Mathematical Modeling - Sailing Simulation Application

Group 08
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Agenda

Introduction

Physics Model

Mathematical Model

Experiments + Discussion

Conclusion

Introduction and Research Questions

How well does the model perform when compared against real-world data?

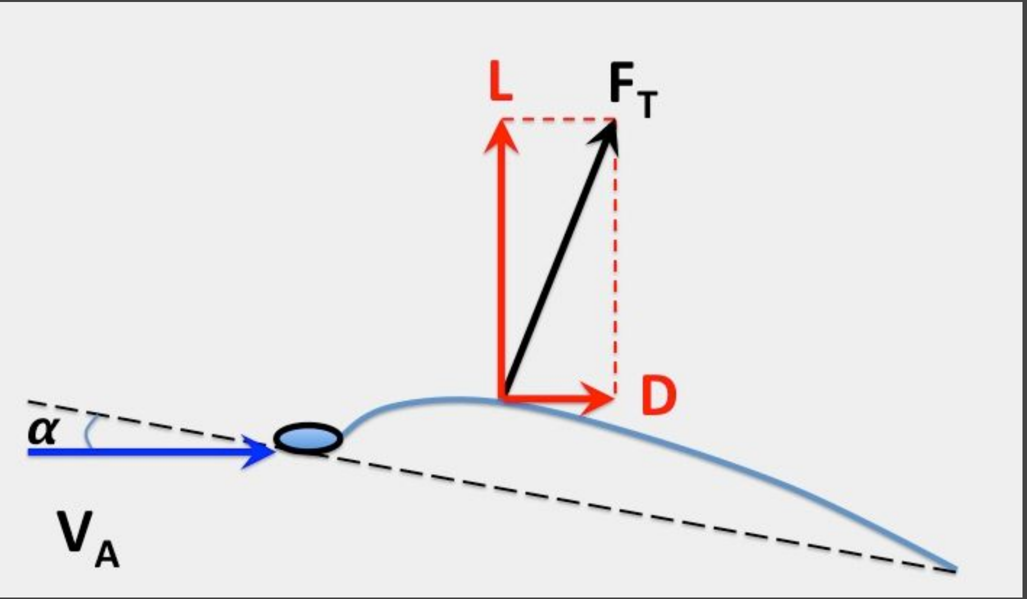
When converting reality into a model, what are the trade-offs we have to take by simplifying?

How does the size of the time-step used to compute the next state of the system impact the performance of the model and what is a good “middle ground” that balances accuracy vs computational effort?

