$$Var[T] = \left(0 - \frac{2}{3}\right)^2 \times \frac{3}{6} + \left(1 - \frac{2}{3}\right)^2 \times \frac{2}{6} + \left(2 - \frac{2}{3}\right)^2 \times \frac{1}{6}$$

$$= \frac{4}{9} \times \frac{3}{6} + \frac{1}{9} \times \frac{2}{6} + \frac{16}{9} \times \frac{1}{6}$$

$$= \frac{12 + 2 + 16}{6 \times 9} = \frac{30}{6 \times 9} = \frac{5}{9}$$

FOLHA 4

ξ experiência aleatória

acontecimento S c/ probabilidade p ($p \in]0,1[$)

X : v.a. que representa o nº de vezes de S ocorre em n repetições independentes de ξ

X v.a. discreta com contradomínio $C_X = \{0, 1, \dots, n\}$

e f.m.p. dada por

$$P(X = k) = \binom{n}{k} p^k (1-p)^{n-k}, \quad k = 0, \dots, n$$

X segue a distribuição Binomial com parâmetros n e p

$$X \sim Bin(n, p)$$

$$E[X] = np \qquad Var[X] = np(1-p)$$

3. $Bin(10, 0.6)$

a) $P(X=9) = \binom{10}{9} \times 0{,}6^9 \times 0{,}4^1$

$$= \frac{10!}{1!\,9!} \times 0{,}6^9 \times 0{,}4 = 10 \times 0{,}6^9 \times 0{,}4$$
$$= 4 \times 0{,}6^9$$

$P(X \geq 9) = P(X=9) + P(X=10) = 4 \times 0{,}6^9 + 0{,}6^{10}$
$= 4{,}6 \times 0{,}6^9$

$$P(X=0) = \binom{10}{0} \times 0{,}6^0 \times 0{,}4^{10}$$
$$= 0{,}4^{10}$$

(b) cara : prob. $\frac{1}{2}$ $\quad p=0{,}5$

$n=10$

$X \sim Bin(10, \frac{1}{2})$

$$P(X=2) = \binom{10}{2} \times \left(\frac{1}{2}\right)^2 \times \left(1-\frac{1}{2}\right)^{10-2}$$
$$= \frac{10!}{8!\,2!} \times \frac{1}{2^2} \times \frac{1}{2^8}$$
$$= \frac{10\times 9}{2} \times \frac{1}{2^2} \times \frac{1}{2^8} = \frac{45}{2^{10}} = 45 \times (0{,}5)^{10}$$

$$P(X \geq 2) = 1 - P(X<2)$$
$$= 1 - P(X=0) - P(X=1)$$
$$= 1 - \binom{10}{0}\times\left(\frac{1}{2}\right)^0 \times \left(\frac{1}{2}\right)^{10} - \binom{10}{1}\times\left(\frac{1}{2}\right)^1 \times \left(\frac{1}{2}\right)^9$$
$$= 1 - \left(\frac{1}{2}\right)^{10} - 10 \times \left(\frac{1}{2}\right)^{10}$$
$$= 1 - 11 \times \left(\frac{1}{2}\right)^{10}$$

(c) : igual a b)

(d) $P(B) = \frac{3}{5} \quad P(V) = \frac{2}{5}$

Binomial $(4, 3/5)$

c/ reposição

X: nº de bolas brancas

$$P(X=4) = \binom{4}{4} \left(\frac{3}{5}\right)^4 \times \left(\frac{2}{5}\right)^0 = \left(\frac{3}{5}\right)^4$$

Todas as bolas vermelhas $\rightsquigarrow X=0$

$$P(X=0) = \binom{4}{0} \left(\frac{3}{5}\right)^0 \left(\frac{2}{5}\right)^4 = \left(\frac{2}{5}\right)^4$$

s/ reposição $\quad P(X=4)=0 \quad P(X=0)=0$

Distribuição de Poisson, com parâmetro λ

Y v.a. discreta

contradomínio $\mathbb{N}_0$

f.m.p. de Y dada por:

$$P(Y=k) = \frac{\lambda^k}{k!} e^{-\lambda}, \quad k \in \mathbb{N}_0$$

Y segue uma distribuição de Poisson com parâmetro λ.

