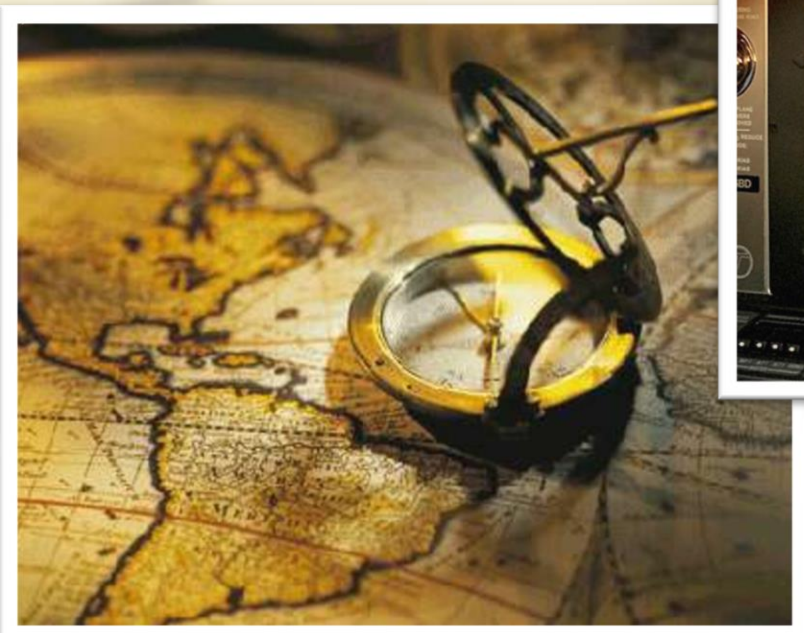
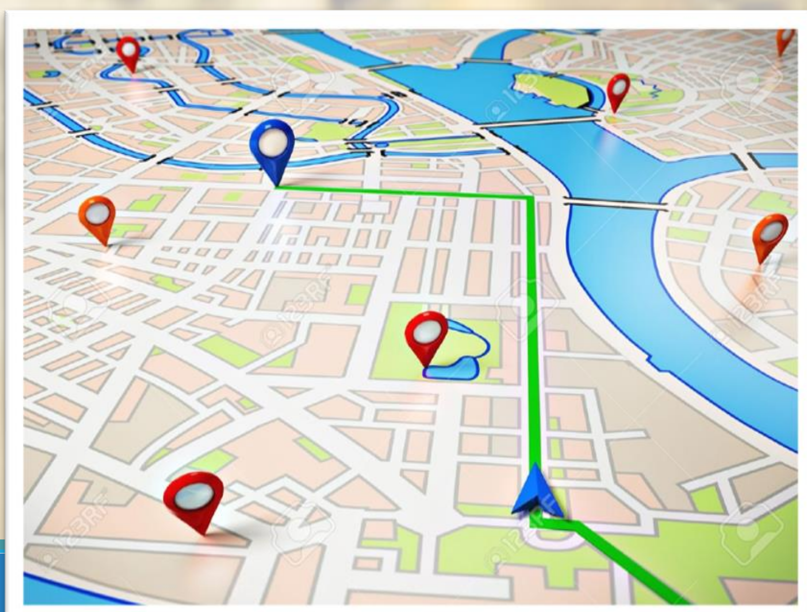




Lecture

KALMAN FILTERING

- INTEGRATION schemes-

Attitude and heading reference systems – AHRS
- Direct model -

$$\dot{\varphi} = \omega_x + \omega_y sin\varphi tan\theta + \omega_z cos\varphi tan\theta$$
$$\dot{\theta} = \omega_y cos\varphi - \omega_z sin\varphi$$
$$\dot{\psi} = \omega_y \frac{sin\varphi}{cos\theta} + \omega_z \frac{cos\varphi}{cos\theta}$$
$$\dot{b} = 0 + w$$

$$x = [\varphi \quad \theta \quad \psi \quad b_{\omega x} \quad b_{\omega y} \quad b_{\omega z}]^T$$
$$u = [\omega_x \quad \omega_y \quad \omega_z]^T$$
$$z = [f_x \quad f_y \quad f_z \quad m_x \quad m_y \quad m_z]^T$$

$$\begin{bmatrix} f_x \\ f_y \\ f_z \end{bmatrix}_{body} = 0 - C_n^b \times \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$$

$$m^b = [\varphi]_1 \times [\theta]_2 \times [\psi]_3 \times m^n \qquad \begin{bmatrix} c\theta & s\theta s\varphi & s\theta c\varphi \\ 0 & c\varphi & -s\varphi \\ -s\theta & c\theta s\varphi & c\theta c\varphi \end{bmatrix} m^b = \begin{bmatrix} c\psi & s\psi & 0 \\ -s\psi & c\psi & 0 \\ 0 & 0 & 1 \end{bmatrix} m^n$$

$m^n = [m_n, 0, m_d]^T$ the vector of the Earth magnetic field in NED

Indirect model – general analysis

$\tilde{\mathbf{y}}^b = \mathbf{r}^b + \mathbf{x}_y + \boldsymbol{\nu}_y$ measured position in B-frame, r^b – true position, x_y-deterministic sensor error, v_y – sensor noise

$\hat{\mathbf{y}}^n = \hat{\mathbf{R}}_b^n \left(\tilde{\mathbf{y}}^b - \hat{\mathbf{x}}_y\right)$ computed value of the measurement in N-frame

$$\begin{aligned}
\delta \mathbf{y}^n &= \mathbf{y}^n - \hat{\mathbf{y}}^n \\
&= \mathbf{y}^n - (\mathbf{I} - \mathbf{P})\mathbf{R}_b^n \left(\mathbf{r}^b + \mathbf{x}_y + \boldsymbol{\nu}_y - \hat{\mathbf{x}}_y\right) \\
&= \mathbf{P}\mathbf{r}^n - \mathbf{R}_b^n \left(\delta \mathbf{x}_y + \boldsymbol{\nu}_y\right) + \mathbf{P}\mathbf{R}_b^n \left(\delta \mathbf{x}_y + \boldsymbol{\nu}_y\right) \\
&= -\left[\mathbf{r}^n \times\right] \boldsymbol{\rho} - \mathbf{R}_b^n \left(\delta \mathbf{x}_y + \boldsymbol{\nu}_y\right)
\end{aligned}$$

TRUE value

$$\hat{\mathbf{R}}_b^n = (\mathbf{I} - \mathbf{P})\mathbf{R}_b^n$$

computed value

$$\mathbf{P} = [\boldsymbol{\rho}\times] = \begin{bmatrix} 0 & -\epsilon_D & \epsilon_E \\ \epsilon_D & 0 & -\epsilon_N \\ -\epsilon_E & \epsilon_N & 0 \end{bmatrix}$$

$$\begin{aligned}
\hat{\mathbf{R}}_n^b &= \mathbf{R}_n^b(\mathbf{I} + \mathbf{P}) \\
\mathbf{R}_n^b &= \hat{\mathbf{R}}_n^b(\mathbf{I} - \mathbf{P}) \\
\mathbf{R}_b^n &= (\mathbf{I} + \mathbf{P})\hat{\mathbf{R}}_b^n
\end{aligned}$$

$$\mathbf{P}\mathbf{r}^n = -\left[\mathbf{r}^n \times\right] \boldsymbol{\rho}$$

$\mathbf{P}\mathbf{R}_b^n \left(\delta \mathbf{x}_y + \boldsymbol{\nu}_y\right)$ dropped away, 2nd order in the error quantities