Physics Model

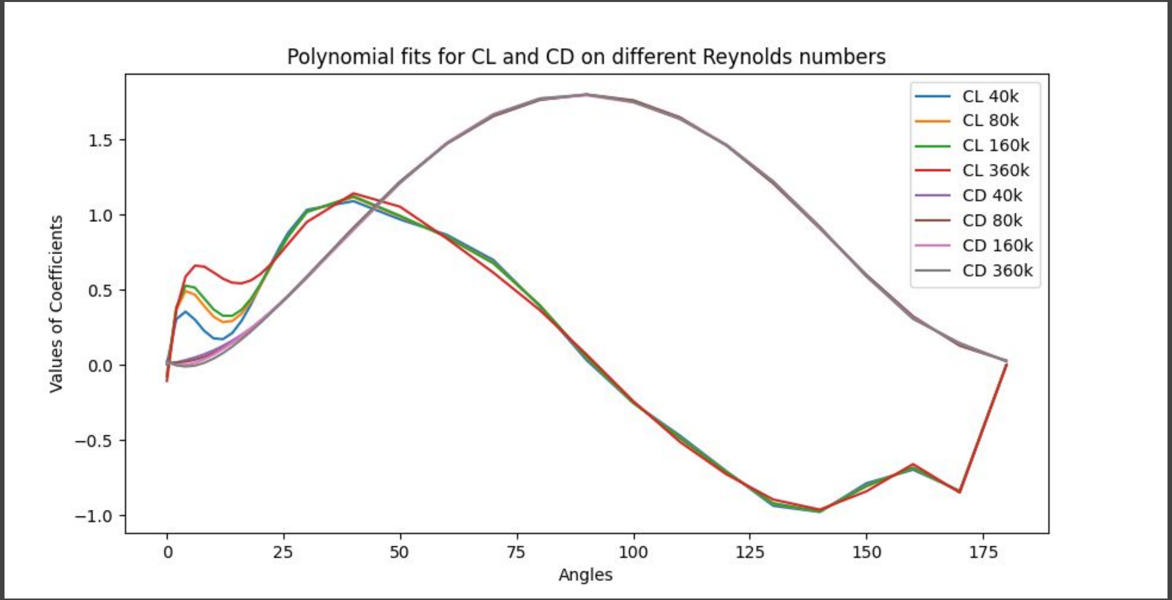
Lift and Drag Force

$$F_D = \frac{1}{2} * \rho * C_D(\beta) * V^2 * S$$
$$F_L = \frac{1}{2} * \rho * C_L(\beta) * V^2 * S$$



Physics Model

Lift and Drag Coefficients

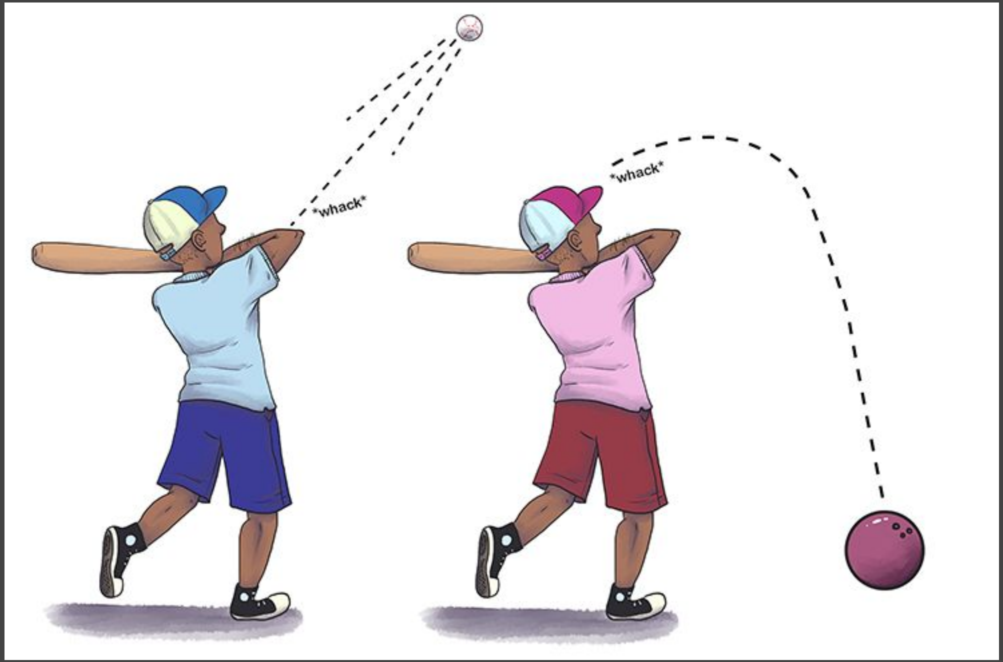


Physics Model

Acceleration

Newton's 2nd Law of Motion:

$$F = F_D + F_L$$
$$a = F/m$$



Mathematical Model

State Space

- Positions:
  px = boat position x
  py = boat position y
  θB = boat angle
  θS = sail angle (relative to boat)

- Velocities:
  vWx = velocity wind x
  vWy = velocity wind y
  vBx = velocity boat x
  vBy = velocity boat y

Differential equations

$$\frac{d}{dx}\begin{pmatrix} pos(x) \\ vel(x) \end{pmatrix} = \begin{pmatrix} vel(x) \\ acc(x) \end{pmatrix}$$

$$\frac{d}{dx}(p_x, p_y, \theta_B, \theta_S) = (v_{Bx}, v_{By}, 0, 0)$$

$$\frac{d}{dx}(v_{Wx}, v_{Wy}, v_{Bx}, v_{By}) = (0, 0, a_x, a_y)$$

Mathematical Model

Solvers

Euler's Method (local error $O(h^2)$)

$$w_{i+1} = w_i + h_i f(t_i, w_i)$$

Two-stage Adams-Moulton Method (predictor-corrector)