$$Y \sim \text{Poisson}(\lambda)$$

$$E[Y] = \lambda \qquad Var[Y] = \lambda$$

4. $Y \sim \text{Poisson}(\lambda)$

$$P(Y=k) = \frac{\lambda^k}{k!} e^{-\lambda}$$

$$P(Y=0) = \frac{1}{e} \Leftrightarrow \frac{\lambda^0}{0!} e^{-\lambda} = \frac{1}{e} \Leftrightarrow \frac{1}{e^\lambda} = \frac{1}{e} \Rightarrow \lambda = 1$$

(a) $E[Y] = \lambda = 1$

(b) i) $P(Y=2) = \frac{1^2}{2!} e^{-1} = \frac{1}{2} e^{-1}$

ii) $$P(Y \leq 2) = P(Y=0) + P(Y=1) + P(Y=2)$$
$$= \frac{1^0}{0!} e^{-1} + \frac{1^1}{1!} e^{-1} + \frac{1}{2} e^{-1}$$
$$= e^{-1} + e^{-1} + \frac{1}{2} e^{-1} = \frac{5}{2} e^{-1}$$

iii) $$P(Y \geq 2) = 1 - P(Y=0) - P(Y=1)$$
$$= 1 - e^{-1} - e^{-1} = 1 - 2e^{-1}$$

5. X : nº de artigos de luxo vendidos diariamente

$$X \sim \text{Poisson}(0,6) \qquad \lambda = E[X] = 0,6$$

$$P(X=k) = \frac{0,6^k}{k!} e^{-0,6}$$

(a) $$P(X=0) = \frac{0,6^0}{0!} e^{-0,6} = e^{-0,6}$$

(b) $X_1, X_2, X_3, X_4, X_5, X_6$ todos com distribuição de Poisson com parâmetro 0,6

$$P(X_1=0, X_2=0, \ldots, X_6=0) \underset{\text{ind.}}{=} \left(P(X=0)\right)^6 =$$
$$= \left(e^{-0,6}\right)^6 = e^{-3,6}$$

(c) X = nº de artigos vendidos diariamente
Y = nº de artigos defeituosos vendidos diariamente

$$P(Y=c) = P(Y=c \mid X=c)\,P(X=c) + P(Y=c \mid X=c+1)\,P(X=c+1) +$$
$$+ \ldots + P(Y=c \mid X=c+m)\,P(X=m) + \ldots$$
$$= \binom{c}{c} p^c (1-p)^0 \times \frac{0,6^c}{c!} e^{-0.6} + \binom{c+1}{c} p^c (1-p)^1 \times \frac{0,6^{c+1}}{(c+1)!} e^{-0.6} +$$
$$+ \ldots + \binom{c+m}{c} p^c (1-p)^m \times \frac{0.6^{c+m}}{(c+m)!} e^{-0.6} + \ldots$$
$$= \frac{\cancel{c!}}{c!\,0!} p^c \frac{0.6^c}{\cancel{c!}} e^{-0.6} + \frac{\cancel{(c+1)!}}{c!\,1!} p^c (1-p) \times \frac{0.6^{c+1}}{\cancel{(c+1)!}} e^{-0,6} +$$
$$+ \ldots + \frac{\cancel{(c+m)!}}{c!\,m!} p^c (1-p)^m \times \frac{0.6^{c+m}}{\cancel{(c+m)!}} e^{-0.6} + \ldots$$

$= \frac{p^c}{c!} \times 0.6^c \times e^{-0.6} \times \Big(1 + \frac{1}{1!}(1-p)\times 0.6^1 + \frac{1}{2!}(1-p)^2 \times 0.6^2 +$

