Longitudinal Trim

Last time: Long. Forces + Moments

$$L = \tfrac{1}{2}\rho V_a^2 S\left(\overset{\text{coordinate def.}}{C_{L_0} \to 0} + C_{L_\alpha}\alpha + C_{L_q}\hat{q} + C_{L_{\delta e}}\delta e\right)$$

$$D = \tfrac{1}{2}\rho V_a^2 S\left(C_{D_{min}} + K\left(C_L(\alpha, q, \delta e) - C_{L_{min}}\right)^2\right)$$

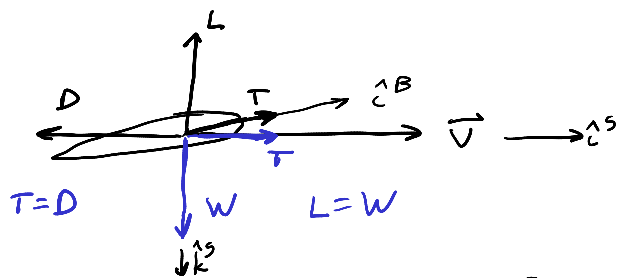
$$M = \tfrac{1}{2}\rho V_a^2 S \bar{c}\left(C_{m_0} + C_{m_\alpha}\alpha + C_{m_q}\hat{q} + C_{m_{\delta e}}\delta e\right)$$

$$C_{a_b} \equiv \frac{\partial C_a}{b}$$

Trim: SLUF: Steady Level Unaccelerated Flight

just "in trim"

For linear trim calc



At trim, forces + moments about C.G. sum to 0

$\hat{i}^S$ dir: $T_{trim} = D_{trim}$

$\hat{k}^S$ dir: $L_{trim} = W$

$M_{trim} = 0$

V_a fixed

$$\cancel{C_{L_{trim}} = C_{L_\alpha}\alpha_{trim} = \frac{W}{\tfrac{1}{2}\rho V_a^2 S}}$$

$$\cancel{C_{m_{trim}} = C_{m_0} + C_{m_\alpha}\alpha_{trim} = 0}$$

$$C_L = C_{L_\alpha}\alpha + C_{L_{\delta e}}\delta e$$

$$C_m = C_{m_0} + C_{m_\alpha}\alpha + C_{m_{\delta e}}\delta e = 0$$

$$\begin{bmatrix} C_{L_\alpha} & C_{L_{\delta e}} \\ C_{m_\alpha} & C_{m_{\delta e}} \end{bmatrix}\begin{bmatrix} \alpha_{trim} \\ \delta e_{trim} \end{bmatrix} = \begin{bmatrix} C_{L_{trim}} \\ -C_{m_0} \end{bmatrix}$$

$$\begin{bmatrix} \alpha_{trim} \\ \delta e_{trim} \end{bmatrix} = \begin{bmatrix} C_{L_\alpha} & C_{L_{\delta e}} \\ C_{m_\alpha} & C_{m_{\delta e}} \end{bmatrix}^{-1}\begin{bmatrix} C_{L_{trim}} \\ -C_{m_0} \end{bmatrix}$$

$$\begin{bmatrix} a & b \\ c & d \end{bmatrix}^{-1} = \frac{1}{ad - bc}\begin{bmatrix} d & -b \\ -c & a \end{bmatrix}$$

(special case of Cramer's Rule)

$$\boxed{\alpha_{trim} = \frac{C_{m_0} C_{L_{\delta e}} + C_{m_{\delta e}} C_{L_{trim}}}{\Delta} \qquad \delta e_{trim} = -\frac{C_{m_0} C_{L_\alpha} + C_{m_\alpha} C_{L_{trim}}}{\Delta} \qquad \Delta = C_{L_\alpha} C_{m_{\delta e}} - C_{L_{\delta e}} C_{m_\alpha}}$$

$$\boxed{C_{m_\alpha} = C_{L_\alpha}(h - h_n)}$$ (from slides)

