2.

a)

$$F_X(c)\begin{cases} \int_{-\infty}^{c} 0\,dx = 0 & \text{se } c \leq 0 \\ \int_{-\infty}^{0} 0\,dx + \int_{0}^{c} 12x^2(1-x)\,dx = \left[4x^3 - 3x^4\right]_0^c = 4c^3 - 3c^4 & \text{se } 0 < c < 1 \\ \int_{-\infty}^{0} 0\,dx + \int_{0}^{1} 12x^2(1-x)\,dx + \int_{1}^{c} 0\,dx = 1 & \text{se } c \geq 1 \end{cases}$$

$$E[X] = \int_{-\infty}^{+\infty} x f(x)\,dx = \int_{-\infty}^{0} x\,0\,dx + \int_{0}^{1} x\,12x^2(1-x)\,dx + \int_{1}^{+\infty} 0\,dx =$$

$$= 0{,}6 = \frac{6}{10} = \frac{3}{5} //$$

$$Var[X] = \left[\int_{-\infty}^{+\infty} x^2 f(x)\,dx\right] - (E[X])^2 =$$

$$= \left[\int_{-\infty}^{0} 0\,dx + \int_{0}^{1} x^2\,12x^2(1-x)\,dx + \int_{1}^{+\infty} 0\,dx\right] - \left(\frac{3}{5}\right)^2 =$$

$$= \frac{4}{10} - \frac{9}{25} = \frac{1}{25} //$$

b)

i. Y: "v.a. que representa o lucro obtido na venda de um litro de composto"

$$Y: \begin{cases} 8 & 13 & 18 \\ \frac{1}{9} & \frac{13}{27} & \frac{11}{27} \end{cases}$$

$$\cdot\ P\left(X < \frac{1}{3}\right) = F_X\left(\frac{1}{3}\right) = 4\left(\frac{1}{3}\right)^3 - 3\left(\frac{1}{3}\right)^4 =$$

$$= \frac{1}{9} //$$

$$\cdot\ P\left(\frac{1}{3} \leq X < \frac{2}{3}\right) = P\left(\frac{1}{3} < X \leq \frac{2}{3}\right) =$$

$$= F_X\left(\frac{2}{3}\right) - F_X\left(\frac{1}{3}\right) =$$

$$= \left[4\left(\frac{2}{3}\right)^3 - 3\left(\frac{2}{3}\right)^4\right] - \frac{1}{9} =$$

$$= \frac{16}{27} - \frac{1}{9} = \frac{13}{27} //$$

$$\bullet\ P\left(X \geq \frac{2}{3}\right) = P\left(X > \frac{2}{3}\right) =$$

uma vez que X é uma v.a contínua

$$= 1 - P\left(X \leq \frac{2}{3}\right) =$$

$$= 1 - F_X\left(\frac{2}{3}\right) = 1 - \frac{16}{27} = \frac{11}{27} //$$

ii.

$$E[Y] = \sum_{x_i \in C_Y} x_i\, P(Y = x_i) =$$

$$= 8 \times \frac{1}{9} + 13 \times \frac{13}{27} + 18 \times \frac{11}{27} = \frac{391}{27} \simeq 14{,}5 \text{ €/L}$$

R:. O lucro médio por litro é $\frac{391}{27}$ €/L.

4.

a)

$N = 100$

$E[N] = 100 \cdot E[X] =$

$= 100\,(0 \cdot 0{,}8 + 1 \cdot 0{,}15 + 2 \cdot 0{,}05) =$

$= 100 \cdot 0{,}25 = 25 //$

$Var[N] = 100 \cdot Var[X] =$

$= 100\ \left[(0^2 \cdot 0{,}8 + 1^2 \cdot 0{,}15 + 2^2 \cdot 0{,}05) - (E[X])^2\right] =$

$= 100\ (0{,}35 - 0{,}25^2) = 28{,}75 //$