$\dots + \frac{1}{m!}(1-p)^m \times 0.6^m + \dots\Big) = \frac{p^c}{c!} \times 0.6^c \times e^{-0.6} \times e^{(1-p)0.6}$

$$\boxed{e^x = \sum_{m=0}^{+\infty} \frac{x^m}{m!} \quad x \in \mathbb{R}}$$

(expansão em série de Taylor da função e^x em torno de zero)

$= \frac{(0.6p)^c}{c!} e^{-0.6p}$

$Y \sim$ Poisson $(0.6p)$

d) $X \sim$ Poisson $(\lambda) \Rightarrow Y \sim$ Poisson (λp)

e)

i) $E[100X] = 100 \times E[X] = 100 \times 0.6 = 60$

$Var[100X] = 100^2 \times Var[X] = 100^2 \times 0.6 = 6000$

ii) Z $\quad 100X_1, 100X_2, \dots, 100X_5$ (indep.)

$E[Z] = E[100X_1] + E[100X_2] + \dots + E[100X_5]$

$= 5 \times 60 = 300$

$Var[Z] \underset{v. indep.}{=} \sum_{i=1}^{5} Var[100X_i] = 5 \times 6000 = 30000$

6. X v.a. contínua

função densidade de probabilidade $\quad f(x) = \begin{cases} 0 & \text{se } x < a \vee x > b \\ \frac{1}{10} & \text{se } a \leq x \leq b \end{cases}$

$a, b \in \mathbb{R}$

$P(X > 8) = 0.4$

(a) $\int_{-\infty}^{+\infty} f(x)\,dx = \int_a^b \frac{1}{10}\,dx = \frac{1}{2}(b-a)$

Sabemos que $\int_{-\infty}^{+\infty} f(x)\,dx = 1$. Logo, $b-a=10$,

ou seja, $b = a+10$

$$f(x) = \begin{cases} 0 & \text{se } x<a \lor x>a+10 \\ \frac{1}{10} & \text{se } a\le x\le a+10 \end{cases}$$

$$P(X>8) = 1-P(X\le 8)$$

$$= 1-F(8)$$

Logo, $1-F(8)=0.4$, donde $F(8)=0.6$

Assim,

$$\int_{-\infty}^{8} f(x)\,dx = 0.6, \text{ donde } \frac{1}{10}(8-a) = 0.6$$

Logo, $8-a=6$, i.e. $a=2$.

Portanto, $b=12$.

$$f(x)=\begin{cases} 0 & \text{se } x<2 \\ \frac{1}{10} & \text{se } 2\le x\le 12 \\ 0 & \text{se } x>12 \end{cases}$$

Temos que, para $c<2$, $F(c)=0$. Para c t.q. $2\le c\le 12$,

$$F(c)=\int_2^c \frac{1}{10}\,dx = \frac{c-2}{10}.$$

Para $c>12$, $F(c) = \int_2^{12}\frac{1}{10}\,dx = 1$.

Assim,

$$F(c) = \begin{cases} 0 & \text{se } c<2 \\ \frac{c-2}{10} & \text{se } 2\le c\le 12 \\ 1 & \text{se } c>12 \end{cases}$$

Diz-se que uma v.a. contínua X segue a distribuição Uniforme no intervalo $[a,b]$ se a função densidade de probabilidade é dada por

$$f(x) = \begin{cases} \frac{1}{b-a} & \text{se } a \leq x \leq b \\ 0 & \text{c.c.} \end{cases}$$

