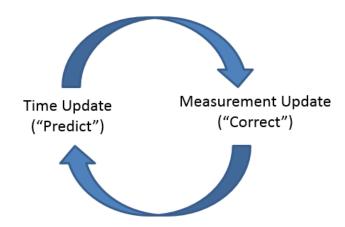
SUMMARY

We have derived all five Kalman Filter equations in matrix notation. Let us put them all together in a single page.

The Kalman Filter operates in a "predict – correct" loop, as shown in the diagram below.



Once initialized, the Kalman Filter will **predict** the system state at the next time step. It also provides the uncertainty of the prediction.

Once the measurement is received, the Kalman Filter updates (or corrects) the prediction and the uncertainty of the current state. As well, the Kalman Filter predicts the next states, and so on.

The following diagram provides a complete picture of the Kalman Filter operation.



Time Update ("Predict")

- 1. Extrapolate the state
 - $\widehat{\boldsymbol{x}}_{n+1,n} = \boldsymbol{F}\widehat{\boldsymbol{x}}_{n,n} + \boldsymbol{G}\boldsymbol{u}_n$
- 2. Extrapolate uncertainty

$$\boldsymbol{P}_{n+1,n} = \boldsymbol{F}\boldsymbol{P}_{n,n}\boldsymbol{F}^T + \boldsymbol{Q}$$

Measurement Update ("Correct")

1. Compute the Kalman Gain

$$K_n = P_{n,n-1}H^T (HP_{n,n-1}H^T + R_n)^{-1}$$

2. Update estimate with measurement

$$\widehat{x}_{n,n} = \widehat{x}_{n,n-1} + K_n(z_n - H\widehat{x}_{n,n-1})$$

3. Update the estimate uncertainty

$$\boldsymbol{P}_{n,n} = (\boldsymbol{I} - \boldsymbol{K}_n \boldsymbol{H}) \boldsymbol{P}_{n,n-1} (\boldsymbol{I} - \boldsymbol{K}_n \boldsymbol{H})^T + \boldsymbol{K}_n \boldsymbol{R}_n \boldsymbol{K}_n^T$$

Initial Estimate: $\widehat{m{x}}_{0,0}$, $m{P}_{0,0}$

	Equation	Equation Name	Alternative names
Predict	$\hat{x}_{n+1,n} = F\hat{x}_{n,n} + Gu_n$	State Extrapolation	Predictor Equation Transition Equation Prediction Equation Dynamic Model State Space Model
	$P_{n+1,n} = FP_{n,n}F^T + Q$	Covariance Extrapolation	Predictor Covariance Equation
Update (correction)	$\hat{x}_{n,n} = \hat{x}_{n,n-1} + K_n(z_n - H\hat{x}_{n,n-1})$	State Update	Filtering Equation
	$P_{n,n} = \left(I - K_n H\right) P_{n,n-1} \left(I - K_n H\right)^T + K_n R_n K_n^T$	Covariance Update	Corrector Equation
	$K_n = P_{n,n-1}H^Tig(HP_{n,n-1}H^T + R_nig)^{-1}$	Kalman Gain	Weight Equation
Auxiliary	$z_n = Hx_n$	Measurement Equation	
	$oldsymbol{R_n} = E\left(oldsymbol{v_n}oldsymbol{v_n^T} ight)$	Measurement Uncertainty	Measurement Error
	$oldsymbol{Q_n} = E\left(oldsymbol{w_n}oldsymbol{w_n^T} ight)$	Process Noise Uncertainty	Process Noise Error
	$oldsymbol{P_{n,n}} = E\left(oldsymbol{e_n} oldsymbol{e_n^T} ight) = E\left(\left(oldsymbol{x_n} - \hat{oldsymbol{x}}_{n,n} ight)\left(oldsymbol{x_n} - \hat{oldsymbol{x}}_{n,n} ight)^T ight)$	Estimation Uncertainty	Estimation Error

The next table summarizes notation (including differences found in the literature) and dimension.

Term	Name	Alternative term	Dimensions
$oldsymbol{x}$	State Vector		$n_x imes 1$
z	Output Vector	y	$n_z imes 1$
$oldsymbol{F}$	State Transition Matrix	Φ,A	$n_x imes n_x$
$oldsymbol{u}$	Input Variable		$n_u imes 1$
G	Control Matrix	B	$n_x imes n_u$
P	Estimate Uncertainty	Σ	$n_x imes n_x$
Q	Process Noise Uncertainty		$n_x imes n_x$
R	Measurement Uncertainty		$n_z imes n_z$
$oldsymbol{w}$	Process Noise Vector	y	$n_x imes 1$
$oldsymbol{v}$	Measurement Noise Vector		$n_z imes 1$
H	Observation Matrix	C	$n_z imes n_x$



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