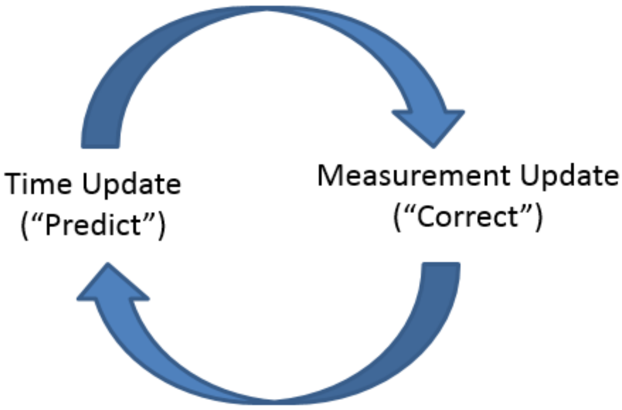
SUMMARY

We have derived all five Kalman Filter equations in matrix notation. Let us put them all together in a single page.

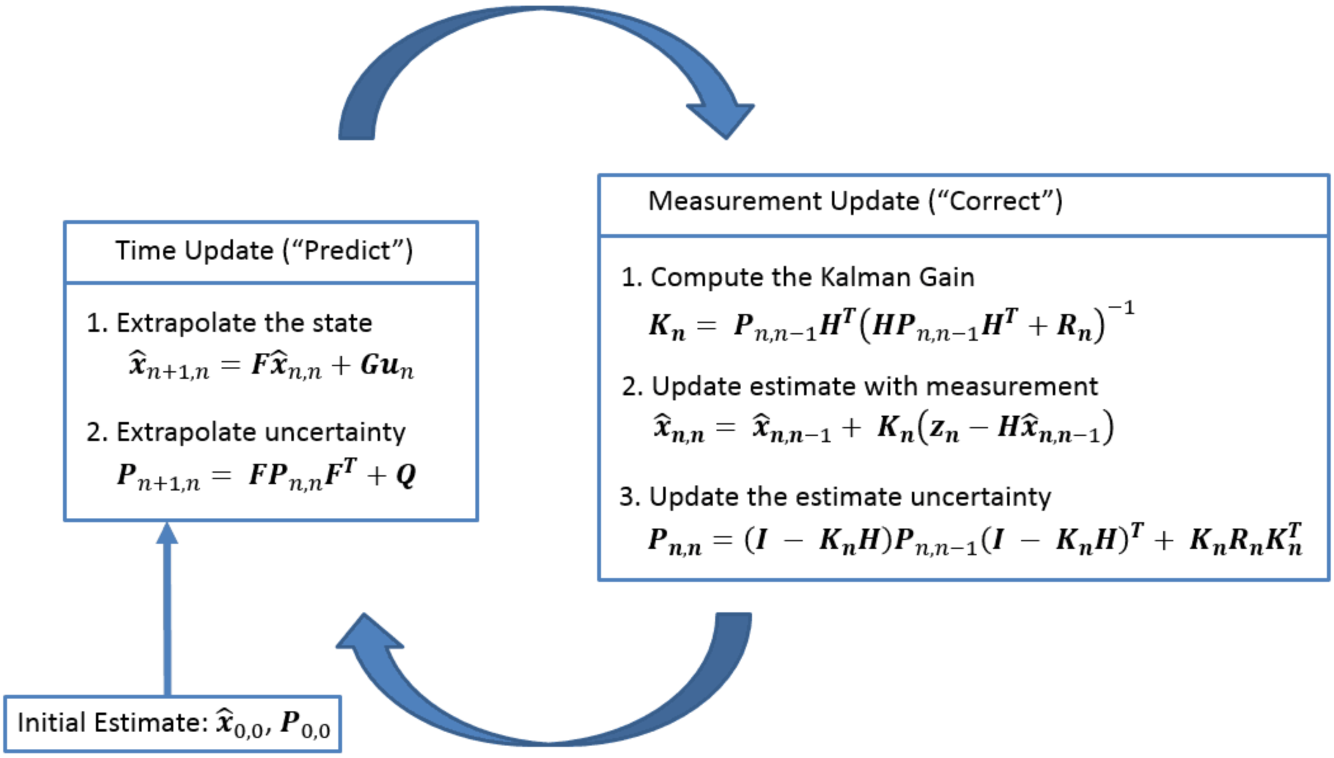
The Kalman Filter operates in a "predict – correct" loop, as shown in the diagram below.



