

Tue 02/01/16

Selection sort

```
SelecionSort(A)
  i = findLargestElement(A)
  SWAP(A[i], A[A.length - 1])
  SelecionSort(A[0:A.length-2])
```

```
findLargestElement(A)
  i = 0
  for j=1 to A.length:
    if A[j] > A[i]
      i = j
  return i
```

How many comparisons does this perform for an array of length n ?

Performs $n-1$ comparisons

How about selection sort?

$T(n) = \# \text{ comparisons performed by selectionSort()}$

$T(1) = 1$

$T(n) = n + T(n-1)$

$T(n-1) = (n-1) + T(n-2)$

Hence $T(n) = n + (n-1) + T(n-2) = n + (n-1) + (n-2) + T(n-3)$
 $\Rightarrow n + (n-1) + (n-2) + (n-3) \dots 1$

$$\Rightarrow n(n-1)/2$$

Merge sort

```
MergeSort(A, B, R)
  nextA = 0; nextB = 0;
  if(nextA < A.length && (nextB >= B.length || A[nextA] <=
B[nextB]))
    R[nextA + nextB] = A[nextA]; nextA++;
  else
    R[nextA + nextB] = B[nextB] nextB++;
```

```
mergeSort(A)
  if A.length <= 1
    return A
  X = mergeSort(A[0 : [A.length / 2 - 1]])
  Y = mergeSort(A[A.length / 2 : A.length - 1])
  R = newArray(A.length)
  MergeSort(X, Y, R)
  return R
```

Number of comparisons in merge w/ output of size n : $\leq 4n$

Let $T(n)$ be the most comparisons performed by mergeSort when given an array of size n

$$T(1) = 1$$

$$T(n) = 1 + 4n + 2T(n/2) = 2T(n/2) + 4n + 1$$

$$T(n/2) = 2T(n/2/2) + 4(n/2) + 1$$

$$\Rightarrow 2T(n/4) + 2n + 1$$

So now

$$\begin{aligned}T(n) &= 2(2T(n/4) + 2n + 1) + 4n + 1 \\&\Rightarrow 4T(n/4) + 4n + 2 + 4n + 1 \\&\Rightarrow 4T(n/4) + 8n + 3\end{aligned}$$

$$\begin{aligned}T(n/4) &= 2T(n/4/2) + 4(n/4) + 1 \\&\Rightarrow 2T(n/8) + n + 1\end{aligned}$$

$$\begin{aligned}\text{So now } T(n) &= 4(2T(n/8) + n + 1) + 8n + 3 \\&\Rightarrow 8T(n/8) + 4n + 4 + 8n + 3 \\&\Rightarrow 8T(n/8) + 12n + 7\end{aligned}$$