Attitude and heading reference systems – AHRS
- Indirect model -
- Accelerometer analysis -

$$\mathbf{y}_a = \mathbf{a}_{ib}^b - \mathbf{g}^b + \mathbf{x}_a + \boldsymbol{\nu}'_a \qquad \mathbf{y}_a = -\mathbf{g}^b + \mathbf{x}_a + \boldsymbol{\nu}_a \qquad \hat{\mathbf{g}}^b = \hat{\mathbf{x}}_a - \mathbf{y}_a$$

$$\delta\mathbf{g}^n = \mathbf{g}^n - \hat{\mathbf{g}}^n$$

$$\begin{aligned}
\delta\mathbf{g}^n &= \mathbf{g}^n - \hat{\mathbf{R}}_b^n \hat{\mathbf{g}}^b \\
&= \mathbf{g}^n - (\mathbf{I} - \mathbf{P})\mathbf{R}_b^n \left(\hat{\mathbf{x}}_a - \mathbf{y}_a\right) \\
&= \mathbf{g}^n + (\mathbf{P} - \mathbf{I})\mathbf{R}_b^n \left(\mathbf{g}^b - \delta\mathbf{x}_a - \boldsymbol{\nu}_a\right) \\
&= -[\mathbf{g}^n \times]\boldsymbol{\rho} + \mathbf{R}_b^n \left(\delta\mathbf{x}_a + \boldsymbol{\nu}_a\right) \\
&= \begin{bmatrix} 0 & g_e & 0 \\ -g_e & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \boldsymbol{\rho} + \mathbf{R}_b^n \left(\delta\mathbf{x}_a + \boldsymbol{\nu}_a\right)
\end{aligned}$$

$$\delta\mathbf{g}^n = \mathbf{H}_a \delta\mathbf{x} + \mathbf{R}_b^n \boldsymbol{\nu}_a$$

$$\mathbf{H}_a = \begin{bmatrix} [-\mathbf{g}^n \times] & \mathbf{0} & \mathbf{R}_b^n \end{bmatrix}$$

Attitude and heading reference systems – AHRS

- Indirect model -

- Magnetometer analysis -

$$\delta\mathbf{m}^n = \mathbf{m}^n - \hat{\mathbf{m}}^n \qquad \hat{\mathbf{m}}^n = \hat{\mathbf{R}}_b^n \mathbf{y}_m$$

$$\delta\mathbf{m}^n = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & m_e \\ 0 & -m_e & 0 \end{bmatrix} \boldsymbol{\rho} - \mathbf{R}_b^n \boldsymbol{\nu}_m$$

$$\delta\mathbf{m}^n = \mathbf{H}_m \delta\mathbf{x} - \mathbf{R}_b^n \boldsymbol{\nu}_m$$

$$\mathbf{H}_m = \begin{bmatrix} -[\mathbf{m}^n \times] & \mathbf{0} & \mathbf{0} \end{bmatrix} \longrightarrow \mathbf{H}_m = \begin{bmatrix} [0, 0, m_e] & \mathbf{0} & \mathbf{0} \end{bmatrix}$$

This model indicates that mag. measurement might allow calibration of $\epsilon_E$and ϵ_D. However, the accuracy of estimation of ϵ_E relies on the accuracy of the vertical component of $\boldsymbol{m}^n$which is location dependent. When position information is not available, the magnetometer measurement matrix will be reduced to Hm which will allow calibration of the yaw ϵ_D.

Attitude and heading reference systems – AHRS

- Indirect model -

- Dynamic model -

$$\dot{\hat{\mathbf{R}}}_b^n = \hat{\mathbf{R}}_b^n(\hat{\boldsymbol{\Omega}}_{ib}^b - \hat{\boldsymbol{\Omega}}_{in}^b) \qquad \hat{\boldsymbol{\Omega}}_{ib}^b = [\hat{\boldsymbol{\omega}}_{ib}^b \times] \quad \hat{\boldsymbol{\omega}}_{ib}^b = \mathbf{u} - \hat{\mathbf{x}}_g$$

for AHRS

$$\hat{\boldsymbol{\omega}}_{in}^n = \mathbf{0}$$

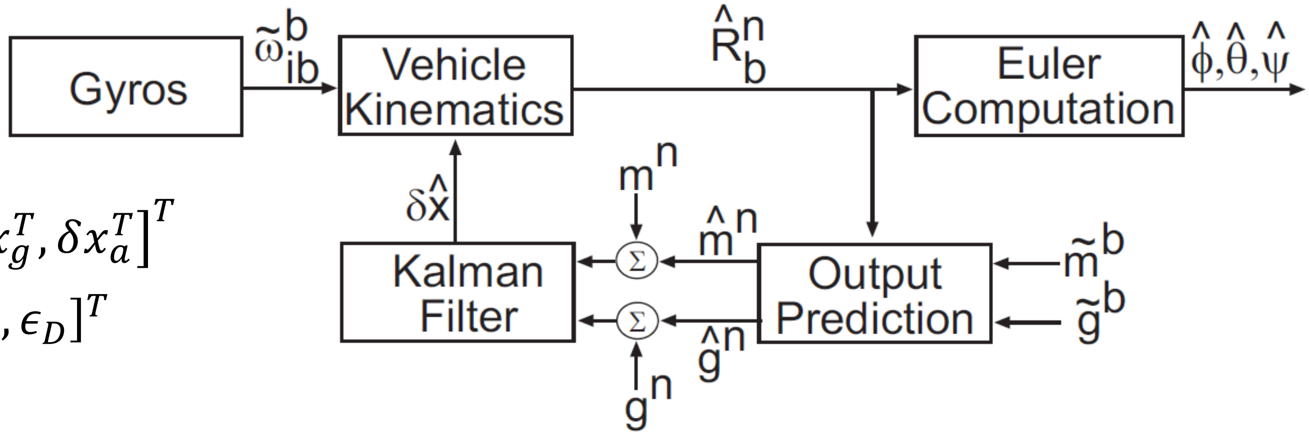
$$\dot{\hat{\mathbf{R}}}_b^n = \hat{\mathbf{R}}_b^n \hat{\boldsymbol{\Omega}}_{ib}^b$$

$$\dot{\boldsymbol{\rho}} = \hat{\mathbf{R}}_b^n \left(\delta\boldsymbol{\omega}_{ib}^b - \boldsymbol{\omega}_{in}^n\right)$$

$$\boxed{\delta\boldsymbol{\omega}_{ib}^b = \boldsymbol{\omega}_{ib}^b - \hat{\boldsymbol{\omega}}_{ib}^b \qquad \delta\boldsymbol{\omega}_{ib}^b = -\delta\mathbf{x}_g - \boldsymbol{\nu}_g \qquad \delta\dot{\mathbf{x}}_g = \mathbf{F}_g \delta\mathbf{x}_g + \boldsymbol{\omega}_g}$$

$$\boxed{\dot{\boldsymbol{\rho}} = -\hat{\mathbf{R}}_b^n \delta\mathbf{x}_g - \hat{\mathbf{R}}_b^n \boldsymbol{\nu}_g - \hat{\mathbf{R}}_b^n \boldsymbol{\omega}_{in}^n}$$

