$$f(x) = \begin{cases} \frac{x}{4} & \text{se } 0 \leq x \leq 1 \\ \frac{7}{8} & \text{se } 1 < x \leq 2 \\ 0 & \text{se c.c.} \end{cases}$$

$$F(c) = P(X \leq c) = \int_{-\infty}^{c} f(x)\, dx$$

$\boxed{c < 0}$

$$F(c) = \int_{-\infty}^{c} \underset{\substack{c<0 \\ (f(x)=0)}}{f(x)\, dx} = \int_{-\infty}^{c} 0\, dx = 0$$

$\boxed{0 \leq c \leq 1}$

$$F(c) = \int_{-\infty}^{c} \underset{c \geq 0}{f(x)\, dx} = \underbrace{\int_{-\infty}^{0} f(x)\, dx}_{=0} + \int_{0}^{c} f(x)\, dx =$$

$$= 0 + \int_{0}^{c} \frac{1}{4} x\, dx = \frac{1}{4}\left[\frac{x^2}{2}\right]_{0}^{c} = \frac{1}{4}\left(\frac{c^2}{2} - 0\right) = \frac{c^2}{8}$$

$\boxed{1 < c \leq 2}$

$$F(c) = \int_{-\infty}^{c} f(x)\, dx = \underbrace{\int_{-\infty}^{1} f(x)\, dx}_{= \frac{1}{8} \text{ pelo caso anterior com } c=1} + \int_{1}^{c} f(x)\, dx =$$

$$= \frac{1}{8} + \int_{1}^{c} \frac{7}{8}\, dx = \frac{1}{8} + \frac{7}{8}[x]_{1}^{c} = \frac{1}{8} + \frac{7}{8}(c-1) = \frac{7}{8}c - \frac{6}{8} = \frac{7c-6}{8}$$

$$\boxed{e > 2}$$

$$F(e) = \int_{-\infty}^{e} f(x)\,dx = \underbrace{\int_{-\infty}^{2} f(x)\,dx}_{=1 \text{ pelo caso anterior}} + \int_{2}^{e} f(x)\,dx =$$

com $e = 2$

$$= 1 + \int_{2}^{e} 0 \, dx = 1$$

$$F(e) = \begin{cases} 0 & \text{se } e < 0 \\[2mm] \dfrac{e^2}{8} & \text{se } 0 \le e \le 1 \\[2mm] \dfrac{7e-6}{8} & \text{se } 1 < e \le 2 \\[2mm] 1 & \text{se } e > 2 \end{cases} \qquad \text{②}$$

$$P(X < 1) = P(X \le 1) = \boxed{F(1) = \frac{1}{8}} \qquad \text{①} \ \frac{1}{8}$$

$$\boxed{P\left(X = \frac{1}{2}\right) = 0} \qquad \text{v.a. contínua} \qquad \text{③} \ 0$$

$$\boxed{F\left(\frac{1}{2}\right) - F(0) = \frac{1}{32}} \qquad \text{④} \ \frac{1}{32}$$

$$\chi_{0,5} = \inf\{e \in \mathbb{R} : F(e) \ge 0,5\}$$

$$\frac{e^2}{8} \ge 0,5 \Leftrightarrow e \ge 2 \qquad \text{N pode ser } fa \ 0 \le c \le 1$$

$$\frac{7e-6}{8} \ge 0,5 \Leftrightarrow 7e \ge 10 \Leftrightarrow \boxed{e \ge \frac{10}{7}} \qquad \text{⑤} \ \frac{10}{7}$$

