

Tue 02/04/16

Merge sort continued

```
So now  $T(n) = 4(2T(n/8) + n + 1) + 8n + 3$ 
 $\Rightarrow 8T(n/8) + 4n + 4 + 8n + 3$ 
 $\Rightarrow 8T(n/8) + 12n + 7$ 
 $\Rightarrow 2^i T(n/2^i) + 4in + 2^i - 1$ 
```

Solve for i such that $n/2^i = 1$

$n = 2^i$

$\log(2)(n) = i$

So

```
 $T(n) = 2^{(\log(2)(n))} T(n/2^{(\log(2)(n))}) + 4n\log(2)(n) + 2^{(\log(2)(n))} - 1$ 
 $\Rightarrow n.T(n/n) + 4n\log(2)(n) + n - 1$ 
 $\Rightarrow n + 4n(\log(2)(n)) + n - 1$ 
 $\Rightarrow 4n\log(2)(n) + 2n - 1$ 
```

Time taken :

$|n|$ Selection Sort (n^2) | MergeSort ($4n\log_2 n$) | $|-|-|$ | 16 | 256 | 256 | | 1024 | ~million | 40000 | |
1m | 1trillion | 80 million |

Quick Sort

```
partition(A, x)
  i=0, j=A.length-1
  while (i<j):
```

```
if(A[i] <= x):  
    i++  
else if(A[j] >= x):  
    j--  
else:  
    SWAP(A[i], A[j])  
return (A[i] > x) ? i : i+1
```

```
QuickSort(A)  
    if A.length <=1 then return  
    t = partition(A[0 : A.length-2], A[A.length-1])  
    QuickSort(A[0 ... t-1])  
    QuickSort(A[t+1 .. A.length-1])
```

Running time

$T(1) = 1$ $T(n) = 3(n+1) + T(t) + T(n-t-1)$

For sorted array : $3n + T(n-1) \Rightarrow n^2$

For unsorted array [1,3,7,8,5]: $2T(n/2) + 3n \Rightarrow 3n \log_3 n$

Cool fact : If quicksort picks the pivot for A, then quicksort runs in $\sim n \log n$ time with high probability