GABARITO

(1) A, B e C (eventos) ou

a) $A \cup B \cup C = A$ ou

b) $A \cap B \cap C = A$

para a) B e C estão contidos em A

$B \subseteq A$. $C \subseteq A$.

não podemos afirmar nada sobre B e C

para b) A está contido em B e C

$A \subseteq B$ e $A \subseteq C$

no caso de B e C:

$B \cap C = A$, $B \supseteq C \supseteq A$, $C \supseteq B \supseteq A$

(2) Se $P(A) = 0,6$; $P(B) = 0,3$; $P(A \cap B^c) = 0,4$

$B \subset C$. calcular $P(A \cup B^c \cup C^c)$

Se $B \subset C \Rightarrow B^c \supset C^c$, de onde

temos que $B^c \cup C^c = B^c$

$$P(A \cup B^c \cup C^c) = P(A \cup B^c)$$
$$= P(A) + P(B^c) - P(A \cap B^c)$$
$$= 0,6 + 0,3 - 0,4$$
$$= 0,5$$

(3) p/ A, B, C. $P(A \cap B) = P(A \cap C)$ e $P(B \cap C) = 0$

mostrar que $P(A \cup B \cup C) = P(A) + P(B) + P(C) - 2P(AB)$

$$P(A \cup B \cup C) = P(A) + P(B) + P(C) - \underbrace{P(AB) - P(AC)}_{=2} - \underbrace{P(BC)}_{0} + \underbrace{P(ABC)}_{0}$$

4: 52 cartas: extraimos $\underline{3}$

qual a probabilidade de pelo menos 1 Ase)?

[o baralho contém 4 ases]

Seja X: nº de ases extraídos

$X = \{0, 1, 2, 3\}$

$$P(X \geq 1) = 1 - P(X=0) = 1 - \frac{\binom{4}{0}\binom{48}{3}}{\binom{52}{3}}$$

5: A, B e C estão relacionados como

$A \subset B \subset C$.

$P(A) = 1/4$

$P(B) = 5/12$

$P(C) = 7/12$

calcular:

a) $P(A^c \cap B) = P(B) - P(A) = 5/12 - 1/4$

b) $P(A^c \cap C) = P(C) - P(A) = 7/12 - 1/4$

c) $P(B^c \cap C) = P(C) - P(B) = 7/12 - 5/12$

d) $P(A^c \cap B^c \cap C^c) \overset{\text{Morgan}}{=} P(A \cup B \cup C)^c = P(C)^c$

$= 1 - P(C) = 1 - 7/12$

⑥ O número de "interruptores" de luz é

$$X \sim f(x) = \begin{cases} c\left(\frac{9}{10}\right)^{x-1}, & x = 1, 2, \ldots \\ 0, & \text{e.c.} \end{cases}$$

a) determinar c.

$$\sum_{x=1}^{\infty} c\left(\frac{9}{10}\right)^{x-1} = c \cdot \sum_{x=1}^{\infty} \frac{9}{10}^{x-1}$$

$$= c\left[\left(\frac{9}{10}\right)^0 + \left(\frac{9}{10}\right)^1 + \left(\frac{9}{10}\right)^2 + \ldots\right] = 1$$

$$= c\,\underbrace{\left[1 + \left(\frac{9}{10}\right)^1 + \left(\frac{9}{10}\right)^2 + \ldots\right]}_{S} = 1$$

$$= c\left[1 + \left(\frac{9}{10}\right)\underbrace{\left[\left(\frac{9}{10}\right)^0 + \left(\frac{9}{10}\right)^1 + \ldots\right]}_{S}\right] = 1$$

$$S = 1 + \frac{9}{10} S \Rightarrow S - \frac{9}{10} S = 1 \Rightarrow$$

$$\frac{S}{10} = 1 \Rightarrow S = 10$$

$$\rightarrow c \cdot S = 1 \Rightarrow \boxed{c = \frac{1}{10}}$$

b) $P(X>10) = 1 - P(X \le 10)$

$$= 1 - \sum_{x=1}^{10} \left(\frac{1}{10}\right)\left(\frac{9}{10}\right)^{x-1}$$

c) $P(X \le [a]) = \sum_{x=1}^{[a]} P(X=x)$

$$= \sum_{x=1}^{[a]} \frac{1}{10} \cdot \left(\frac{9}{10}\right)^{x-1} \quad x = 1, 2, \dots, [a]$$

