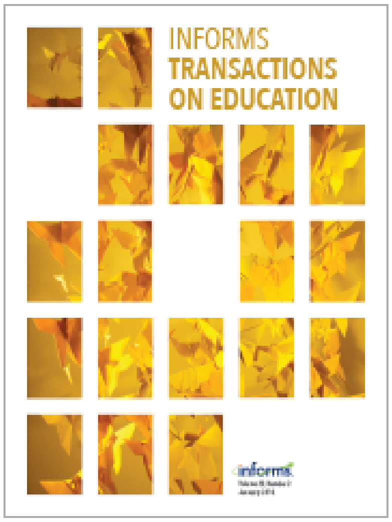

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Puzzle

Logic Grid Puzzles

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A type of logic-based puzzle, now referred to as a logic grid puzzle, began to appear in magazines sometime in the nineteen-eighties. In these puzzles the solver is provided with sets of attributes with an equal number of members in each set. The goal is to figure out which attributes are linked together based on a series of given clues. The attributes in each set are used once and only once and each puzzle has a unique solution which can be found using simple logic.

A specific example of this type of puzzle is modeled and solved using integer programming and the lessons learned are used produce a set of generic rules that may be applied in solving a wide range of these puzzles.

Keywords: puzzle; integer programming; logic puzzles
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A type of logic-based puzzle began to appear in puzzle magazines sometime in the nineteen-eighties. These have now become popular on the internet (for example, http://www.logic-puzzles.org) and are commonly referred to as logic grid puzzles. In these puzzles the solver is provided with sets of attributes with an equal number of members in each set. The goal is to figure out which attributes are linked together based on a series of given clues. The attributes in each set are used once and only once and each puzzle has a unique solution which can be found using simple logic. Perhaps the most famous of this type of puzzle is known as the *Einstein puzzle* which is discussed in depth and modeled in Yeomans (2004).

The grid in Figure 1 represents five sets of attributes with each set containing five members. This is accompanied by a set of clues which collectively contain enough information to uniquely match attributes to individuals.

The clues are:

1. Michael Tompkins studies mathematics.
2. Patricia does not study history or physics.
3. Johnson does not study physics.
4. Lucas is at Cardiff.
5. The student at Oxford studies economics.
6. Arthur is not Lucas.
7. The student at Reading does not study physics.
8. Of Johnson and Beasley, one studies economics and the other is at York.
9. Of Veronica and Mason, one is at Oxford and the other studies Business.
10. Of the mathematics and physics students, one is at York and the other is at Reading.

The standard method of solving this type of puzzle is to read through the clues adding ticks and crosses to the grid as appropriate until the full solution is obtained. At no point is the solver required to resort to guesswork. The reader who is not familiar with this type of puzzle is encouraged to solve the example before continuing. The solution is in Appendix A (see Figure A.1).

To model this as an integer program, we define the set $M = 1, \ldots, m$ where m is the number of attributes in each set. Also define variables $x_{i,j} = 1$ if first name i has surname j, otherwise 0, $y_{i,j} = 1$ if first name i studies subject j, otherwise 0, and $z_{i,j} = 1$ if first name i studies at university j, otherwise 0, $\forall\, i \in M,\ j \in M$.

A set of constraints that apply to all logic grid puzzles and enforce the one-to-one correspondences of attributes are as follows:

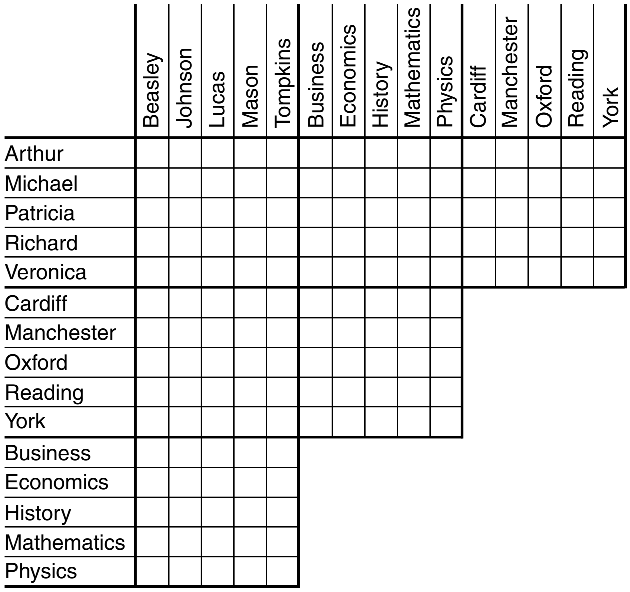
$$\sum_{j\in M} x_{i,j} = 1 \quad \forall\, i \in M,$$

$$\sum_{i\in M} x_{i,j} = 1 \quad \forall\, j \in M,$$

$$\sum_{j\in M} y_{i,j} = 1 \quad \forall\, i \in M,$$

$$\sum_{i\in M} y_{i,j} = 1 \quad \forall\, j \in M,$$

Figure 1 **Typical Logic Grid Puzzle**



$$\sum_{j\in M} z_{i,j}=1 \quad \forall\, i\in M,$$
$$\sum_{i\in M} z_{i,j}=1 \quad \forall\, j\in M.$$

These generic conditions are supplemented by constraints that model the specific conditions of the puzzle at hand.

