Conventional A/C Dynamics

Longitudinal

- Altitude
- Airspeed
- Pitch

Lateral-Directional

- Roll
- Yaw
- Sideslip
- Turning

Lift
Drag
(Pitch Moment)

density, airspeed, wing Area

$$L = \frac{1}{2}\rho V_a^2 S C_L$$

$$D = \frac{1}{2}\rho V_a^2 S C_D$$

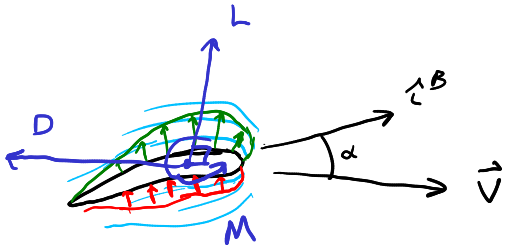
$$M = \frac{1}{2}\rho V_a^2 S c C_m$$ (c: chord)

$C_L(\alpha, q, \delta_e)$ nonlinear

$C_D(\alpha, q, \delta_e)$

$C_m(\alpha, q, \delta_e)$

Lift



1st order Taylor Series of $C_L(\alpha, q, \delta_e)$

$$L = \frac{1}{2}\rho V_a^2 S\left(C_{L_0} + \frac{\partial C_L}{\partial \alpha}\alpha + \frac{\partial C_L}{\partial q}q + \frac{\partial C_L}{\partial \delta_e}\delta_e\right)$$

$$= \frac{1}{2}\rho V_a^2 S\left(C_{L_0} + C_{L_\alpha}\alpha + C_{L_q}\frac{c}{2V_a}q + C_{L_{\delta_e}}\delta_e\right)$$

$\hat{q}$

C_{L_α}: Nondimensional Stability Derivative

$C_{L_{\delta_e}}$: Nondim. control derivative

Stability Derivatives

- based linear assumptions / linearization
- Main tool connecting aerodynamics to dynamics
- Functions of A/C geometry

Example

Pitch Stiffness C_{m_α}

$$\ddot{\alpha} \approx \frac{k C_{m_\alpha}}{I}\alpha$$

$$\ddot{x} = -\frac{k_{spring}}{m}x$$

- Only valid in a linear region e.g. small α

$$AR = \frac{b^2}{S}$$ (b: span)

Estimated Using
- Geometric Data
- Wind Tunnel
- Flight Test
- CFD
- Other Aircraft

eg. $$C_{L_\alpha} = \frac{\pi AR}{1 + \sqrt{1 + (AR/2)^2}}$$

Drag

nonlinear

α only

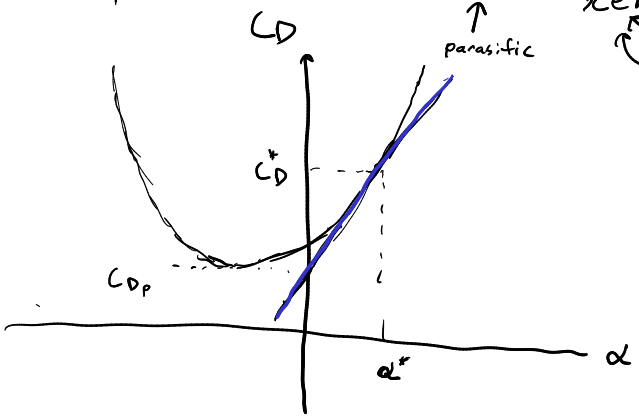
$$C_D(\alpha) = C_{D_p} + \frac{C_L(\alpha)^2}{\pi e AR}$$

C_{D_p}: parasitic

e: Oswalds efficiency / induced drag

linearize

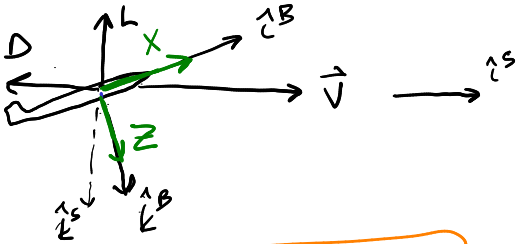
$$C_{D_0} + C_{D_\alpha}\alpha + C_{D_q}\hat{q} + C_{D_{\delta e}}\delta e$$



In this class:

$$K = \frac{1}{\pi e AR}$$

$$C_D = C_{D_{min}} + K\left(C_L(\alpha, q, \delta e) - C_{L_{min}}\right)^2$$



"stability frame"

aerodynamic forces in $\hat{i}^B$ and $\hat{k}^B$

$$\begin{bmatrix} X \\ Z \end{bmatrix} = \begin{bmatrix} \cos\alpha & -\sin\alpha \\ \sin\alpha & \cos\alpha \end{bmatrix} \begin{bmatrix} -D \\ -L \end{bmatrix}$$

Pitching Moment

$$M = \frac{1}{2}\rho V_a^2 S c \left[C_{m_0} + C_{m_\alpha}\alpha + C_{m_q}\hat{q} + C_{m_{\delta e}}\delta e\right]$$

- C_{m_0}: trim coefficient of moment
- C_{m_α}: pitch stiffness coefficient
- C_{m_q}: pitch damping coefficient