CS320 Fall 2014

Homework#1 Solutions

Problem 1: Use DeMorgan's Law to write an expression for F' where
a) $F(x, y, z) = x(y' + z)$
b) $F(x, y, z) = xy + x'z + yz'$
c) $F(w, x, y, z) = xyz'(y'z + x)' + (w'yz + x')$

Solution: a) $F(x, y, z) = x(y' + z)$
$F' = (x(y' + z))' = x' + (y' + z)' = x' + yz'$

b) $F(x, y, z) = xy + x'z + yz'$
$F' = (xy + x'z + yz')' = (xy)'(x'z)'(yz')' = (x' + y')(x + z')(y' + z)$

c) $F(w, x, y, z) = xyz'(y'z + x)' + (w'yz + x')$
$F' = (xyz'(y'z + x)' + (w'yz + x'))'$
$F' = (xyz'(y'z)'x' + (w'yz + x'))'$
$F' = (w'yz)'(x')' = (w + y' + z')x$

Problem 2: Functions F, G, and H are defined in the following way:

$$F = A'C' + A'B'C$$
$$G = A'B' + A'C'$$
$$H = A'B'C' + A'C' + B'C$$

Which of the functions are equivalent?

Solution:

A	B	C	F	G	H
0	0	0	1	1	1
0	0	1	1	1	1
0	1	0	1	1	1
0	1	1	0	0	0
1	0	0	0	0	0
1	0	1	0	0	1
1	1	0	0	0	0
1	1	1	0	0	0

Functions F and G are equivalent.

By Algebra:

$$\begin{array}{lcl}
F = A'C' + A'B'C & ?= & A'B' + A'C' = G \\
F = A'C' + A'B'C & ?= & A'B'(C + C') + A'C' = G \\
F = A'C' + A'B'C & ?= & A'B'C + A'B'C' + A'C' = G \\
F = A'C' + A'B'C & ?= & A'B'C + A'C'(B' + 1) = G \\
F = A'C' + A'B'C & == & A'B'C + A'C' = G
\end{array}$$

$$\begin{array}{lcl}
H = A'B'C' + A'C' + B'C & ?= & A'C' + A'B'C = F \\
H = A'C'(B + 1) + (A' + A)B'C & ?= & A'C' + A'B'C = F \\
H = A'C' + A'B'C + AB'C & != & A'C' + A'B'C = F
\end{array}$$

Can not simplify anymore. To ensure that, you can't simplify anymore, the best way is to write out each side to minterms or maxterms and compare minterms.

Problem 3: For each of the following functions:

$$J = ((A' + B)' + C')' + DC' + AB'$$
$$K = x(y + w'z) + (w' + x' + z')'$$

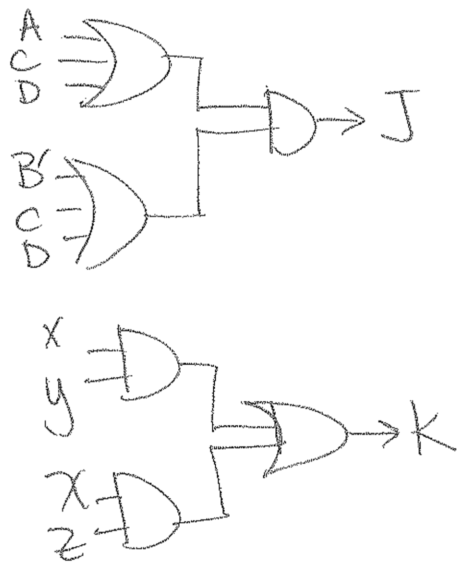
a) Create the truth table for each expression.
b) Name the minterms (Σ m(?)) and maxterms (Π M(?)) for each expression.
c) Simplify each expression using boolean logic.
d) Draw a 2-level gate network (AND-OR or OR-AND) for the expression in c), whichever form uses the fewest gates. DO NOT SIMPLIFY the expression first.
e) How many literals in each expression from (c)? How many terms in the expression from (c)?
f) Draw 2-level gate networks for the simplified expressions in NAND-NAND or NOR-NOR form.
g) Write the Boolean expressions for the complement of each expression (eg. J') in product of sums form. DO NOT SIMPLIFY.

