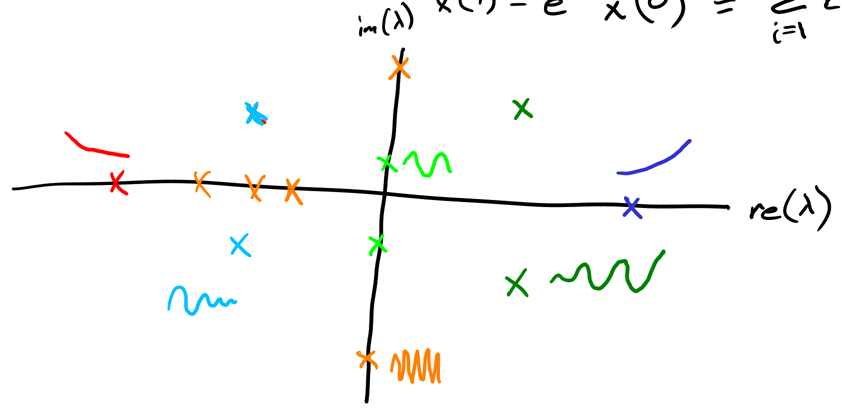
Review: State space representation of a Linear Dynamical System

$$\dot{\vec{x}} = A\vec{x} + B\vec{u}$$
$$\dot{\vec{y}} = C\vec{x} + D\vec{u}$$

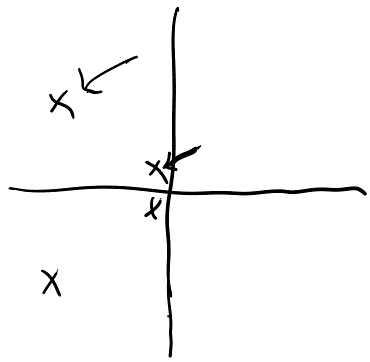
Eigenvectors Eigenvalues

$$A\vec{v}_i = \lambda_i \vec{v}_i$$
$$\vec{x}(t) = e^{At}\vec{x}(0) = \sum_{i=1}^{n} k_i e^{\lambda_i t}\vec{v}_i$$



a+bi
a-bi

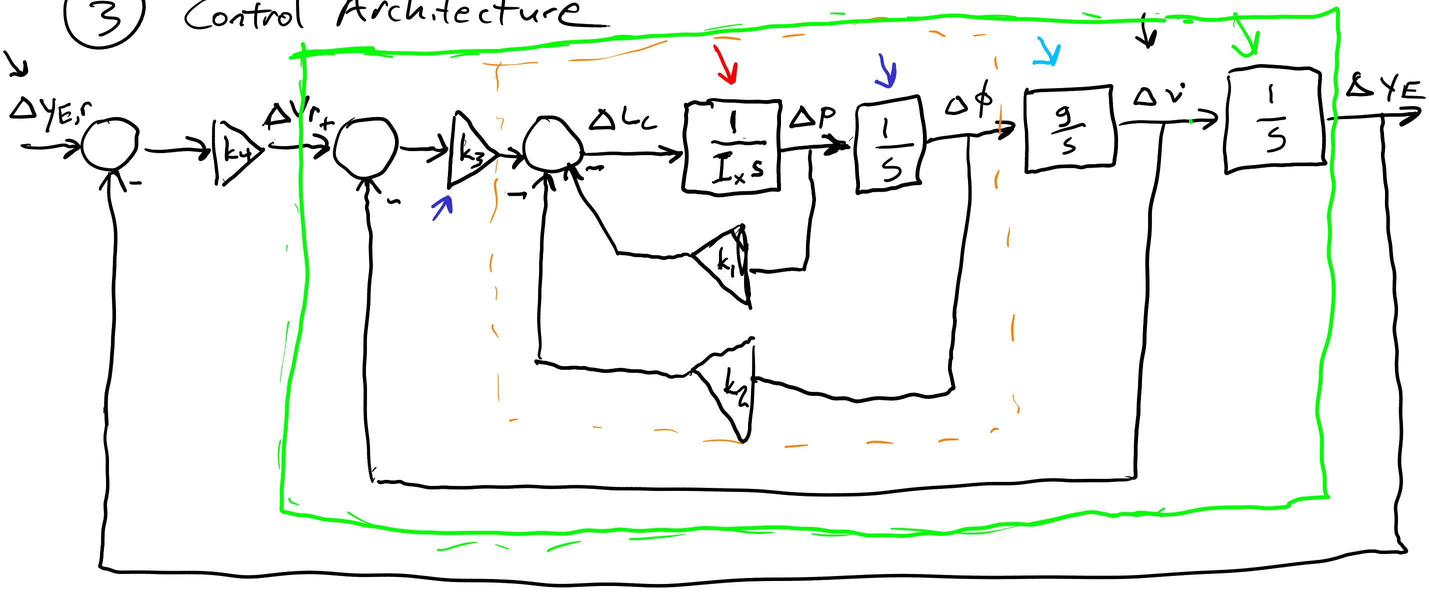
$$\vec{u} = -K\vec{x}$$
$$\dot{\vec{x}} = \underbrace{(A-BK)}_{A^{cl}}\vec{x}$$



