Longitudinal Modes

$$x(t) = \sum_i k_i \vec{v}_i e^{\lambda_i t}$$

Dynamics of Flight, Eq. (4.9,18) $\qquad \dot{\mathbf{x}}_{lon} = \mathbf{A}_{lon}\mathbf{x}_{lon} + \mathbf{c}_{lon}$

$$\mathbf{x}_{lon} = \begin{pmatrix} \Delta u \\ \Delta w \\ \Delta q \\ \Delta\theta \end{pmatrix} \qquad \mathbf{c}_{lon} = \begin{pmatrix} \dfrac{\Delta X_c}{m} \\ \dfrac{\Delta Z_c}{m - Z_{\dot{w}}} \\ \dfrac{\Delta M_c}{I_y} + \dfrac{M_{\dot{w}}}{I_y}\dfrac{\Delta Z_c}{(m - Z_{\dot{w}})} \\ 0 \end{pmatrix}$$

$$\mathbf{A}_{lon} = \begin{pmatrix} \dfrac{X_u}{m} & \dfrac{X_w}{m} & 0 & -g\cos\theta_0 \\ \dfrac{Z_u}{m - Z_{\dot{w}}} & \dfrac{Z_w}{m - Z_{\dot{w}}} & \dfrac{Z_q + mu_0}{m - Z_{\dot{w}}} & \dfrac{-mg\sin\theta_0}{m - Z_{\dot{w}}} \\ \dfrac{1}{I_y}\left[M_u + \dfrac{M_{\dot{w}}Z_u}{m - Z_{\dot{w}}}\right] & \dfrac{1}{I_y}\left[M_w + \dfrac{M_{\dot{w}}Z_w}{m - Z_{\dot{w}}}\right] & \dfrac{1}{I_y}\left[M_q + \dfrac{M_{\dot{w}}(Z_q + mu_0)}{m - Z_{\dot{w}}}\right] & \dfrac{-M_{\dot{w}}mg\sin\theta_0}{I_y(m - Z_{\dot{w}})} \\ 0 & 0 & 1 & 0 \end{pmatrix}$$

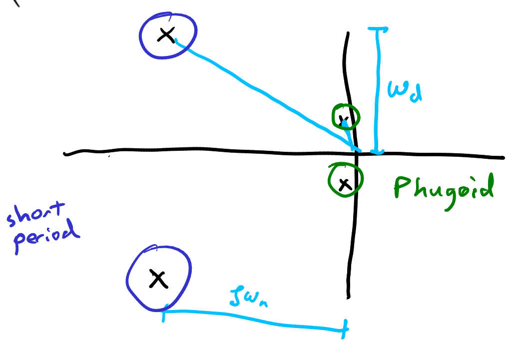
$h_0 = 40\text{k ft}$
$V_0 = 774\text{ ft/s}$
$\gamma_0 = \theta_0 = \alpha_0 = 0$

$$\mathbf{A}_{lon} = \begin{pmatrix} -0.006868 & 0.01395 & 0 & -32.2 \\ -0.09055 & -0.3151 & 773.98 & 0 \\ 0.0001187 & -0.001026 & -0.4285 & 0 \\ 0 & 0 & 1 & 0 \end{pmatrix}$$

$\omega_n = \sqrt{a^2 + b^2}$

$$\lambda_{1,2} = \overset{a}{-0.37} \overset{\pm}{\pm} \overset{bi}{0.89i}$$
$\omega_n = 0.96, \ \zeta = 0.38$

$$\lambda_{3,4} = -0.0033 \pm 0.067i$$
$\omega_n = 0.067$
$\zeta = 0.049$



Eigenvectors

$A\vec{v}_i = \vec{v}_i \lambda_i$

$(A - \lambda_i I)v_i = 0$

$$A = \begin{bmatrix} 0 & 1 \\ -2 & -3 \end{bmatrix}$$

$$|A - \lambda_i I| = 0 = \begin{vmatrix} 0-\lambda & 1 \\ -2 & -3-\lambda \end{vmatrix} = \lambda^2 + 3\lambda + 2 = 0 \qquad \lambda_1 = -1, \ \lambda_2 = -2$$

