* 1) $\int \frac{x^2+2x-1}{x^3-x}\,dx = \int \frac{x^2+2x-1}{x(x^2-1)}\,dx = \int \frac{1}{x} + \frac{1}{x-1} - \frac{1}{x+1}\,dx =$

$\frac{x^2+2x-1}{x(x^2-1)} = \frac{x^2+2x-1}{x(x-1)(x+1)} = \frac{A}{x} + \frac{B}{x-1} + \frac{C}{x+1} = \frac{\overset{Ax^2-A}{A(x-1)(x+1)}}{x(x+1)(x-1)} + \frac{\overset{Bx^2+Bx}{Bx(x+1)}}{x(x+1)(x-1)} +$

$+ \frac{\overset{Cx^2-Cx}{Cx(x-1)}}{x(x+1)(x-1)} = \frac{(A+B+C)x^2+(B-C)x-A}{x(x+1)(x-1)} = \frac{1}{x} + \frac{1}{x-1} - \frac{1}{x+1}$

$\left|\begin{array}{ll} A+B+C & C=-1 \\ B-C=2 & B=1 \\ A=1 & \end{array}\right.$

$\Rightarrow \int \frac{1}{x}dx + \int \frac{1}{x-1}\,dx - \int \frac{1}{x+1}\,dx$ $\begin{array}{l} u=x+1 \\ dx=du \\ z=x-1 \\ dx=dz \end{array}$ $= \ln x + \int \frac{1}{z}dz - \int \frac{1}{u}du =$

$= \ln x + \ln z - \ln u = \ln(x) + \ln(x-1) - \ln(x+1) + C$

* 2) $\int \frac{x^2-5x+16}{(2x+1)(x-2)^2}\,dx = \int \frac{3}{2x+1} + \frac{2}{(x-2)^2} - \frac{1}{x-2}\,dx = 3\int \frac{1}{2x+1}\,dx + 2\int \frac{1}{(x-2)^2}\,dx - 1\int \frac{1}{x-2}\,dx =$

$\frac{x^2-5x+16}{(2x+1)(x-2)^2} = \frac{A}{2x+1} + \frac{B}{(x-2)^2} + \frac{C}{x-2} \Rightarrow x^2-5x+16 = A(x-2)^2 + B(2x+1) + C(2x+1)(x-2)$

$\Rightarrow Ax^2 - 4Ax + 4A + 2Bx + B + 2Cx^2 - 3Cx - 2C = (A+2C)x^2 - (4A-2B+3C)x + 4A+B-2C$

$\left|\begin{array}{ll} A+2C=1 & A=3 \\ 4A-2B+3C=5 & B=2 \\ 4A+B-2C=16 & C=-1 \end{array}\right.$ $\left\{\begin{array}{lll} u=2x+1 & z=(x-2) & p=(x-2) \\ du=2\,dx & dz=dx & dp=dx \\ \frac{du}{dx}=2x+1 & \frac{dz}{dx}\ x-2 & \end{array}\right.$

$= \frac{3}{2}\int \frac{2}{u}\,du + 2\int \frac{1}{z^2}dz - \int \frac{1}{p}dp = \frac{3}{2}\ln(u) + 2\cdot\frac{1}{z} - \ln(p) =$

$= \frac{3}{2}\ln(2x+1) - \frac{2}{x-2} - \ln(x-2) + C$