Dúvidas

Ex 2.1.6 Resolução:

2.1.6

$A = PDP^{-1}$ $\quad A^2 = (PDP^{-1})\cdot(PDP^{-1}) \Rightarrow A^2 = (P\cdot D)\overbrace{P^{-1}\cdot P}^{I}(DP^{-1})$

$A^2 = P\cdot D\cdot I\cdot D\cdot P^{-1} \Rightarrow A^2 = P\cdot D^2\cdot P^{-1}$ $\quad A^3 = A^2\cdot A = PD^2\overbrace{P^{-1}P}^{I}DP^{-1}$

$A^3 = PD^3P^{-1}$ assim para todo $K \in \mathbb{Z}$, então $A^K =$

$$= PD^KP^{-1} = \begin{bmatrix} 1 & 1 \\ -2 & 2 \end{bmatrix}\begin{bmatrix} 3^K & 0 \\ 0 & (-1)^K \end{bmatrix}\begin{bmatrix} 1 & 1 \\ -2 & 2 \end{bmatrix}^{-1} =$$

$$\begin{bmatrix} 1 & 1 \\ -2 & 2 \end{bmatrix}\begin{bmatrix} 3^K & 0 \\ 0 & (-1)^K \end{bmatrix}\begin{bmatrix} 1/2 & \frac{-1}{4} \\ 1/2 & 1/4 \end{bmatrix} = \begin{bmatrix} 3^K & (-1)^K \\ -2\cdot 3^K & 2\cdot(-1)^K \end{bmatrix}\cdot\begin{bmatrix} 1/2 & \frac{-1}{4} \\ 1/2 & 1/4 \end{bmatrix} =$$

$$= \begin{bmatrix} \frac{(-1)^K+3^K}{2} & \frac{(-1)^K-3^K}{4} \\ (-1)^K-3^K & \frac{(-1)^K+3^K}{2} \end{bmatrix} = A^K \text{ para } K = 1, 2, 3\ldots$$

Ex 2.1.11
Resolução:

$$(I_n - A)^{-1}$$

$$(I_n - A)\cdot(I_n + A + A^2 + \dots + A^{k-1}) = I_n$$ → tentar provar

$$(I_n - A)\cdot I_n + (I_n - A)\cdot A + \dots + (I_n - A)\cdot A^{k-1}$$

$$(\overbrace{I_n^2}^{I} - \overbrace{AI_n}^{A}) + (\overbrace{I_nA}^{A} - A^2) + (\overbrace{I_nA^2}^{A^2} - A^3) + \dots (I_nA^{k-1} - A^k) =$$

$$I_n \underbrace{- A + A}_{0} \underbrace{- A^2 + A^2}_{0} \underbrace{- A^3}_{0} \dots \underbrace{+ A^{k-1}}_{0} - A^k = I_n - \underset{0}{A^k} = I_n$$

então $(I_n - A^{-1}) = (I_n + A + A^2 + \dots + A^{k-1})$

Ex 2.1.13:

2.1.13 A invertível e A+B invertível

$$(A+B)^{-1} = A^{-1}(I_n + BA^{-1})^{-1}$$

$$(A+B)\cdot A^{-1}(I_n+BA^{-1})^{-1} = I_n \longrightarrow \text{provar}$$

$$(A\cdot A^{-1} + BA^{-1})\cdot(I_n+BA^{-1})^{-1} \Rightarrow \underbrace{(I_n+BA^{-1})}_{X}\cdot\underbrace{(I_n+BA^{-1})^{-1}}_{X^{-1}}$$

$X\cdot X^{-1} = I_n$ então $A^{-1}(I_n+BA^{-1})^{-1}$ é inversa de $(A+B)$

Ex 2.2.6 c e d:

2.2.6) $\det(A - \lambda I_n) = 0$

c)

$$A = \begin{bmatrix} 2 & -2 & 3 \\ 0 & 3 & -2 \\ 0 & -1 & 2 \end{bmatrix} \quad A - \lambda I_n = \begin{bmatrix} 2 & -2 & 3 \\ 0 & 3 & -2 \\ 0 & -1 & 2 \end{bmatrix} - \begin{bmatrix} \lambda & 0 & 0 \\ 0 & \lambda & 0 \\ 0 & 0 & \lambda \end{bmatrix} =$$

$$= \begin{bmatrix} 2-\lambda & -2 & 3 \\ 0 & 3-\lambda & -2 \\ 0 & -1 & 2-\lambda \end{bmatrix}$$ usando a propriedo transpomos a

matriz $$\begin{bmatrix} 2-\lambda & 0 & 0 \\ -2 & 3-\lambda & -1 \\ 3 & -2 & 2-\lambda \end{bmatrix}$$ faremos o cofator da 1ª linha

$$\det(A - \lambda I_n) = (2-\lambda) \det \begin{bmatrix} 3-\lambda & -1 \\ -2 & 2-\lambda \end{bmatrix} =$$

$$= (2-\lambda)\left[(2-\lambda)\cdot(3-\lambda) - 2\right] = (2-\lambda)(\lambda^2 - 5\lambda + 4) \rightarrow$$

fatorando $\rightarrow -(\lambda-4)(\lambda-2)(\lambda-1) = 0 \quad \lambda = 4, \ \lambda = 2$ ou $\lambda = 1$

2.26 d)

$$A-\lambda I_n = \begin{bmatrix} 2 & 2 & 3 \\ 1 & 2 & 1 \\ 2 & -2 & 1 \end{bmatrix} - \begin{bmatrix} \lambda & 0 & 0 \\ 0 & \lambda & 0 \\ 0 & 0 & \lambda \end{bmatrix} = \begin{bmatrix} 2-\lambda & 2 & 3 \\ 1 & 2-\lambda & 1 \\ 2 & -2 & 1-\lambda \end{bmatrix}$$

$$\det A-\lambda I_n = \det \begin{bmatrix} 2-\lambda & 2 & 3 \\ 1 & 2-\lambda & 1 \\ 2 & -2 & 1-\lambda \end{bmatrix} \overset{\text{troca}}{=} -\det \begin{bmatrix} 1 & 2-\lambda & 1 \\ 2-\lambda & 2 & 3 \\ 2 & -2 & 1-\lambda \end{bmatrix}$$

(1^a linh $\times (2-\lambda)$ e subtrai na 2^a)

$$-\det \begin{bmatrix} 1 & 2-\lambda & 1 \\ 0 & 2-(2-\lambda)^2 & 1+\lambda \\ 2 & -2 & 1-\lambda \end{bmatrix} \overset{L3-2L1}{=} -\det \begin{bmatrix} 1 & 2-\lambda & 1 \\ 0 & 2-(2-\lambda)^2 & 1+\lambda \\ 0 & -6+2\lambda & -(1+\lambda) \end{bmatrix}$$

Usando cofatores por col 1 $= -\left(1 \cdot \det \begin{bmatrix} 2-(2-\lambda)^2 & 1+\lambda \\ -6+2\lambda & -(1+\lambda) \end{bmatrix}\right)$

$$= -(1+\lambda) \det \begin{bmatrix} 2-(2-\lambda)^2 & 1 \\ -6+2\lambda & -1 \end{bmatrix} = -(\lambda-4)(\lambda-2)(\lambda+1)$$

$\det A-\lambda I_n = 0 \iff \lambda=4,\ \lambda=2$ ou $\lambda=-1$