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Invited Review

# A review of the contribution of Operational Research to Project Management

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**Abstract**

Project Management is a professional domain receiving growing attention during the last decades and now it is considered a key concept by Management Sciences to understand and to develop organizations. Operational Research has given essential scientific contributions to the success of Project Management not just through multiple models to understand and to represent projects but also by the development of algorithms and aids to support the decisional role of project manager. In this paper, the major contributions of OR are presented and discussed. © 2002 Elsevier Science B.V. All rights reserved.

*Keywords:* Project Management; Scheduling

## 1. The concept of Project Management

In modern management, *organizations* are considered *open* and *complex systems* interacting with the environment and pursuing *objectives* according to their specific *mission* and *nature* (see, e.g. Ackoff, 1970; Simon, 1977). The achievement of such objectives implies structuring the activities of the organization through *projects* with specific *targets* that should be consistent with the adopted organizational objectives. This explains why *project management* has become such a key subject in modern world of private or public organizations (see, e.g., Drucker, 1974).

The notion of *project* can be defined (Tavares and Weglarz, 1990) as any purposeful transformation leading a system, $\Omega$, from an initial state, $s$, to a specific state, $s'$ and so $s'$ should represent the targets to be achieved. This means that the concept of project implies:

(A) The identification of the system, $\Omega$, to be transformed.

(B) The description of the initial state, $s$.

(C) The description of the new state, $s'$, that should represent the targets of the project.

*Project Management* is the *process* of *conceiving*, *preparing*, *organizing*, *driving* and *controlling* such transformations in order that $s'$ will be achieved

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under the most convenient conditions. Therefore, this process is carried out in terms of

(D) An organizational framework.

(E) A set of resources.

(F) A methodological support.

Any taxonomy about project management includes a very wide variety of cases and problems as they can be described by the six perspectives already presented ((A)–(F)).

A wide range of case-studies has been discussed:

- Development of a new missile Polaris by the US Navy (in US Navy, 1958).
- Maintenance shutdowns of a DUPONT factory (in Kelley, 1961).
- Development of a new aircraft: CONCORDE (1964–1972) (in Hayward, 1983).
- Development of a European spacecraft (GIOTTO) to observe Halley's comet, March 86 (in Jenkins and Link, 1984).
- Development of a new railways network in Oporto, Portugal (in Tavares, 1984).
- Construction of a new refinery in New Zealand, including the cooperation of three engineering teams in Yokohama (Japan), the Hague (The Netherlands) and Whangarci (New Zealand), (in Bishop and Gembey, 1985).
- Preparation of the school education systems at the commencement of each academic year (in Palmer, 1985a).
- Planning of rural development centers in Pakistan (in Palmer, 1985b).
- Development of a new Concert Hall for the European Culture Capital, Porto 2001 (in Tavares, 2000).

Project Management, PM, has been considered as a specific domain of professional activity since the second world war because of the challenges coming from the economic development, the complexity of new technologies and the significant methodologic advances to support PM which have been mainly offered by OR.

Such OR contributions include an extremely wide variety of methods, techniques, algorithms, programs, etc. explaining why most reviews focus more on one or another sub-domain rather than on the whole picture of OR know-how serving PM.

The objective of this review is presenting such an overall perspective and therefore a systemic approach will be adopted based on the three major types of questions required by any systemic study:

(I) How can a project be *modelled?*

(II) How can a project be *evaluated?*

(III) How can a project be *scheduled* and *monitored?*

Major findings on I, II and III are presented in the following Sections 2, 3, 4, respectively. A final section (Section 5) presents more advanced and recent results.

## 2. Project modelling

### 2.1. General model

The general model (Battersby, 1967) developed to represent projects is quite a basic concept in OR: a directed and acyclic network. Actually, each project can be modelled by:

(a) A discrete and finite set of entities, $A$, usually, called *jobs or activities* with $A = \{A_i\colon i = 1, \dots, N\}$.

(b) A set of *precedence conditions*, $J$, with, $J = \{J_i\colon i = 1, \dots, N\}$ where $J_i$ is the set of activities immediately preceding $i$. $J_i$ can be defined by $J_i = \{k\colon (k \in J'_i) \cap (k \in J'_m) = \emptyset \text{ for any } m \in J'_i)$ where $J'_i$ or $J'_m$ is the whole set of activities which have to be completed before starting $i$ or before starting $m$. Similarly, the set of activities which are the immediate successors of $i$, $K_i$, can be defined by $K_i\{k\colon i \in J_k\}$.

(c) A discrete and finite set of *attributes* $\{B_1(i), \dots, (B_p(i)\}$ with $p \geqslant 1$ defined for each activity and describing its properties relevant for project management such as duration, cost, consumption required of each resource, etc.

(d) A discrete and finite set of *criteria* $\{V_1, \dots, V_q\}$ which should express the values and the preferences of the project manager to compare alternative decisions concerning the management of the project. The most common criteria are the total duration, the total cost, a cost–benefit function, a measure of the project risk, the net present value (NPV).

The available models to describe each of these components have been improved by a large number of OR researchers.

### 2.2. Progress in network modelling

The representation of a project by a network in terms of (a) and (b) can be done adopting the usual assumption ($\alpha$) "activity-on-arc" (AoA) where each arc describes an activity and where each node represents the completion of the activities converging on it, but it also can be produced using the alternative hypotheses ($\beta$) "activity-on-node" (AoN) where each node represents an activity and each arc between two nodes describes a precedence relationship between the activities associated to such nodes (see an example in Fig. 1).

The adoption of $\alpha$ is more common in the OR literature as it was used by the popular methods of PERT/CPM, but an alternative method proposed by Roy (1964) (The Method of Potentials, Roy, 1964) has adopted $\beta$. In this case, each node represents an activity, each arc describes a precedence relation and the duration of each arc is equal to the minimal time span between the pair of starting times of two adjacent activities. In any case, the computation of the critical path is straightforward but the second representation is more flexible to describe overlapping activities. For instance, if A precedes B and A has duration equal to 10 units but B can start when half of A is carried out, then the arc between A and B will have a duration equal to 5.

The improvement of network models for project management has been pursued along seven different lines:

(A) Construction of "generalized networks" according to Kauffmann and Desbazeille (1964) where some activities just occur with specific probabilities or in terms of the outcomes of previous activities (probabilistic networks).

(B) Construction of "logical networks" where the occurrence of each activity is conditioned by logical relationships between precedent activities (Battersby, 1967).

(C) Modelling of "overlapping activities", in terms of the time domain (more easily achieved using "activity-on-node" networks, as it was shown by the Method of Potentials and by the "Precedence Diagramming Method", Leashman and Kun, 1993) or in terms of the consumed resources expressed by progress lag constraints for activities carried out at each time (see Leashman et al., 1990).

(D) Introduction of "hammock" activities. These activities are associated to the time span occurring between events concerning other activities. For instance, the use of an equipment between the start (or end) of an activity and the start (or end) of another activity. The duration of these activities is equal to the difference of times between the two specified events. The construction of networks with these activities was studied by a few authors (see Harhalakis, 1990).