⑥ Nenhuma das anteriores

Gruppe II

① $Y \sim N(0,4) \qquad Z = \dfrac{Y-0}{2} \sim N(0,1)$

$P(Y < -4) = P(2Z < -4) = P(Z < -2) = 1 - P(Z < 2)$

$= 1 - (P(Z \leq 0) + P(0 < Z < 2))$

$= 1 - (0,5 + 0,4772) = \boxed{0,0228}$

② $P(|2Z| \leq 2) = 2P(0 < 2Z \leq 2) = 2P(0 < Z \leq 1)$

$= 2(0,3413) = \boxed{0,6826}$

③ $T \sim N(3,9)$

$V \sim N(3 + 0, 9 + 4 \times 4)$

$\boxed{V \sim N(3, 25)}$

④ $P(\ldots Y > 5) = P(200 Z > 5) = 1 - P(Z \leq \ldots)$

Grupo III

$P(M_1) = 0,6$ $P(M_2) = 0,3$ $P(M_3) = 0,1$

$P(D|M_1) = 0,4$ $P(D|M_2) = 0,2$ $P(D|M_3) = 0,1$

① Artigo tem defeito ou não tem defeito   (SIM)

② Artigo tem defeito ou artigo é da M_1 (NÃO)

③ $P(D|M_1) = \dfrac{P(D \cap M_1)}{P(M_1)} \Longrightarrow P(D \cap M_1) = P(D|M_1) \times P(M_1)$

$$= \boxed{0,4 \times 0,6}$$

④ $P(D) = P(D|M_1) P(M_1) + P(D|M_2) P(M_2) + P(D|M_3) P(M_3)$

$$= \boxed{0,4 \times 0,6 + 0,2 \times 0,3 + 0,1 \times 0,1}$$

⑤ $P(M_3|D) = \dfrac{P(D|M_3) P(M_3)}{P(D)} = \boxed{\dfrac{0,1 \times 0,1}{0,4 \times 0,6 + 0,2 \times 0,3 + 0,1 \times 0,1}}$

⑥ $P(M_2 \cup M_3 | \bar{D}) = \dfrac{P((M_2 \cup M_3) \cap \bar{D})}{P(\bar{D})} = 1 - \dfrac{P(M_1 \cap \bar{D})}{1 - P(D)} =$

$= 1 - \dfrac{P(M_1) P(\bar{D}|M_1)}{1 - P(D)} = 1 - \dfrac{P(M_1)(1 - P(D|M_1))}{1 - P(D)} =$
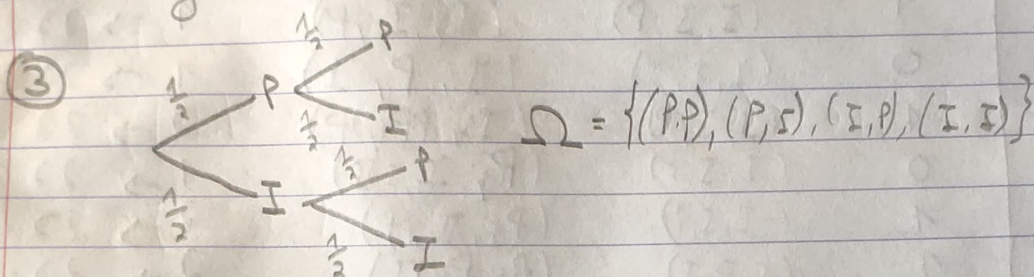
$= 1 - \dfrac{0,6 \times (1 - 0,4)}{1 - P(D)} = 1 - \dfrac{0,6 \times 0,6}{1 - P(D)}$

Grupo IV

① $\Omega = \{1,2,3,4,5,6\}^3$

② subconjunto $= \{(1,1,1), (2,2,2), (3,3,3), (4,4,4), (5,5,5), (6,6,6)\}$

Não é um acontecimento elementar porque o subconjunto tem mais do que um elemento

③

$\Omega = \{(P,P), (P,I), (I,P), (I,I)\}$

$P(A) = \frac{1}{2}$
$P(B) = \frac{1}{2}$
$P(C) = \frac{1}{2}$

$P(A \cap B) = \frac{1}{4} \rightarrow (P, I) = P(A) \times P(B)$ ⎤ A, B e C são
$P(A \cap C) = \frac{1}{4} \rightarrow (P, P) = P(A) \times P(C)$ ⎬ independentes
$P(B \cap C) = \frac{1}{4} \rightarrow (I, I) = P(B) \times P(C)$ ⎦ 2 a 2

$P(A \cap B \cap C) = 0 \neq \frac{1}{2} \times \frac{1}{2} \times \frac{1}{2} = P(A)P(B)P(C)$