$(A - \lambda_1 I)v_1 = 0$

$\rightarrow \boxed{\begin{bmatrix} 1 & 1 \\ -2 & -2 \end{bmatrix} v_1} = 0$

let $v_1[1] = 1$
$v_1[2] = -1$

$\boxed{v_1[1] + v_1[2] = 0}$

$\vec{v}_1 = \begin{bmatrix} 1 \\ -1 \end{bmatrix} \alpha$

$\vec{v}_2 = \begin{bmatrix} 1 \\ -2 \end{bmatrix}$

$$\vec{v}_{1,2} = \begin{array}{c c} \begin{bmatrix} 0.02 \pm 0.016i \\ 0.9996 \\ -0.0001 \pm 0.001i \\ 0.0011 \mp 0.0004i \end{bmatrix} & \begin{array}{c} \Delta u \\ \Delta w \\ \Delta q \\ \Delta \theta \end{array} \end{array} \qquad \vec{v}_{3,4} = \begin{bmatrix} -0.9983 \\ -0.057 \pm 0.0097 \\ -0.0001 \mp 0.000i \\ 0.0001 \pm 0.0021i \end{bmatrix}$$

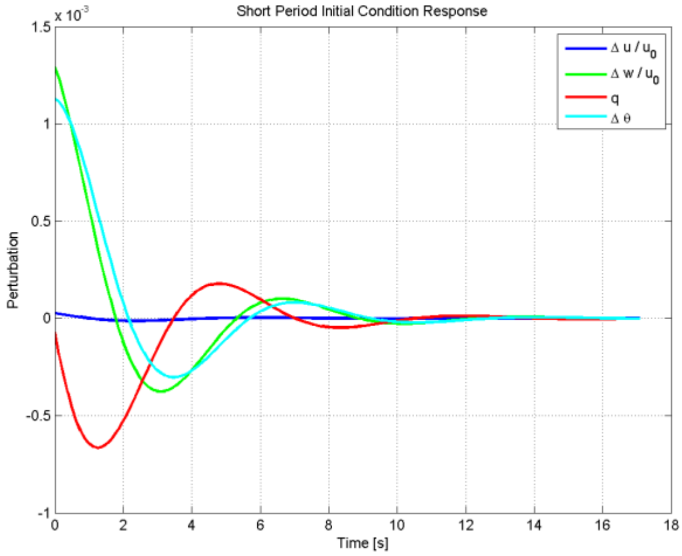
$$x(t) = \sum_i k_i \vec{v}_i e^{\lambda_i t}$$

$$x(0) = \mathrm{Re}(\vec{v}_1) = \sum_i k_i \vec{v}_i \cancel{e^{\lambda_i 0}} \to 1 = 0.5\vec{v}_1 + 0.5\vec{v}_2$$

$$x(t) = 0.5\vec{v}_1 e^{\lambda_1 t} + 0.5\vec{v}_2 e^{\lambda_2 t}$$

$$\lambda_{1/2} = -.372 + .888i$$
$$\zeta = 0.387$$
$$\omega_n = 0.962$$

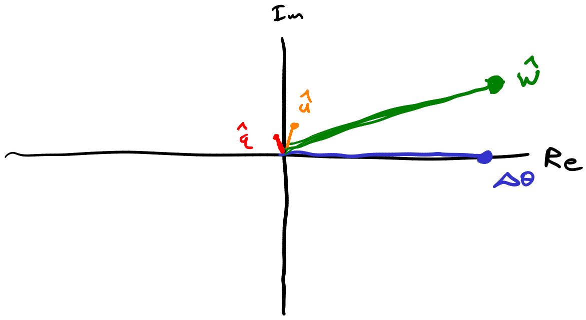
$$\mathbf{x}(0) = Re(\mathbf{v}_1) = \begin{pmatrix} 0.0211 \\ 0.9996 \\ -0.0001 \\ 0.0011 \end{pmatrix}$$



$$\hat{v}_{1,2} = \begin{bmatrix} 0.016 \pm 0.024i \\ 1.02 \pm 0.36i \\ -0.0066 \pm 0.016i \\ 1.0 \end{bmatrix}$$

$$\hat{u} = \frac{\Delta u}{u_0}$$
$$\hat{w} = \frac{\Delta w}{u_0} \approx \alpha$$
$$\hat{q} = \frac{\Delta q\, 2\bar{c}}{u_0}$$
$$\Delta\theta$$

Phasor Plot ($\hat{v}_1$, Short Period)



$\vec{v}$ { α and $\Delta\theta$ change in-phase; $\hat{u}$ relatively constant }