$X \sim U[2,12]$

(b) X: consumo diário de água em m^3

i) $P(X < 8) = F(8) = 0.6$

ii) X_1, X_2, X_3, X_4, X_5 v.a. independentes dist. $U[2,12]$

$Y = Bin(5, 0.6)$ $\qquad$ Y: nº de dias c/ consumo < 8

$$\begin{aligned} P(Y \leq 2) &= P(Y=0) + P(Y=1) + P(Y=2) \\ &= \binom{5}{0} 0.6^0\, 0.4^5 + \binom{5}{1} 0.6^1\, 0.4^4 + \binom{5}{2} 0.6^2 \cdot 0.4^3 \\ &= 0.4^5 + 3 \times 0.4^4 + 10 \times 3.6 \times 0.4^3 \\ &= 0.4^3 \left(0.4^2 + 3 \times 0.4 + 36\right) \\ &= 0.4^3 \left(1.6 + 1.2 + 36\right) \\ &= 0.4^3 \times 38.8 \end{aligned}$$

7. X v.a. contínua

$X \sim U[340, 360]$

$$f(x) = \begin{cases} \frac{1}{20} & \text{se } 340 \leq x \leq 360 \\ 0 & \text{c.c.} \end{cases}$$

$$F(c) = \begin{cases} 0 & \text{se } c < 340 \\ \frac{c-340}{20} & \text{se } 340 \leq c \leq 360 \\ 1 & \text{se } c \geq 360 \end{cases}$$

(a) $P(X < 345) = P(X \leq 345) = \frac{345-340}{20} = \frac{5}{20} = \frac{1}{4}$

(b) $P(X > 355) = 1 - P(X \leq 355) = 1 - F(355) =$

$= 1 - \frac{355-340}{20} = 1 - \frac{15}{20} = 1 - \frac{3}{4} = \frac{1}{4}$

(c) aceite se $348 \leq X \leq 352$

é rejeitado se $X < 348 \vee X \geq 352$

$1 - P(348 \leq X \leq 352) = 1 - F(352) + F(348) =$

$= 1 - \frac{12}{20} + \frac{8}{20} = 1 - \frac{1}{5} = \frac{4}{5}$

(d) QUANTIS $p \in \,]0,1[$

$x_p = \inf\{c \in \mathbb{R} : F(c) \geq p\} = 340 + 20p$

$\frac{c-340}{20} \geq p \wedge c \in [340, 360] \Leftrightarrow$

$\Leftrightarrow c \geq 340 + 20p \wedge c \in [340, 360]$

$340 + 20p \leq 360 \Leftrightarrow 20p \leq 20 \Leftrightarrow p \leq 1$

8. $F(c) = \begin{cases} 0 & \text{se } c < 0 \\ 1 - e^{-\lambda c} & \text{se } c \geq 0 \end{cases}$ $E[X] = \frac{1}{\lambda}$, $Var[X] = \frac{1}{\lambda^2}$

$X \sim Exp(\lambda)$

(a) $E[X] = \frac{1}{\lambda} \Leftrightarrow 10 = \frac{1}{\lambda} \Leftrightarrow \lambda = 0.1$

(b) $P(X<8) = P(X\leq 8) = F(8) = 1-e^{-0.1\times 8} =$

$= 1- e^{-0.8}$

(c) $P(X\geq 10) = 1- P(X<10) =$

$= 1- P(X\leq 10) = 1- F(10) =$

$= 1- (1-e^{-0.1\times 10}) = e^{-1}$

9. $X \sim Exp(\lambda)$

$P(X > E[X]) = 1 - P(X \leq \frac{1}{\lambda})$

$= 1 - F(\frac{1}{\lambda}) = 1-(1-e^{-\lambda\times\frac{1}{\lambda}})$

$= e^{-1}$

FOLHA 5

X v.a. contínua

média, σ desvio padrão

$X \sim N(\mu, \sigma^2)$ $(\mu\in\mathbb{R}, \sigma\in\mathbb{R}^+)$

se a função densidade de probabilidade é dada por

$$f(x) = \frac{1}{\sigma\sqrt{2\pi}} \exp\left\{-\frac{1}{2}\left(\frac{x-\mu}{\sigma}\right)^2\right\}, \quad x\in\mathbb{R}$$