$\quad\quad\quad\quad \hookrightarrow A, B e C$ ñ são independentes

④ Falso, tal como deu para ver no último exercício A, B e C eram independentes 2 a 2, mas mesmo assim não eram independentes porque $P(A \cap B \cap C) \neq P(A)P(B)P(C)$

⑤ $\Omega = \{1,2,3,4,5,6\}^3 = \{(1,1,1),(1,1,2),(1,1,3),\dots\}$

a) $\#\Omega = 216$

X: nº de faces pares obtidas

$w \in \Omega$	$X_1(w)$
$(1,1,1)$	0
$(1,1,2)$	1
$(1,1,3)$	0
$(1,1,4)$	1
$(1,1,5)$	0
$(1,1,6)$	1
$(1,2,1)$	1
$(1,2,2)$	2
$(1,2,3)$	1
$(1,2,4)$	2
$(1,2,5)$	1
$(1,2,6)$	2
$(1,3,1)$	0

(I,I,I) (I,I,I) $0 \to 27$ casos
(S,I,P) (I,I,P) $1 \to 27$ casos
(I,I,I) (I,P,I) $1 \to 27$ casos
(I,I,P) (I,P,P) $2 \to 27$ casos
(I,I,I) (P,I,I) $1 \to 27$ casos
(I,I,P) (P,I,P) $2 \to 27$ casos
(S,P,S) (P,P,I) $2 \to 27$ casos
(I,P,P) (P,P,P) $3 \to 27$ casos

$X:$

0	1	2	3
$\frac{1}{8}$	$\frac{3}{8}$	$\frac{3}{8}$	$\frac{1}{8}$

$X \sim \text{Bin}(3,\,0,5)$

$$P(X=0) = \binom{3}{0}\,0,5^0(1-0,5)^{3-0} = \frac{1}{8}$$

$$P(X=1) = \binom{3}{1}\,0,5^1(1-0,5)^{3-1} = \frac{3}{8}$$

$$P(X=2) = \binom{3}{2}\,0,5^2(1-0,5)^{3-2} = \frac{3}{8}$$

$$P(X=3) = \binom{3}{3}\,0,5^3(1-0,5)^{3-3} = \frac{1}{8}$$

b) $F: \mathbb{R} \to [0,1]$

$$e \longmapsto F(e) = P(X \le e)$$

$P(X < 0) = 0$
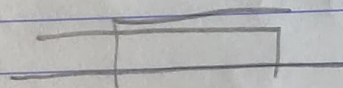
$P(X \le 0) = P(X_1 = 0) = \frac{1}{8}$

$P(X < 1) = P(X_1 = 0) = \frac{1}{8}$

$P(X \le 1) = P(X = 0) + P(X = 1) = \frac{1}{8} + \frac{3}{8} = \frac{4}{8} = \frac{1}{2}$

$P(X \le 2) = P(X = 0) + P(X = 1) + P(X = 2) = \frac{1}{8} + \frac{3}{8} + \frac{3}{8} = \frac{7}{8}$

$P(X \le 3) = P(X = 0) + P(X = 1) + P(X = 2) + P(X = 3) = \frac{1}{8} + \frac{3}{8} + \frac{3}{8} + \frac{1}{8} = 1$

$$F_X(e) = \begin{cases} 0 & \text{si } e < 0 \\ \frac{1}{8} & \text{si } 0 \le e < 1 \\ \frac{1}{2} & \text{si } 1 \le e < 2 \\ \frac{7}{8} & \text{si } 2 \le e < 3 \\ 1 & \text{si } e \ge 3 \end{cases}$$

c) $P(X < 3 \mid X \ge 1) = \dfrac{P(X < 3 \wedge X \ge 1)}{P(X \ge 1)} = \dfrac{P(X = 2) + P(X = 1)}{1 - P(X = 0)} =$

$= \dfrac{\frac{3}{8} + \frac{3}{8}}{1 - \frac{1}{8}} = \dfrac{\frac{6}{8}}{\frac{7}{8}} = \dfrac{6}{7}$