ASEN 3728 Aircraft Dynamics
Written Homework 2

Due date listed on Gradescope.

Question 1. Consider the following simplified quadrotor translational dynamics equation,

$$m\dot{u}_E = Z_c \sin\theta - \nu|u_E|u_E,$$

where u_E is the velocity in the *inertial* x axis*, and the control force Z_c is managed by an automatic control system to maintain zero vertical acceleration, that is $Z_c = -\frac{mg}{\cos\theta}$. This equation can be used to solve for a steady forward-flight trim condition characterized by $\dot{u}_E = 0$. Assume $\psi = \phi = 0$ and that there is no wind.

a) Draw a diagram of the quadrotor in forward flight from the side (i.e. the y axis is pointing directly toward you). Label the angle θ, the direction of positive u_E, and the aerodynamic, control, and gravity force vectors, ${}^a\mathbf{f}$, ${}^c\mathbf{f}$, and ${}^g\mathbf{f}$ assuming u_E is positive.†

b) Write an equation for the pitch angle θ_0 as a function of the forward flight velocity $u_{E,0}$, in a forward flight trim state.

c) Linearize these dynamics about the trim state, that is, find a linear equation for $\Delta\dot{u}_E$ in terms of Δu_E and $\Delta\theta$ assuming that u_E is positive. Simplify so that every term in the equation has a disturbance variable (one that begins with Δ) in it.‡

*u_E should not be confused with u^E, the earth-relative velocity component in the *body* x axis.

†You can label $-\theta$ instead of θ if you would like to.

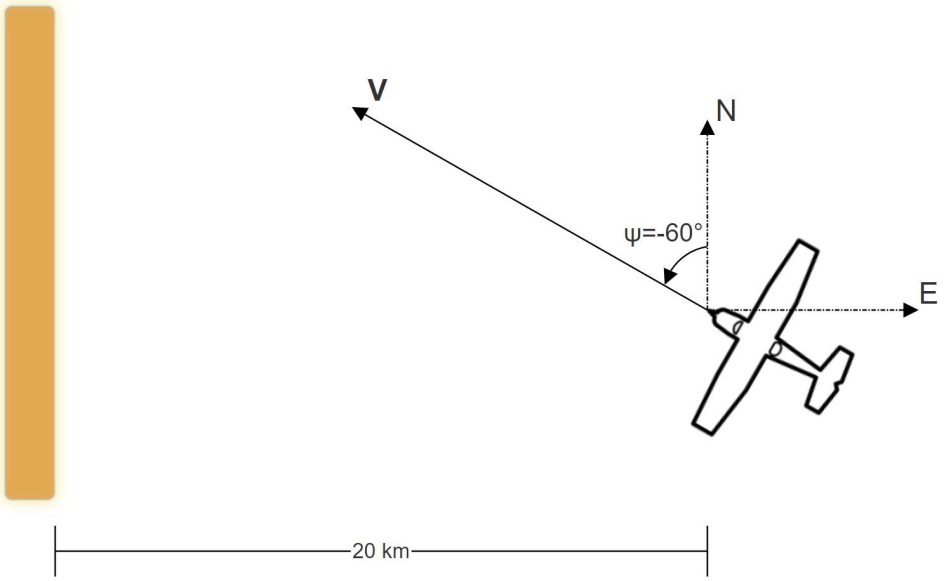
‡Hint: Use a Taylor series expansion to linearize $\tan\theta$ about θ_0.

Question 2. A 10 kg quadrotor has state vector $\vec{x} = [0$ m, 0 m, 0 m, 0 rad, $-\pi/6$ rad, 0 rad, 11 m/s, 4 m/s, 1 m/s, -0.1 rad/s, 0.2 rad/s, 0.4 rad/s]. There is wind with velocity $\vec{W}_B^E = [2, 2, 0]^T$ m/s. Each rotor applies a control force of magnitude 30 N to the aircraft, and $\nu = 10^{-2}$ kg/m. Considering forces due to gravity, aerodynamic drag, and the rotor control, what is $\dot{\vec{V}}_B^E$?

Question 3. Consider a aircraft flying near a mountain ridge as shown in the diagram. The aircraft's pitot tube is used to measure the air-relative velocity in body coordinates:

$$\vec{V}_B^W = [100\, m/s, 0, 0]^T$$

The background wind is known to be blowing due East from a recent weather report at 30 m/s. The aircraft is flying at an altitude $z_E = -3000$ m, with a heading angle $\psi = -60°$. The ridge is located located 20 km West of the current position and has a height of 4000 m.



(a) How long will it take for the aircraft to reach the ridge?

(b) If the pilot wants to clear the ridge, how fast must the aircraft climb? Assume no change in W or the internal x and y components of $\vec{V}^E$.