(local error $O(h^4)$)

$$w_{i+1} = w_i + (h/12)(5f(t_{i+1}, w_{i+1}) + 8f(t_i, w_i) - f(t_{i-1}, w_{i-1})))$$

Mathematical Model

Solvers

Fourth Order Runge-Kutta Method (local error $O(h^5)$)

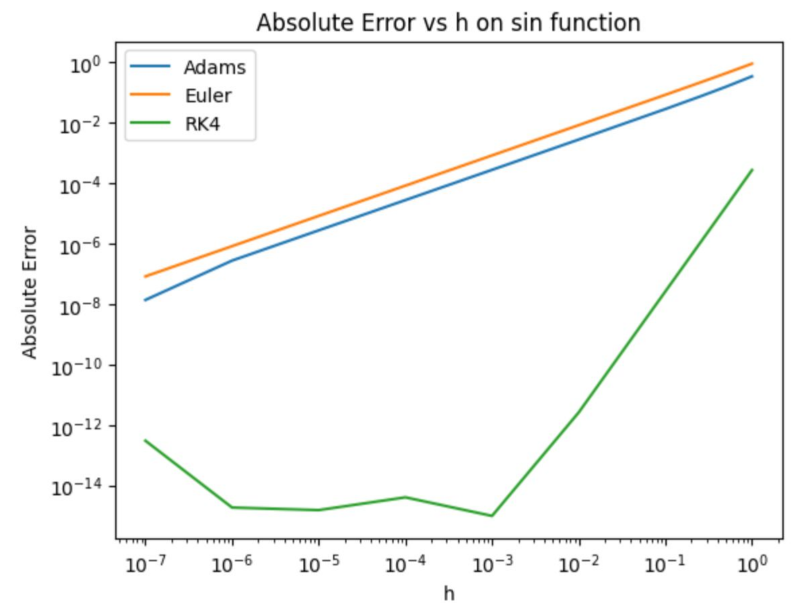
$$w_{i+1} = w_i + \frac{1}{6}\left(k_{i,1} + 2k_{i,2} + 2k_{i,3} + k_{i,4}\right)$$

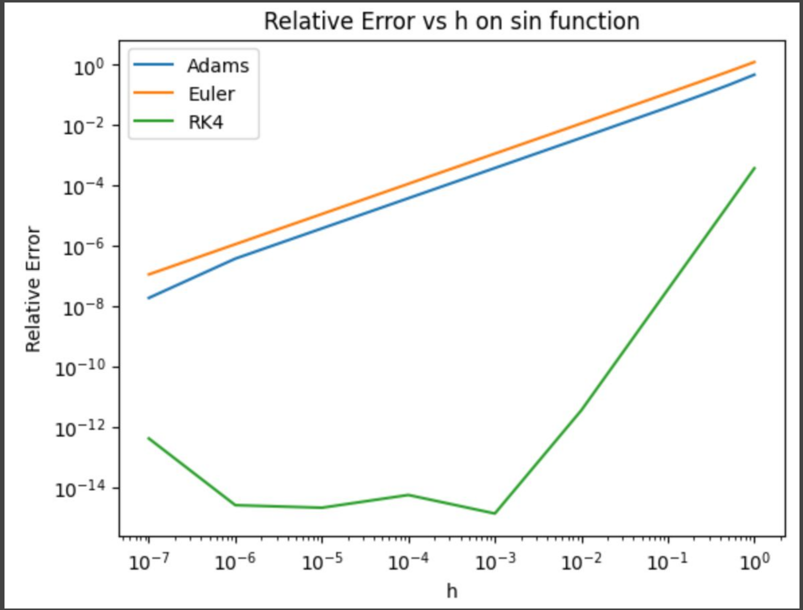
$$k_{i,1} = h_i f\left(t_i, w_i\right); k_{i,2} = h_i f\left(t_i + \frac{1}{2}h_i, w_i + \frac{1}{2}k_{i,1}\right)$$
$$k_{i,3} = h_i f\left(t_i + \frac{1}{2}h_i, w_i + \frac{1}{2}k_{i,2}\right); k_{i,4} = h_i f\left(t_i + h_i, w_i + k_{i,3}\right)$$