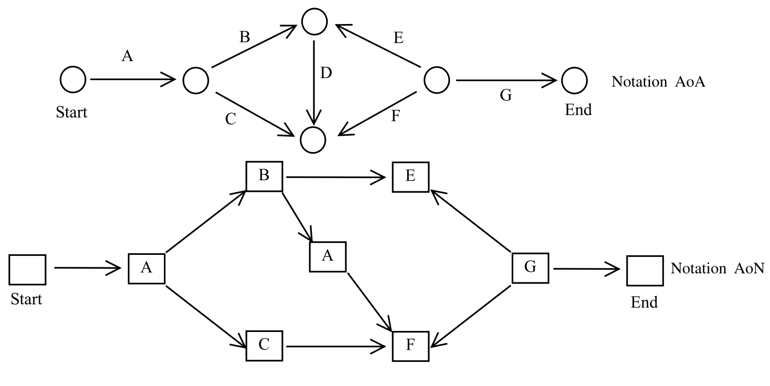


Fig. 1. The network model of a project using the activity-on-arc (I) and the activity-on-node (II) notation.

(E) Morphologic modelling of project networks. The morphologic study of networks (see Tavares et al., 1997; Tavares, 1998) is important to classify or to simulate networks and is based on two important concepts of project networks, often ignored, the progressive and the regressive levels (Elmaghraby, 1977).

- The progressive level of activity $i$ is $m(i)$ given by

$$m(i) = \mathrm{Max}_{j \in J_i} m(j) + 1 \text{ if } J_i \neq \emptyset$$
$$\text{and making } m(i) = 1 \text{ if } J_i = \emptyset.$$

The maximal $m(i)$ is denoted by $M$.

- The regressive level of activity $i$ is $n(i)$ given by

$$n(i) = \min_{k \in K_i} n(k) - 1 \text{ if } K_i \neq \emptyset$$
$$\text{and making } n(i) = M \text{ if } K_i = \emptyset.$$

The level float, $\Delta(i)$, can be defined for each activity $i$ by $\Delta(i) = n(i) - m(i)$.

The level length of any precedence link between $i$ and $k$ is given by $L(i,k) = m(k) - m(i)$.

The morphology of the network depends on several indicators which can be built in terms of the number of activities, $M$, of the number of activities per level and of the level length of the precedence links. As an example, a network is shown in Fig. 2 using AoN and presenting $(m(i),\ n(i))$ for each activity $i$ as well as the level length of each precedence link.

The serial (parallel) type of networks has a high (low) ratio $M/N$ and the depth (shallow) type of networks has a high (low) average level length of the precedence links. Obviously, a wide range of types can be constructed to describe the morphology of each network.

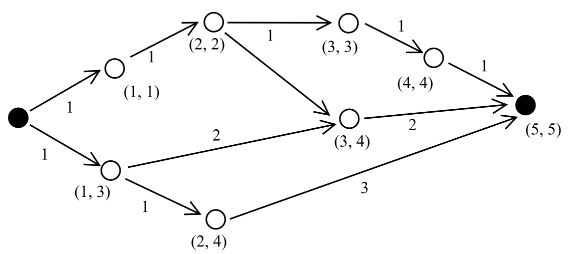


Fig. 2. The progressive and regressive order of the activities of a network.

(F) Construction of hierarchical networks. Each project can be viewed as a set of interconnected sub-projects ("macro-activities") and each of these macro-activities can be modelled by another network constructed in terms of more detailed activities. This process of modelling can be studied using multiple hierarchical levels as it was presented by Speranza and Vercellis (1993) or partitioning methods which have also been proposed (Rafael, 1990).

(G) Aggregation of projects networks to be transformed into simpler and more synthetic networks. Two major approaches have been proposed: the decomposition and the reduction methods.

The method of modular decomposition is based on the identification of "modules" which can be synthetized by equivalent macro-activities (Muller and Spinrad, 1989).

A module ($\alpha$) is defined as a subset of activities of the project network satisfying the following properties:

- The set of precedent activities $j$ of any activity $i \in \alpha$ with $j \notin \alpha$ is the same for any $i$.
- The set of succedent activities $k$ of any activity $i \in \alpha$ with $k \notin \alpha$ is the same for any $i$.

An example to two modules ($\alpha$ and $\alpha'$) is given in Fig. 3, using AoN and successive levels of modules can be now identified within $\alpha$ and $\alpha'$.

Muller and Spinrad (1989) proposed on algorithm to carry out this identification with a computational time proportional to the square of the number of activities.

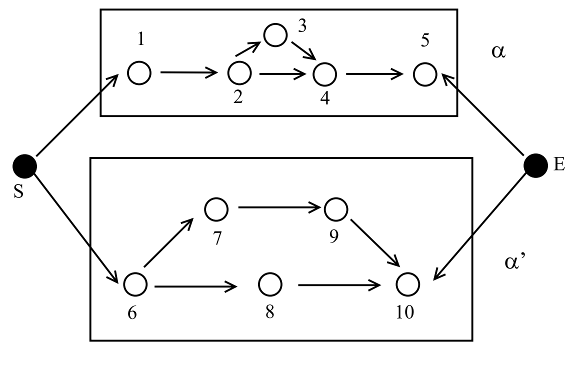


Fig. 3. Modular decomposition.

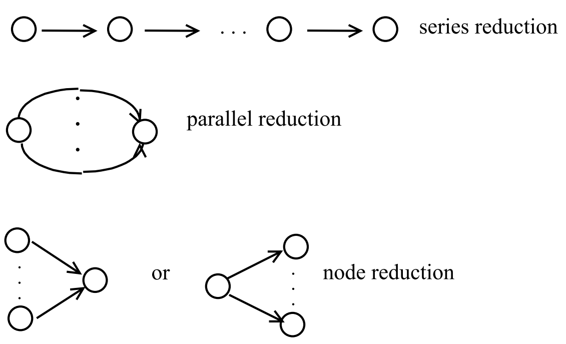


Fig. 4. Network reduction.

The method of network reduction (Bein et al., 1992) is based on three different types of reduction (see Fig. 4, using AoA):

(a) *Series reduction*. A sequence of activity is substituted by an equivalent activity;

(b) *Parallel reduction*. A set of parallel activities is substituted by an equivalent activity;

(c) *Node reduction*. A set of arcs converging to a node with just one out-arc can be substituted by a set of equivalent arcs. Similarly, a set of arcs diverging from a node just receiving one arc can be substituted by a set of equivalent arcs.

The "reduction complexity" is the number of node reductions required to transform the given network into just one arc from start to end.

### 2.3. Progress in activity modeling

#### 2.3.1. On the description of activities

The description of each activity $i$ should be done in terms of the attributes mentioned in Section 2.1: $\{B_1(i), \ldots, B_p(i)\}$.

Too few features imply unrealistic models but too complex attributes are responsible for untreatable models demanding too much data and hardly solvable by the existing methods.

This line of research has been mainly oriented to study the formulation of the activity duration and of its resource requirements.

#### 2.3.2. Modelling the activity's duration

The duration has been formulated as a non-deterministic magnitude (Clark, 1962). Initially, the Beta distribution was assumed by the authors of PERT adopting:

$$\mu = \frac{A + 4M + B}{6},$$
$$\sigma = \frac{(B - A)}{6},$$

where $\mu$ and $\sigma^2$ are the mean and the variance of the duration, $A$ and $B$ are the minimal and the maximal durations. However, most often the distribution adopted has been the Gaussian law which can model the duration of a path either because the duration of each activity is normal or due to the application of the central limit theorem.

