

As *regras de inferência* do sistema formal de Dedução Natural para o Cálculo Proposicional (DNP) são as seguintes:

Regras de Introdução

$$\frac{\varphi \quad \psi}{\varphi \wedge \psi} \wedge I$$

$$\frac{\begin{array}{c} \not\vdots \\ \psi \end{array}}{\varphi \rightarrow \psi} \rightarrow I$$

$$\frac{\begin{array}{c} \not\vdots \\ \bot \\ \neg\varphi \end{array}}{\neg\varphi} \neg I$$

$$\frac{\varphi}{\varphi \vee \psi} \vee_1 I \quad \frac{\psi}{\varphi \vee \psi} \vee_2 I$$

$$\frac{\begin{array}{c} \not\vdots \\ \psi \end{array} \quad \begin{array}{c} \not\vdots \\ \varphi \end{array}}{\varphi \leftrightarrow \psi} \leftrightarrow I$$

$$\frac{\bot}{\varphi} (\bot)$$

Regras de Eliminação

$$\frac{\varphi \wedge \psi}{\varphi} \wedge_1 E \quad \frac{\varphi \wedge \psi}{\psi} \wedge_2 E$$

$$\frac{\psi \quad \psi \rightarrow \varphi}{\varphi} \rightarrow E$$

$$\frac{\varphi \quad \neg\varphi}{\bot} \neg E$$

$$\frac{\psi \vee \sigma \quad \begin{array}{c} \not\vdots \\ \varphi \end{array} \quad \begin{array}{c} \not\vdots \\ \varphi \end{array}}{\varphi} \vee E$$

$$\frac{\varphi \quad \varphi \leftrightarrow \psi}{\psi} \leftrightarrow_1 E \quad \frac{\psi \quad \varphi \leftrightarrow \psi}{\varphi} \leftrightarrow_2 E$$

$$\frac{\not\vdots}{\varphi} (RAA)$$