Vector Clarity

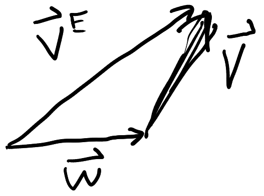
$$\vec{V}^{E}_{B}$$

E: "frame of reference", "with respect to", "relative to"

B: "coordinate frame", "expressed in", "written in"

$$\vec{V}^{E} = \vec{V}^{(W)} + \vec{W}^{(E)}$$

by convention



True in <u>Any</u> coordinate system as long as all in <u>same</u> coordinate system

$$\vec{V}^{E}_{B} = \vec{V}_{B} + \vec{W}_{B} \qquad \vec{V}^{E}_{E} = \vec{V}_{E} + \vec{W}_{E}$$

$$\begin{bmatrix} u^E \\ v^E \\ w^E \end{bmatrix} \quad \begin{bmatrix} u \\ v \\ w \end{bmatrix} + \cancel{\begin{bmatrix} \; \end{bmatrix}}$$

Kinematics

Kinematics : "Geometry of Motion" (no forces) $<$ Translational, Rotational

Dynamics : Effects of forces and moments on objects

Vector Derivatives

$\rightarrow \frac{d}{dt}\vec{p} \equiv$ time rate of change of $\vec{p}$

$\frac{d}{dt}\vec{p} = \vec{v}^E$

$$\vec{p}_B = \begin{bmatrix} x_B \\ y_B \\ z_B \end{bmatrix} \quad \nearrow \quad \dot{\vec{p}}_B \equiv \begin{bmatrix} \dot{x}_B \\ \dot{y}_B \\ \dot{z}_B \end{bmatrix}$$

$$\vec{V}_B^E = \begin{bmatrix} u^E \\ v^E \\ w^E \end{bmatrix} \overset{?}{=} \dot{\vec{p}}_B = \begin{bmatrix} \dot{x}_B \\ \dot{y}_B \\ \dot{z}_B \end{bmatrix}$$

Not Always True

$\hat{\imath}^B$

$\hat{\imath}_B^B = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$

$\hat{\imath}_B \equiv \hat{\imath}_B^B$

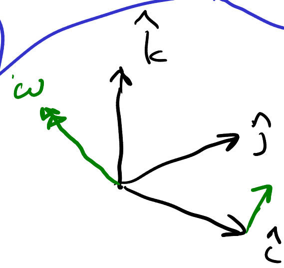
$$\vec{p}_B = p_x \hat{\imath}_B + p_y \hat{\jmath}_B + p_z \hat{k}_B$$

$$\dot{\vec{p}}_B = \dot{p}_x \hat{\imath}_B + \dot{p}_y \hat{\jmath}_B + \dot{p}_z \hat{k}_B$$

$$\frac{d}{dt}(uv) = u\frac{d}{dt}v + v\frac{d}{dt}u$$

$$\left(\frac{d}{dt}\vec{p}\right)_B = \dot{p}_x \hat{\imath}_B + p_x \frac{d}{dt}\hat{\imath}_B + \dot{p}_y \hat{\jmath}_B + p_y \frac{d}{dt}\hat{\jmath}_B + \dot{p}_z \hat{k}_B + p_z \frac{d}{dt}\hat{k}_B$$

What is $\frac{d}{dt}\hat{\imath}_B$



$\frac{d}{dt}\hat{\imath} = \vec{\omega} \times \hat{\imath}$

$\vec{\omega}_B = \vec{\omega}_B^E = \begin{bmatrix} p \\ q \\ r \end{bmatrix}$

$$= p_x(\vec{\omega}_B \times \hat{\imath}_B) + p_y(\vec{\omega}_B \times \hat{\jmath}_B) + p_z(\vec{\omega}_B \times \hat{k}_B)$$

$$= \vec{\omega}_B \times \vec{p}_B$$

$$\boxed{\frac{d}{dt}\vec{p}_B = \dot{\vec{p}}_B + \vec{\omega}_B \times \vec{p}_B} = \dot{\vec{p}}_B + \tilde{\omega}_B \vec{p}_B \qquad \tilde{\omega}_B = \begin{bmatrix} 0 & -r & q \\ r & 0 & -p \\ -q & p & 0 \end{bmatrix}$$

Kinematic Transport Theorem

Equations of motion for a 3D body

$\dot{\vec{x}} = f(t, \vec{x})$ → Need

Aircraft State

$$\vec{x} = \begin{bmatrix} x_E \\ y_E \\ z_E \\ \phi \\ \theta \\ \psi \\ u^E \\ v^E \\ w^E \\ p \\ q \\ r \end{bmatrix} \quad \begin{array}{l} \vec{p}_E = \vec{p}_E^{\,E} \\ \vec{\Theta} \text{ pseudo-vector} \\ \vec{V}_B^E \\ \vec{\omega}_B^E \end{array}$$