7: $X \sim F(x)$

$$F(x) = \begin{cases} 0 & x \le 0 \\ 2c\left(x^2 - \frac{1}{3}x^3\right) & 0 < x \le 2 \\ 1 & x > 2 \end{cases}$$

a) $f(x) = F'(x) = \left[2c\left(x^2 - \frac{1}{3}x^3\right)\right]'$

$$= \left[2cx^2 - \frac{2}{3}cx^3\right]' = 4cx - 2cx^2$$

$$= \begin{cases} 2c[2x - x^2] & 0 < x \le 2 \\ 0 & \text{o.w.} \end{cases}$$

b) $\int_0^2 f(x)\,dx = 1 \Rightarrow \int_0^2 2c[2x - x^2]\,dx = 1$

$$\Rightarrow 2c\left[x^2 - \frac{x^3}{3}\Big|_0^2\right] = 2c\left[4 - \frac{8}{3}\right] = 1$$

$$\Rightarrow 2c\left[\frac{4}{3}\right] = 1 \Rightarrow c = \frac{3}{8}$$

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8.- $X \sim F$; $F(x) = \begin{cases} 0 & x \leq 0 \\ x^3 - x^2 + x & 0 < x \leq 1 \\ 1 & x > 1 \end{cases}$

a) $f(x) = [F(x)]' = [x^3 - x^2 + x]'$

$= 3x^2 - 2x + 1 \qquad 0 \leq x \leq 1$

$f(x) = \begin{cases} 3x^2 - 2x + 1 & 0 \leq x \leq 1 \\ 0 & \text{c.c.} \end{cases}$

b) $P(X > 1/2) = \int_{1/2}^{1} (3x^2 - 2x + 1)\, dx$

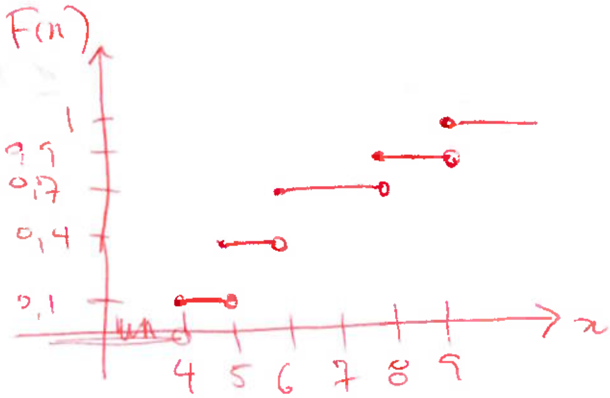
$= \frac{3x^3}{3} - \frac{2x^2}{2} + x \Big|_{1/2}^{1} = \frac{3}{3} - \frac{2}{2} + 1 + $

$- \frac{1}{8} + \frac{1}{4} - \frac{1}{2}$

$= 1 + \frac{2}{8} - \frac{5}{8} = \frac{5}{8}$

(9) $X \sim F$ $\qquad F(x) = \begin{cases} 0 & x < 4 \\ 0{,}1 & 4 \le x < 5 \\ 0{,}4 & 5 \le x < 6 \\ 0{,}7 & 6 \le x < 8 \\ 0{,}9 & 8 \le x < 9 \\ 1 & x \ge 9 \end{cases}$

a)



b) i) $P(X \le 6{,}5) = F(6{,}5) = 0{,}7$

ii) $P(X > 8{,}1) = 1 - F(8{,}1) = 1 - 0{,}9 = 0{,}1$

c) $P(5 < X < 8) = P(X < 8) - P(X \le 5)$
$= 0{,}7 - 0{,}4 = 0{,}3$

[Se construirmos a fp, teríamos:]

