Folha 7.

1. a) $f(x,y) = x^2 y$, $R = [0,2]\times[1,2]$

$$\int_1^2\int_0^2 x^2 \cdot y\,dx\,dy = \int_1^2 2y\,dy \cdot \int_0^2 3x^2\,dx \cdot \frac{1}{3}\cdot\frac{1}{2} = \frac{1}{6}\cdot\left[y^2\right]_1^2\cdot\left[x^3\right]_0^2 =$$

$$= \frac{1}{6}\cdot(4-1)\cdot 8 = \underline{4}$$

d) $f(x,y) = ye^{xy}$, $R = [0,1]^2$ $\qquad e^{u}{}' = u'\cdot e^{u}$

$$\int_0^1\int_0^1 ye^{xy}\,dx\,dy = \int_0^1\left[e^{xy}\right]_0^1 dy = \int_0^1\left(e^{y}-1\right)dy = \left[e^{y}-y\right]_0^1 = e-1-1 = \underline{e-2}$$

g) $f(x,y) = \dfrac{1+x^2}{1+y^2}$ $\quad R = [0,1]^2$

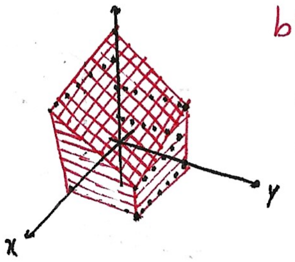
$$\int_0^1\int_0^1 \frac{1+x^2}{1+y^2}\,dy\,dx = \int_0^1(1+x^2)\,dx \cdot \int_0^1\frac{1}{1+y^2}\,dy = \left([x]_0^1 + \frac{1}{3}[x^3]_0^1\right)\cdot\left[\operatorname{arctg} y\right]_0^1$$

$$= \frac{4}{3}\cdot\frac{\pi}{4} = \frac{\pi}{3}$$

2. a) $f(x,y) = 4 - x - 2y$

$f(0,0) = 4$ $\qquad f(0,1) = 2$

$f(1,0) = 3$ $\qquad f(1,1) = 1$



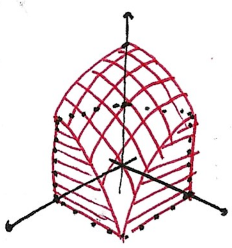
b) $f(x,y) = 2 - x^2 - y^2$

$f(0,0) = 2$

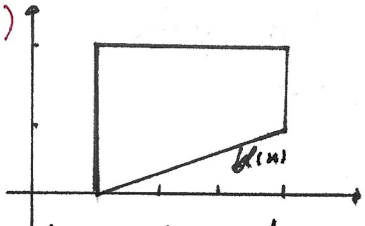
$f(1,0) = 1$

$f(0,1) = 1$

$f(1,1) = 0$



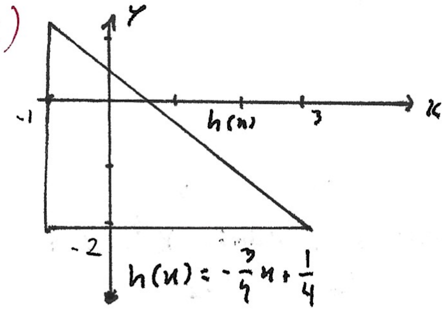
3. b)



$$\int_1^4\int_{h(x)}^2 f(x,y)\,dy\,dx$$

$h(x) = 1/3\,x - \frac{1}{3}$

c)

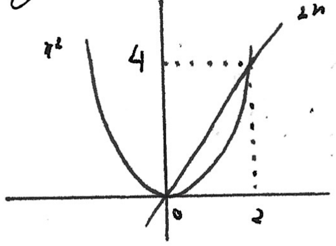


$$\int_{-1}^3\int_{-2}^{h(x)} f(x,y)\,dy\,dx$$

ou é escapado para a integrada maior.

