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Comparison of Initial Alignment Methods for SINS*

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***Abstract*— Six standard analytic coarse alignment methods and a direct method are compared via error analyses for strapdown inertial navigation system on stationary base. Although these methods are derived from the same measurements of the local gravity vector and the Earth rate, their error characteristics are not completely identical. The susceptibility of these methods to geographical latitude is examined, and a criterion for the election of an appropriate algorithm is established. Due to its high accuracy, computation efficiency and immunity to latitude uncertainty, the direct method is more suitable for practical applications. The results obtained from both simulation and real data test give a visual demonstration.**

***Index Terms*— SINS, Analytic coarse alignment, Initial alignment, Error analysis, Reference vector**

I. INTRODUCTION

Initial alignment process is of vital importance to inertial navigation system, and poor initial alignment accuracy will end up with poor navigation result [1]. Normally, initial alignment process is divided into two phases, namely, coarse alignment and fine alignment [2]. There is an essential difference between the coarse and fine alignment. For the coarse alignment there is no initial conditions knowledge of a priori. The purpose of coarse alignment algorithm is to accomplish the coarse estimation of strapdown attitude matrix, so as to provide a fairly good initial condition for the fine alignment processing.

A lot of literature has been devoted to coarse alignment methods. Two different methods for coarse alignment have been discussed in [3]. One of them, refered to as direct method, utilizes directly both accelerometers and gyros measurement for attitude estimation. Coarse alignment method has been explained in detail in [4] under the heading of ground alignment methods. Autonomous alignment method mentioned in [5] also starts with the coarse alignment method. The chapter 6 of [6] provides a detailed derivation of equation for the analytical coarse alignment of SINS. Coarse alignment is also a subject of [7], [8].

This paper studies the properties of analytic alignment methods for strapdown systems with content of the coarse alignment. Theoretically, an analytic self-alignment method for strapdown system is functionally equal to the physical gyrocompassing in gimbaled system. The error characteristics of both systems seem to be identical at steady state in a stationary base [9]. However, precise geographic latitude is demanded in the standard alignment methods in spite of the fact that the local latitude is not readily available or not necessary required in certain situations. Whether the geographic latitude uncertainty will affect the accuracy of initial alignment is also analyzed.

The remainder of this paper is organized as follows. The standard algorithm approaches for analytic coarse alignment are compared via error analyses in Section II. Section III provides an evaluation of the direct method by proving its equivalence with the classical method. Section IV examines the sensitivity of coarse alignment accuracy to the latitude uncertainty. The conclusion makes up Section V.

II. STANDARD ALIGNMENT METHODS

In this section, six standard analytic coarse alignment methods and the associated error analyses are provided for SINS. Although these methods are derived from the same measurements, the local gravity and the Earth rate, in both body and navigation frames, when choosing different reference vectors will result in different misalignment angles.

A. Alignment Basis Vectors

For the purpose of this study, use the ENU-system (East-North-Up) as external reference. Based on the fact that inertial navigation systems are entirely self-contained, they can align themselves by using the outputs of the accelerometers and gyros along their own axes, which are the measurements of the local gravity vector and the Earth rate vector known in the Earth-fixed frame.

Assume that the geographical latitude L can be accurately measured at the given place, the gravity acceleration vector and the Earth rate vector projected on navigation frame are

$$g^n = \begin{bmatrix} 0 & 0 & -g \end{bmatrix}^T,\ \omega_{ie}^n = \begin{bmatrix} 0 & \Omega_N & \Omega_U \end{bmatrix}^T,$$

where $\Omega_N = \omega_{ie}\cos L,\ \Omega_U = \omega_{ie}\sin L$, g and ω_{ie} represent the magnitude of local gravity vector and the Earth rate vector, respectively.