λ { well-damped; high frequency }

Phugoid Mode

phugoid = "flight"

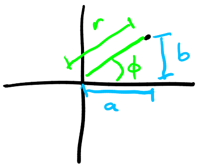
side note:

$z = a + bi$

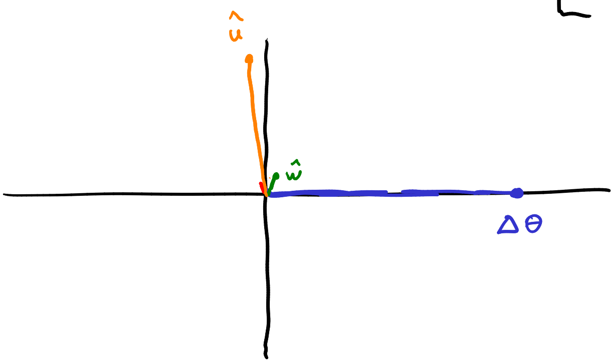
$= r \angle \phi$

$= r e^{i\phi}$

$r = \sqrt{a^2+b^2}\ ;\ \phi = \mathrm{atan2}(b, a)$



$$\hat{v}_{3,4} = \begin{array}{cc} \left[\begin{array}{c} 0.62 \angle 92^\circ \\ 0.036 \angle 83^\circ \\ 0.0012 \angle 93^\circ \\ 1.0 \angle 0 \end{array}\right] & \begin{array}{l} \hat{u} \\ \hat{w} = \alpha \\ \hat{q} \\ \Delta\theta \end{array} \end{array}$$

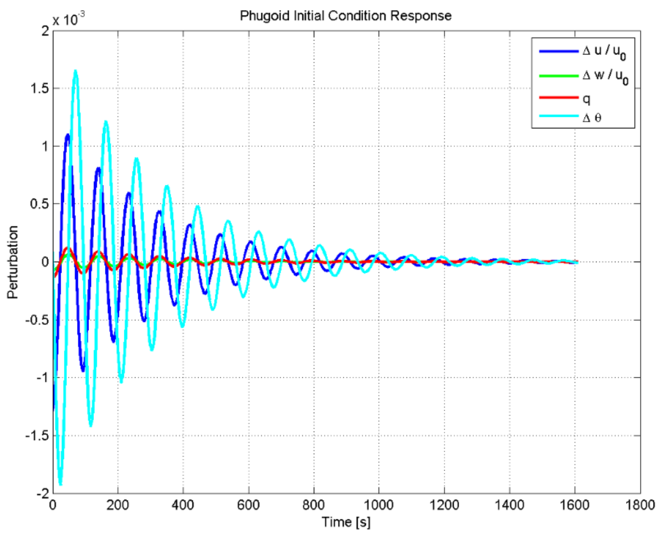


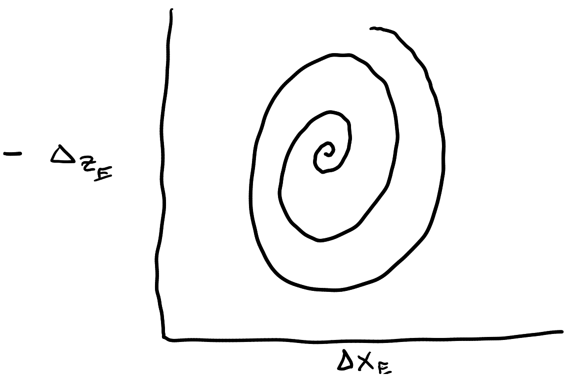
$\vec{v}$ $\begin{cases} \text{Large } \hat{u}, \Delta\theta \text{ out-of phase oscillations (offset} \sim 90^\circ) \\ \text{small } \alpha \end{cases}$

λ $\begin{cases} \text{low frequency} \\ \text{low damping} \end{cases}$

$$\lambda_{3/4} = -3.29e-03 + 6.72e-02i$$
$\zeta = 0.0489$ ← poorly damped
$\omega_n = 0.0673$ ← slow response

$$\mathbf{x}(0) = Re(\mathbf{v}_3) = \begin{pmatrix} -0.9983 \\ -0.0573 \\ -0.0001 \\ 0.0001 \end{pmatrix}$$





General Response

$$\mathbf{x}_0 = 0.8 \cdot Re(\mathbf{v}_{sp}) + 0.2 \cdot Re(\mathbf{v}_{ph})$$