[$\mu=0, \sigma=1$: dist. Normal standard]

$f(\mu+a) = f(\mu-a)$

$E[X] = \mu$ $\quad Var[X] = \sigma^2$

$X\sim N(\mu,\sigma^2) \Rightarrow Z = \frac{X-\mu}{\sigma} \sim N(0,1)$

$Z\sim N(0,1) \Rightarrow Y = \mu+\sigma Z \sim N(\mu,\sigma^2), \quad \forall_{\substack{\mu\in\mathbb{R}\\ \sigma\in\mathbb{R}^+}}$

TABELA $\quad P(0 < Z \leq c) \quad c \in \mathbb{R}_0^+ \quad , \quad Z \sim N(0,1)$

$$F_{N(0,1)}(-c) = 1 - F_{N(0,1)}(c)$$

$$x_p = -x_{1-p}$$

$Z \sim N(0,1)$

$\Downarrow$

$$P(|Z| \leq b) = 2P(0 < Z \leq b)$$

1. $\quad X \sim N(0,1)$

(a) $\quad$ F

$P(X \leq a) + P(X \geq -a) =$

$= F(a) + 1 - P(X < -a) = F(a) + 1 - P(X \leq -a)$

$= F(a) + 1 - F(-a) = F(a) + 1 - (1 - F(a)) =$

$= F(a) + F(a) = 2F(a)$

(b) $\quad P(X \leq a) + P(X \geq -a) = 1 \quad$ F

(c) $\quad P(X \leq a) = P(X > a)$

$\Leftrightarrow F(a) = 1 - F(a) \Leftrightarrow 2F(a) = 1 \Leftrightarrow F(a) = \frac{1}{2}$

F

(d) $\quad P(X \leq a) = P(X \geq -a) \quad$ V

$P(X \leq a) = F(a)$

$P(X \geq -a) = 1 - P(X \leq -a)$

$= 1 - F(-a) = 1 - (1 - F(a))$

$= F(a)$

2. $Z \sim N(0,1)$

(a) $P(Z \leq c) = 0.975$

(b) $P(Z \leq c) = 0.025$

(c) $P(Z > c) = 0.05$

(d) $P(|Z| \geq c) = 0.1$

(a) $P(Z \leq c) = P(Z \leq 0) + P(0 < Z \leq c)$
$= 0.5 + P(0 < Z \leq c)$

$P(Z \leq c) = 0.975 \Leftrightarrow P(0 < Z \leq c) = 0.975 - 0.5 = 0.475$

$\Rightarrow c = 1.96$

(b) $P(Z < c) = 0.025$ $c = -1.96$

$1 - 0.975 = 0.025$ $F_{N(0,1)}(-c) = 1 - F_{N(0,1)}(c)$

(c) $P(Z > c) = 0.05 \Leftrightarrow 1 - P(Z \leq c) = 0.05$
$\Leftrightarrow P(Z \leq c) = 0.95$
$\Leftrightarrow P(0 < Z \leq c) = 0.95 - 0.5 = 0.45$
$\therefore c = 1.645$

(d) $P(|Z| \geq c) = 0.1 \Leftrightarrow 1 - P(|Z| < c) = 0.1$
$\Leftrightarrow 1 - 2P(0 < X \leq c) = 0.1$
$\Leftrightarrow P(0 < X \leq c) = \frac{1 - 0.1}{2}$
$\Leftrightarrow P(0 < X \leq c) = 0.45$
$c \simeq 1.645$