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Based on vector g^n and ω_{ie}^n, other three vectors, $g^n \times \omega_{ie}^n$, $(g^n \times \omega_{ie}^n) \times g^n$ and $\omega_{ie}^n \times (g^n \times \omega_{ie}^n)$, can be generated by vector cross product. The orientation relationships among these five vectors are shown in [10] as,

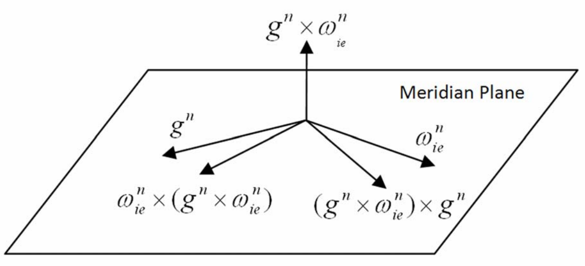

Fig. 1. Orientation relationships between basis vectors.

As shown in Fig. 1, four vectors lie in the meridian plane, while the other one is perpendicular to the plane. In general, there are six possible sets can be used to compute an estimate of the transformation matrix, which are defined as

$$
\begin{aligned}
s_1 &= \{\ g^n \quad \omega_{ie}^n \quad g^n \times \omega_{ie}^n\ \} \\
s_2 &= \{\ g^n \quad g^n \times \omega_{ie}^n \quad (g^n \times \omega_{ie}^n) \times g^n\ \} \\
s_3 &= \{\ g^n \quad \omega_{ie}^n \times (g^n \times \omega_{ie}^n) \quad g^n \times \omega_{ie}^n\ \} \\
s_4 &= \{\ \omega_{ie}^n \quad g^n \times \omega_{ie}^n \quad \omega_{ie}^n \times (g^n \times \omega_{ie}^n)\ \} \\
s_5 &= \{\ \omega_{ie}^n \quad (g^n \times \omega_{ie}^n) \times g^n \quad g^n \times \omega_{ie}^n\ \} \\
s_6 &= \{\ (g^n \times \omega_{ie}^n) \times g^n \quad \omega_{ie}^n \times (g^n \times \omega_{ie}^n) \quad g^n \times \omega_{ie}^n\ \},
\end{aligned}
$$

where vector $g^n \times \omega_{ie}^n$, the only one perpendicular to the meridian plane, is contained in every set.

It is quite obvious that $\{s_1, \ldots, s_6\}$ are suitable to set up different analytic coarse alignment methods except at the Earth poles. Generally, only the simple and significant sets, s_1 and s_2, are analyzed in literature [9], [11], [12], but not all the standard methods.

B. Transformation Matrix Calculation

Considering a set of basis vectors $\{V_1^n, V_2^n, V_3^n\}$ in navigation frame, it can be transformed from its body frame projection form by

$$
\begin{bmatrix} V_1^n & V_2^n & V_3^n \end{bmatrix} = C_b^n \begin{bmatrix} V_1^b & V_2^b & V_3^b \end{bmatrix}, \tag{1}
$$

where C_b^n is the transformation matrix defined in [13].

Thus, the analytic alignment problem becomes to choose a proper set which consists of three linearly independent basis vectors $\{V_1, V_2, V_3\}$ upon body and navigation frames, so that the solution of C_b^n can be directly derived from (1) as

$$
C_b^n = [\ V_1^n \quad V_2^n \quad V_3^n\ ][\ V_1^b \quad V_2^b \quad V_3^b\ ]^{-1} \tag{2}
$$

As we know, C_b^n is an orthogonal matrix. For the sake of computational convenience, it can also be written as

$$
C_b^n = \begin{bmatrix} (V_1^n)^T \\ (V_2^n)^T \\ (V_3^n)^T \end{bmatrix}^{-1} \begin{bmatrix} (V_1^b)^T \\ (V_2^b)^T \\ (V_3^b)^T \end{bmatrix} \tag{3}
$$

1) Transformation Matrix based on s_1: By substituting the three vectors of s_1 into (3), C_b^n becomes

$$
C_b^n = \begin{bmatrix} (g^n)^T \\ (\omega_{ie}^n)^T \\ (g^n \times \omega_{ie}^n)^T \end{bmatrix}^{-1} \begin{bmatrix} (g^b)^T \\ (\omega_{ie}^b)^T \\ (g^b \times \omega_{ie}^b)^T \end{bmatrix}, \tag{4}
$$