$h(x) = -\frac{3}{4}x + \frac{1}{4}$

4. $f(x,y) = x^2 + y^2$



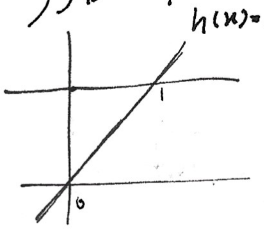
$x^2 = 2x \Leftrightarrow x(x-2) = 0$

$$\int_0^2\int_{x^2}^{2x} x^2+y^2\,dy\,dx = \int_0^2\left[x^2y+\frac{y^3}{3}\right]_{x^2}^{2x}dx =$$

$$= \int_0^2 x^2\cdot 2x + \frac{(2x)^3}{3} - x^2\cdot x^2 + \frac{(x^2)^3}{3} = \int_0^2 2x^3+\frac{8x^3}{3}-x^4+\frac{x^6}{3} =$$

$$= \left[\frac{2x^4}{4}+\frac{8x^4}{4\cdot 3}-\frac{x^5}{5}+\frac{x^7}{7\cdot 3}\right]_0^2 = \frac{2\times 16}{4}+\frac{8\times 16}{12}-\frac{32}{5}+\frac{128}{21}=\frac{1928}{105}$$

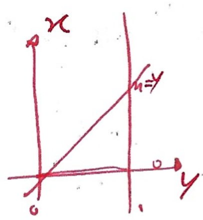
5. $\iint \sin y^2$ $f(x,y) = \sin(y^2)$



$$\int_0^1\int_x^1 \sin y^2\,dy\,dx$$ ← agente troca!!

⇓

$$\int_0^1\int_0^y \sin y^2\,dx\,dy$$

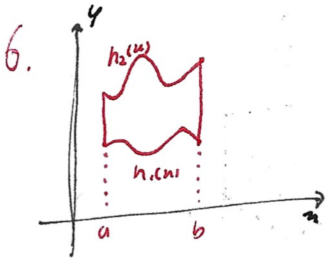


$\int \sin x^2\,dx$

$\int 2x\sin(x^2) = \sin(x^2)$

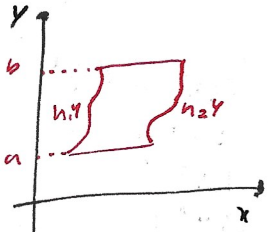
$$\int_0^1\left[x\cdot\sin y^2\right]_{x=0}^{x=y}dy = \frac{1}{2}\int_0^1 2y\cdot\sin y^2\,dy = \frac{1}{2}\cdot\left[\sin y^2\right]_0^1 = \frac{1}{2}\cdot$$

6.



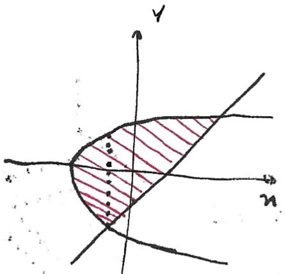
V. simples

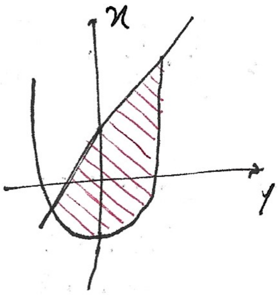
$$\int_a^b\int_{h_1}^{h_2} f(x,y)\,dy\,dx$$



h. simples

$$\int_a^b\int_{h_1}^{h_2} f(x,y)\,dx\,dy$$





$$\int_{-3}^{-1}\int_{-\sqrt{2x+6}}^{\sqrt{2x+6}} xy\,dy\,dx + \int_{-1}^{5}\int_{x-1}^{\sqrt{2x+6}} xy\,dy\,dx$$ V. S.

$$\int_{-2}^{4}\int_{\frac{y^2}{2}-3}^{y+1} xy\,dx\,dy$$ h. S.