Attitude and heading reference systems – AHRS
- Indirect model -



$$\delta x = [\rho^T, \delta x_g^T, \delta x_a^T]^T$$

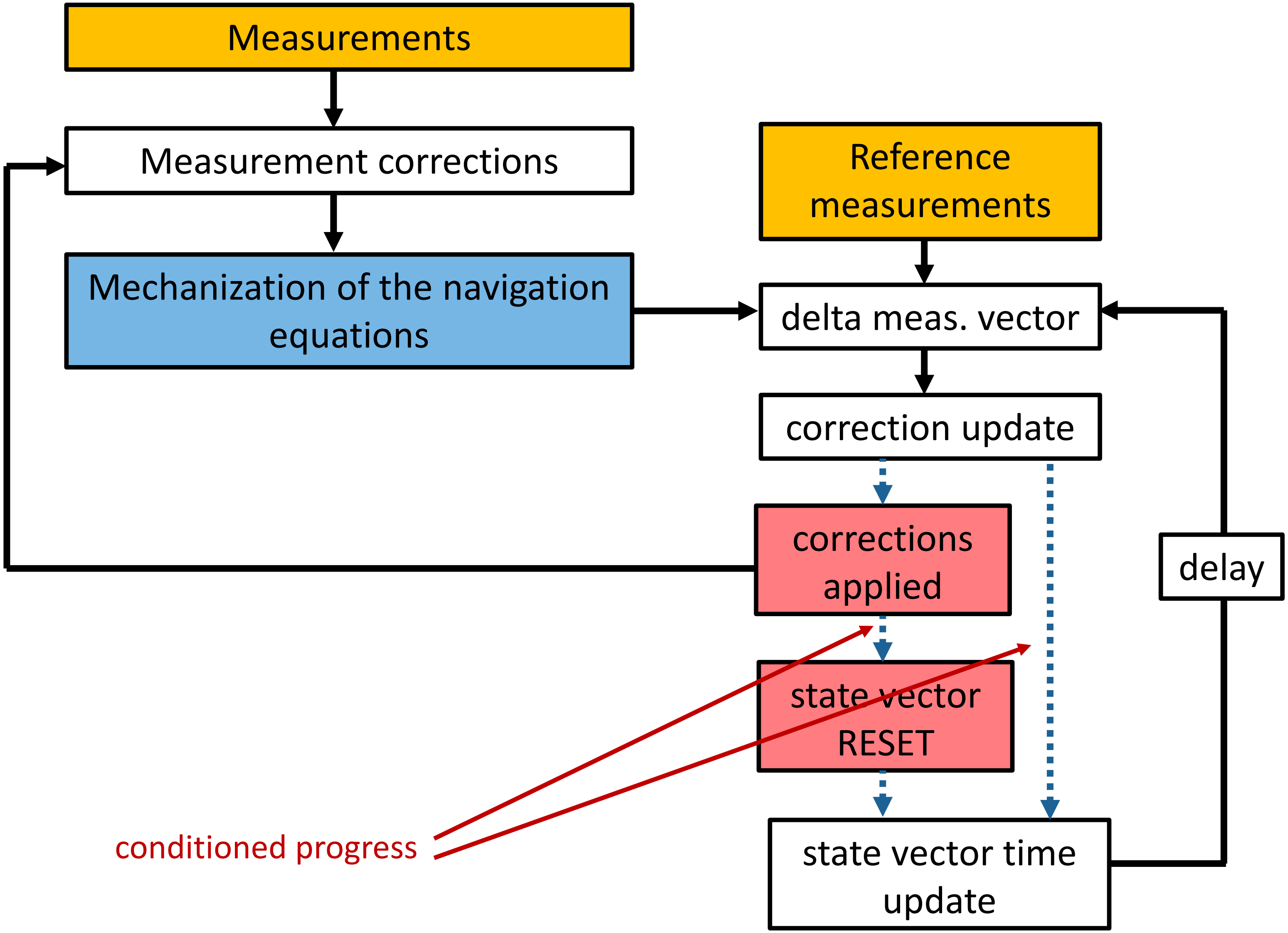
$$\rho = [\epsilon_N, \epsilon_E, \epsilon_D]^T$$

$$\begin{bmatrix} \dot{\boldsymbol{\rho}} \\ \delta\dot{\mathbf{x}}_g \\ \delta\dot{\mathbf{x}}_a \end{bmatrix} = \begin{bmatrix} \mathbf{0} & -\hat{\mathbf{R}}_b^n & \mathbf{0} \\ \mathbf{0} & \mathbf{F}_g & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{F}_a \end{bmatrix} \begin{bmatrix} \boldsymbol{\rho} \\ \delta\mathbf{x}_g \\ \delta\mathbf{x}_a \end{bmatrix} + \begin{bmatrix} -\hat{\mathbf{R}}_b^n & \mathbf{0} & -\hat{\mathbf{R}}_b^n & \mathbf{0} \\ \mathbf{0} & \mathbf{I} & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{I} \end{bmatrix} \begin{bmatrix} \boldsymbol{\omega}_{in}^n \\ \boldsymbol{\omega}_g \\ \boldsymbol{\nu}_g \\ \boldsymbol{\omega}_a \end{bmatrix}$$

$$\begin{aligned} \delta\mathbf{g}^n &= \mathbf{H}_a\delta\mathbf{x} + \mathbf{R}_b^n\boldsymbol{\nu}_a \\ \delta\mathbf{m}^n &= \mathbf{H}_m\delta\mathbf{x} - \mathbf{R}_b^n\boldsymbol{\nu}_m \end{aligned}$$

$$\hat{C}_b^n = (I - [\rho\,\times])C_b^n$$

Indirect model – close loop system



GNSS vs. application in INS/GNSS integrated solution

Commercial GPS receivers have a Kalman filter built into them, and their output is the filtered estimate created by that filter. So, suppose you have a steady stream of output from the GPS consisting of a position and position error. Can you not pass those two pieces of data into your own filter?

Well, what are the characteristics of that data stream, and more importantly, what are the fundamental requirements of the input to the Kalman filter?

- Inputs to the Kalman filter must be Gaussian and time independent. This is because we imposed the requirement of the Markov property: the current state is dependent only on the previous state and current inputs. This makes the recursive form of the filter possible.
- The output of the GPS is time dependent because the filter bases its current estimate on the recursive estimates of all previous measurements. Hence, the signal is not white, it is not time independent, and if you pass that data into a Kalman filter you have violated the mathematical requirements of the filter.