Unfortunately, reality may have quite different features. The uncertainty of the duration can be due to the occurrence of additional works which have not been foreseen but then their compensation is hardly conceivable by the suppression of other works. In this case, the distribution of the duration tends to be positively skewed and a lower bound can be realistically assumed. The negative exponential law or the lognormal law has been proposed to model this distribution (Dean et al., 1969).

Within the framework of projects including activities carried out by sub-contractors, the duration of such activities is usually a condition stated in the contracts and quite often there is no advantage for the sub-contractor to have them finished before the contracted duration. A mixed discrete-continuous model was proposed by Tavares (1986) to cope with this common case (see Fig. 5).

Another feature of the adopted model for the activity duration concerns the possibility (or not)

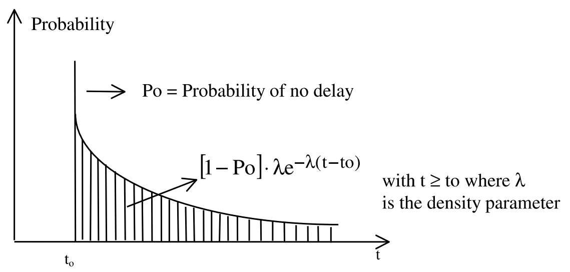


Fig. 5. A mixed model for the duration of an activity.

of interrupting its development, the so-called preemptive (or non-preemptive) case.

The preemption assumption gives more freedom to the manager as it increases the number of alternative feasible schedules.

*2.3.3. Modelling the activities resources*

Usually, a deterministic formulation of the resources required by each activity has been proposed. Resources can be classified into renewable, non-renewable and doubly constrained ones (Blazewicz et al., 1985). Non-renewable resources are those constrained in terms of their cumulative consumption such as energy, raw materials, etc. The renewable case is described by a restriction concerning the maximal (and, or the minimal) rate of use of a resource per time unit which means that some capacity is available under bounds to be respected. Manpower, electric tension, temperature or the capacity of a specific equipment are common examples. Doubly constrained resources include those restricted by cumulative and capacity constraints.

The utilization factor, $U$, is defined by the ratio between the total resources employed and the total resources provided during the total duration of the project. For instance, if a renewable resource is constrained by a function $\alpha$ and if the diagram of consumption is given by $\beta$, $U$ will be determined by the quotient between the areas bounded by $\alpha$ and $\beta$, $A(\beta)/A(\alpha)$ as shown in Fig. 6.

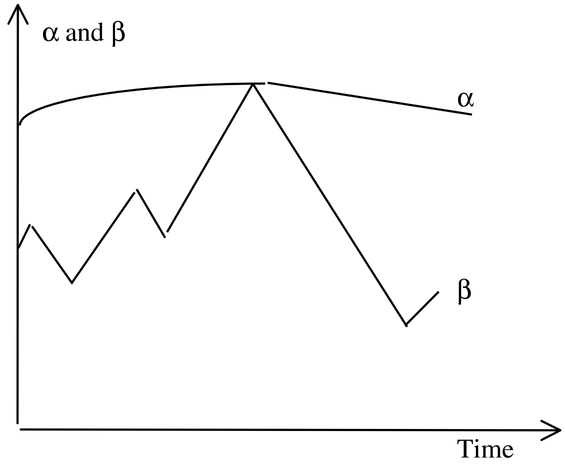


Fig. 6. The utilization factor, $U$.

Several formulations have been adopted to model the resources required by the activities:

(A) *Single resource or multiple resources*. Obviously, the second case is more realistic but most goods and services are available if money exists when we are within a market economy. Hence, the financial resource often can be used as a single resource representing a synthesis of multiple requirements.

(B) *Constant or variable requirements*. The basic model assumes that the resources required by each activity are constant but more sophisticated models consider such requirements in terms of the activity's duration (Tavares, 1987, 1990; Elmaghraby, 1993).

(C) *Single-mode or multi-mode formulations*. The second case assumes that each activity can be performed following different production modes which means that the project manager can choose the most convenient one. Each mode is described by a set of attributes, namely, duration and resources (see Boctor, 1994).

(D) *Stochastic correlation between the duration and the resource requirements*. Often, the uncertainty of duration is due to the occurrence of unforeseen difficulties and additional works. This means that the increase of the duration will be positively correlated with the amounts of consumed resources. A model is proposed by Tavares (1994) to describe this situation.

## 3. Project evaluation

Traditionally, the evaluation of projects has been studied by economists in the area of project appraisal using standard indexes such as net present value (NPV), return period, etc. Unfortunately, they tend to give little importance to long-term effects ("myopic nature") and do not consider other important non-monetary criteria. Furthermore, they have been developed before the rise of the Multi-criteria Decision Theory and hence now OR can enrich this domain with new contributions. A multi-criteria model in terms of NPV, duration and risk of delay was developed and applied by Tavares (1984) to a real case-study. Another model (MACMODEL) is now available as a decision-aid to support the process of multi-criteria evaluation of a project (Tavares, 1998). This model helps the decision-maker:

(a) To construct the most appropriate value-tree (see, for instance, Fig. 7). The adopted tree can include (cost, time, etc.) and also the probability of having outcomes much worse than those expected (risk).

(b) To assess the relative importance (weight) of each criteria.

(c) To carry out a sensitivity analysis on the weights or on the data of alternative projects (TRIDENT analysis). As an example, the sub-domain of each ranking of alternatives is presented on the weights space ($\lambda_1, \lambda_2, \lambda_3$ with $\lambda_1 + \lambda_2 + \lambda_3 = 1$) in Fig. 8.

Also, the discussion of the bidding and of the negotiation problems is receiving innovative contributions by Elmaghraby (1990) and Daynand and Padman (1994).

Unfortunately, most of the work on project evaluation has not considered the relationship between the evaluation and the adopted schedule. However, it is quite clear that indicators such as NPV or the risk of delay strongly depend on the schedule as early (late) starting times tend to be responsible for lower (higher) NPV and risk of delay. Therefore, this is another reason to increase the importance of the problem of project scheduling, which will be studied in following sections.

## 4. Project scheduling and monitoring

### *4.1. Project scheduling without restrictions on resources*

Project scheduling has been a major objective of most models and methods proposed to aid planning and management of projects.

Initially, the study of project scheduling has been done considering just the duration and precedence conditions and ignoring the resource requirements. Two basic methods proposed to schedule a project assuming deterministic durations are the well-known CPM – Critical Path Method and the very much ignored MP – Method of Potential. They both determine the critical path which gives the minimal total duration of the project, $T$, and the slacks for each node and activity.