Demo

Experiments and Discussion

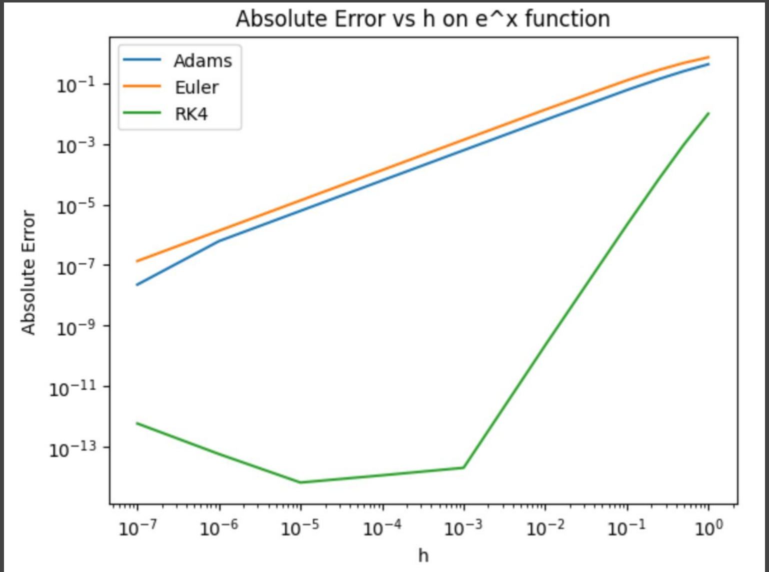
Solvers Experiments - Sin Function

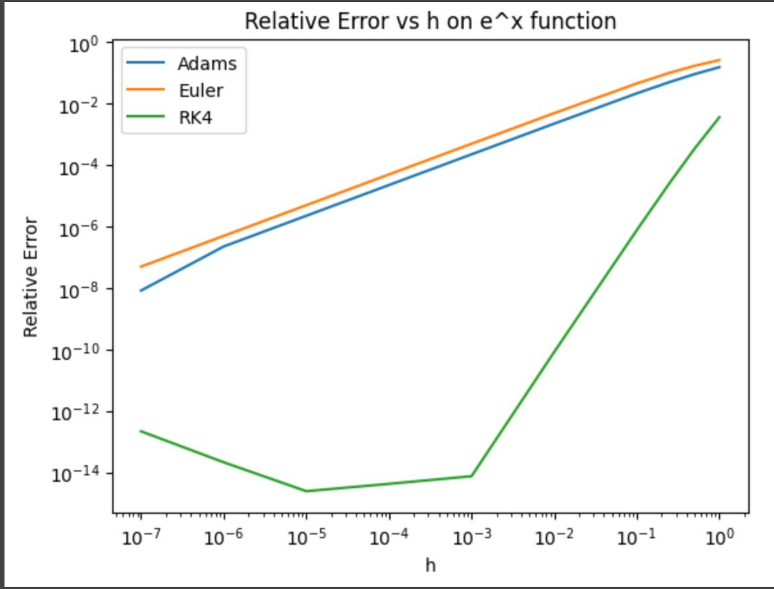




Experiments and Discussion

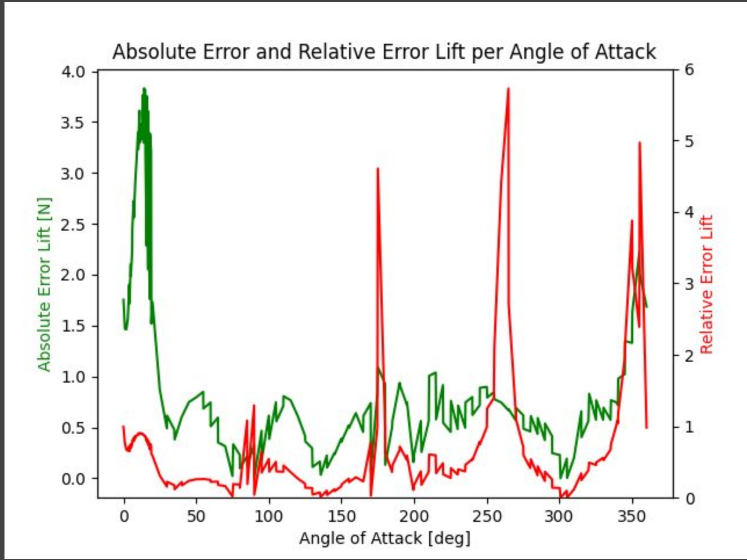
Solvers Experiments - Exponential Function