Let a^b and ω^b represent the outputs of accelerometers and gyros in the body frame, respectively, and define their components as

$$
a^b = \begin{bmatrix} a_x & a_y & a_z \end{bmatrix}^T,\ \omega^b = \begin{bmatrix} \omega_x & \omega_y & \omega_z \end{bmatrix}^T
$$

Alternatively, in practical applications, there are

$$
g^b = -a^b,\ \omega_{ie}^b = \omega^b
$$

When g^n, ω_{ie}^n, a^b and ω^b are actually substituted into (4), the components of C_b^n can be computed accurately as

$$
C_b^n = \begin{bmatrix} \frac{a_z\omega_y - a_y\omega_z}{g\Omega_N} & \frac{a_x\omega_z - a_z\omega_x}{g\Omega_N} & \frac{a_y\omega_x - a_x\omega_y}{g\Omega_N} \\ \frac{g\omega_x - a_x\Omega_U}{g\Omega_N} & \frac{g\omega_y - a_y\Omega_U}{g\Omega_N} & \frac{g\omega_z - a_z\Omega_U}{g\Omega_N} \\ \frac{a_x}{g} & \frac{a_y}{g} & \frac{a_z}{g} \end{bmatrix}
$$

2) Transformation Matrix based on s_2: Similarly, if the basis vectors of s_2 are substituted into (3), there is

$$
C_b^n = \begin{bmatrix} (g^n)^T \\ (g^n \times \omega_{ie}^n)^T \\ [(g^n \times \omega_{ie}^n) \times g^n]^T \end{bmatrix}^{-1} \begin{bmatrix} (g^b)^T \\ (g^b \times \omega_{ie}^b)^T \\ [(g^b \times \omega_{ie}^b) \times g^b]^T \end{bmatrix}
$$

In this case, the first and third rows of C_b^n are the same as that of case s_1, while the elements of the second row are changed to

$$
\begin{cases} C_{21} = \dfrac{a_z^2\omega_x - a_z a_x \omega_z - a_y a_x \omega_y + a_y^2 \omega_x}{g^2\Omega_N} \\ C_{22} = \dfrac{a_z^2\omega_y - a_z a_y \omega_z - a_x a_y \omega_x + a_x^2 \omega_y}{g^2\Omega_N} \\ C_{23} = \dfrac{a_y^2\omega_z - a_y a_z \omega_y - a_x a_z \omega_x + a_x^2 \omega_z}{g^2\Omega_N} \end{cases}
$$

Let a be the magnitude of a^b, the above equations become

$$
\begin{cases} C_{21} = \dfrac{a^2\omega_x - a_x(a^b \cdot \omega^b)}{g^2\Omega_N} \\ C_{22} = \dfrac{a^2\omega_y - a_y(a^b \cdot \omega^b)}{g^2\Omega_N} \\ C_{23} = \dfrac{a^2\omega_z - a_z(a^b \cdot \omega^b)}{g^2\Omega_N} \end{cases}
$$

It can be easily seen that, by using the equalities, $a = g$ and $a^b \cdot \omega^b = g\Omega_U$, the second rows of C_b^n based on s_1 and s_2 they are identical in the ideal situation [9].

In fact, the first rows of C_b^n for every basis are consistent with each other; the second rows in case s_5 and s_6 agree with that of set s_2; and the third rows in case s_1, s_2 and s_3 are identical. The easy proof of this relationship is left to the reader.

C. Error Analysis

In practical strapdown system, it is inevitable that the inertial measurements will be contaminated with sensing noises. The average value of the outputs of accelerometers and gyros are described as

$$a^b = g^b + \delta a^b, \; \omega^b = \omega_{ie}^b + \delta\omega^b,$$

where δa^b is the noise contributing to the accelerometer measurements, and $\delta\omega^b$ is the noise contributing to the gyro measurements.