Solution:

a)

A	B	C	D	J
0	0	0	0	0
0	0	0	1	1
0	0	1	0	1
0	0	1	1	1
0	1	0	0	0
0	1	0	1	1
0	1	1	0	1
0	1	1	1	1
1	0	0	0	1
1	0	0	1	1
1	0	1	0	1
1	0	1	1	1
1	1	0	0	0
1	1	0	1	1
1	1	1	0	1
1	1	1	1	1

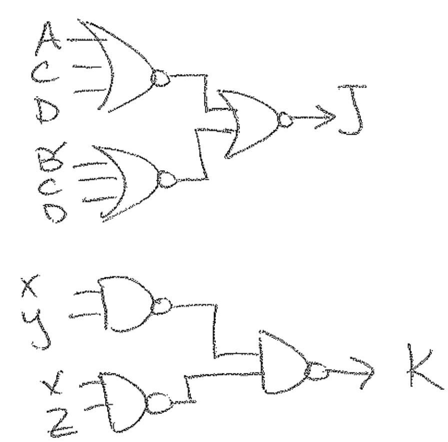
d)



b) Minterms: J(A,B,C,D) $= \sum m(1, 2, 3, 5, 6, 7, 8, 9, 10, 11, 13, 14, 15)$
Maxterms: J(A,B,C,D) $= \prod M(0, 4, 12)$

c) $J = (A + B + C + D)(A + B' + C + D)(A' + B' + C + D)$
$J = (A + B + C + D)(A + B' + C + D)(A + B' + C + D)(A' + B' + C + D)$
$J = ((B + B')(A + C + D))((A + A')(B' + C + D))$
$J = (A + C + D)(B' + C + D)$

f)



~~d)~~
e) J has 6 literals and 2 terms.

f)
g) $J'(A, B, C, D) = A'B'C'D' + A'BC'D' + ABC'D'$

a)

w	x	y	z	K
0	0	0	0	0
0	0	0	1	0
0	0	1	0	0
0	0	1	1	0
0	1	0	0	0
0	1	0	1	1
0	1	1	0	1
0	1	1	1	1
1	0	0	0	0
1	0	0	1	0
1	0	1	0	0
1	0	1	1	0
1	1	0	0	0
1	1	0	1	1
1	1	1	0	1
1	1	1	1	1

b) Minterms: K(w,x,y,z) $= \sum m(5, 6, 7, 13, 14, 15)$
Maxterms: K(w,x,y,z) $= \prod M(0, 1, 2, 3, 4, 8, 9, 10, 11, 12)$

c) $K = x(y + w'z) + (w' + x' + z')'$
$K = xy + w'xz + wxz$
$K = xy + (w' + w)xz$
$K = xy + xz$

d) See above

e) K has 4 literals and 2 terms.

g) $K'(w, x, y, z) = (w + x' + y + z')(w + x' + y' + z)(w + x' + y + z')(w' + x' + y + z')(w' + x' + y' + z)(w' + x' + y' + z')$

Problem 4: For each of the following functions:

x	y	z	P	Q
0	0	0	0	1
0	0	1	1	1
0	1	0	0	0
0	1	1	0	0
1	0	0	0	1
1	0	1	1	1
1	1	0	1	1
1	1	1	1	0

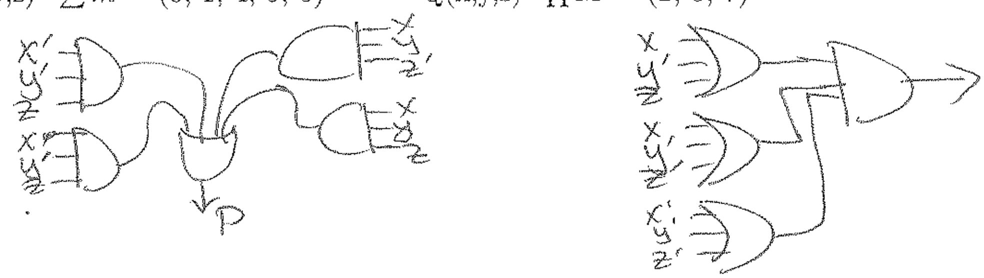
a) Name the minterms (Σ m(?)) and maxterms (Π M(?)) for P and Q.
b) Draw a 2-level gate network (AND-OR or OR-AND). Do NOT simplify the expression first.
c) Write the minterm and maxterm expression in boolean logic.
d) Simplify each expression from (c) using boolean logic.
e) Draw 2-level gate networks for the simplified expressions in NAND-NAND or NOR-NOR form.
f) Write the Boolean expressions for the complement of each P and Q (ie. P' and Q') in product of sums form. DO NOT SIMPLIFY.