So, the answer is no, you cannot get better estimates by running a KF on the output of a commercial GPS.

Another way to think of it is that KFs are optimal in a least squares sense. There is no way to take an optimal solution, pass it through a filter, any filter, and get a 'more optimal' answer because it is a logical impossibility. At best the signal will be unchanged, in which case it will still be optimal, or it will be changed, and hence no longer optimal.

Vehicle dynamic models for GNSS

Model name	State vector	F	Q
Unknown constant	$x = [pos]$	$F = [0]$	$Q = [0], {\sigma_{pos}}^2(0)$
TYPE2	$x = \begin{bmatrix} pos \\ vel \end{bmatrix}$	$F = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}$	$Q = \begin{bmatrix} 0 & 0 \\ 0 & {\sigma_{acc}}^2 \Delta t^2 \end{bmatrix}$
TYPE3	$x = \begin{bmatrix} pos \\ vel \\ acc \end{bmatrix}$	$F = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{bmatrix}$	$Q = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & {\sigma_{jerk}}^2 \Delta t^2 \end{bmatrix}$
DAMP1	$x = \begin{bmatrix} pos \\ vel \end{bmatrix}$	$F = \begin{bmatrix} 0 & 1 \\ 0 & \frac{-1}{\tau_{vel}} \end{bmatrix}$	$Q = \begin{bmatrix} 0 & 0 \\ 0 & {\sigma_{acc}}^2 \Delta t^2 \end{bmatrix}$
DAMP2	$x = \begin{bmatrix} pos \\ vel \\ acc \end{bmatrix}$	$F = \begin{bmatrix} 0 & 1 & 0 \\ 0 & \frac{-1}{\tau_{vel}} & 1 \\ 0 & 0 & \frac{-1}{\tau_{acc}} \end{bmatrix}$	$Q = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & {\sigma_{jerk}}^2 \Delta t^2 \end{bmatrix}$
DAMP3	$x = \begin{bmatrix} pos \\ vel \\ acc \end{bmatrix}$	$F = \begin{bmatrix} \frac{-1}{\tau_{pos}} & 1 & 0 \\ 0 & \frac{-1}{\tau_{vel}} & 1 \\ 0 & 0 & \frac{-1}{\tau_{acc}} \end{bmatrix}$	$Q = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & {\sigma_{jerk}}^2 \Delta t^2 \end{bmatrix}$

Vehicle dynamic models for GNSS – cont.

Model	x	F	Q	Independent variable	Dep. variable
TYPE2	$\begin{bmatrix} pos \\ vel \end{bmatrix}$	$\begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & 0 \\ 0 & {\sigma_{acc}}^2 \Delta t^2 \end{bmatrix}$	${\sigma_{acc}}^2$	${\sigma_{pos}}^2 \to \infty$ ${\sigma_{vel}}^2 \to \infty$

The model is driven with zero-mean white noise acceleration, unbounded steady-state mean-squared velocity and unbounded steady-state mean-squared position variation.

WHEN GNSS signal is lost the velocity uncertainty variance grows without bound. Thus, this model is useful for cases when GNSS signal is present.

Adjustable parameter $\boldsymbol{\sigma_{acc}}^{\mathbf{2}}$ according to experimenting

$$\Phi = \begin{bmatrix} 1 & 0 & 0 & \Delta t & 0 & 0 \\ 0 & 1 & 0 & 0 & \Delta t & 0 \\ 0 & 0 & 1 & 0 & 0 & \Delta t \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix} \qquad Q_{k\ vel,pos} = q \begin{bmatrix} \frac{\Delta t^3}{3} & \frac{\Delta t^2}{2} \\ \frac{\Delta t^2}{2} & \Delta t \end{bmatrix}$$

Vehicle dynamic models for GNSS – cont.

Model	x	F	Q	Independent variable	Dep. variable
TYPE3	$\begin{bmatrix} pos \\ vel \\ acc \end{bmatrix}$	$\begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & {\sigma_{jerk}}^2 \Delta t^2 \end{bmatrix}$	${\sigma_{acc}}^2$	${\sigma_{pos}}^2 \to \infty$ ${\sigma_{vel}}^2 \to \infty$ ${\sigma_{jerk}}^2$

WHEN GNSS signal is lost the velocity and position uncertainty variance grows without bound. Thus, this model is useful for cases when GNSS signal is present.

$$\Phi = \begin{bmatrix} 1 & \Delta t & \frac{\Delta t^2}{2} \\ 0 & 1 & \Delta t \\ 0 & 0 & 1 \end{bmatrix}$$

$$\Gamma = \begin{bmatrix} \frac{\Delta t^2}{2} \\ \Delta t \\ 1 \end{bmatrix}$$

$$Q_{k\ vel,pos} = \Gamma * q * \Gamma^T = q \begin{bmatrix} \frac{\Delta t^4}{4} & \frac{\Delta t^3}{2} & \frac{\Delta t^2}{2} \\ \frac{\Delta t^3}{2} & \Delta t^2 & \Delta t \\ \frac{\Delta t^2}{2} & \Delta t & 1 \end{bmatrix}$$

$$0.5 * \Delta a_M \le \sigma_{jerk} \le \Delta a_M$$

(Piecewise constant) **Discrete Wiener process acceleration model**

Vehicle dynamic models for GNSS – cont.

Model	x	F	Q	Independent variable	Dependent variable
DAMP1	$\begin{bmatrix} pos \\ vel \end{bmatrix}$	$\begin{bmatrix} 0 & 1 \\ 0 & \frac{-1}{\tau_{vel}} \end{bmatrix}$	$\begin{bmatrix} 0 & 0 \\ 0 & {\sigma_{acc}}^2 \Delta t^2 \end{bmatrix}$	${\sigma_{vel}}^2$ τ_{vel}	${\sigma_{pos}}^2 \to \infty$ ${\sigma_{acc}}^2$

It differs from TYPE2 in that it includes a velocity damping time constant τ_{vel} which allows a steady-state variance for velocity → **more realistic because of vehicle fine speed capabilities**.

WHEN τ_{vel} is set it distinguishes a type of vehicle or dynamics of the vehicle respectively = **measure of persistence of the velocity**.

$$\Phi = \begin{bmatrix} 1 & 0 & 0 & \Delta t & 0 & 0 \\ 0 & 1 & 0 & 0 & \Delta t & 0 \\ 0 & 0 & 1 & 0 & 0 & \Delta t \\ 0 & 0 & 0 & \varepsilon & 0 & 0 \\ 0 & 0 & 0 & 0 & \varepsilon & 0 \\ 0 & 0 & 0 & 0 & 0 & \varepsilon \end{bmatrix}$$

$${\sigma_{acc}}^2 = {\sigma_{vel}}^2 \left(1 - exp(-2\Delta t/\tau_{vel})\right)/\Delta t^2$$

where $\varepsilon = \exp(-\Delta t/\tau_{vel})$

Condition: $\Delta t/\tau_{vel} \ll 1$

Vehicle dynamic models for GNSS – cont.