Obviously, these methods provide a very useful aid for the project manager to schedule each activity of the project, assuming a total duration, $T'$, to be respected (equal to, or greater than, the minimal total duration) because they compute the minimal (or maximal) starting and finishing times

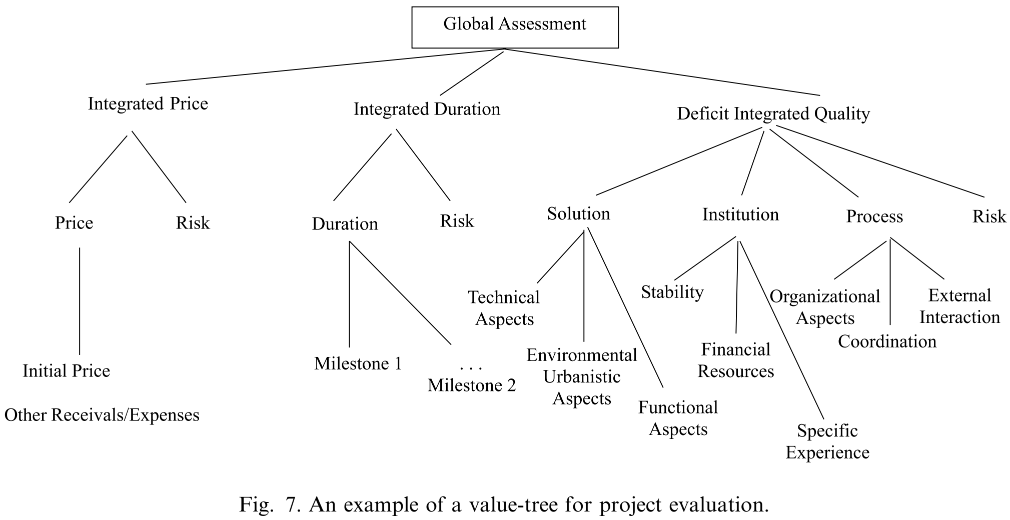


Fig. 7. An example of a value-tree for project evaluation.

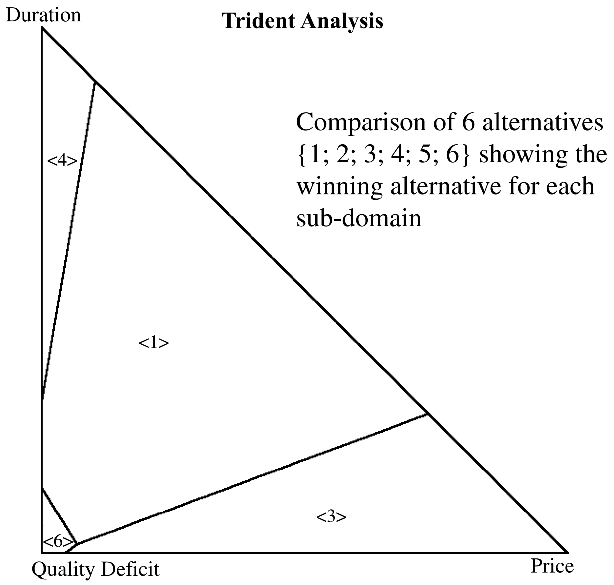


Fig. 8. An example of the results produced by the TRIDENT model.

for each activity. If $T = T'$, the scheduling of the critical activities offers no choice to the manager but this is not the case if $T' > T$.

However, most durations have a random nature and therefore PERT was proposed to determine the distribution of the total duration, $T$. This method is based on the substitution of the network by the CPAD – critical path assuming that each activity has a fixed duration equal to its mean ("critical path using average durations").

The mean and the variance of the CPAD is given by the sum of the means and of the variances of its activities, respectively, and therefore these results are considered the mean and the variance of the total duration of the network.

Unfortunately, this is an optimistic assumption as the real mean, $E(T)$, is greater than or equal to such estimate. Thus, many authors have studied:

(A) Analytical approximations of the cumulative distribution function of $T$, $F(T)$ (Alesarea and Drezner, 1986).

(B) Upper or lower bounds of $F(T)$ (Dodin, 1985).

(C) Monte-Carlo simulations to estimate $F(t)$ (Ragsdale, 1989).

(D) Analytical bounds for $E(T)$ (Elmaghraby, 1967; Robillard and Trahan, 1976).

However, in the world of applications, PERT results are still the most popular ones.

### 4.2. Project scheduling with restrictions on resources

#### 4.2.1. Formulation

In general terms, this problem can be formulated by a mathematical model where:

(a) The decision variables are the scheduled starting times of the activities.

(b) The constraints include the precedence conditions and the maximal (and or minimal) bounds concerning the available resources.

(c) The objective function describes the main criteria such as minimization of the total duration, levelling of resources or minimization of the risk of delay as well as maximization of the net present value or of other cost–benefit indicators. Quite often, the objective function is a weighted average of some of these criteria. These criteria can be expressed by deterministic measures or by stochastic functions defined in terms of expectations or of extreme quantiles (risk analysis).

A wide range of alternative models can be adopted for each of these variables or functions and therefore several taxonomies can be proposed.

The author believes that an appropriate classification can be based just on three dimensions as is shown in Fig. 9 (in Tavares, 1998).

A few key alternative assumptions should be underlined:

(A) *Implementation of the activities*. The implementation of the activities can be assumed having to follow just one pattern or, alternatively, the intensity of its implementation on each time unit can be considered as a discrete or continuous variable under specific bounds.

The former problem (*single-mode*) is easier but the latter is more realistic and it includes the discrete assumption (*multimode-model*) and the continuous case (*continuous model*).

Obviously, the second assumption implies that the duration of each activity depends on the intensity of the implementation allocated to each time unit.

Another option about the implementation of activities concerns the preemption assumption already mentioned: (non) preemptive activities can (not) be interrupted and continued later on.

A more realistic formulation assumes the preemptive hypothesis but it allocates an additional fixed charge every time an interruption occurs.

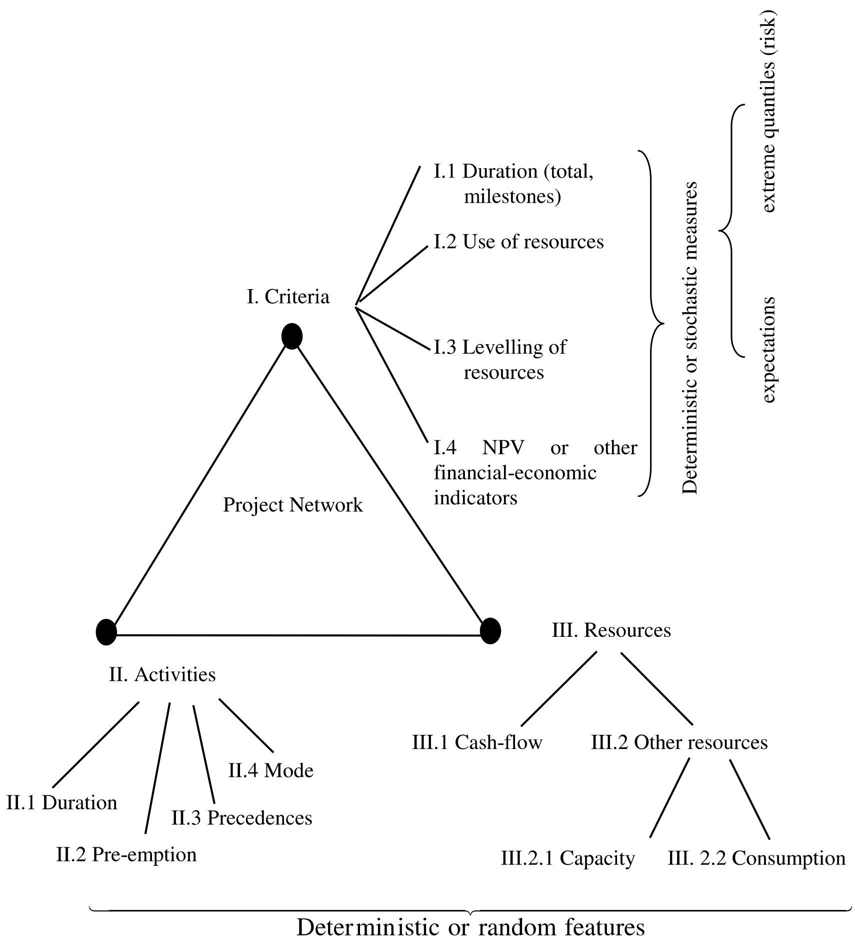


Fig. 9. Typology of the scheduling problems.

