\bigg|_0 \frac{\partial \hat{p}}{\partial p}\bigg|_0$$

$$\boxed{L_p = \frac{1}{4}\rho u_0 b^2 S C_{\ell_p}}$$

Table 4.5
Lateral Dimensional Derivatives

	Y	L	N
v	$\frac{1}{2}\rho u_0 S C_{y_\beta}$	$\frac{1}{2}\rho u_0 b S C_{l_\beta}$	$\frac{1}{2}\rho u_0 b S C_{n_\beta}$
p	$\frac{1}{4}\rho u_0 b S C_{y_p}$	$\frac{1}{4}\rho u_0 b^2 S C_{l_p}$	$\frac{1}{4}\rho u_0 b^2 S C_{n_p}$
r	$\frac{1}{4}\rho u_0 b S C_{y_r}$	$\frac{1}{4}\rho u_0 b^2 S C_{l_r}$	$\frac{1}{4}\rho u_0 b^2 S C_{n_r}$

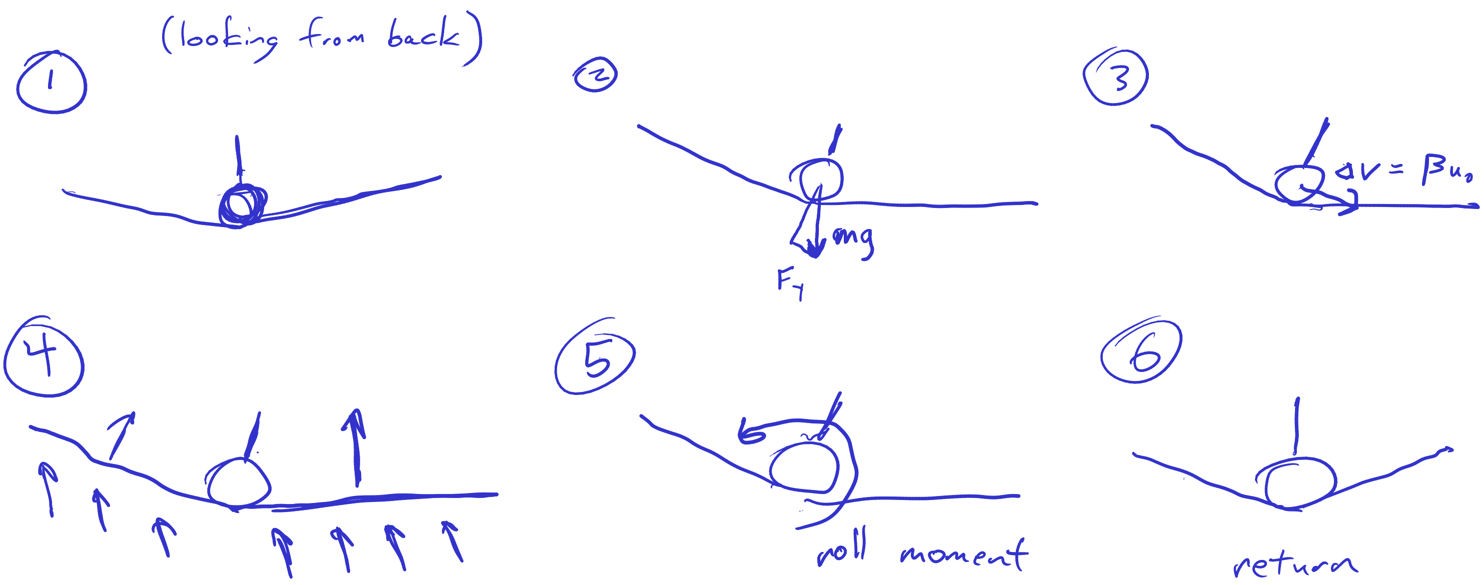
Table 5.2
Summary—Lateral Derivatives

	C_y	C_l	C_n
β	$*-a_F \frac{S_F}{S}\left(1 - \frac{\partial\sigma}{\partial\beta}\right)$	N.A.	$*a_F V_V\left(1 - \frac{\partial\sigma}{\partial\beta}\right)$
$\hat{p}$	$*-a_F \frac{S_F}{S}\left(2\frac{z_F}{b} - \frac{\partial\sigma}{\partial\hat{p}}\right)$	N.A.	$*a_F V_V\left(2\frac{z_F}{b} - \frac{\partial\sigma}{\partial\hat{p}}\right)$
$\hat{r}$	$*a_F \frac{S_F}{S}\left(2\frac{l_F}{b} + \frac{\partial\sigma}{\partial\hat{r}}\right)$	$*a_F \frac{S_F}{S}\frac{z_F}{b}\left(2\frac{l_F}{b} + \frac{\partial\sigma}{\partial\hat{r}}\right)$	$*-a_F V_V\left(2\frac{l_F}{b} + \frac{\partial\sigma}{\partial\hat{r}}\right)$

*means contribution of the *tail only*, formula for wing-body not available; $V_F/V = 1$.
N.A. means no formula available.