X	4	5	6	8	9
P(X)	0,1	0,3	$\boxed{0{,}3}$	0,2	0,1

$\longrightarrow P[5 < X < 8]$

(10) $X \sim f(x) = c\,x^{-(c+1)}$, $x \ge 1$ e $c > 0$

a) p/ f(x) ser f.d.p. $\Rightarrow \int_1^{\infty} f(x)\,dx = 1$

$\lim_{t\to\infty} \int_1^{t} c x^{-(c+1)}\,dx = \lim_{t\to\infty} -\frac{c\,x^{-c}}{c}\Big|_1^t = \lim_{t\to\infty} -t^{-c} + 1 = 1$

$\therefore \lim_{t\to\infty} t^{-c} = 0$ pelo que $c > 1$

b) $F(a) = \int_1^{a} c\,x^{-(c+1)}\,dx = -x^{-c}\Big|_1^a = 1 - a^{-c}$, $c > 1$

11. $X = \{0,1,2,\dots\}$; $P(X=j) = \frac{c}{3^j}$; $j=0,1,2\dots$

determine

a) c.

P o domin.: $\sum_{j=0}^{\infty} P(X=j) = 1$

$\Rightarrow \sum_{j=0}^{\infty} \frac{c}{3^j} = 1 \Rightarrow c \underbrace{\boxed{\sum_{j=0}^{\infty} \frac{1}{3^j}}}_{S} = 1$

$S = \underbrace{\frac{1}{3^0} + \frac{1}{3^1} + \frac{1}{3^2} + \dots}_{S_0} = 1 + \frac{1}{3^1}\underbrace{\left[\frac{1}{3^1} + \frac{1}{3^2} + \dots\right]}_{S_0}$

~~$S = 1 + S_0 = 1 + \frac{1}{3}S_0 \Rightarrow S_0\left[1-\frac{1}{3}\right] =$~~

$\frac{1}{3^0} + \frac{1}{3^1} + \frac{1}{3^2} + \dots = 1 + \frac{1}{3^1}\left[\frac{1}{3^0} + \frac{1}{3^1} + \frac{1}{3^2} \dots\right]$

$S = 1 + \frac{1}{3} S$

$S\left(1 - \frac{1}{3}\right) = 1 \Rightarrow S = \frac{3}{2}$

$c \cdot \frac{3}{2} = 1 \Rightarrow c = \frac{2}{3}$

b) $P(X \geq 3) = 1 - P(X<3) = 1 - [P(X=0) + P(X=1) + P(X=2)]$

c) $P($~~$X=2k+1$~~ $, k=0,1,\dots)$

$P(X=2k+1)$

$P(X=1) = \frac{2}{3} \cdot \frac{1}{3^{2\cdot0+1}} = \frac{2}{3} \cdot \frac{1}{3} = \frac{2}{3^2}$

$P(X=3) = \frac{2}{3} \cdot \frac{1}{3^3} = \frac{2}{3^4}$

$P(X=5) = \frac{2}{3} \cdot \frac{1}{3^5} = \frac{2}{3^6}$

$P(X=2k+1) = \frac{2}{3^{2k+2}}$ $k=0,1,\dots$

12: c: $f(x) = c\,\alpha^x \quad x = 0, 1, \ldots$ i fdp

$$\sum_{x=0}^{\infty} c\,\alpha^x = 1 \Rightarrow c \sum_{x=0}^{\infty} \alpha^x = 1$$

$$S = \alpha^0 + \alpha^1 + \alpha^2 + \ldots$$
$$= \alpha^0 + \alpha^1 [\alpha^0 + \alpha^1 + \ldots]$$
$$= \alpha^0 + \alpha^1 S \Rightarrow S(1-\alpha) = 1$$
$$S = \frac{1}{1-\alpha}$$

$$c = \frac{1}{\frac{1}{1-\alpha}} = 1-\alpha$$

$$1 \geq c \geq 0$$

13: $f(x) = \quad f(x) = c\,e^{-cx} \quad x > 0$

a) $$\int_0^{\infty} c\,e^{-cx}\,dx = 1 \Rightarrow c \cdot \frac{e^{-cx}}{-c} \Big|_0^{\infty} = 1$$

