Lateral Stability Derivatives
Sideslip, Coordinated Turn

Table 5.2
Summary—Lateral Derivatives

	C_y	C_l	C_n
β	$*-a_F \frac{S_F}{S}\left(1-\frac{\partial\sigma}{\partial\beta}\right)$	N.A.	$*a_F V_V\left(1-\frac{\partial\sigma}{\partial\beta}\right)$
$\hat{p}$	$*-a_F \frac{S_F}{S}\left(2\frac{z_F}{b}-\frac{\partial\sigma}{\partial\hat{p}}\right)$	N.A.	$*a_F V_V\left(2\frac{z_F}{b}-\frac{\partial\sigma}{\partial\hat{p}}\right)$
$\hat{r}$	$*a_F \frac{S_F}{S}\left(2\frac{l_F}{b}+\frac{\partial\sigma}{\partial\hat{r}}\right)$	$*a_F \frac{S_F}{S}\frac{z_F}{b}\left(2\frac{l_F}{b}+\frac{\partial\sigma}{\partial\hat{r}}\right)$	$*-a_F V_V\left(2\frac{l_F}{b}+\frac{\partial\sigma}{\partial\hat{r}}\right)$

*means contribution of the *tail only*, formula for wing-body not available; $V_F/V = 1$.
N.A. means no formula available.

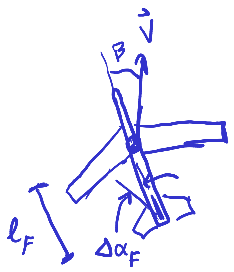
β derivatives

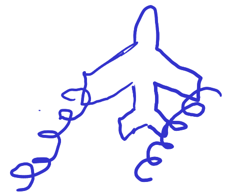
C_{l_β}: Dihedral Effect

1) Wing Height
2) Dihedral Angle
3) Vertical Tail
4) Wing Sweep

C_{n_β}: Weathervane Derivative Sign? (+)

Main Component: Vertical Tail





$$\alpha_F = -\beta + \sigma \leftarrow \text{sidewash}$$

$$\rightarrow C_{L_F} = a_F(-\beta+\sigma)$$

$$N_F = -C_{L_F}\tfrac{1}{2}\rho V_F^2 S_F l_F$$

$$C_{n_F} = -C_{L_F}\frac{S_F l_F}{S\,b}\left(\frac{V_F}{V}\right)^2$$

$$= -V_V C_{L_F}\left(\frac{V_F}{V}\right)^2$$

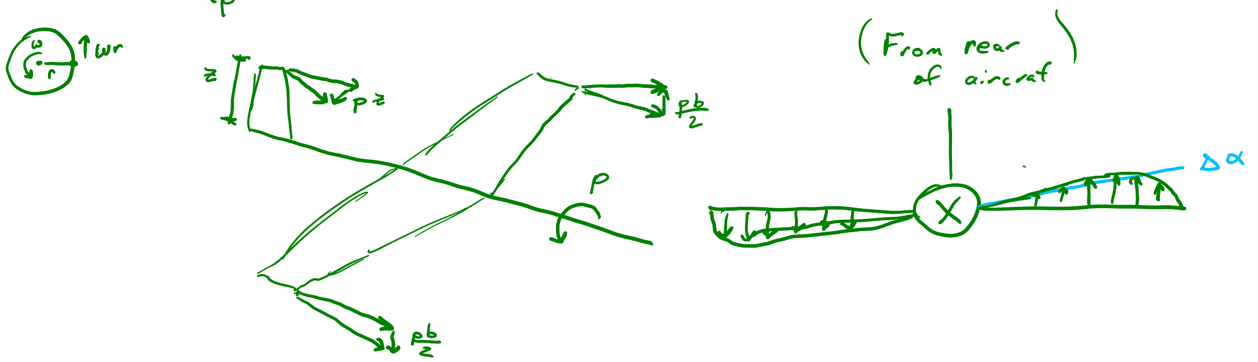
$$V_V \equiv \frac{S_F l_F}{S\,b}$$

$$C_{n_\beta} \approx \left.\frac{\partial C_{n_F}}{\partial\beta}\right|_0 = -V_V\left(\frac{V_F}{V}\right)^2\left.\frac{\partial C_{L_F}}{\partial\beta}\right|_0 = \boxed{V_V a_F\left(\frac{V_F}{V}\right)^2\left(1-\frac{\partial\sigma}{\partial\beta}\right)}$$

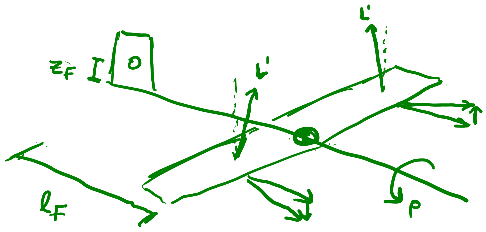
C_{y_β} (usually small) (similar derivation to C_{n_β})

p derivatives

C_{l_p} roll damping derivative (−)



C_{n_p} Wing Effects



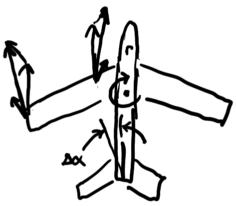
Tail Effect $\Delta\alpha_F = -\frac{p z_F}{u_0} + p\frac{\partial\sigma}{\partial p} = -\hat{p}\left(2\frac{z_F}{b} - \frac{\partial\sigma}{\partial\hat{p}}\right)$

$$(\Delta C_n)_{tail} = -\Delta C_{Y_F}\frac{S_F}{S}\frac{\ell_F}{b} = a_F V_V \hat{p}\left(2\frac{z_F}{b} + \frac{\partial\sigma}{\partial\hat{p}}\right)$$

$$(C_{n_p})_{tail} = a_F V_V\left(2\frac{z_F}{b} + \frac{\partial\sigma}{\partial\hat{p}}\right)$$

