CSE320
Practice Problems #1

Problem 1: For the following functions:

$$F(x, y, z) = xyz + xy'z' + x'y'z'$$
$$G(x, y, z) = (x + y)(x + z)(x' + z)$$

a) How many literals in each expression? How many terms?
b) Create the truth table.
c) Name the minterms ($\sum m(?)$) and maxterms ($\prod M(?)$).
d) Draw a 2-level gate network (AND-OR or OR-AND). Do NOT simplify the expression first.
e) Simplify each expression using boolean logic.
f) Draw 2-level gate networks for the simplified expressions in NAND-NAND or NOR-NOR form.
g) Write the Boolean expressions for the complement of each (eg. F') in product of sums form.

Problem 2: Functions F through L are defined in the following way:

$$F = B + AC'$$
$$G = AB'C' + BC' + BC + AB$$
$$H = A + A'BC + BC'$$
$$I = A'B + BC + AC'$$
$$J = A'C' + A'B'C$$
$$K = A'B' + A'C'$$
$$L = A'B'C' + A'C' + B'C$$

Which of the functions are equivalent (same output for all input values)?

Problem 3: Find the complement for each expression. Apply DeMorgan's Law to simplify.
F(x,y,z) = x(y' + z)
G(x,y,z) = xy + x'z + yz'
H(w,x,y,z) = xyz'(y'z+x)' + (w'yz + x')

Problem 4: For each of the following functions f, g, and h:

x	y	z	f	g	h
0	0	0	0	1	0
0	0	1	0	1	0
0	1	0	1	0	0
0	1	1	1	1	1
1	0	0	0	0	0
1	0	1	1	0	1
1	1	0	1	0	1
1	1	1	1	1	1

a) Name the minterms ($\sum m(?)$) and maxterms ($\prod M(?)$) of f, g, and h.
b) Give the boolean expressions in the sum of products form for f and g. Simplify. Give the boolean expression in the product of sums form for h. Simplify.
c) Name the minterms ($\sum m(?)$) and maxterms ($\prod M(?)$) for f' and g'.
d) Draw 2-level gate networks for the simplified expression of f, g, and h in NAND-NAND or NOR-NOR form.

Problem 5: For the following truth table:

x	y	z	P	Q
0	0	0	0	1
0	0	1	1	1
0	1	0	0	0
0	1	1	0	0
1	0	0	0	1
1	0	1	1	1
1	1	0	1	1
1	1	1	1	0

a) Name the minterms (Σ m(?)) and maxterms (Π M(?)) for P and Q.
b) Draw a 2-level gate network (AND-OR or OR-AND). Do NOT simplify the expression first.
c) Write the minterm and maxterm expression in boolean logic.
d) Simplify each expression from (c) using boolean logic.
e) Draw 2-level gate networks for the simplified expressions in NAND-NAND or NOR-NOR form.
f) Write the Boolean expressions for the complement of each P and Q (ie. P' and Q') in product of sums form. DO NOT SIMPLIFY.

Problem 6: For each of the following functions:

$$J = ((A' + B)' + C')' + DC' + AB'$$
$$K = x(y + w'z) + (w' + x' + z')'$$

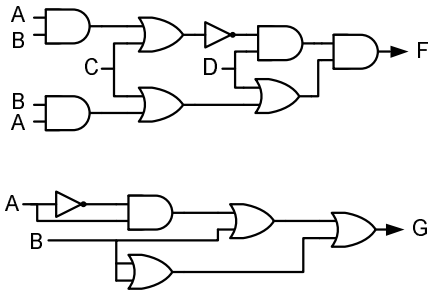
a) Create the truth table for each expression.
b) Name the minterms (Σ m(?)) and maxterms (Π M(?)) for each expression.
c) Simplify each expression using boolean logic.
d) Draw a 2-level gate network (AND-OR or OR-AND) for the expression in c), whichever form uses the fewest gates. DO NOT SIMPLIFY the expression first.
e) How many literals in each expression from (c)? How many terms in the expression from (c)?
f) Draw 2-level gate networks for the simplified expressions in NAND-NAND or NOR-NOR form.
g) Write the Boolean expressions for the complement of each expression (eg. J') in product of sums form. DO NOT SIMPLIFY.

Problem 7: Using the postulates of Boolean algebra, prove the following formula:
$A(xy) = x'y' + x'y + xy = x' + y$
$B(x, y, z) = y + x'z + xy' = x + y + z$
$C(x, y, z) = x'y + y'z' + xy + y'z = 1$
$D(x, y, z) = x'y'z' + x'y'z + x'yz + xy'z + xyz = x'y' + z$
$Y(A, B, C, D) = ((A' + B')' + A'B')(C'D' + CD) + (AC)' = A' + C' + BD$
$E(w, x, y, z) = wxy + w'xy + x'(zw + zy') + z(x'w' + y'x) = xy + z$
$G(x, y, z) = x'y + xyz' + xyz = y$
$G(w, x, y, z) = (xy' + w'z)(wx' + yz') = 0$
$H(x, y) = (x + y)(x' + y')' = xy$
$J(x, y, z) = x'y'z' + x'y'z + x'yz + xy'z + xyz = x'y' + z$
$K(X, Y, Z) = X'Y' + Y'Z + XZ + XY + YZ' = X'Y' + XZ + YZ'$
$Z(A, B, C, D) = ((A' + B')' + A'B')(C'D' + CD) + (AC)' = A' + C' + BD$