Therefore only the computational transformation matrix and the noise contaminated measurements are available. As finishing the alignment process, (3) has to be rewritten as

$$\hat{C}_b^n = \begin{bmatrix} (\hat{V}_1^n)^T \\ (\hat{V}_2^n)^T \\ (\hat{V}_3^n)^T \end{bmatrix}^{-1} \begin{bmatrix} (\hat{V}_1^b)^T \\ (\hat{V}_2^b)^T \\ (\hat{V}_3^b)^T \end{bmatrix} \tag{5}$$

As described in [14], $\hat{C}_b^n$ and C_b^n can be related with

$$\hat{C}_b^n = (I + S - U)C_b^n \ ,$$

where I is the identity matrix; S is a symmetric matrix, whose diagonal components are the scale errors, while the off-diagonal components are the skew errors; U is an anti-symmetric matrix, the elements of which represent the drift errors.

The computational transformation matrix $\hat{C}_b^n$ needs to be refined to meet the orthogonality conditions by

$$(\hat{C}_b^n)_0 = \hat{C}_b^n \left[(\hat{C}_b^n)^T(\hat{C}_b^n)\right]^{-1/2} \tag{6}$$

The orthonormalization process can eliminate the scale and skew errors, but leaves the drift errors unaffected. Hence, when the alignment process has completed, we have

$$(\hat{C}_b^n)_0 = (I - U)C_b^n$$

Formulas calculating $\hat{C}_b^n$ can be obtained as

$$\hat{C}_b^n = M\hat{Q} = M(Q + \delta Q), \tag{7}$$

where M is related to g^n and ω_{ie}^n; the perturbed matrix $\hat{Q}$ can be computed from a^b and ω^b; δQ represents the departure of Q from its ideal value.

By using Taylor series expansion for (6), the first-order approximation is obtained as

$$(\hat{C}_b^n)_0 = \left[I + \frac{1}{2}(M\delta Q C_n^b - C_b^n \delta Q^T M^T)\right] C_b^n$$

Then, the antisymmetric matrix U, representing the drift errors, is derived as

$$U = \frac{1}{2}\left[C_b^n \delta Q^T M^T - (C_b^n \delta Q^T M^T)^T\right], \tag{8}$$

This is the basic error equation for analysis of analytic coarse alignment methods.

D. Error Calculation

In order to analyze the alignment errors associated with the computational methods, one can individually substitute the six sets of basis vectors into (5) for error analysis.

1) Error analysis for basis s_1: Let us begin with set s_1 by considering the effect of sensor noise. Now, substituting g^b and ω_{ie}^b with $-a^b$ and ω^b, respectively, and denoting the effective sensing errors in the navigation frame as

$$\begin{aligned} \delta a^n &= C_b^n \delta a^b = \begin{bmatrix} \delta a_E & \delta a_N & \delta a_U \end{bmatrix} \\ \delta\omega^n &= C_b^n \delta\omega^b = \begin{bmatrix} \delta\omega_E & \delta\omega_N & \delta\omega_U \end{bmatrix} \end{aligned}$$

Comparing (4) with (7) obtains

$$M = \begin{bmatrix} 0 & 0 & \frac{1}{g\omega_{ie}\cos L} \\ \frac{\tan L}{g} & \frac{1}{\omega_{ie}\cos L} & 0 \\ -\frac{1}{g} & 0 & 0 \end{bmatrix} \tag{9}$$

$$\delta Q = \begin{bmatrix} \delta a^b & \delta\omega^b & \delta(a^b \times \omega^b) \end{bmatrix}^T$$

Now, to the first-order approximation,

$$(C_b^n \delta Q^T)^T = \begin{bmatrix} \delta a^n & \delta\omega^n & (\delta a^n \times \omega_{ie}^n + g^n \times \delta\omega^n) \end{bmatrix}^T$$

In the present case, the analytic alignment errors can be uniquely determined. Applying (8), the drift misalignment angles are obtained as:

$$\begin{aligned} \phi_{E1} &= \frac{1}{2}\left(\frac{\delta a_N}{g} + \frac{\delta a_U}{g}\tan L + \frac{\delta\omega_U}{\omega_{ie}\cos L}\right) \\ \phi_{N1} &= -\frac{\delta a_E}{g} \\ \phi_{U1} &= -\frac{\delta a_E}{g}\tan L - \frac{\delta\omega_E}{\omega_{ie}\cos L} \end{aligned}$$

This error analysis had been done by Britting [15]. It is obvious that not only the north accelerometer error but also the vertical accelerometer and azimuth gyro uncertainties will induce the east level error, and the azimuth gyro uncertainty is the dominant component in practice. This situation is quite different from that occurred in a gimbaled system.

