Lateral Dynamic Modes

$$\dot{\mathbf{x}}_{lat} = \mathbf{A}_{lat}\mathbf{x}_{lat} + \mathbf{c}_{lat}$$

$$\mathbf{x}_{lat} = \begin{pmatrix} \Delta v \\ \Delta p \\ \Delta r \\ \Delta \phi \end{pmatrix} \qquad \mathbf{c}_{lat} = \begin{pmatrix} \frac{\Delta Y_c}{m} \\ \Gamma_3 \Delta L_c + \Gamma_4 \Delta N_c \\ \Gamma_4 \Delta L_c + \Gamma_8 \Delta N_c \\ 0 \end{pmatrix}$$

$$\mathbf{A}_{lat} = \begin{pmatrix} \frac{Y_v}{m} & \frac{Y_p}{m} & \left(\frac{Y_r}{m} - u_0\right) & g\cos\theta_0 \\ \Gamma_3 L_v + \Gamma_4 N_v & \Gamma_3 L_p + \Gamma_4 N_p & \Gamma_3 L_r + \Gamma_4 N_r & 0 \\ \Gamma_4 L_v + \Gamma_8 N_v & \Gamma_4 L_p + \Gamma_8 N_p & \Gamma_4 L_r + \Gamma_8 N_r & 0 \\ 0 & 1 & \tan\theta_0 & 0 \end{pmatrix}$$

$$\mathbf{A}_{lat} = \begin{pmatrix} -0.0558 & 0 & -774 & 32.2 \\ -0.003865 & -0.4342 & 0.4136 & 0 \\ 0.001086 & -0.006112 & -0.1458 & 0 \\ 0 & 1 & 0 & 0 \end{pmatrix}$$

$$x(t) = \sum_i c_i e^{\lambda_i t} \vec{v}_i$$

↑ determined by initial condition

λ_i	ζ	ω_n
$-7.30e-03$	$1.00e+00$	$7.30e-03$
$-5.62e-01$	$1.00e+00$	$5.62e-01$
$-3.30e-02+9.47e-01i$	$3.49e-02$	$9.47e-01$
$-3.30e-02-9.47e-01i$	$3.49e-02$	$9.47e-01$

Δv
Δp
Δr
$\Delta \phi$

$\Delta \psi$
Δy_E

$\mathbf{v}_1$	$\mathbf{v}_2$	$\mathbf{v}_{3/4}$
0.9821	-0.9972	-1.0000
-0.0014	-0.0367	$0.0019 \mp 0.0032i$
0.0078	0.0021	$-0.0001 \pm 0.0011i$
0.1880	0.0652	$-0.0035 \mp 0.0019i$

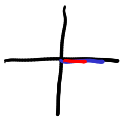
Flight Path State Space Dynamics Matrix (also exists for lon)

Δz_E
Δx_E

$$\rightarrow \quad \Delta\dot{\psi} = \Delta r \sec\theta_0$$
$$\Delta \dot{y}_E = u_0 \cos\theta_0 \, \Delta\psi + \Delta v$$

$$\begin{bmatrix} \Delta\dot{v} \\ \vdots \end{bmatrix} = \left[\begin{array}{cccc|cc} & & & & 0 & 0 \\ & A_{lat} & & & 0 & 0 \\ & & & & 0 & 0 \\ & & & & 0 & 0 \\ \hline 0 & 0 & \sec\theta_0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & u_0\cos\theta_0 & 0 \end{array}\right] \begin{bmatrix} \Delta v \\ \Delta p \\ \Delta r \\ \Delta\phi \\ \Delta\psi \\ \Delta y_E \end{bmatrix}$$

