

$$1) f(x,y) = \frac{x}{x+y} \quad (2,1) \quad g(x,y) = \sqrt{x+e^{4y}} = (x+e^{4y})^{\frac{1}{2}} \quad (3,0)$$

$$a) \frac{\partial f}{\partial x} = \frac{1(x+y) - x(1)}{(x+y)^2} = \frac{x+y-x}{(x+y)^2} = \frac{y}{(x+y)^2}$$

$$\frac{\partial f}{\partial y} = \frac{0(x+y) - x(1)}{(x+y)^2} = -\frac{x}{(x+y)^2}$$

$$\frac{\partial f}{\partial x} = \frac{1}{2\sqrt{x+e^{4y}}} \cdot (1+0) = \frac{1}{2\sqrt{x+e^{4y}}}$$

$$\frac{\partial g}{\partial y} = \frac{1}{2\sqrt{x+e^{4y}}} \cdot (0+e^{4y} \cdot 4) = \frac{2e^{4y}}{\sqrt{x+e^{4y}}}$$

$$2) z - z_0 = f_x(x_0, y_0)(x - x_0) + f_y(x_0, y_0)(y - y_0), \text{ sendo } z = f(x, y)$$

$$z_0 = \frac{2}{2+1} = \frac{2}{3}$$

$$f_x(2,1) = \frac{1}{(2+1)^2} = \frac{1}{3^2} = \frac{1}{9}$$

$$f_y(2,1) = \frac{-2}{(2+1)^2} = \frac{-2}{3^2} = \frac{-2}{9}$$

$$z - \frac{2}{3} = \frac{1}{9} \cdot (x-2) - \frac{2}{9} \cdot (y-1) \Rightarrow z = \frac{x-2}{9} - \frac{2(y-1)}{9} + \frac{2}{3} \Rightarrow$$

$$z = \frac{(x-2) - 2(y-1) + 6}{9}$$

$$z - z_0 = g_x(x_0, y_0)(x - x_0) + g_y(x_0, y_0)(y - y_0), \text{ sendo } z = g(x, y)$$

$$z_0 = \sqrt{3+e^{4 \cdot 0}} = \sqrt{4} = 2 \quad f_x(3,0) = \frac{1}{2\sqrt{3+e^{4 \cdot 0}}} = \frac{1}{2\sqrt{4}} = \frac{1}{4} \quad f_y(3,0) = \frac{2 \cdot e^{4 \cdot 0}}{\sqrt{3+e^{4 \cdot 0}}} = \frac{2 \cdot 1}{\sqrt{4}} = \frac{2}{2} = 1$$

$$z - 2 = \frac{1}{4}(x-3) + 1(y-0) \Rightarrow z - 2 = \frac{x-3}{4} + y \Rightarrow$$

$$z = \frac{x-3}{4} + y + 2$$

$$c) L(x,y) = f(x_0, y_0) + f_x(x_0, y_0)(x - x_0) + f_y(x_0, y_0)(y - y_0), \text{ usando os resultados do item "b", temos:}$$

$$L_f(x,y) = \frac{2}{3} + \frac{1}{9}(x-2) - \frac{2}{9}(y-1) = \frac{(x-2) - 2(y-1) + 6}{9}$$

$$L_g(x,y) = 2 + \frac{1}{4}(x-3) + 1(y-0) = \frac{x-3}{4} + y + 2$$