(1)
(2)

$$\underbrace{\begin{bmatrix}\Delta\dot{y}_E\\ \Delta\dot{v}\\ \Delta\dot{\phi}\\ \Delta\dot{p}\end{bmatrix}}_{\dot{\vec{x}}} = \begin{bmatrix}\Delta v\\ g\Delta\phi\\ \Delta p\\ \frac{1}{I_x}\Delta L_c\end{bmatrix} = \underbrace{\begin{bmatrix}0 & 1 & 0 & 0\\ 0 & 0 & g & 0\\ 0 & 0 & 0 & 1\\ 0 & 0 & 0 & 0\end{bmatrix}}_{A}\underbrace{\begin{bmatrix}\Delta y_E\\ \Delta v\\ \Delta\phi\\ \Delta p\end{bmatrix}}_{\vec{x}} + \underbrace{\begin{bmatrix}0\\ 0\\ 0\\ \frac{1}{I_x}\end{bmatrix}}_{B}\underbrace{\Delta L_c}_{u}$$

(3) Control Architecture



(4) Choosing Gains

1) $k_1 + k_2$
2) k_4
3) k_3 Root Locus

$$\Delta L_c = -k_1 \Delta p - k_2 \Delta \phi - k_3 \Delta v - k_3 k_4 \Delta y_E + k_3 k_4 \Delta y_{E,r}$$

$$\begin{bmatrix} \Delta \dot{y}_E \\ \Delta \dot{v} \\ \Delta \dot{\phi} \\ \Delta \dot{p} \end{bmatrix} = \underbrace{\begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & g & 0 \\ 0 & 0 & 0 & 1 \\ \frac{-k_3 k_4}{I_x} & \frac{-k_3}{I_x} & \frac{-k_2}{I_x} & \frac{-k_1}{I_x} \end{bmatrix}}_{A^{cl}} \begin{bmatrix} \Delta y_E \\ \Delta v \\ \Delta \phi \\ \Delta p \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 0 \\ \frac{k_3 k_4}{I_x} \end{bmatrix} \Delta y_{E,r}$$

$$\rightarrow \begin{bmatrix} \Delta \dot{\phi} \\ \Delta \dot{p} \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ \frac{-k_2}{I_x} & \frac{-k_1}{I_x} \end{bmatrix} \begin{bmatrix} \Delta \phi \\ \Delta p \end{bmatrix}$$

$$\rightarrow \quad |A - \lambda I| = 0$$

$$\lambda^2 + \frac{k_1}{I_x}\lambda + \frac{k_2}{I_x} = 0$$

$$\cancel{\rightarrow} \quad \lambda = -\frac{k_1}{2 I_x} + \sqrt{\frac{k_1^2}{4 I_x^2} - \frac{k_2}{I_x}}$$

Desired:

$$\zeta = 0.7$$

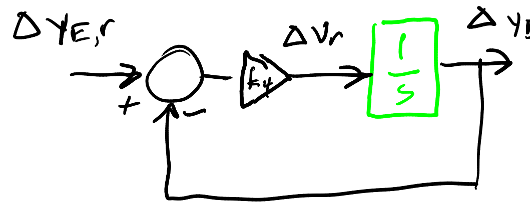
$$\omega_n = 16 \text{ rad/s} \qquad T = \frac{2\pi}{\omega_n}$$

$$\frac{T}{2} = 0.2\text{s}$$

$$\lambda = -\zeta \omega_n \pm i \omega_n \sqrt{1-\zeta^2}$$

$$\lambda = -11.2 + 11.4i$$

$$\boxed{\begin{aligned} k_1 &= 0.0016 \\ k_2 &= 0.0179 \end{aligned}}$$



$$\Delta \dot{y}_E = k_4 (y_{E,r} - \Delta y_E)$$

$$\Delta y_E(t) = \Delta y_{E,r} \left(1 - e^{-k_4 t}\right)$$

choose k_4 1 order of magnitude slower

$$\tau = \frac{1}{k_4} > 10 \cdot \frac{2\pi}{\zeta \omega_n} = 5.6 \qquad \boxed{k_4 = 0.17}$$

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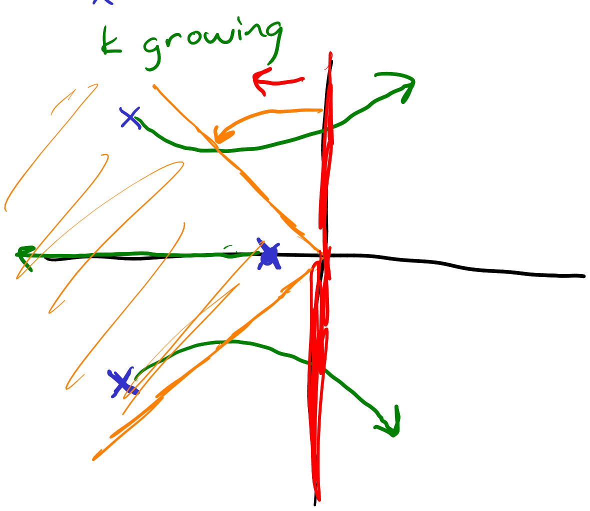
x: k=0

Choose k_3 via Root Locus



Root Locus: plot of the poles/eigenvalues of a system as a parameter (usually a gain) changes