(B) *Uncertainty and randomness*. The formulated variables (duration or resources requirements) can be assumed to be deterministic magnitudes or, alternatively, random or stochastic variables.

(C) *Static or dynamic decision-making*. The process of decision-making concerning the scheduling of activities and the allocation of resources to the implementation of activities can be considered *static* or *dynamic*: static if the decision should be made before starting the project without the acceptance of any later correction or change; *dynamic* if the decision can be changed along the process of implementing the project.

The real problem faced by most project managers corresponds to the dynamic formulation and the adaptive control of the scheduling decisions is a major skill for successful project management.

*4.2.2. The deterministic static single-mode problem*

A very large number of methods have been proposed to solve the problem of project scheduling under resources constraints but unfortunately most of them address the deterministic, static, single-mode non-preemptive formulations.

Most of these contributions are based on a model defined in terms of $X_i(t) = 0$ (or 1) if the activity $i$ is (or is not) carried out at time unit $t$. Less explicit formulations are based on $Y_i(t)$ which can be equal to 1 (or to 0) if the finishing or starting time of $i$ is equal (or not equal) to $t$.

The objective of minimizing the total duration can be formulated in terms of the maximal $[t \cdot X_i(t)]$, or of $[t \cdot Y_i(t)]$ for the last activity to be completed.

The objective of levelling the consumption of resources can be expressed by the minimization of the sum of squares of the resources allocated at each time unit, $t$, over the duration of the project.

The precedence conditions can be formulated in terms of linear functions of $t \cdot X_i(t)$ or $t \cdot Y_i(t)$.

This problem has been studied by binary optimization methods since late 1960s (Pritsker et al., 1969; Davies, 1972; Patterson, 1984; Demeulemeester and Herroelen, 1997) and every year several algorithms are proposed claiming better results than the competitors. They belong to two major groups:

(a) *Exact methods*. These results explore the full space of the scheduling alternatives. Usually they are based on branch and bound procedures to avoid full enumeration. Recent proposals were presented by Brucker et al. (1998) and Sprecher and Drexl (1998).

(b) *Heuristics*. These algorithms do not guarantee the obtention of the optimum but they tend to be faster (Boctor, 1990). Recently, several procedures based on tabu search, simulated annealing or genetic methods have been applied to the scheduling problems.

Recent contributions can be quoted (Ahn and Erengue, 1998; Bianco et al., 1998).

These methods have to be tested using the experimental approach based on generated sets of networks.

Unfortunately, the developed methods have important shortcomings:

(a) Usually, they have not been tested for project networks with a medium or large size ($N > 50$).

(b) A numeric solution is produced for each set of data providing little understanding about the structure of the problem. No sensitivity analysis is available.

(c) Their performance tends to be unstable and sensitive to the features of the project network. However, this relationship has not been studied.

*4.2.3. Deterministic continuous mode problem*

In this case, the decisions concerning the implementation of each activity include its starting time and also its intensity in terms of time.

Therefore, the total resource consumed by each activity and its duration are continuous variables.

All the magnitudes are assumed to be deterministically known.

It seems that the first contribution to study this problem was given by Fulkerson (1962), using a network flow model.

Weglarz (1981) has studied this problem using Optimal Control Theory and assuming that the processing speed of each activity at time $t$ is a continuous, non-decreasing function of the amount resource allocated to the activity at that instant of time. This means that also time is here considered as a continuous variable. Unfortunately, it seems that this approach is not applicable to networks with a reasonable size (>10).

A powerful approach to solve this problem is based on Dynamic Programming (Elmaghraby, 1993, 1995). The author has presented a general model based on the decomposition of the project into a sequence of stages (Tavares, 1987, 1989) and the optimal solution can be easily computed for each practical problem as it is shown for a real case-study. Also, the theoretical optimal resource profile can be deduced for some specific cases using the Calculus of Variations (see Fig. 10).

This formulation also can be extended to the dynamic case because at each stage of DP one has the time of occurrence of that stage as a state variable and therefore the optimal decision is made in terms of such state which means that the model can consider eventual delays or advances.

Recently, new models were proposed to study this problem (the continuous allocation problem) and are presented in the following Section 5.1.

The stochastic assumption introduces additional difficulties and then the experimental approach has to be adopted.

Actually, the study of advanced deterministic or stochastic scheduling methods has to be based on generated sets of network which explains why generation methods have become so important.

New developments on generation of networks and on scheduling models are presented in Section 5.

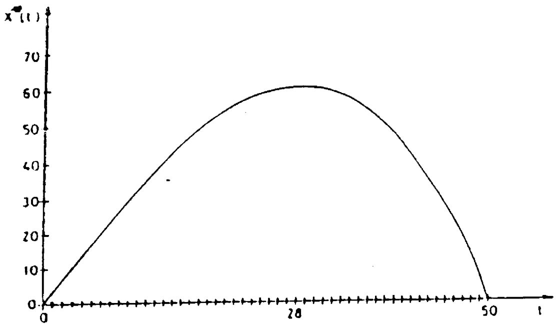


Fig. 10. An example of the optimal resource curve (in Tavares, 1987).

### 4.3. Project monitoring

The monitoring and control of projects has been heavily supported by three types of instruments:

(a) Impressive development of information systems under several labels such as Management Information Systems or Executive Information Systems to produce updated pictures of how the project is progressing in terms of completion of activities, consumption of resources, delays, quality control, etc. (Drigani, 1989).

(b) Multivariate data analysis of completed activities or of previous projects to learn how to improve and to correct initial estimates adopted for the project evaluation and scheduling (Kelley, 1982).

(c) Decision Support Systems to assess the progress of the project and to update the adopted models for project management (Mitra, 1986).

For instance, PERT or scheduling models can be updated in terms of the information given by the systems mentioned in (a) and using the knowledge produced by models included in (b).

## 5. New results

### 5.1. The continuous allocation model

This model solves the deterministic continuous mode problem (Tavares, 1998) and follows the line of development started by Ferreira and Antunes (1989) and Tavares (1990).

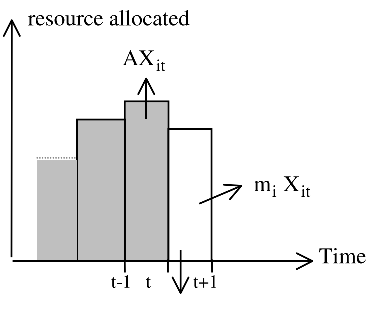


Fig. 11. Notation adopted by the continuous allocation model.