2) Error analysis for basis s_2: Similarly, the associated alignment errors of the second computational method can be obtained by substituting the three vectors of set s_2 into (5).

After some calculations, we have

$$M = \begin{bmatrix} 0 & \frac{1}{g\omega_{ie}\cos L} & 0 \\ 0 & 0 & \frac{1}{g^2\omega_{ie}\cos L} \\ -\frac{1}{g} & 0 & 0 \end{bmatrix} \tag{10}$$

$$\delta Q = \begin{bmatrix} \delta a^b & \delta(a^b \times \omega^b) & \delta[(a^b \times \omega^b) \times a^b] \end{bmatrix}^T$$

Ignoring the second order terms,

$$(C_b^n \delta Q^T)^T = \begin{bmatrix} \delta a^n \\ (\delta a^n \times \omega_{ie}^n + g^n \times \delta\omega^n) \\ (\delta a^n \times \omega_{ie}^n + g^n \times \delta\omega^n) \times g^n + (g^n + \omega_{ie}^n) \times \delta a^n \end{bmatrix}$$

Then the drift misalignment angles are obtained as

$$\phi_{E2} = \frac{\delta a_N}{g}$$
$$\phi_{N2} = -\frac{\delta a_E}{g}$$
$$\phi_{U2} = -\frac{\delta a_E}{g}\tan L - \frac{\delta \omega_E}{\omega_{ie}\cos L}$$

It can be easily seen that the drift errors according to basis s_2 are identical to those obtained in a gimbaled system. The east level error is not corrupted by vertical accelerometer and azimuth gyro uncertainties. Hence, the alignment method based on s_2 is superior to that based on s_1.

Similarly, the error analyses for the methods based on set s_3, s_4, s_5 and s_6 are separately performed. It is confirmed that the drift misalignment angles about north and vertical axes are identical to each other, the same as those obtained with physical gyrocompassing, but a distinct difference exists about the east axis.

The east level errors are presented here for completeness.

$$\phi_{E3} = \frac{\delta \omega_U}{\omega_{ie}}\cos L - \frac{\delta \omega_N}{\omega_{ie}}\sin L$$
$$\phi_{E4} = -\frac{\delta \omega_N}{\omega_{ie}\sin L}$$
$$\phi_{E5} = \frac{\delta a_N}{g} - \frac{\delta a_U}{g\tan L}$$
$$\phi_{E6} = \frac{1}{2}\left(\frac{\delta a_N}{g} + \frac{\delta a_U}{g}\tan L + \frac{\delta \omega_U}{\omega_{ie}\cos L}\right)$$

Therefore, only the east level errors need to be considered when to establish a criterion for algorithm selection.

Generally, set s_2 is the optimal basis to compute the transformation matrix. Set s_5 takes the second place, but it has the same alignment accuracy as set s_2 when at high latitudes. The drift misalignment angles for set s_1 and s_6 are identical, respectively. Set s_3 and s_4 are too much affected by the gyro drifts.

However, the method based on s_2 is not always the most accurate one. In the case that the gyro can achieve very high accuracy while the accelerometer is of general precision, the methods based on s_3 and s_4 will become the better alignment methods, instead, because their east level errors are only influenced by the gyro drifts.

E. Simulation Verification

To test the performance of the analytic coarse alignment methods, a computer simulation is carried out.

In the simulation, the local latitude is $L = 40°$. The heading angle ψ, pitch angle θ and roll angle γ are chosen as $-45°$, $30°$ and $20°$ respectively. The constant and random drifts of each gyro are chosen as $\varepsilon = 0.1°/h$ and $\sigma_\varepsilon = 0.01°/h$, respectively. The constant and random biases of each accelerometer are chosen as $\nabla = 100\mu g$ and $\sigma_\nabla = 50\mu g$, respectively.

The sample period for initial sensors is 20 ms. Each coarse alignment lasts 20 s, and the simulation runs 50 times. The east level errors at the end of each coarse alignment are shown in Fig. 2 and their statistics are listed in Table I.