3. $E[X_i] = 10$ $X_i \sim N(10, (0.5)^2)$
$\sigma_{X_i} = 0.5$

$Y = N(150, 8^2)$

$W = X_1 + \dots + X_{12} + Y$

$W = X_1 + \dots + X_{12} + Y \quad E[W] = E[X_1] + \dots + E[X_{12}] + E[Y]$

$Var[W] = Var[X_1] + \dots + Var[X_{12}] + Var[Y]$

$= 0,5^2 + \dots + 0,5^2 + 8^2$

(a) $E[W] = 12 \times 10 + 150 = 270$

$Var[W] = 12 \times (0.5)^2 + 8^2$

$= 3 + 8^2 = 3 + 64 = 67$

$W \sim N(270, 67)$

(b) $Z = \frac{W - \mu}{\sqrt{\sigma}} \Leftrightarrow W = \sqrt{67}\, Z + 270$

$P(W > 285) = P(\sqrt{67}\, Z + 270 > 285)$

$= P\left(Z > \frac{15}{\sqrt{67}}\right) = 1 - P\left(Z \le \underbrace{\frac{15}{\sqrt{67}}}_{\simeq 1,83}\right)$

$= 1 - 0.5 - P\left(0 < Z \le \frac{15}{\sqrt{67}}\right)$

$= 1 - 0.5 - 0,4664$

$= 0,0336$

4. $X \sim N(3.2, (1.8)^2)$

amostra aleatória $X_1, \dots, X_{10}$ proveniente de $X \sim N(3.2, (1.8)^2)$

(média amostral)

$\overline{X}_{10} \sim N\left(3.2, \frac{(1.8)^2}{10}\right) = N\left(3.2, \left(\frac{1.8}{\sqrt{10}}\right)^2\right)$

$Z = \frac{\overline{X}_{10} - 3.2}{\frac{1.8}{\sqrt{10}}} \Leftrightarrow \frac{1.8}{\sqrt{10}} Z = \overline{X}_{10} - 3.2 \Leftrightarrow$

$\Leftrightarrow \overline{X}_{10} = 3.2 + \frac{1.8}{\sqrt{10}} Z$

$P(\overline{X}_{10} > 3.5) = P\left(3.2 + \frac{1.8}{\sqrt{10}} Z > 3.5\right) =$

$= P\left(\frac{1.8}{\sqrt{10}} Z > 0.3\right) = P\left(Z > \frac{0.3\sqrt{10}}{1.8}\right) =$

$$= P\left(Z > \frac{-\frac{2}{10}}{\frac{12}{10}}\sqrt{10}\right) = P\left(Z > \frac{\sqrt{10}}{6}\right) = 1 - P\left(Z \leq \frac{\sqrt{10}}{6}\right)$$

$$= 1 - 0.5 - \underbrace{P\left(0 < Z \leq \frac{\sqrt{10}}{6}\right)}_{\substack{0.5270 \\ \simeq 0.53}} = 0.5 - 0.2019 = 0.2981$$

5. $X_1, X_2, \ldots, X_{100} \sim \text{Poisson}\ (0.02)$

$X_1, \ldots, X_{100}$ v.a.'s i.i.d.'s
(independentes e identicamente distribuidas)

X_i variância : 0.02 —— desvio padrão $\sqrt{0.02}$
valor médio : 0.02

$$S_{100} \underset{\sim}{\sim} Y \sim N\left(100 \times \mu,\ 100 \times \sigma^2\right)$$

$$S_{100} \simeq Y \sim N(2, 2)$$

$$S_{100} \simeq \sqrt{2}\,Z + 2$$

$$P(S_{100} > 2) \simeq P(\sqrt{2}Z + 2 > 2)$$
$$= P(Z > 0) = 0.5$$

6.

