Linearization and Quadrotors

Aircraft Dynamics
UNIVERSITY OF COLORADO **BOULDER**

Equilibrium and Static Stability

Equilibrium = A steady state that does not change without a disturbance or control action

- Usually, the derivatives of the states are zero

Static stability = Consider the equilibrium condition and whether aircraft returns to condition after small perturbation

Dynamic Stability = Details of how a system returns to equilibrium

For aircraft we call the equilibrium condition the "**trim condition**"

For fixed-wing aircraft the simplest trim condition is

- "Steady symmetric flight"
- "Straight, level flight"
- "Straight, level, unaccelerating flight (SLUF)"

For a quadrotor the simplest trim state is

- "Hovering"
- "Steady hover"

Linearized Control Forces and Moments

$$Z_c = -f_1 - f_2 - f_3 - f_4 \quad \Longrightarrow \quad \Delta Z_c = -\Delta f_1 - \Delta f_2 - \Delta f_3 - \Delta f_4$$

$$\begin{bmatrix} L_c \\ M_c \\ N_c \end{bmatrix} = \begin{bmatrix} \frac{d}{\sqrt{2}}(-f_1 - f_2 + f_3 + f_4) \\ \frac{d}{\sqrt{2}}(f_1 - f_2 - f_3 + f_4) \\ k_m(f_1 - f_2 + f_3 - f_4) \end{bmatrix}$$

$$\Downarrow$$

$$\begin{bmatrix} \Delta L_c \\ \Delta M_c \\ \Delta N_c \end{bmatrix} = \begin{bmatrix} -\frac{d}{\sqrt{2}} & -\frac{d}{\sqrt{2}} & \frac{d}{\sqrt{2}} & \frac{d}{\sqrt{2}} \\ \frac{d}{\sqrt{2}} & -\frac{d}{\sqrt{2}} & -\frac{d}{\sqrt{2}} & \frac{d}{\sqrt{2}} \\ k_m & -k_m & k_m & -k_m \end{bmatrix} \begin{bmatrix} \Delta f_1 \\ \Delta f_2 \\ \Delta f_3 \\ \Delta f_4 \end{bmatrix}$$

Linearized Quadrotor EOM

$$\begin{pmatrix} \Delta\dot{x}_E \\ \Delta\dot{y}_E \\ \Delta\dot{z}_E \end{pmatrix} = \begin{pmatrix} \Delta u \\ \Delta v \\ \Delta w \end{pmatrix}$$

$$\begin{pmatrix} \Delta\dot{\phi} \\ \Delta\dot{\theta} \\ \Delta\dot{\psi} \end{pmatrix} = \begin{pmatrix} \Delta p \\ \Delta q \\ \Delta r \end{pmatrix}$$

$$\begin{pmatrix} \Delta\dot{u} \\ \Delta\dot{v} \\ \Delta\dot{w} \end{pmatrix} = g \begin{pmatrix} -\Delta\theta \\ \Delta\phi \\ 0 \end{pmatrix} + \frac{1}{m} \begin{pmatrix} 0 \\ 0 \\ \Delta Z_c \end{pmatrix}$$

$$\begin{pmatrix} \Delta\dot{p} \\ \Delta\dot{q} \\ \Delta\dot{r} \end{pmatrix} = \begin{pmatrix} \frac{1}{I_x}\Delta L_c \\ \frac{1}{I_y}\Delta M_c \\ \frac{1}{I_z}\Delta N_c \end{pmatrix}$$

Linearized Quadrotor EOM

These four variables only depend on one another, and do not influence any other variables

$$\begin{pmatrix} \Delta\dot{x}_E \\ \Delta\dot{y}_E \\ \Delta\dot{z}_E \end{pmatrix} = \begin{pmatrix} \Delta u \\ \Delta v \\ \Delta w \end{pmatrix}$$

$$\begin{pmatrix} \Delta\dot{\phi} \\ \Delta\dot{\theta} \\ \Delta\dot{\psi} \end{pmatrix} = \begin{pmatrix} \Delta p \\ \Delta q \\ \Delta r \end{pmatrix}$$

$$\begin{pmatrix} \Delta\dot{u} \\ \Delta\dot{v} \\ \Delta\dot{w} \end{pmatrix} = g \begin{pmatrix} -\Delta\theta \\ \Delta\phi \\ 0 \end{pmatrix} + \frac{1}{m} \begin{pmatrix} 0 \\ 0 \\ \Delta Z_c \end{pmatrix}$$

$$\begin{pmatrix} \Delta\dot{p} \\ \Delta\dot{q} \\ \Delta\dot{r} \end{pmatrix} = \begin{pmatrix} \frac{1}{I_x}\Delta L_c \\ \frac{1}{I_y}\Delta M_c \\ \frac{1}{I_z}\Delta N_c \end{pmatrix}$$

Linearized Quadrotor EOM

Longitudinal

$$\begin{pmatrix} \Delta \dot{x}_E \\ \Delta \dot{u} \\ \Delta \dot{\theta} \\ \Delta \dot{q} \end{pmatrix} = \begin{pmatrix} \Delta u \\ -g\Delta\theta \\ \Delta q \\ \frac{1}{I_y}\Delta M_c \end{pmatrix}$$

Lateral

$$\begin{pmatrix} \Delta \dot{y}_E \\ \Delta \dot{v} \\ \Delta \dot{\phi} \\ \Delta \dot{p} \end{pmatrix} = \begin{pmatrix} \Delta v \\ g\Delta\phi \\ \Delta p \\ \frac{1}{I_x}\Delta L_c \end{pmatrix}$$

Vertical

$$\begin{pmatrix} \Delta \dot{z}_E \\ \Delta \dot{w} \end{pmatrix} = \begin{pmatrix} \Delta w \\ \frac{1}{m}\Delta Z_c \end{pmatrix}$$

Spin

$$\begin{pmatrix} \Delta \dot{\psi} \\ \Delta \dot{r} \end{pmatrix} = \begin{pmatrix} \Delta r \\ \frac{1}{I_z}\Delta N_c \end{pmatrix}$$