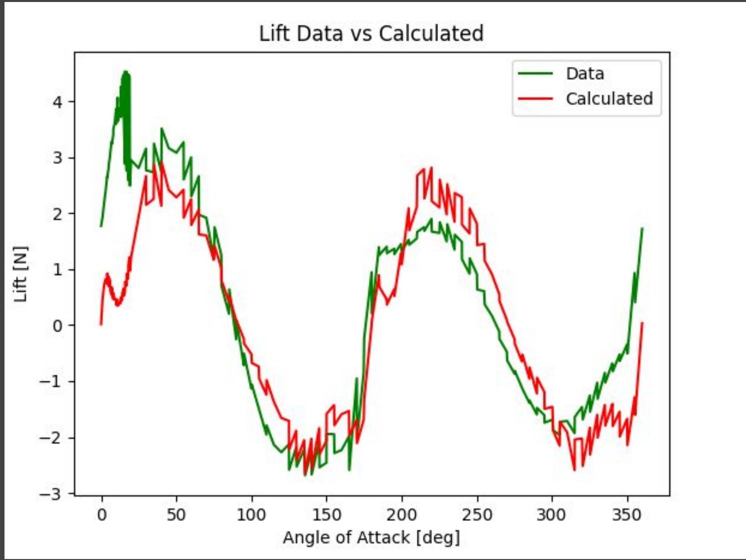




Experiments and Discussion

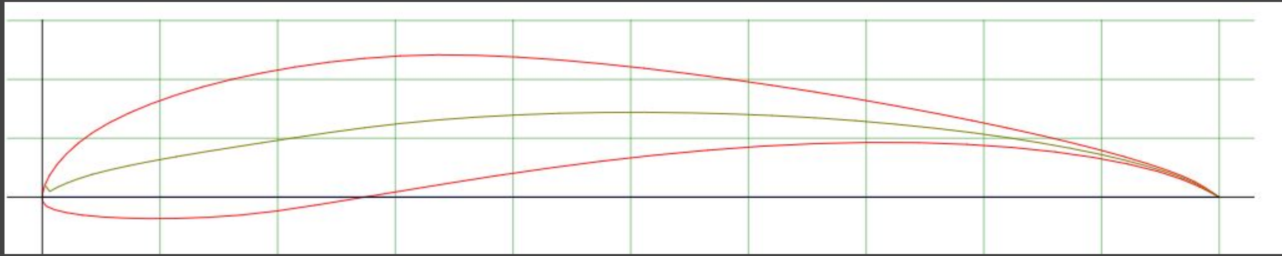
Physics Model Experiments - Lift



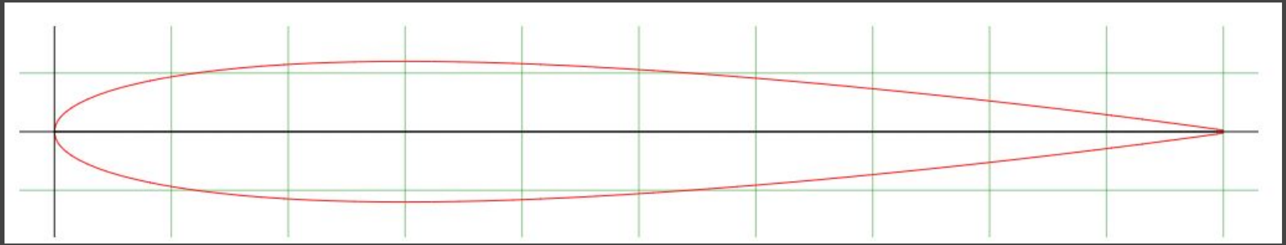


Experiments and Discussion

Airfoil comparison:



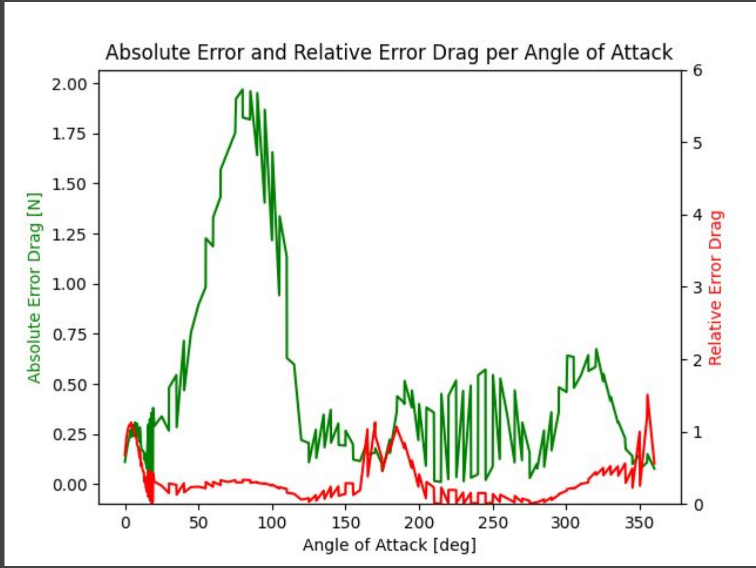
S1210
- Data set from windtunnel

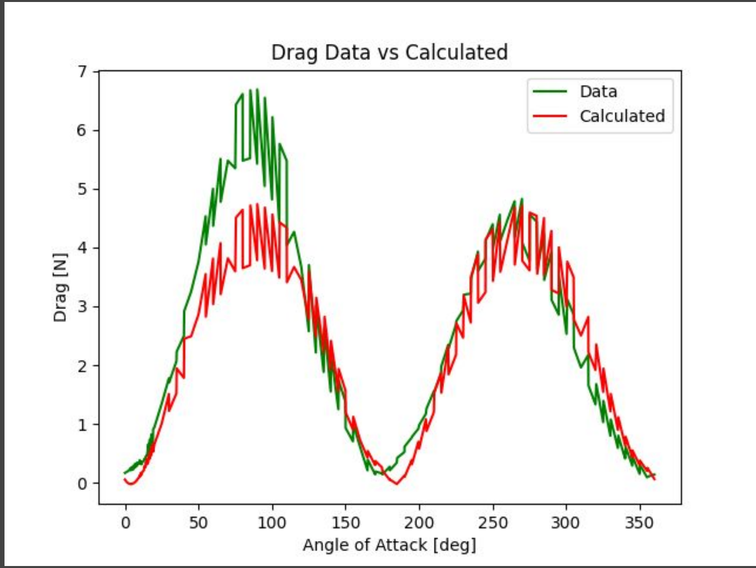


NACA 0012
- Coefficient approximation

Experiments and Discussion

Physics Model Experiments - Drag





Conclusion

The model performs reasonably well, with some discrepancies at specific angles.

2D model and interpolating coefficients introduces some degree of simplification while still maintaining the ability to approximate the real world.

The size of the time-step in numerical solvers critically impacts the model's accuracy and computational efficiency. The experiments show that smaller time-steps generally yield more accurate results, but at the cost of increased computational effort. The Runge-Kutta method, in particular, achieves a good balance.

What comes next

Transition from wind tunnel to real water body - forces acting on the hull and the hull-sail interactions.

Movement in the real world using geographical coordinates.

Captivating and a rich graphical interface, able to convey and improve intuition on some of the advanced sailing concepts.

Thank you!

Q&A