$$\cancel{-e^{-\infty}} + e^{-c \cdot 0} = 1$$

$\Rightarrow$

$c > 0$

b) $$P(X > 10) = \int_{10}^{\infty} c\,e^{-cx}\,dx = c \cdot \frac{e^{-cx}}{-c} \Big|_{10}^{\infty}$$

$$= -e^{-\infty} + e^{-10c} = e^{-10c}$$

c) $e^{-10c} = 0{,}5 \Rightarrow c = ?$

$$-10c = \ln 0{,}5 \Rightarrow \boxed{c = \frac{\ln 0{,}5}{-10}}$$

5

14: $X \sim$ Pareto, $f(x) = \dfrac{1+\alpha}{x^{2+\alpha}}$, $x > 1$ $\alpha > 0$

a) $f(x)$ d fd, ?

$$\int_1^\infty \frac{1+\alpha}{x^{2+\alpha}}\,dx = 1 \Rightarrow \lim_{t\to\infty} \int_1^t \frac{1+\alpha}{x^{2+\alpha}}\,dx = (1+\alpha) \lim_{t\to\infty} \int_1^t x^{-(2+\alpha)}\,dx$$

$$= (1+\alpha) \lim_{t\to\infty} \frac{x^{-(2+\alpha)+1}}{-(2+\alpha)+1}\Bigg|_1^t = \frac{(1+\alpha)}{-(1+\alpha)} \cdot \lim_{t\to\infty} x^{-(\alpha+1)}\Bigg|_1^t$$

$$= 1 - \lim_{t\to\infty} t^{-(\alpha+1)} = 1$$

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16 X: temperatura $X \sim f(x) = n(1-x)^{n-1}$, $0 < x < 1$, $n \geq 1 \in \mathbb{Z}$

calcular x_0

$P(X > x_0) = \dfrac{1}{10^{2n}}$.

$$\int_{x_0}^{1} -\frac{\partial(1-x)^n}{\partial x} = -(1-x)^n\Big|_{x_0}^{1} = (1-x_0)^n = \frac{1}{10^{2n}}$$

de onde $1 - x_0 = \sqrt[n]{\dfrac{1}{10^{2n}}} = \dfrac{1}{10^2}$

$\therefore x_0 = 1 - \dfrac{1}{10^2}$

17) Se $P(A|B) > P(A) \Rightarrow P(B|A) > P(B)$

$$\frac{P(AB)}{P(B)} > P(A), \text{ supondo } P(A) \text{ e } P(B) > 0$$

$$\frac{P(B/A).P(A)}{P(B)} > P(A) \Rightarrow P(B/A).P(A) > P(A)P(B)$$

$$P(B|A) > P(B)$$

18) $A \cap B = \phi$ e $P(A \cup B) > 0$

$$P(A|A\cup B) = \frac{P(A \cap (A\cup B))}{P(A\cup B)} = \frac{P(A)}{P(A)+P(B)}$$

$$P(B|A\cup B) = \frac{P(B\cap(A\cup B))}{P(A\cup B)} = \frac{P(B)}{P(A)+P(B)}$$

19) $N = 20$ 16 Bons (B)
4 Def. (D)

$n = 3$. Sejam B_i (Bom na i-ésima) $i = 1,2,3$
D_i (Defeituosa na i-ésima) $i = 1,2,3$

a) $P(B_3 | B_1 B_2)$

$$= \frac{P(B_1 B_2 B_3)}{P(B_1 B_2)}$$

b) $P(D_3 | B_1 D_2)$

c) $P(D_3 | D_1 B_2)$

d) $$P(D_3 | B_1 D_2 \cup D_1 B_2) = \frac{P(D_3 \cap [B_1 D_2 \cup D_1 B_2])}{P(B_1 D_2) + P(D_1 B_2)}$$

