

# Thu 02/11/16

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## Find Kth largest element

Claim: There exists atleast  $3n/10$  elements of  $A$  that are smaller than  $x$  and  $3n/10$  elements of  $A$  that are larger than  $x$

Proof:

- there are  $n/10$  elements in  $m$  that are smaller than  $x$
- For each element of  $m$  smaller than  $x$  there are 2 more elements of  $A$  smaller than  $x$

Recursive call will be a problem of size  $\leq 7n/10$

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(refer to previous notes for this)
findRankKthElement(A, k):
    if(|A| < 5):
        sort(A)
        return A[k]
    1. build m // |m| = n/5
    2. x = findRankKthElement(m, |m|/2)
    3. partition(a, x)
    4. Do a recursive call on one side of the partition, size of partition
    <= 7n/10
```

$T(n)$  = running time of findRankKthElement on input of size  $n$

$T(1) = 1$

$T(n) = 9n + T(n/5) + T(7n/10)$

$T(n) = 5n + 4n + T(2n/10) + T(7n/10)$

Lets guess that  $T(n) \leq cn$

for some  $c$  that we will figure out later

For  $n \leq 5$ , we know that  $T(n) \leq 5n$  so that statement is true for  $n \leq 5$  as long as  $c \geq 5$

So now suppose  $T(n') \leq cn'$  for all  $n' < n$  then

$$\begin{aligned}
 T(n) &= 5n + 4n + T(2n/10) + T(7n/10) \\
 &\leq 9n + 2cn/10 + 7cn/10 \\
 &= (9 + 9c/10)n \leq cn
 \end{aligned}$$

If  $c \geq 90$ , this is true

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qSort(A):
  x = findRankKthElement(A, |A|/2)
  partition(A, x)
  qSort(A[0, .. , |A|/2])
  qSort(A[|A|/2, .. , |A| - 1])

```

$$T(1) = 1$$

$$T(n) = 94n + 2T(n/2) = 94n \log n$$

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If 2 input arrays lead to the same leaf of the execution tree then the arrays are equal.

Since there are  $n!$  distinct input arrays then there must be  $\geq n!$  leaves. Thus there must exist at least 1 path through the tree with length  $\geq \log n!$

Stirlings Approximation

$$n! \sim \sqrt{2 \pi n} \left(\frac{n}{e}\right)^n$$

$$\log n! \sim \log(\sqrt{2\pi n}) + (\log n)/2 + n(\log n - 1/e) \Rightarrow \theta(n \log n)$$