C_{Y_p} (usually small) (similar derivation to $(C_{n_p})_{tail}$)

r-derivatives



$$\Delta\alpha_F = \frac{r\ell_F}{u_0} + r\frac{\partial\sigma}{\partial r} = \hat{r}\left(2\frac{\ell_F}{b} + \frac{\partial\sigma}{\partial\hat{r}}\right)$$

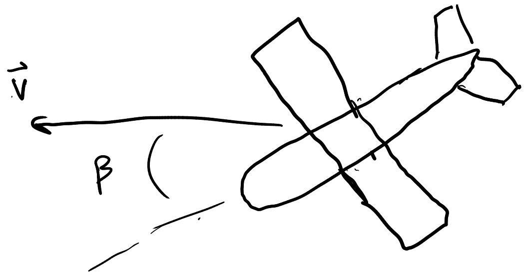
$$(C_{Y_r})_{tail} = a_F\frac{S_F}{S}\left(2\frac{\ell_F}{b} + \frac{\partial\sigma}{\partial\hat{r}}\right)$$

$$(C_{\ell_r})_{tail} = a_F\frac{S_F}{S}\frac{z_F}{b}\left(2\frac{\ell_F}{b} + \frac{\partial\sigma}{\partial\hat{r}}\right)$$

$$(C_{n_r})_{tail} = -a_F V_V\left(2\frac{\ell_F}{b} + \frac{\partial\sigma}{\partial\hat{r}}\right)$$ ← yaw damping derivative

2 steady flight conditions (not "level")

Sideslip



$$\Delta Y + mg\,\Delta\phi = 0$$

$$\Delta L = 0$$
$$\Delta N = 0$$

← can incorporate 1-engine out

$$\Delta Y = Y_v \Delta v + \cancel{Y_p p}^{\,0} + \cancel{Y_r r}^{\,0} + Y_{\delta_a}\delta_a + Y_{\delta_r}\delta_r$$

$$\underbrace{\begin{bmatrix} Y_{\delta_r} & 0 & mg \\ L_{\delta_r} & L_{\delta_a} & 0 \\ N_{\delta_r} & N_{\delta_a} & 0 \end{bmatrix}}_{\text{invert}} \begin{bmatrix} \delta_r \\ \delta_a \\ \Delta\phi \end{bmatrix} = -\begin{bmatrix} Y_v \\ L_v \\ N_v \end{bmatrix} \Delta v$$

Δv ← βu_0

Steady Sideslip

For Piper Cherokee

inv

$$\begin{bmatrix} 280.7 & 0 & 2400 \\ 755.7 & -3821.9 & 0 \\ -3663.5 & 359 & 0 \end{bmatrix} \begin{bmatrix} \delta_r \\ \delta_a \\ \phi \end{bmatrix} = \begin{bmatrix} 2.991 \\ 102.93 \\ -19.394 \end{bmatrix} \cancel{v} \qquad (7.8,4)$$

$\cancel{v}$ → βu_0

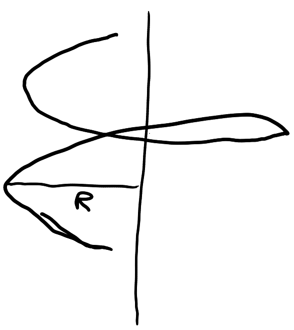
It is convenient to express the sideslip as an angle instead of a velocity. To do so we recall that $\beta \doteq v/u_0$, with u_0 given above as 112.3 fps. The solution of (7.8,4) is found to be

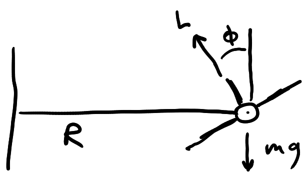
$$\delta_r/\beta = .303$$
$$\delta_a/\beta = -2.96$$
$$\phi/\beta = .104$$

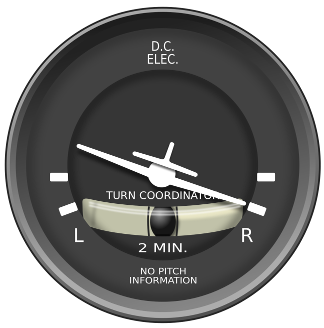
We see that a positive sideslip (to the right) of say 10° would entail left rudder of 3° and right aileron of 29.6°. Clearly the main control action is the aileron displacement, without which the airplane would, as a result of the sideslip to the right, roll to the left. The bank angle is seen to be only 1° to the right so the sideslip is almost flat.

Coordinated Turn

- angular velocity vector is constant and aligned with inertial z
- No aerodynamic forces in Y direction







$$\omega = \frac{u_0}{R}$$

$$a_n = \omega^2 R = \frac{u_0^2}{R}$$

$$\vec{\omega}_E = \begin{bmatrix} 0 \\ 0 \\ \omega \end{bmatrix}$$

$$\vec{\omega}_B = R_E^B \vec{\omega}_E = \begin{bmatrix} p \\ q \\ r \end{bmatrix} = \begin{bmatrix} -\sin\Theta \\ \sin\phi\cos\Theta \\ \cos\phi\cos\Theta \end{bmatrix} \omega$$

Since there is increased need for lift due to bank angle, involves both lat and lon dynamics

$$L\cos\phi = mg\cos\theta$$

$$L\sin\phi = m\frac{u_0^2}{R}$$

$$= m\omega u_0$$

$$\boxed{\tan\phi} = \frac{L\sin\phi}{L\cos\phi} = \frac{m\omega u_0}{mg\cos\theta} = \boxed{\frac{\omega u_0}{g\cos\theta}}$$