This model (Fig. 11) is based on:

(a) *Variables*

- $X_{it}$ $\rightarrow$ intensity of implementation of activity $i$ at time unit $t$ with $t = 0, 1, \ldots, H$ where $H$ is the maximal time limit to complete the project with $0 \leqslant X_{it} \leqslant 1$.
- $m_{it}$ $\rightarrow$ consumption or use of resource by the activity $i$ at time unit $t$ if $X_{it} = 1$. Therefore, the actual consumption of resource by activity $i$ at time unit $t$ is given by $(m_{it} \cdot X_{it})$.
- $M_i$ $\rightarrow$ total resource required to complete $i$.
- $AX_{it}$ $\rightarrow$ cumulative resource used until time unit $(t-1)$ and including $(t-1)$ which is defined by $AX_{ti} = \sum_{\tau=0}^{t-1} X_{i\tau} \cdot m_{i\tau}$.
- $\Delta X_{it}$ $\rightarrow$ remaining amount of resource not yet used until the beginning of the time unit $t$: $AX_{it} = M_i - AX_{it}$.
- Finally,

$$\Delta X_t = \sum_i \Delta X_{it}.$$

(b) *Restrictions*

- Resource bounds

$$\sum_{i=1}^{N} X_{it} m_{it} \leqslant L_t$$

or

$$\sum_{\tau=0}^{t-1} X_{i\tau} m_{i\tau} \leqslant LA_t,$$

where $L_t$ is the maximal amount of resource available at time unit $t$ and where $LA_t$ is the maximal cumulative consumption of resource until the start of $t$.
- Precedence conditions $(i \rightarrow j) X_{jt} \cdot \Delta X_{it} = 0$ for any pair $(i, j)$ where $i$ directly precedes $j$ and for any possible $t$.

(c) *Objectives*. The economic or financial objectives such as the Present Cost, PC, can be formulated very easily as there is an explicit definition of the resource allocated to each time unit: $\mathrm{PC} = \sum_i f^t \cdot m_i X_{it} \cdot C_i$ , where $C_i$ is the unit cost of $i$. The criteria defined in terms of the duration such as the total duration can be expressed by defining auxiliary variables, $Y_t$ with $0 \leqslant Y_t \leqslant 1$ and making $Y_t \cdot \Delta_t = 0$ with $t = 0, \ldots, H$. Then, an auxiliary objective function $\sum_{t=0}^{H} Y_t$ has to be maximized and one obtains the total duration $T$ given by

$$T = (H + 1) - \sum_{t=0}^{H} Y_t.$$

Therefore, an objective like the maximization of the net present value (see Elmaghraby and Herroelen, 1990) can be expressed by: $\mathrm{Max}\,[Bf^T - \mathrm{PC}]$ where $B$ is the benefit received immediately after completing the project.

This model includes a set of complementary conditions $(X_{jt} \cdot \Delta X_{it} = 0$ and $Y_t \cdot \Delta_t = 0)$ which can be relaxed by introducing the corresponding Lagrangan terms in the objective function to be minimized

$$\left[\alpha \sum_{(i,j)} \sum_t X_{jt} \cdot \Delta X_{it} + \beta \sum_t Y_t \cdot \Delta_t\right],$$

where $\alpha$ and $\beta$ are appropriate penalty coefficients.

The presented model assumes the preemptive assumption and can be easily extended to several resources.

Extensive computational experiments show that this model can be easily solved for large networks using standard software for non-linear optimization such as GAMS-MINOS (see Tavares, 1998).

Examples of networks with several hundreds activities can be solved by this approach in a few minutes of computing time using a Pentium 166.

An example of a schedule for 150 activities is shown in Fig. 12 (Tavares, 1998). This model has the main advantages of considering all the alternative ways concerning the implementation of activities, allowing a direct and easy formulation of economic or financial objective functions and being quickly solved by standard non-linear optimization software.

### 5.2. A model to generate project networks

A few models have been proposed to generate project networks but often they adopt the AoA assumption to avoid problems of arc redundancy.

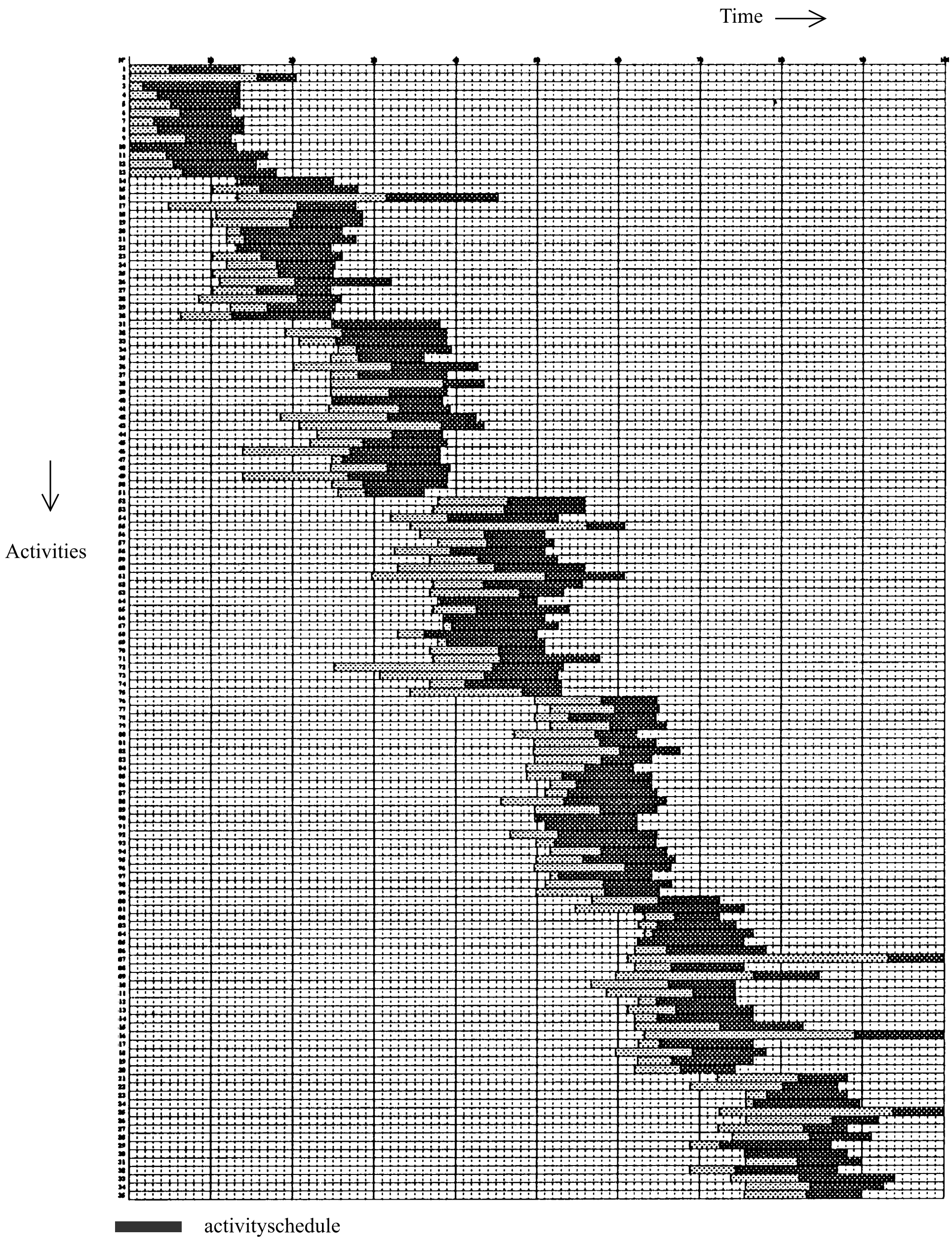


Fig. 12. An example of the latest starting time schedule for a network with 150 activities.