$$= \frac{P(D_3 B_1 D_2) + P(D_3 D_1 B_2)}{P(B_1 D_2) + P(D_1 B_2)}$$

20) $P(A)>0$, $P(B)>0$ e $P(C)>0$

a) $P(A^c|B) = 1 - P(A|B)$

$$P(A^c|B) = \frac{P(A^cB)}{P(B)} = \frac{P(B)-P(AB)}{P(B)} = 1 - \frac{P(AB)}{P(B)}$$

$$= 1 - P(A/B)$$

b) $P(A \cup B|C) = P(A|C) + P(B|C) - P(A \cap B|C)$

La probabilidade condicional é uma probabilidade

$$P(A \cup B|c) = \frac{P((A \cup B)\cap C)}{P(C)} = \frac{P(AC \cup BC)}{P(C)}$$

$$= \frac{P(AC) + P(BC) - P(AC \cap BC)}{P(C)}$$

$$= \frac{P(AC)}{P(C)} + \frac{P(BC)}{P(C)} - P\left(\frac{AC \cap BC}{P(C)}\right)$$

$$= P(A|C) + P(B|C) - P(A \cap B|C)$$

21) $N = 15$ 10 vermelhas (V)
5 pretas (P)

V_i: Vermelha na i-ésima extração

P_i: Preta na i-ésima extração

$P(V_1 V_4)$: $\underline{V_1}\ \underline{P_2}\ \underline{P_3}\ \underline{V_4}$: $\frac{10}{15} \cdot \frac{5}{14} \cdot \frac{4}{13} \cdot \frac{9}{12} +$

$\underline{V_1}\ \underline{P_2}\ \underline{V_3}\ \underline{V_4}$ ....

$V_1\ \underline{V_2}\ \underline{V_3}\ V_4$

$V_1\ V_2\ \underline{P_3}\ V_4$: $\frac{10}{15} \cdot \frac{9}{14} \cdot \frac{5}{13} \cdot \frac{9}{12}$

(22) $A \perp A \iff P(A)=0$ ou $P(A)=1$

$\Rightarrow P(A\cap A) = P(A)P(A) = [P(A)]^2 = P(A)$

$\Updownarrow$

$P(A)=0$ ou

$P(A)=1$

$\Leftarrow P(A)=0 \Rightarrow P(A\cap A) = P(A).P(A) = 1$

$P(A)=1 \Rightarrow P(A\cap A) = P(A).P(A) = 1$

(23) W: ~~Ap~~ Aprovar teste escrito

R: Aprovar teste de direção

Aprovar na n-ésima tentativa

a) Aprovar na

X_1: Nº de testes escritos

X_2: Nº de testes de direção.

$X_1 + X_2 = n$

$P(W_i) = 0{,}9 = p$ (probabilidade de Aprovar o teste escrito)

$P(R_i) = 0.8 = q$ (o teste de Direção)

p/ aprovar na n-ésima tentativa $(n>2)$ temos que se i representa o Nº de testes escrito reprovado e j o nº de testes de direção reprovados, então

$i + j = n-2$ [pois necessariamente aprova 1 vez c/ teste]

$j = n-2-i$

$$P(X_1+X_2=n) = pq \sum_{i=0}^{n-2} (1-p)^i (1-q)^{n-2-i}$$

$$= pq \sum_{i=0}^{n-2} (1-p)^i (1-q)^{n-2} \cdot (1-q)^{-i}$$

23) ~~cont.~~

$$= pq\cdot(1-q)^{n-2}\sum_{i=0}^{n-2}\left(\frac{1-p}{1-q}\right)^{i}$$

b) para $n=5$

$$p(X_1+X_2=5) = pq\sum_{i=0}^{5-2}(1-q)^{3}\left(\frac{1-p}{1-q}\right)^{i}$$

$$= pq(1-q)^{3}\left[\left(\frac{1-p}{1-q}\right)^{0}+\left(\frac{1-p}{1-q}\right)^{1}+\left(\frac{1-p}{1-q}\right)^{2}+\left(\frac{1-p}{1-q}\right)^{3}\right]$$

