ASEN 3728 Aircraft Dynamics
Written Homework 3

Due date listed on Gradescope.

Question 1. Consider a quadrotor with $I_y = 3$ kg m^2. Perform a closed-loop modal analysis of the quadrotor's longitudinal $\Delta\theta$ and Δq motion using the information below. Subscripts on the natural frequencies and damping ratios correspond to the eigenvalues of the state space model for the motion.

$$\begin{aligned} \Delta M_c &= -k_1 \Delta q - k_2 \Delta\theta \\ \omega_{n,1,2} &= 1.8 \quad \text{[rad/s]} \\ \zeta_{1,2} &= 0.5 \end{aligned}$$

1. Calculate the values of k_1 and k_2, as well as the eigenvalues λ_1 and λ_2 of the size 2×2 state space model $\mathbf{A}$ matrix.
2. At $t = 0$, the quadrotor's state is $\mathbf{x}(0) = 5\mathbf{v}_1 + \mathbf{v}_2$. The vectors $\mathbf{v}_1$ and $\mathbf{v}_2$ are the unit-length eigenvectors of the $\mathbf{A}$ matrix which correspond to the eigenvalues λ_1 and λ_2. Write the solution $\mathbf{x}(t) = (\Delta\theta(t), \Delta q(t))^T$ in terms of t, λ_1, λ_2, $\mathbf{v}_1$, and $\mathbf{v}_2$.
3. Calculate the eigenvectors $\mathbf{v}_1$ and $\mathbf{v}_2$.
4. Describe the behavior of $\Delta\theta$ over time using the eigenvalues you calculated in Part (1).

Question 2. Consider the following linear ordinary differential equation:

$$a\ddot{y} + b\dot{y} + cy = du.$$

Write the A, B, C, and D matrices for a state space representation of this system with input u and output y.

Question 3. Suppose that the control forces and moments for a quadrotor with arm length $d = 10cm$ and rotor moment coefficient $k_m = 0.003$ are given by

$$\begin{bmatrix} Z_c \\ L_c \\ M_c \\ N_c \end{bmatrix} = \begin{bmatrix} -5N \\ 0Nm \\ 0.2Nm \\ 0.01Nm \end{bmatrix}.$$

If the quadrotor has the standard rotor configuration described in class, what are the thrust forces generated by each rotor?

Question 4. Consider the longitudinal dynamics of the linearized quadrotor EOM:

$$\begin{pmatrix} \Delta \dot{x}_E \\ \Delta \dot{u} \\ \Delta \dot{\theta} \\ \Delta \dot{q} \end{pmatrix} = \begin{pmatrix} \Delta u \\ -g\Delta\theta \\ \Delta q \\ \frac{1}{I_u}\Delta M_c \end{pmatrix}$$

where ΔM_c is defined in terms of k_1 and k_2 as in question 1.

1. Suppose a closed-loop modal analysis of the system was performed and you were given only the following values:

$$\lambda_1 = -1.5 + 4.2i \quad \lambda_2 = -0.0023 + 0.037i$$

$$\mathbf{v}_1 = \begin{pmatrix} 0.005 + 0.0021i \\ 0.075 + 0.0019i \\ 0.0085 + 0.0065i \\ 0.05 \end{pmatrix} \quad \mathbf{v}_2 = \begin{pmatrix} 0.006 + 0.0089i \\ 0.095 + 0.0030i \\ 0.0075 + 0.0015i \\ 0.0120 + 0.0030i \end{pmatrix}$$

   True or False: It is possible to determine λ_3, λ_4, $\mathbf{v}_3$, and $\mathbf{v}_4$ if k_1 or k_2 are unknown. Explain your answer.

   ☐ TRUE ☐ FALSE

2. Consider the following eigenvalues for another quadrotor system with the same dynamics described above:

$$\lambda_1 = -1.5 \quad \lambda_2 = -0.0023 + 0.037i$$

   True or False: It is possible to determine λ_3, λ_4 if k_1 or k_2 are unknown. Explain your answer.

   ☐ TRUE ☐ FALSE