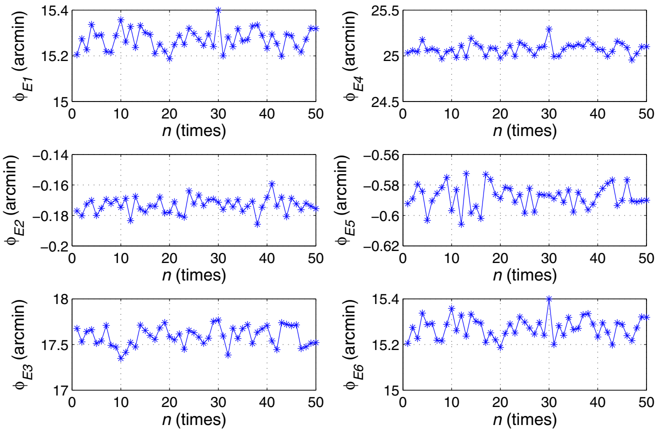

Fig. 2. Alignment results of the 50 simulations.

TABLE I
STATISTICS FOR SIMULATION RESULT

Basis	Statistics			
	MEAN (′)	STD (′)	MAX (′)	MIN (′)
s_1	15.2729	0.0463	15.3992	15.1870
s_2	-0.1733	0.0050	-0.1592	-0.1858
s_3	17.5911	0.1063	17.7671	17.3474
s_4	25.0745	0.0665	25.2932	24.9545
s_5	-0.5879	0.0077	-0.5725	-0.6058
s_6	15.2729	0.0463	15.3992	15.1870

It is observed from Fig. 2 and Table I that the simulation results are consistent with those obtained by the previous theoretical analysis.

III. THE DIRECT METHOD

A. Transformation Matrix Calculation

To facilitate presentation, let us define a matrix as

$$C_n^b = \begin{bmatrix} X & Y & Z \end{bmatrix},$$

where X, Y and Z are the matrix columns, and they are represent orthogonal unit vectors; C_n^b is the transformation matrix from navigation frame to body frame.

In this way, the body frame projection form of the gravity vector and the Earth rate vector can be expressed as

$$g^b = C_n^b g^n = gZ$$
$$\omega_{ie}^b = C_n^b \omega_{ie}^n = (\omega_{ie}\cos L)Y + (\omega_{ie}\sin L)Z$$

The measurements at arbitrary time t are perturbed as

$$a^b(t) = -gZ + \delta a^b(t)$$
$$\omega^b(t) = (\omega_{ie}\cos L)Y + (\omega_{ie}\sin L)Z + \delta\omega^b(t),$$

where $t \in [0, T]$, T is the overall coarse alignment time, and $\delta a^b(t)$ and $\delta\omega^b(t)$ are the noise vectors.

Using the least square method under the orthonormalization constraint, the direct algorithm is implemented by obtaining the three columns of C_n^b [3].

$$\begin{aligned}\hat{Z} &= -\frac{1}{\lambda}\sum_{t=1}^{N} a^b(t)\\ \hat{Y} &= \frac{1}{\eta}\left(\sum_{t=1}^{N}\omega^b(t)+\rho\hat{Z}\right)\\ \hat{X} &= \hat{Y}\times\hat{Z} = \frac{1}{\eta}\left(\sum_{t=1}^{N}\omega^b(t)\right)\times\hat{Z}\ ,\end{aligned}$$

where N is the total number of measurements, and

$$\begin{aligned}\lambda &= \left[\left(\sum_{t=1}^{N}a^b(t)\right)^T\left(\sum_{t=1}^{N}a^b(t)\right)\right]^{1/2}\\ \rho &= \frac{1}{\lambda}\left(\sum_{t=1}^{N}a^b(t)\right)^T\left(\sum_{t=1}^{N}\omega^b(t)\right)\\ \eta &= \left[\left(\sum_{t=1}^{N}\omega^b(t)\right)^T\left(\sum_{t=1}^{N}\omega^b(t)\right)-\rho^2\right]^{1/2}\end{aligned}$$

Finally the estimation of strapdown matrix is obtained as

$$\hat{C}_b^n = \left[\begin{array}{ccc}\hat{X} & \hat{Y} & \hat{Z}\end{array}\right]^T$$

According to its computational formulas, $\hat{C}_b^n$ is already orthogonal, and there is no need for orthogonalization.