$$C_{L_\alpha} = a = a_{wb}\left[1 + \frac{a_t S_t}{a_{wb} S}\left(1 - \frac{\partial \epsilon}{\partial \alpha}\right)\right]$$

$$C_{m_0} = C_{m_{ac_{wb}}} + C_{m0_p} + a_t \bar{V}_H \left(\epsilon_0 + i_t\right)\left[1 - \frac{a_t S_t}{aS}\left(1 - \frac{\partial \epsilon}{\partial \alpha}\right)\right]$$

$$h_n = h_{n_{wb}} + \frac{a_t}{a}\bar{V}_H\left(1 - \frac{\partial \epsilon}{\partial \alpha}\right) - \frac{1}{a}\frac{\partial C_{m_p}}{\partial \alpha}$$

$$C_{m_\alpha} = C_{L_\alpha}\left(h - h_n\right)$$

Direct dependence on CG location h

$$C_{L_{\delta_e}} = \frac{\partial C_{L_t}}{\partial \delta_e}\frac{S_t}{S} = a_e \frac{S_t}{S}$$

$$C_{m_{\delta_e}} = -a_e \bar{V}_H + C_{L_{\delta_e}}\left(h - h_{n_{wb}}\right)$$

Some mistakes on hw

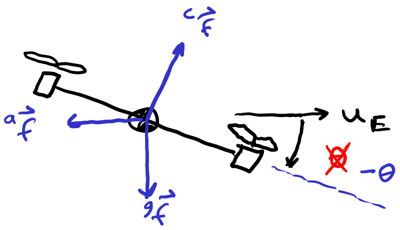
if $x(0) = \sum_i k_i \vec{v}_i$ $\qquad x(t) = \sum_i k_i e^{\lambda_i t} \vec{v}_i$

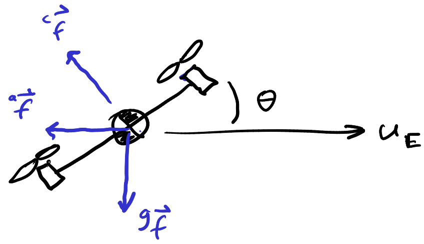
not gains

Rate of climb $= \cancel{\dot{z}_E} = -\dot{z}_E$

$\dot{h} = -\dot{z}_E$

Diagram





Linearization

2 methods

Method 1: f.o. Taylor Series

$$f(x,y) = f(x_0, y_0) + \frac{\partial f}{\partial x}\Big|_0 \Delta x + \frac{\partial f}{\partial y}\Big|_0 \Delta y$$

$$xy = x_0 y_0 + y_0 \Delta x + x_0 \Delta y$$

$$\sin(\theta) = \sin(\theta_0) + \frac{\partial \sin\theta}{\partial \theta}\Big|_0 \Delta\theta$$

$$= \sin(\theta_0) + \cos(\theta_0)\Delta\theta$$

may include x_0

$$\Delta\dot{x} = a\,\Delta x + b$$

Method 2

$f(x,y) \rightarrow f(x_0+\Delta x, y_0+\Delta y) \longrightarrow$ expand out/cancel out $\longrightarrow$ apply small disturbance approx

$$xy \longrightarrow (x_0+\Delta x)(y_0+\Delta y) \longrightarrow x_0 y_0 + y_0\Delta x + x_0 \Delta y + \cancel{\Delta y \Delta x}^{\,0}$$

$$\sin\theta \longrightarrow \sin(\theta_0+\Delta\theta) \longrightarrow \sin(\theta_0)\cancel{\cos(\Delta\theta)}^{\,1} + \cos(\theta_0)\cancel{\sin(\Delta\theta)}^{\,\Delta\theta}$$

$$\longrightarrow \sin(\theta_0) + \cos(\theta_0)\Delta\theta$$

W2 Q2

$$\vec{x} = \begin{bmatrix} \\ \\ \\ \\ \end{bmatrix} \qquad \dot{\vec{V}}_B^E \qquad \vec{W}_E$$