Need

$\dot{\vec{p}}_E$, $\dot{\vec{\Theta}}$ — Kinematic

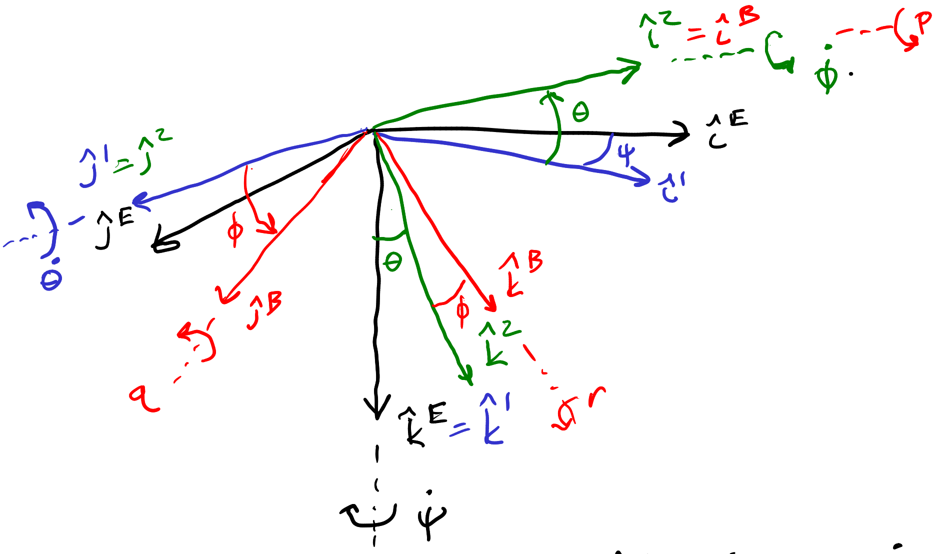
$\dot{\vec{V}}_B^E$, $\dot{\vec{\omega}}_B$ — Dynamics

Translational Kinematics

$$\dot{\vec{p}}_E^{\,E} = \frac{d}{dt}\vec{p}_E - \cancelto{0}{{}^E\vec{\omega}_E^E} \times \vec{p}_E^{\,E} = \vec{V}_E^E = R_B^E \vec{V}_B^E$$

$$\boxed{\dot{\vec{p}}_E^{\,E} = R_B^E \vec{V}_B^E}$$

Rotational Kinematics



$$\vec{\omega}_B = \begin{bmatrix} p \\ q \\ r \end{bmatrix}$$

$$\vec{\omega} = \dot\psi \hat{k}^E + \dot\theta \hat{j}^1 + \dot\phi \hat{i}^B$$

$$\vec{\omega}_B = \dot\psi \hat{k}^E_B + \dot\theta \hat{j}^1_B + \dot\phi \hat{i}^B_B$$

$$= \dot\psi R_E^B \underbrace{\hat{k}^E_E}_{\begin{bmatrix}0\\0\\1\end{bmatrix}} + \dot\theta R_1^B \underbrace{\hat{j}^1_1}_{\begin{bmatrix}0\\1\\0\end{bmatrix}} + \dot\phi \hat{i}^B_B$$

$$= R_1(\phi) R_2(\theta) R_3(\psi) \begin{bmatrix} 0 \\ 0 \\ \dot\psi \end{bmatrix} + R_1(\phi) R_2(\theta) \begin{bmatrix} 0 \\ \dot\theta \\ 0 \end{bmatrix} + \begin{bmatrix} \dot\phi \\ 0 \\ 0 \end{bmatrix}$$

$$R_E^B = R_1(\phi) R_2(\theta) R_3(\psi)$$

$$R_1^B = R_1(\phi) R_2(\theta)$$

$$\begin{bmatrix} p \\ q \\ r \end{bmatrix} = \underline{\begin{bmatrix} 1 & 0 & -s\theta \\ 0 & c\phi & s\phi\, c\theta \\ 0 & -s\phi & c\phi\, c\theta \end{bmatrix}} \begin{bmatrix} \dot\phi \\ \dot\theta \\ \dot\psi \end{bmatrix}$$

invert

$$\begin{bmatrix} \dot\phi \\ \dot\theta \\ \dot\psi \end{bmatrix} = \begin{bmatrix} 1 & \sin\phi\tan\theta & \cos\phi\tan\theta \\ 0 & \cos\phi & -\sin\phi \\ 0 & \sin\phi\sec\theta & \cos\phi\sec\theta \end{bmatrix} \begin{bmatrix} p \\ q \\ r \end{bmatrix}$$

$\sec x = \frac{1}{\cos x}$

$\dot{\vec{o}} = T \vec{\omega}_B$

attitude influence matrix