Solution:

a)

P(x,y,z)=$\sum m(1, 5, 6, 7)$ P(x,y,z)=$\prod M(0, 2, 3, 4)$
Q(x,y,z)=$\sum m = (0, 1, 4, 5, 6)$ Q(x,y,z)=$\prod M = (2, 3, 7)$

b)



c)
minterm: $P = x'y'z + xy'z + xyz' + xyz$
$Q = x'y'z' + x'y'z + xy'z' + xy'z + xyz'$

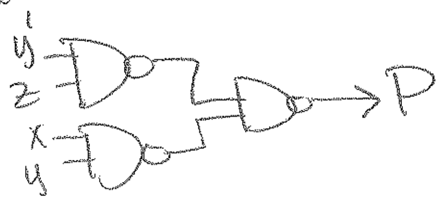
maxterm: $P = (x + y + z)(x + y' + z)(x + y' + z')(x' + y + z)$
$Q = (x + y' + z)(x + y' + z')(x' + y' + z')$

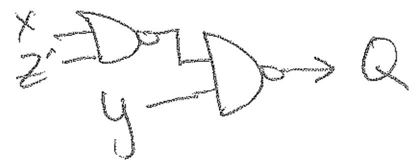
d)
$P = x'y'z + xy'z + xyz' + xyz$
$P = (x' + x)y'z + xy(z + z')$
$P = y'z + xy$

$Q = x'y'z' + x'y'z + xy'z' + xy'z + xyz'$
$Q = x'y'z' + x'y'z + xy'z + xy'z'(1 + 1) + xyz'$
$Q = (x'y'z' + x'y'z + xy'z + xy'z') + (xy'z' + xyz')$
$Q = y'(x'z' + x'z + xz' + xz) + xz'(y + y')$
$Q = y'(x + x')(z + z') + xz'$

$Q = y' + xz'$

e)



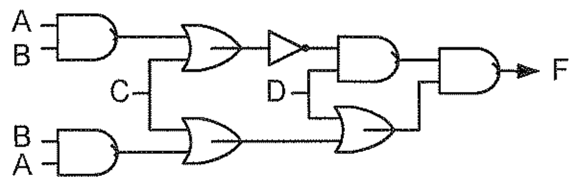


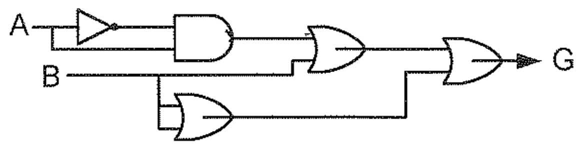
f)

$P' = (x + y + z')(x' + y + z')(x' + y' + z)(x' + y' + z')$
$Q' = (x + y + z)(x + y + z')(x' + y + z)(x' + y + z')(x' + y' + z)$

d)

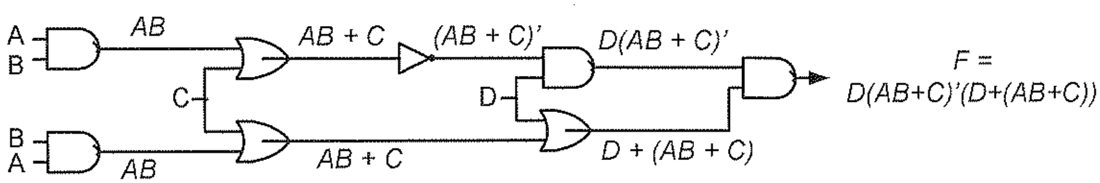
Problem 5: For the following diagrams, give boolean expressions. Simplify and redraw the system.