Model	x	F	Q	Independent variable	Dependent variable
DAMP2	$\begin{bmatrix} pos \\ vel \\ acc \end{bmatrix}$	$\begin{bmatrix} 0 & 1 & 0 \\ 0 & \frac{-1}{\tau_{vel}} & 1 \\ 0 & 0 & \frac{-1}{\tau_{acc}} \end{bmatrix}$	$\begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & {\sigma_{jerk}}^2 \Delta t^2 \end{bmatrix}$	${\sigma_{vel}}^2$ ${\sigma_{acc}}^2$ τ_{acc}	${\sigma_{pos}}^2 \to \infty$ τ_{vel} ${\sigma_{jerk}}^2$

It differs from DAMP1 in that it includes even acceleration damping time constant τ_{acc} → **more realistic because of vehicle fine speed and acceleration capabilities**.

WHEN τ_{acc} is set it characterizes how much a vehicle is lively.

$$\Phi = \begin{bmatrix} \Phi_{11} & \Phi_{12} \\ \Phi_{21} & \Phi_{22} \end{bmatrix}$$

$$\Phi_{11} = \exp\left(-\frac{\Delta t}{\tau_{vel}}\right)$$

$$\Phi_{12} = \frac{\tau_{vel}\tau_{acc}\left[\exp\left(-\frac{\Delta t}{\tau_{vel}}\right) - \exp\left(-\frac{\Delta t}{\tau_{acc}}\right)\right]}{\tau_{vel} - \tau_{acc}}$$

$$\Phi_{21} = 0$$

$$\Phi_{22} = \exp\left(-\frac{\Delta t}{\tau_{acc}}\right)$$

$${\sigma_{jerk}}^2 = {\sigma_{acc}}^2 \left(1 - exp(-2\Delta t / \tau_{acc})\right) / \Delta t^2$$

Vehicle dynamic models for GNSS – cont.

Model	x	F	Q	Independent variable	Dependent variable
DAMP3	$\begin{bmatrix} pos \\ vel \\ acc \end{bmatrix}$	$\begin{bmatrix} \frac{-1}{\tau_{pos}} & 1 & 0 \\ 0 & \frac{-1}{\tau_{vel}} & 1 \\ 0 & 0 & \frac{-1}{\tau_{acc}} \end{bmatrix}$	$\begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & {\sigma_{jerk}}^2 \Delta t^2 \end{bmatrix}$	${\sigma_{pos}}^2$ ${\sigma_{vel}}^2$ ${\sigma_{acc}}^2$ τ_{acc}	τ_{pos} τ_{vel} ${\sigma_{jerk}}^2$

This model is designed for vehicles with limited but nonzero position variation, such as watercrafts (e.g. riverboats) and land vehicles. Ships that remain a sea level are at zero altitude by definition, thus they need no vertical navigation (unless tides estimated)

Host vehicles	Filter models	
	Horizontal direction	Vertical direction
Ships	DAMP1, DAMP2	Unknown constant
Land vehicles	DAMP1, DAMP2	DAMP3
Aircraft and missiles	DAMP1, DAMP2	DAMP2, DAMP3

Complementarity of GNSS and INS

GNSS **provides** a deterministic solution for both position and velocity,

BUT has its own shortcomings, i.e.:

- low data rate (typically 1 Hz up to 10 Hz),
- susceptibility to jamming (even unintentional interference),
- lack of precision attitude information.

Reasons for systems' **complementarity**:

- their error characteristics are different
- they are measured based on different quantities and principles
- GNSS is long-term stable, INS provides a short-term stable solution
- GNSS provides measures of position and velocity.
- an accelerometer measures specific force. The gyroscopes provide a measure of attitude rate, and after initial alignment, they allow the accelerometer measurements to be resolved into a known coordinate frame.

Complementarity of GNSS and INS – cont.

GNSS based position accuracy is limited due to a combination of :

- low signal strength,
- the length of the pseudo-random code,
- errors in the code tracking loop.
- multipath - the phenomenon whereby several delayed copies of the signal arrive at the antenna after being reflected from nearby surfaces, is a source of correlated noise, especially for a moving vehicle.
- constant or slowly changing bias due to satellite ephemeris and clock errors (bounded and not integrated since they are already at the position level)

GNSS based velocity is also noisy due to:

- variations in signal strength
- the effects of changing multipath
- user clock instability.

