Step-by-Step Guide to Generating an Ephemeris
SSP, July 2005

1. Get the classical orbital elements (a,e,T,i,Ω,ω) and determine M at ephemeris time:

Using k=0.01720209894 and μ=1.000000 for an asteroid orbiting the sun, with time units in days:

$$M = n(t - T) \quad \text{where:} \quad n = k\sqrt{\frac{\mu}{a^3}}$$

2. Get E from M inverting the Kepler equation, $M = E - e\sin E$ using the Newton method as follows: Your first guess for E is M. You then use $f = M - (E - e\sin E)$ you find f', and so your second guess of E would be:
   $E_1 = E_0 - \dfrac{f(E_0)}{f'(E_0)} = E_0 - \left(\dfrac{M - (E_0 - e\sin(E_0))}{e\cos(E_0) - 1}\right)$. Repeat this step several times for E_2 etc. until the M you get in Kepler's equation differs little from the M you had to begin with.
3. Get the Cartesian coordinates of the position vector:

$$\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} a\cos E - ae \\ a\sqrt{1-e^2}\sin E \\ 0 \end{pmatrix}$$

That's the vector describing the asteroid position in orbit coordinates with the x axis coinciding with the perihelion.

4. Now start spinning that vector. First rotation is by (-ω), then by (i) and then by (Ω) to get the **r** vector in ecliptic coordinates:

$$\begin{pmatrix} x_1 \\ y_1 \\ z_1 \end{pmatrix} = \begin{pmatrix} \cos(\Omega) & -\sin(\Omega) & 0 \\ \sin(\Omega) & \cos(\Omega) & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 \\ 0 & \cos(i) & -\sin(i) \\ 0 & \sin(i) & \cos(i) \end{pmatrix} \begin{pmatrix} \cos(\omega) & -\sin(\omega) & 0 \\ \sin(\omega) & \cos(\omega) & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} x \\ y \\ z \end{pmatrix}$$

5. Convert r to equatorial by rotating the coordinates by the tilt of earth relative to the ecliptic ε:

$$\begin{pmatrix} x_2 \\ y_2 \\ z_2 \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & \cos(\varepsilon) & -\sin(\varepsilon) \\ 0 & \sin(\varepsilon) & \cos(\varepsilon) \end{pmatrix} \begin{pmatrix} x_1 \\ y_1 \\ z_1 \end{pmatrix}$$

6. Determine the earth-sun vector from the JPL site
7. Determine the range vector **ρ** from $\vec{\rho} = \vec{r} + \vec{R}$, and also $\hat{\rho} = \vec{\rho}/|\vec{\rho}|$.
8. Solve for RA and Dec from : $\sin(\delta) = \hat{\rho}_z$, and $\cos(\alpha) = \hat{\rho}_x/\cos(\delta)$