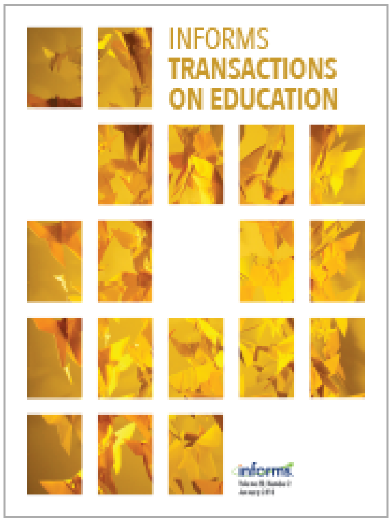

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Martin J. Chlond,

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Puzzle

Minesweeper Puzzles

Martin J. Chlond
Lancashire Business School, University of Central Lancashire, Preston PR1 2HE, United Kingdom,
mchlond@uclan.ac.uk

Introduction

Minesweeper is a computer puzzle game whose objective is to determine the location of several mines hidden within a rectangular grid. The player is required to click on cells. If a cell containing a mine is clicked, the game is lost. Otherwise, the total number of mines in neighbouring cells is displayed, and this information may be used to guide future decisions as to which cells to click. The game is won when all mines have been located. Stewart (2010) has a more detailed description for readers who are unfamiliar with the game.

This classical version of Minesweeper is unsatisfactory from a puzzlist's viewpoint because there is an element of chance involved. For example, the first move is made in complete ignorance of the underlying configuration of mines, and throughout the game situations can arise for which it is impossible to proceed without resorting to guesswork.

A modification of the standard Minesweeper game described in du Sautoy (2010) leads to an interesting and more satisfying puzzle. A partially played grid is given that contains information regarding cells that are known to contain mines and also the number of mines surrounding exposed cells. The objective is to identify the location of all mines that are consistent with the partial information given. Steinruecken (2010) provides several small but challenging examples. Each example has a unique solution that is consistent with the information provided.

Kaye (2000) discusses a closely related problem known as the *Minesweeper Consistency Problem* in which the objective is to determine whether or not a feasible solution exists. He has shown that this modified problem is equivalent to the SAT problem and hence is NP-complete. It is therefore of interest to OR/MS practitioners and teachers. A useful and accessible discussion of this equivalence is in Stewart (2010).

Examples

A simple example taken from du Sautoy (2010) is in Figure 1. A second, more challenging example taken from Kaye (2007) is in Figure 2. An asterisk is used to represent a mine that has been located. Solutions are given in Appendix A.

IP Formulation

There follows an IP formulation to solve Steinruecken's puzzles, which contain information regarding mine counts but not the location of any mines.

Decision variables are defined as follows.

$x_{i,j} = 1$ if cell $\{i, j\}$ contains a mine, 0 otherwise.

The initial state of the minefield is described by matrix $\mathbf{R}$, where $r_{i,j}$ is equal to the number of mines surrounding cell $\{i, j\}$ and where -1 represents an unexposed cell (this will be referred to later as the *mine count matrix*). Also, we define set $N = \{1, \ldots, n\}$, where $n =$ size of the square grid.

The aim of the puzzles is to find solutions that are consistent with the information given, and hence no objective function is required.

Constraints are required in order to ensure that all exposed squares have the correct number of surrounding mines, as given in $\mathbf{R}$.

$$\sum_{p=i-1 \;\; 1\le p\le n}^{i+1} \; \sum_{q=j-1 \;\; 1\le q\le n}^{j+1} x_{p,q} = r_{i,j} \quad \forall\, i \in N,\; j \in N \mid r_{i,j} \ge 0.$$

Figure 1 du Sautoy's Puzzle

	2	2	2	2	
	2	0	0	2	
	2	0	0	2	
	2	2	2	2	

Figure 2 Kaye's Puzzle

2	3	*	2	2	*	2	1
*	*	5			4	*	2
		*	*	*		4	*
*	6		6	*	*		2
2	*	*		5	5		2
1	3	4		*	*	4	*
0	1	*	4			*	3
0	1	2	*	2	3	*	

Also, exposed squares do not contain a mine.

$$x_{i,j} = 0 \quad \forall\, i \in N,\ j \in N \mid r_{i,j} \geq 0.$$

Kaye's example, above, includes information regarding a number of mines whose locations are known. The above formulation may be extended to accommodate this modification by defining an additional matrix **M** where $m_{i,j} = 1$ if cell $\{i, j\}$ is known to contain a mine, 0 otherwise. A valid starting configuration would require that $r_{i,j} = -1$ if $m_{i,j} = 1$. The following additional constraints complete the formulation:

$$x_{i,j} \geq m_{i,j} \quad \forall\, i \in N,\ j \in N.$$

Results

An Mathprog (2010) model of the above formulation is included (minesweeper.mod) and may be used to solve the puzzles and also further examples on Steinruecken's website. Christian Steinruecken also kindly provided the author with a couple of larger examples, and these are included in the Mathprog file.

An R (2010) function to generate random minefields of arbitrary size and compute associated mine count matrices is included (make.mine.r). These mine count matrices may be used as input to the Mathprog model. A $1{,}000 \times 1{,}000$ puzzle requiring one million binary variables was generated using the function and was solved in 38 seconds on a 2.16 GHz laptop. The solution was obtained using the presolve facilities provided in GLPK. Thus, the NP-hardness of these puzzles is not readily apparent in such large, randomly generated minefields. Kaye (2007) notes that in some cases, as with the puzzle in Figure 2, the entire minefield must be considered in order to determine consistent solutions, and it is these cases where the difficulties emerge. Hence the puzzles must be carefully constructed to ensure they are sufficiently challenging for human or computer solvers.

Appendix A

Solution to puzzle in Figure 1.

		*	*		
	2	2	2	2	
*	2	0	0	2	*
*	2	0	0	2	*
	2	2	2	2	
		*	*		

Solution to puzzle in Figure 2.

2	3	*	2	2	*	2	1
*	*	5		*	4	*	2
	*	*	*	*		4	*
*	6	*	6	*	*	*	2
2	*	*		5	5		2
1	3	4	*	*	*	4	*
0	1	*	4		*	*	3
0	1	2	*	2	3	*	

Supplemental Files

An electronic companion to this paper is available at http://ite.pubs.informs.org.

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