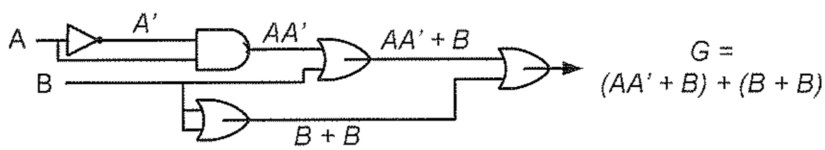




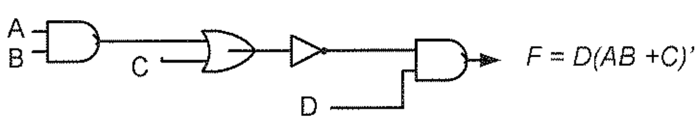
Solution:

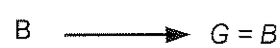
Circuit Function:





Simplified Circuits:





Problem 6: Convert the following truth table to a switching expression (Boolean Algebra) and simplify the expression as much as possible.

x	y	z	F
0	0	0	1
0	0	1	1
0	1	0	1
0	1	1	0
1	0	0	0
1	0	1	1
1	1	0	0
1	1	1	0

Solution:

x	y	z	F	minterm
0	0	0	1	x'y'z'
0	0	1	1	x'y'z
0	1	0	1	x'yz'
0	1	1	0	
1	0	0	0	
1	0	1	1	xy'z
1	1	0	0	
1	1	1	0	

$F = x'y'z' + x'y'z + x'yz' + xy'z$
$F = x'z'(y' + y) + y'z(x' + x)$
$F = x'z' + y'z$

Problem 7: Using the postulates of Boolean algebra, prove the following formulae:
a) $x'y'z' + x'y'z + x'yz + xy'z + xyz = x'y' + z$
b) $ABC' + A'C'D + AB'C' + BC'D + A'D = AC' + A'D$
c) $wxy + w'xy + x'(zw + zy') + z(x'w' + y'x) = xy + z$

Solution:

a) $x'y'z' + x'y'z + x'yz + xy'z + xyz = x'y' + z$

$$\begin{aligned}
x'y'z' + x'y'z(1+1) + x'yz + xy'z + xyz &= x'y' + z \\
x'y'z' + x'y'z + x'y'z + x'yz + xy'z + xyz &= x'y' + z \\
x'y'(z' + z) + z(x'y' + x'y + xy' + xy) &= x'y' + z \\
x'y' + z((x + x')y' + (x + x')y) &= x'y' + z \\
x'y' + z(y' + y) &= x'y' + z \\
x'y' + z &= x'y' + z
\end{aligned}$$

b) $ABC' + A'C'D + AB'C' + BC'D + A'D = AC' + A'D$

$$
\begin{aligned}
ABC' + A'C'D + AB'C' + BC'D + A'D &= AC' + A'D \\
ABC' + AB'C' + BC'D + A'D(C + C') &= AC' + A'D \\
ABC' + AB'C' + BC'D + A'D &= AC' + A'D \\
AC'(B + B') + BC'D + A'D &= AC' + A'D \\
AC' + BC'D(A + A') + A'D &= AC' + A'D \\
AC' + ABC'D + A'BC'D + A'D &= AC' + A'D \\
AC'(1 + BD) + A'D(1 + BC') &= AC' + A'D \\
AC' + A'D &= AC' + A'D
\end{aligned}
$$

c) $wxy + w'xy + x'(zw + zy') + z(x'w' + y'x) = xy + z$

$$
\begin{aligned}
(w + w')xy + x'(zw + zy') + z(x'w' + y'x) &= xy + z \\
xy + z(x'w + x'y') + z(x'w' + y'x) &= xy + z \\
xy + z(x'w + x'y' + x'w' + y'x) &= xy + z \\
xy + z(x'(w + w') + y'(x + x')) &= xy + z \\
xy + z(x' + y') &= xy + z \\
xy + z(xy)' &= xy + z \\
xy(1 + z) + z(xy)' &= xy + z \\
xy + xyz + z(xy)' &= xy + z \\
xy + z(xy + (xy)') &= xy + z \\
xy + z &= xy + z
\end{aligned}
$$