$$= pq\left[(1-q)^{3}+(1-p)(1-q)^{2}+(1-p)^{2}(1-q)+(1-p)^{3}\right]$$

24) $100X$: % de alcool

$0<X<1 \sim f(x) = 20x^{3}(1-x)$

a) $F(a) = \int_{0}^{a} f(x)\,dx = \int_{0}^{a}(20x^{3}-20x^{4})\,dx$

$$= \frac{20x^{4}}{4}\Big|_{0}^{a} - \frac{20x^{5}}{5}\Big|_{0}^{a} = a^{4}(5-4a)$$

b) $p(X\leq 2/3) = \left(\frac{2}{3}\right)^{4}\left(5-4\left(\frac{2}{3}\right)\right)$

c) $U: \begin{cases} c_1-c_3 & \frac{1}{3}<X<\frac{2}{3} \\ c_2-c_3 & 0<X\leq\frac{1}{3} \text{ or } \frac{2}{3}<X<1 \end{cases}$

$$p(U=c_1-c_3) = \int_{1/3}^{2/3} f(x)\,dx = F\left(\tfrac{2}{3}\right)-F\left(\tfrac{1}{3}\right)$$

$$p(U=c_2-c_3) = \int_{0}^{1/3} f(x)\,dx + \int_{2/3}^{1} f(x)\,dx = F(1/3)+\left(1-F\left(\tfrac{2}{3}\right)\right)$$

25: $X \sim U.a \quad f(x)$:

$$f(x) = \begin{cases} ax & 0 \le x \le 1 \\ a & 1 \le x \le 2 \\ -ax + 3x^2 & 2 \le x \le 3 \end{cases}$$

a) determinar a.

$$\int_{-\infty}^{\infty} f(x)dx = 1 \Rightarrow \int_0^1 ax\,dx + \int_1^2 a\,dx + \int_2^3 (-ax+3x^2)dx = 1$$

$$\Rightarrow a\frac{x^2}{2}\Big|_0^1 + ax\Big|_1^2 + \frac{3x^3}{3} - a\frac{x^2}{2}\Big|_2^3 = 1$$

$$\Rightarrow \frac{a}{2} + \frac{2a}{2} + \frac{81}{3} - \frac{9a}{2} - \frac{24}{3} + \frac{4a}{2} = 1$$

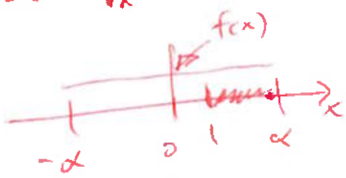
$$-\frac{2a}{2} = 1 + \frac{24}{3} - \frac{81}{3}$$

$$-a = -\frac{54}{3} \Rightarrow a = \frac{54}{3}$$

b) $F(x) = \int_{-\infty}^{x} f(x)dx$ $\quad F(b) = \begin{cases} \int_0^b ax\,dx & 0 \le x < b \\ \int_0^1 ax\,dx + \int_1^b a\,dx & 1 \le x < b \\ \int_0^1 ax\,dx + \int_1^2 a\,dx + \int_2^b (-ax+3x^2)dx & 2 \le x < b \\ 1 & x \ge 3 \end{cases}$

26: $X \sim U[-\alpha, \alpha]$, $\alpha > 0$

a) det α p/ $P(X>1) = 1/3$



$f(x) = \frac{1}{2\alpha} \quad -\alpha \le x \le \alpha$

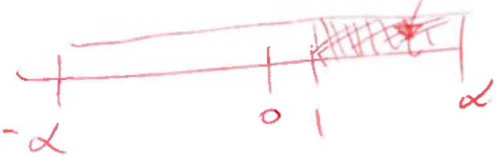
$$P(X>1) = \int_1^{\alpha} \frac{1}{2\alpha}dx = \frac{1}{3}$$

$$\Rightarrow \frac{1}{2\alpha}x\Big|_1^{\alpha} = \frac{(\alpha-1)}{2\alpha} = \frac{1}{3}$$

de onde $\frac{1}{2} - \frac{1}{2\alpha} = \frac{1}{3} \Rightarrow \alpha = 3$

26) cont.