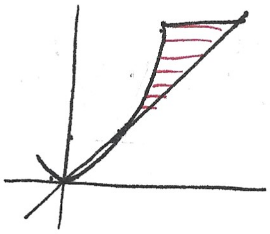
7. b) $\int_{-2}^0\int_{-\sqrt{9-x^2}}^0 2xy\,dy\,dx$

$y^2 = 9-x^2 \Leftrightarrow y^2+x^2 = 9$

$$\int_{-2}^0\int_{-\sqrt{9-x^2}}^0 2xy\,dy\,dx = \int_{-2}^0\left[xy^2\right]_{-\sqrt{9-x^2}}^0 dx = \int_{-2}^0 x\cdot(9-x^2)\,dx = \int_{-2}^0 9x\,dx - \int_{-2}^0 x^3\,dx =$$

$$= \left[\frac{9x^2}{2}\right]_{-2}^0 - \left[\frac{x^4}{4}\right]_{-2}^0 = -\frac{9\times 4}{2} - \frac{2^4}{4} = -\frac{44}{2}$$

c) $\int_1^4 \int_{\sqrt{y}}^{y} x \, dx \, dy = \int_1^4 \left[\frac{x^2}{2}\right]_{\sqrt{y}}^{y} dy = \int_1^4 \frac{y^2 - y}{2} dy = \frac{1}{2} \cdot \left(\int_1^4 y^2 \, dy - \int_1^4 y \, dy \right) = \frac{1}{2} \left(\left[x^3/3 \right]_1^4 - \left[x^2/2 \right]_1^4 \right)$

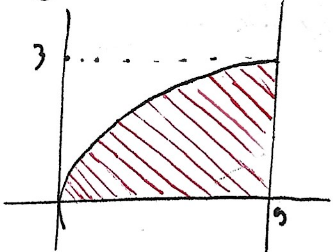


$= \frac{1}{2} \cdot \left(\frac{4^3 - 1}{3} - \frac{4^2 - 1}{2} \right) = \frac{1}{2} \cdot \left(\frac{63 \times 2 - 15 \times 3}{6} \right) = \frac{81}{12}$

??
tá certo oh romae
ele anda da parab.
pra a reta, da esq-
uerda pra a direita

$\sqrt{y} = x \Leftrightarrow y = x^2$

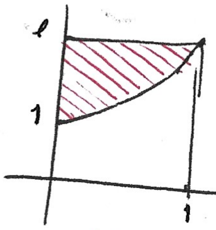
8) $f(x,y) = x + y$



$\int_0^9 \int_0^{\sqrt{x}} x + y \, dy \, dx = \int_0^9 \left[xy + \frac{y^2}{2} \right]_0^{\sqrt{x}} dx = \int_0^9 x \cdot \sqrt{x} + \frac{x}{2} =$

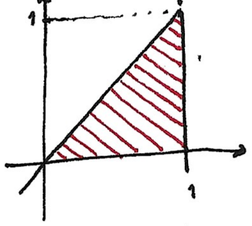
$= \int_0^9 x^{\frac{3}{2}} + \frac{x}{2} \, dx = \left[\frac{x^{5/2}}{5/2} + \frac{x^2}{4} \right]_0^9 = \frac{9^{5/2}}{5/2} + \frac{9^2}{4} \ldots$

9. a) $\int_0^1 \int_{e^x}^{e} \frac{y}{\log y} \, dy \, dx \Leftrightarrow \int_1^e \int_0^{\ln y} \frac{y}{\ln y} \, dx \, dy = \int_1^e \left[x \cdot \frac{y}{\ln y} \right]_0^{\ln y} dy = \int_1^e y \, dy = \left[y^2/2 \right]_1^e$

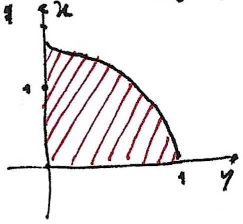


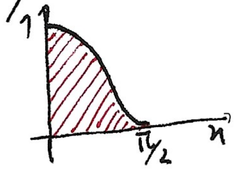
$= \frac{e^2 - 1}{2}$

b) $\int_0^1 \int_y^1 e^{x^2} dx \, dy = \int_0^1 \int_0^x e^{x^2} dy \, dx = \int_0^1 x e^{x^2} = \frac{1}{2} \cdot \left[e^{x^2} \right]_0^1 = \frac{e - 1}{2}$



c) $\int_0^1 \int_0^{\cos^{-1} y} e^{\sin x} \, dx \, dy = \int_0^{\pi/2} \int_0^{\cos x} e^{\sin x} \, dy \, dx = \int_0^{\pi/2} \cos x \cdot e^{\sin x} \, dx = \left[e^{\sin x} \right]_0^{\pi/2} =$