1. Michael Tompkins studies mathematics,

$$x_{2,5}=1,$$
$$y_{2,4}=1.$$

2. Patricia does not study history or physics,

$$y_{3,3}=0,$$
$$y_{3,5}=0.$$

3. Johnson does not study physics,

$$x_{i,2}+y_{i,5}\le 1 \quad \forall\, i\in M.$$

4. Lucas is at Cardiff,

$$\sum_{i\in M} i x_{1,3}=\sum_{i\in M} i z_{i,1}.$$

5. Either economics or physics is studied at Oxford,

$$y_{i,2}+y_{i,5}\le z_{i,3} \quad \forall\, i\in M.$$

6. Arthur is not Lucas,

$$x_{1,3}=0.$$

7. The student at Reading does not study physics,

$$y_{i,1}+z_{i,4}\le 1 \quad \forall\, i\in M.$$

8. Of Johnson and Beasley, one studies economics and the other is at York,

$$x_{i,1}+x_{i,2}\le y_{i,2}+z_{i,5} \quad \forall\, i\in M.$$

9. Of Veronica and Mason, one is at Oxford and the other studies business,

$$y_{5,1}+z_{5,3}=1,$$
$$y_{i,1}+z_{i,3}=x_{i,4} \quad \forall\, i\in M \mid i\ne 5.$$

10. Of the mathematics and physics students, one is at York and the other is at Reading,

$$y_{i,4}+y_{i,5}=z_{i,4}+z_{i,5} \quad \forall\, i\in M.$$

A Mathprog model (studies.mod) and an Excel spreadsheet (studies.xlsx) are included with this article (available as supplemental material at http://dx.doi.org/10.1287/ited.2014.0129). Also included are templates (logicgrid.mod and logicgrid.xlsx) that implement the generic conditions. These provide a starting point to which a user may insert attributes and constraints for a specific puzzle to obtain a solution.

Some useful rules. We now generalize what has been learned from this specific example and produce a set of rules that may be applied in solving a wide range of these puzzles.

Define attributes A, B, C, D, E where A is referred to as a *key attribute*. In the grid layout the key attributes are members of the set listed on the top left. The key attributes of the example in Figure 1 therefore are the first names.

A number of common clue types together with constraint examples are now presented.

A is B,

$$x_{2,3}=1.$$

A is not B,

$$x_{1,4}=0.$$

A is either B or C,

$$x_{2,2}+x_{2,4}=1.$$

A is neither B nor C,

$$x_{2,2}+x_{2,4}=0.$$

Of A and B, one is C and the other is D,

$$x_{2,1}+x_{2,4}=1,$$
$$x_{i,1}+z_{i,4}=y_{i,3} \quad \forall\, i\in N \mid i\ne 2.$$

B is C,

$$\sum_{i=1}^{n} i x_{i,2}=\sum_{i=1}^{n} i y_{i,2}.$$

B is not C,

$$x_{i,3} + y_{i,1} \leq 1 \quad \forall\, i \in N.$$

B is either C or D,

$$y_{i,2} + y_{i,5} \leq z_{i,3} \quad \forall\, i \in M.$$

B is neither C nor D,

$$z_{i,2} + z_{i,4} \leq y_{i,3} \quad \forall\, i \in M.$$

Of B and C, one is D and the other is E,

$$y_{i,3} + z_{i,1} = y_{i,2} + z_{i,4} \quad \text{for } i = 1, \ldots, n.$$

Appendix B contains an example taken from http://www.mathsisfun.com/puzzles/meeting-the-challenge.html. Despite the daunting complexity of this puzzle it may be modeled and solved almost entirely using the techniques outlined in this article. The exception is clue 20 which proves to be considerably more of a challenge and is left as an exercise for the reader. A Mathprog model (couples.mod) is included with this article.

Supplemental Material

Supplemental material to this paper is available at http://dx.doi.org/10.1287/ited.2014.0129.

Appendix A

Figure A.1 Solution to Puzzle in Figure 1

Arthur	Beasley	Physics	York
Michael	Tompkins	Mathematics	Reading
Patricia	Mason	Business	Manchester
Richard	Lucas	History	Cardiff
Veronica	Johnson	Economics	Oxford

Appendix B

1. Daniella Black and her husband work as Shop-Assistants.
2. The book "The Seadog" was brought by a couple who drive a Fiat and love the color red.
3. Owen and his wife Victoria like the color brown.
4. Stan Horricks and his wife Hannah like the color white.
5. Jenny Smith and her husband work as Warehouse Managers and they drive a Wartburg.
6. Monica and her husband Alexander borrowed the book "Grandfather Joseph."
7. Matthew and his wife like the color pink and brought the book "Mulatka Gabriela."
8. Irene and her husband Oto work as Accountants.
9. The book "We Were Five" was borrowed by a couple driving a Trabant.
10. The Cermaks are both Ticket-Collectors who brought the book "Shed Stoat."
11. Mr. and Mrs. Kuril are both Doctors who borrowed the book "Slovacko Judge."
12. Paul and his wife like the color green.
13. Veronica Dvorak and her husband like the color blue.
14. Rick and his wife brought the book "Slovacko Judge" and they drive a Ziguli.
15. One couple brought the book "Dame Commissar" and borrowed the book "Mulatka Gabriela."
16. The couple who drive a Dacia, love the color violet.
17. The couple who work as Teachers borrowed the book "Dame Commissar."
18. The couple who work as Agriculturalists drive a Moskvic.
19. Pamela and her husband drive a Renault and brought the book "Grandfather Joseph."
20. Pamela and her husband borrowed the book that Mr. and Mrs. Zajac brought.
21. Robert and his wife like the color yellow and borrowed the book "The Modern Comedy."
22. Mr. and Mrs. Swain work as Shoppers.
23. "The Modern Comedy" was brought by a couple driving a Skoda.

Reference

Yeomans JS (2004) Solving Einstein's riddle using spreadsheet optimization. *INFORMS Trans. Ed.* 3(2):55–63.