B. Error Analysis of Direct Method

The error characteristics of the direct method can be evaluated by proving its equivalence with the method based on set s_2, which has higher accuracy in all standard coarse alignment methods [12].

In order to compare these two methods, $\hat{X}$, $\hat{Y}$ and $\hat{Z}$ can be expressed by the time average value a^b and ω^b as follows

$$\begin{aligned}\hat{Z} &= -\frac{a^b}{|a^b|}\\ \hat{X} &= \frac{1}{\eta}\left(\omega^b\times\hat{Z}\right) = \frac{a^b\times\omega^b}{|a^b\times\omega^b|}\\ \hat{Y} &= \hat{Z}\times\hat{X} = \frac{\left(a^b\times\omega^b\right)\times a^b}{\left|\left(a^b\times\omega^b\right)\times a^b\right|}\end{aligned}$$

We can see that the magnitudes of $\hat{Z}$, $\hat{X}$ and $\hat{Y}$ are equal to the normalized forms of the three vectors in set s_2.

On the other hand, the estimation of C_n^b based on s_2 can be obtained as

$$\hat{C}_n^b = \left[\begin{array}{ccc}\dfrac{a^b\times\omega^b}{g\omega_{ie}\cos L} & \dfrac{(a^b\times\omega^b)\times a^b}{g^2\omega_{ie}\cos L} & -\dfrac{a^b}{g}\end{array}\right]$$

Then $\hat{C}_n^b$ is orthogonalized using (6), and its three normalized unit vectors are in the following

$$\hat{X}_0 = \frac{a^b\times\omega^b}{|a^b\times\omega^b|}\ ,\hat{Y}_0 = \frac{(a^b\times\omega^b)\times a^b}{|(a^b\times\omega^b)\times a^b|}\ ,\hat{Z}_0 = -\frac{a^b}{|a^b|}$$

It can be clearly seen that $\hat{X} = \hat{X}_0$, $\hat{Y} = \hat{Y}_0$ and $\hat{Z} = \hat{Z}_0$, which means that the direct method is equivalent to the method based on set s_2.

C. Simulation and Real Data Test

By using the same simulation conditions in section II, the drift misalignment angles of the methods based on s_1, s_2 and the direct method are shown in table II. The simulation results agree well with the previous error analyses.

TABLE II
DRIFT MISALIGNMENT ANGLES

Methods	Misalignment Angles		
	$\phi_E(')$	$\phi_N(')$	$\phi_U(')$
s_1	18.0804	-0.3935	-34.5847
s_2	-0.1728	-0.3935	-34.5873
M_D	-0.1728	-0.3935	-34.5873

A real data test is conducted to verify the performance of the coarse alignment methods presented in this paper. The method based on set s_6 is replaced by the direct method in this test since its has the same alignment accuracy as that of s_1. With the laser strapdown IMU kept completely motionless, the real data is sampled in an interval of 8 ms. The test is implemented in real-time for 8 seconds, and the convergence trajectories of pitch angles are shown in Fig. 3.

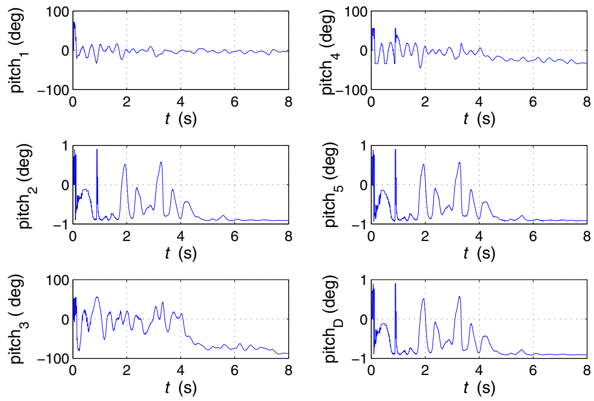

Fig. 3. Convergence trajectory of pitch angle.