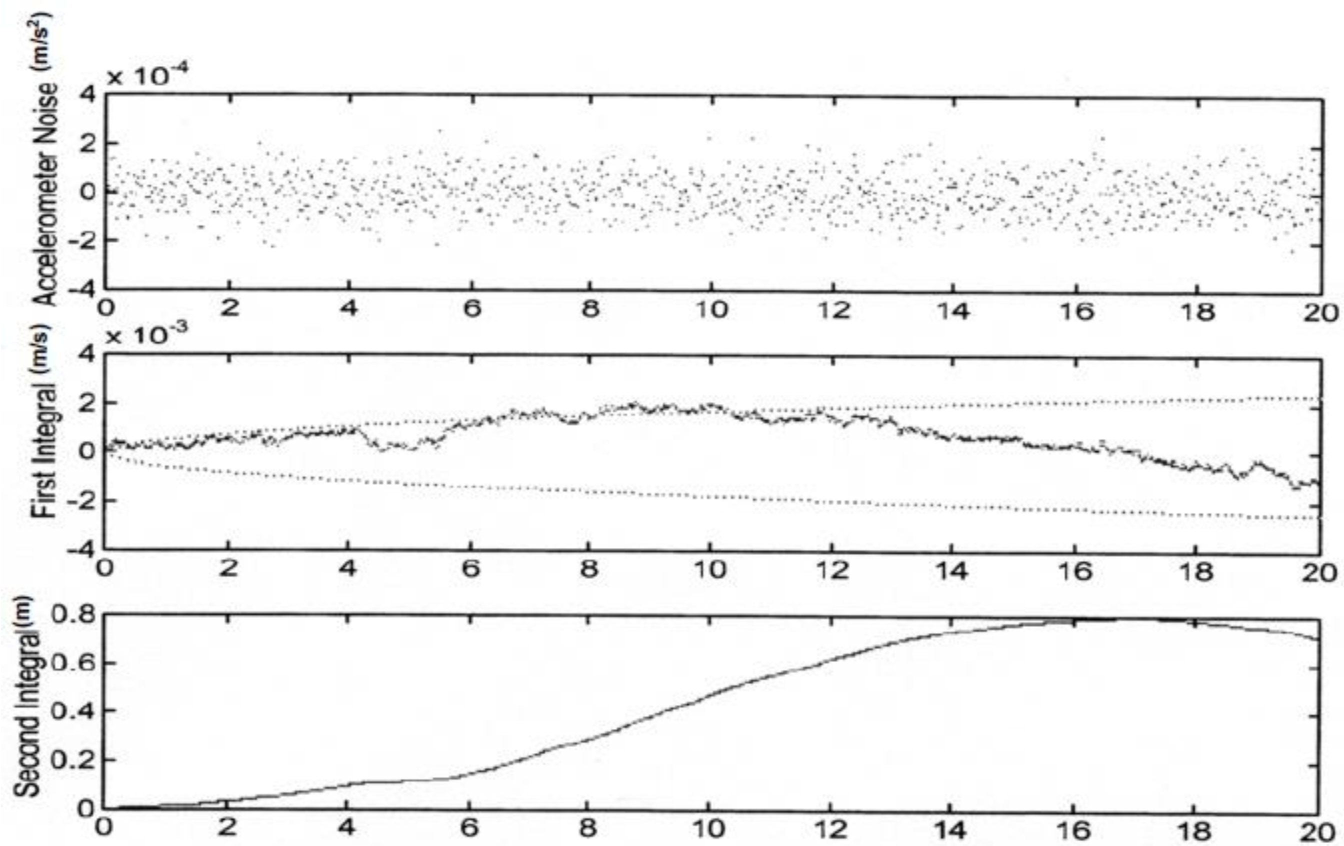
IN CONTRAST

Accelerometers in an INS:

- have relatively low noise characteristics when compared with GPS measurements
- must be compensated for gravity & integrated twice before providing position estimates.

This fundamental difference in radio navigation measurements and inertial measurements leads to the difference in the behavior of INS and GPS navigators.

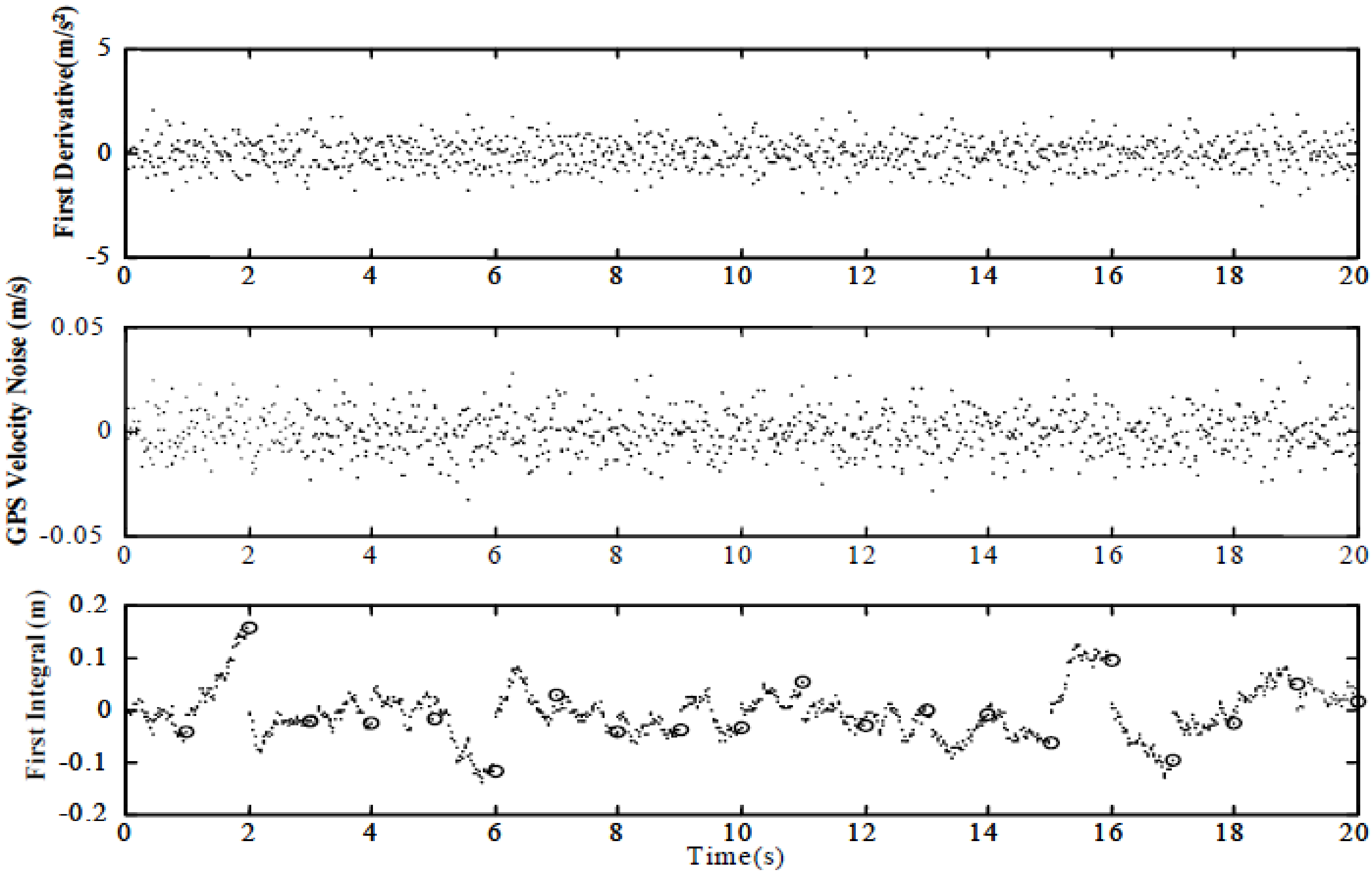
Accelerometer noise and its first two integrals



The integral of acceleration → "random walk".
The dotted lines = 1σ of expected errors.

The noise level is at 56 μg/√Hz, typical of a 10-nmi/h inertial system.

GPS velocity measurement noise and its first derivative & integral



GPS receivers typically produce solutions at 1 Hz to 10 Hz. The data bit rate of 50 Hz sets a "natural" minimum of 20 ms between position and velocity determinations.

The standard deviation of the velocity measurement is 0.01 m/s, typical of a good GPS receiver and strong signals in a benign environment.

Features of inertial and GPS navigation systems

	Attributes	Shortcomings
GPS	Self-initializing Errors are bounded	Low data rate Lower attitude accuracy Susceptible to interference (intentional and unintentional) Expensive infrastructure
INS	High data rate Both translational and rotational information Self-contained (not susceptible to jamming)	Unbounded errors Requires knowledge of gravity field Requires initial conditions

The standard deviation of the velocity measurement is 0.01 m/s, typical of a good GPS receiver and strong signals in a benign environment.