However, this assumption implies that the user will give the number of nodes which is usually unknown and even undetermined as it has been proved that different graphs can represent the same network if AoA is adopted.

The published models do not allow the generation of networks preserving their morphologic features (Demeulemeester and Herroelen, 1993; Agrawal et al., 1996) but, recently, a model was proposed using AoN and generating networks in

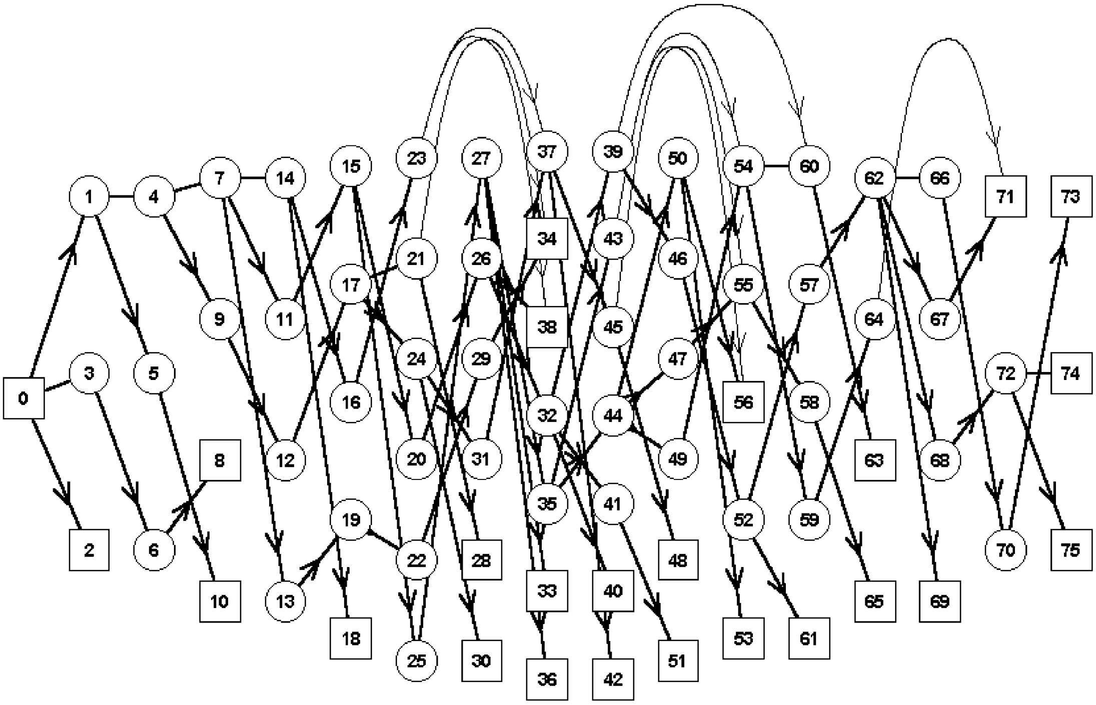


Fig. 13. Example of the graphical representation of a network with 75 activities (AoN).

terms of their morphology described by Tavares et al. (1997):

- Size, $I_1 = N$.
- Length,

$$I_2 = \frac{M-1}{N-1} \quad \text{for } N > 1 \text{ with } 0 \leqslant I_2 \leqslant 1.$$

- Width,

$$I_3(m) = \frac{W_{(m)} - 1}{N - M} \quad \text{with } m = 1, \ldots, M,$$

where $W_{(m)}$ is the number of activities with the progressive level $m$ with $m = 1, \ldots, M$. Also, this indicator can be defined in terms of the maximal width,

$$\mathrm{MW} = \max_m W(m), \quad I'_3 = \frac{MW - 1}{N - M}.$$

Obviously, one has $0 \leqslant I_3$ or $I'_3 \leqslant 1$

- Number of non-redundant precedence links with length equal to one, $n(1)$, $I_4 = [n(1) - N]/[D - N]$ where $D$ is the maximal $n(1)$ and again $0 \leqslant I_4 \leqslant 1$.The length of a precedence link $(j \to i)$ is defined by $[m(i) - m(j)]$ where $m(\cdot)$ is the progressive level of.
- Rate of decrease of the number of links in terms of their length, $I_5 = p$. Assuming an exponential decrease, one has $S(v+1) = S(v).\ p$ where $S(\cdot)$ is the number of non-redundant links with length $(\cdot)$ and $v = 1, 2, \ldots, V$ with $V$ being the maximal length.
- Maximal length of the precedence links, $I_6 = (V-1)/(M-1)$ with $0 \leqslant I_6 \leqslant 1$.

A test set of 216 networks was generated with $N = 10, \ldots, 1000$. An example of a network with 75 activities is presented in Fig. 13. AoN is adopted and any activity $i$ with $K(i) = \emptyset$ is represented by a square instead of a circle to avoid the need to represent the links between such nodes and the end of the project network.

### 5.3. The stochastic modelling of project risk

The most realistic and useful models of project management adopt the stochastic assumption to represent project uncertainty. Such uncertainty concerns mainly the duration and the resources or cost required by each activity but, in general, the proposed models just consider the randomness of the durations. However, often, there is a significant correlation between duration and cost as delays are often a consequence of additional works or of unexpected difficulties which are also inevitably responsible for an increase of cost. This was studied by the analytical model already mentioned (Tavares, 1994) and also by an experimental model (Tavares et al., 1998).

This model assumes that the duration of each activity, $D_i$, follows a lognormal law and that its cost, $C_i$, follows a linear regression in terms of the duration:

$$C_i = \alpha + \beta D_i + \varepsilon_i,$$

where $\alpha$, $\beta$ are appropriate constants and $\varepsilon_i$ is a random normal deviate with zero mean.

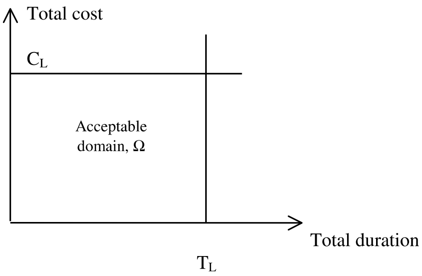


Fig. 14. Acceptable domain.