The experiment results are in accordance with the simulation results and the theoretical analysis. Now, let us make a conclusion that the direct method has the same alignment accuracy as the method based on set s_2, but it does not use the complicate trigonometric expressions and actually avoid the orthogonalization process.

IV. ALIGNMENT WITH LATITUDE UNCERTAINTY

As per the above analysis, the direct method utilizes directly both accelerometer and gyro measurements for attitude estimation. This is distinct from the classic coarse alignment methods, which assume that the geographical latitude can usually be accurately measured at the given place.

It is well known that dense tree coverage, urban canyons and deep tunnels can occlude the satellite signals, so dose the underwater, indoors and underground environment. Under these circumstances, the local latitude is not readily available. Some other applications, such as communications satellite tracking or weapon platform stabilization, the local latitude is not necessary required.

In order to verify the sensitivity of coarse alignment accuracy to the latitude uncertainty, further error analysis is performed [11]. Let us define the local latitude as

$$\hat{L} = L + \delta L,$$

where δL is the measurement error.

Therefore, the navigation frame projection form of the Earth rate vector becomes

$$\hat{\omega}_{ie}^{n} = \begin{bmatrix} 0 & \omega_{ie}\cos\hat{L} & \omega_{ie}\sin\hat{L} \end{bmatrix}^{T}$$

Errors in latitude cause a departure of the matrix M in (7) from its ture value as

$$\hat{C}_b^n = \hat{M}\hat{Q} = (M + \delta M)(Q + \delta Q)$$

Employing (6),

$$(\hat{C}_b^n)_0 = \left\{ I + \frac{1}{2}\left[(M\delta Q + \delta M Q)C_n^b - C_b^n (M\delta Q + \delta M Q)^T \right] \right\} C_b^n$$

Similarly to (8), we then have

$$U = \frac{1}{2}\left\{ \left[C_b^n \delta Q^T M^T - (C_b^n \delta Q^T M^T)^T \right] + \left[C_b^n Q^T \delta M^T - (C_b^n Q^T \delta M^T)^T \right] \right\} \quad (11)$$

The first part of the right side of (11) is the same as (8).

Let us taking the method based on set s_2 for example, and retaining the first-order terms only, we have

$$\delta M = \begin{bmatrix} 0 & \dfrac{\delta L \tan L}{g\omega_{ie}\cos L} & 0 \\ 0 & 0 & \dfrac{\delta L \tan L}{g^2\omega_{ie}\cos L} \\ 0 & 0 & 0 \end{bmatrix}$$

Now consider the second part of the right side of (11).

$$C_b^n Q^T = \begin{bmatrix} 0 & g\omega_{ie}\cos L & 0 \\ 0 & 0 & g^2\omega_{ie}\cos L \\ -g & 0 & 0 \end{bmatrix}$$

and also

$$C_b^n Q^T \delta M^T = \begin{bmatrix} \delta L \tan L & 0 & 0 \\ 0 & \delta L \tan L & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

Specifically,

$$C_b^n Q^T \delta M^T - (C_b^n Q^T \delta M^T)^T = 0$$

Therefore, it is found that, when using the method based on set s_2 or the direct method, the coarse alignment accuracy is immune to the latitude uncertainty. Further analysis indicated that whether the latitude error will affect the alignment accuracy depends on the basis we employed. We leave the details of the further analysis to the reader.

V. CONCLUSION

This paper discusses six standard analytic coarse alignment methods and a direct method for SINS on stationary base. The associated error analyses verify that the drift misalignment angles about north and vertical axes are identical, but a difference exists about the east axis. It is found that, when properly choosing the basis s_2, the method has higher accuracy in all alignment methods under the same sensors precision, because its east level error is not corrupted by gyro uncertainty and its drift misalignment angles are identical to those obtained with physical gyrocompassing. Moreover, a direct method is evaluated by proving its equivalence with the method based on set s_2, and the insensitivity of these methods to the latitude uncertainty is also examined.

Generally, the direct method does not use the complicate trigonometric expressions and actually avoids the orthogonalization process. Due to its high accuracy, computation efficiency and immunity to the latitude uncertainty, the direct method is more suitable for practical applications than other six coarse alignment methods.

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