

$$2) \int \frac{x^2 - 5x + 16}{(2x+1)(x-2)^2} dx =$$

$$\frac{x^2 - 5x + 16}{(2x+1)(x-2)^2} = \frac{A}{x-2} + \frac{B}{(x-2)^2} + \frac{C}{2x+1} = \frac{A(2x+1)(x-2) + B(2x+1) + C(x-2)^2}{(2x+1)(x-2)^2}$$

$$= \frac{2Ax^2 - 3Ax - 2A + 2Bx + B + Cx^2 - 4Cx + 4C}{(2x+1)(x-2)^2}$$

$$= \frac{x^2(2A+C) + x(-3A+2B-4C) + B-2A+4C}{(x-2)^2(2x+1)} = \frac{x^2 - 5x + 16}{(x-2)^2(2x+1)}$$

$$\begin{cases} 2A+C=1 \\ -3A+2B-4C=-5 \\ B-2A+4C=16 \end{cases} \quad \begin{cases} A=-1 \\ B=2 \\ C=3 \end{cases}$$

Substituindo na integral:

$$\int \frac{x^2 - 5x + 16}{(2x+1)(x-2)^2} = \int \frac{-1}{x-2} dx + \int \frac{2}{(x-2)^2} dx + \int \frac{3}{2x+1} dx$$

$$= - \int \frac{1}{x-2} dx + 2 \int \frac{1}{(x-2)^2} dx + 3 \int \frac{1}{2x+1} dx = -\ln|x-2| + \frac{2}{x-2} + \frac{3}{2} \ln|2x+1| + C$$

$$\textcircled{1} \quad u = x-2 \\ du = 1 dx \Rightarrow \int \frac{1}{u^2} du = -\frac{1}{u} = -\frac{1}{x-2} + C$$

$$\textcircled{2} \quad u = 2x+1 \\ du = 2 dx \Rightarrow \int \frac{1}{u} \frac{du}{2} = \frac{1}{2} \int \frac{1}{u} du = \frac{1}{2} \ln|2x+1| + C$$

$$1) \int \frac{x^2 + 2x - 1}{x^3 - x}$$

$$\textcircled{1} \quad \frac{x^2 + 2x - 1}{x^3 - x} = \frac{x^2 + 2x - 1}{x(x^2 - 1)} = \frac{x^2 + 2x - 1}{x(x-1)(x+1)} =$$

$$= \frac{A}{x} + \frac{B}{x+1} + \frac{C}{x-1} = \frac{A(x-1)(x+1) + B(x-1)x + C(x+1)x}{(x+1)(x-1)x}$$

$$\frac{A(x^2 - 1) + B(x^2 - x) + C(x^2 + x)}{(x+1)(x-1)x} = \frac{Ax^2 - A + Bx^2 - Bx + Cx^2 + Cx}{(x+1)(x-1)x} =$$

$$\frac{x^2(A+B+C) + x(C-B) - A}{(x+1)(x-1)x} = \frac{x^2 + 2x - 1}{(x+1)(x-1)x}$$

$$\begin{cases} A+B+C=1 \\ C-B=2 \\ -A=-1 \end{cases} \Rightarrow \begin{matrix} A=1 \\ B=-1 \\ C=1 \end{matrix}$$

2) Substituindo na integral:

$$\int \frac{x^2 + 2x - 1}{x^3 - x} = \int \frac{1}{x} + \int \frac{-1}{x+1} + \int \frac{1}{x-1} =$$

$$\ln|x| - \int \frac{1}{x+1} + \int \frac{1}{x-1} = \ln|x| - \ln|x+1| + \ln|x-1| + C$$

RESPOSTA

$$\int \frac{1}{x+1} = \int \frac{1}{u} du =$$

$$u = x+1$$

$$du = 1 dx$$

$$\ln|u| du = \ln|x+1| + C$$