Once initialized, the Kalman Filter will **predict** the system state at the next time step. It also provides the uncertainty of the prediction.

Once the measurement is received, the Kalman Filter updates (or corrects) the prediction and the uncertainty of the current state. As well, the Kalman Filter predicts the next states, and so on.

The following diagram provides a complete picture of the Kalman Filter operation.



	Equation	Equation Name	Alternative names
Predict	$\boldsymbol{\hat{x}_{n+1,n} = F\hat{x}_{n,n} + Gu_n}$	State Extrapolation	Predictor Equation Transition Equation Prediction Equation Dynamic Model State Space Model
	$\boldsymbol{P_{n+1,n} = FP_{n,n}F^T + Q}$	Covariance Extrapolation	Predictor Covariance Equation
Update (correction)	$\boldsymbol{\hat{x}_{n,n} = \hat{x}_{n,n-1} + K_n(z_n - H\hat{x}_{n,n-1})}$	State Update	Filtering Equation
	$\boldsymbol{P_{n,n} = (I - K_nH)\,P_{n,n-1}(I - K_nH)^T + K_nR_nK_n^T}$	Covariance Update	Corrector Equation
	$\boldsymbol{K_n = P_{n,n-1}H^T\left(HP_{n,n-1}H^T + R_n\right)^{-1}}$	Kalman Gain	Weight Equation
Auxiliary	$\boldsymbol{z_n = Hx_n}$	Measurement Equation	
	$\boldsymbol{R_n} = E\left(\boldsymbol{v_n v_n^T}\right)$	Measurement Uncertainty	Measurement Error
	$\boldsymbol{Q_n} = E\left(\boldsymbol{w_n w_n^T}\right)$	Process Noise Uncertainty	Process Noise Error
	$\boldsymbol{P_{n,n}} = E\left(\boldsymbol{e_n e_n^T}\right) = E\left(\left(\boldsymbol{x_n - \hat{x}_{n,n}}\right)\left(\boldsymbol{x_n - \hat{x}_{n,n}}\right)^T\right)$	Estimation Uncertainty	Estimation Error

The next table summarizes notation (including differences found in the literature) and dimension.

Term	Name	Alternative term	Dimensions
$\boldsymbol{x}$	State Vector		$n_x \times 1$
$\boldsymbol{z}$	Output Vector	$\boldsymbol{y}$	$n_z \times 1$
$\boldsymbol{F}$	State Transition Matrix	$\boldsymbol{\Phi, A}$	$n_x \times n_x$
$\boldsymbol{u}$	Input Variable		$n_u \times 1$
$\boldsymbol{G}$	Control Matrix	$\boldsymbol{B}$	$n_x \times n_u$
$\boldsymbol{P}$	Estimate Uncertainty	$\boldsymbol{\Sigma}$	$n_x \times n_x$
$\boldsymbol{Q}$	Process Noise Uncertainty		$n_x \times n_x$
$\boldsymbol{R}$	Measurement Uncertainty		$n_z \times n_z$
$\boldsymbol{w}$	Process Noise Vector	$\boldsymbol{y}$	$n_x \times 1$
$\boldsymbol{v}$	Measurement Noise Vector		$n_z \times 1$
$\boldsymbol{H}$	Observation Matrix	$\boldsymbol{C}$	$n_z \times n_x$

$\boldsymbol{K}$	Kalman Gain		$n_x \times n_z$
$\boldsymbol{n}$	Discrete Time Index	$\boldsymbol{k}$	