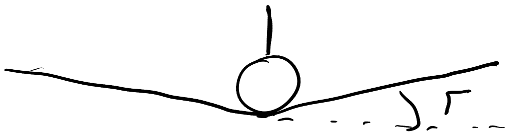
C_{l_β} : Dihedral Effect

sign : negative

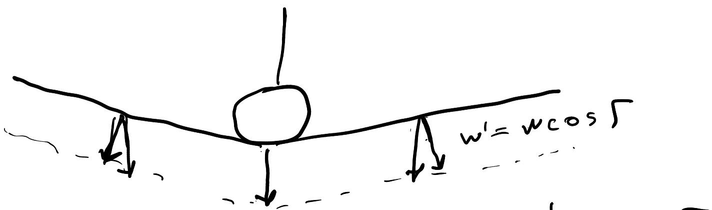


4 significant factors

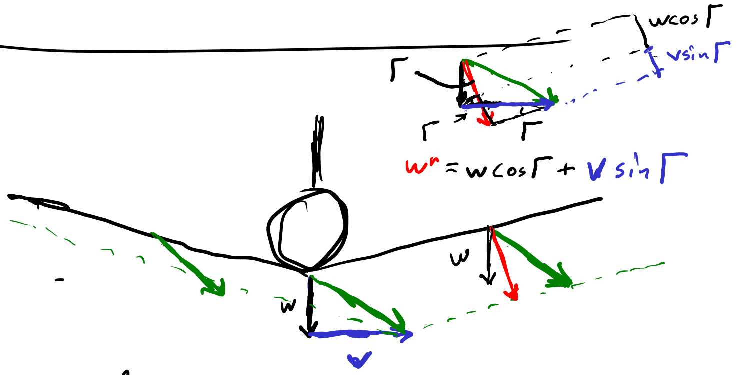
1. Dihedral Angle
2. Wing Height
3. Wing Sweep
4. Vertical Tail



1. Dihedral Angle



$w = V\alpha$ $\qquad \alpha' = \frac{w'}{V} \alpha \cos\Gamma$



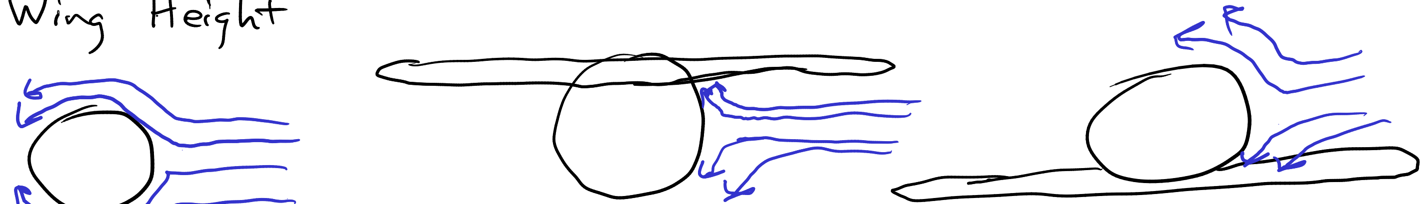
$\alpha^l = \alpha - \Gamma\beta$

$w^r = w + v\Gamma$

$\alpha^r = \alpha + \Gamma\beta$

$$\boxed{C^{\Gamma}_{l_\beta} \propto \Gamma}$$

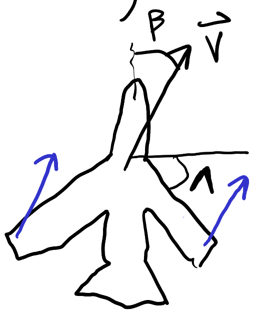
2. Wing Height



high wing $\Rightarrow -C_{l_\beta}$

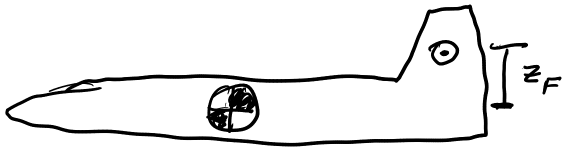
low wing $\Rightarrow +C_{l_\beta}$

3. Wing Sweep



$$C_{l_\beta}^{\Lambda} \propto 2 C_L V^2 \sin 2\Lambda$$

4. Tail



$$\Delta C_l^F = C_{L_F}\frac{S_F z_F}{Sb} = a_F(-\beta+\sigma)\frac{S_F z_F}{Sb}$$

$$C_{l_\beta}^F = -a_F\left(1-\frac{\partial\sigma}{\partial\beta}\right)\frac{S_F z_F}{Sb}\left(\frac{V_F}{V}\right)^2$$