b) $p(x > 1) = \frac{1}{2}$



não é possível ter um valor α conhecido

c) $p(x < \frac{1}{2}) = 0,7$

$$\int_{-\alpha}^{1/2} \frac{1}{2\alpha}\,dx = \frac{x}{2\alpha}\Big|_{-\alpha}^{1/2} = \frac{1/2+\alpha}{2\alpha} = 0,7$$

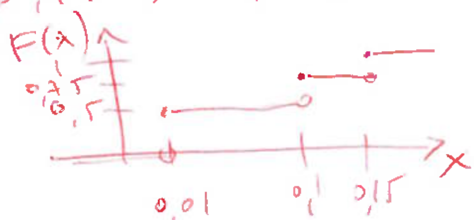
$$\Rightarrow \frac{1}{4\alpha} + \frac{1}{2} = 0,7 \Rightarrow \alpha = \frac{5}{4}$$

27) $X \sim F$:

$F(0,01) - F(0,01^{(-)}) = 0,5$

$F(0,1) - F(0,1^{(-)}) = 0,25$

$F(0,15) - F(0,15^{(-)}) = 0,25$



a) $p(X=x) = \begin{cases} 0,50 & x = 0,01 \\ 0,25 & x = 0,1 \\ 0,25 & x = 0,15 \end{cases}$

b) $E(X) = 0,01 * 0,5 + 0,1 * 0,25 + 0,15 * 0,25$

c)

28: $X \sim F(x)$:

$$F(x) = \begin{cases} 0 & x < 10 \\ 0,2 & 10 \le x < 12 \\ 0,5 & 12 \le x < 13 \\ 0,9 & 13 \le x < 25 \\ 1 & x \ge 25 \end{cases}$$

a) f.p. de x : $f_p(x) \Rightarrow$

$X = \{10, 12, 13, 25\}$

X	10	12	13	25
$p(X=x)$	0,2	0,3	0,4	0,1

b) $P(X = 12) = 0,3$

c) $P(12 \le X \le 20) = 0,7$

d) $E(X) = 10 \times 0,2 + 12 \times 0,3 + 13 \times 0,4 + 25 \times 0,1$

29: $E(X) = 1$ e $Var(X) = 5$ $\left(Var(X) = E(X^2) - (E(X))^2 \right)$

a) $E(2 + X^2) = 2 + E(X^2) = 2 + Var(X) + (E(X))^2$

$= 2 + 5 + 1^2 = 8$

b) $Var(4 + 3X) = 3^2 Var(X) = 9 \cdot 5 = 45$

30: $X \sim f(x) = 0,1\, e^{-0,1x}$ $x > 0$

a) Garantia 1 ano $\Rightarrow$ $P(X < 1)$ é de componentes trocados

$$P(X < 1) = \int_0^1 0,1\, e^{-0,1x}\, dx = -e^{-0,1x} \Big|_0^1 =$$

$$= 1 - e^{-0,1}$$

b) $F(a) = P(X \le a) = \int_{-\infty}^{a} f(x)\, dx = \int_{-\infty}^{a} 0,1\, e^{-0,1x}\, dx$

$$= \int_0^a 0,1\, e^{-0,1x}\, dx = -e^{-0,1x} \Big|_0^a = 1 - e^{-0,1a}$$

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30) cont.

d) $g(x) = \begin{cases} -100 & x \le 1 \\ 200 & x > 1 \end{cases}$

$$E(g(x)) = \int_{-\infty}^{\infty} g(x) f(x)\, dx = \int_0^1 -100 \cdot 0{,}1\, e^{-0{,}1x} dx + \int_1^{\infty} 200 \cdot 0{,}1\, e^{-0{,}1x} dx$$

$$= -100 \cdot \left[-e^{-0{,}1x} \Big|_0^1 \right] + 200 \left[-e^{-0{,}1x} \Big|_1^{\infty} \right]$$