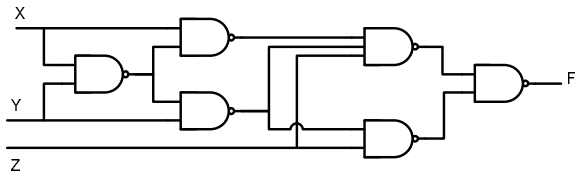
Problem 8: Using the Boolean identities, simplify each of the following formula.

$$F(x, y, z) = x'yz + x'yz' + xyz + zyz'$$
$$G(w, x, y, z) = w'x'y'z' + w'xy'z' + xx'y'z'$$
$$H(x, y, z) = x'(y'z + yz) + xyz$$

Problem 9: For the following diagrams, give the boolean expressions for F and G. Simplify and redraw the gate network.



Problem 10: Obtain the truth table for the circuit shown. Give the proper minimal boolean expression for this circuit (either sum of products or products of sums). Draw an equivalent circuit for F with fewer NAND gates.



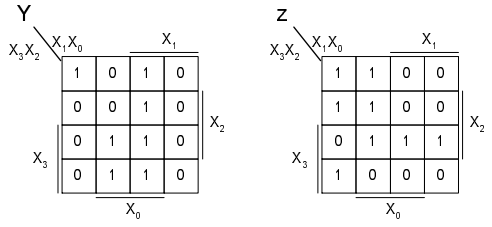
Problem 11: Consider a logic circuit (Boolean expression) with 3 inputs (x, y, and z) representing the binary values 0-7, and three outputs (a,b, and c) also representing the binary values 0-7. When the input is 2, 3 or 4 the binary output should be 2 less than the input. When the input is 1, 5 or 6 the output should be 1 greater than the input value. When the input is 0 or 7, the output is the same value as the input.

a) Create the truth table for the circuit

b) Write the Boolean expression for each output in POS or SOP form. Think efficiency.

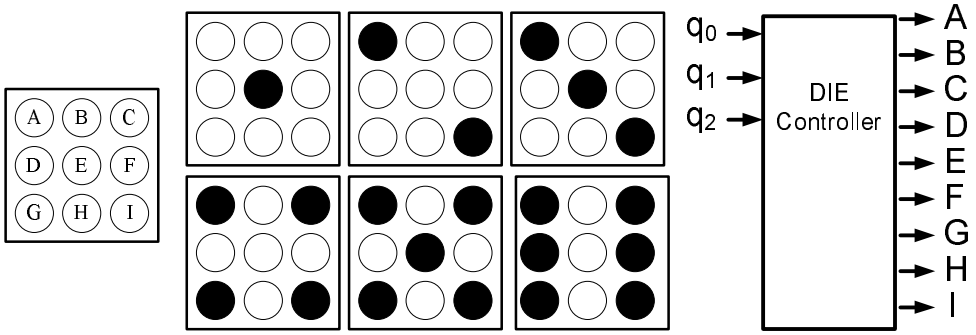
c) Simplify each expression and draw the corresponding NAND-NAND or NOR-NOR network for each.

Problem 12: Find the minimal sum of products and products of sums for the functions shown in the k-maps. Implement Y using a NAND-NAND network, and implement Z using NOR-NOR network.



Problem 13: Design a controller to display an electronic die as shown below. Inputs to the controller represent a 3-bit number, which indicates what is to be displayed. There are nine outputs, A - I, one for each of the dots. A dot will light when given a 1, and is unlit when given a 0. If value 0 or 7 is given, no dots should light.

a) Create the truth table.
b) Write the boolean expression for each output (A-I) using the minimal number of terms (ie. use SOP or POS depending which form has less terms) Do not simplify the expressions.
c) How many literals are in each expression? How many minterms/maxterms?
d) Draw the 2-level gate networks for output A, D, and E in AND-OR or OR-AND form.
e) Draw the 2-level gate networks for outputs A, D, and E in NAND-NAND or NOR-NOR form.



Problem 14: A logic circuit realizes the function F(a,b,c,d) = a'b'+a'cd+ac'd+ab'd'. Assuming that a=c never occurs when b = d = 1, find a simplified expression for F. (Hint: When can Don't-Cares be used?)

Problem 15: Given two inputs $x = (x_1, x_0)$ and $y = (y_1, y_0)$, the output is the absolute value of their difference: $z = |2y - x|$.
a) Write the truth table for the output values (hint: how many bits are needed to represent the values).
b) Write the POS or SOP boolean expressions for each output bit.
c) Simplify the expressions in b) and draw the corresponding gate networks using NAND-NAND or NOR-NOR

Problem 16: A circuit will either subtract 2-bit unsigned X from 2-bit unsigned Y or Y from X, depending on the value of A. If A = 1, the output should be $|X-Y|$, and if A = 0, the output should be $|Y-X|$.

a) Write the truth table for the output values (hint: how many bits are needed to represent the values).
b) Draw k-maps for each output.
c) Draw the corresponding gate networks using NAND-NAND or NOR-NOR for each output.