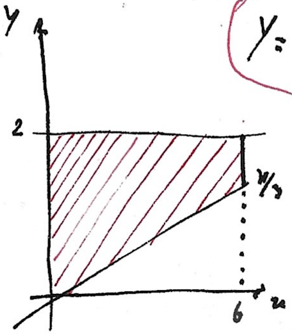


$= e$

$x = \arccos y$

$y = \cos x$

10 $\int_0^6\int_{x/3}^2 x\sqrt{y^3+1}\,dy\,dx = \int_0^2\int_{3y}^6 x\sqrt{y^3+1}\,dx\,dy = \int_0^2 \sqrt{y^3+1}\cdot\left[\frac{x^2}{2}\right]_{3y}^6 dy =$



$y = x/3 \Leftrightarrow x = 3y$

$= \int_0^2 (y^3+1)^{\frac{1}{2}}\cdot\left(18-\frac{9y^2}{2}\right)dy = \int_0^2 \frac{3}{2}\cdot 3y^2(y^3+1)^{\frac{1}{2}} + 18\int_0^2 (y^3+1)^{\frac{1}{2}} =$

$\left[(y^3+1)^{\frac{3}{2}}\right]_0^2 + 18\int_0^2 (y^3+1)^{\frac{1}{2}}\,dy$ preguiça...

$\int f\cdot g = f'\int g - \int f'\int g$

$u^{\frac{3}{2}}{}' = \frac{3}{2}u^{\frac{1}{2}}$

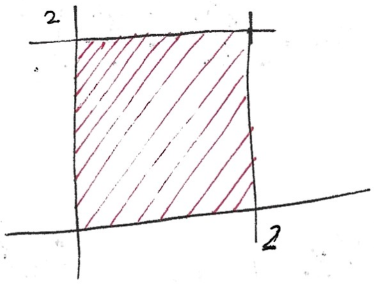
11 $f(x,y) = 16 - x^2 - 2y^2$

$16 - x^2 - 2y^2 = 0$

$x^2 + 2y^2 = 16$

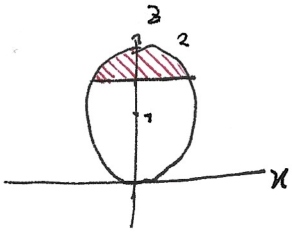
$x = 0 \Rightarrow y = 2\sqrt{2}$

$y = 0 \Rightarrow x = 4$



$\int_0^2\int_0^2 16 - x^2 - 2y^2\,dy\,dx$ ...

12.

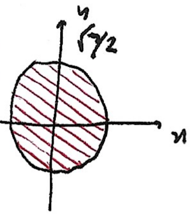


$R = x^2+y^2 = 3/4 \Leftrightarrow x^2+y^2 = \frac{\sqrt{3}}{2}$

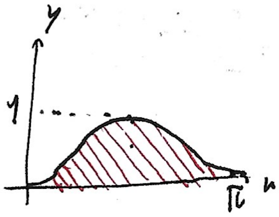
$\int_{-\frac{\sqrt{3}}{2}}^{\frac{\sqrt{3}}{2}}\int_{-\sqrt{\frac{\sqrt{3}}{2}-x^2}}^{\sqrt{\frac{\sqrt{3}}{2}-x^2}} f(x,y)\,dy\,dx$

$f(x,y) = \sqrt{x^2+y^2-1}+1$

R



13. $A = \{(x,y)\in\mathbb{R}^2 : 0 \le x \le \pi \wedge 0 \le y \le \sin x\}$



$\int_0^\pi \sin x\,dx = \int_0^\pi\int_0^{\sin x} 1\,dy\,dx$