ACH2011 - Cálculo I

Sistema de Informação - EACH

Regra do quociente: Se f e g são funções deriváveis, então

$$\frac{d}{dx}\left(\frac{f(x)}{g(x)}\right) = \frac{g(x)\frac{df(x)}{dx} - f(x)\frac{dg(x)}{dx}}{g(x)^2}.$$

Prova: Usando a definição de derivada

$$\frac{d}{dx}\left(\frac{f(x)}{g(x)}\right) = \lim_{h\to 0}\frac{\frac{f(x+h)}{g(x+h)} - \frac{f(x)}{g(x)}}{h}.$$

Vamos a trabalhar primeiro com o numerador

$$\frac{f(x+h)}{g(x+h)} - \frac{f(x)}{g(x)} = \frac{f(x+h)g(x) - f(x)g(x+h)}{g(x+h)g(x)}$$

$$= \frac{f(x+h)g(x) - g(x)f(x) + g(x)f(x) - f(x)g(x+h)}{g(x+h)g(x)} \qquad -f(x)$$

$$= \frac{g(x)\left(f(x+h) - f(x)\right) - f(x)\left(g(x+h) - g(x)\right)}{g(x+h)g(x)}$$

Portanto,

$$\frac{d}{dx}\left(\frac{f(x)}{g(x)}\right) = \lim_{h\to 0}\frac{\frac{f(x+h)}{g(x+h)} - \frac{f(x)}{g(x)}}{h}$$

$$\lim_{h\to 0}\frac{\frac{g(x)(f(x+h)-f(x))-f(x)(g(x+h)-g(x))}{g(x+h)g(x)}}{h}$$

$$= \lim_{h\to 0} \frac{g(x)\left(f(x+h)-f(x)\right) - f(x)\left(g(x+h)-g(x)\right)}{(g(x+h)g(x))h}$$

$$= \lim_{h\to 0} \frac{1}{g(x+h)g(x)} \left(\frac{g(x)\left(f(x+h)-f(x)\right) - f(x)\left(g(x+h)-g(x)\right)}{h} \right)$$

$$= \lim_{h\to 0} \frac{1}{g(x+h)g(x)} \left(\frac{g(x)(f(x+h)-f(x))}{h} - \frac{f(x)(g(x+h)-g(x))}{h} \right)$$

$\frac{ab}{c} = a \cdot \frac{b}{c}$

$\to g(x)$ $\qquad \to f'(x)$ $\qquad \to f(x)$ $\qquad \to g'(x)$

$$= \lim_{h\to 0} \underbrace{\frac{1}{g(x+h)g(x)}}_{\frac{1}{g(x)^2}} \left(g(x)\frac{f(x+h)-f(x)}{h} - f(x)\frac{g(x+h)-g(x)}{h} \right)$$

$$= \frac{g(x)\frac{df(x)}{dx} - f(x)\frac{dg(x)}{dx}}{g(x)^2}.$$

Teorema: Se f é contínua em b e $\lim_{x\to a} g(x) = b$, então

$$\lim_{x\to a} f(g(x)) = f(b)$$

$$\lim_{h\to 0} g(\underbrace{x+h}_{\xrightarrow{h\to 0} x}) = g(x)$$