This model allows the study of the stochastic concept of *project risk*, $R$, as the probability of obtaining a total cost and a total duration outside the acceptable domain, $\Omega$ (Fig. 14):

$$R = 1 - P[C \leqslant C_L \cap T \leqslant T_L].$$

This research assumes that the project manager is attempting to implement a pre-defined schedule,

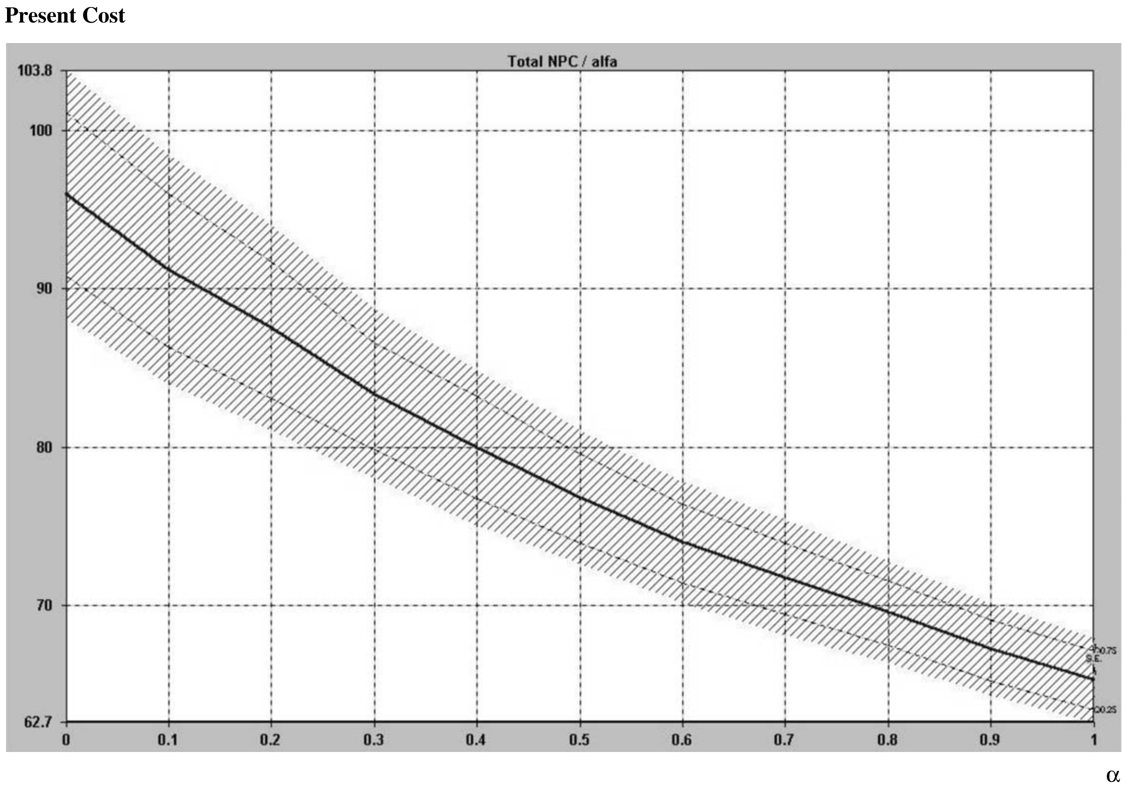


Fig. 15. Present cost in terms of $\alpha$.

**Total Duration**

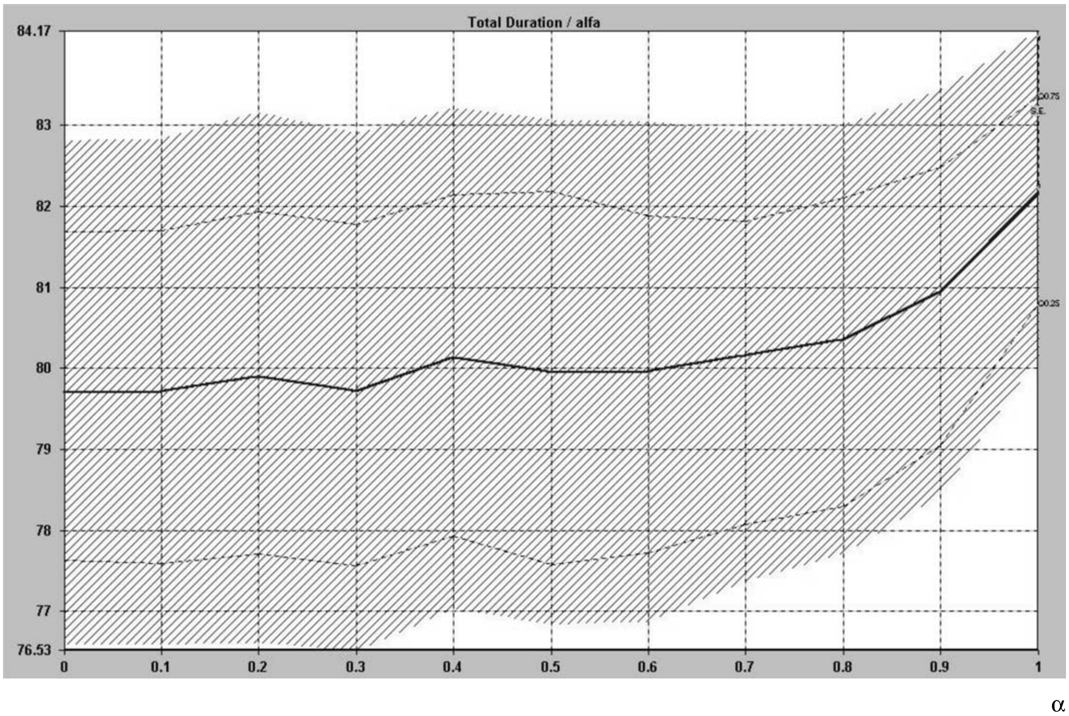


Fig. 16. Total duration in terms of $\alpha$.

$\{t^{s^*}(j)\}$, and hence, in each generated instance, $k$, the starting time of each activity, $j$, $t^s_k(j)$ is determined by

$$t^s_k(j) = \max\left\{t^{s^*}(j);\ \max_{i\in I(j)}\left[t^s_k(i) + D_k(i)\right]\right\}$$

for $I(j) \neq \emptyset$ and where $D_k(i)$ is the duration of $i$ for the instance $k$.

If $I(j) = \emptyset$, one has $t^s_k(j) = t^{s^*}(j)$.

A *theorem* on the *float management* is proved in (Tavares et al., 1998) showing that the manager can adopt for each instance

$$t^s(i) = t^{\mathrm{E}}(i) + \alpha\left[t^{\mathrm{L}}(i) - t^{\mathrm{E}}(i)\right]$$

with $0 \leqslant \alpha \leqslant 1$ and where $t^{\mathrm{E}}(i)$ and $t^{\mathrm{L}}(i)$ are the earliest and latest starting times of $i$.

A large $\alpha$ tends to increase the duration and to reduce the present cost due to the discount factor and a lower $\alpha$ has the opposite effect.

Therefore, an optimal $\alpha$ has to be determined with the objective of minimizing the project risk.

An example of the present cost and of the total duration computed for a large number of generated instances is presented in Figs. 15 and 16.

These results illustrate the new type of decision-aids which can be offered by OR stochastic models to help project managers to make better decisions.

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