$$= -100 \left[1 - e^{-0{,}1} \right] + 200 \left[e^{-0{,}1} - 0 \right]$$

$$= 300 e^{-0{,}1} - 100$$

e) $P(X > t+h \mid X > t) = P(X > h) \quad \forall h > 0 \quad \forall t > 0$

$$\frac{P(X > t+h \cap X > t)}{P(X > t)} = \frac{P(X > t+h)}{P(X > t)} = \frac{e^{-\lambda(t+h)}}{e^{-\lambda t}} = e^{-\lambda h}$$

pois $P(X > a) = 1 - F(a) = 1 - \int_0^a \lambda e^{-\lambda x} dx$

$$= 1 - \left[-e^{-\lambda x} \Big|_0^a \right] = 1 - \left[1 - e^{-\lambda a} \right]$$

$$= \boxed{e^{-\lambda a}}$$

31). Voltagem $X_t = a\cos(\omega t + \Theta)$

a e ω constantes e

$\Theta \sim U[-\pi, \pi]$

calcular $E(X_t)$.

lembre que X_t é função de Θ $\therefore$ $E(X_t) = \int x_t f_\Theta$

$$f(\theta) = \begin{cases} \frac{1}{2\pi} & -\pi \le \theta \le \pi \\ 0 & \text{c.c.} \end{cases}$$

$$E(X_t) = \int_{-\pi}^{\pi} a\cos(\omega t + \theta)\frac{1}{2\pi}\, d\theta = \frac{a}{2\pi}\int_{-\pi}^{\pi} \cos(\omega t + \theta)\, d\theta$$

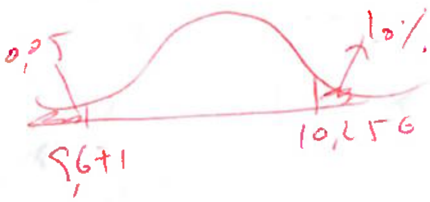
$$= \frac{a}{2\pi}\sin(\omega t + \theta)\Big|_{-\pi}^{\pi} = \frac{a}{2\pi}\left[\sin(\omega t + \pi) - \sin(\omega t - \pi)\right]$$

$= 0$.

32) $X \sim N$.

$P(X > 10{,}256) = 0{,}1$

$P(X < 9{,}671) = 0{,}05$



$P(X > 10{,}256) = P\left(Z > \frac{10{,}256 - \mu}{\sigma}\right) = 0{,}1$

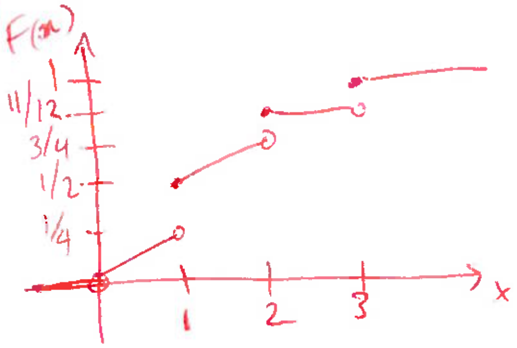
de onde (1) $z_1 = 1{,}28 = \frac{10{,}256 - \mu}{\sigma}$

$P(X < 9{,}671) = P\left(Z < \frac{9{,}671 - \mu}{\sigma}\right)$

(2) $z_2 = -1{,}64 = \frac{9{,}671 - \mu}{\sigma}$

de (1) e (2) a solução.

33) $$F(b) = \begin{cases} 0 & b < 0 \\ \frac{b}{4} & 0 \le b < 1 \\ \frac{1}{2} + \frac{b-1}{4} & 1 \le b < 2 \\ \frac{11}{12} & 2 \le b < 3 \\ 1 & b \ge 3 \end{cases}$$



a) p/ os pontos $x = i$, $i = 1, 2, 3$

X	1	2	3
$P(X)$	$1/4$	$\left(\frac{11}{12} - \frac{3}{4}\right)$	$\frac{1}{12}$

b) $P\left(\frac{1}{2} < X < \frac{3}{2}\right) = F\left(\frac{3}{2}\right) - F\left(\frac{1}{2}\right) = \frac{1}{2} + \frac{1}{8} - \frac{1}{8} = 1/2$