INS/GPS integration

provides the redundancy of two systems and their synergy

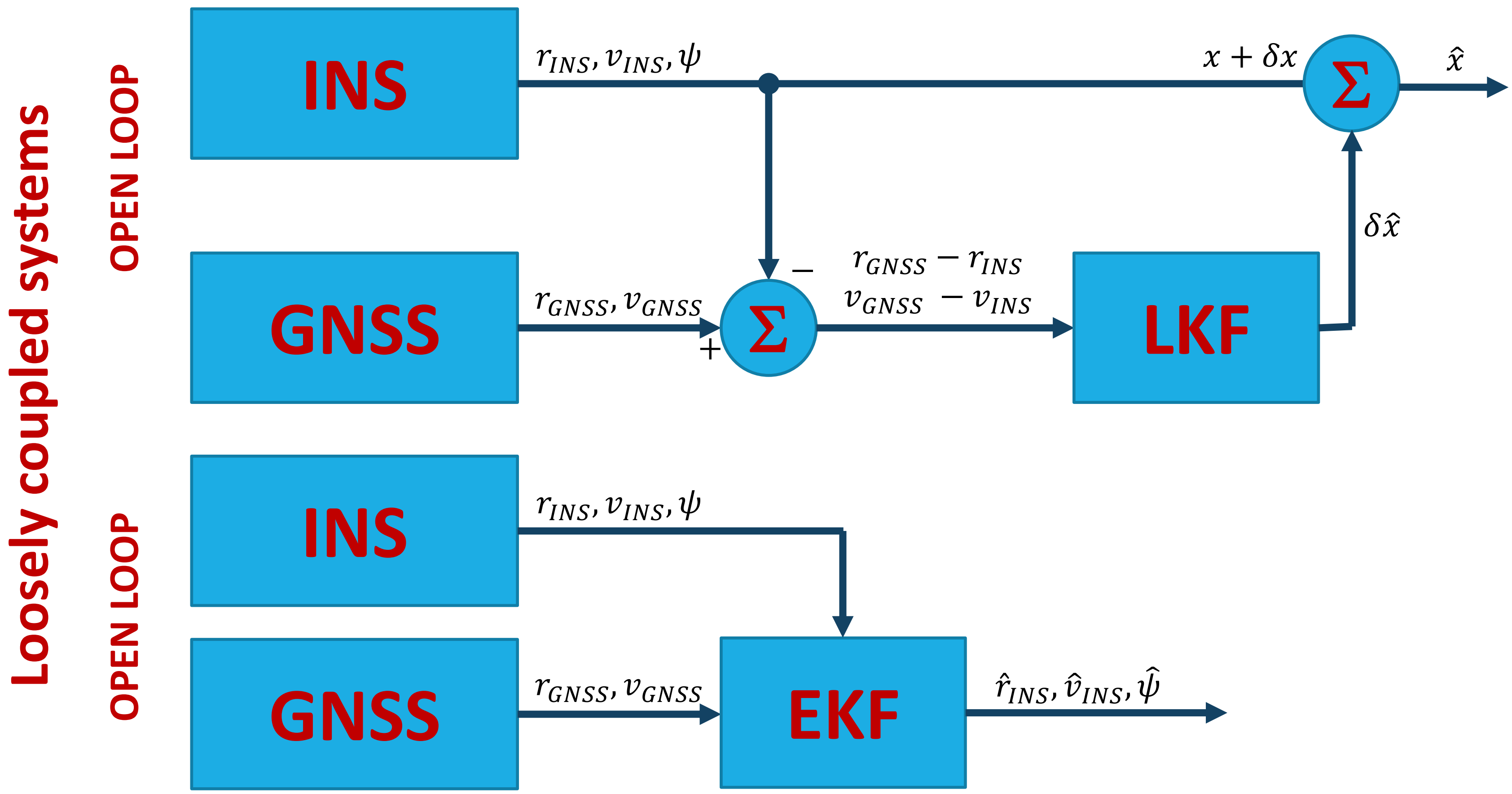
- aiding the receiver's carrier and code tracking loops with inertial sensor information allows the effective bandwidth of these loops to be reduced, even in the presence of severe vehicle maneuvers, thereby improving the ability of the receiver to track signals in a noisy environment such as caused by a jammer. The more accurate the inertial information, the narrower the bandwidth of the loops that can be designed;
- in a jamming environment, this allows the vehicle to more closely approach a jammer-protected target before losing GPS tracking;
- an accurate navigation solution in situations where "GPS only" navigation would be subject to "natural" short-term outages caused by signal blockage and antenna shading;

The accuracy of the solution, the resistance to jamming, and the ability to calibrate the biases in low-noise inertial system components depend on the **system architecture**, i.e.:

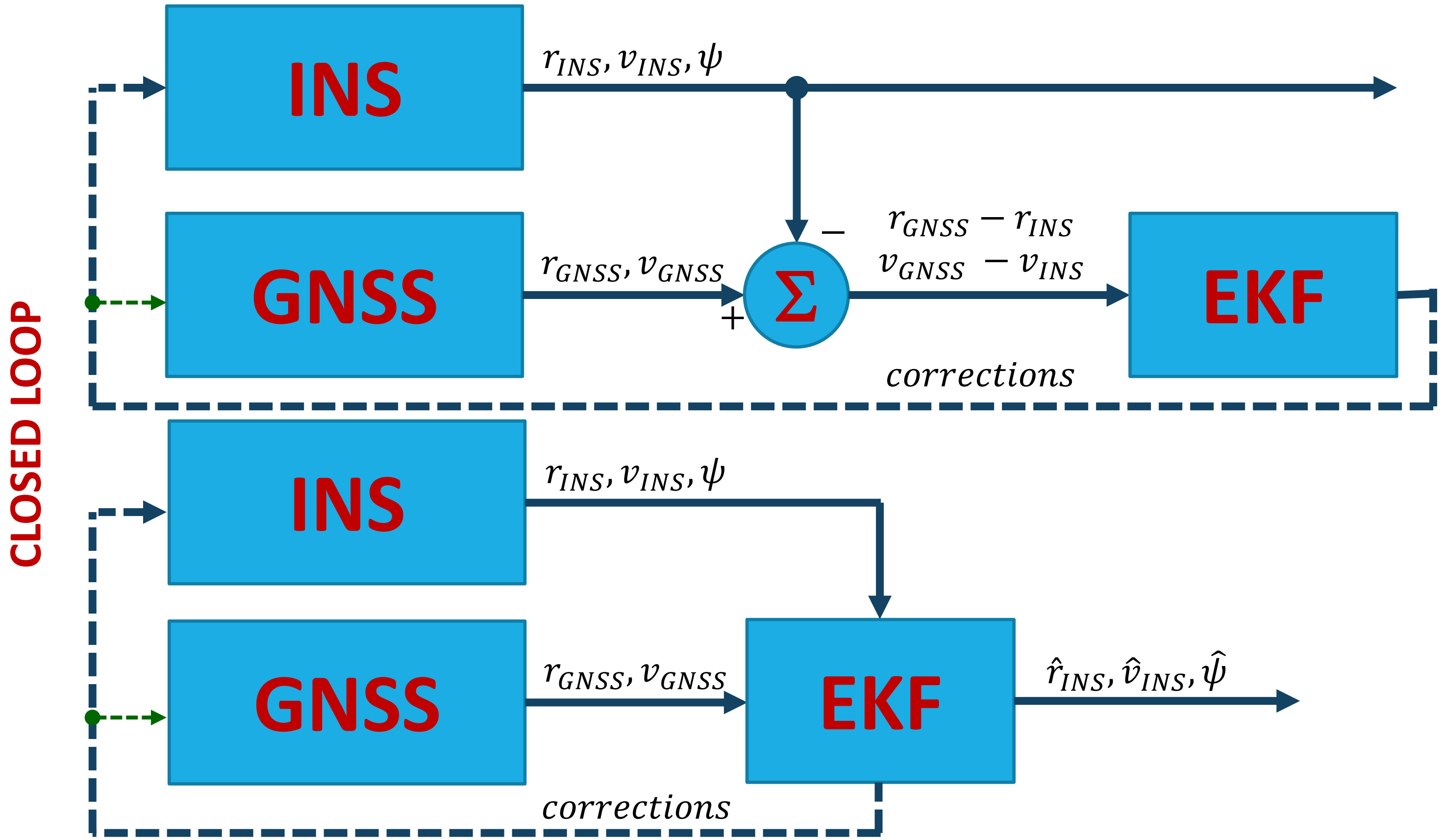
- **separate systems / uncoupled systems**
- **loosely coupled systems**
- **tightly coupled systems**
- **ultra-tightly coupled systems**

