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1] $\int \frac{x^2+2x-1}{x^3-x}\,dx$

$P(x) = \frac{x^2+2x-1}{x(x^2-1)}$

$= \frac{A}{x} + \frac{B}{x^2-1}$

$= \frac{A(x^2-1)+Bx}{x(x^2-1)}$

$\Rightarrow A(x^2-1)+Bx = x^2+2x-1$

$Ax^2 - A + Bx$

$= Ax^2 + Bx - A = 1x^2 + 2x - 1$

$\begin{cases} A = 1 \\ B = 2 \end{cases}$

$\therefore \frac{x^2+2x-1}{x^3-x} = \frac{1}{x} + \frac{2}{x^2-1}$

$\int \frac{1}{x} + \frac{2}{x^2-1}\,dx$

$= \int \frac{1}{x}\,dx + 2\cdot\int \frac{1}{x^2-1}\,dx$

$= \ln|x| + 2\cdot\int Q(x)\,dx$

onde

$Q(x) = \frac{1}{x^2-1}$

$Q(x) = \frac{1}{x^2-1} = \frac{1}{(x-1)(x+1)}$

$= \frac{A}{(x-1)} + \frac{B}{(x+1)}$

$\Rightarrow A(x+1) + B(x-1) = 1$

$Ax + A + Bx - B = 1$

$Ax + Bx + A - B = 1$

$\begin{cases} A + B = 0 \\ A - B = 1 \end{cases} \Rightarrow \begin{cases} A = +\frac{1}{2} \\ B = -\frac{1}{2} \end{cases}$

$\therefore \frac{1}{x^2-1} = \frac{+\frac{1}{2}}{x-1} + \frac{-\frac{1}{2}}{x+1}$

$\int \frac{1}{x^2-1}\,dx = \int \frac{1}{2}\cdot\frac{1}{x-1} - \frac{1}{2}\cdot\frac{1}{x+1}\,dx$

$= \frac{1}{2}\int -\frac{1}{x+1} + \frac{1}{x-1}\,dx$

$= \frac{1}{2}\left(-\int \frac{1}{x+1}\,dx + \int \frac{1}{x-1}\,dx\right)$

$= \frac{2}{2}\cdot\left(-\ln|x+1| + \ln|x-1|\right)$

$\therefore \int \frac{x^2+2x-1}{x^3-x}\,dx$

$= \ln|x| - \ln|x+1| + \ln|x-1| + C$

2] $\int \frac{x^2-5x+16}{(2x+1)(x-2)^2}\,dx$

$\frac{P(x)}{Q(x)} = \frac{A}{2x+1} + \frac{B}{(x-2)^2} + \frac{C}{(x-2)}$

$= \frac{A(x-2)^2+B(2x+1)+C(x-2)(2x+1)}{(2x+1)(x-2)^2}$

$\Rightarrow x^2-5x+16$
$= A(x-2)^2+B(2x+1)+C(x-2)(2x+1)$

$A(x^2-4x+4)+B(2x+1)+C(2x^2-3x-2)$
$= Ax^2-4Ax+4A+2Bx+B+2Cx^2-3Cx-2C$

$= Ax^2+2Cx^2$
$+ -4Ax+2Bx-3Cx$
$+ 4A+B-2C$

$= 1x^2-5x+16$

$\therefore \begin{cases} A + 2C = 1 \\ -4A+2B-3C = -5 \\ 4A+B-2C = 16 \end{cases}$

$* \ A = 1-2C$

$\begin{cases} -4\cdot(1-2C)+2B-3C = -5 \\ 4\cdot(1-2C)+B-2C = 16 \end{cases}$

$\Leftrightarrow \begin{cases} 5C+2B = -1 \\ -10C+B = 12 \end{cases} + 2x$

$\Leftrightarrow \begin{cases} 5C+2B = -1 \\ 0 + 5B = 10 \end{cases}$

$B = 2$

$C = -1$

$A = 3$

$\therefore \frac{P(x)}{Q(x)} = \frac{3}{2x+1} + \frac{2}{(x-2)^2} - \frac{1}{x-2}$

$\int \frac{3}{2x+1}\,dx + \int \frac{2}{(x-2)^2}\,dx - \int \frac{1}{x-2}\,dx$

$= \int \frac{3}{2} u^{-1}\,du + \int \frac{2}{u^2}\,du - \int \frac{1}{u}\,du$

$= \frac{3}{2} \cdot \ln|2x+1| - \frac{2}{x-2} - \ln|x-2|$

$\therefore \int \frac{x^2-5x+16}{(2x+1)(x-2)^2}\,dx$

$= \frac{3}{2}\ln|2x+1| - \ln|x-2| - \frac{2}{x-2}$

$+ C$

Credeal