Spiral Mode



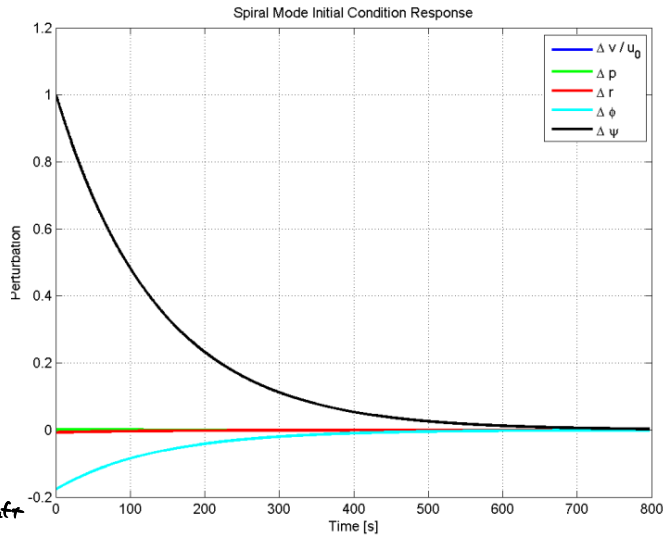
$$\tilde{V}_1 = \begin{bmatrix} -0.0012 \\ 0.0013 \\ -0.0073 \\ -0.1768 \\ 1.0 \end{bmatrix} \begin{matrix} \hat{v} = \beta \\ p \\ r \\ \phi \\ \psi \end{matrix} \begin{matrix} \leftarrow \text{small} \\ \leftarrow \text{small} \\ \leftarrow \text{some} \\ \leftarrow \text{large} \\ \leftarrow \text{large} \end{matrix}$$

nondimensionalize velocities, normalize so that $\Delta\psi = 1$

$\lambda_1 = -0.0073$ ← stable for 747

$\tau = 137$ s

unstable for many aircraft

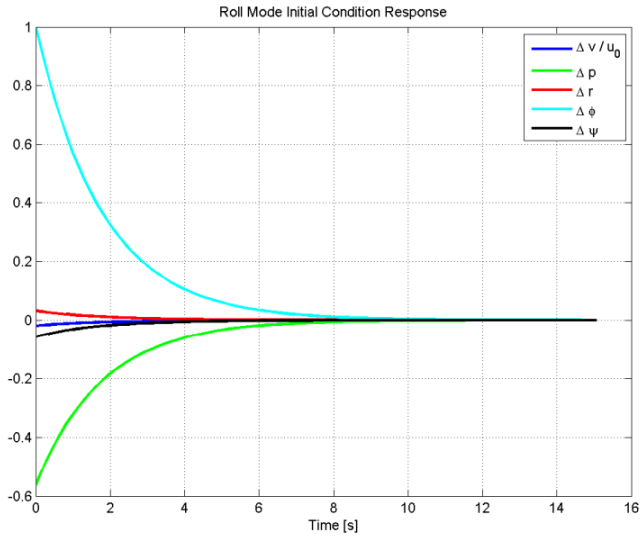


Roll

$$\tilde{V}_2 = \begin{bmatrix} -0.0198 \\ -0.5625 \\ 0.0316 \\ 1.0 \\ -0.0562 \end{bmatrix} \begin{matrix} \hat{v} = \beta \\ p \\ r \\ \phi \\ \psi \end{matrix} \begin{matrix} \leftarrow \text{small} \\ \leftarrow \text{large} \\ \leftarrow \text{small} \\ \leftarrow \text{large} \\ \leftarrow \text{small} \end{matrix}$$

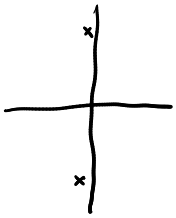
$\lambda_2 = -0.5625$

$\tau = 1.78$ s



Dutch Roll

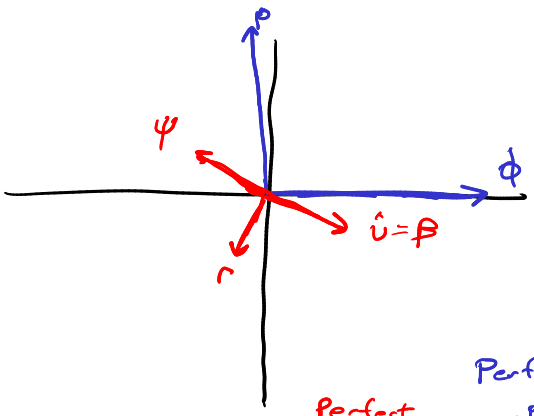
$$\tilde{V}_3 = \begin{bmatrix} 0.3271 \angle -28^\circ \\ 0.9471 \angle 92^\circ \\ 0.2915 \angle -112.3^\circ \\ 1.0 \\ 0.3078 \angle 155^\circ \end{bmatrix} \begin{matrix} \hat{v} = \beta \\ p \\ r \\ \phi \\ \psi \end{matrix}$$