$X_i \quad \mu_i = 20 \quad \sigma_i^2 = 350$

$$\overline{X}_{100} \sim N\left(20, \frac{350}{100}\right) = N(20, 3.5)$$

$$\overline{X}_{100} = \sqrt{3.5}\,Z + 20$$

$$P(\overline{X}_{100} > 18) = P(\sqrt{3.5}\,Z + 20 > 18)$$

$$= P\left(Z > \frac{-2}{\sqrt{3.5}}\right) = \frac{1}{2} + P\left(0 < Z \leq \overbrace{\frac{2}{\sqrt{3.5}}}^{\substack{\simeq 1.07 \\ 1.0690 \\ \simeq}}\right)$$

$$= \frac{1}{2} + 0.3577 = 0.8577$$

7. 100 baterias

$\mu_i = 3$

$\sigma_i = 3$

$$\overline{X}_{100} \simeq Y \sim N\left(\mu, \frac{\sigma^2}{100}\right) = N\left(3, \left(\frac{3}{10}\right)^2\right)$$

$$\overline{X}_{100} \simeq 3 + 0.3 Z$$

$$P(\overline{X}_{100} > 2.5) \simeq P(3 + 0.3 Z > 2.5) = P\left(Z > \frac{0.5}{0.3}\right)$$

$$= 0.5 + P\left(0 < Z \le \frac{0.5}{0.3}\right)$$

$$= 0.5 + 0.4525 = 0.9525$$

8. $\mu_i = 4$ $\quad \sigma_i^2 = (1.2)^2$

$$\overline{X}_{70} \simeq Y \sim N\left(4, \frac{(1.2)^2}{70}\right) = N\left(4, \left(\frac{1.2}{\sqrt{70}}\right)^2\right)$$

$$\overline{X}_{70} \simeq \frac{1.2}{\sqrt{70}} Z + 4 \qquad \frac{1}{12} \times 1.2 = 0.1$$

$$P(3.9 \le \overline{X}_{70} \le 4.1) \simeq P(3.9 \le Y \le 4.1) =$$

$$= P\left(-0.1 \le \frac{1.2}{\sqrt{70}} Z \le 0.1\right) =$$

$$= P\left(-\frac{\sqrt{70}}{12} \le Z \le \frac{\sqrt{70}}{12}\right) =$$

$$= 2 P\Bigg(0 < Z \le \underbrace{\frac{\sqrt{70}}{12}}_{\simeq 0.697}\Bigg) = 2 \times 0.2580 = 0.516$$

ii) X: nº de vezes que saiu face 6 nos 100 lançamentos

Bin(n, p)

$X \sim Bin\left(100, \frac{1}{6}\right)$ $\quad E[X] = 100 \times \frac{1}{6}$ $\quad (np)$

$Var[X] = 100 \times \frac{1}{6} \times \frac{5}{6}$ $\quad (np(1-p))$

$= \frac{500}{36}$

TLC

↳ X tem aproximadamente dist. $N\left(\frac{100}{6}, \frac{500}{36}\right)$

$$X \simeq W \sim N\left(\frac{100}{6}, \frac{500}{36}\right)$$

$$W = \sqrt{\frac{500}{36}}\, Z + \frac{100}{6} = \frac{\sqrt{500}}{6} Z + \frac{100}{6}$$

$$P(X \leq 30) \simeq P(W \leq 30) = P\left(\frac{\sqrt{500}}{6} Z \leq 30 - \frac{100}{6}\right)$$

$$= P\left(Z \leq \frac{\frac{80}{6}}{\frac{\sqrt{500}}{6}}\right) = P\left(Z \leq \frac{8}{\sqrt{5}}\right)$$

$$= \frac{1}{2} + P\left(0 < Z \leq \underbrace{\frac{8}{\sqrt{5}}}_{\simeq 3{,}58}\right) = 0.5 + 0.4998 = 0.9998$$

iii) Se X = 100 então Y = 600

$$P(Y = 300 \mid X = 100) = 0$$

$P(X = 100) \neq 0$

$P(Y = 300) \neq 0$

$P(Y = 300, X = 100)$

$= P(Y = 300 \mid X = 100) \times P(X = 100)$

$= 0$

Logo, $P(Y = 300, X = 100) \neq P(X = 100)\, P(Y = 300)$

X, Y não são independentes.