

Derivação Logarítmica

$$\frac{d}{dx}(\ln(f(x))) = \frac{f'(x)}{f(x)}$$

$$\frac{d}{dx} \ln|x| = \frac{1}{x}$$

$$\ln(ab) = \ln(a) + \ln(b)$$

$$\ln(a^b) = b \ln(a)$$

$$\ln\left(\frac{a}{b}\right) = \ln(a) - \ln(b)$$

$$y = \frac{x^{3/2} \sqrt{x^2-1}}{(3x+1)^{10}}$$

$$\ln|y| = \ln\left|\frac{x^{3/2} \sqrt{x^2-1}}{(3x+1)^{10}}\right| = \ln|x^{3/2} \sqrt{x^2-1}| - \ln|(3x+1)^{10}|$$

$$\begin{aligned}\ln|y| &= \ln|x^{3/2}| + \ln|(x^2-1)^{1/2}| - 10 \ln|3x+1| \\ &= \frac{3}{2} \ln|x| + \frac{1}{2} \ln|x^2-1| - 10 \ln|3x+1|\end{aligned}$$

$$\frac{y'}{y} = \frac{3}{2} \frac{1}{x} + \frac{1}{2} \frac{2x}{x^2-1} - 10 \frac{3}{3x+1}$$

$$y' = \left(\frac{3}{2x} + \frac{x}{x^2-1} - \frac{30}{3x+1} \right) y$$

$$= \left(\frac{3}{2x} + \frac{x}{x^2-1} - \frac{30}{3x+1} \right) \frac{x^{3/2} \sqrt{x^2-1}}{(3x+1)^{10}}$$

$$y = x^n \quad \ln|y| = \ln|x^n| = n \ln|x|$$

$$\Rightarrow \frac{y'}{y} = n \frac{1}{x} \Rightarrow y' = \frac{n}{x} y = \frac{n}{x} x^n = n x^{n-1} \Rightarrow \underline{\underline{y' = n x^{n-1}}}$$

$$y = f(x)^{g(x)}$$

$$\ln |y| = \ln |f(x)^{g(x)}| = g(x) \ln |f(x)|$$

$$y = x^{\sqrt{x}}$$

$$\ln |y| = \ln |x^{\sqrt{x}}| = \sqrt{x} \ln |x|$$

$$\frac{y'}{y} = \frac{1}{2\sqrt{x}} \ln |x| + \sqrt{x} \cdot \frac{1}{x}$$

$$\Rightarrow y' = \left(\frac{1}{2\sqrt{x}} \ln |x| + \frac{\sqrt{x}}{x} \right) \cdot x^{\sqrt{x}}$$