Separate/uncoupled systems

By using a GPS "reset" or correction, the inertial system errors are kept bounded, but after the first reset, the INS solution is no longer independent of the GPS system. This mode of operation or coupling has the advantage of leaving the two systems independent and redundant.

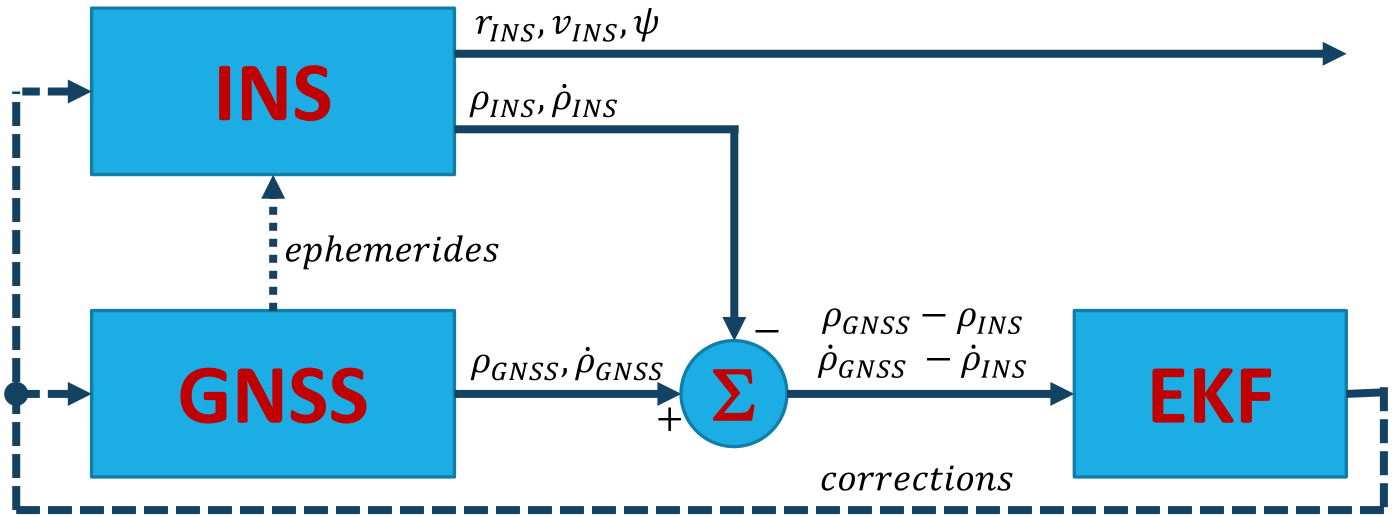


Loosely coupled systems with a feedback



The GNSS receiver only uses INS data for the purpose of aiding in acquisition. Knowing the position and velocity of the vehicle enables the code generator and oscillator to make good initial guesses of the frequency and code phase of the incoming signal. The search time during acquisition can be reduced significantly depending on the accuracy of these estimates.

Tightly coupled systems



The inertial "system" now simply provides raw measurements. The GPS receiver does not have its own Kalman filter, but it does still have independent tracking loops that provide the values for pseudo-range and range rate. The tracking loops in the receiver are aided by data from the INS/GPS state estimator.

The tightly coupled navigation systems are more accurate. The integration filter can make optimal use of any and all satellites that are being tracked, even if there are less than four of them. It should be said that GPS-only solutions can be maintained with either three or two satellites if one or two or both of the following assumptions are made: 1) the clock bias is constant and 2) the altitude is constant or is known by some other means (e.g. baro-altimeter).

Cumulative benefits of increasingly tight coupling

Coupling Level	Benefit
Uncoupled/reset INS to GPS (Sum of system attributes)	Position, velocity, acceleration, attitude, and attitude rate information Redundant systems - A drift-free GPS - A high-bandwidth INS
Loosely coupled	More rapid GPS acquisition In-flight calibration and alignment Better inertial instrument calibration and alignment - Better attitude estimates - Longer operation after jamming
Tightly coupled	Better navigation performance Better instrument calibration Reliable tracking under high dynamics Reduced tracking loop bandwidth (jamming resistance) Optimum use of however many SVs available

INS/GPS integration architecture – No. of states

Table 1: State Elements for the Unaided GPS Receiver.

State Element	Components
Position	3
Velocity	3
Acceleration	3
User clock bias	1
User clock drift	1
Altimeter bias	1
Total	12

Table 2: State Elements for the Loosely Coupled Integration Filter.

Error State Element	Components
Position	3
Velocity	3
Misalignment	3
Gyro drift	3
Gyro scale factor	3
Accel. bias	3
Accel. scale factor	3
Altimeter bias	1
Total	22

INS/GPS integration architecture – No. of states – cont.

Table 3: State Elements for the Tightly Coupled Integration Filter.

Error State Element	Components
Position	3
Velocity	3
User clock bias	1
User clock drift	1
Misalignment	3
Gyro drift	3
Gyro scale factor	3
Accel. bias	3
Accel. scale factor	3
Altimeter bias	1
Total	24

IMU error sources

	IMU Quality (All errors except random walk are 1σ biases)			
Error Source	**10 nmi/h**	**1.0 nmi/h**	**0.5 nmi/h**	**0.2 nmi/h**
Accel. bias	223 μg	37 μg	19 μg	4.2 μg
Accel. scale factor	223 ppm	179 ppm	90 ppm	21 ppm
Input axis misalign.	22 arcsec	3 arcsec	1.5 arcsec	0.4 arcsec
Random walk	56 μg/√Hz	15 μg/√Hz	7.5 μg/√Hz	4.2 μg/√Hz
Gyro bias	0.11 deg/h	0.0045 deg/h	0.0022 deg/h	0.00084 deg/h
Gyro scale factor	112 ppm	7.5 ppm	3.5 ppm	1.67 ppm
Input axis misalign.	22 arcsec	2.2 arcsec	1.1 arcsec	0.4 arcsec
Random walk	4.7 deg/h/√Hz	0.13 deg/h/√Hz	0.066 deg/h/√Hz	0.03 deg/h/√Hz