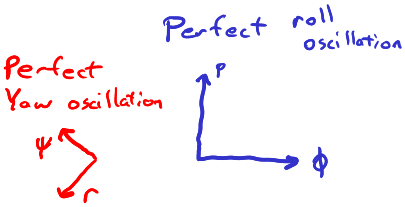


$\lambda_{3/4} = -0.033 \pm 0.947i$

$\zeta = 0.0349$ ← not well-damped

$\omega_n = 0.947$ ← fast



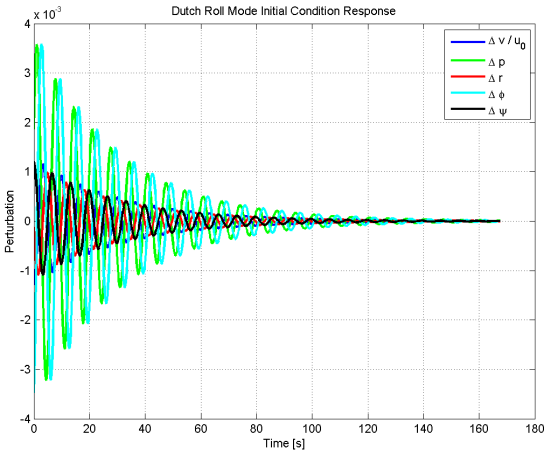


Phugoid (for comparison)

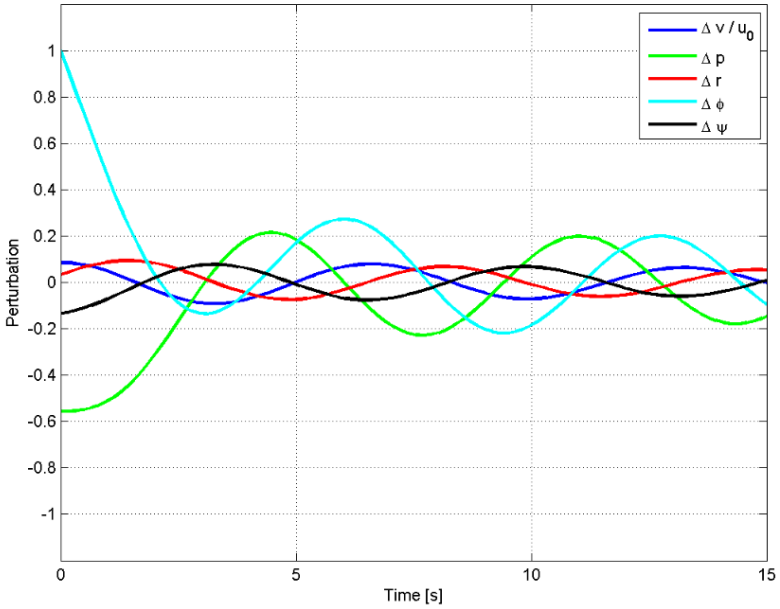
$\lambda = -0.00329 \pm 0.0672i$

$\zeta = 0.0491$

$\omega_n = 0.0673$



$$\mathbf{x}(0) = 0.4 \cdot Re(\mathbf{v}_r) + 0.4 \cdot Re(\mathbf{v}_{dr}) + 0.2 \cdot Re(\mathbf{v}_{spi})$$



Review: All modes

Name	Primary Variables	Fast / Slow	Damping
Short Period	α	Fast	Well-damped
Phugoid	speed/altitude	Slow	Poorly
Roll	Roll rate	Fast	(over)
Spiral	Yaw, Roll	Slow	(over)/unstable
Dutch Roll	β, Roll	Fast	Poorly

Trust the mathematical properties of A over your intuition!